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Patterns of Dipole Antenna
on
Stratified Medium

by

W. W. Cooper

TR 71-3

June 1971

CENTER FOR SPACE RESEARCH
MASSACHUSETTS INSTITUTE OF TECHNOLOGY



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TE & TM SOLUTIONS
IN PLANAR STRATIFIED MEDIA

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ABSTRACT

The choice of an antenna for a lunar surface electrical properties experiment depends heavily on the radiation characteristics of the antenna as placed on the surface of a lunar medium. For engineering purposes, it is useful to simplify the model to a Hertzian dipole (x-, y-, or z-directed) on the surface of a planar-stratified medium. A general formulation for the fields in a piece-wise isotropic, stratified medium, with a finite number of layers, is derived using the method of TE & TM decomposition of the fields into transverse-harmonic (plane) waves, without use of any electromagnetic potentials.

The medium will be further simplified to a half-space problem (which was treated by Sommerfeld,¹ van der Pol,² Bãnos⁷) for numerical comparison of dipole characteristics. Dipoles on the surface between two half-space isotropic media will be compared on the basis of: (i) surface fields (non-radiating), (ii) sub-surface radiation patterns, of which we are interested in: (a) antenna gain and total power radiated within various lobes, (b) radiation peaks in certain directions and radiation nulls in other directions, (c) azimuthal symmetry of radiation patterns.

Further special results for a 3-layer medium will be discussed.

ACKNOWLEDGEMENT

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I also wish to thank John Groener for the computer programming that made the many graphs in this report possible.

Finally, as this is the last technical work I shall have an opportunity to do at the Center because I am entering graduate school at the University of Rhode Island, I want to thank all my friends at LSE and especially my long time associate and friend Dick Baker for his active support of this work.

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CHAPTER 1

Basic Conceptual Framework

This chapter is intended: (1) to give a general introduction to the characteristics of the various dipole solutions and (2) to illustrate the general method of decomposition into TE and TM waves, without use of any electromagnetic potentials.

This report summarizes the first six lectures given by the author at the inception of a seminar at M.I.T., which was organized by Professor J. A. Kong for the purpose of studying electromagnetic problems in a lunar medium. These lectures were given from December 9, 1970 to January 12, 1971.

1.1 Antenna and Field Properties

The design of an antenna for a lunar surface electrical properties experiment is influenced heavily by two problems: (1) antenna gain and radiation patterns, normalized on the basis of constant radiated power and (2) antenna impedance and efficiency. Problem (2), which is important in the design of a finite-sized, practical antenna, is not considered in this report. Problem (1), for a Hertzian dipole in a stratified medium has been solved conceptually in the literature. (See Sommerfeld,¹ van der Pol,² Wait,³ Collin,⁴ Bahar.⁵) The specific TE/TM formulation showing all six E and H field components for all six Hertzian dipoles (3 electric, 3 magnetic), and specific gain and radiation patterns for a half-space problem, appears to be new and should aid in a system design.

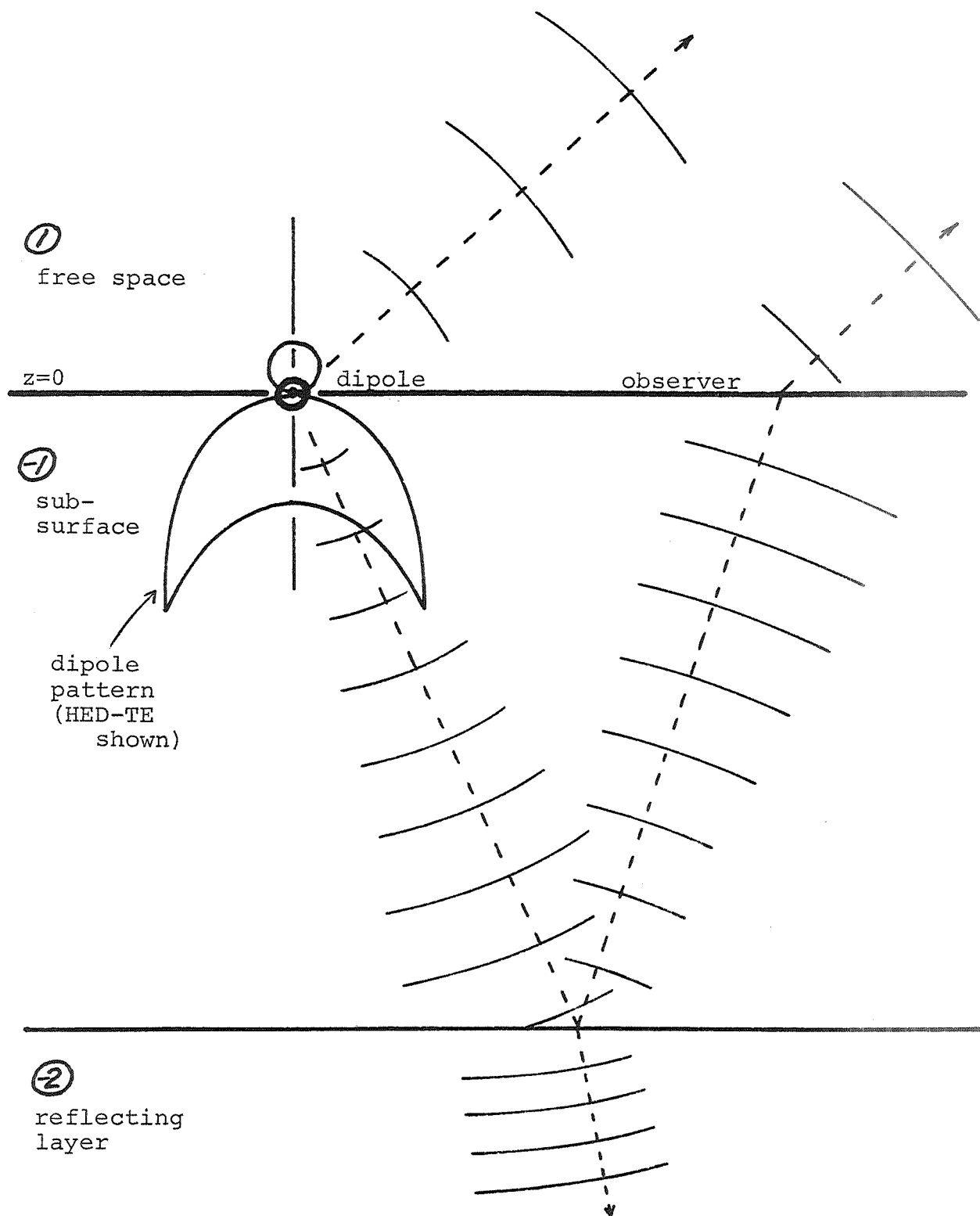


Fig. 1-1 Reflection Geometry

An antenna on the surface of a lunar medium (Figure 1-1) will generate electromagnetic fields, which can be interpreted as a combination of near fields (non-radiating), far fields (radiating), and reflected far fields (from sub-surface discontinuities). The purpose of an antenna design is to produce an antenna which gives the best likelihood of discriminating between certain effects in the lunar medium. Certain antenna properties are desirable:

- (a) to determine near-surface electrical properties:
 - (i) near fields giving near surface dielectric constant, and
 - (ii) minimal radiation near vertical incidence into the medium to minimize sub-surface reflected fields near the antenna.
- (b) to determine sub-surface properties of reflecting layers:
 - (i) strong sub-surface radiation peak(s) in certain direction(s), which give a good indication of sub-surface layers, after the radiation peaks are reflected back to the surface at some distance from the transmitter.
 - (ii) strong total power coupling into the sub-surface medium.

(c) to determine azimuthal symmetry of the lunar materials:

(i) azimuthal symmetry of radiated fields.

In addition to power radiation patterns, it is desirable to compare polarizations of fields, which, in conjunction with a measurement of relative phase of received signals, gives additional information regarding the sub-surface material properties.

Antenna are assumed to be orthogonal Hertzian dipoles. Electrical dipoles are considered; the fields produced by magnetic dipoles are obtained by duality. For a more general antenna, it is only necessary to decompose the transmitting antenna currents into a set of Hertzian dipole currents, relative to an observer's frame of coordinates. This technique gives very simply, for example, the solution for a generalized infinitesimal turnstile (with arbitrary amplitudes and phases for the different dipole driving currents).

The medium is assumed to be piece-wise isotropic, planar-stratified, with a finite number of layers. The solution for an arbitrary Hertzian dipole can be written in closed form in k-space. The fields in real space are obtained by a Fourier transform. Special techniques for performing an approximate transform valid only in special domains of real space will be the basis for the special half-space and surface-field problems to be treated in later chapters.

The solutions for Hertzian dipoles on the surface of a stratified medium will exhibit the following properties:

- (a) Surface fields asymptotically proportional to $1/r^2$ for large r .
- (b) For a horizontal dipole, both TE and TM fields with radiation lobes in orthogonal azimuthal planes. For a vertical dipole, azimuthally symmetric TE or TM field only (TM for elec., TE for mag. dipole).
- (c) Possible "parallel-plate" waveguide mode with low radial attenuation, depending on material constants.

The total power radiated into various lobes, both above and below the surface, will give some indication of the efficiency of the antenna in coupling power into the medium in useful directions. The azimuthal symmetry of a pattern should facilitate the discrimination of azimuthal and vertical inhomogeneities in the medium.

1.2 Maxwell and Wave Equations

My intention is to present an engineer's solution of a classical field problem. For engineering purposes, it is of interest to obtain expressions for the E and H fields directly, without going through the unnecessary machinery of vector or scalar potentials.

The problem is to find a solution to the time-harmonic Maxwell's equations in a planar-stratified medium, for a given source distribution. Time-harmonic fields are given by:

$$\underline{a}(t) = \text{Re} \left[\underline{A}(\omega) e^{j\omega t} \right] \quad (1-1)$$

For this convention, Maxwell's equations are:

$$\underline{\nabla} \times \underline{E} = -j\omega \underline{B} - \underline{J}_m \quad (\text{with magnetic sources}) \quad (1-2)$$

$$\underline{\nabla} \times \underline{H} = j\omega \underline{D} + \underline{J} \quad (1-3)$$

$$\underline{\nabla} \cdot \underline{D} = \rho(\omega) \quad (1-4)$$

$$\underline{\nabla} \cdot \underline{B} = \rho_m(\omega) \quad (\text{with magnetic sources}) \quad (1-5).$$

An equation of continuity relates J and ρ :

$$-j\omega \rho = \underline{\nabla} \cdot \underline{J} \quad (1-6),$$

where charge is conserved. (And a similar equation of continuity holds for $\underline{J}_m$ and ρ_m .)

Within each stratum, the material is assumed to be homogeneous and isotropic, with:

$$\underline{D} = \epsilon_i \underline{E}, \quad i = \dots, -2, -1, 1, \dots \quad (1-7)$$

$$\underline{B} = \mu_i \underline{H}, \quad i = \dots, -2, -1, 1, \dots \quad (1-8).$$

The sources are assumed to exist in the plane $z = 0$, between media +1 and -1. (Figure 1-2). The general approach is to transform the sources into spatial harmonic current sheets in x and y (transverse coordinates), to find the solution for an arbitrary current sheet, and then to synthesize the spatial fields as a superposition of all the existing spatially-harmonic fields. (This is a spatial Fourier synthesis.)

For a preliminary development, let us note that within a single stratum, $\underline{E}$ and $\underline{H}$ satisfy the wave equation:

$$\nabla^2 \underline{E} + \omega^2 \mu_i \epsilon_i \underline{E} = j\omega \mu_i \underline{J} - \underline{\nabla} (\underline{\nabla} \cdot \underline{J}) / j\omega \epsilon_i \quad (1-9)$$

$$\nabla^2 \underline{H} + \omega^2 \mu_i \epsilon_i \underline{H} = - \underline{\nabla} \times \underline{J} \quad (1-10).$$

Magnetic sources, $\underline{J}_m$, are omitted from equations (1-9) and (1-10)

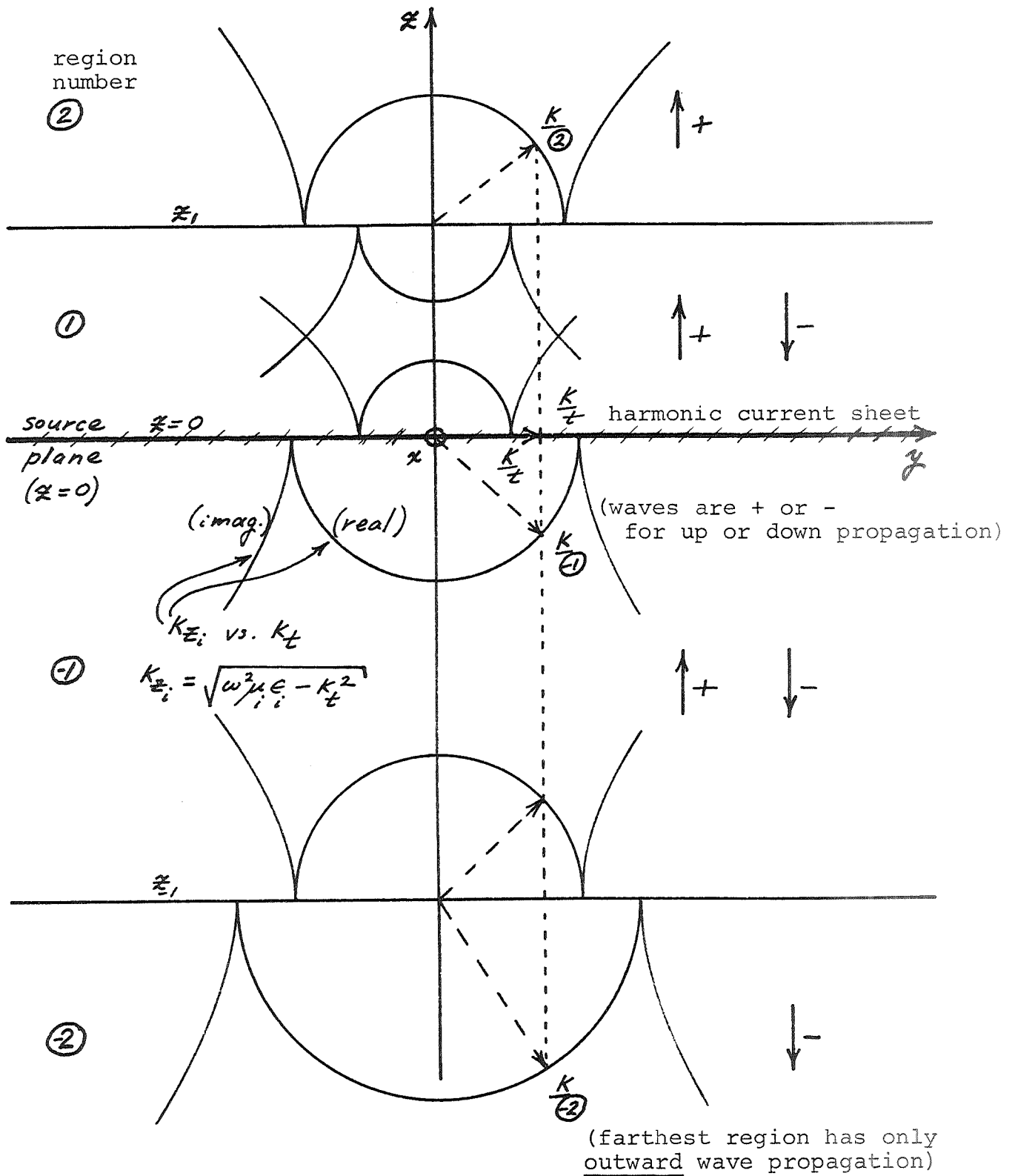


Fig. 1-2 Plane-Wave Transmission and Reflection

because the equations are symmetrical and the solutions can be obtained by application of the principle of duality.

Electrical current sheets are assumed to exist in the plane $z = 0$:

$$\underline{J}(\underline{r}) = \underline{a} e^{-j \underline{k} \cdot \underline{r}} \delta(z) \quad (1-11)$$

with the notation:

$$\underline{a} = \underline{a}_t + \underline{u}_z \underline{a}_z \quad (1-12)$$

$$\underline{k}_t = (k_x, k_y, 0) \quad (1-13)$$

$$\underline{r} = (x, y, z) \quad (1-14)$$

All fields are assumed to have the same transverse harmonicity in every stratum, i :

$$\underline{H} = \underline{H}_i^+ e^{-j(\underline{k}_t \cdot \underline{r} + k_{z_i} z)} + \underline{H}_i^- e^{-j(\underline{k}_t \cdot \underline{r} - k_{z_i} z)} \quad (1-15)$$

$$\underline{E} = \underline{E}_i^+ e^{-j(\underline{k}_t \cdot \underline{r} + k_{z_i} z)} + \underline{E}_i^- e^{-j(\underline{k}_t \cdot \underline{r} - k_{z_i} z)} \quad (1-16)$$

Waves are "+" or "-", for upward ($\uparrow$) or downward ($\downarrow$) propagation.

In order to satisfy the source-free wave equations (1-9) and (1-10), it is only necessary to satisfy the dispersion relation:

$$k_i \triangleq k_{z_i} = \sqrt{k_{c_i}^2 - k_t^2} \quad (1-17) (a)$$

with $k_{c_i} = \omega \sqrt{\mu_i \epsilon_i}$ (1-17) (b).

The radiation condition, allowing only outgoing waves as $z \rightarrow \pm \infty$, and dissipativeness of the media require that:

$$\text{Im}(k_{z_i}) \leq 0 \quad (1-18)$$

$$\text{Re}(k_{z_i}) \geq 0 \quad (1-19).$$

Excitation of waves by electric current sheet.

Excitation of E and H fields at the source-current boundary ($z = 0$) will be derived from limiting forms of Maxwell's equations (1-2), (1-3), (1-4), (1-5). A limiting form of the continuity equation (1-6) will be useful. This method will show a direct decomposition of the current sheet vector J into components which excite TE and TM waves directly, and independently. The advantage of using a TE/TM decomposition is that TE & TM fields are decoupled at planar boundaries between isotropic media.

For a transverse electrical current sheet at $z = 0$, the

appropriate limiting Maxwell's equation is:

$$\underline{\nabla} \times \underline{H} \rightarrow \underline{a}_z \times \left(\underline{H}_+ - \underline{H}_- \right) e^{-j \underline{k}_t \cdot \underline{r}} \delta(z) \quad (1-20)$$

which must equal

$$\underline{J} = \frac{a}{t} e^{-j \underline{k}_t \cdot \underline{r}} \delta(z) \quad \text{from (1-11).}$$

This current excites TE waves perpendicular to $\underline{a}_t$, and TM waves in the plane of $\underline{a}_t$ and $\underline{a}_z$, as illustrated by Figures (1-3) and (1-4). For both TE and TM waves, application of Maxwell's equations, (1-2) and (1-3) implies that, for the propagation vector:

$$\frac{\underline{k}^\pm}{i} = \frac{\underline{k}}{t} \pm \underline{a}_z k_{zi} \quad (1-21),$$

we have $\frac{\underline{k}^\pm}{i} \times \underline{E}_i^\pm = \omega \mu_i \underline{H}_i^\pm \quad (1-22)$

$$\frac{\underline{k}^\pm}{i} \times \underline{H}_i^\pm = -\omega \epsilon_i \underline{E}_i^\pm \quad (1-23).$$

These further imply that $\underline{E}_i^+$, $\underline{H}_i^+$ and $\underline{k}_i$ are mutually orthogonal in medium "i" and that:

$$\frac{|\underline{E}_i|}{|\underline{H}_i|} = \sqrt{\frac{\mu_i}{\epsilon_i}} \quad (1-24).$$

The ratio between transverse $|\underline{E}|$ and $|\underline{H}|$ will be most useful for future matching of transverse boundary conditions:

$$\left. \begin{aligned} n_i \\ \left\{ \begin{array}{l} TE \\ TM \end{array} \right\} \end{aligned} \right\} = \frac{|\underline{E}_{i,transverse}|}{|\underline{H}_{i,transverse}|} = \left\{ \begin{array}{l} \omega \mu_i / k_{z_i}, TE \text{ wave} \\ k_{z_i} / \omega \epsilon_i, TM \text{ wave} \end{array} \right. \quad \begin{array}{l} (1-25) \\ (1-26) \end{array}$$

A vertical electrical current sheet excites only a TM (vertically polarized) wave, as shown in Figure (1-5). Using appropriate limiting processes, as sketched in Section 5.1, equation (1-28) will give the wave excitation condition.

Conclusion

The general excitation equations for a transverse-harmonic electrical current sheet at $z = 0$ are:

$$\frac{H}{+i,t} - \frac{H}{-i,t} = -\frac{a}{z} \times \frac{a}{t} \quad (1-27)$$

$$\frac{E}{+i,t} - \frac{E}{-i,t} = -\frac{k}{t} \frac{a}{z} / \omega \epsilon_i \quad (i = \pm 1) \quad (1-28)$$

from equation (1-20) and Section 5.1. These equations exhibit

some interesting features:

- (a.) If the source terms are zero, then both $\underline{E}_t$ and $\underline{H}_t$ are continuous across a planar boundary. Therefore a pure TE field can be matched only by a pure TE field across a boundary, and similarly for TM fields. Thus TE and TM fields are decoupled at planar boundaries. In addition, as illustrated by Figures (1-3) and (1-4) the TE and TM fields are orthogonal for each $\underline{k}$. Therefore the fields are decoupled and their powers are additive in integrals giving the total power radiated at all (solid angles). This additivity will be used in the half-space power integrals below.
- (b.) The field amplitudes for a vertical current sheet, $\underline{u}_z a_z$, change discontinuously, depending on whether the current sheet is "thought of" as existing in the upper medium ($i = +1$) or the lower medium ($i = -1$). Furthermore, a constant current source ($a_z = \text{const.}$) generates the greatest field amplitudes when placed in the less dense medium (normally the upper medium). However, the radiation patterns, which are normalized for a constant total radiated power, are not affected by the scale factor ϵ_i .

Synthesis of Fields

The harmonic $\underline{E}$ & $\underline{H}$ fields generated by harmonic current sheets will be synthesized into spatial $\underline{E}$ & $\underline{H}$ fields by means

of a 2-dimensional spatial Fourier transform. The harmonic current sheets are obtained by a Fourier transform of a given source current density $\underline{J}(\underline{r})$ (assumed for this analysis to exist in the plane $z = 0$). Defining some terms:

$$\text{spatial current density} = \underline{J}(x, y) \delta(z) \quad (1-29)$$

$$\text{transformed current density} = \underline{\tilde{J}}(k_x, k_y) \delta(z) \quad (1-30)$$

$$\text{infinitesimal current vector} =$$

(cf. eq. 1-11)

$$\underline{\tilde{J}}(k_x, k_y) e^{-j \underline{k} \cdot \underline{r}} \delta(z) dk_x dk_y \quad (1-31).$$

For this infinitesimal current sheet, the infinitesimal harmonic $\underline{E}$ & $\underline{H}$ fields are determined by the method of Chapter 2, and are:

$$\left\{ \begin{array}{l} \underline{\tilde{E}}(k_x, k_y, z) \\ \underline{\tilde{H}}(k_x, k_y, z) \end{array} \right\} e^{-j \underline{k} \cdot \underline{r}} dk_x dk_y \quad (1-32).$$

The spatial transforms giving the total fields are defined:

$$\underline{\tilde{J}}(k_x, k_y) = \iint_{x, y = -\infty}^{\infty} \underline{J}(x, y) e^{j \underline{k} \cdot \underline{r}} dx dy \quad (1-33)$$

$$\left\{ \begin{array}{l} \underline{E}(\underline{r}) \\ \underline{H}(\underline{r}) \end{array} \right\} = \frac{1}{(2\pi)^2} \iint_{k_x, k_y = -\infty}^{\infty} \left\{ \begin{array}{l} \underline{\tilde{E}}(k_x, k_y, z) \\ \underline{\tilde{H}}(k_x, k_y, z) \end{array} \right\} e^{-j \underline{k} \cdot \underline{r}} dk_x dk_y \quad (1-34).$$

For a Hertzian dipole, symmetry reduces integral (1-34) to a single integral including Bessel functions (or, equivalently, Hankel functions) of the order 0 and 1, as formulated in Chapter 3.

CHAPTER 2

Solutions for Plane Waves in Stratified Media

Formal solutions for vertically and horizontally polarized plane waves (respectively called TM and TE) in isotropic media have a considerable history in the literature. (See the work by Brekhovskikh,⁶ Wait,³ Collin,⁴ Bahar.⁵)

This chapter will give a concise solution to this problem in our own notation.

2.1 Transformation of Wave Amplitudes to Source Plane

For a medium with a finite number of layers, with a source at $z = 0$, the radiation condition requires that in the farthest layers away from the source (layers $i = I + 1$ and $i = -J - 1$) only outgoing waves can exist (see Figure 1-2). The outgoing wave amplitudes comprise a pair of unknown vectors which can be transformed back to $z = 0$, using a cascade of propagation matrices. Then the outgoing wave amplitudes can be solved from the 2x2 excitation equations (1-27) and (1-28).

To simplify our notation, the following simplifications are made in this chapter:

- (a) dropping off $e^{-jk_t \cdot \underline{r}}$ from all expressions,
- (b) including only transverse field vectors,
- (c) referring wave (H) amplitudes to innermost boundary of each layer, and
- (d) using only H vector, knowing that

$$\underline{E}_i^{\pm} = \pm n_i \frac{H_i^{\pm}}{(TE, TM)} \times \underline{u}_z \quad (2-1)$$

with n_i given by (1-25), (1-26).

For $z > 0$, a linear relation between transverse wave vectors in layers i and $i+1$ are derived from source-free boundary conditions (1-27) and (1-28), at $z=z_i$ (See Figure 2-1.)

$$\underline{H}_i^+ t_{\odot} + \underline{H}_i^- / t_{\odot} = \underline{H}_{i+1}^+ + \underline{H}_{i+1}^- \quad (2-2)$$

$$n_i \underline{H}_i^+ t_{\odot} - n_i \underline{H}_i^- / t_{\odot} = n_{i+1} \underline{H}_{i+1}^+ - n_{i+1} \underline{H}_{i+1}^- \quad (2-3)$$

in which $t_{\odot} = e^{-jk_z(z_i - z_{i-1})} = e^{-jk_z \Delta_i}, \quad i > 0 \quad (2-4).$

Equations (2-2) and (2-3) are equivalent to a 2x2 matrix equation which can be inverted to give:

$$\begin{bmatrix} \underline{H}_i^+ \\ \underline{H}_i^- \end{bmatrix} = \underline{M}_{\odot}^+ \begin{bmatrix} \underline{H}_{i+1}^+ \\ \underline{H}_{i+1}^- \end{bmatrix} = \frac{1}{2n_i} \begin{bmatrix} (n_i + n_{i+1})/t_{\odot} & (n_i - n_{i+1})/t_{\odot} \\ (n_i - n_{i+1})t_{\odot} & (n_i + n_{i+1})t_{\odot} \end{bmatrix} \begin{bmatrix} \underline{H}_{i+1}^+ \\ \underline{H}_{i+1}^- \end{bmatrix} \quad (2-5)$$

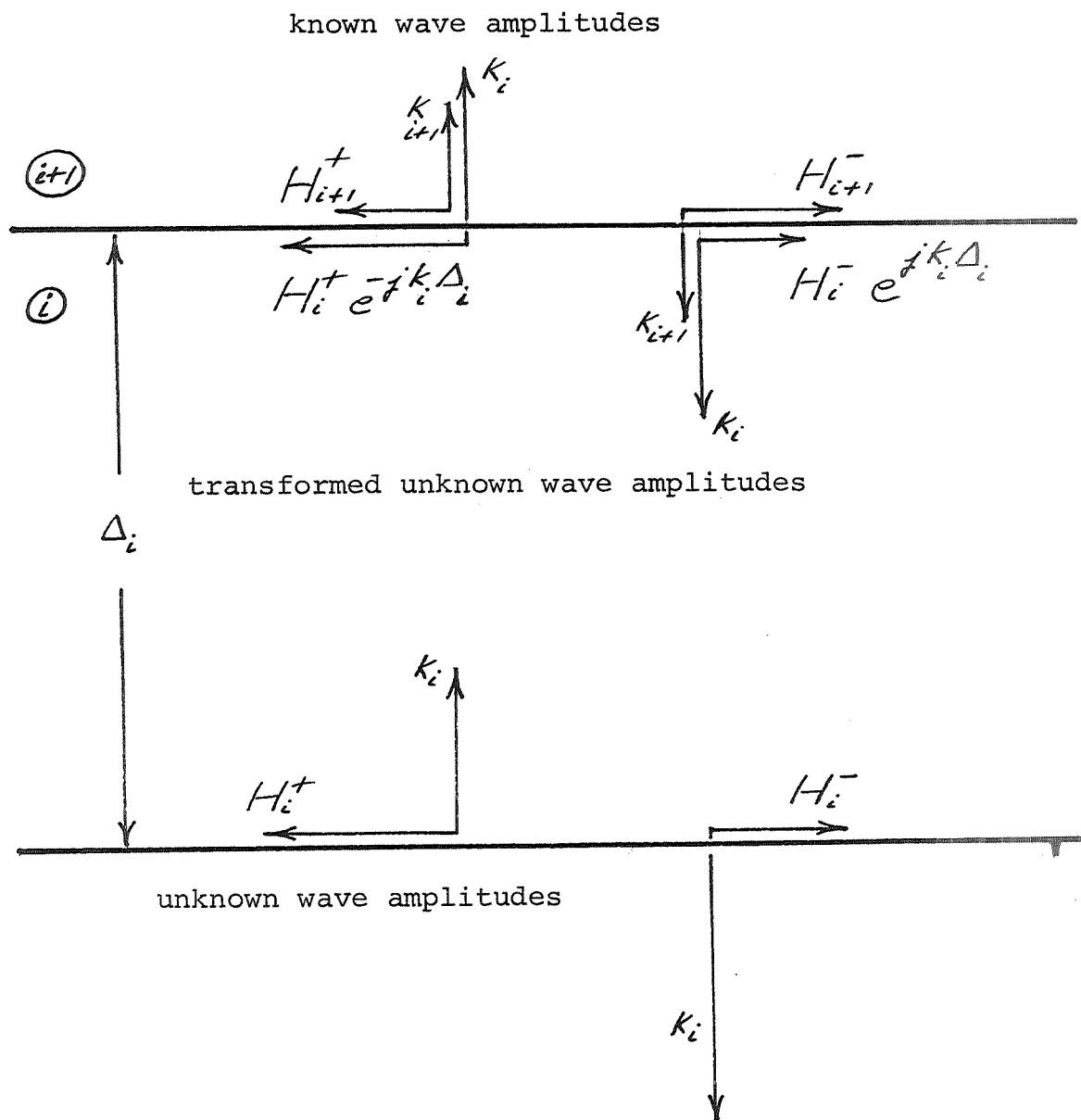


Fig. 2-1 Transformation of Wave Amplitudes

At $z = 0$

$$\begin{bmatrix} \underline{H}_I^+ \\ \underline{H}_I^- \end{bmatrix} = M_{\textcircled{1}}^+ \cdots M_{\textcircled{I}}^+ \begin{bmatrix} \underline{H}_{I+1}^+ \\ \underline{0} \end{bmatrix} \quad (2-6).$$

At $z_i < 0$, relating wave vectors in layers i and $i-1$,

$$\begin{bmatrix} \underline{H}_i^+ \\ \underline{H}_i^- \end{bmatrix} = M_{\textcircled{i}}^- \begin{bmatrix} \underline{H}_{i-1}^+ \\ \underline{H}_{i-1}^- \end{bmatrix} = \frac{1}{2n_i} \begin{bmatrix} (n_i + n_{i-1})/t_{\textcircled{i}} & (n_i - n_{i-1})/t_{\textcircled{i}} \\ (n_i n_{i-1})t_{\textcircled{i}} & (n_i + n_{i-1})t_{\textcircled{i}} \end{bmatrix} \begin{bmatrix} \underline{H}_{i-1}^+ \\ \underline{H}_{i-1}^- \end{bmatrix} \quad (2-7)$$

$$\text{in which } t_{\textcircled{i}} = e^{-jk_{z_i}(z_i - z_{i+1})} = e^{+jk_{z_i}\Delta_i}, \quad i < 0 \quad (2-8)$$

At $z = 0$, as in equation (2-6):

$$\begin{bmatrix} \underline{H}_{-I}^+ \\ \underline{H}_{-I}^- \end{bmatrix} = M_{\textcircled{1}}^- \cdots M_{\textcircled{J}}^- \begin{bmatrix} \underline{0} \\ \underline{H}_{-J-1}^- \end{bmatrix} \quad (2-9).$$

Equations (2-6) and (2-9) show that the wave amplitudes at $z = 0^+$ and $z = 0^-$ are linearly proportional to $\underline{H}_{I+1}^+$ and $\underline{H}_{-J-1}^-$.

2.2 Solution of 2 x 2 Source Equations for Wave Amplitudes

At $z = 0$, the source excitation equations (1-27) and (1-28) are written

$$(\underline{H}_{+1}^+ + \underline{H}_{+1}^-) - (\underline{H}_{-1}^+ + \underline{H}_{-1}^-) = -\frac{u}{z} X \underline{a}_t \quad (2-10)$$

$$n_i(\underline{H}_{+1}^+ - \underline{H}_{+1}^-) - n_i(\underline{H}_{-1}^+ - \underline{H}_{-1}^-) = -\frac{u}{z} X \frac{k a}{z} / \omega \epsilon_i \quad (2-11),$$

with the latter equation for a vertical current, a_z at $z = 0^+$ ($i = +1$) or at $z = 0$ ($i = -1$). Combined with (2-6) and (2-9), (2-10) and (2-11) become

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \underline{H}_{-I+1}^+ \\ \underline{H}_{-J-1}^- \end{bmatrix} = \begin{bmatrix} \underline{b}_1 \\ \underline{b}_2 \end{bmatrix} \quad (2-12)$$

solved by

$$\begin{bmatrix} \underline{H}_{-I+1}^+ \\ \underline{H}_{-J-1}^- \end{bmatrix} = \frac{\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}}{(a_{11}a_{22} - a_{12}a_{21})} \begin{bmatrix} \underline{b}_1 \\ \underline{b}_2 \end{bmatrix} \quad (2-13).$$

Formally, the spatially-harmonic field vectors in any layer, i , now are given by

$$\begin{bmatrix} \underline{H}_i^+(z) \\ \underline{H}_i^-(z) \end{bmatrix} = \begin{bmatrix} e^{-jk_{z_i}(z-z_i)} & 0 \\ 0 & e^{+jk_{z_i}(z-z_i)} \end{bmatrix} \begin{matrix} M_{\textcircled{i}}^+ \cdots M_{\textcircled{I}}^+ \\ \underline{0} \end{matrix} \begin{bmatrix} \underline{H}_{I+1}^+ \\ \underline{0} \end{bmatrix} \quad (2-14)$$

and similarly for $z < 0$. The transverse $\underline{E}$ - fields are

$$\begin{bmatrix} \underline{E}_i^+(z) \\ \underline{E}_i^-(z) \end{bmatrix} = \begin{bmatrix} n_i & 0 \\ 0 & -n_i \end{bmatrix} \begin{bmatrix} \underline{H}_i^+(z) \\ \underline{H}_i^-(z) \end{bmatrix} \times \underline{u}_z \quad (2-15)$$

The vertical field component, which is E_z for a TM wave or H_z for a TE wave, is equal to

$$\left. \begin{matrix} \underline{E}_i^{\pm}(z) \\ \underline{H}_i^{\pm}(z) \end{matrix} \right\} = \mp \frac{k_t}{k_{z_i}} \cdot \begin{cases} \text{sign}[\underline{E}_i(z) \cdot \underline{k}_t] & , \text{ TM} \\ \text{sign}[\underline{H}_i(z) \cdot \underline{k}_t] & , \text{ TE} \end{cases} \quad (2-16)$$

(2-17)

derived from equations (1-22), (1-23) and the permutation

property of a vector triple product $\left[\underline{u}_z \cdot (\underline{u}_t \times \underline{H}_t) = -\underline{u}_t \cdot (\underline{u}_z \times \underline{H}_t) \text{ or } \underline{u}_z \cdot (\underline{u}_t \times \underline{E}_t) = -\underline{u}_t \cdot (\underline{u}_z \times \underline{E}_t) \right]$.

CHAPTER 3

Fourier-Bessel Synthesis of Fields

For a Hertzian dipole, approximate spatial fields will be obtained from the Fourier transforms (1-33) and (1-34), in which $\underline{J}(k_x, k_y)$ reduces to a constant. Then the transforms for $\underline{E}(\underline{r})$ & $\underline{H}(\underline{r})$, in a piecewise isotropic, stratified medium, reduce to single integrals. These single integrals will be approximated in the special cases of a half-space and a three-layer medium.

3.1 General Formulation As a Single Integral

The Hertzian dipole current and its transform (from equation (1-33)) are given by:

$$\underline{J}(x, y) = \underline{a} \delta(x) \delta(y) \delta(z) \quad (3-1)$$

$$\underline{\tilde{J}}(k_x, k_y) = \underline{a} = \frac{a}{t} + \frac{u}{z} \frac{a}{z} \quad (3-2).$$

The inverse transform (1-34) giving the spatial fields is written in cylindrical coordinates, (k_t, θ) , for an observer at (r, ϕ) . See Figure (3-1).

(3-3)

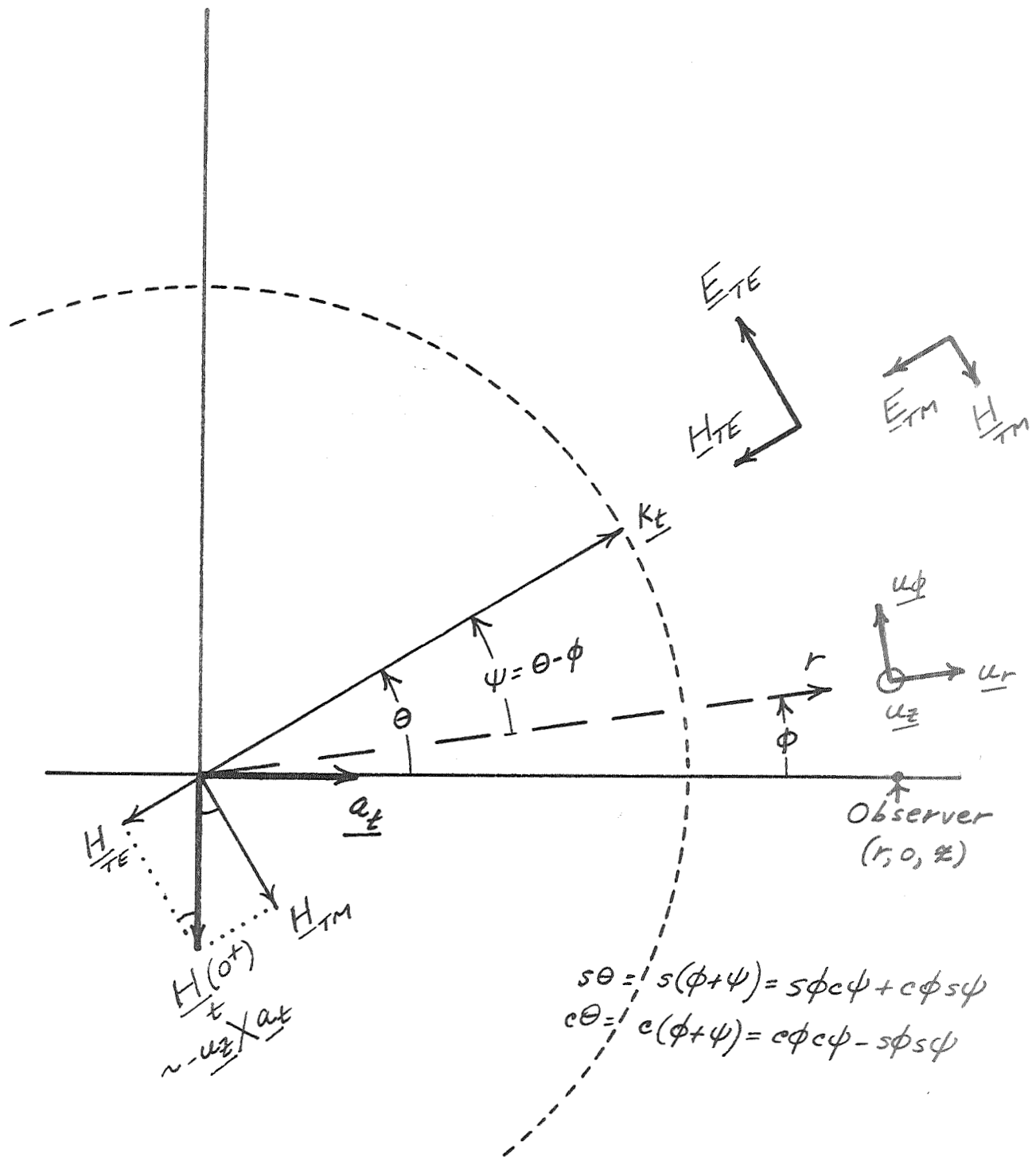


Fig. 3-1 Angular Dependence of Wave Excitation

Examining the harmonic field expressions (2-14), (2-15), (2-16), (2-17), one sees that all the matrices and scalars relating components of $\underline{E}$ & $\underline{H}$ to the excitation factors are independent of θ , except for a sign (± 1) in (2-16), (2-17). The excitation factors, (2-10) and (2-11), depend simply on ϕ, θ as shown in Table (3-1). These factors, which appear in equation (3-3), are integrated over θ , with results as shown in Table (3-1), by aid of the following identities, (3-4) - (3-7):

$$e^{-jkr_t \cos \psi} = J_0(kr_t) - j2J_1(kr_t) \cos \psi - 2J_2(kr_t) \cos 2\psi - \dots \quad (3-4)$$

$$\left. \begin{aligned} \theta &= \phi + \psi \\ \cos \theta &= c\phi c\psi - s\phi s\psi \\ \sin \theta &= s\phi c\psi + c\phi s\psi \end{aligned} \right\} \quad (3-5)$$

$$\left. \begin{aligned} \cos^2 \psi \\ \sin^2 \psi \end{aligned} \right\} = \frac{1 \pm \cos 2\psi}{2} \quad (3-6)$$

$$\frac{1}{2\pi} \int_{\psi=-\pi}^{\pi} d\psi \cos m\psi \cos n\psi = \begin{cases} 0, & m \neq n \\ \frac{1}{2}, & m = n \neq 0 \\ 1, & m = n = 0 \end{cases} \quad (3-7)$$

The following identities will be used in Section 3.2:

$$\frac{J_0 - J_2}{2} = J_0 - \frac{J_1}{kr_t} \quad (3-8)$$

$$\frac{\overline{J_0} + \overline{J_2}}{2} = \frac{\overline{J_1}}{k_t r} \quad (3-9).$$

After integrating the angular part of transform (3-3), the spatial fields are given as single integrals.

$$\left. \begin{matrix} \underline{E}(z) \\ \underline{H}(z) \end{matrix} \right\} = \frac{1}{2\pi} \int_{k_t=0}^{\infty} dk_t k_t \left\{ \begin{matrix} \underline{\tilde{E}}'(k_t, z) \\ \underline{\tilde{H}}'(k_t, z) \end{matrix} \right\} \quad (3-10),$$

where $\underline{E}^{\pm}$ and $\underline{H}^{\pm}$ contain Bessel functions (Table 3-1) from the angular integration; and reflection-transmission factors from equations (2-14) - (2-17). Where a $\pm$ or $\mp$ occurs, as in equations (2-1), (2-15), (2-16) and Table (3-1), the upper sign is for "+" ($\uparrow$) wave and the lower sign for a "-" ($\downarrow$) wave.

An alternative form of equation (3-10) is easily written as a contour integral containing Hankel functions instead of Bessel functions. Such methods will not be developed in detail in this report.

	Component of <u>E</u> or <u>H</u>	Dependence on $\phi, \psi = \theta - \phi$	Integrated on $\psi = \theta - \phi$
a_z - horizontal dipole TM fields	H_r	$\sim c \theta s \psi$	$-s \phi (J_0 + J_2)/2$
	H_ϕ	$\sim -c \theta c \psi$	$-c \phi (J_0 - J_2)/2$
	H_z	$= 0$	0
	E_r	$\sim c \theta c \psi$	$-c \phi (J_0 - J_2)/2$
	E_ϕ	$\sim -c \theta s \psi$	$+s \phi (J_0 + J_2)/2$
	E_{z_i}	$\sim \pm k_t / k_{z_i} c \theta$	$\mp \frac{k_t}{k_{z_i}} c \phi j J_1(k_t r)$
TE fields	H_r	$\sim -s \theta c \psi$	$-s \phi (J_0 - J_2)/2$
	H_ϕ	$\sim -s \theta s \psi$	$-c \phi (J_0 + J_2)/2$
	H_{z_i}	$\sim \pm \frac{k_t}{k_{z_i}} s \theta$	$\mp \frac{k_t}{k_{z_i}} s \phi j J_1(k_t r)$
	E_r	$\sim -s \theta s \psi$	$-c \phi (J_0 + J_2)/2$
	E_ϕ	$\sim +s \theta c \psi$	$+s \phi (J_0 - J_2)/2$
	E_{z_i}	$= 0$	0
a_z - vertical dipole TM fields only	H_r	$\sim s \psi$	0
	H_ϕ	$\sim -c \psi$	$j J_1(k_t r)$
	H_z	$= 0$	0
	E_r	$\sim -c \psi$	$j J_1(k_t r)$
	E_ϕ	$\sim -s \psi$	0
	E_{z_i}	$\sim \pm \frac{k_t}{k_{z_i}}$	$\pm \frac{k_t}{k_{z_i}} J_0(k_t r)$

Table 3-1. Integration of Factors in Inverse Transform
($\sim$ denotes proportionality)

3.2 Special Half-Space Integral

For a Hertzian dipole on the surface between two semi-infinite isotropic media, as shown in Figure (3-2), the solution of excitation equations (2-10) and (2-11) for the harmonic wave vectors is given by:

$$\begin{bmatrix} \underline{H}_+^+ \\ \underline{H}_-^- \end{bmatrix} = \frac{\begin{bmatrix} n_+ & 1 \\ -n_- & 1 \end{bmatrix}}{(n_+ n_-)} \begin{bmatrix} \frac{a}{z} \times \frac{a}{t} \\ \frac{a}{z} \times \frac{k_{tz} a}{\omega \epsilon_i} \end{bmatrix} \quad (3-11).$$

The transverse impedances, n_i , are for TE or TM waves, as illustrated in Figure (3-1). The angular factors in equation (3-3) are integrated as shown in Table (3-1). The remaining single integral, equation (3-10), takes a particularly simple form, for the half space $z > 0$:

$$\left. \begin{matrix} E_i(r) \\ H_i(r) \end{matrix} \right\} = \frac{1}{2\pi} \int_{k_t=0}^{\infty} dk_t k_t e^{-jk_{z_i} z} \left\{ \begin{matrix} \tilde{E}_i''(k_t) \\ H_i''(k_t) \end{matrix} \right\} \quad (3-12)$$

and similarly for the halfspace $z < 0$.

$\tilde{E}_i''$ and $\tilde{H}_i''$ reduce to the following for a horizontal electric dipole:

(TM fields)

(TE fields)

$$\tilde{H}_r'' = -a_s \phi \left[\frac{\epsilon_i k_i}{(\epsilon_i k_i + \epsilon_i k_i)} \frac{J_i}{k_i r} + \frac{\mu_i k_i}{(\mu_i k_i + \mu_i k_i)} \left(J_0 - \frac{J_i}{k_i r} \right) \right] \quad (3-13)$$

$$\tilde{H}_\phi'' = -a_c \phi \left[\frac{\epsilon_i k_i}{(\epsilon_i k_i + \epsilon_i k_i)} \left(J_0 - \frac{J_i}{k_i r} \right) + \frac{\mu_i k_i}{(\mu_i k_i + \mu_i k_i)} \frac{J_i}{k_i r} \right] \quad (3-14)$$

$$\tilde{H}_z'' = -a_s \phi \cdot \left[\frac{\mu_i k_i}{(\mu_i k_i + \mu_i k_i)} j J_i(k_i r) \right] \quad (3-15)$$

$$\tilde{E}_r'' = -a_c \phi \left[\frac{k_i k_i}{\omega(\epsilon_i k_i + \epsilon_i k_i)} \left(J_0 - \frac{J_i}{k_i r} \right) + \frac{\omega \mu_i \mu_i}{(\mu_i k_i + \mu_i k_i)} \frac{J_i}{k_i r} \right] \quad (3-16)$$

$$\tilde{E}_\phi'' = +a_s \phi \left[\frac{k_i k_i}{\omega(\epsilon_i k_i + \epsilon_i k_i)} \frac{J_i}{k_i r} + \frac{\omega \mu_i \mu_i}{(\mu_i k_i + \mu_i k_i)} \left(J_0 - \frac{J_i}{k_i r} \right) \right] \quad (3-17)$$

$$\tilde{E}_z'' = -a_c \phi \left[\frac{k_i k_i}{\omega(\epsilon_i k_i + \epsilon_i k_i)} j J_i(k_i r) \right] \quad (3-18),$$

for $z > 0$, abbreviating k_{zi} to k_i (1-17a).

For a vertical electric dipole, (3-12) contains

(TM fields only)

$$\tilde{H}_{\phi}'' = a_z \frac{\epsilon_z k_z}{(\epsilon_z k_z + \epsilon_z k_z)} j J_1(k_z r) \quad (3-19)$$

$$\tilde{E}_r'' = a_z \frac{\epsilon_z k_z k_r}{\omega \epsilon_z (\epsilon_z k_z + \epsilon_z k_z)} j J_1(k_z r) \quad (3-20)$$

$$\tilde{E}_z'' = a_z \frac{\epsilon_z k_z^2}{\omega \epsilon_z (\epsilon_z k_z + \epsilon_z k_z)} J_0(k_z r) \quad (3-21).$$

These expressions will be integrated approximately, in the next two sections.

3.3 Special Half-Space Far Fields and Power Gain

Far from the source and far from the boundary plane, the outgoing fields approximate horizontally and vertically polarized plane waves (called TE & TM, respectively), as illustrated in Figures (1-3) and (1-4). E_{ϕ} (for TE) and H_{ϕ} (for TM) are sufficient to specify the vector fields. The other components are determined from equations (1-22) - (1-24). Neglecting second order terms [all terms $\sim J_1/K_z r$ in equations (3-14), (3-17), (3-20)] reduces integral (3-12) to one term per field component. Then a "stationary-phase" approximation, to be derived in Section 5.2, gives asymptotic far fields:

$$\left. \begin{matrix} E_{\phi}(z) \\ H_{\phi}(z) \end{matrix} \right\} = \frac{j2k_{c_i} z}{R} \frac{e^{-jk_{c_i} R}}{4\pi R} \cdot \left\{ \begin{matrix} E'_{\phi}, TE \\ H'_{\phi}, TM \end{matrix} \right\} \quad (3-22)$$

Expressions for E'_{ϕ} and H'_{ϕ} are listed below in Table (3-2) (a) and (b), and k_{c_i} is given by eq. (1-17b). In eq. (3-22):

$$R = \sqrt{r^2 + z^2} = \sqrt{x^2 + y^2 + z^2} \quad (3-23)$$

$z > 0$	k -space Field Approximation $z > 0$	$\frac{dP}{d\Omega}_\ell$ proportional to
Hertzian electric dipoles horizontal - a_t $H'_{\phi_1} (TM)$ $E'_{\phi_2} (TE)$ vertical - a_z $H'_{\phi_3} (TM)$	$-a_t c \phi \frac{\epsilon_1 k_{-1}}{(\epsilon_1 k_{-1} + \epsilon_1 k_{-1})}$ $+a_t s \phi \frac{\omega \mu_1 \mu_{-1}}{(\mu_1 k_{-1} + \mu_1 k_{-1})}$ $+a_z \frac{\epsilon_1 k_t}{(\epsilon_1 k_{-1} + \epsilon_1 k_{-1})}$	$\sqrt{\frac{\mu_1}{\epsilon_1}} \left \frac{k_{-1}}{K_c} \frac{\epsilon_1 k_{-1}}{(\epsilon_1 k_{-1} + \epsilon_1 k_{-1})} \right ^2$ $\sqrt{\frac{\epsilon_1}{\mu_1}} \left \frac{k_{-1}}{K_c} \frac{\omega \mu_1 \mu_{-1}}{(\mu_1 k_{-1} + \mu_1 k_{-1})} \right ^2$ $\sqrt{\frac{\mu_1}{\epsilon_1}} \left \frac{k_{-1}}{K_c} \frac{\epsilon_1 k_t}{(\epsilon_1 k_{-1} + \epsilon_1 k_{-1})} \right ^2$
Hertzian magnetic dipoles (obtained by duality) horizontal - b_t $E'_{\phi_4} (TE)$ $H'_{\phi_5} (TM)$ vertical - b_z $E'_{\phi_6} (TE)$	$+b_t c \phi \frac{\mu_1 k_{-1}}{(\mu_1 k_{-1} + \mu_1 k_{-1})}$ $+b_t s \phi \frac{\omega \epsilon_1 \epsilon_{-1}}{(\epsilon_1 k_{-1} + \epsilon_1 k_{-1})}$ $-b_z \frac{\mu_{-1} k_t}{(\mu_{-1} k_{-1} + \mu_{-1} k_{-1})}$	$\sqrt{\frac{\epsilon_1}{\mu_1}} \left \frac{k_{-1}}{K_c} \frac{\mu_1 k_{-1}}{(\mu_1 k_{-1} + \mu_1 k_{-1})} \right ^2$ $\sqrt{\frac{\mu_1}{\epsilon_1}} \left \frac{k_{-1}}{K_c} \frac{\omega \epsilon_1 \epsilon_{-1}}{(\epsilon_1 k_{-1} + \epsilon_1 k_{-1})} \right ^2$ $\sqrt{\frac{\epsilon_1}{\mu_1}} \left \frac{k_{-1}}{K_c} \frac{\mu_{-1} k_t}{(\mu_{-1} k_{-1} + \mu_{-1} k_{-1})} \right ^2$

Table 3-2 (a). Far-Field Approximations, $z > 0$ ($i = 1$)

$$\ell = 1, 2, 3, 4, 5, 6$$

$z < 0$	k -space Field Approximation $z < 0$	$\frac{dP^-}{d\Omega_\ell}$ proportional to
Hertzian electric dipoles horizontal - a_t $H'_{\phi_7} (TM)$ $E'_{\phi_8} (TE)$ vertical - a_z $H'_{\phi_9} (TM)$	$-a_t c \phi \frac{\epsilon_i k_i}{(\epsilon_i k_i + \epsilon_{-i} k_{-i})}$ $+a_t s \phi \frac{\omega \mu_i \mu_{-i}}{(\mu_i k_i + \mu_{-i} k_{-i})}$ $+a_z \frac{\epsilon_i k_z}{(\epsilon_i k_i + \epsilon_{-i} k_{-i})}$	$\sqrt{\frac{\mu_i}{\epsilon_i}} \left \frac{k_i}{k_{-i}} \frac{\epsilon_i k_i}{(\epsilon_i k_i + \epsilon_{-i} k_{-i})} \right ^2$ $\sqrt{\frac{\epsilon_i}{\mu_i}} \left \frac{k_i}{k_{-i}} \frac{\omega \mu_i \mu_{-i}}{(\mu_i k_i + \mu_{-i} k_{-i})} \right ^2$ $\sqrt{\frac{\mu_i}{\epsilon_i}} \left \frac{k_i}{k_{-i}} \frac{\epsilon_i k_z}{(\epsilon_i k_i + \epsilon_{-i} k_{-i})} \right ^2$
Hertzian magnetic dipoles (obtained by duality) horizontal - b_t $E'_{\phi_{10}} (TE)$ $H'_{\phi_{11}} (TM)$ vertical - b_z $E'_{\phi_{12}} (TE)$	$+b_t c \phi \frac{\mu_i k_i}{(\mu_i k_i + \mu_{-i} k_{-i})}$ $+b_t s \phi \frac{\omega \epsilon_i \epsilon_{-i}}{(\epsilon_i k_i + \epsilon_{-i} k_{-i})}$ $-b_z \frac{\mu_i k_z}{(\mu_i k_i + \mu_{-i} k_{-i})}$	$\sqrt{\frac{\epsilon_i}{\mu_i}} \left \frac{k_i}{k_{-i}} \frac{\mu_i k_i}{(\mu_i k_i + \mu_{-i} k_{-i})} \right ^2$ $\sqrt{\frac{\mu_i}{\epsilon_i}} \left \frac{k_i}{k_{-i}} \frac{\omega \epsilon_i \epsilon_{-i}}{(\epsilon_i k_i + \epsilon_{-i} k_{-i})} \right ^2$ $\sqrt{\frac{\epsilon_i}{\mu_i}} \left \frac{k_i}{k_{-i}} \frac{\mu_i k_z}{(\mu_i k_i + \mu_{-i} k_{-i})} \right ^2$

Table 3-2 (b). Far-Field Approximations, $z < 0$ ($i < 0$)

$\ell = 7, 8, 9, 10, 11, 12$

The power radiated/unit of solid angle is proportional to:

$$\frac{dP}{d\Omega} \sim \sqrt{\frac{\mu_1}{\epsilon_1}} \left| \frac{k_1}{k_c} H'_{\phi, TM} \right|^2 + \sqrt{\frac{\epsilon_1}{\mu_1}} \left| \frac{k_1}{k_c} E_{\phi, TE} \right|^2 \quad (3-24)$$

from Poynting's theorem, using plane wave impedance (1-24). The twelve power terms in Table (3-2) are directly comparable, assuming $\mu_1 = \mu_{-1}$, for different values of a single parameter:

$$C = \frac{\epsilon_1}{\epsilon_{-1}} \quad (3-25).$$

The results are graphed in Figures (3-2), (3-3) and (3-4). The graphs compare fractional power radiated (in different lobes) and antenna gain. The dipole currents must be adjusted to give the same total radiated power for each type of dipole. This requires an integration of $dP/d\Omega$ over all solid angles, using equation (3-26), which was approximated by a 100-point trapezoidal sum.

$$P_{\pm} = \left\{ \frac{\pi}{2\pi} \right\} \int_{k_t/k_{c\pm}=0}^1 \frac{dk_t}{k_{c\pm}} \frac{k_t}{k_{c\pm}} \frac{k_{c\pm}}{k_{\pm\pm}} \frac{dP_{\pm}}{d\Omega_{\pm}} \quad (3-26)$$

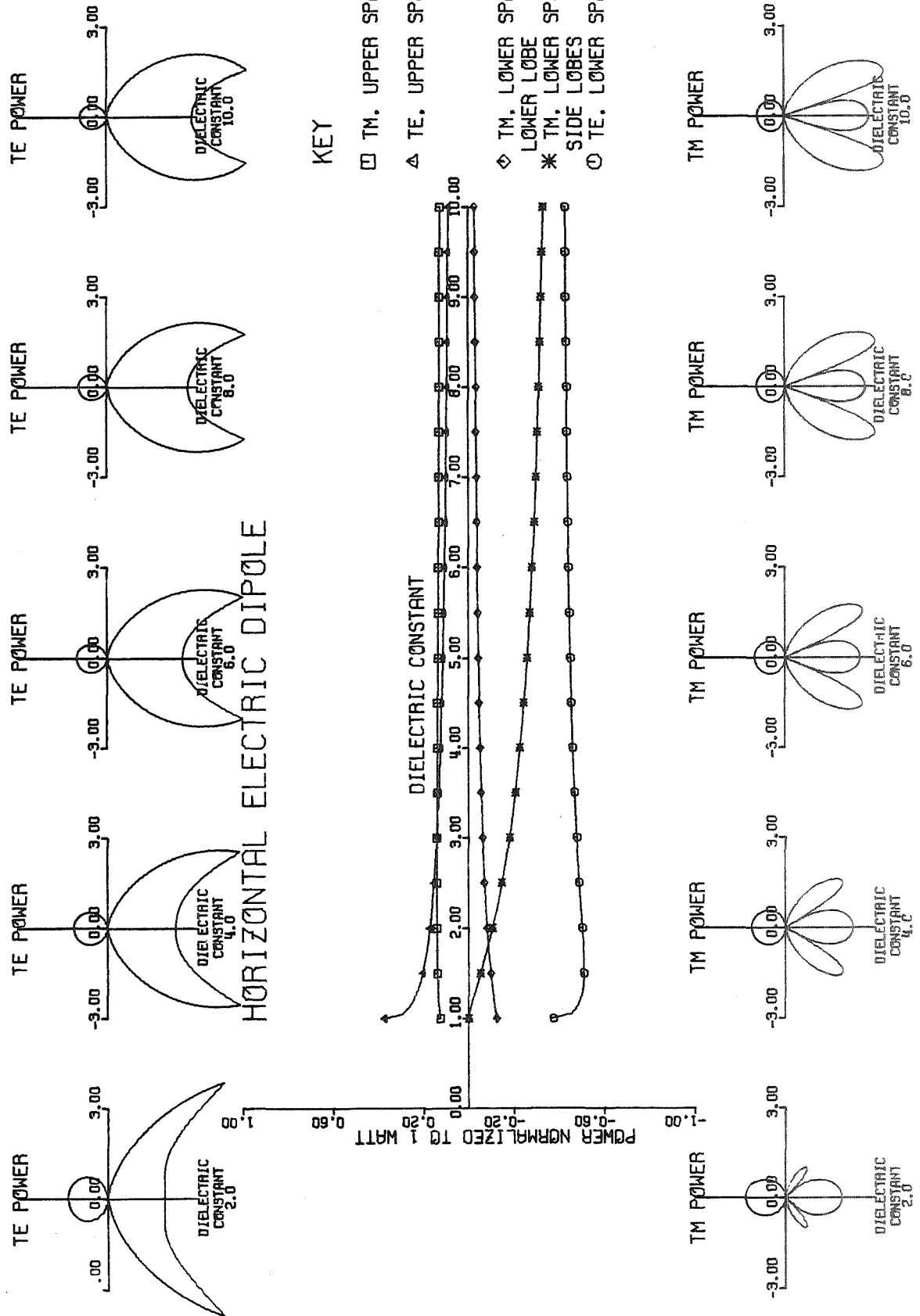


FIG. 3-2A HORIZONTAL ELECTRIC DIPOLE

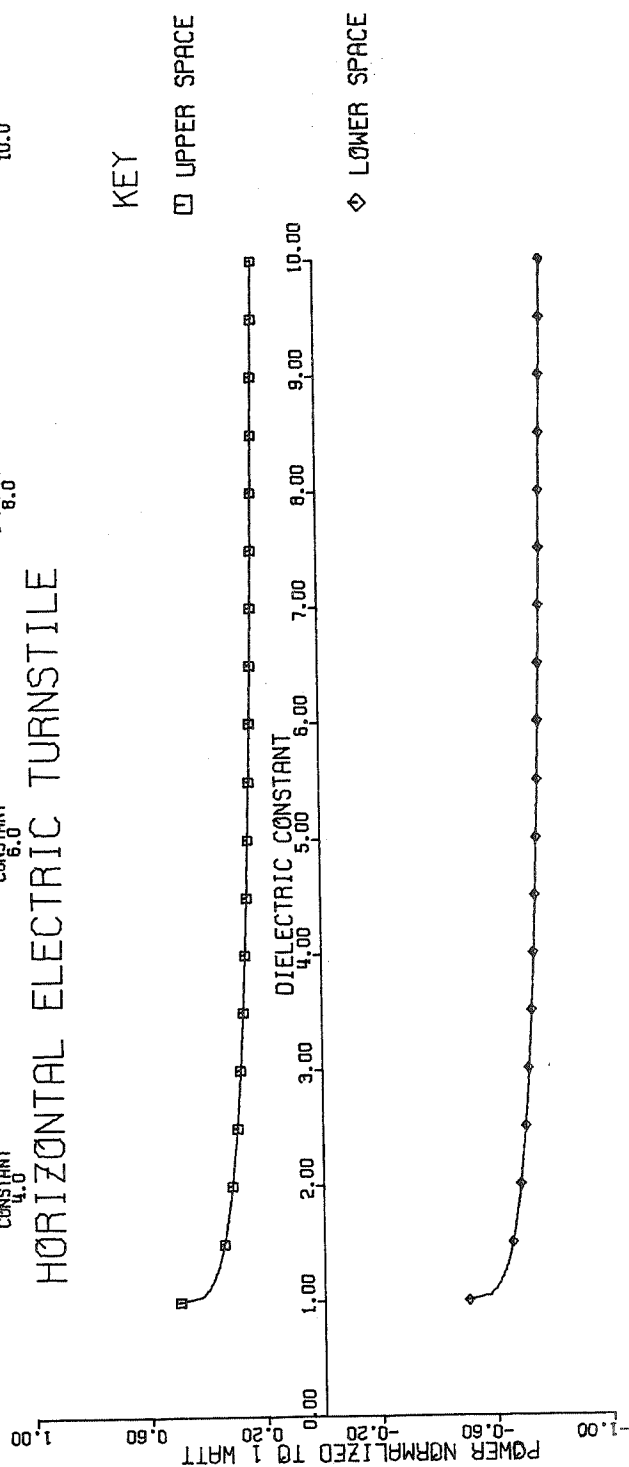
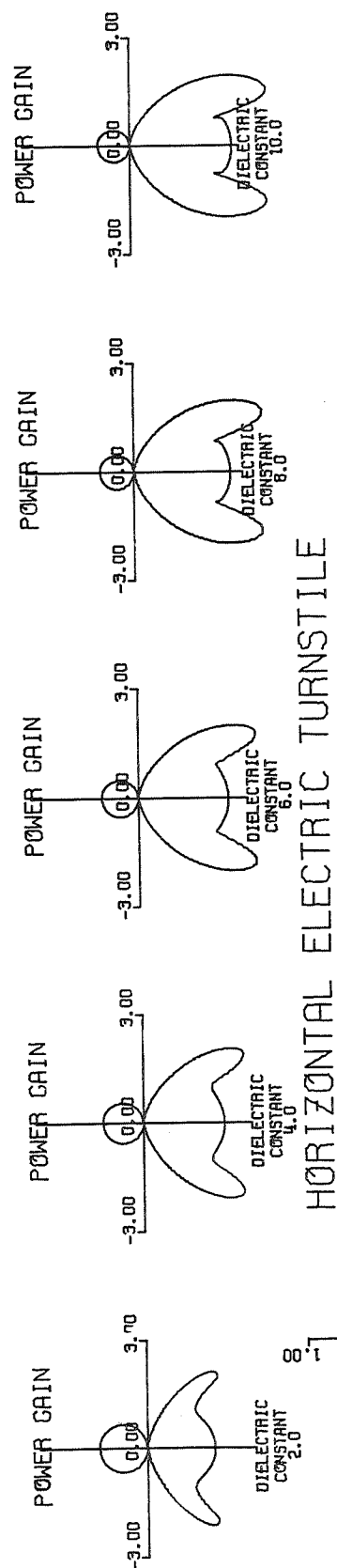


FIG. 3-2c HORIZONTAL ELECTRIC TURNSTILE

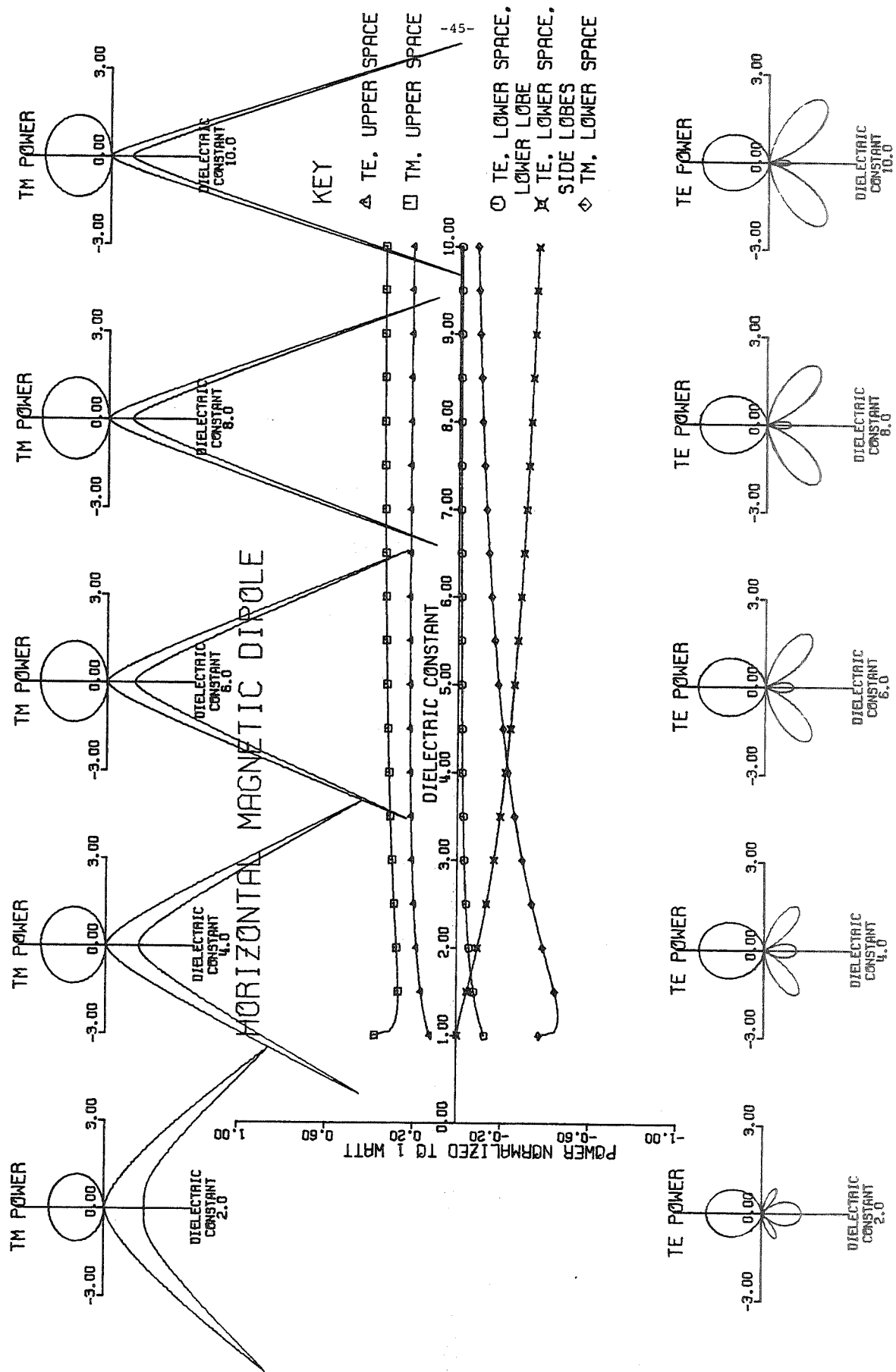
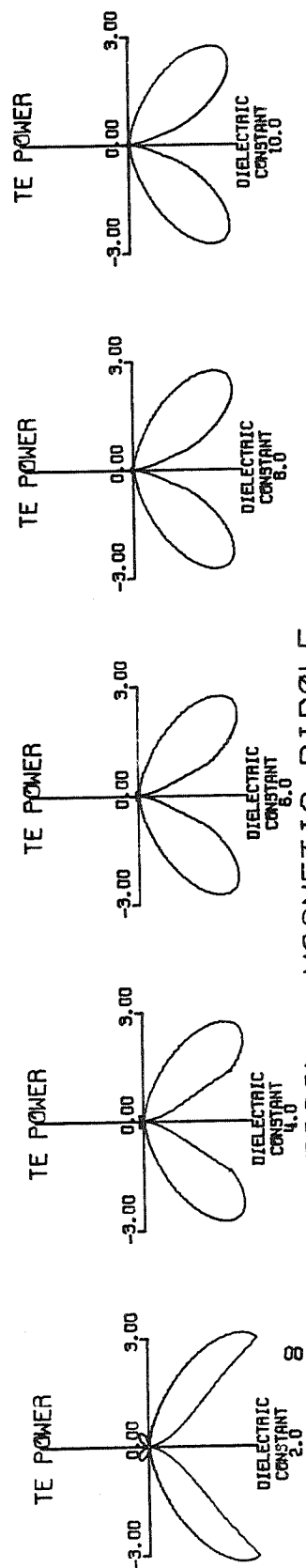


FIG. 3-2D HORIZONTAL MAGNETIC DIPOLE



VERTICAL MAGNETIC DIPOLE

KEY

▲ TE. UPPER SPACE

○ TE. LOWER SPACE

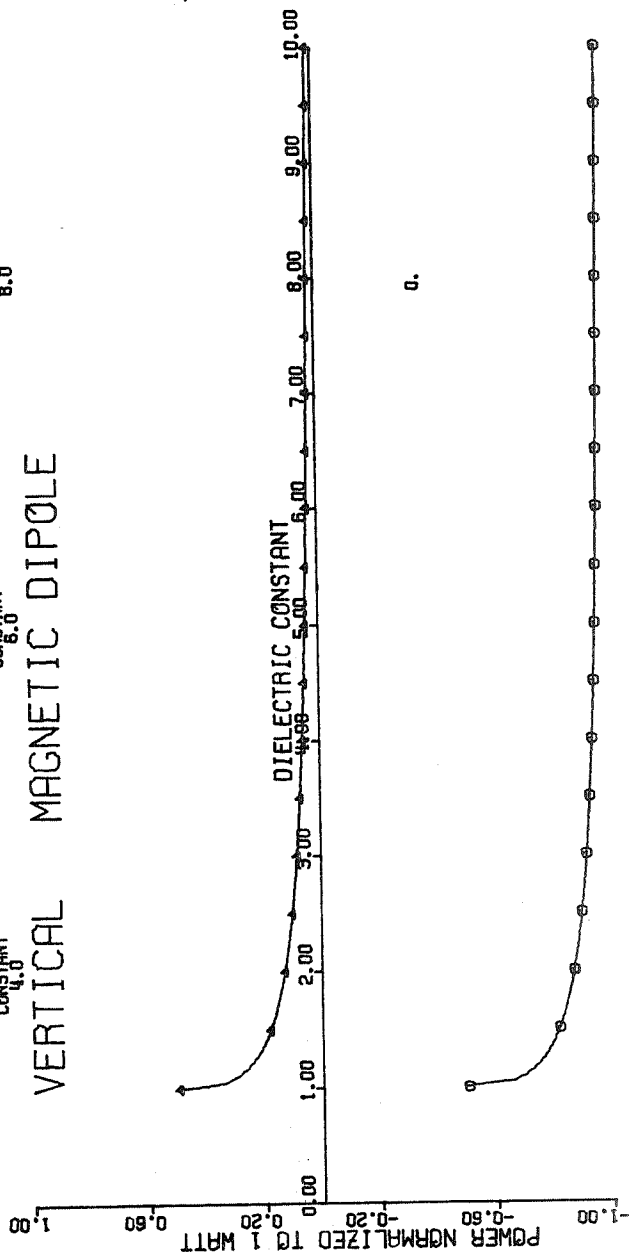


FIG. 3-2E VERTICAL MAGNETIC DIPOLE

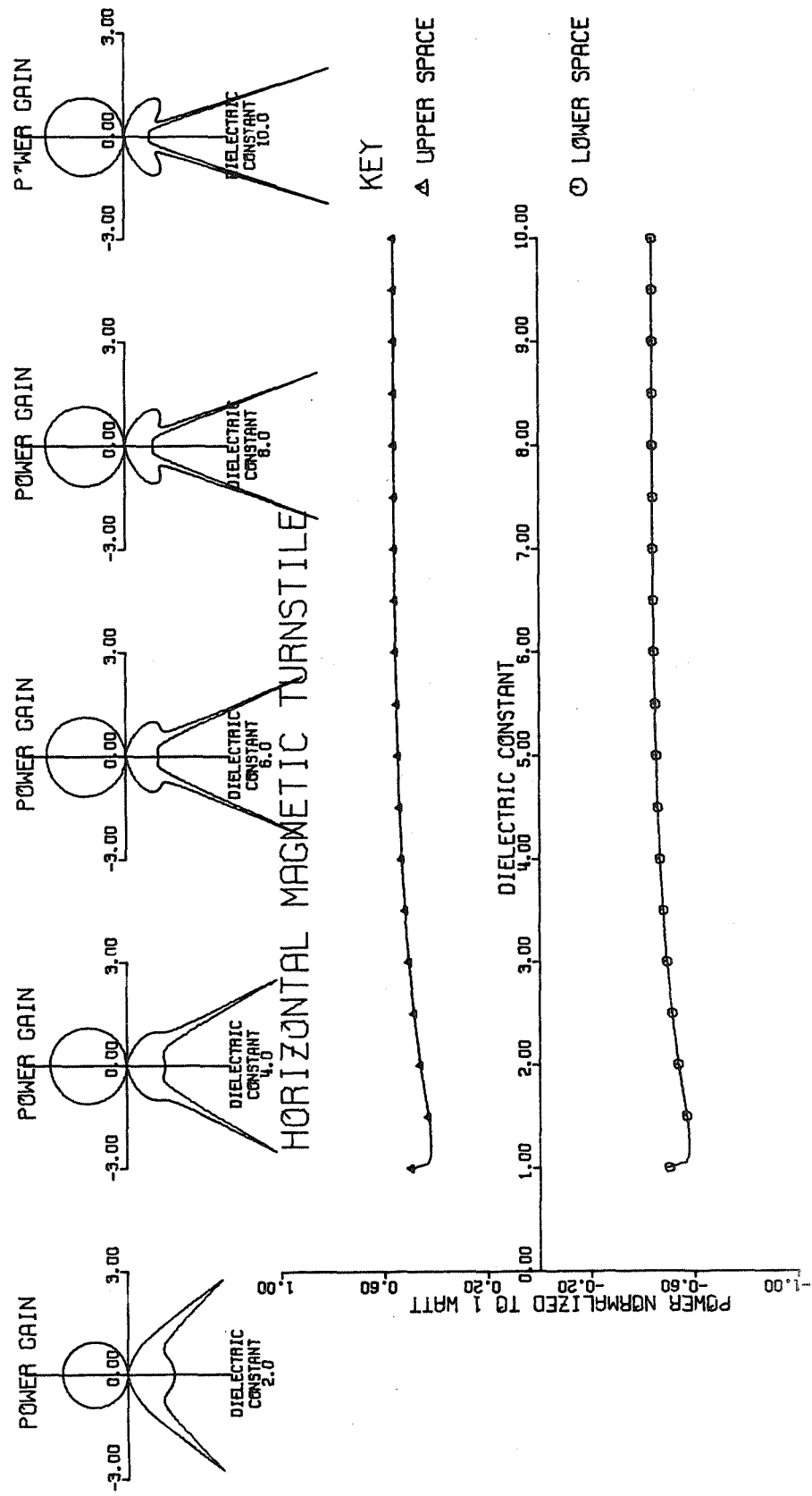


FIG. 3-2F HORIZONTAL MAGNETIC TURNSTILE

HORIZONTAL ELECTRIC DIPOLE

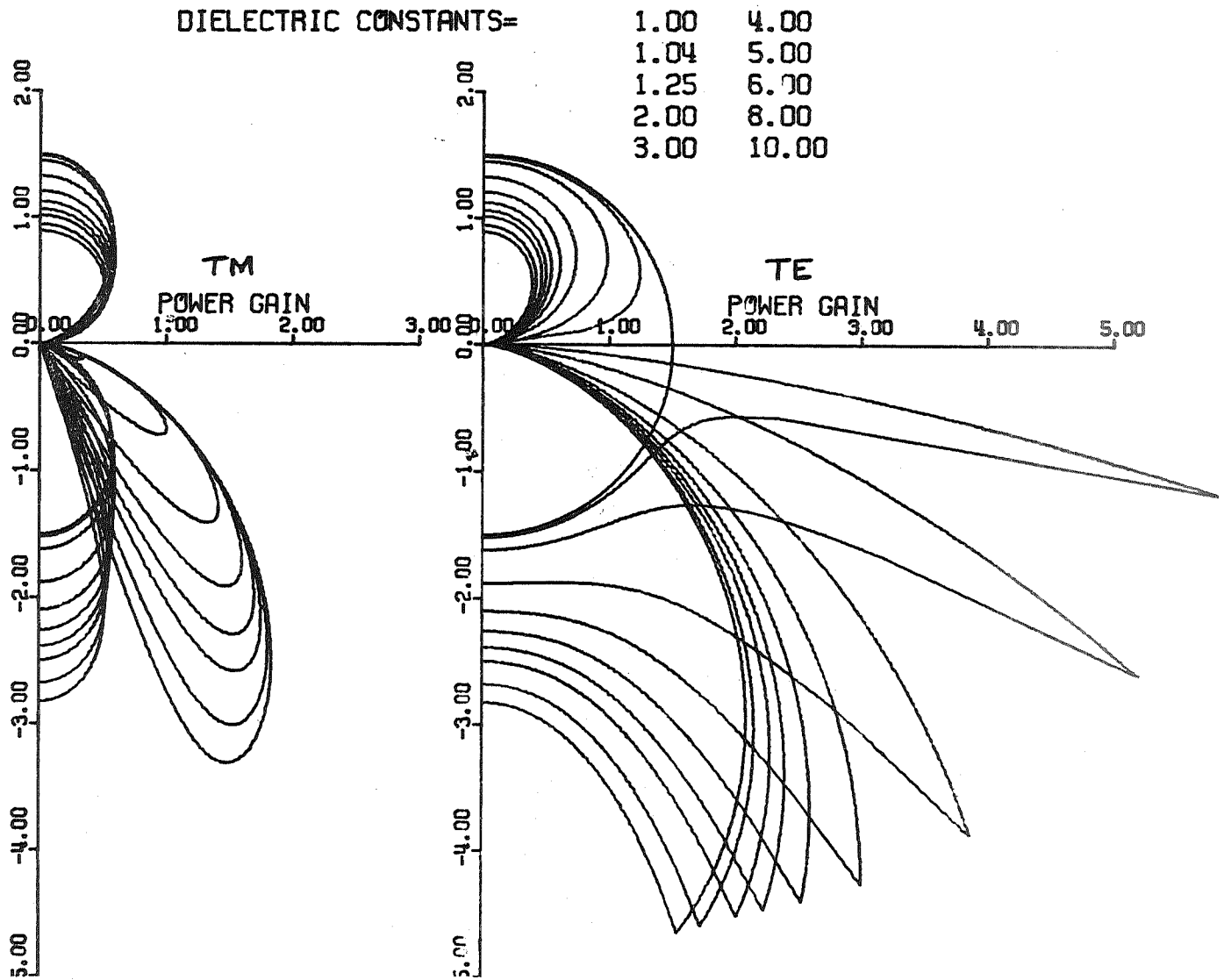


FIG. 3-3A HORIZONTAL ELECTRIC DIPOLE

VERTICAL ELECTRIC DIPOLE

DIELECTRIC CONSTANTS=

1.00
1.04
1.25
2.00
3.00
4.00
5.00
6.00
8.00
10.00

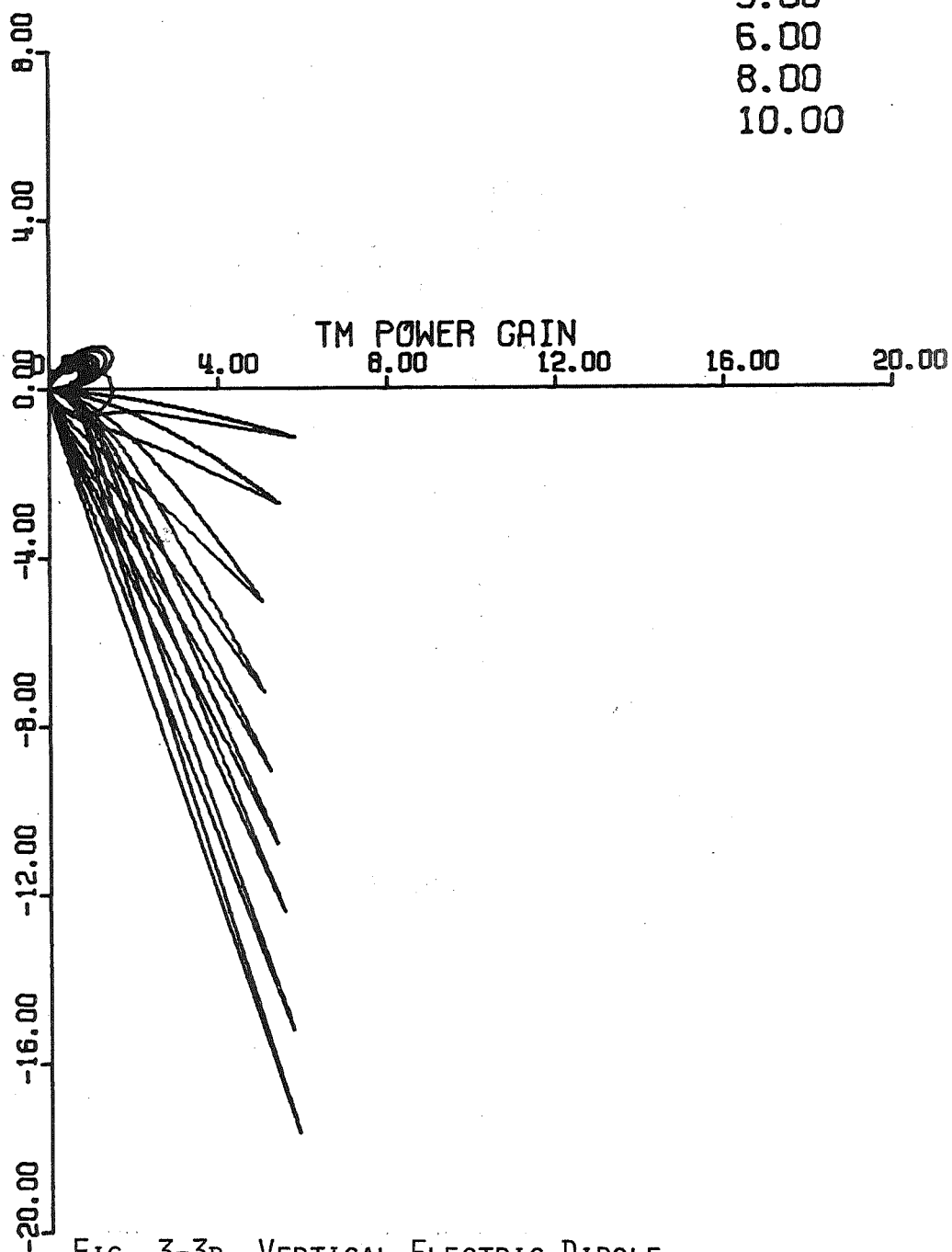


FIG. 3-3B VERTICAL ELECTRIC DIPOLE

HORIZONTAL ELECTRIC TURNSTILE

DIELECTRIC CONSTANTS=

1.00
1.04
1.25
2.00
3.00
4.00
5.00
6.00
8.00
10.00

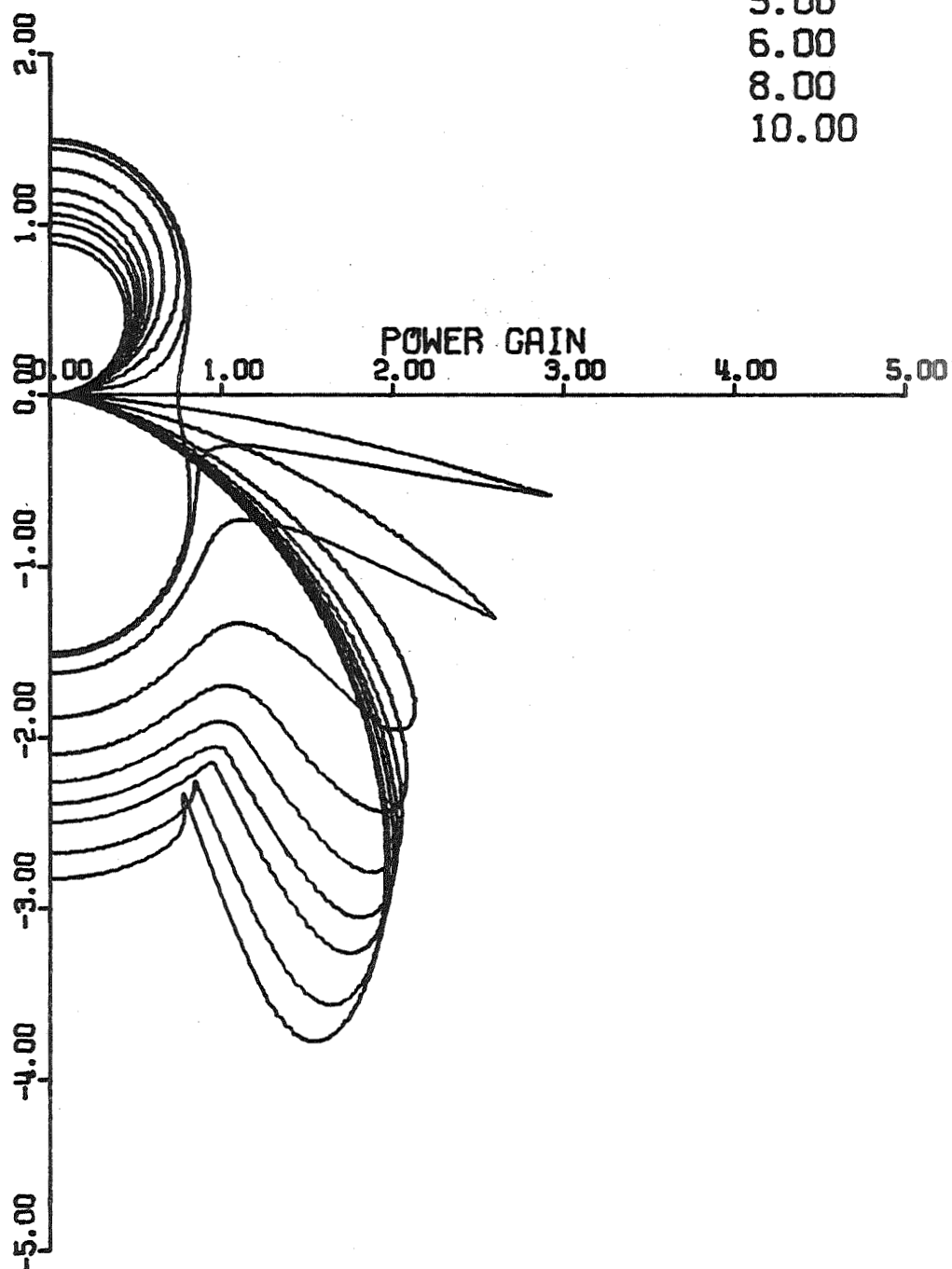


FIG. 3-3C HORIZONTAL ELECTRIC TURNSTILE

HORIZONTAL MAGNETIC DIPOLE

DIELECTRIC CONSTANTS=

1.00
1.04
1.25
2.00
3.00
4.00
5.00
6.00
8.00
10.00

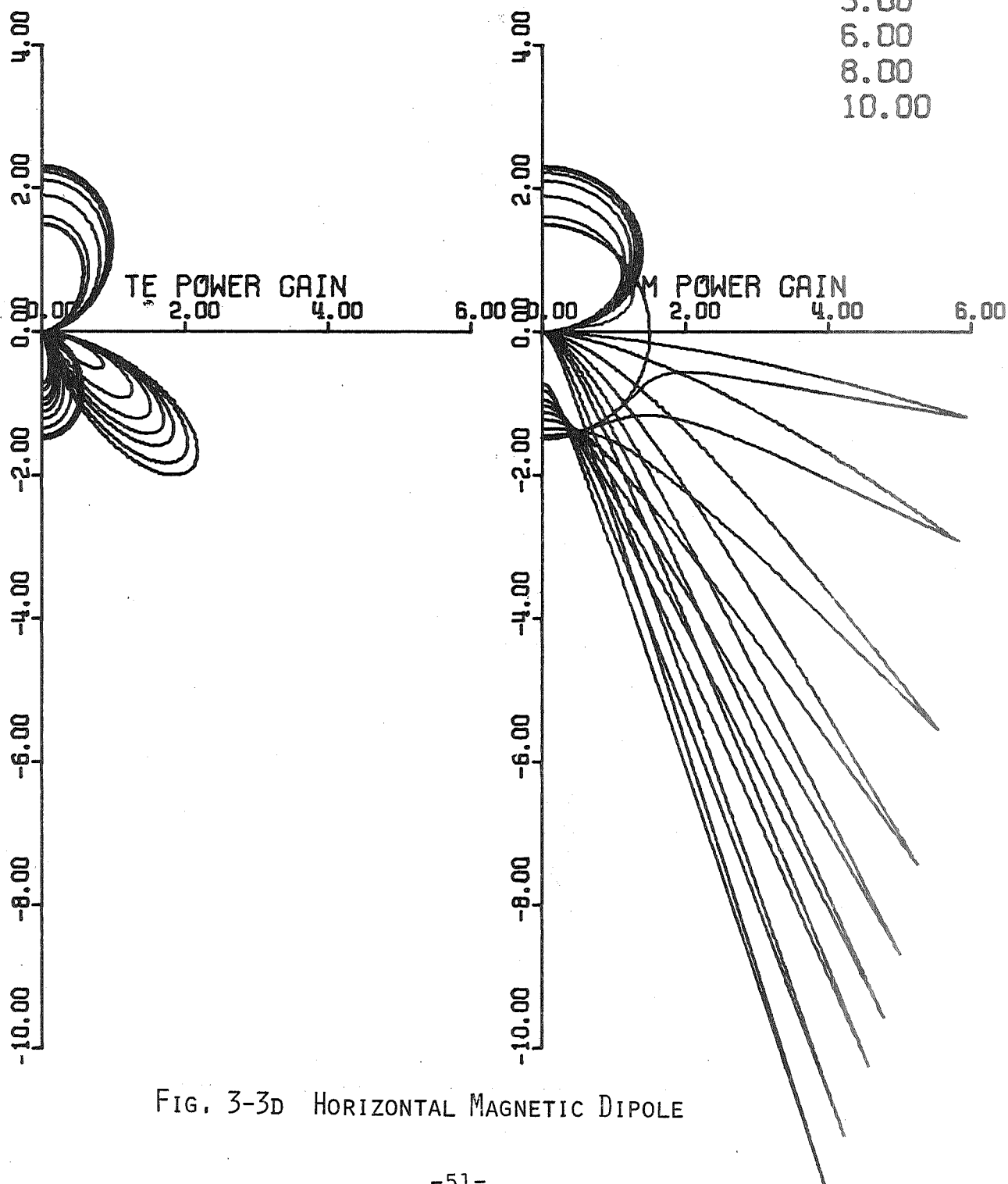


FIG. 3-3D HORIZONTAL MAGNETIC DIPOLE

VERTICAL MAGNETIC DIPOLE

DIELECTRIC CONSTANTS=

1.00
1.04
1.25
2.00
3.00
4.00
5.00
6.00
8.00
10.00

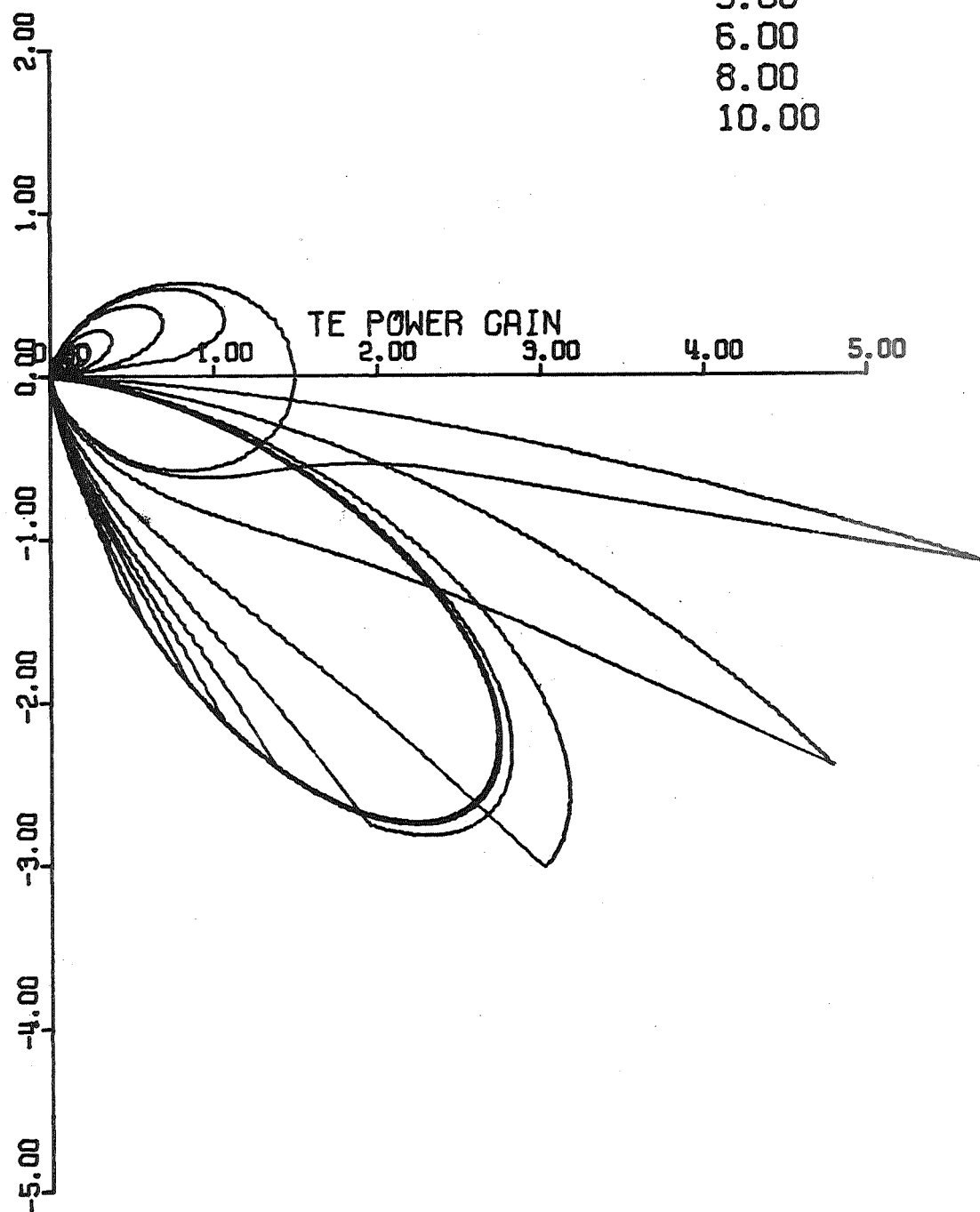


FIG. 3-3E VERTICAL MAGNETIC DIPOLE

HORIZONTAL MAGNETIC TURNSTILE

DIELECTRIC CONSTANTS=

1.00
1.04
1.25
2.00
3.00
4.00
5.00
6.00
8.00
10.00

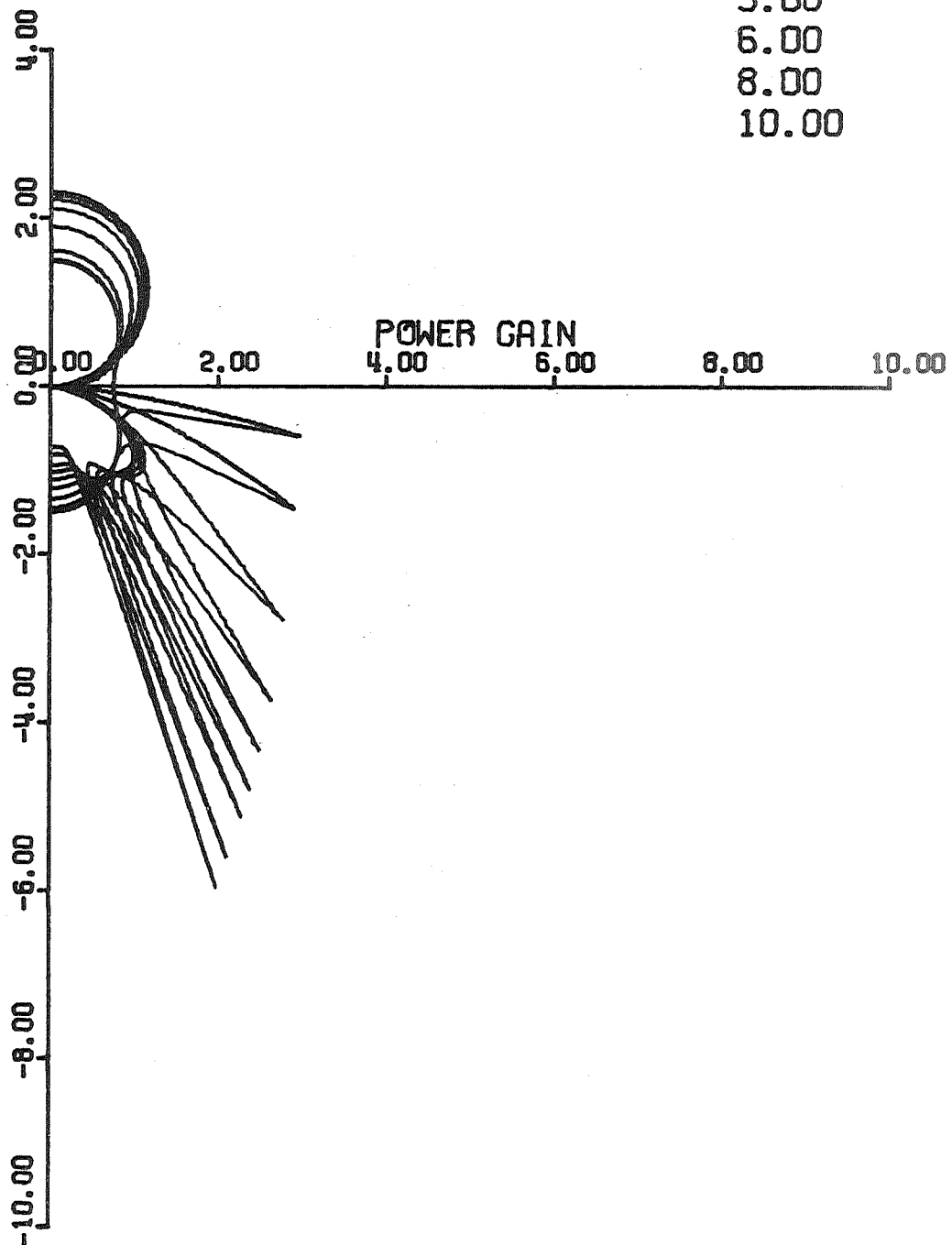


FIG. 3-3F HORIZONTAL MAGNETIC TURNSTILE

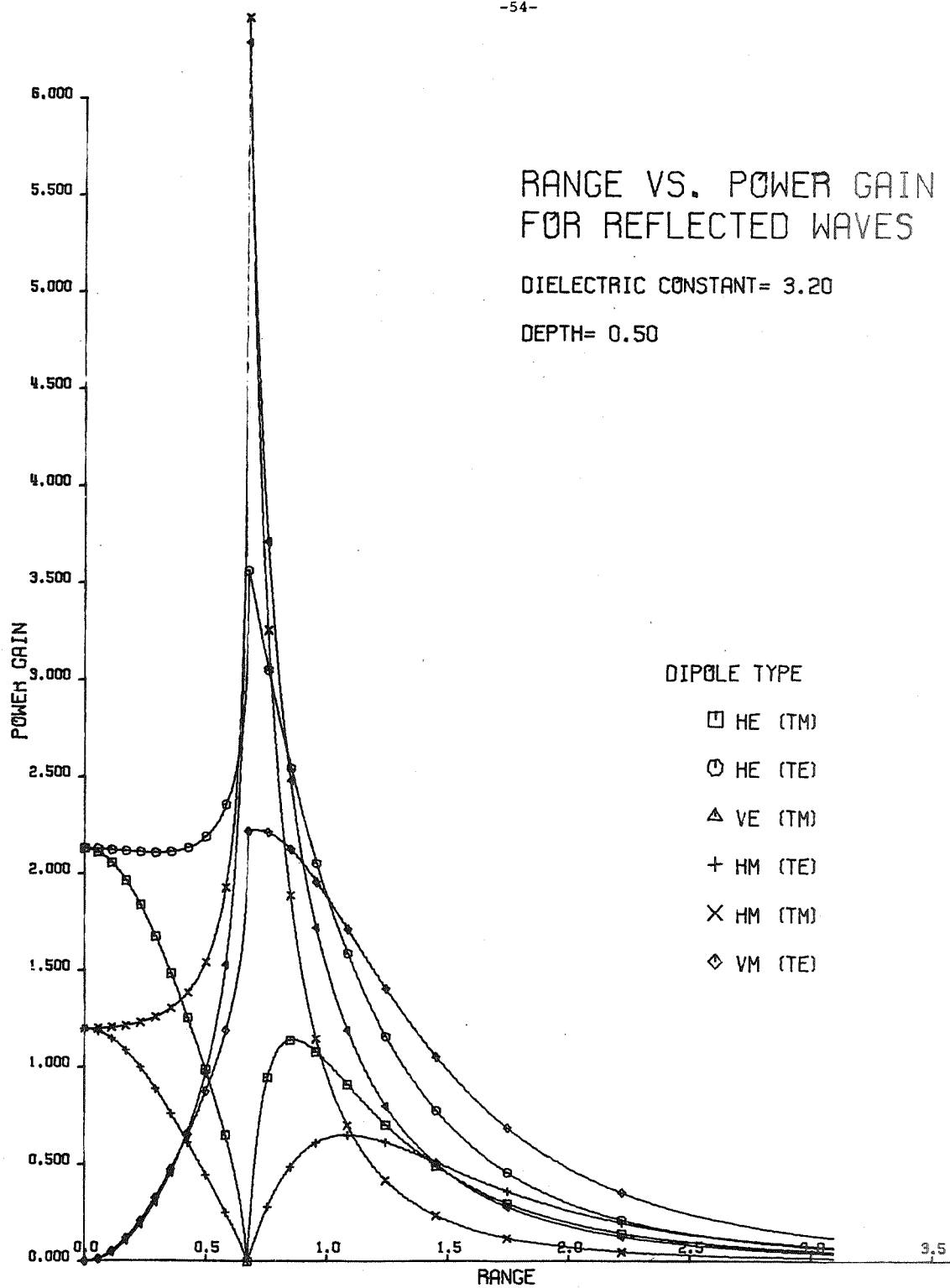


FIG. 3-4A RANGE VS. POWER GAIN FOR REFLECTED WAVES

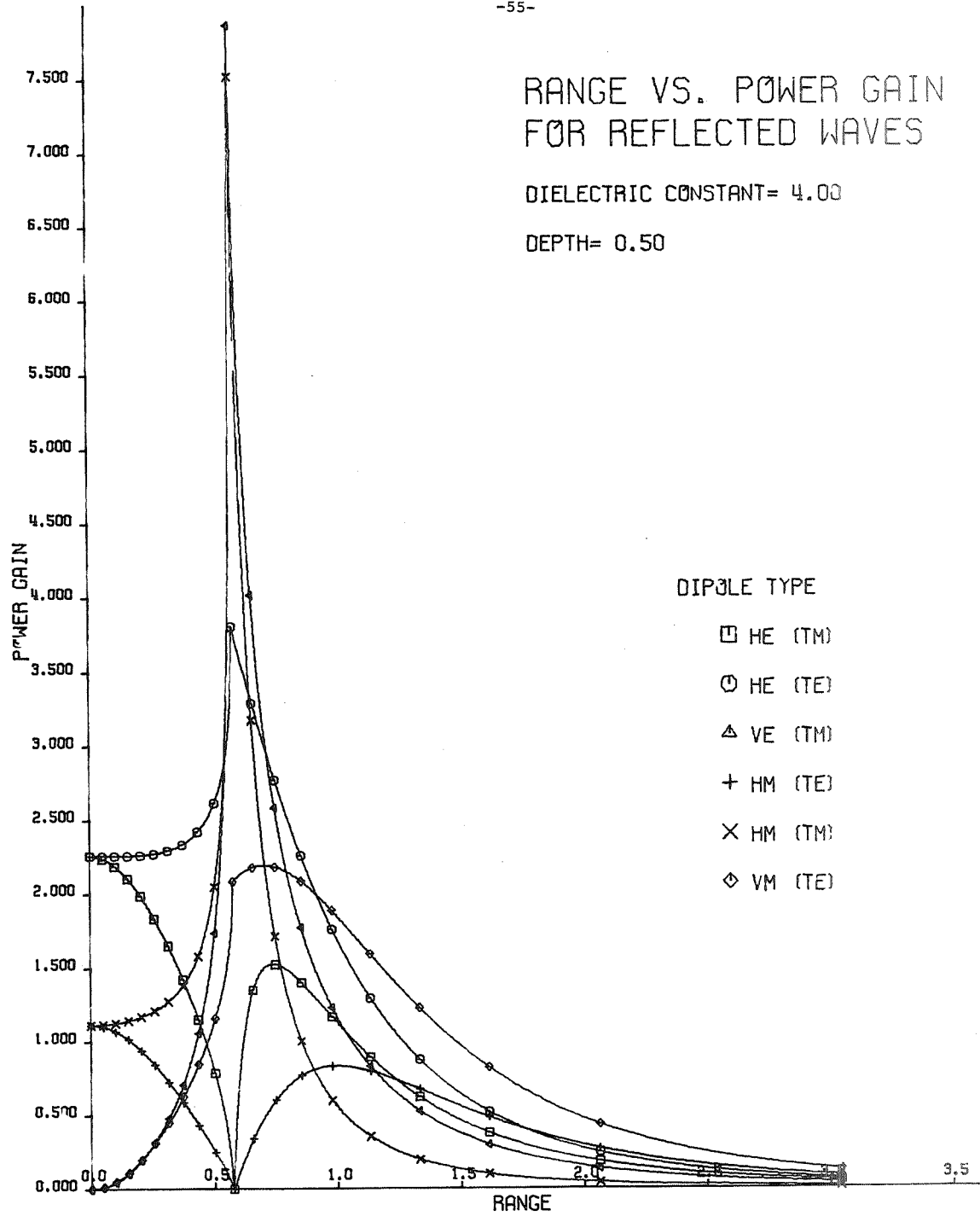


FIG. 3-4B RANGE VS. POWER GAIN FOR REFLECTED WAVES

with $\pm$ for upper/lower half-space, and

with $\begin{cases} \pi & \text{for horizontal dipole (integral of } \sin^2 \phi, \cos^2 \phi) \\ 2\pi & \text{for vertical dipole (integral of } l \cdot d\phi). \end{cases}$

$Kc_{\pm}$ is given by eq. (1-17(a)), with $i = \pm 1$. Antenna gain is then given by:

$$\text{Gain} = \frac{\frac{dP}{d\Omega}_{\ell}^{\pm}(\theta, \phi)}{\frac{1}{4\pi} \sum_{\ell} \sum_{\pm} P_{\ell}^{\pm}} \quad (3-27)$$

The sum contains the following terms, for:

horizontal electric dipole,	$\ell = 1, 2, 7, 8$
vertical electric dipole,	$\ell = 3, 9$
horizontal magnetic dipole,	$\ell = 4, 5, 10, 11$
vertical magnetic dipole,	$\ell = 6, 12$

The resulting curves are compared in Chapter 4.

3.4 Special Half-Space Surface Fields

In the boundary plane ($z = 0$) between two semi-infinite isotropic media, integral expressions (3-12) and (3-13) to (3-21) are used to derive asymptotic field expressions for $r \rightarrow \infty$. Certain field vectors are shown to have large radial components. All field components are $O(1/r^2)$.

Assuming that most of the integral (3-12) is contributed near the branch-point singularities kc_i , where $k_i \sim (kc_i - k_t)^{1/2}$, the following mathematical identity (5-22), derived in Section 5.2, is used:

$$\frac{1}{2\pi} \int_{k_t=0}^{\infty} dk_t k_t e^{-jk_i z} \left\{ \begin{matrix} J_0(k_t r) \\ jJ_1(k_t r) \end{matrix} \right\} \approx j\partial_z^2 G_i \approx \frac{k_i}{R} G_i = \frac{k_i}{r} G_i \quad (3-28),$$

with

$$G_i = \frac{e^{-jk_i R}}{4\pi R} \quad (3-29)$$

and $R = \sqrt{r^2 + z^2}$ from (3-23),

assuming in (3-28) the real-part convention (1-1).

The mathematical form of each integrand, (3-13) - (3-21), near each branch point is derived by a geometric series method, using $\tilde{H}_r$ for example:

$$\begin{aligned} \tilde{H}_r \sim & -a_s \phi \left\{ \frac{\mu(k_i)}{\mu, k_i} \left[1 - \left(\frac{\mu, k_i}{\mu, k_i} \right) - \left(\frac{\mu, k_i}{\mu, k_i} \right)^2 - \dots \right] \right. \\ & \left. + \left[1 - \left(\frac{\mu(k_i)}{\mu, k_i} \right) - \left(\frac{\mu, k_i}{\mu, k_i} \right)^2 - \dots \right] \right\} \quad (3-30) \end{aligned}$$

Only the leading terms proportional to K_1 and $\underline{K}_1$ (as circled) will be integrated. Only these leading terms give fields $\sim 1/r^2$. Correspondingly, all terms proportional to $J_1/k_t r$ will be neglected.

Using (3-26) and the leading terms of (3-30) one obtains the leading field terms for H_r :

$$H_r \sim -2a_s \phi \left\{ \frac{\mu_{-1} k_{c1}}{\mu_{-1} \underline{K} R} G_1 - \frac{\mu_{-1} k_{c1}}{\mu_{-1} (-i \underline{K}) R} \underline{G}_1 \right\} \quad (3-31)$$

where $\underline{K} \triangleq \sqrt{k_{c1}^2 - k_{c1}^2} = k_{c1} k \quad (3-32),$

$$k \triangleq \sqrt{c-1} \quad (3-33).$$

(c is given in (3-24).)

Similar algebraic manipulations give all other leading ($\sim 1/r^2$) $\underline{E}$ & $\underline{H}$ field components, as listed in Table (3-3). In Table (3-3), the $\underline{E}$ & $\underline{H}$ components are normalized, denoted by primes ('). The normalization is done separately for electric and magnetic dipoles. For each type of dipole, the $\underline{E}$ & $\underline{H}$ components are normalized to $E_{z,+1,t}$ and $H_{\phi,+1,t}$ such that:

$$E'_{z,+1,t} = H'_{\phi,+1,t} \triangleq 1 \quad (3-34),$$

	H_i'	G_i	G_{-i}	E_i'	G_i	G_{-i}
a_t	H_r'	$-c^{-1}$	$j\bar{c}^{-1/2}$	E_r'	$-\bar{c}'k$	$-\bar{c}^{-3/2}k$
	H_ϕ'	$(+1)$	$-j\bar{c}^{-3/2}$	E_ϕ'	$-\bar{c}'k^{-1}$	$+\bar{c}^{-1/2}k^{-1}$
	H_z'	$+\bar{c}^{-3/2}$	$-k^{-1}$	E_z'	$(+1)$	$-j\bar{c}^{-2}$
a_z	H_ϕ'	$+ck^{-1}$	$+\bar{c}'k^{-1}$	E_r'	$+1$	$-j\bar{c}'$
				E_z'	$-ck^{-1}$	$+\bar{c}^{-1/2}k^{-1}$
b_t	H_r'	$+\bar{c}^{-2}k^2$	$+\bar{c}^{-3/2}k^2$	E_r'	$-\bar{c}'k$	$j\bar{c}^{-5/2}k$
	H_ϕ'	$(+1)$	$-\bar{c}^{-5/2}$	E_ϕ'	$+\bar{c}^{-2}k$	$-j\bar{c}^{-3/2}k$
	H_z'	$-\bar{c}^{-2}k^2$	$j\bar{c}'k$	E_z'	$(+1)$	$-\bar{c}^{-2}$
b_z	H_r'	$+\bar{c}^{-2}k$	$-j\bar{c}'k$	E_ϕ'	$+\bar{c}^{-2}$	$-\bar{c}^{-1}$
	H_z'	$-\bar{c}^{-2}$	$+\bar{c}^{-1/2}$			

Table 3-3 Normalized Surface Fields, $K = \sqrt{C-1}$

as circled in Table (3-3), where "t" denotes "horizontal dipoles," and "+1" denotes the term, as in eq. (3-31), proportional to G_1 .

Coincidentally, the un-normalized components satisfy:

$$\frac{E_{z_{\pm 1, t}}}{H_{\phi_{\pm 1, t}}} = \sqrt{\frac{\mu_{\pm 1}}{\epsilon_{\pm 1}}} \triangleq \xi_{\pm 1} \quad (3-35).$$

Prior to plotting the surface field components, the components in Table (3-3) are further scaled such that each dipole radiates the same amount of power into all of space. The plots in Figures (3-5) (a) - (d) show relative magnitudes of such field components, for comparison of the different types of dipoles.

In general, the E-field or the H-field will be elliptically polarized on the surface. This fact is obvious from the appearance of imaginary terms in Table 3-3 and in Figure 3-5.

Again, comparison of the different dipoles will be made in Chapter 4.

SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

HORIZONTAL ELECTRIC DIPOLE TE

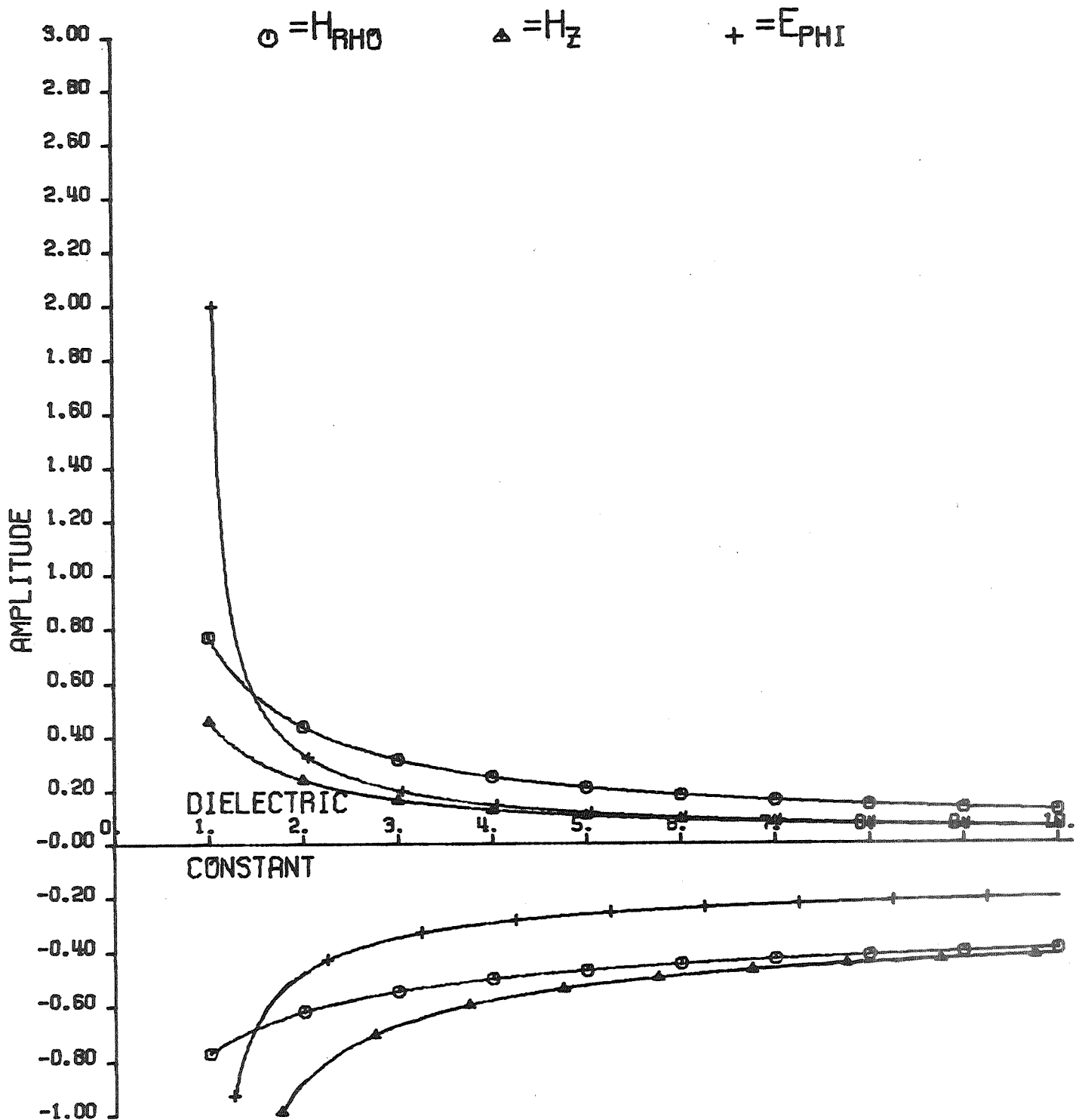


FIG. 3-5A SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

HORIZONTAL ELECTRIC DIPOLE TM

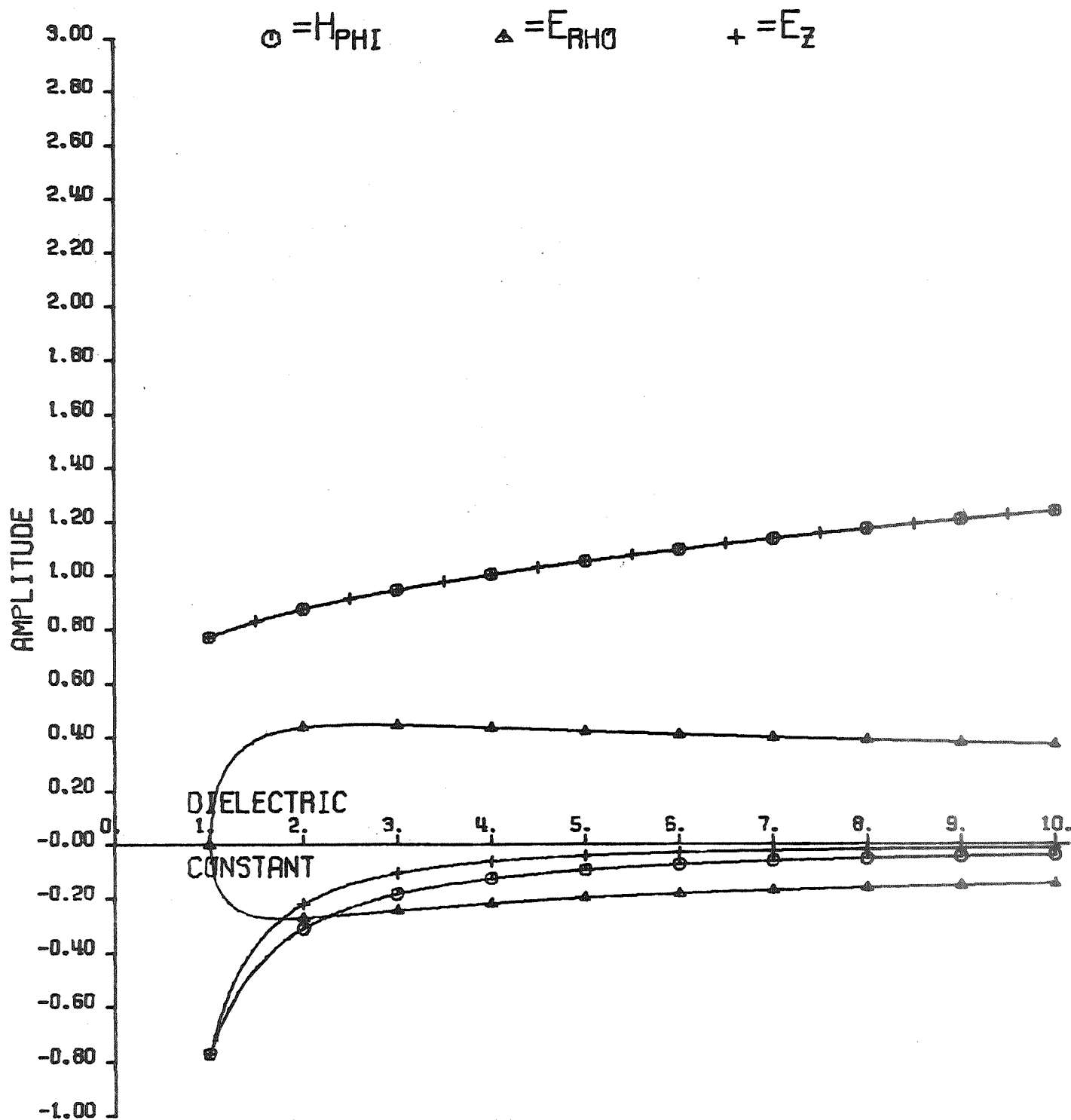


FIG. 3-5B SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

VERTICAL ELECTRIC DIPOLE TM

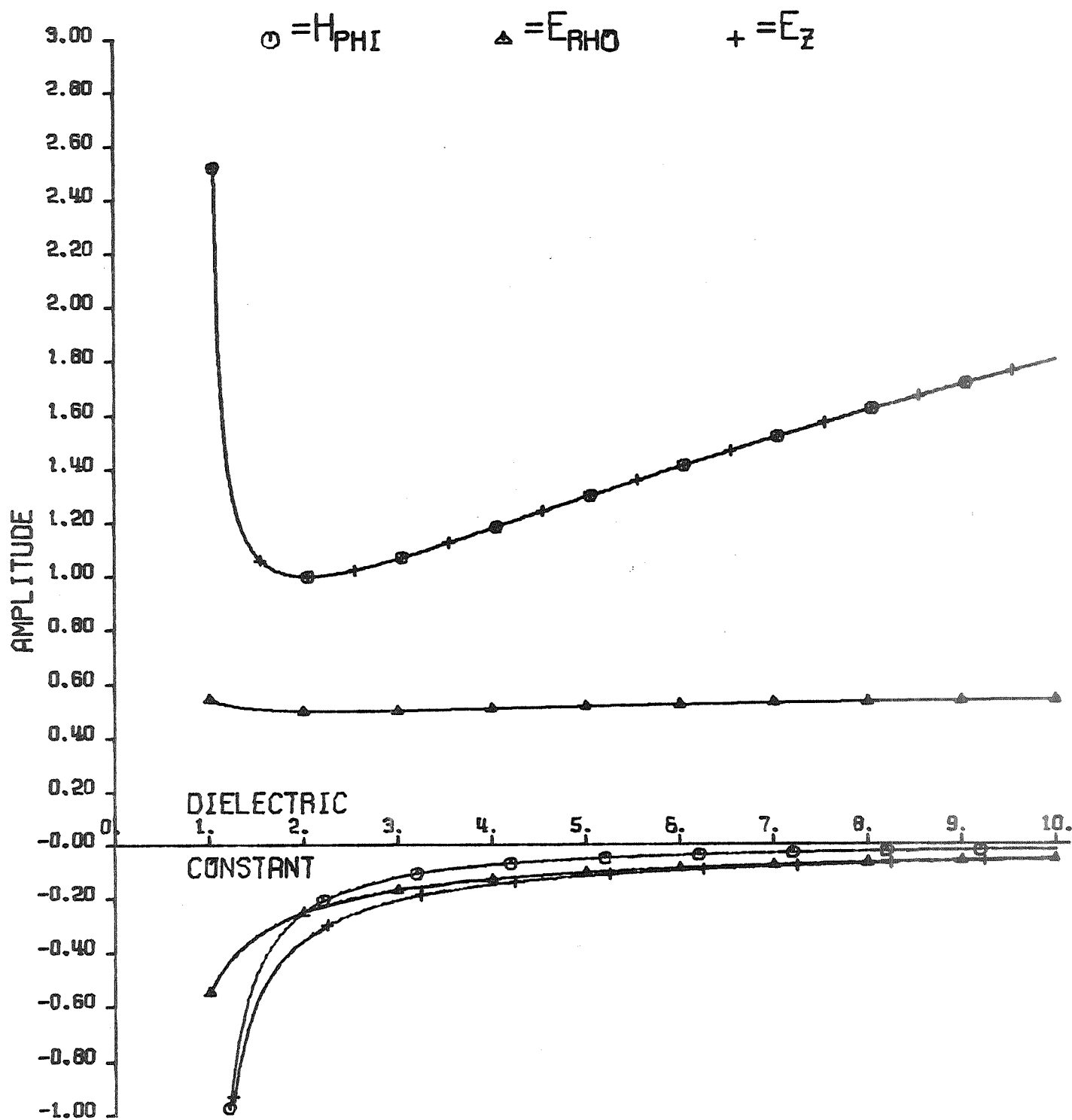


FIG. 3-5c SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

HORIZONTAL MAGNETIC DIPOLE TM

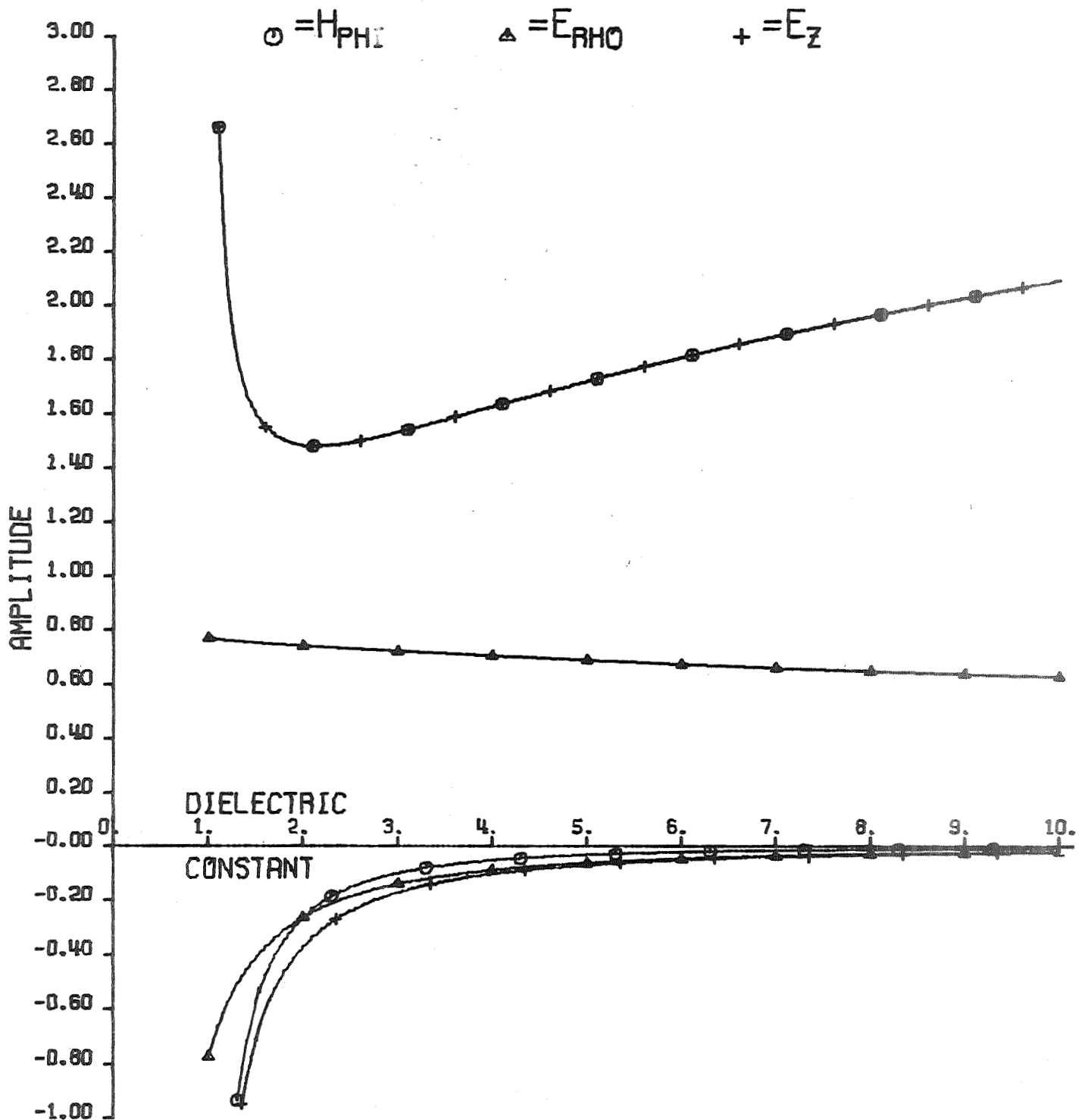


FIG. 3-5D SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

HORIZONTAL MAGNETIC DIPOLE TE

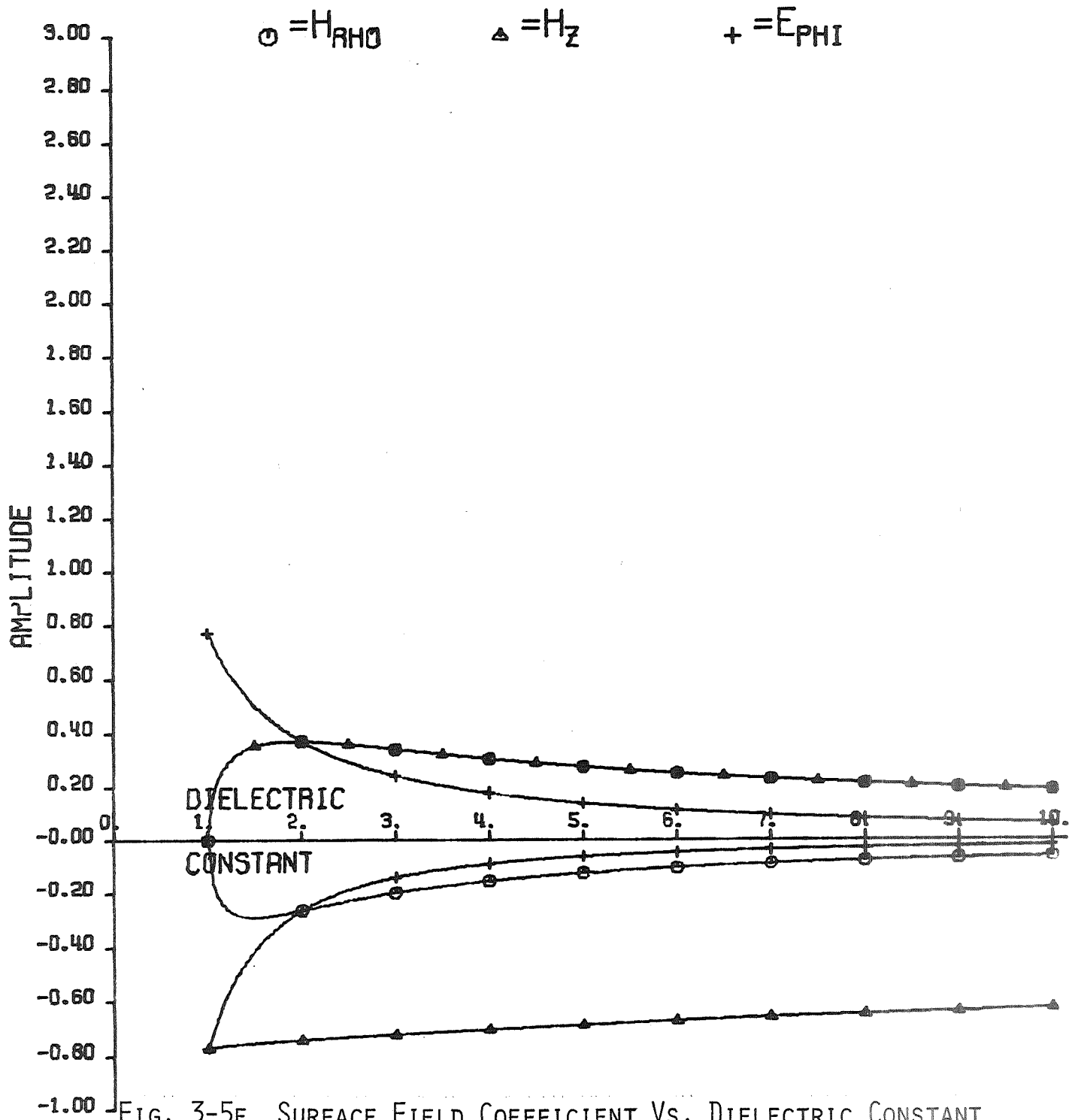


FIG. 3-5E SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

VERTICAL MAGNETIC DIPOLE TE

$$\circ = H_{RH0}$$

$$\Delta = H_z$$

$$+ = E_{PHI}$$

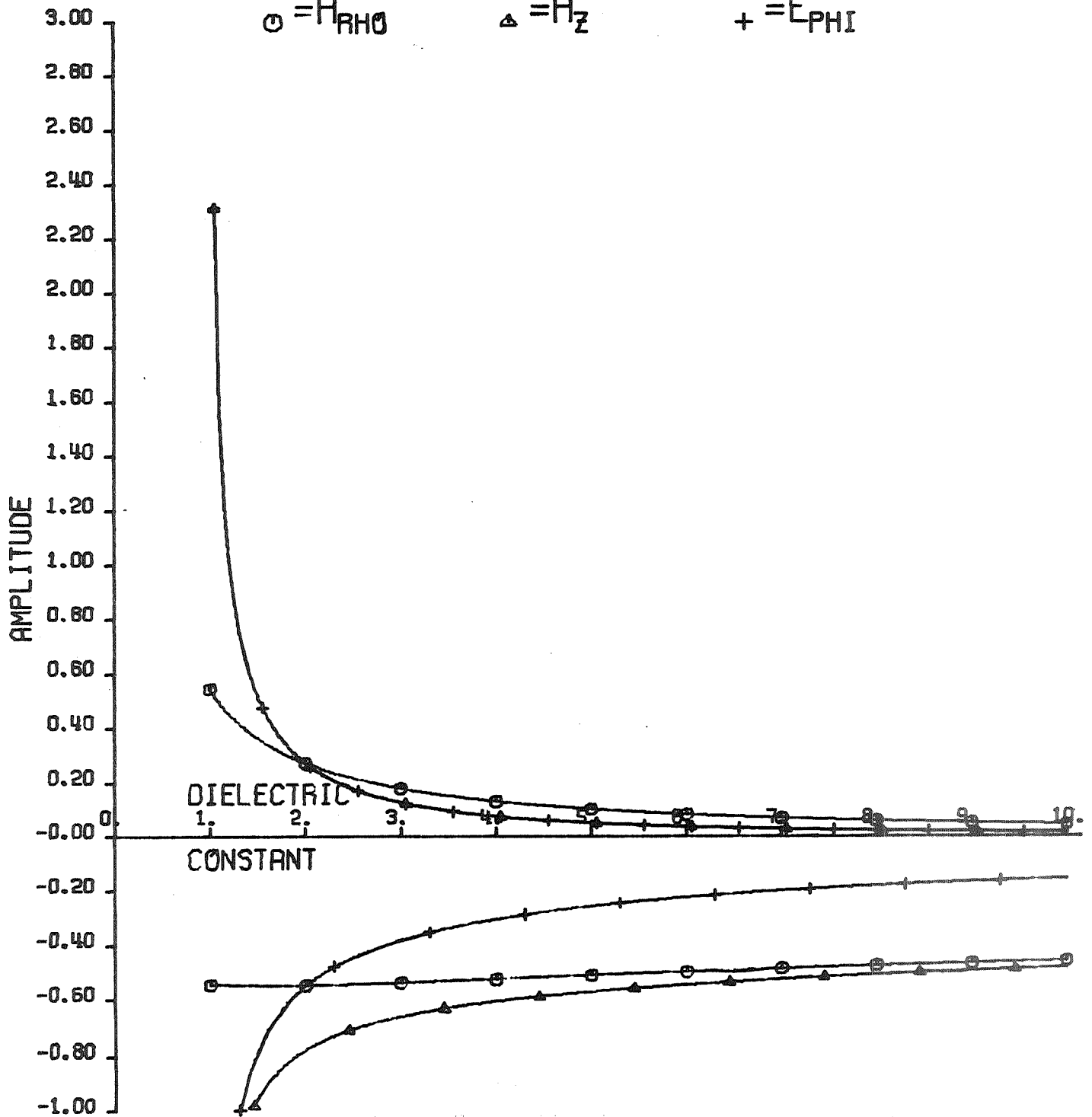


FIG. 3-5F SURFACE FIELD COEFFICIENT VS. DIELECTRIC CONSTANT

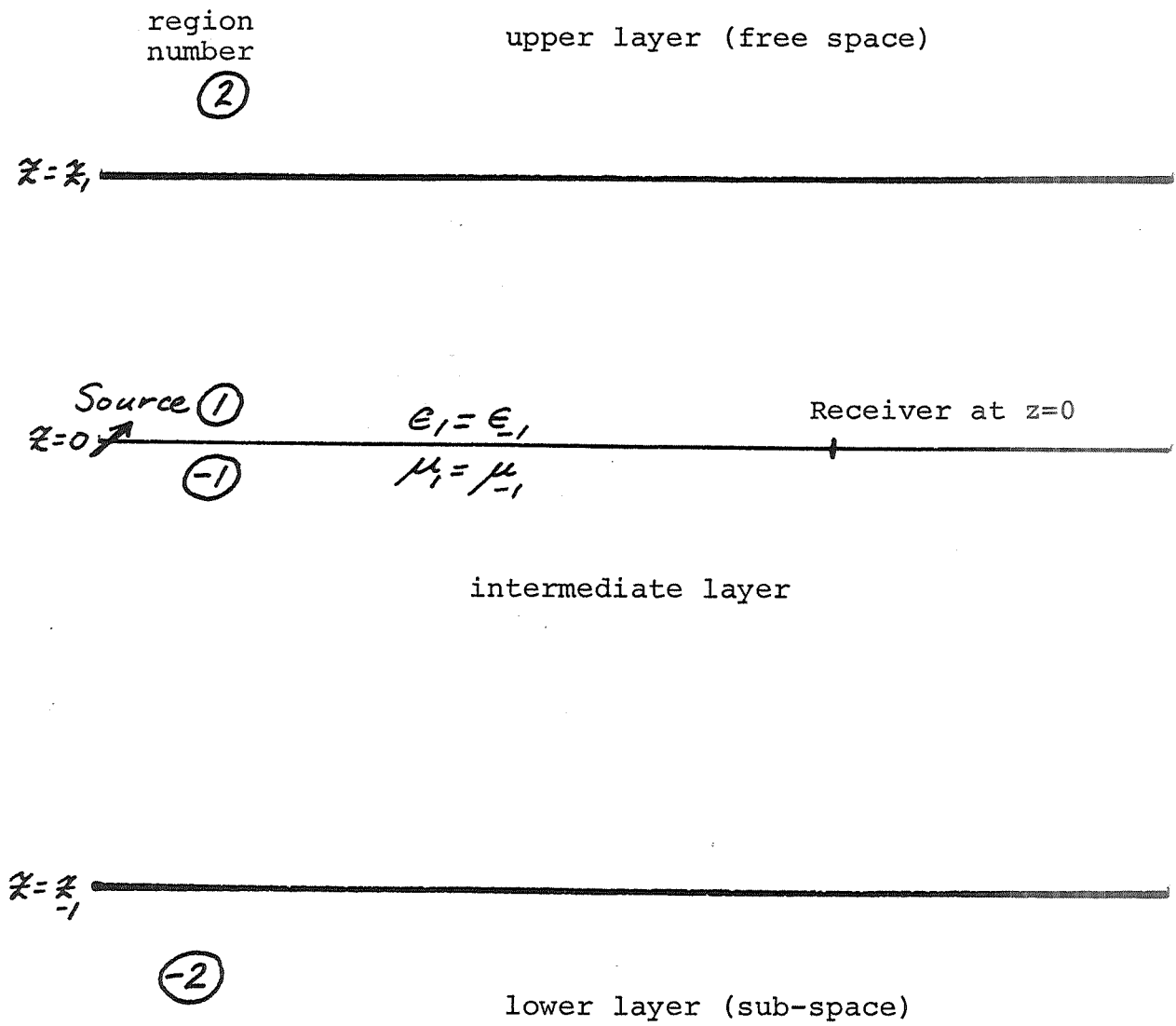


Fig. 3-6 3-Layer Medium Geometry

3.5 Perturbation by a Thin Surface Layer

The addition of a thin layer of dielectric material between two semi-infinite dielectric half-spaces results in a perturbation of the half-space electromagnetic fields. In practice, such a thin perturbing layer might represent a photoelectron layer directly above the lunar surface or a thin layer of rubble directly below the lunar surface.

It is easily seen that the half-space far fields and power gain patterns, as calculated in section 3.3, are simply perturbed by terms on the order of $k_i \Delta_i$. Here, k_i (1-17) is the z-propagation constant of the intermediate layer; and Δ_i is its thickness. Similarly, the asymptotic surface fields, which were calculated in section 3.4, are simply perturbed by the addition of similar terms, $O(k_i \Delta_i)$, added to the coefficients of the leading field approximations, which were found to be proportional to:

$$\frac{e^{-jk_i r}}{r^2}$$

as in (3-31).

For a medium with 3 layers (or N layers), a fundamentally new type of field appears: the parallel-layer dielectric waveguide mode. For a thin layer, it is easily seen that all of the waveguide modes are highly attenuated. (The attenuation is due to radiation into the upper and

HORIZONTAL ELECTRIC DIPOLE PERTURBED ANTENNA PATTERNS

PERTURBED LAYER OF 0.0500λ
DIELECTRIC CONSTANTS

SURFACE	1.000
PERTURBED LAYER	0.250
SUBSURFACE	4.000

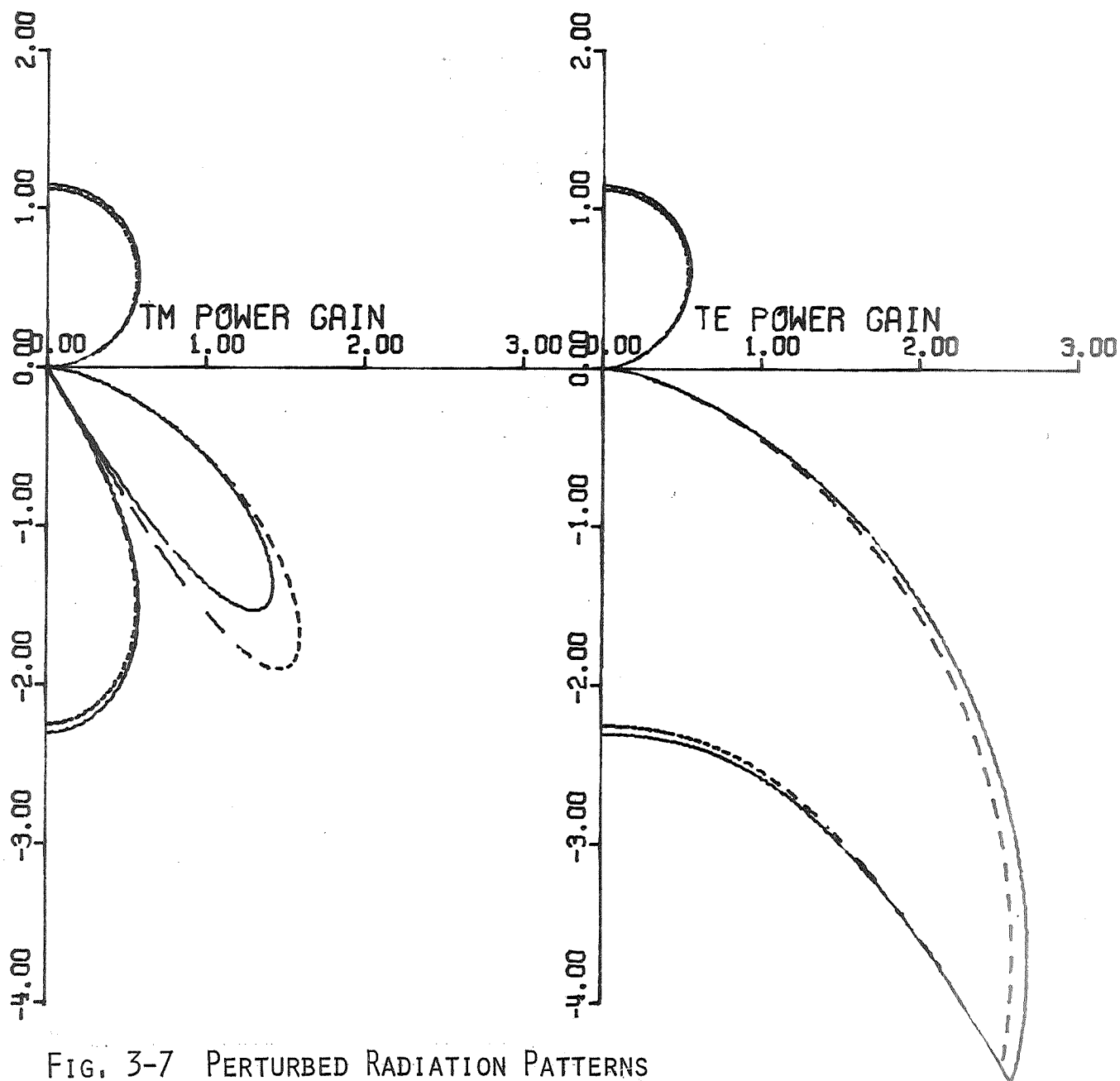


FIG. 3-7 PERTURBED RADIATION PATTERNS

HORIZONTAL ELECTRIC DIPOLE PERTURBED ANTENNA PATTERNS

PERTURBED LAYER OF 0.0100λ
DIELECTRIC CONSTANTS

SURFACE	1.000
PERTURBED LAYER	16.000
SUBSURFACE	4.000

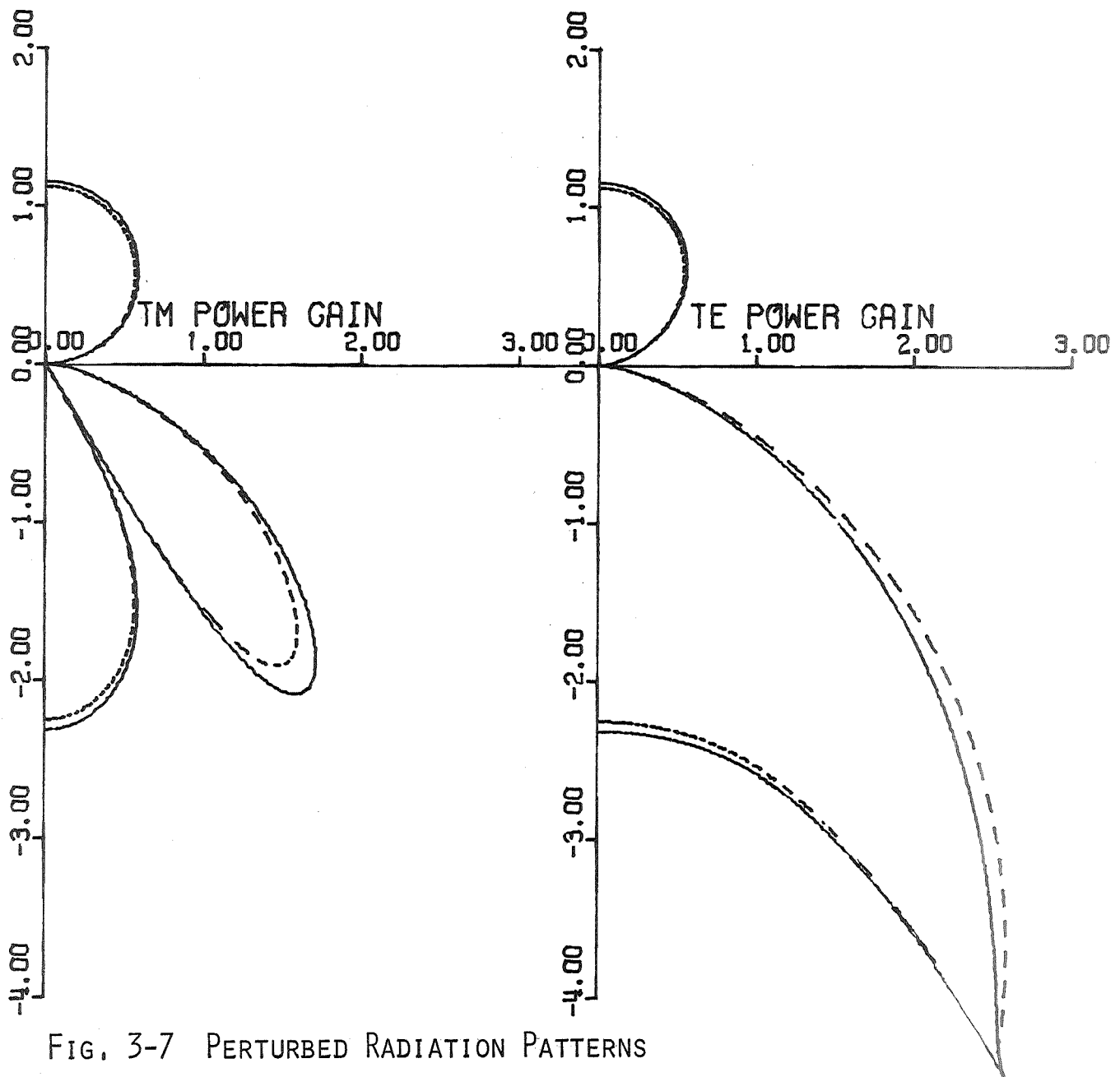


FIG. 3-7 PERTURBED RADIATION PATTERNS

lower half-spaces.) However, it is possible that the conditions for a waveguide mode can be approximately satisfied for real k_t . At such a k_t , there exists a "pseudo-waveguide mode" for which: (i) power is ducted near the surface from the source out to several wavelengths, but (ii) does not propagate to infinity with the $\sim 1/r$ dependence of a lossless waveguide mode.

The mathematical condition for existence of a waveguide mode in a 3-layer medium is:

$$e^{j2k_{-1}\Delta_{-1}} = \frac{(\eta_{-1}-\eta_1)(\eta_{-1}-\eta_{-2})}{(\eta_{-1}+\eta_1)(\eta_{-1}+\eta_{-2})} \quad (3-36)$$

where

$\underline{k}_1$ = z-propagation constant of surface layer (1-17)

Δ_{-1} = thickness of surface layer

$\underline{\eta}_1$ = transverse wave impedance of surface layer (1-25, 1-26)

η_1 = transverse wave impedance of upper half-space (1-25, 1-26)

$\underline{\eta}_2$ = transverse wave impedance of lower half-space (1-25, 1-26)

From (3-36), it is easy to verify the following facts:

- (a) There are an infinite number of both TE and TM waveguide modes. All but a finite number are exponentially attenuated.
- (b) Disregarding material losses; if and only if the following two conditions are satisfied: (i) Δ_1 is sufficiently thick and (ii) $k_{c1} \gg k_{c1}, k_{c2}$; then there exist lossless TE and TM waveguide modes. For these, k_t is real.
- (c) If Δ_1 is thin, and only if $k_{c1} \approx k_{c2} \neq k_{c1}$, then there is a TM "pseudo-waveguide mode" for a real k_t . No other "pseudo-waveguide modes" can exist. Kc_i is defined in (1-17b).

The conditions under which waveguide phenomena can be important are defined in (b) and (c) above. These conditions are not thought to be sufficiently likely to warrant a further investigation of waveguide modes.

3.6 Three Layers-Geometrical Optics Limit

The Fourier-transformed fields for a Hertzian dipole in a 3-layer medium are found directly from equations (2-5), (2-7), (2-10) and (2-11). The angle-dependent factors in the inverse transform are integrated directly, as given in equation (3-3) and Table (3-1). The fields now are given by a single integral (3-10) with components from equations (2-14) - (2-17) and Table (3-1).

For a 3-layer medium as labeled in Figure (3-6); the components for $z > 0$ or $z < 0$, respectively are proportional to $\underline{H}_2^+$ (k_t) or $\underline{H}_2^-$ (k_t), which are given by:

$$\frac{\underline{H}_2^+}{\underline{H}_2^-} = \frac{\begin{vmatrix} -(d_1/t_1 - s_1/t_1) & (d_1/t_1 + s_1/t_1) \\ -(s_1/t_1 - d_1/t_1) & (s_1/t_1 + d_1/t_1) \end{vmatrix} \begin{vmatrix} b_1 \\ b_2/n_1 \end{vmatrix}}{(s_1 s_1/t_1/t_1 - d_1 d_1/t_1/t_1)} \quad (3-37),$$

where:

$k_1 = \underline{k}_1$ = z-propagation const. (1-17) of middle layer

$\Delta = z_1 - \underline{z}_1$ = thickness of middle layer

$$t_1 = e^{-jk_1 z_1} \quad (3-38) \quad (a)$$

$$\underline{t}_1 = e^{-jk_1 \underline{z}_1} = e^{-jk_1 z_1} \quad (b)$$

$$t = \underline{t}_1/t_1 = e^{jk_1 (z_1 - \underline{z}_1)} = e^{jk_1 \Delta} \quad (c)$$

η_2 = transverse (TE or TM) wave impedance of layer ②

$\eta_1 = \underline{\eta}_1$ = transverse (TE or TM) wave impedance of layer ①

$\underline{\eta}_2$ = transverse (TE or TM) wave impedance of layer ②

(from equation (1-25) or (1-26))

$$s_1 = 1 + \eta_2/\eta_1 \quad (a)$$

$$d_1 = 1 - \eta_2/\eta_1 \quad (b)$$

(3-39)

$$\underline{s}_1 = 1 + \underline{\eta}_2/\eta_1 \quad (c)$$

$$\underline{d}_{-1} = 1 - \underline{\eta}_2/\eta_1 \quad (d)$$

$$\underline{b}_1 = - \underline{u}_z \underline{x} \underline{a}_t \quad (a)$$

(3-40)

$$\underline{b}_2 = - \underline{u}_z \underline{x} \underline{k}_t \underline{a}_z / \omega \epsilon_1 \quad (b)$$

(as in equations (2-10), (2-11).)

Application of equations (3-37), (2-14) - (2-17) to the following specific problems can be made directly:

- (a) perturbation of far-field radiation patterns by terms, $O(k_{c1}\Delta)$, for a thin layer.
- (b) perturbation of asymptotic surface fields by terms, $O(k_{c1}\Delta)$, for a thin layer.
- (c) derivation of mathematical condition (equation (3-36)) for existence of a 3-layer waveguide mode.
- (d) calculation of surface-subsurface multiply-reflected "rays", in geometrical optics limit.

The perturbed fields, (a) and (b) above, are obtained from equation (3-37) by a first-order expansion of the exponentials (3-38), resulting in:

$$\begin{bmatrix} \underline{H}_2^+ \\ \underline{H}_{-2}^- \end{bmatrix} \approx \frac{\begin{bmatrix} (-jkz + n_2/n_1) & (1 - jkz n_2/n_1) \\ (-jkz - n_2/n_1) & (1 + jkz n_2/n_1) \end{bmatrix} \begin{bmatrix} \underline{b}_1 \\ \underline{b}_2/n_1 \end{bmatrix}}{(n_2 + n_{-2})/n_1 + jk(z - \underline{z}_1)(1 + n_2 n_{-2}/n_1^2)} \quad (3-41)$$

Assuming $kc_i \Delta \ll 1$, for $i = -2, 1, 2$; clearly, $\underline{H}_2^+$ and $\underline{H}_{-2}^-$ will be perturbed only slightly from their half-space values for all k_t of interest. The perturbing terms are first-order in $\Delta = (z_1 - \underline{z}_1)$; $O(kc_i \Delta)$. Correspondingly, the far radiated fields, which are proportional to $\underline{H}_2^+$ and $\underline{H}_{-2}^-$, from equations (2-14) to (2-17) and (3-22), are perturbed by similar terms, $O(kc_i \Delta)$. Perturbed radiation patterns, for a thin layer, with $z_1 = 0$ and $\underline{z}_1 = -\Delta$, are shown in Fig. 3-7, (a) and (b).

The coefficients of the asymptotic surface terms, G_{+2} and G_{-2} from eq. (3-29), similarly are perturbed by small increments, $O(kc_i \Delta)$; $i = -2, 1, 2$. The existence of a third surface term, $\sim G_1$, is precluded by the lack of any odd powers of k_1 in the expansion of (3-37) or (3-41) near $k_t = kc_1$.

When $kc_1 \Delta \gg 1$, 3-layer waveguide modes (c) and "geometrical optics" reflections (d) can be important. As discussed in Sec. 3-5, the waveguide modes probably can be ignored. The reflected "rays" consist of plane waves travelling near those

sub-vertical angles which result in single and multiple specular reflections. Under simple assumptions, a stationary-phase method shows the existence and magnitude of such rays.

To simplify matters for illustration, assume: (i) both source and receiver at $z = 0$, (ii) $z_1 = 0$, (iii) $\underline{z}_1 = -\Delta$. Then $t_1 = 1$, $\underline{t}_1 = t$ and (3-37) becomes:

$$\left[\begin{array}{c} \underline{H}^+ \\ \underline{H}^- \\ -2 \end{array} \right] = \frac{\left[\begin{array}{cc} (\underline{s}_1, t - \underline{d}_1/t) & (\underline{s}_1, t + \underline{d}_1/t) \\ -2n_2/n_1 & 2 \end{array} \right]}{(s_1, s_1, t - d_1, d_1/t)} \left[\begin{array}{c} \underline{b}_1 \\ \underline{b}_2/n_1 \end{array} \right] \quad (3-42).$$

Assuming $kc_{-1} < kc_{-2}$; for $0 < k_t < kc_{-1}$, it is permissible to expand the denominator of (3-42) geometrically:

$$\frac{1}{s_1 s_1 t - d_1 d_1/t} = \frac{1}{s_1 s_1 t} \left\{ 1 + \left(\frac{d_1 d_1}{s_1 s_1 t^2} \right) + \left(\frac{d_1 d_1}{s_1 s_1 t^2} \right)^2 + \dots \right\} \quad (3-43)$$

Accordingly, $\underline{H}_2^+$ is expanded in even powers of t : $1, t^{-2}, t^{-4}, \dots$. $\underline{H}_2^-$ is expanded in odd powers of t : $t^{-1}, t^{-3}, t^{-5}, \dots$.

For example, for a horizontal electric dipole (HED), $\underline{b}_2 = 0$ and (3-42), (3-43) yield:

$$\underline{H}_2^+ = \underline{b}_1 \left\{ \frac{1}{s_1} + \left(\frac{d_1 - s_1}{s_1} \right) \left(\frac{d_{-1}}{s_1 s_{-1} t^2} \right) \left[1 + \left(\frac{d_1 d_{-1}}{s_1 s_{-1} t^2} \right) + \left(\frac{d_1 d_{-1}}{s_1 s_{-1} t^2} \right)^2 + \dots \right] \right\} \quad (3-44)$$

$$\underline{H}_{-2}^- = \underline{b}_1 \left\{ \left(\frac{d_1 - s_1}{s_1} \right) \left(\frac{1}{s_{-1} t} \right) \left[1 + \left(\frac{d_1 d_{-1}}{s_1 s_{-1} t^2} \right) + \left(\frac{d_1 d_{-1}}{s_1 s_{-1} t^2} \right)^2 + \dots \right] \right\} \quad (3-45)$$

Notice that the terms can be interpreted as follows:

$\underline{b}_1/s_1$ = half-space wave vector as in (3-11)

$$\underline{b}_1 \frac{(d_1 - s_1)}{(s_1)} = \underline{b}_1 \frac{(-2\eta_2)}{(\eta_1 + \eta_2)} = \text{lower-space excitation factor} \quad (3-46)$$

$$\frac{1}{s_1} = \frac{\eta_1}{\eta_1 + \eta_2} = \text{transmission factor for surface, } z = 0 \quad (3-47)$$

$$\frac{1}{s_{-1}} = \frac{\eta_1}{\eta_1 + \eta_{-2}} = \text{transmission factor for surface, } z = -\Delta \quad (3-48)$$

$$\frac{d_{-1}}{s_{-1}} = \frac{\eta_1 - \eta_{-2}}{\eta_1 + \eta_{-2}} = \text{reflection factor for surface, } z = -\Delta \quad (3-49)$$

$$\frac{d_1}{s_1} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \text{reflection factor for surface, } z = 0 \quad (3-50)$$

To obtain the stationary-phase contributions to the integral (3-10) or (3-12), as derived in Section 5.3, the phase of the integrand is written as

$$\Phi \approx -(k_t r + m k_{+1} \Delta) \quad m = 1, 2, 3, \dots \quad (3-51),$$

for $k_t r \gg 1$. Differentiating by k_t gives:

$$\frac{d\Phi}{dk_t} = -r + m \frac{k_t}{k_1} \Delta = 0 \implies \frac{k_t}{k_1} = \frac{r}{m\Delta} \quad (3-52),$$

$$\frac{d^2\Phi}{dk_t^2} = m\Delta \left(\frac{1}{k_1} + \frac{k_t^2}{k_1^3} \right) = \frac{R_m^3}{k_c (m\Delta)^2} \quad (3-53)$$

$$R_m = \sqrt{r^2 + (m\Delta)^2} \quad (3-54).$$

Thus the stationary-phase points (3-52) correspond to multiply-reflected geometrical-optics "rays" in the sub-surface layer.

Under two assumptions (i) $k_t r \gg 1$, (ii) $(kc_1 - |k_1|)\Delta \gg 1$, the method of Sec. 5.3 shows a typical stationary-phase contribution to the fields:

$$\left. \frac{H_2^+}{b_2=0} \right|_{m=2} \approx \underline{b}_1 \left(\frac{d_1 - s_1}{s_1} \right) \left(\frac{d_1}{s_1 s_1} \right) \frac{j k_c 2\Delta}{R_2} \frac{e^{-j k_c R_2}}{4\pi R_2} \quad (3-55),$$

where $R_2 = \sqrt{r^2 + (2\Delta)^2}$, from (3-54). This H_2^+ term illustrates the geometrical optics behavior of all such terms:

- (i) Field strength decaying as $1/R_n$ (power as $1/R_n^2$).
 "n", which equals the number of segments of the ray, must be even for fields at $z = 0$, odd for fields at $z = -\Delta$.
- (ii) Reflected (TE & TM) wave amplitudes multiplied by plane-wave reflection and transmission factors given in (3-47) to (3-50).

As a final note, the condition (3-36) for existence of a 3-layer dielectric waveguide mode is obtained simply by equating the denominator of (3-37) or (3-41) to zero:

$$s_1 \underline{s}_1^t - d_1 \underline{d}_1 / t = 0 \quad (3-56).$$

Detailed calculations of 3-layer surface fields, derived from a Hertzian potential and calculated using a saddle-point method in the geometrical optics limit, are given by Annon⁸ and Sinha.⁹

CHAPTER 4

Comparison of Dipoles Over Half-Space

Conclusions as to the best dipole for a lunar electrical experiment will not be made, for several reasons: (i) conflicting merits of different infinitesimal dipoles, (ii) practical considerations of construction, antenna matching, etc. (which are not considered in this report), and (iii) poor correspondence between theoretical solutions and lunar fields, due to non-stratification, azimuthal asymmetry, and size of antennas.

A summary of half-space field properties of the different infinitesimal dipoles is shown in Table 4.1 below. The properties summarized are: (a) peak directivity, from Sec. 3.3, (b) azimuthal symmetry, (c) fraction of power radiated into sub-surface medium, from Sec. 3.3, and (d) surface field strengths, from Sec. 3.4. This comparison is for a sub-surface/above-surface dielectric ratio of $\epsilon_2/\epsilon_1 = 4$. A discussion of the importance of these properties (a) - (d) is in Sec. 1.1.

Dipole Type	Max. Gain	Symmetry	Power Rad.	Surface Fields $\sim r^{-2}$					
				H_ρ	H_ϕ	H_z	E_ρ	E_ϕ	E_z
VED	10.6		.661		1.174 -.073		+.508 -.127		1.174 -.147
HED-TM	2.4		.276		1.000 -.125		+.433 -.217		1.000 -.062
HED-TE	5.1		.460 .736	+.250 -.500		+.125 -.577		+.143 -.289	
VMD	3.7		.938	+.131 -.523		+.075 -.604		+.075 -.302	
HMD-TM	10.1		.227		1.624 -.050		+.704 -.088		1.624 -.101
HMD-TE	1.9		.239 .466	+.304 -.152		+.304 -.704		+.176 -.088	
HET	2.5		.736	+.177 -.353	+.707 -.085	+.085 -.408	+.306 -.153	+.101 -.204	+.707 -.044
HMT	5.0		.466	+.215 -.107	1.149 -.035	+.215 -.487	+.487 -.062	+.124 -.062	1.149 -.071

Table 4.1 Properties of Hertzian Dipole over Half-Space

In Table 4.1:

VED = vertical electric dipole

HED = horizontal electric dipole

VMD = vertical magnetic dipole

HMD = horizontal magnetic dipole

HET = horizontal electric turnstile

HMT = horizontal magnetic turnstile

 = cylindrical symmetry

 : cylindrical asymmetry with TE and TM lobes 90°
apart in azimuth, power $\sim \cos^2 \phi$

Surface fields are given in relative units, however each dipole radiates the same total power. The +/- prefixes to the surface field coefficients denote upper/lower-space coefficients, for field terms $\sim G_1$ or $\sim G_{-1}$, respectively.

4.1 Based on Total Coupling Efficiency to Lower Medium

The VMD, for $\epsilon_1/\epsilon_2 = 4$, radiates 94% of its power into the lower half-space. However, the HED and VED radiate almost as much (74% and 66%, respectively). None of the dipoles radiates such a small fraction of power into the lower half-space that would be a serious disadvantage for any of the dipoles.

4.2 Based on Azimuthal Symmetry and Polar Peaks in Radiation Pattern

Asymmetry of the horizontal electric and magnetic dipole patterns complicates the theoretical problem of discriminating between azimuthal field variations caused by the transmitting antenna and those caused by irregular, azimuthally asymmetric reflecting layers and scattering centers in the medium.

Sharp polar peaks in the half-space patterns should help in distinguishing reflecting layers. The VED and HMD-TM subspace lobes are most directive, as is obvious from Fig. 3-3.

4.3 Based on Surface Fields

Strong and closely-matched above-surface and sub-surface components, giving strong interference phenomena, can be found for at least one field component for each dipole. As seen in Table 4-1, reasonably strong H-components are found for the HED-TE field, VMD, and HMD-TE field.

Lack of sub-vertical radiation favors the vertical dipoles.

CHAPTER 5

Special Mathematical Points

This chapter contains mathematical derivations which were too lengthy to include in chapters 1 and 3.

5.1 Boundary Conditions for Vertically-Directed Current Sheet

A vertical electric current sheet excites only a TM wave. Using appropriate limiting processes, the remarkable result is found that the field amplitudes vary discontinuously depending on whether the current sheet is "thought of" as existing in the upper medium (+1) or the lower medium (-1). A justification of this statement is sketched below.

Starting with a vertically-directed current at a boundary across which $\epsilon_1 = \epsilon_1$,

$$\underline{J} = \underline{a_z} e^{-jk_z z} \delta(x) \quad (5-1),$$

continuity (1-6) requires

$$-j\omega\rho = \underline{a_z} e^{-jk_z z} \delta(x) \quad (5-2).$$

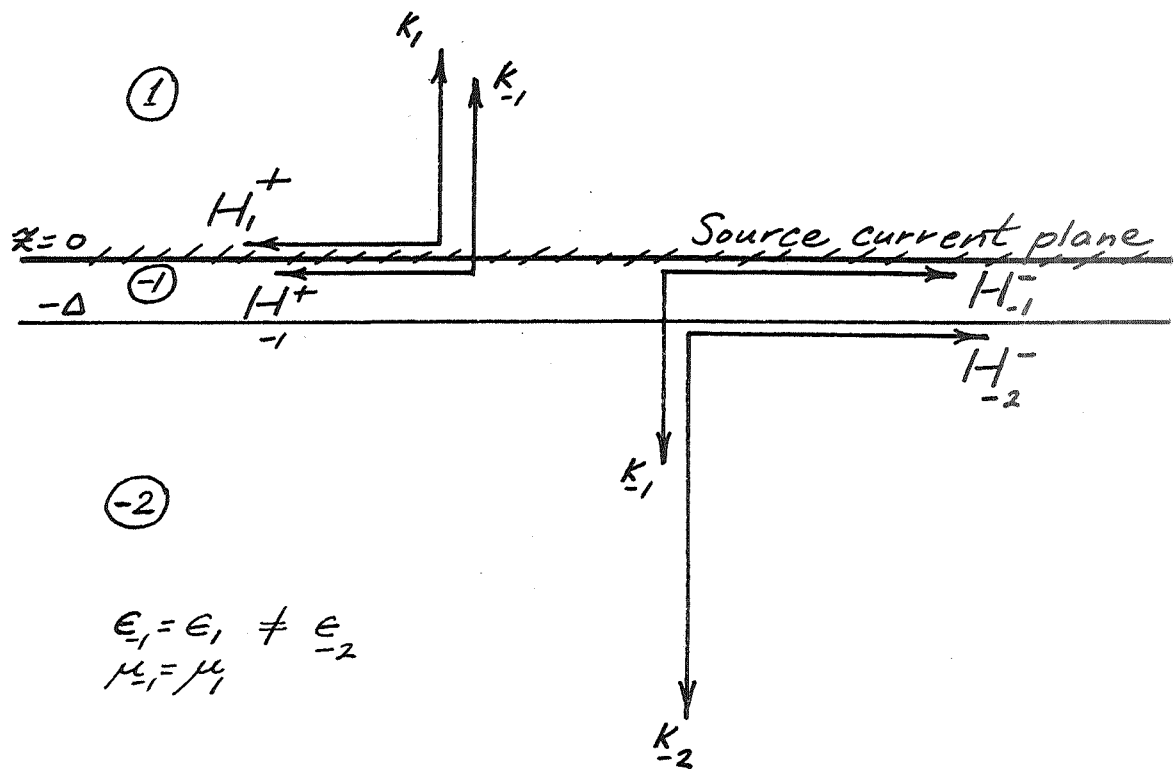


Fig. 5-1 Vertical Current Sheet $z \sim 0$

Maxwell's equation (1-4) requires:

$$-j\omega\rho = -j\omega \underline{\nabla} \cdot \underline{D} \quad (5-3),$$

for which a possible solution is for D to include

$$\underline{D}_o = O(1) + \frac{a_z}{-j\omega} e^{-j\frac{k_z}{t} \cdot r} \delta(x) \underline{u}_z \quad (5-4)$$

This tentative form of D, in conjunction with Maxwell's curl equation (1-3), implies that H is continuous across the boundary. Then, using the condition that $\epsilon_1 = \epsilon_{-1}$ in equation (1-7), a limiting form of Maxwell's curl equation (1-2) gives the source current excitation equation:

$$O(1) = \underline{\nabla} \times \underline{E} \longrightarrow - \left[j \left(\frac{k_z}{t} \times \underline{E}_o \right) + \underline{u}_z \times \left(\underline{E}_{+1} - \underline{E}_{-1} \right) \right] \delta(x) e^{-j\frac{k_z}{t} \cdot r} \quad (5-5)$$

where

$$\underline{E}_o = \frac{\underline{D}_o}{\epsilon_1} = \frac{a_z \underline{u}_z}{j\omega \epsilon_1} e^{-j\frac{k_z}{t} \cdot r} \quad (5-6),$$

giving

$$\frac{E}{+1} - \frac{E}{-1} = \frac{-k_t a_z}{\omega \epsilon_i} \quad (5-7).$$

Now it is a simple matter to verify that a tentative solution satisfying these conditions also satisfies all the basic equations.

The final limiting process starts with the vertical current sheet imbedded a small distance from a boundary across which $\epsilon_1 \neq \epsilon_{-1}$. (Figure 5-1). Letting the distance go to zero, including the contributions of reflected waves in the "small layer" between source and boundary, it is easily seen that the limiting excitation equation is of the form (5-7) but with ϵ_1 replaced by ϵ_i ($i = \pm 1$), for source in upper or lower medium, respectively).

5.2 Free-Space Dipole Used to Derive Bessel Integral Identity

The fields of a Hertzian dipole in free space are derived simply from the vector wave equations (1-9) and (1-10) by use of a scalar Green's function. Then by taking the free-space values $\epsilon_1 = \epsilon_1$, $\mu_1 = \mu_1$, $k_1 = k_1$ in equations (3-12), (3-13) - (3-18); it is a simple matter to derive a number of exact integrals involving the Bessel functions J_0 and $J_1(k_t r)$. In fact, it will be seen that only one special integral is independent; the other integrals can be derived from the one.

The free-space fields are first derived in Cartesian

coordinates; then the identification $r = x, y = 0$ puts the results in correspondence with the integrals from Sec. 3.2, which are in cylindrical coordinates.

$$\nabla^2 \underline{E} + k_c^2 \underline{E} = j\omega\mu_0 \underline{J} - \underline{\nabla}(\underline{\nabla} \cdot \underline{J}) / j\omega\epsilon_0 \quad (1-9)$$

$$\nabla^2 \underline{H} + k_c^2 \underline{H} = -\underline{\nabla} \times \underline{J} \quad (1-10)$$

$$k_c^2 = \omega^2 \mu_0 \epsilon_0 \quad (1-176)$$

$$\underline{J} = \underline{a}_t \delta(x) \delta(y) \delta(z) = \underline{a}_t \delta \quad (3-1)$$

$$\underline{a}_t = a_t (c\phi, s\phi, 0)$$

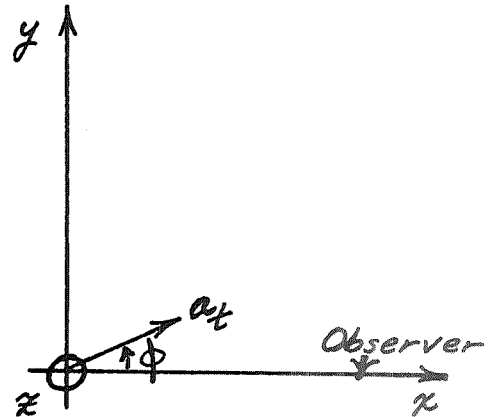


Fig. 5-2 Free-Space Dipole

(5-8)

$$\underline{\nabla} \cdot \underline{J} = a_t (c\phi \partial_x \delta + s\phi \partial_y \delta)$$

(5-9)

$$\underline{\nabla}(\underline{\nabla} \cdot \underline{J}) = \underline{u}_x a_t (c\phi \partial_x^2 \delta + s\phi \partial_x \partial_y \delta)$$

$$+ \underline{u}_y a_t (c\phi \partial_x \partial_y \delta + s\phi \partial_y^2 \delta)$$

$$+ \underline{u}_z a_t (c\phi \partial_z^2 \delta + s\phi \partial_y \partial_z \delta)$$

(5-10)

$$\underline{\nabla} \times \underline{J} = -\underline{u}_x a_t s\phi \partial_z \delta + \underline{u}_y a_t c\phi \partial_z \delta$$

$$+ \underline{u}_z a_t (s\phi \partial_x \delta - c\phi \partial_y \delta)$$

(5-11)

Scalar free-space Green's function, G, satisfies:

$$\nabla^2 G + k_c^2 G = \delta = \delta(x) \delta(y) \delta(z) \quad (5-12)$$

As in (3-29),

$$G = \frac{e^{-jk_c R}}{4\pi R} \quad (5-13)$$

$$R = \sqrt{x^2 + y^2 + z^2} \quad (5-14)$$

$$\frac{\partial R}{\partial x} = \frac{x}{R} \quad (5-15)$$

$$\partial_x G = \left(\frac{-jk_c x}{R} - \frac{x}{R^2} \right) G \quad (5-16)$$

$$\partial_x^2 G = \left(\frac{-jk_c}{R} - \frac{1}{R^2} - \frac{k_c^2 x^2}{R^2} + \frac{j3k_c x^2}{R^3} + \frac{3x^2}{R^4} \right) G \quad (5-17)$$

Substituting G and its partial derivatives for corresponding derivatives of δ on the right sides of (1-9), (1-10), (5-10) (5-11) gives the solution at (x,0,x):

$$\begin{aligned}
 \underline{E}(x, 0, z) = & \underline{u_x} \left\{ j\omega\mu_0 \frac{a_c \phi}{t} \frac{z^2}{R^2} \right. \\
 & + \frac{j}{\omega\epsilon_0} \frac{a_c \phi}{t} \left[\frac{j k_c (2x^2 - z^2)}{R^3} + \frac{(2x^2 - z^2)}{R^4} \right] \Big\} G \\
 & + \underline{u_y} \frac{a_s \phi}{t} \left[j\omega\mu_0 + \frac{1}{j\omega\epsilon_0} \left(\frac{j k_c}{R} + \frac{1}{R^2} \right) \right] G \\
 & - \underline{u_z} \frac{a_c \phi}{j\omega\epsilon_0} \left[\frac{-k_c^2 x z}{R^2} + \frac{j 3 k_c x z}{R^3} + \frac{3 x z}{R^4} \right] G
 \end{aligned}$$

(5-18)

$$\begin{aligned}
 \underline{H}(x, 0, z) = & \underline{u_x} \frac{a_s \phi}{t} \left(\frac{j k_c z}{R} + \frac{z}{R^2} \right) G \\
 & - \underline{u_y} \frac{a_c \phi}{t} \left(\frac{j k_c z}{R} + \frac{z}{R^2} \right) G \\
 & + \underline{u_z} \frac{a_s \phi}{t} \left(\frac{j k_c x}{R} + \frac{x}{R^2} \right) G
 \end{aligned}$$

(5-19)

To verify that these solutions agree with the standard solutions in spherical coordinates, as in Kraus, Antennas, (1950), Eq.

(5-38) - (5-40), set $z = 0$:

$$\begin{aligned} \underline{E}(x,0,0) = & -\underline{u}_x \frac{a_t c \phi}{j \omega \epsilon_0} \left(\frac{j 2 k_c}{x} + \frac{2}{x^2} \right) \frac{e^{-j k_c x}}{4 \pi x} \\ & + \underline{u}_y \frac{a_t s \phi}{j \omega \mu_0} \left[j \omega \mu_0 + \frac{1}{j \omega \epsilon_0} \left(\frac{j k_c}{x} + \frac{1}{x^2} \right) \right] \frac{e^{-j k_c x}}{4 \pi x} \end{aligned}$$

(5-20)

$$\underline{H}(x,0,0) = -\underline{u}_z \frac{a_t s \phi}{j \omega \mu_0} \left(-j k_c - \frac{1}{x} \right) \frac{e^{-j k_c x}}{4 \pi x}$$

(5-21)

Basic Integrals Involving Bessel Function

Our principal use of the free-space dipole solution (5-18) and (5-19) is to derive special integrals of the form (3-12) or (3-28) with Bessel-function kernels such as (3-13) - (3-18). The principal exact integral is derived from H_z :

$$\frac{1}{2\pi} \int_{k_z=0}^{\infty} dk_t k_t e^{-j k_t x} J_0(k_t r) = -2 \frac{\partial G}{\partial z} = 2 \left(\frac{j k_c x}{r} + \frac{x}{r^2} \right) G$$

(5-22)

Using this integral, identities (5-27) and (5-28) for $J_0(k_t r)$ and $J_1(k_t r)$, and differentiating freely under the integrals, it is trivial to show the integral arising from (3-14) - (3-18) follow from (5-22). For example, from $\tilde{H}_z$ (3-15):

$$\frac{k_t J_1(k_t r)}{-j k_t} = \frac{-\partial_r J_0}{-j k_t} \longrightarrow -2 \partial_r G \quad (5-23)$$

From $\tilde{E}_x$ (3-16):

$$\frac{k_t}{k_c^2} \left(J_0 - \frac{J_1}{k_t r} \right) + \frac{1}{k_t} \frac{J_1}{k_t r} \longrightarrow \frac{-j2}{k_c^2} \left(\partial_z^2 + \frac{1}{r} \partial_r \right) G = -j2 \left(1 + \frac{\partial_r^2}{k_c^2} \right) G$$

at $\phi = 0$ (5-24)

Similarly, from $\tilde{E}_\phi$ (3-17):

$$\frac{k_t}{k_c^2} \frac{J_1}{k_t r} + \frac{1}{k_t} \left(J_0 - \frac{J_1}{k_t r} \right) \longrightarrow -j2 \left(1 + \frac{1}{k_c^2 r} \partial_r \right) G = -j2 \left(1 + \frac{1}{k_c^2} \partial_y^2 \right) G$$

at $y = 0$ (5-25)

From E_z (3-18):

$$\frac{k_t}{k_c^2} j J_1(k_t r) \longrightarrow -j \frac{2}{k_c^2} \partial_r \partial_z G \quad (5-26)$$

where G is given previously (5-13).

The Bessel function identities used in the preceeding derivation are:

$$\frac{d}{dr} J_0(k_t r) = -k_t J_1(k_t r) \quad (5-27)$$

$$\frac{d}{dr} J_1(k_t r) = k_t J_0(k_t r) - k_t \frac{J_1(k_t r)}{k_t r} \quad (5-28).$$

5.3 Fresnel Integrals Derived from "Stationary-Phase" Approximation

Expressions (3-22) and (3-55) are obtained by application of a "stationary-phase" approximation to integrals of the form (3-12). Specifically, the integrals of interest are of the form:

$$I = \frac{1}{2\pi} \int_{k_t=0}^{\infty} dk_t k_t e^{-jk_t z} J_m(k_t r) F(k_t) \quad ; m=0,1 \quad (5-29).$$

"z" can be replaced by "mΔ" to obtain the expressions integrated in Sec. 3-6, such as the terms in (3-44), (3-45). The exponential and $J_n(k_t r)$ are assumed to vary so rapidly that $F(k_t)$ can be considered constant within the most important range of integration. $J_n(k_t r)$ is approximated by its asymptotic form for large arguments:

$$J_n(k_t r) \sim \sqrt{\frac{2}{\pi k_t r}} \cos\left(k_t r - \frac{n\pi}{2} - \frac{\pi}{4}\right) \quad (5-30).$$

Now, by expanding $J_n(k_t r) e^{-jk_1 z}$ a stationary-phase term is obtained:

$$J_n(k_t r) e^{-jk_1 z} \sim \frac{1}{2} e^{j\left(\frac{\pi}{4} + \frac{n\pi}{2}\right)} \cdot e^{j\bar{\Phi}} \sqrt{\frac{2}{\pi k_t r}} \quad (5-31)$$

with

$$\bar{\Phi} = -(k_t r + k_1 z) \quad (5-32)$$

$$\frac{d\bar{\Phi}}{dk_t} = -r + \frac{k_t}{k_1} z = 0 \implies \frac{k_t}{k_1} = \frac{r}{z} \quad (5-33)$$

$$\frac{d^2 \bar{\Phi}}{dk_t^2} = \approx \left(\frac{1}{k_i} + \frac{k_t^2}{k_i^3} \right) = \frac{R^3}{k_c z^2}$$

(5-34)

$$R = \sqrt{r^2 + z^2}$$

(5-35)

Now, integral (5-29) can be approximated by

$$I \sim \frac{1}{2\pi} \int_{-\infty}^{\infty} d(\delta k_t) \left(k_c \frac{r}{R} \right) \frac{1}{2} \sqrt{\frac{2R}{\pi k_c r^2}} \cdot \frac{1}{2} e^{j\left(\frac{\pi}{4} + \frac{\pi r}{2}\right)} \cdot e^{j \frac{R^3 (\delta k_t)^2}{2 k_c z^2}}$$

(5-36)

$$\delta k_t = k_t - k_{t_0} = -\infty$$

At this point the Fresnel integral formulas are applied:

$$\int_{-\infty}^{\infty} \cos k^2 dk = \int_{-\infty}^{\infty} \sin k^2 dk = \sqrt{\frac{\pi}{2}}$$

(5-37)

and (5-36) becomes:

$$I \sim \frac{j2k_c z}{R} \frac{e^{-jk_c R}}{4\pi R} \cdot e^{j\frac{m\pi}{2}}$$

(5-38).

With $n = 0$, (5-38) gives (3-22). With $n = 0$ and $z = 2\Delta$, (5-38) gives (3-55).

5.4 Modification of Numerical Integration Formulas for Singular End Point ($\sim \Delta k^{1/2}$)

To obtain an approximate integral of a function in a small segment, at one endpoint of which the function approaches $a_0 + a_1(\Delta k)^{1/2}$ (which has an infinite derivative as $\Delta k \rightarrow 0$), one simply replaces the original function by the limiting function $a_0 + a_1(\Delta k)^{1/2}$. Such singularities occur in the half-space power integrals, Sec. 3.3. See eq. (3-26), (3-24), Table (3-2). The integral of the limiting function simply is:

$$I_{\Delta} = \int_{\Delta k=0}^{\Delta} \{a_0 + a_1(\Delta k)^{1/2}\} d(\Delta k) = \Delta \left\{ a_0 + \frac{2}{3} a_1 \Delta^{1/2} \right\}$$

(5-39)

Therefore, the value of the function at the singular point is weighted $1/3$ and the value at the other end-point of the segment is weighted $2/3$. Similar formulas exist for other known singularities; e.g. $a_2(\Delta k)^{-1/2}$.

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