Final Report Vol. IV
Contract No. NAS8-21131
Covering Period Nov. 4, 1969 - April 4, 1971
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION

User's Guide for Computer Program COMPDES

by

Lutz Willner

Rensselaer Polytechnic Institute

Troy, New York

Rensselaer Polytechnic Institute
Troy, New York

Final Report Vol. IV
Contract No. NAS8-21131
Covering Period Nov. 4, 1969 - April 4, 1971
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION

User's Guide for Computer Program COMPDES

by

Lutz Willner

Submitted on behalf of

Rob J. Roy
Professor
Systems Engineering Division

User's Guide for the Computer Program COMPDES

1. Introduction

In reference 1 the theories and algorithms for two digital computer programs, namely, a compensator design program and a pole placement program, are developed. The two programs are written in FORTRAN IV and are listed in appendices A and B of reference 1. The present user's guide describes the use of the compensator design program, COMPDES, only; the necessary instructions for the useage of the pole placement program are given in the form of comments at the beginning of that program.

The purpose of COMPDES is the design of a low-order compensator to stabilize a given controllable and observable linear time-invariant system in the presence of parameter uncertainties. The theoretical development for the algorithm can be found in reference 1.

2. Program Outline

COMPDES can be broken down into two major sections. The first section consists of a gradient procedure which tries to increase the stability of a system by computing appropriate feedback gains. The resulting closed-loop system is tested for stability with respect to the maximum possible parameter variations. The program terminates if stability can be guaranteed. If not, the program proceeds to program section two. Section one can be by-passed.

Program section two carries out the actual compensator design and sensitivity reduction. The compensator is composed of a state estimator and a set of 'all-state' feedback gains. The sensitivity function is minimized by means of the Davidon function minimization method. The

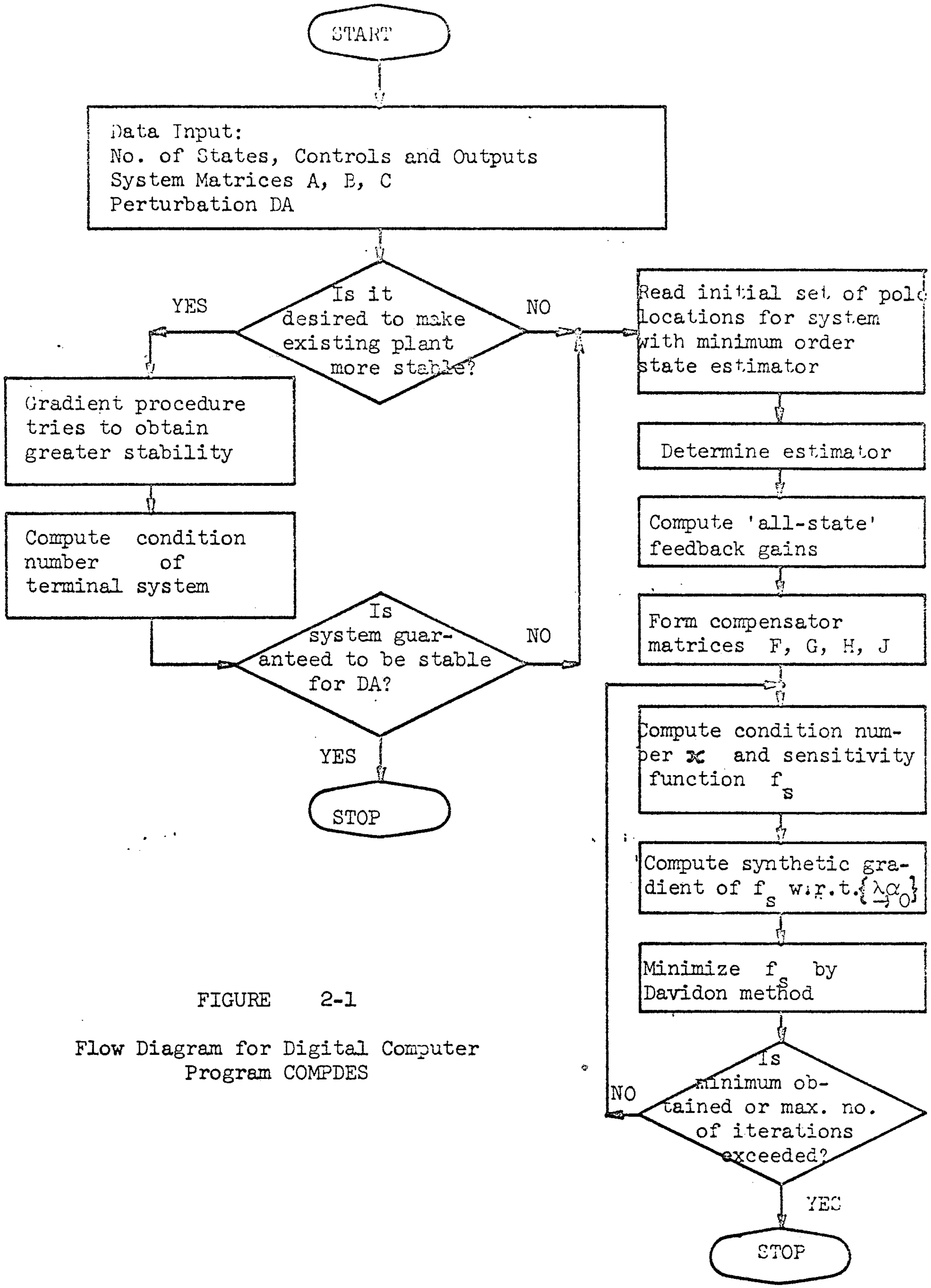

FIGURE 2-1

Flow Diagram for Digital Computer Program COMPDES gradient of the sensitivity function with respect to the eigenvalues and ALO is required for the Davidon method and is synthetically generated. Thus the computed gradient may not be zero where the function actually has a minimum. To avoid numerical oscillation about such a minimum, the total number of iterations is limited. A flow chart of the COMPDES computer program is depicted in Figure 2-1.

3. Table of Input Parameters

Parameter	Input Format	Type	Explanation
NS	8I10	Int., simple	No. of System States
NC		" "	No. of Control Inputs
NF		" "	No. of Outputs
NFF		" "	Desired order of compensator, used in gradient method to increase stability.
IDELR		" "	≤ 0 DELR = .2 by default > 0 read DELR
ISTOP		" "	≤ 0 STOPR is computer from other input data >0 Read STOPR
IMATJ		" "	≤ 0 Matrix [FJ] = [0.] by default >0 Read matrix [FJ]
NU1		" "	≥ 0 ; >0 to generate a more conservative STOPR and condition no. χ.
IPASS	8I10	Int., simple	≤ 0 Program tries to determine an appropriate compensator or order NFF via gradient method >0 Gradient method is by-passed, compensator will be of order NFF=NS-NF

Parameter	Input Format	Type	Explanation
DELR	7F10.4	Real, simple	Only if IDELR > 0 ; DELR is the increment for the gradient method
STOPR	7F10.4	Real, simple	Only if ISTOP > 0 ; STOPR determines the minimum required system stability for termination of the gradient method.
A	5F15.6	Real, matrix	NS * NS System matrix; input by rows.
DA	5F15.6	" "	NS * NS System uncertainty matrix; input by rows.
B	5F15.6	" "	NS * NC System input matrix; input by rows.
C	5F15.6	Real, matrix	NF * NS System output matrix; input by rows.
ACC	5F15.6	Real, simple	Min. stability required for nominal system plus compensator.
AC		Real, simple	Min. Stability required for perturbed system plus compensator.
FJ	5F15.6	Real, matrix	Only if IMATJ > 0; NC * NF matrix of initial direct feedback gains to start gradient method.
IPOLE	8I10	Int., simple	≤ 0 Program generates initial set of stable system and compensator poles >0 Read system and compensator poles.
IAREA		" "	≤ 0 No region constraint for pole locations >0 Read region constraint DR and weighting GA
LIMIT		" "	Maximum no. of iterations (for any given set of initial poles) to determine valid compensator.
KSHIFT		" "	≥ 1, no. of different initial pole sets. If KSHIFT > 1, program will generate new initial sets.

Parameter	Input Format	Type	Explanation
DR	4F20.10	Real, simple	Only if IAREA > 0: Region height 2*DR, width DR. Final design should have poles in or close to this region.
GA	4F20.10	" "	Only if IAREA > 0: Weighting of region constraints; choose GA $\leq$ .001
EMS	4F20.10	Real, matrix	Only if IPOLE > 0, NS*2 matrix (i.e., real and im. part of poles). NS poles initially to be realized by system with 'all-state' feedback.
EMF	4F20.10	" "	Only if IPOLE>0,NFF*2 matrix.NFF poles initially to be realized by the compensator.

4. Example

The following Computer Print-out represents a typical example. The example chosen describes a 3rd order system with one input and two outputs (Note: for the program to work the system has to be in observer canonical form, i.e., the first part of the NF*NS output matrix C represents the identity matrix of rank NF, the remainder of the matrix C is zero). The card set-up for the input data is shown in Fig. 4-1. Fig. 4-2 presents the actual computer print-out.

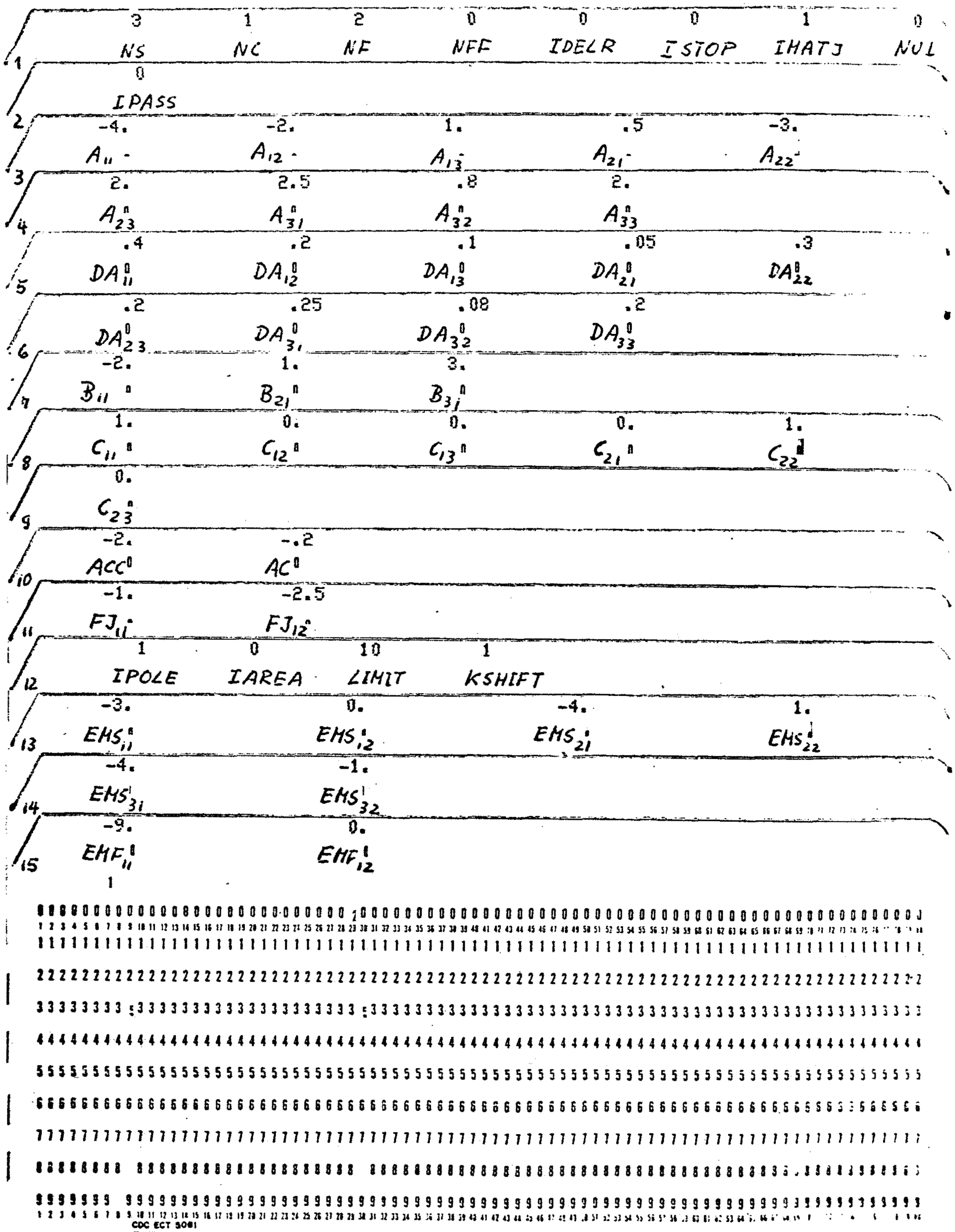

FIGURE 4-1

Card Set-Up for 3rd Order Example

```
 STATES     INPUTS     OUTPUTS    COMP-ORD  IDELR      ISTOP      IMATJ      NUI
 NS =  3    NC =  1    NF =  2    NFF =  0       0          0          1          0


 SYSTEM MATRIX  A.
   -0.4000000D 01     -0.2000000D 01     0.1000000D 01     0.5000000D 00
   -0.3000000D 01      0.2000000D 01     0.2500000D 01     0.8000000D 00
    0.2000000D 01


 MAXIMUM ABSOLUTE CHANGE  DA  OF THE ELEMENTS OF THE SYSTEM MATRIX  A
    0.4000000D 00      0.2000000D 00     0.1000000D 00     0.5000000D-01
    0.3000000D 00      0.2000000D 00     0.2500000D 00     0.8000000D-01
    0.2000000D 00


 CONTROL INPUT MATRIX B
   -0.2000000D 01      0.1000000D 01     0.3000000D 01


 OUTPUT MATRIX  C
    0.1000000D 01      0.0               0.0               0.0
    0.1000000D 01      0.0


 ACCEPTABLE LARGEST REAL PART OF THE EIGENVALUES FOR THE WORST CASE OF
 PARAMETER UNCERTAINTY HAS TO BE LESS THAN  AC =  -0.20000
 EIGENVALUES OF NOMINAL SYSTEM ARE KEPT TO THE LEFT OF  ACC =  -2.00000


 MINIMUM STABILITY REQUIRED FOR TERMINATION           -4.09999943


 STABILITY INCREASE INCREMENT =          0.20000000



 GRADIENT PROCEDURE TRIES TO DETERMINE A COMPENSATOR OF ORDER  NFF =  0
 SUCH THAT THE CLOSED-LOOP SYSTEM DOES NOT BECOME UNSTABLE FOR THE GIVEN
 MAXIMUM PARAMETER UNCERTAINTIES (SEE MATRIX  DA).


 DIRECT FEEDBACK MATRIX  FJ
   -0.1000000D 01     -0.2500000D 01


ROOTS      REAL PART          IMAG. PART
       -0.251839D 01       0.165062D 01
       -0.251839D 01      -0.165062D 01
       -0.463227D 00       0.0


MAXIMUM REAL PART OF ROOTS
   -0.4632269D 00


 CLOSED-LOOP SYSTEM MATRIX  A(C.-L.).
   -0.2000000D 01      0.3000000D 01     0.1000000D 01    -0.5000000D 00
   -0.5500000D 01      0.2000000D 01    -0.5000000D 00    -0.6700000D 01
    0.2000000D 01
```

FIGURE 4-2, i

Computer Print-Out for 3rd Order Example

```
EIGVEC ERROR MESSAGES
SW1=0.3692D-11          ITER=    2          DIF=C.1866E-10


EIGENVECTORS CORRES TO MRP EIGENVALUE
   ROW REAL PART     ROW IMAG PART       COL REAL PART      COL IMAG PART
  -0.7049734D-01     0.C                 C.1CCCCCCD C1      0.0
   0.1000000D 01     0.C                 C.2331792D C0      0.0
  -0.7833232D C0     0.C                 C.8372354D C0      0.0


GRADIENT MATRIX DUE TO FJ
   0.2451565D 01     0.5716541D CC


DIRECT FEEDBACK MATRIX  FJ
  -0.1309494D 01    -0.2572168D C1


ROOTS      REAL PART            IMAG. PART
      -0.159931D 01          0.C
      -0.167694D 01          C.184924D C1
      -0.167694D 01         -0.184924D C1


MAXIMUM REAL PART OF ROOTS
  -0.1599307D 01


CLOSED-LOOP SYSTEM MATRIX  A(C.-L.).
  -0.1381012D 01      0.2144335D C1      C.1CCCCCCD C1      -0.8094941D 00
  -0.5572168D 01      0.2CCCCCCD C1     -C.1428482D C1      -0.6916503D 01
   0.2000000D 01


EIGVEC ERROR MESSAGES
SW1=0.2651D-14          ITER=    2          DIF=C.1485E-14


EIGENVECTORS CORRES TO MRP EIGENVALUE
   ROW REAL PART     ROW IMAG PART       COL REAL PART      COL IMAG PART
   0.2558633D-01     0.C                 C.1CCCCCCD C1      0.0
   0.1000000D 01     0.C                -C.1214326D C0      0.0
  -0.5627712D 00     0.C                 C.1635296D C0      0.0


GRADIENT MATRIX DUE TO FJ
   0.3936033D 01    -0.4779628D CC


DIRECT FEEDBACK MATRIX  FJ
  -0.1324045D 01    -0.2570401D C1


ROOTS      REAL PART            IMAG. PART
      -0.165629D 01          0.C
      -0.163301D 01          C.186884D C1
      -0.163301D 01         -0.186884D C1


MAXIMUM REAL PART OF ROOTS
  -0.1633008D 01
```

FIGURE 4-2, ii

Computer Print-Out for 3^rd Order Example (continued)

```
CLCSED-LOOP SYSTEM MATRIX  A(C.-L.).
  -0.1351909D 01     0.314C8C1D C1     C.1CCCCCCD C1     -0.8240453C 00
  -0.5570401C 01     0.2CCCCCCD C1    -C.1472136D C1     -0.6911202C 01
   0.2000000D 01

EIGVEC ERROR MESSAGES
SW1=0.7713D-11           ITER=    2           DIF=C.7487E-12

EIGENVECTORS CORRES TO MRP EIGENVALLE
   RCW REAL PART      ROW IMAG PART     CCL REAL PART     CCL IMAG PART
   0.2057685D C0      0.3487345C-C1     C.1CCCCCCD C1     0.0
   0.1000000D 01     -0.1387779D-16    -C.6653863D-C1     0.3779265C 00
  -0.4762001D 00     -0.2545559CD CC   -C.721137CD-C1     0.6818475C 00

GRACIENT MATRIX CUE TO FJ
  -0.2885096C 01     0.7649145C CC

CIRECT FEEDBACK MATRIX  FJ
  -0.1321017C 01    -0.25712C3D C1

RCCTS     REAL PART           IMAG. PART
      -0.164440C 01        C.C
      -0.164238C 01        C.1665C8D C1
      -0.164238C 01       -C.1665C8D C1

MAXIMUM REAL PART OF ROOTS
   -0.1642382C 01

CLCSED-LOOP SYSTEM MATRIX  A(C.-L.).
  -0.1357965C 01     0.31424C7D C1     C.1CCCCCCD C1     -0.8210174C 00
  -0.5571203D 01     0.2CCCCCCD C1    -C.1463052D C1     -0.6913610D 01
   0.2000000D C1

EIGVEC ERROR MESSAGES
SW1=0.7376D-10           ITER=    2           DIF=C.4553E-11

EIGENVECTORS CORRES TO MRP EIGENVALLE
   RCW REAL PART      ROW IMAG PART     COL REAL PART     CCL IMAG PART
   0.2042519D 00      0.361C422D-C1     C.1CCCCCCD C1    -0.4163336D-16
   0.1000000C 01     -0.2775558D-16    -C.671487CD-C1     0.3774747C 00
  -0.4754359D C0     -0.2533584D CC    -C.734C831D-C1     0.6788966C 00

GRACIENT MATRIX CUE TO FJ
  -0.2902577D 01     0.7597035D CC

CIRECT FEEDBACK MATRIX  FJ
  -0.1320730D 01    -0.2571275D C1

RCCTS     REAL PART           IMAG. PART
      -0.164327C 01        C.C
      -0.164327C 01        C.186472D C1
      -0.164327C 01       -C.186472D C1
```

FIGURE 4-2, iii

Computer Print-Out for 3rd Order Example (continued)

```
MAXIMUM REAL PART OF ROOTS
   -0.1643272D 01

 CLOSED-LOOP SYSTEM MATRIX  A(C.-L.).
   -0.1358539D 01      0.3142557D 01      0.1000000D 01      -0.8207304D 00
   -0.5571279D 01      0.2000000D 01     -0.1462191D 01      -0.6913836D 01
    0.2000000D 01

 EIGVEC ERROR MESSAGES
 SW1=0.9037D-10            ITER=    2          DIF=0.5437E-11

 EIGENVECTORS CORRES TO MRP EIGENVALUE
    ROW REAL PART      ROW IMAG PART      COL REAL PART       COL IMAG PART
    0.2041088D 00      0.3622100D-01      0.1000000D 01       0.4163336D-16
    0.1000000D 01      0.1387779D-16     -0.6720672D-01       0.3774326D 00
   -0.4753631D 00     -0.2532450D 00     -0.7353225D-01       0.6786178D 00

 GRADIENT MATRIX DUE TO FJ
   -0.2904210D 01      0.7591988D 00

 DIRECT FEEDBACK MATRIX  FJ
   -0.1320730D 01     -0.2571279D 01

ROOTS     REAL PART           IMAG. PART
      -0.164327D 01        0.0
      -0.164327D 01        0.186472D 01
      -0.164327D 01       -0.186472D 01

MAXIMUM REAL PART OF ROOTS
   -0.1643272D 01

 CLOSED-LOOP SYSTEM MATRIX  A(C.-L.).
   -0.1358539D 01      0.3142557D 01      0.1000000D 01      -0.8207304D 00
   -0.5571279D 01      0.2000000D 01     -0.1462191D 01      -0.6913836D 01
    0.2000000D 01

 EIGVEC ERROR MESSAGES
 SW1=0.9037D-10            ITER=    2          DIF=0.5437E-11

 EIGENVECTORS CORRES TO MRP EIGENVALUE
    ROW REAL PART      ROW IMAG PART      COL REAL PART       COL IMAG PART
    0.2041088D 00      0.3622100D-01      0.1000000D 01       0.4163336D-16
    0.1000000D 01      0.1387779D-16     -0.6720672D-01       0.3774326D 00
   -0.4753631D 00     -0.2532450D 00     -0.7353225D-01       0.6786178D 00

 GRADIENT MATRIX DUE TO FJ
   -0.2904210D 01      0.7591988D 00

 DIRECT FEEDBACK MATRIX  FJ
   -0.1320730D 01     -0.2571279D 01
```

FIGURE 4-2, iv

Computer Print-Out for 3rd Order Example (continued)

```
RCCTS      REAL PART           IMAG. PART
      -0.164327C 01         C.C
      -0.164327C 01         C.1εε472D C1
      -0.164327C 01        -C.1εε472D C1


 CETERMINE COMPENSATOR OF ORCER  NFF =  1
 AND ITERATE ON CONCITION-NUMBER.

 MINIMUM STABILITY REQUIREC FOR TERMINATICN           -4.79999924


 CCMPENSATOR CESIGN - INITIAL VALLES.   ISHIFT =  1

 CIRECT FEEDBACK MATRIX  FJ
   -0.1231167C 01    -0.15548740 C2

 CCMPENSATOR OUTPUT MATRIX  FH
   -0.2834564C 01

 CCMPENSATOR INPUT MATRIX  FG
    0.2827917C 01     0.εε71861D C1

 COMPENSATOR MATRIX  FF
   -0.1913589C 01

RCCTS      REAL PART           IMAG. PART
      -0.900000C 01         C.C
      -0.400000C 01         C.1CCCCCD C1
      -0.400000C 01        -C.1CCCCCD C1
      -0.300000C 01         C.C

 ALC =     1.000000COND.-NUMBER =  222.9C9CC3

 ALC =      1.00000  ROOT1 =  -C.3CCCCCCD C1  CCND.-NLMBER =   0.2229090C 03  FUNCTION VALUE =    0.9937

 ITERATION NUMBER  KOUNT=  1

 ALC =      1.00200  ROOT1 =  -C.2999431D C1  CCND.-NLMBER =   0.2212218C 03  FUNCTION VALUE =    C.9936

 ITERATION NUMBER  KOUNT=  1

 ALC =      1.00400  ROOT1 =  -C.299E662D C1  CCND.-NLMBER =   0.2195591C 03  FUNCTION VALUE =    0.9936

 ITERATION NUMBER  KOUNT=  1

 ALC =      1.00800  ROOT1 =  -C.2997724D C1  CCND.-NLMBER =   0.2163055C 03  FUNCTION VALUE =    C.9935

 ITERATION NUMBER  KOUNT=  1
```

FIGURE 4-2, v

Computer Print-Out for 3rd Order Example (continued)

```
ALO =     1.01601  ROOT1 =  -0.2995448D 01  COND.-NUMBER =  0.2100730D 03  FUNCTION VALUE =   0.9933

ITERATION NUMBER  KOUNT=  1

ALO =     1.03202  ROOT1 =  -0.2990897D 01  COND.-NUMBER =  0.1986163D 03  FUNCTION VALUE =   0.9930

ITERATION NUMBER  KOUNT=  1

ALO =     1.06403  ROOT1 =  -0.2981793D 01  COND.-NUMBER =  0.1791212D 03  FUNCTION VALUE =   0.9923

ITERATION NUMBER  KOUNT=  1

ALO =     1.12807  ROOT1 =  -0.2963586D 01  COND.-NUMBER =  0.1501772D 03  FUNCTION VALUE =   0.9910

ITERATION NUMBER  KOUNT=  1

ALO =     1.25614  ROOT1 =  -0.2927172D 01  COND.-NUMBER =  0.1154399D 03  FUNCTION VALUE =   0.9888

ITERATION NUMBER  KOUNT=  1

ALO =     1.51228  ROOT1 =  -0.2854344D 01  COND.-NUMBER =  0.8297166D 02  FUNCTION VALUE =   0.9858

ITERATION NUMBER  KOUNT=  1

ALO =     1.46258  ROOT1 =  -0.2868476D 01  COND.-NUMBER =  0.8750232D 02  FUNCTION VALUE =   0.9863

ITERATION NUMBER  KOUNT=  1

ALO =     1.42769  ROOT1 =  -0.2878396D 01  COND.-NUMBER =  0.9094286D 02  FUNCTION VALUE =   0.9866

ITERATION NUMBER  KOUNT=  1

ALO =     1.40311  ROOT1 =  -0.2885383D 01  COND.-NUMBER =  0.9353388D 02  FUNCTION VALUE =   0.9869

ITERATION NUMBER  KOUNT=  1

ALO =     1.38567  ROOT1 =  -0.2890343D 01  COND.-NUMBER =  0.9547476D 02  FUNCTION VALUE =   0.9871

ITERATION NUMBER  KOUNT=  1

ALO =     1.37315  ROOT1 =  -0.2893903D 01  COND.-NUMBER =  0.9692715D 02  FUNCTION VALUE =   0.9872

ITERATION NUMBER  KOUNT=  1
```

FIGURE 4-2, vi

Computer Print-Out for 3rd Order Example (continued)

```
ALO =      1.36405  ROOT1 =  -C.2E5645CD C1  COND.-NUMBER =   0.9806241C 02  FUNCTION VALUE =    C.9873

ITERATION NUMBER  KOUNT=  1

ALO =      1.37261  ROOT1 =  -C.2E54C57D C1  COND.-NUMBER =   0.9699118C 02  FUNCTION VALUE =    C.9872

ITERATION NUMBER  KOUNT=  1

ALO =      1.37211  ROOT1 =  -C.2E542CCD C1  COND.-NUMBER =   0.9705061C 02  FUNCTION VALUE =    C.9872

ITERATION NUMBER  KOUNT=  1

ALO =      1.37164  ROOT1 =  -C.2E94332D C1  COND.-NUMBER =   0.9710586C 02  FUNCTION VALUE =    0.9872

ITERATION NUMBER  KOUNT=  1

ALO =      1.37121  ROOT1 =  -C.2E94455D C1  COND.-NUMBER =   0.9715729C 02  FUNCTION VALUE =    C.9872

ITERATION NUMBER  KOUNT=  1

ALO =      1.37080  ROOT1 =  -C.2E9457CD C1  COND.-NUMBER =   0.9720521C 02  FUNCTION VALUE =    0.9872
      1

THE BELOW COMPENSATOR GUARANTEES STABILITY ONLY FOR A TOTAL
UNCERTAINTY OF        0.C277644E


COMPENSATOR DESIGN - FINAL VALUES.

DIRECT FEEDBACK MATRIX  FJ
  -0.1691622C 01    -0.152C581D C2

COMPENSATOR OUTPUT MATRIX  FH
  -0.2269640D 01

COMPENSATOR INPUT MATRIX  FG
   0.4328633C 01     0.E397799D C1

COMPENSATOR MATRIX  FF
  -0.1968502D 01

ROOTS      REAL PART           IMAG. PART
      -0.758312C 01        C.C
      -0.416168C 01        C.241976D C1
      -0.416168C 01       -C.241976D C1
      -0.288457C 01        C.C
```

FIGURE 4-2, vii

Computer Print-Out for 3rd Order Example (continued)

5. Explanation of Computer Print-Out, Fig. 4-2

The top half of example page i gives a partial listing of the input parameter.

The bottom half shows the beginning of the iterative gradient procedure to obtain a more stable system. The initial direct feedback gain matrix was inputted as -1. -2.5 resulting in a closed loop system whose least stable root has a maximum real part (MRP) of -.463. The following pages list the iterations produced by the gradient method in order to improve the system stability. The stability is improved such that the least stable closed-loop system root has a MRP = -1.643.

Since the closed-loop system without compensator (recall NFF was chosen 0), could not guarantee the required minimum stability -AC = -.2 in the presence of the chosen parameter uncertainty DA the program tries to determine a low-order compensator to obtain the required minimum stability. The compensator order is NF = NF - NF = 1. The compensator matrices FJ, FH, FG and FF, to realize the inputted set of initial system-plus compensator poles (EMS, EMF), are computed and listed in the top half of example page v.

The closed-loop system formed by the original system and the compensator yields a condition number of $\varkappa = 222.909$ for ALO = 1 (ALO multiplies matrix FG and divides matrix FH).

In order to achieve a high relative stability the function

$$f_s\,(\underline{\lambda},\ \text{ALO}) = 1 + \frac{\text{Re}\,(\lambda_{max}) - \text{ACC}}{\varkappa\ \|\text{DA}\| - \text{ACC}} \quad ,$$

where $\underline{\lambda}$ are the eigenvalues of the closed-loop system, is minimized.

The iterations on ALO, the COND. - NUMBER and the FUNCTION VALUE are listed on pages v, vi and vii. It should be noted that the program terminates because of too many iterations. Although the function f_s achieves at some iteration step a value that is lower than the terminal FUNCTION VALUE the minimization procedure will in general, not be able to return to the lowest function value because the function gradient is synthetically computed.

The terminal design values and closed-loop system eigenvalues are listed at the bottom of example page vii.

6. Changes in Program

The COMPDES program taken to compute the 3^{rd} order example is the same as listed in Appendix A of reference 1, with 2 exceptions. In the MAIN program,

a) insert: READ (1, 10) IPASS between ISN0077 and ISN0078,

b) take out ISN0140 (IF(IMATJ.EQ.0)GO TO 1099) and substitute IF(IPASS.GT.0)GO TO 1099.

Reference:

1. Lutz Willner, "Compensator Design for Low-Sensitivity Linear Time-Invariant Systems," Final Report - Vol. II, Contract No. NAS8-21131, Covering the period Nov. 4, 1969 - April 4, 1971, NASA.