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ON ELECTRON-CYCLOTRON WAVE RESONANCE IN INDUCTIVELY-COUPLED HF DISCHARGES†

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ABSTRACT

The theory of Pfeiffer on electron-cyclotron wave resonance phenomenon in inductively-coupled hf ring discharges has been extended by Sager to include electron temperature, non-uniform density distributions and the finite dimensions of a plasma of rectangular cross section. A scheme is herein proposed for adapting Sager's results to the experimentally important case of circular cross section. In order to test the scheme, an experiment was conducted in a low-pressure hf (27 MHz) argon discharge of cylindrical geometry with plasma densities in the range $1.0 - 3.5(10^{11})/cm^3$ and static magnetic fields less than 60 gauss. The agreement between theory and experiment is found to be good and lends credence to the scheme as a means for predicting resonance values of magnetic field in cylindrical hf discharges.

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I. INTRODUCTION

The superposition of a static magnetic field perpendicular to the axis of a hf ring discharge not only optimizes carrier production but has also been observed to demonstrate a resonance-like behavior. These phenomena were first reported by Neuert[1] and theoretically explained by Pfeiffer[2,3] as a result of a modified skin effect in anisotropic plasmas and a resonant excitation of electron-cyclotron waves.

The theory of Pfeiffer[2] has been extended by Sager[4] to include a non-homogeneous density distribution as well as temperature influences and the finite dimensions of a plasma of rectangular cross section. A scheme is herein proposed for adapting Sager's results to the experimentally important case of circular cross section and data are presented which support the scheme.

II. THEORETICAL CONSIDERATIONS

To completely determine the electromagnetic field in a cylindrical hf discharge (hf magnetic field applied axially at the surface of the cylindrical plasma volume) with a static magnetic field superimposed in a direction normal to the cylinder axis is indeed very difficult. Pfeiffer's approach to the problem[2] was to restrict his considerations to a cold, homogeneous plasma slab which had its thickness aligned with the superimposed static magnetic field.

The electron-cyclotron resonance phenomenon was then theoretically described by Pfeiffer by starting with the wave equation for the hf magnetic field $\vec{H}$, Eq. (1),

$$(\nabla \times \nabla \times + k_o^2 \epsilon_{ij} \cdot) \vec{H} = 0 \tag{1}$$

and requiring that the solution be linearly polarized in the plane of the plasma boundary. His solution to Eq. (1), where k_o is the vacuum wave number and ϵ_{ij} is the dielectric tensor of the plasma, was a superposition of a standing right- and left-hand polarized wave which resulted from the Faraday effect. By imposing the condition that the hf electromagnetic-field production of carriers in the plasma be at an optimum when the amplitude of the right-hand polarized wave (which propagates into the plasma along the static magnetic field $\vec{B}_o$) is large compared to that of the left-hand polarized wave, Pfeiffer obtained the relationship between the plasma density and the applied static magnetic field. For the ground resonance state this relationship is given in Eq. (2)

$$N_{res} = (c^2 \pi^2 \epsilon_o m_e / e^2)\ (\beta_{res} - 1)\ t_{\parallel}^{-2} \tag{2}$$

where N is the plasma density, β is the electron-cyclotron frequency ω_c normalized to the angular frequency of the generator ω, i.e. $\beta = \omega_c/\omega = (eB_o/m_e)/\omega$, $t_{\|}$ is the thickness of the plasma slab and the subscript "res" refers to the specific values under conditions of electron-cyclotron resonance. The previously undefined quantities in Eq. (2) are the vacuum speed of light c, the permittivity of free space ϵ_o, and the mass and charge of an electron m_e and e, respectively.

Like Pfeiffer, Sager[4] began with the wave equation for the hf magnetic field (in a form different from that shown in Eq. (1)) but through a series of approximations has included the effects of non-zero plasma temperature, non-homogeneous density profiles, as well as a plasma geometry with rectangular cross-section. Since his work is relatively recent and has not been widely distributed a brief outline of his plasma model and theoretical considerations will be presented here. (It should be noted that the full scope of his approximation procedures and associated justifications can best be appreciated by reading the original work.)

Figure 1 presents the geometric configuration of Sager's plasma model where $t_{\|}$ is the plasma dimension parallel to the superimposed magnetic field $\vec{B}_o$ and $t_{\perp}$ is the corresponding perpendicular dimension. (The dimension of the plasma normal to the plane of the figure is assumed to be infinite.) The net result of his approximation procedures is represented by Eq. (3).

$$N_{res} = \left(\frac{c^2 \omega^2 \epsilon_o m_e}{e^2} \right) p^2(\lambda_i) \; t^* \left[\frac{(\beta_{res} - t^*)^3 \; t_{\|}^{-2}}{(\beta_{res} - t^*)^2 + p^2(\lambda_i) t^* \left(\frac{1}{t_{\|}} \right)^2 kT_e / m_e \omega^2} \right] \tag{3}$$

The temperature correction term appears in the denominator of Eq. (3) and was introduced by Sager through the dependence of the plasma dielectric tensor on the electron temperature T_e. An approximate dependence of ϵ_{ij} on T_e was determined by Sager by employing the results of the investigation of Mower[5] on the conductivity of warm plasmas.

The correction term for a non-homogeneous density distribution appears in Eq. (3) as $p^2(\lambda_i)$ and has been calculated by Sager in two limiting cases of ion mean-free-path λ_i. When λ_i is much greater than any plasma dimension t, Sager found $p^2(\lambda_i >> t) = 1.47$; in the opposing limit he found $p^2(\lambda_i << t) = 2.09$. To arrive at these results Sager employed straight-line approximations (shown as dashed lines in Fig. 2) to the cosine-like profiles described in the early work of Tonks and Langmuir[6] for the aforementioned limiting cases of ion mean-free-path. Shown as a solid curve in Fig. 2 is an experimental density profile observed in the cylindrical discharge which will be discussed in Section III. The experimental conditions were near resonance and λ_i was approximately equal to d/10, where d is the cylinder diameter.

Of prime importance to the present investigation is the correction term which accounts for the rectangular cross section of the plasma volume. In Eq. (3) this term is designated by t^* and is defined in Eq. (4) in terms of the plasma cross sectional dimensions.

$$t^* = \sqrt{1 + (t_{\parallel}/t_{\perp})^2} \tag{4}$$

It was introduced by Sager in conjunction with his assumption that Eq. (5) represented the solution for the right-hand polarized wave H_r which propagated along the field $\vec{B}_o$.

$$H_r = H_{or} \text{ Cos } (k_r z) \text{ Cos } (\pi \frac{y}{t_{\perp}}) \tag{5}$$

In Eq. (5) k_r and H_{or} are the wave number and peak value, respectively, of the right-hand polarized wave. The coordinate system is illustrated in Fig. 1 and defined so that $\vec{y}/|\vec{y}| \perp \vec{B}_o$ and $\vec{z}/|\vec{z}| \parallel \vec{B}_o$.

To adapt the results of Sager's work to the experimentally important case of circular cross section, appropriate selections must be made for $t_{\parallel}$ and $t_{\perp}$ in terms of the cylinder diameter d. A reasonable approach is to have the circular cross section of a cylindrical discharge approximated by a square. With this assumption $t_{\parallel} = t_{\perp}$ and $t^* = 1.41$.

The most important consideration revolves about the proper selection of $t_{\parallel}$ since in a circular cross section the thickness of the plasma in the direction parallel to $\vec{B}_o$ takes on continuous values from 0 up to a maximum which is equal to the diameter d. This is illustrated in Fig. 3 where it can readily be seen that $t_{\parallel})_a > t_{\parallel})_b > t_{\parallel})_c$ and that $t_{\parallel})_{max} = t_{\parallel})_a = d$. The scheme proposed here, which differs from that offered by Sager[7], involves a simple averaging of the values of $t_{\parallel}$ over the circumference of the discharge. This averaging process is presented in Eq. (6) where the variable of integration is defined in Fig. 3 and where R is defined as the radius of the circular cross section.

$$t_{\parallel} = \frac{2}{\pi} \int_{0}^{\pi/2} 2R \operatorname{Cos} \theta \, d\theta = \frac{2d}{\pi} \qquad (6)$$

The resulting adaption then approximates the circular cross section of diameter d by a square of side $2d/\pi$.

It has been implicit in the scheme of adaptation that the excited hf waves travel along the static magnetic field lines. This is certainly a reasonable consideration in the rectangular model of Sager for all waves initiated at the plasma boundary $z = \pm t_{\parallel}/2$. In the case of the circular cross section however the predominant wave-normal at the plasma boundary is aligned with the radius and forms an angle θ with $\vec{B}_o$. Now a characteristic of propagation in anisotropic plasmas is the fact that the direction of the wave normal is not the same direction in which the mean intensity of energy flows. In a way analogous to the propagation of ionospheric whistlers[8] the waves excited in the cylindrical plasma volume will be "guided" by the magnetic field and will tend to travel along the lines of force. This will be illustrated by employing the concept of a refractive index surface whose normal gives the direction of energy flow.

The conditions of the experiments to be described in Section III are such that $\omega_p/\omega >> \beta_{res} > 1$ where ω_p is the electron plasma frequency. Neglecting electron-neutral collisions the Appleton-Hartree formula then gives Eq. (7) as the index of refraction for the single allowable mode of wave propagation.

$$n = + \frac{\omega_p/\omega}{\sqrt{\beta_{res} \cos\theta - 1}} \qquad (7)$$

As before β_{res} refers to the electron-cyclotron wave resonance value of β whose relationship to the plasma density is given in Eq. (3). The refractive index surface corresponding to Eq. (3) is shown in Fig. 4 for β_{res} = 2 and 3. Illustrated in the figure is an example of the energy flow direction with and without the plasma, $\hat{s}$ and $\hat{s}_o$ respectively. $\hat{s}$ and $\hat{s}_o$ are unit vectors with $\hat{s}$ being normal to the refractive index surface and $\hat{s}_o$ being aligned with the radius vector. $\hat{s}\cdot\hat{s}_o = \cos\gamma$ and the positive sense of γ is defined in the figure. The resultant angle which $\hat{S}$ makes with B_0 is [illegible]. For [illegible]$_{res}$ [illegible] 2 and θ [illegible] 40^{0} ([illegible]$_{res}$ [illegible] 3 and [illegible] 60^{0}) it can be seen that for all practical purposes $\hat{S} \parallel \vec{B}_0$. For [illegible] 40^{0} (θ [illegible] 60^{0}) the energy of the wave is increasingly directed out of the plasma until finally at a cut-off angle of 60^{0} ($70^{0}33'$), shown as a dashed line, the waves can no longer penetrate within the plasma. These results point out that the physical situation is indeed quite conpatible with the adaptation scheme employed in the investigation and that the averaging process is justified.

III. EXPERIMENT

In order to test the aforementioned scheme of adaptation a series of measurements was carried out in an inductively-coupled hf discharge of cylindrical geometry with length and diameter equal to 9.5 cm. The discharge was maintained by a single-turn coil (see Fig. 5) which was grounded at its point of symmetry and inductively coupled to a hf generator operating at 27 MHz. The plasma volume was bounded along its length by a metal plate on one end and by a wire grid on the other. The gas was argon and the operating pressure was maintained at 1.0 (10^{-3}) torr. A pair of Helmholtz coils of mean diameter 46.8 cm provided the homogeneous static magnetic field which was varied in this experiment from 0 to approximately 60 gauss.

The technique employed in determining β_{res} was the same as that originally employed by Pfeiffer[3] and the results are presented in Fig. 6 where U_{hf} is the hf voltage measured across the gap of the single-turn secondary coil (Fig. 5). N_{res} has been previously defined and its value was experimentally determined by employing the calculations of Laframboise[9] to analize the ion-saturation current collected by a Langmuir probe which was immersed in the plasma. The Langmuir probe was inserted along the axis of the plasma cylinder and was positioned in the geometrical center of the plasma volume. The curves shown in Fig. 6 were generated by maintaining the middle-point plasma density constant and measuring the hf gap voltage as a function of β. The minimum in the response of U_{hf} vs β then identifies β_{res} for a particular value of N_{res}.

Figure 7 includes two curves which result from Eq. (3) as adapted in this paper to the circular cross section of a cylindrical plasma volume. The curves have been calculated for two diameters, 6.4 cm and 9.5 cm, under conditions which set $\lambda_i << d$ and $T_e = 8(10^4)^oK$.

The (N_{res}, β_{res}) results of Fig. 6 appear in Fig. 7 as solid dots and should be compared with the curve identified by d = 9.5 cm. The experimental conditions were such that

λ_i = 9 mm and T_e = $8(10^4)$ °K and the error bars represent the maximum possible inaccuracy in the experimental technique.

In the experiments carried out in this investigation no value of β_{res} less than 2 was found for values of N_{res} $1.0(10^{11})/cm^3$. Oechsner[10] however measured resonance values of β less than 2 and his results are represented in Fig. 7 by the open squares. His experiment was conducted in a cylindrical argon discharge of diameter 9.5 cm with the pressure at $1.7(10^{-3})$ torr and $T_e = 6.5(10^4)$ °K. It can be seen that the measurements of Oechsner and of this investigation compare well with the appropriate theoretical curve (d = 9.5 cm) thus indicating that the scheme of adaptation yields a reasonable representation of the data for cylindrical hf dischrages.

As additional support of the scheme the data of Pfeiffer[3], which were collected in argon at $1.5(10^{-2})$ torr argon and which did not fit his original theoretical model, are plotted in Fig. 7 as open triangles and should be compared with the curve calculated for his experimental conditions at d = 6.4 cm. As in the case of the larger diameter, the agreement between the data and theory is good.

IV COMMENTS & CONCLUSIONS

A scheme has been proposed for adapting the theoretical results of Sager[4] to the experimentally important case of an inductively-coupled cylindrical hf discharge. A prime consideration is the thickness of the plasma in the direction of the superimposed static magnetic field. Since the field is applied in a direction perpendicular to the cylinder axis a simple averaging process over the circumference of the circular cross section has been employed. This process yields an average plasma thickness equal to $2d/\pi$ and the net result of the proposed scheme approximates the circular cross section by a square of side $2d/\pi$.

The experimental results of this investigation as well as those of Pfeiffer[3] and Oechsner[10] agree well with the proposed scheme of adaptation and lend credence to the method as a means for establishing the relationship between plasma density and the static magnetic field in a cylindrical hf discharge under conditions of electron-cyclotron wave resonance.

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FIGURE CAPTIONS

Figure 1. Cross section of the hf plasma configuration assumed by Sager (Ref. 4).

Figure 2. Plasma density profiles along the superimposed static magnetic field. Dashed lines are those assumed in the plasma model of Sager (Ref. 4) and the solid curve is a profile observed in the cylindrical hf discharge studied in this investigation.

Figure 3. An illustration of the variation of plasma thickness along the magnetic field $\vec{B}_o$ which is superimposed perpendicularly to the axis of a cylindrical plasma volume.

Figure 4. Refractive index surface. The length of the vector from the origin to the curve is $n\omega/\omega_p$ and θ is measured from the axis of the magnetic field $\vec{B}_o$.

Figure 5. Schematic representation of the experimental arrangement employed in the study of the cylindrical hf discharge (shown in cross section). U_{hf} is the hf (27 MHz) voltage measured across the gap of the secondary coil.

Figure 6. High-frequency voltage U_{hf} measured across the gap of the secondary coil (Fig. 5) as a function of β, the electron-cyclotron frequency normalized to the generator frequency, at an argon pressure of $1.0(10^{-3})$ torr. The running parameter, N_{res}, is the middle-point plasma density which was maintained constant for the individual curves.

Figure 7. Comparison between experimental results and the theory of Sager (Ref. 4) as adapted to the case of a cylindrical plasma volume. The data in this investigation have been taken from Fig. 6 and the theoretical curves have been calculated from Eq. (3) with $p^2(\lambda_i) = 2.09$, $T_e = 8(10^4)$ °K, and $t_{\parallel} = t_{\perp} = 2d/\pi$. The results of this investigation as well as those of Oechsner (Ref. 10) should be compared with the case d = 9.5 cm while the results of Pfeiffer (Ref. 3) should be compared with d = 6.4 cm.

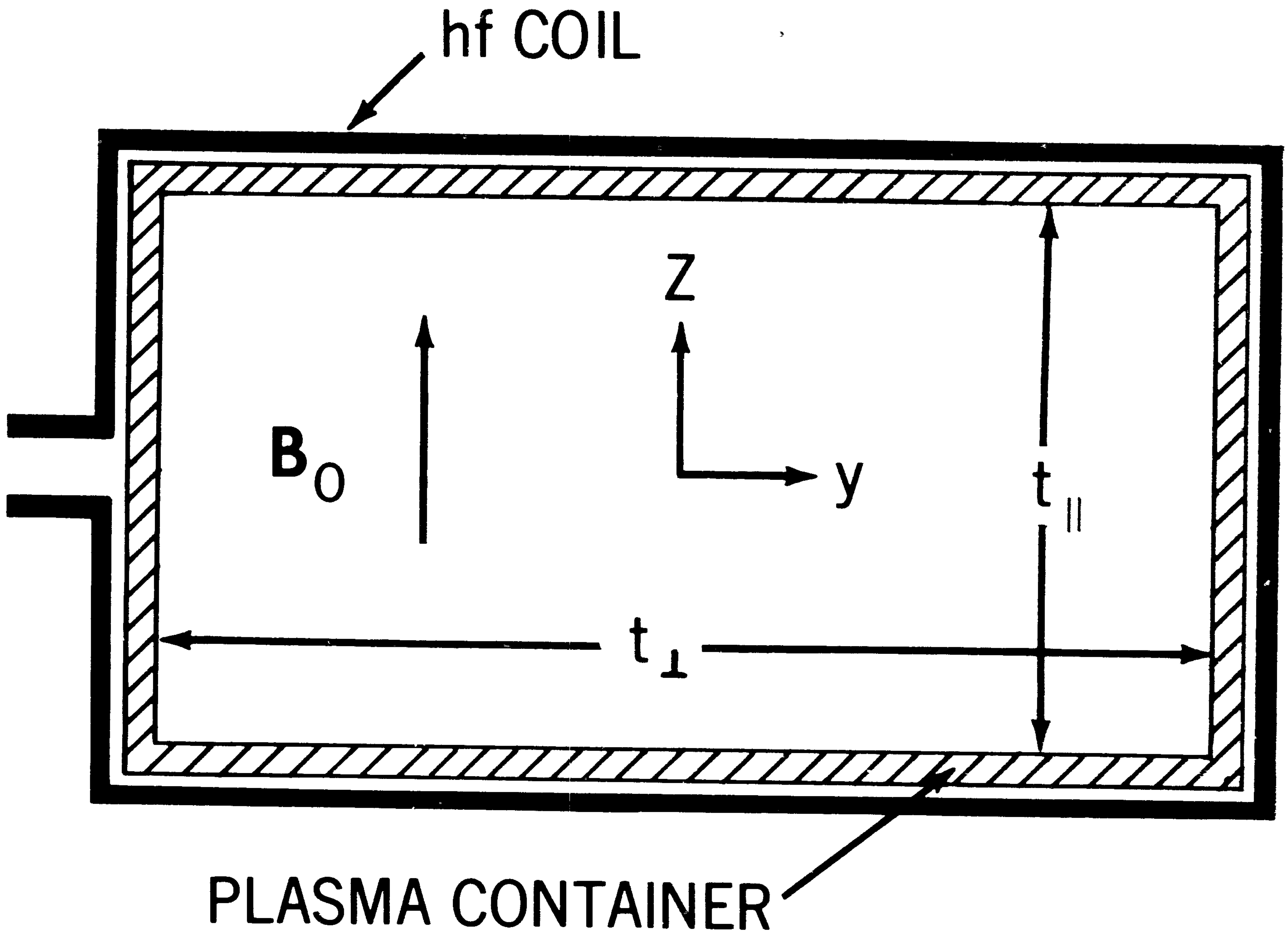


Figure 1

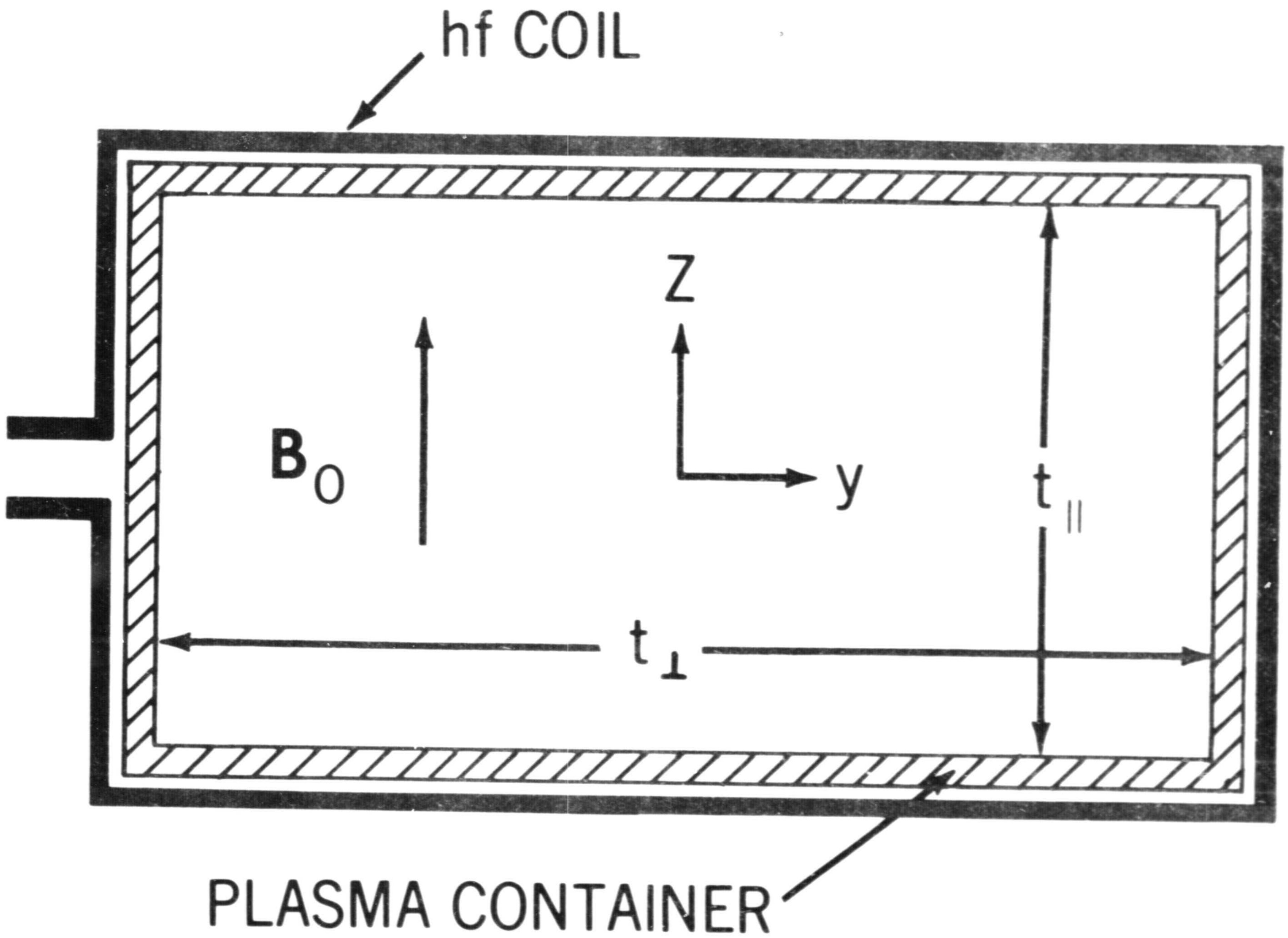


Figure 1

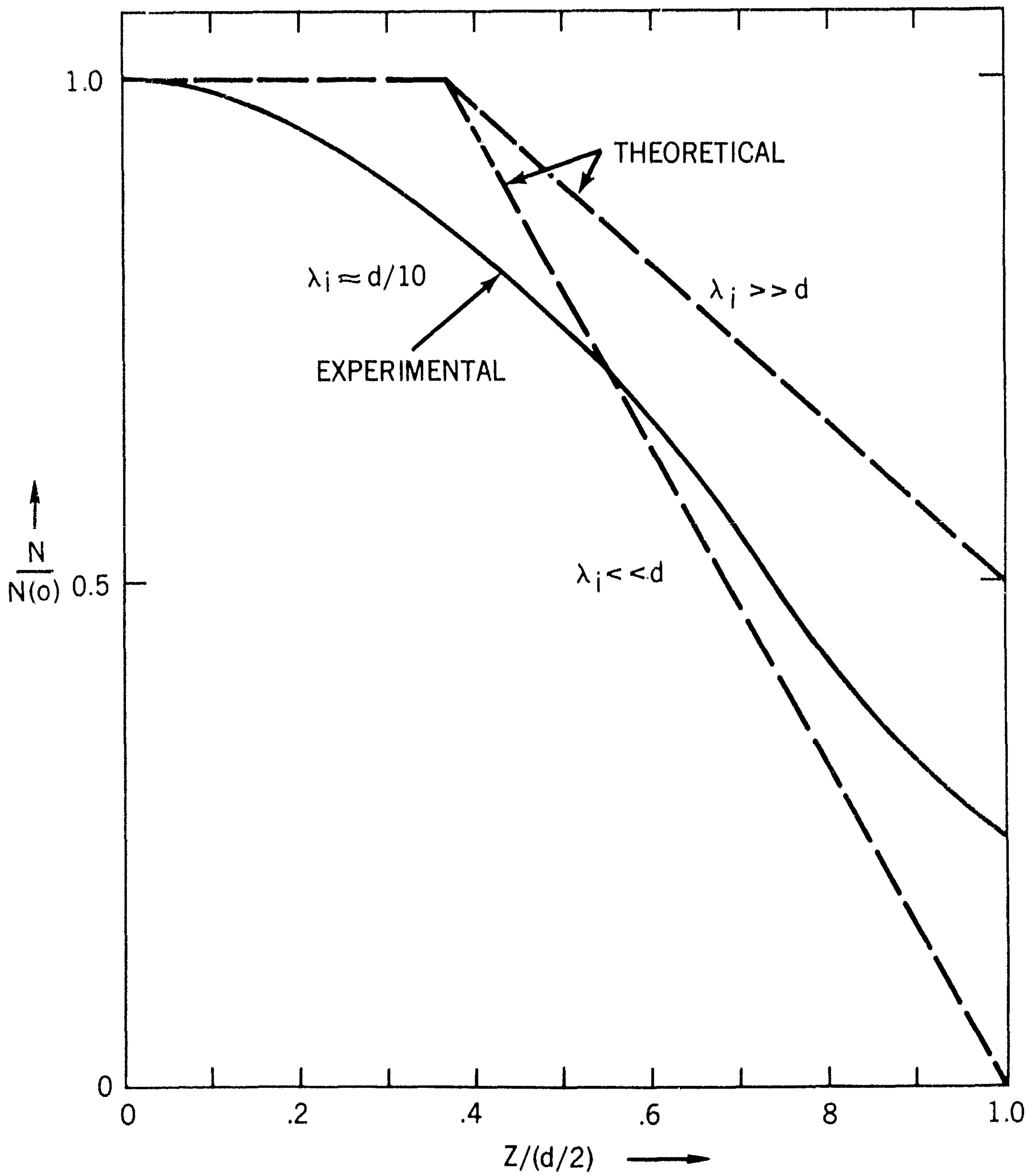

1.0
0.5
0
$\frac{N}{N(o)}$
THEORETICAL
$\lambda_i \approx d/10$
$\lambda_i >> d$
EXPERIMENTAL
$\lambda_i << d$
0
.2
.4
.6
8
1.0
Z/(d/2)

Figure 2

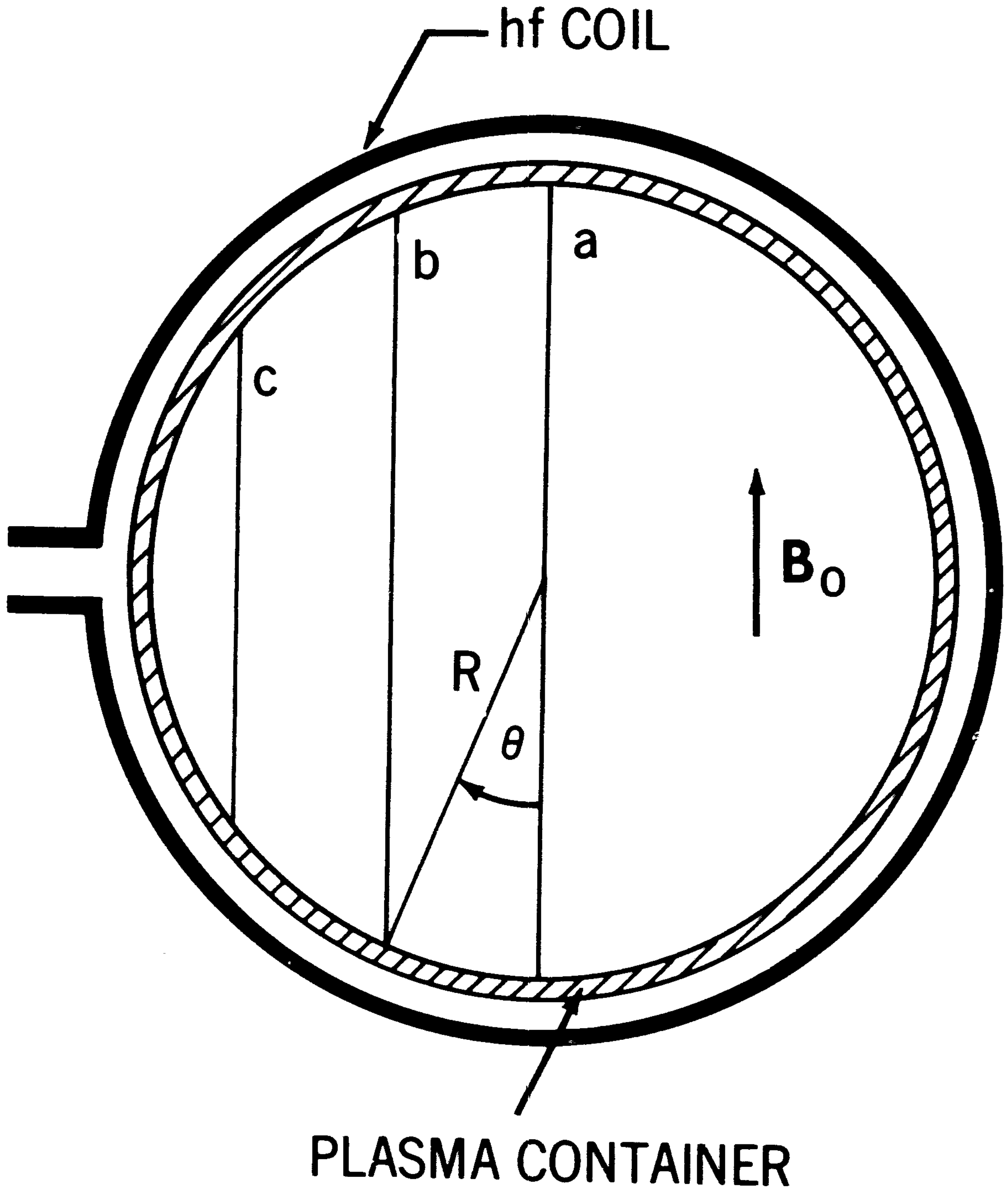


Figure 3

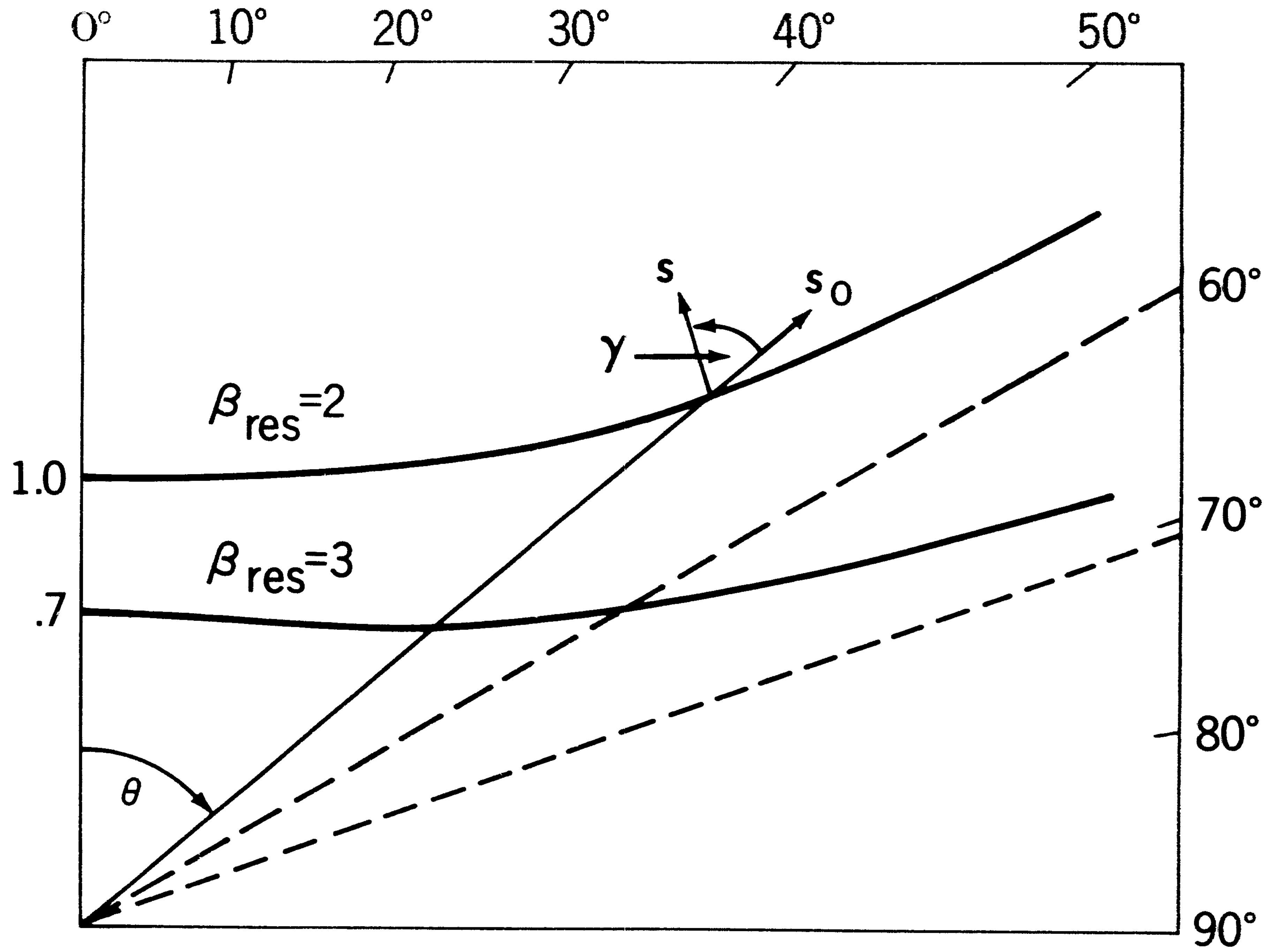


Figure 4

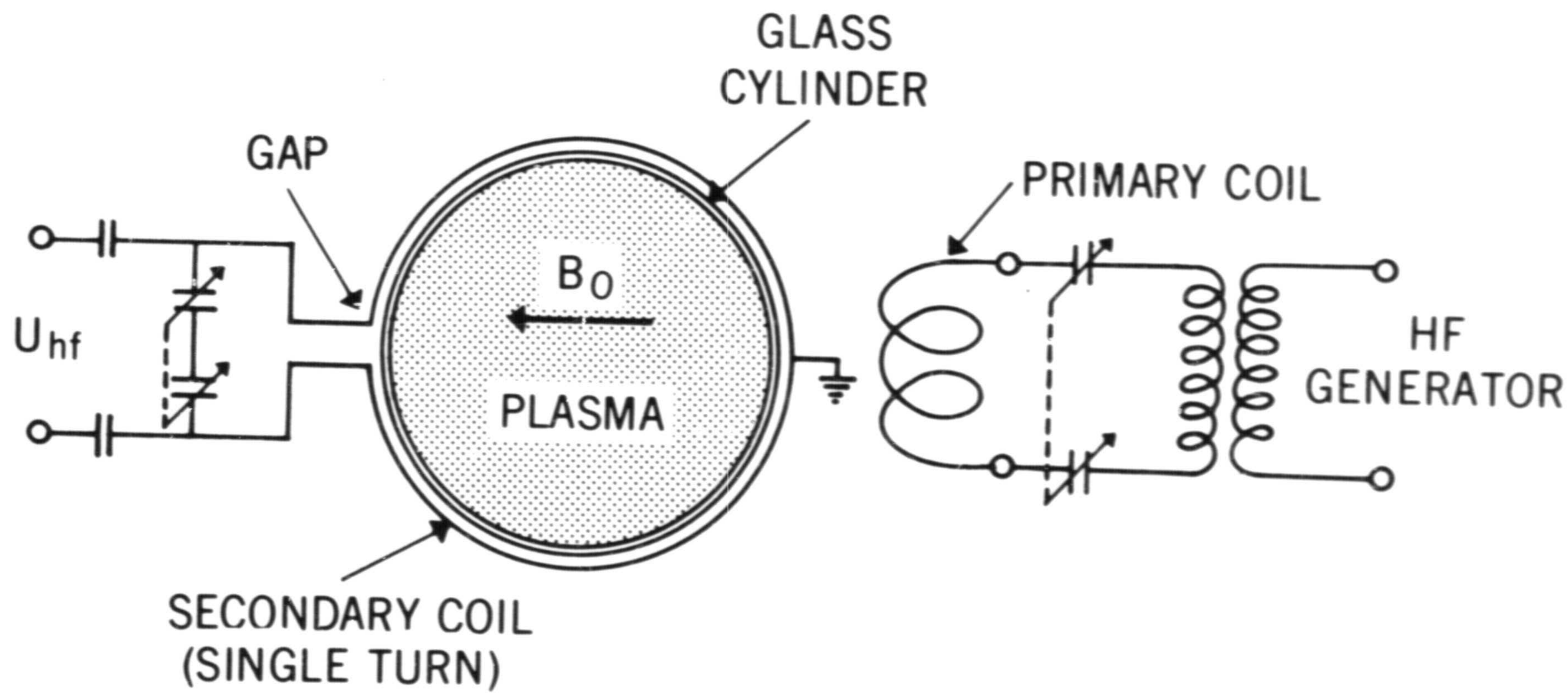
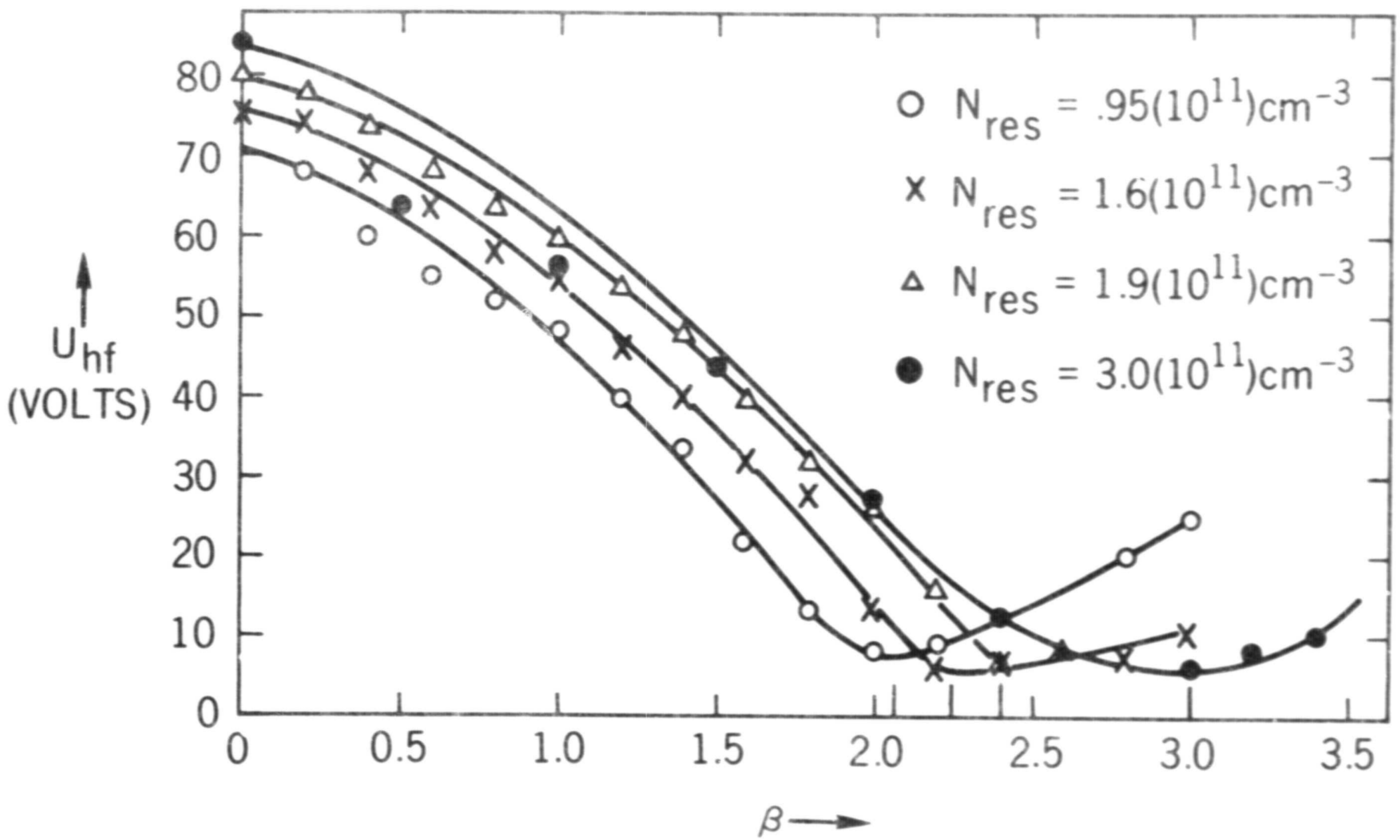
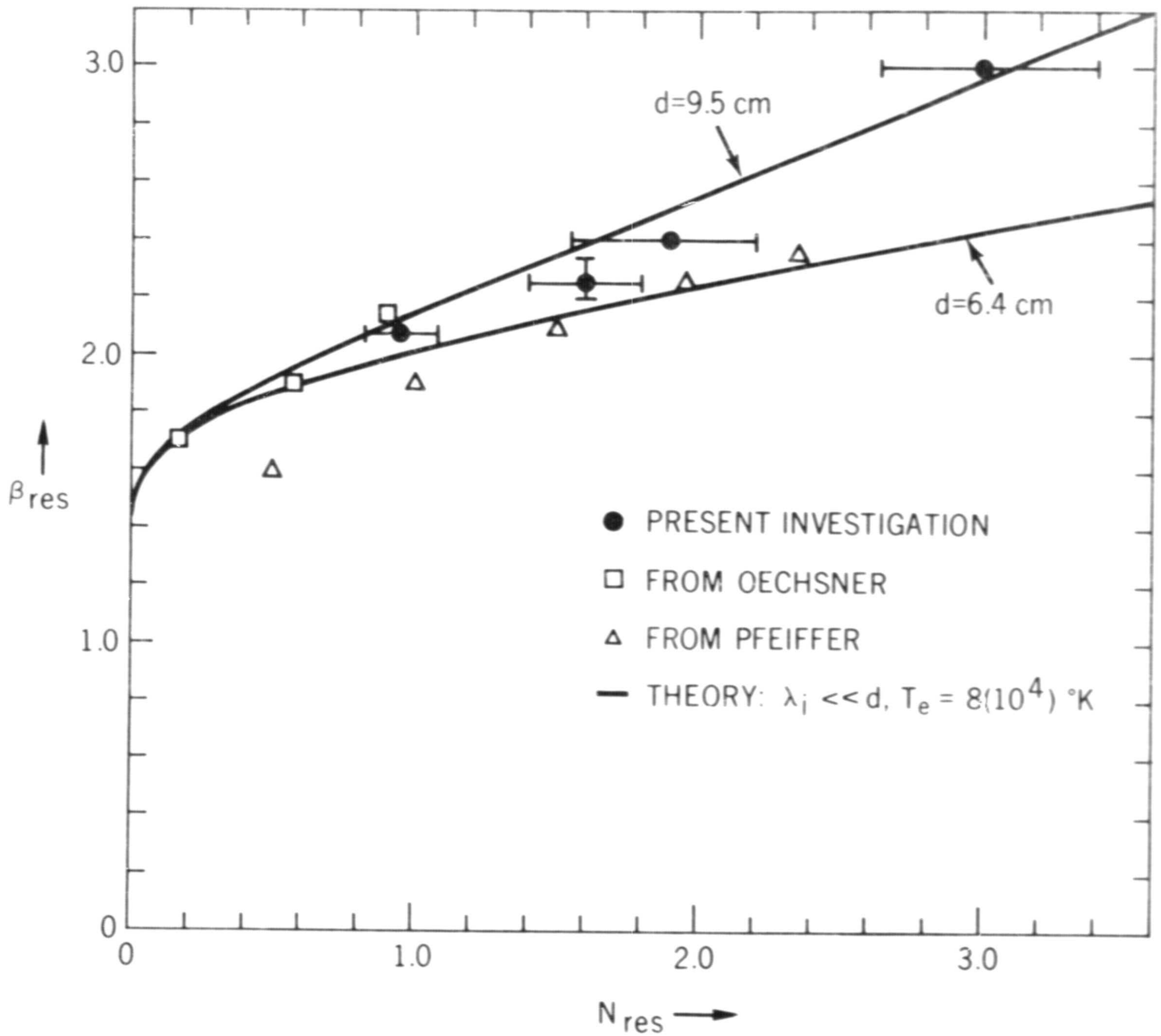

GLASS
CYLINDER
GAP
PRIMARY COIL
B_0
U_{hf}
HF
GENERATOR
PLASMA
SECONDARY COIL
(SINGLE TURN)

Figure 5



N_res = .95(10^11)cm^-3
N_res = 1.6(10^11)cm^-3
N_res = 1.9(10^11)cm^-3
N_res = 3.0(10^11)cm^-3
U_hf (VOLTS)
β
0
10
20
30
40
50
60
70
80
0.5
1.0
1.5
2.0
2.5
3.0
3.5

Figure 6



3.0
2.0
1.0
0
β_{res}
0
1.0
2.0
3.0
N_{res}
d=9.5 cm
d=6.4 cm
PRESENT INVESTIGATION
FROM OECHSNER
FROM PFEIFFER
THEORY: $\lambda_i << d$, $T_e = 8(10^4)$ °K

Figure 7