NASA TECHNICAL NOTE

NASA TN D-6430

c.1

NASA TN D-6430

THE INFLUENCE OF VERTICAL WIND GRADIENTS ON THE LONGITUDINAL MOTION OF AIRPLANES

by Joseph Gera

Langley Research Center
Hampton, Va. 23365

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • SEPTEMBER 1971

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-6430		

4. Title and Subtitle

THE INFLUENCE OF VERTICAL WIND GRADIENTS ON THE LONGITUDINAL MOTION OF AIRPLANES

5. Report Date
September 1971

6. Performing Organization Code

7. Author(s)

Joseph Gera

8. Performing Organization Report No.
L-7671

9. Performing Organization Name and Address

NASA Langley Research Center
Hampton, Va. 23365

10. Work Unit No.
136-62-01-02

11. Contract or Grant No.

12. Sponsoring Agency Name and Address

National Aeronautics and Space Administration
Washington, D.C. 20546

13. Type of Report and Period Covered
Technical Note

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The present study is an attempt to make an assessment of the influence of wind shear on the longitudinal motion of airplanes. It was assumed that the wind is completely horizontal and its speed varies linearly with altitude. It is shown quantitatively that both glide and climb performance are influenced by wind shear and that trimmed flight at constant airspeed, attitude, and with fixed controls is along a parabolic path relative to the ground. The problem of the landing approach in a wind shear is examined in some detail. Small-disturbance theory indicates no wind-shear effect on the short-period motion and the time for the phugoid to damp to half amplitude but the phugoid frequency and damping ratio vary considerably with wind shear. A nondimensional quantity which depends on the wind shear and airspeed is shown to be a fundamental parameter influencing the longitudinal dynamic behavior of the airplane.

17. Key Words (Suggested by Author(s))

Wind shear
Vertical wind gradients

18. Distribution Statement

Unclassified – Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	18	$3.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

THE INFLUENCE OF VERTICAL WIND GRADIENTS ON THE LONGITUDINAL MOTION OF AIRPLANES

By Joseph Gera
Langley Research Center

SUMMARY

The present study is an attempt to make an assessment of the influence of wind shear on the longitudinal motion of airplanes. It was assumed that the wind is completely horizontal and its speed varies linearly with altitude. It is shown quantitatively that both glide and climb performance are influenced by wind shear and that trimmed flight at constant airspeed, attitude, and with fixed controls is along a parabolic path relative to the ground. The problem of the landing approach in a wind shear is examined in some detail. Small-disturbance theory indicates no wind-shear effect on the short-period motion and the time for the phugoid to damp to half amplitude but the phugoid frequency and damping ratio vary considerably with wind shear. A nondimensional quantity which depends on the wind shear and airspeed is shown to be a fundamental parameter influencing the longitudinal dynamic behavior of the airplane.

INTRODUCTION

All-weather operation on a routine basis in civil aviation is one of the major goals of the aviation industry. The most critical phase of all-weather operation is the landing, as attested to by the establishment of the various weather-minimum categories for landing. While progress is being made in the design of ground-based and airborne landing aids, adverse weather conditions – especially variations in wind direction, strength, and gustiness – remain a major factor affecting safety during landing (refs. 1 and 2). In the consideration of wind effects a distinction should be made between random gusts and a velocity gradient in the mean wind or wind shear. Random-gust or turbulence effects are important with respect to the structural design and the riding qualities of the airplane. In contrast, wind-shear effects become significant when the airplane is to be controlled with a high degree of precision as, for instance, during an automatic landing. According to Henley (ref. 2), wind shear was the main cause of touchdown error and of the large variations in rate of descent during the evaluation of a particular automatic landing system. It seems probable that other landing systems will be similarly affected unless control-system designers take wind-shear effects into consideration during the selection of the control laws for their system.

Although the effects of wind shear on the dynamics of the airplane have been considered in numerous engineering analyses, experimental data are scarce. Some early NACA flight tests reported in reference 3 conclude that landing approaches made by conventional airplanes while deliberately disregarding wind effects are unduly hazardous. Reference 3 also includes actual measurements of wind shear near the ground. Reference 4 presents the frequency of wind-shear occurrence and magnitude in a particular locality along with the results of a flight test showing that the effect of wind shear on the climb performance of the B-29 aircraft was significant.

Analytical studies of the effect of wind shear on the longitudinal dynamics include references 5 and 6. Additional information including the effects of other atmospheric gradients such as density and temperature variations can be found in references 7 and 8. In some relatively recent publications there is a lack of agreement as to what actually takes place during an approach to landing in a headwind which is decreasing with altitude. According to references 9 and 10 an airplane trimmed for a glide at constant airspeed and constant attitude would touch down short of the initial aim point if no control action is taken by the pilot. In contrast, reference 11 asserts that a trimmed airplane executing an instrument approach in decreasing headwind would overshoot the intended point of touchdown unless the pilot reduces power.

The present study is a simple attempt to make an assessment of the influence of wind shear on the longitudinal dynamics of airplanes. Improved knowledge of wind-shear effects is important in the future design of approach and landing aids.

SYMBOLS

g gravitational acceleration, 9.80665 m/sec^2

$i = \sqrt{-1}$

$\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ orthogonal unit vectors

I_y mass moment of inertia about $\underset{\sim}{j}$, kg-m^2

m mass, kg

M moment about $\underset{\sim}{j}$, m-N

$M_u = \frac{1}{I_y}\frac{\partial M}{\partial u}, \frac{1}{\text{m-sec}}$

$M_q = \frac{1}{I_y}\frac{\partial M}{\partial \dot{\theta}}, \frac{1}{\text{rad-sec}}$

$M_\alpha = \frac{1}{I_y}\frac{\partial M}{\partial \alpha}, \frac{1}{\text{rad-sec}^2}$

$M_{\dot{\alpha}} = \frac{1}{I_y}\frac{\partial M}{\partial \dot{\alpha}}, \frac{1}{\text{rad-sec}}$

$M_\delta = \frac{1}{I_y}\frac{\partial M}{\partial \delta}, \frac{1}{\text{rad-sec}^2}$

q — rate of change of pitch angle, rad/sec

s — complex variable

t — time, sec

U — velocity relative to air, m/sec

u — perturbation velocity, m/sec

u_w — horizontal wind velocity, m/sec

u'_w — wind velocity gradient, $\frac{du_w}{dz}$, sec^{-1}

$\underset{\sim}{V}_{earth}$ — inertial velocity, tailwind positive, m/sec

X,Z — aerodynamic and propulsive force components in the $\underset{\sim}{i}$ and $\underset{\sim}{k}$ direction, respectively, N

x,y,z — linear displacements in earth-fixed axes, m

$X_u = \frac{1}{m}\frac{\partial X}{\partial u}, \text{sec}^{-1}$

$X_\alpha = \frac{1}{m}\frac{\partial X}{\partial \alpha}, \frac{m}{\text{rad-sec}^2}$

$X_\delta = \frac{1}{m}\frac{\partial X}{\partial \delta}, \frac{m}{\text{rad-sec}^2}$

$Z_q = \frac{1}{m}\frac{\partial Z}{\partial \dot{\theta}}, \frac{m}{\text{rad-sec}}$

$Z_u = \frac{1}{m}\frac{\partial Z}{\partial u}, \text{sec}^{-1}$

$Z_\alpha = \frac{1}{m}\frac{\partial Z}{\partial \alpha}$, $\frac{m}{\text{rad-sec}^2}$

$Z_{\dot{\alpha}} = \frac{1}{m}\frac{\partial Z}{\partial \dot{\alpha}}$, $\frac{m}{\text{rad-sec}}$

$Z_\delta = \frac{1}{m}\frac{\partial Z}{\partial \delta}$, $\frac{m}{\text{rad-sec}^2}$

α	angle of attack, rad
β	exponent in the wind-velocity relation
Γ	angle between earth-fixed x-axis and the unit vector $\underset{\sim}{i}$, rad
γ	perturbation from the equilibrium value of Γ, rad
γ_o	flight-path angle seen by an observer moving with steady wind, rad
δ	generalized control displacement, rad
ζ	damping ratio
Θ	pitch angle of the airplane relative to earth-fixed axes, rad
θ	pertubation from equilibrium value of Θ, rad
ρ	atmospheric density, kg/m^3
σ	nondimensional wind-shear parameter, $\frac{U_o u_w'}{g}$
ω	circular frequency, rad/sec

Subscript:

o	reference condition

A dot denotes differentiation with respect to time.

ANALYSIS

Wind Shear

In the technical sense the term wind shear means a spatial variation in the mean wind speed. According to reference 12, even in the presence of mechanically induced turbulence the mean-wind profile is adequately represented by the law

$$\frac{(u_w)_1}{(u_w)_2} = \left(\frac{z_1}{z_2}\right)^{\beta}$$

where $(u_w)_1$ and $(u_w)_2$ are mean wind speeds at heights z_1 and z_2, and β usually takes a value of 1/7 in an open terrain. In this report it is assumed that β equals 1, so that the mean wind speed varies linearly with altitude according to the simple law

$$u_w = u_w' z$$

u_w' being the wind velocity gradient whose algebraic sign depends on the orientation of the airplane in the moving atmosphere. Figure 1 is a sketch which illustrates what is meant by positive and negative wind shear. Absolute values of u_w' up to 0.03 sec^{-1} are fairly common in the lower layers of the atmosphere with extreme values reaching 0.15 sec^{-1}.

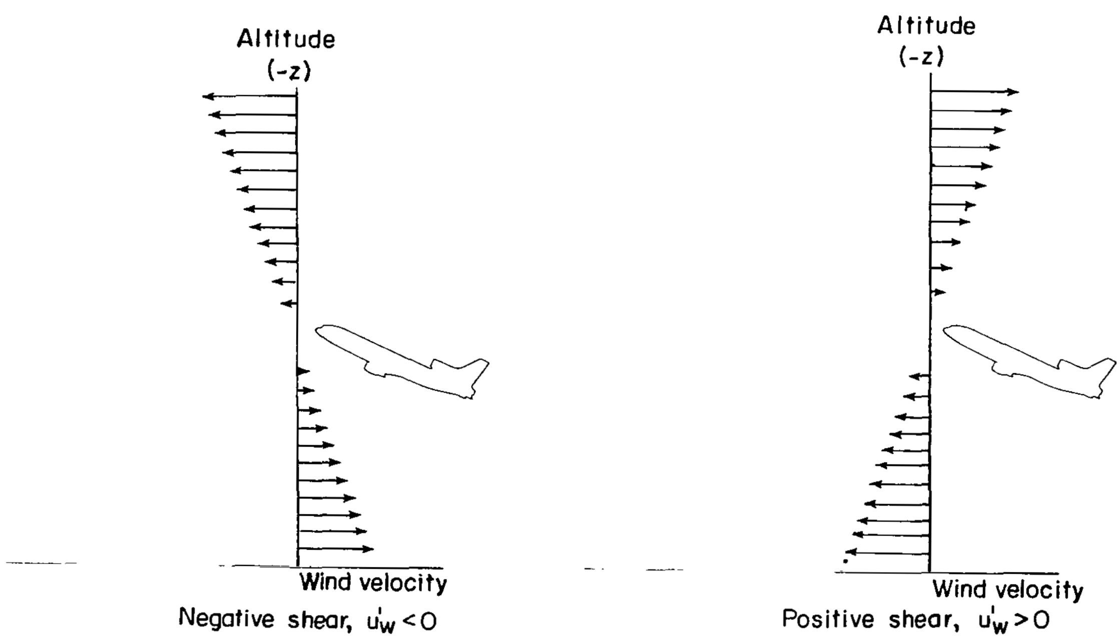

Figure 1.- Definition of positive and negative wind shear.

Equations of Motion

This investigation is concerned with the effects of wind shear on longitudinal dynamics of airplanes; consequently symmetrical flight of the airplane as a rigid body will be assumed. Figure 2 shows the coordinate systems used in the analysis.

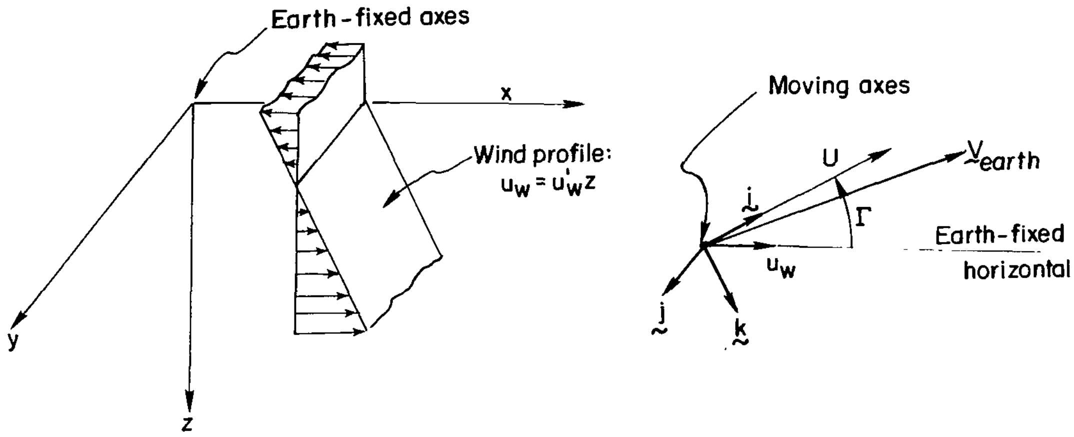

Figure 2.- Coordinate systems.

It is assumed that the earth-fixed axes are fixed in space. The moving system of unit vectors $(\underset{\sim}{i},\underset{\sim}{j},\underset{\sim}{k})$ are defined as follows: the unit vector $\underset{\sim}{i}$ points in the direction of the relative wind, the unit vector $\underset{\sim}{k}$ is directed opposite of the lift vector, and $\underset{\sim}{j}$ is defined by the requirement that $(\underset{\sim}{i},\underset{\sim}{j},\underset{\sim}{k})$ be a right-handed system. The absolute angular velocity of the moving coordinate system is seen to be $\dot{\Gamma}\underset{\sim}{j}$.

The absolute linear acceleration expressed in the moving coordinate system is

$$\dot{\underset{\sim}{V}}_{earth} = \frac{d}{dt}\left(U + u_w \cos \Gamma\right)\underset{\sim}{i} + \frac{d}{dt}\left(u_w \sin \Gamma\right)\underset{\sim}{k} + \dot{\Gamma}\underset{\sim}{j} \times \left[\left(U + u_w \cos \Gamma\right)\underset{\sim}{i} + u_w \sin \Gamma\underset{\sim}{k}\right]$$

where × denotes vector cross product. This expression can be simplified to

$$\dot{\underset{\sim}{V}}_{earth} = \left(\dot{U} + \dot{u}_w \cos \Gamma\right)\underset{\sim}{i} + \left(-U\dot{\Gamma} + \dot{u}_w \sin \Gamma\right)\underset{\sim}{k}$$

The time derivative of u_w may be eliminated from this expression by noting that

$$\dot{u}_w = \frac{d}{dz} u_w \frac{d}{dt} z = -u_w' U \sin \Gamma$$

so that the final form of the linear acceleration becomes

$$\dot{\underset{\sim}{V}}_{earth} = \left(\dot{U} - Uu_w' \sin \Gamma \cos \Gamma\right)\underset{\sim}{i} + \left(-U\dot{\Gamma} - Uu_w' \sin^2\Gamma\right)\underset{\sim}{k}$$

Figure 3 illustrates the external forces acting on the airplane. All aerodynamic and propulsive forces are included in the X and Z components. As is shown, the

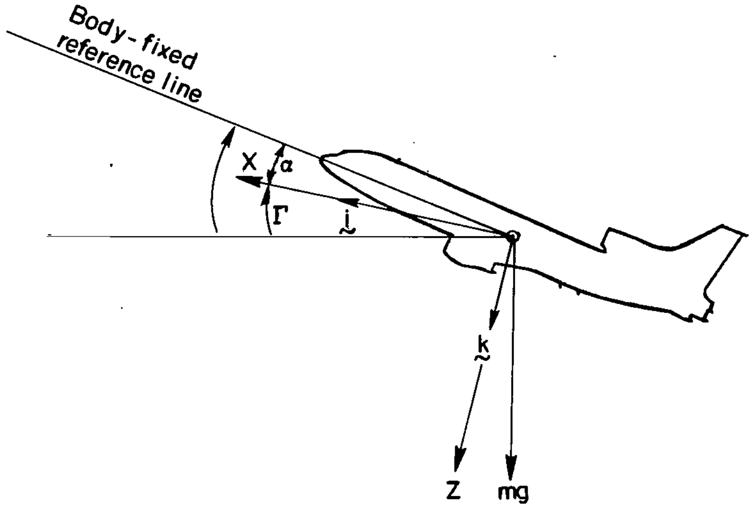

Figure 3.- Forces acting on the airplane.

origin of the moving coordinate system coincides with the center of mass of the airplane. The complete nonlinear longitudinal equations of motion are

$$m\left(\dot{U} - Uu_w' \sin \Gamma \cos \Gamma + g \sin \Gamma\right) = X$$

$$m\left(-U\dot{\Gamma} - Uu_w' \sin^2\Gamma - g \cos \Gamma\right) = Z$$

$$I_y\ddot{\Theta} = M$$

where Θ is the angle of pitch and M is the total external moment acting on the airplane. It should be noted that for zero wind shear the above equations become the standard equations of longitudinal motion referenced to the wind-axis system.

Equilibrium Flight Path

As is usual in airplane stability and control analysis, it is assumed that the motion of the airplane consists of small perturbations from a reference condition. In the present report the reference condition denotes flight at zero external moment and constant airspeed, angle of attack, and pitch angle. It should be noted that according to this definition of reference condition, the airplane may be accelerating along a curvilinear path when viewed by an inertial observer. Denoting the reference condition by the subscript o, the equations of motion become

$$\left.\begin{aligned} m\left(-U_ou_w' \sin \Gamma_o \cos \Gamma_o + g \sin \Gamma_o\right) &= X_o \\ m\left(-U_ou_w' \sin^2\Gamma_o - g \cos \Gamma_o\right) &= Z_o \\ 0 &= M_o \end{aligned}\right\} \qquad (1)$$

Thus, on the reference path one obtains the condition

$$\tan \Gamma_o = -\frac{X_o}{Z_o + mU_o u_w'} \tag{2}$$

Equation (2) may be used to show the influence of wind shear on glide or climb performance on the reference flight path by relating Γ_o to the flight-path angle γ_o that the airplane would have in still air at the same airspeed, angle of attack, and power setting. With the assumption that the atmospheric shear has no influence on the local flow around the various components of the airplane, equation (2) can be rewritten as

$$\tan \Gamma_o = -\frac{X_o}{Z_o}\left(\frac{1}{1 + \frac{mU_o u_w'}{Z_o}}\right) = \frac{\tan \gamma_o}{1 - \sigma \sec \gamma_o} \tag{3}$$

where σ is the nondimensional quantity $U_o u_w'/g$. In the remainder of this report σ will be referred to as the wind-shear parameter. Figure 4 is a plot of $\sin \Gamma_o$ against

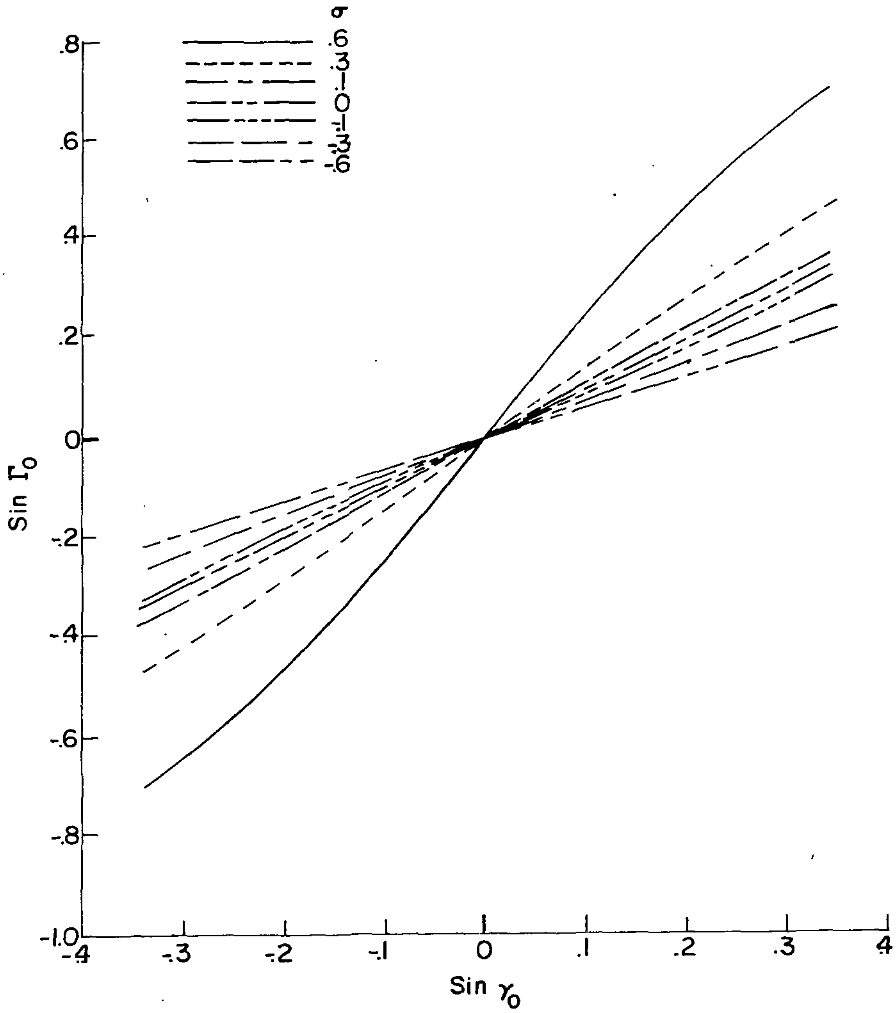

Figure 4.- Effect of wind shear on the nondimensional rate of climb and sink.

$\sin \gamma_o$ for various values of σ. The sine of the respective angles is chosen since it is a direct measure of the normalized rate of climb or sink:

$$\frac{\dot{z}}{U_o} = \begin{cases} \sin \gamma_o & \text{(no wind)} \\ \sin \Gamma_o & \text{(wind shear)} \end{cases}$$

Equation (3) shows that for positive values of σ the angle Γ_o is greater in absolute value than γ_o. Thus, an airplane which climbs into an increasing headwind or decreasing tailwind experiences a rate of climb higher than the value it could achieve in perfectly still or uniformly moving air. Negative values of σ would be reflected in an opposite effect. This phenomenon is well known and the utilization of wind gradients by certain kinds of birds is called dynamic soaring. Wind-shear effects in airplane climb performance were observed and measured during the flight tests reported in reference 4. From the wind-velocity measurements which accompanied the flight tests of reference 4, the value of σ may be calculated. Based on the true airspeed of the test airplane the value of σ is calculated to be ± 0.105, the algebraic sign depending on the direction of flight relative to the wind. With the known value of σ, the net increase or decrease of climb rate due to wind shear may be calculated from equation (3). The results of this calculation are presented in figure 5 along with two flight-test data points taken from reference 4. Instead of using the exact calculations in equation (3), reference 4 proposed that the net increase or decrease of climb rate due to shear may be obtained approximately by multiplying the rate of climb in still air with the value of σ. For small climb rates this is a reasonable approximation as shown by the broken lines in figure 5; for large rates of climb, however, the use of equation (3) is recommended.

Examining the equilibrium flight path in wind shear, one finds that it is not rectilinear in the earth-fixed coordinate system. This can be seen by integrating and comparing the equations

$$\dot{z} = \begin{cases} -U_o \sin \gamma_o & \text{(steady wind)} \\ -U_o \sin \Gamma_o & \text{(wind shear)} \end{cases}$$

$$\dot{x} = \begin{cases} U_o \cos \gamma_o - u_w & \text{(steady wind)} \\ U_o \cos \Gamma_o - u_w(z) & \text{(wind shear)} \end{cases}$$

in which a steady headwind of magnitude u_w is assumed and in the case of wind shear, the dependence of the headwind on altitude is expressed by writing $u_w(z)$. It is interesting to compare the trajectory of an airplane in a steady glide into a uniform headwind with the flight path of the same airplane flown in a wind shear. In order to make the comparison more meaningful, let it be supposed that the initial height above the ground is Δz and that at this height there is a headwind of velocity $u_w' \Delta z$ in both cases,

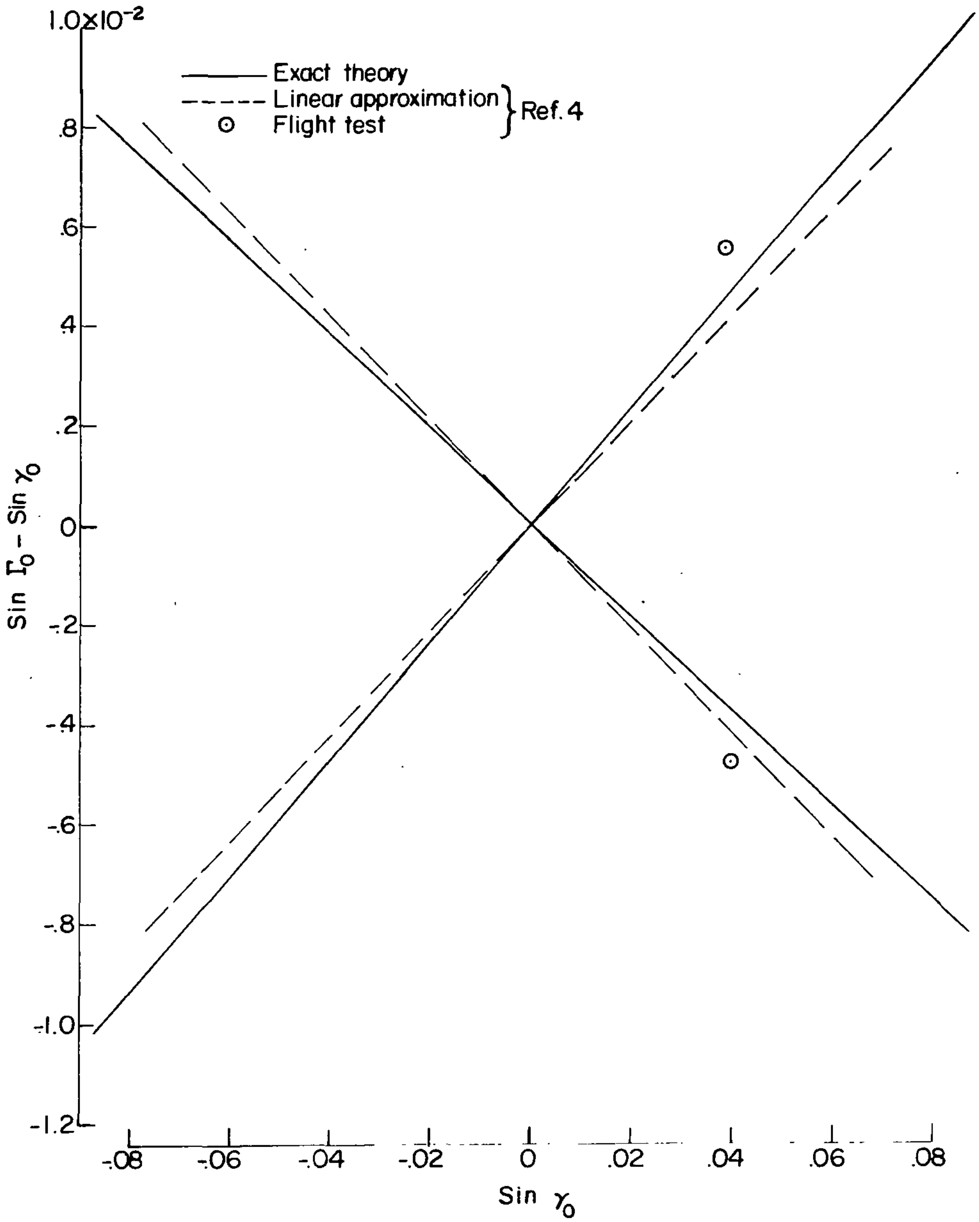

Figure 5.- Net effect of wind shear on the rate of climb.

but in the case of wind shear the wind diminishes to a no-wind condition at ground level. With these assumptions the equation for horizontal velocity becomes

$$\dot{x} = \begin{cases} U_o \cos \gamma_o - u_w' \Delta z & \text{(steady wind)} \\ U_o \cos \Gamma_o + u_w'(z - z_o) - u_w' \Delta z & \text{(wind shear)} \end{cases}$$

This pair of equations can readily be integrated along with the equations for $\dot{z}$, so the horizontal displacement x can be evaluated at both conditions for the same height loss. After some manipulation the following expression is obtained:

$$\Delta x = (x)_{\text{shear}} - (x)_{\text{steady wind}} = \left(\frac{\cos \gamma_o}{\sin \gamma_o} - \frac{\cos \Gamma_o}{\sin \Gamma_o}\right)\Delta z + \frac{g\sigma}{2U_o^2}\left(\frac{1}{\sin \Gamma_o} - \frac{2}{\sin \gamma_o}\right)(\Delta z)^2 \quad (4)$$

Making use of equation (3) results in the simpler form

$$\Delta x = \frac{\sigma}{\sin \gamma_o}\left[\Delta z + \frac{g}{2U_o^2}\left(\frac{\sin \gamma_o}{\sin \Gamma_o} - 2\right)(\Delta z)^2\right]$$

This expression gives the amount of overshoot or undershoot at ground level relative to the flight path in steady wind. Thus, in a headwind decreasing with altitude there is an undershoot ($\Delta x < 0$) as long as the inequality

$$\Delta z < \frac{2U_o^2}{g\left(2 - \dfrac{\sin \gamma_o}{\sin \Gamma_o}\right)}$$

is true. If the height loss during the descent through the shear layer exceeds the right-hand side of the inequality, an overshoot will occur. It is important to note, however, that for moderate values of wind shear there is always an undershoot initially regardless of the thickness of the shear layer.

In the above analysis the undershoot was calculated for the case in which the airspeed, angle of attack, and throttle setting were the same in both the steady wind and the wind shear. It was found that these conditions were possible if the airplane in wind shear assumed a pitch attitude different from the steady-wind value.

Another comparison is possible by requiring that the airspeed, angle of attack, and pitch angle be the same for both the conditions of steady wind and wind shear. This is possible, at least in theory, by having different control deflections and throttle settings under the two conditions. Since now $\Gamma_o = \gamma_o$, equation (4) can be simplified to give

$$\Delta x = (x)_{\text{shear}} - (x)_{\text{steady wind}} = -\frac{g\sigma}{2U_o^2}\frac{(\Delta z)^2}{\sin \gamma_o}$$

This value of Δx in a normal glide is always positive, hence an overshoot will occur.

Figure 6 is a summary of these results for the case of a hypothetical STOL airplane. Thus, the question whether an overshoot or undershoot occurs can be answered by analyzing the method the pilot chooses in making an approach in wind shear. Relative to the steady-wind case, an undershoot will occur if in a wind shear the airspeed, angle of attack, and throttle setting are kept the same. If on the other hand, the pilot changes the throttle and control settings so that the airspeed, angle of attack, and pitch attitude are the same as in the steady wind, an overshoot takes place.

Qualitatively these conclusions remain valid for wind shears with other than the zero surface wind considered here.

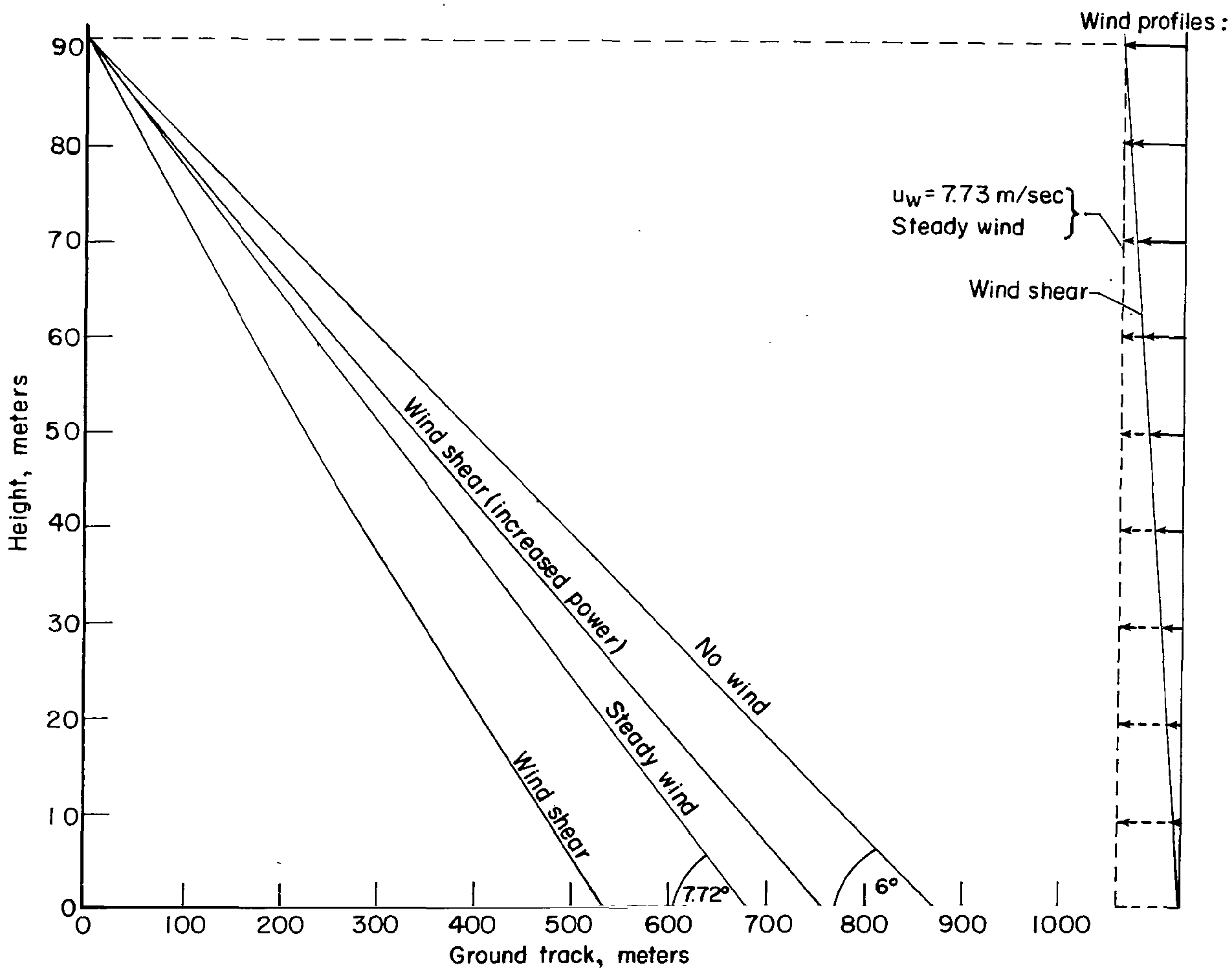

Figure 6.- Approach profiles of a STOL airplane. Controls fixed; airspeed = 36.04 m/sec; wind shear = 0.0845 sec^{-1}.

Wind-Shear Effects on Longitudinal Stability

The assumption of constant airspeed, constant angle of attack, and pitch angle is basic to the concept of trimmed flight. In the preceding analysis it was found that these conditions of trimmed flight in wind shear resulted in a flight path which was not rectilinear for nonzero Γ_o but appeared as a parabolic segment when viewed by an observer on the ground. (It should be pointed out that for the pilot without accurate attitude reference the curvilinear path would appear as the normal condition of equilibrium flight.) The tendency of an airplane to remain on the curvilinear flight path in the presence of disturbances is now examined by the aid of the usual small perturbation theory of airplane stability. Thus, it is assumed that $U = U_o + u$, $\Gamma = \Gamma_o + \gamma$, and $\Theta = \Theta_o + \theta$ in the equations of motion and that the perturbation angles γ and θ are small along with the products of all perturbation quantities. If the aerodynamic and propulsive forces and moments are expanded in the usual manner and the substitution $\alpha = \theta - \gamma$ is employed the linearized equations of motion are obtained as follows:

$$\begin{bmatrix} \frac{d}{dt} - \frac{1}{2}u_w' \sin 2\Gamma_o - X_u & -X_\alpha & g\left(\cos \Gamma_o - \sigma \cos 2\Gamma_o\right) \\ -Z_u - u_w' \sin^2\Gamma_o & -\left(Z_{\dot{\alpha}} + Z_q\right)\frac{d}{dt} - Z_\alpha & -\left(U_o + Z_q\right)\frac{d}{dt} + g \sin \Gamma_o - U_o u_w' \sin 2\Gamma_o \\ -M_u & \frac{d^2}{dt^2} - \left(M_{\dot{\alpha}} + M_q\right)\frac{d}{dt} - M_\alpha & s\left(s - M_q\right) \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ \gamma \end{bmatrix} = \begin{bmatrix} X_\delta \\ Z_\delta \\ M_\delta \end{bmatrix} \delta$$

Stability derivatives appearing in the matrix equation are standard and their definitions are given in the list of symbols. The derivation of the above equation is straightforward, and since it is done entirely within the framework of conventional airplane stability theory it is not given here.

On the right-hand side all control-dependent terms are grouped together. There is seldom a need for the complete form of the above equations; usually the rotary derivatives $Z_{\dot{\alpha}}$ and Z_q are small enough to be neglected. The perturbed motion can therefore be characterized by the stability determinant

$$\begin{vmatrix} s - \frac{1}{2}u_w' \sin 2\Gamma_o - X_u & -X_\alpha & g\left(\cos \Gamma_o - \sigma \cos 2\Gamma_o\right) \\ -Z_u - u_w' \sin^2\Gamma_o & -Z_\alpha & -U_o s + g\left(\sin \Gamma_o - \sigma \sin 2\Gamma_o\right) \\ -M_u & s^2 - \left(M_{\dot{\alpha}} + M_q\right)s - M_\alpha & \dot{s}\left(s - M_q\right) \end{vmatrix}$$

where s is the complex variable. In case of no wind shear the determinant reduces to the conventional longitudinal stability determinant written in terms of the motion variables u, α, and γ.

It will be assumed that even in the presence of wind shear the perturbed longitudinal motion consists of the usual normal modes; i.e., a lightly damped phugoid mode and a well-damped short-period mode. In the latter mode airspeed changes are considered negligible and the motion is approximately characterized by the determinant

$$\begin{vmatrix} Z_\alpha & U_o s - g\left(\sin \Gamma_o - \sigma \sin 2\Gamma_o\right) \\ s^2 - \left(M_{\dot{\alpha}} + M_q\right)s - M_\alpha & s\left(s - M_q\right) \end{vmatrix} = 0 \tag{5}$$

Thus, the presence of wind shear is reflected only in the gravity term in the upper right-hand corner. Since gravity effects are not of great importance in considering the short-period mode regardless of the orientation of the gravity vector, it can be concluded that the influence of wind shear on the short-period mode is a minor one in comparison to the influence of the derivatives $M_{\dot{\alpha}}$, M_q, and M_α.

The long-period or phugoid mode is approximated by assuming that the inertial and aerodynamic damping contributions to the total applied moment are much smaller than those due to changes in u and α. The approximate stability determinant of the phugoid mode is

$$\begin{vmatrix} s - \frac{1}{2} u_w' \sin 2\Gamma_o - X_u & -X_\alpha & g\left(\cos \Gamma_o - \sigma \cos 2\Gamma_o\right) \\ -Z_u - u_w' \sin^2\Gamma_o & -Z_\alpha & -U_o s + g\left(\sin \Gamma_o - \sigma \sin 2\Gamma_o\right) \\ -M_u & -M_\alpha & 0 \end{vmatrix} = 0$$

Or, in an expanded form the determinant may be written as

$$s^2 + 2\zeta_p \omega_p s + \omega_p{}^2 = 0 \tag{6}$$

where

$$2\zeta_p \omega_p = -X_u + M_u \frac{X_\alpha}{M_\alpha} - \frac{g \sin \Gamma_o}{U_o}\left(1 + \sigma \cos \Gamma_o\right)$$

and

$$\omega_p{}^2 = \frac{g}{U_o}\left(\sin \Gamma_o - \sigma \sin 2\Gamma_o\right)\left(X_u - M_u \frac{X_\alpha}{M_\alpha}\right) - \frac{g}{U_o}\left(\cos \Gamma_o - \sigma \cos 2\Gamma_o\right)\left(Z_u - M_u \frac{Z_\alpha}{M_\alpha}\right)$$

The time required for the phugoid to damp to half amplitude increases in a climb and decreases in a dive as expected. Positive wind shear ($\sigma > 0$) is seen to amplify this effect. For sufficiently large positive values of the wind-shear parameter, $\omega_p{}^2$ becomes negative, regardless of the sign of Γ_o, and the subsequent motion is characterized by two real roots of equation (6). One of these roots can become positive indicating a diverging aperiodic mode. It is interesting to note that wind shear affects the phugoid mode even in level flight ($\Gamma_o = 0$). The level flight case was used to verify the assumption of the two normal modes for a typical light aircraft whose aerodynamic characteristics and parameters appear below:

$$X_u = -0.0451 \text{ sec}^{-1}$$

$$Z_u = -0.3697 \text{ sec}^{-1}$$

$$M_u = 0 \frac{1}{\text{m-sec}}$$

$$X_\alpha = -7.8729 \frac{\text{m}}{\text{rad-sec}^2}$$

$$Z_\alpha = -108.79 \frac{\text{m}}{\text{rad-sec}^2}$$

$$M_\alpha = -8.811 \frac{1}{\text{rad-sec}^2}$$

$$M_{\dot{\alpha}} = -0.90904 \frac{1}{\text{rad-sec}}$$

$$M_q = -2.0767 \frac{1}{\text{rad-sec}}$$

$$U_o = 53.64 \text{ m/sec}$$

$$\Gamma_o = 0 \text{ rad}$$

This set of data was used to solve for the roots of the characteristic equation associated with the above 3×3 determinant as well as for the roots of the approximate equations (eq. (5)) for the short period and phugoid modes while allowing σ to vary between -2 and +2. The short-period roots were found to be approximately $-2.51 \pm 2.60i$ with negligible variations due to changes in σ. The short period approximation yielded roots very close to the exact values. The location of the exact and approximate phugoid roots are shown in figure 7. For values of σ greater than 1 an unstable root appears; for the particular airplane this would happen if the wind shear exceeded the rather unlikely value

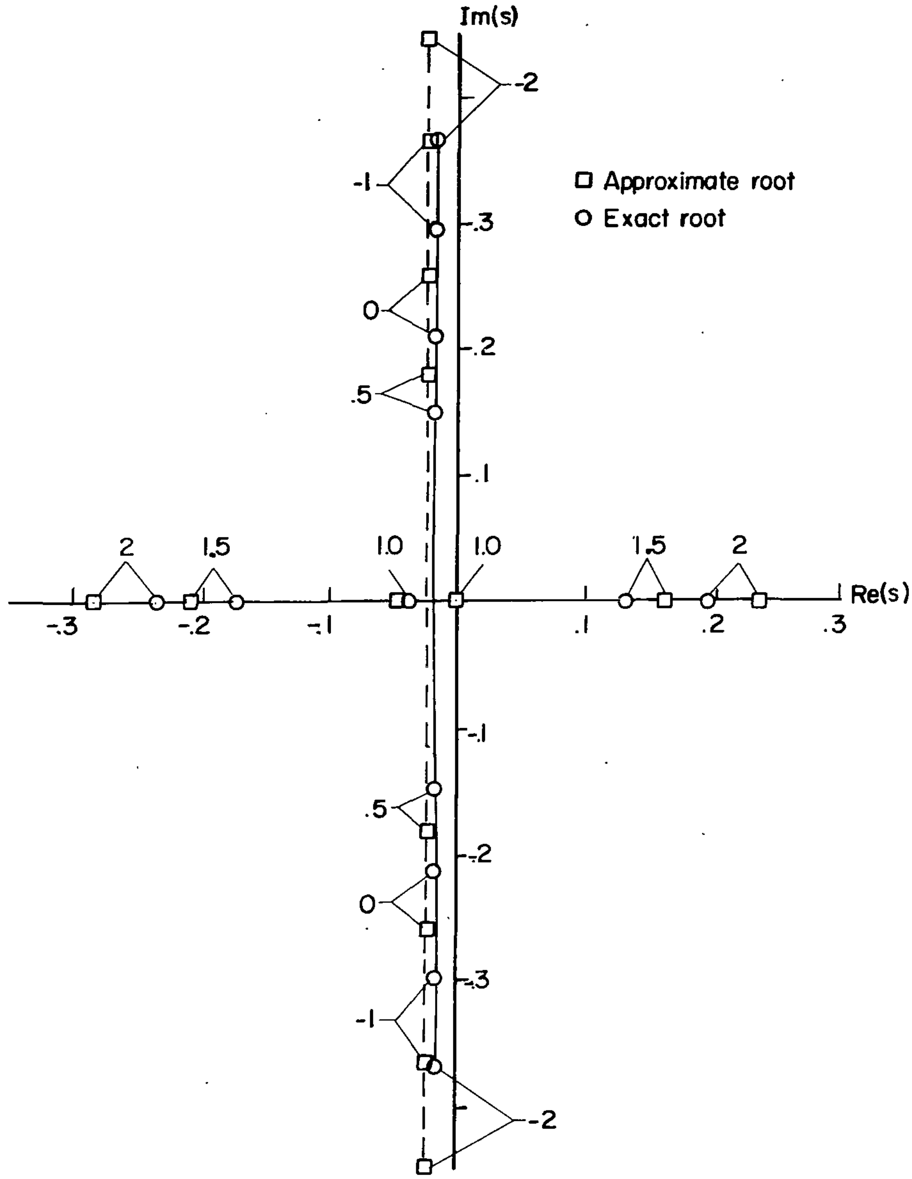

Figure 7.- Root locus plot showing the effect of wind shear on the phugoid roots.

of 0.183 sec^{-1}. It should be noted, however, that for higher reference velocities or at positive values of Γ_0 phugoid roots would appear at lower values of u_W'. For certain configurations and flight conditions the appearance of this diverging first-order root may be important and warrant an analysis.

CONCLUDING REMARKS

A study made to assess the influence of wind shear on the longitudinal dynamics of airplanes found that a linear variation of wind with altitude affects the longitudinal motion of the airplane in two ways. First, the path flown at constant airspeed, pitch angle, and fixed controls is not rectilinear with respect to the ground but rather it is a parabolic path. Second, the longitudinal stability characteristics of the airplane about the reference conditions vary as a function of wind shear.

Associated with the first effect is the apparent ability of the airplane to climb faster in an increasing headwind in comparison to flight in still air at the same airspeed, angle of attack, and power setting. Similarly, glide in decreasing headwind results in a higher rate of sink than what would be experienced in uniform wind or still air. If an airplane is trimmed during an approach in headwind decreasing linearly with altitude, it would normally touch down short of the point it would reach if the wind had remained at its initial value. It is shown in the report that an overshoot could occur if the shear layer were thick enough. An overshoot takes place, also, regardless of the thickness of the shear layer, if the pilot uses the same airspeed, angle of attack, and pitch attitude that he would normally establish for a steady- or no-wind condition. It follows from these results that some elevator and throttle activity would be necessary to "rectify" the curvilinear path of trimmed flight as would be required, for instance, on the glide slope of an instrument landing system.

Longitudinal stability of the airplane is influenced by wind shear even in level flight. The results of the investigation show that while wind shear has negligible effects on the short-period mode and the phugoid damping, the frequency of the phugoid motion and the damping ratio vary considerably with wind shear. In the situation of increasing headwind or decreasing tailwind with altitude the phugoid mode may become unstable if the nondimensional wind-shear parameter is sufficiently large.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., August 9, 1971.

REFERENCES

1. Gorham, J. A.; Richter, H. K.; and Lee, F. B.: Where Are We Now With All-Weather Landing? AIAA Paper No. 67-572, Aug. 1967.

2. Henley, M. G.: Tolerances in Automatic Landing: An Analysis of the Effect of Equipment Tolerances, Changes in Aircraft Configuration and Variations in Wind Conditions. Radio Electron. Eng., vol. 29, no. 6, June 1965, pp. 378-386.

3. Thompson, F. L.; Peck, W. C.; and Beard, A. P.: Air Conditions Close to the Ground and the Effect on Airplane Landings. NACA Rep. 489, 1934.

4. Bikle, Paul F.: The Effect of a Vertical Gradient in Wind Velocity on Climb Tests. Mem. Rep. No. TSFTE-1981, U.S. Army, Apr. 26, 1946.

5. Etkin, Bernard: Effect of Wind Gradient on Glide and Climb. J. Aeronaut. Sci., vol. 14, no. 6, June 1947, pp. 365-367.

6. Neumark, S.: Dynamic Longitudinal Stability in Level Flight, Including the Effects of Compressibility and Variations of Atmospheric Characteristics With Height. Rep. No. Aero.2265, Brit. R.A.E., May 1948.

7. Zbrożek, J. K.: Aircraft Behavior in a Vertical Gradient of Wind Velocity. Aeronaut. Quart., vol. XIV, pt. 3, Aug. 1963, pp. 265-278.

8. Ashkenas, Irving L.: Effects of Atmospheric Gradients on Longitudinal Control. Analysis of Several Handling Quality Topics Pertinent to Advanced Manned Aircraft, AFFDL-TR-67-2, U.S. Air Force, June 1967, pp. 1-27.

9. Browne, J. A.: Wind Shear Effects on Air Speed. Tech. Rep. 163, AWS Sci. Serv., U.S. Air Force, Mar. 1962.

10. Snyder, C. Thomas: Analog Study of the Longitudinal Response of a Swept-Wing Transport Airplane to Wind Shear and Sustained Gusts During Landing Approach. NASA TN D-4477, 1968.

11. Melvin, William W.: Effects of Wind Shear on Approach With Associated Faults of Approach Couplers and Flight Directors. AIAA Paper No. 69-796, July 1969.

12. Brook, R. R.; and Spillane, K. T.: On the Variation of Maximum Wind Gusts With Height. J. Appl. Meteorol., vol. 9, no. 1, Feb. 1970, pp. 72-78.

NASA-Langley, 1971 —— 2 L-7671

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546

OFFICIAL BUSINESS
PENALTY FOR PRIVATE USE $300

FIRST CLASS MAIL

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION

001 001 C1 U 02 710903 S00903DS
DEPT OF THE AIR FORCE
AF SYSTEMS COMMAND
AF WEAPONS LAB (WLOL)
ATTN: E LOU BOWMAN, CHIEF TECH LIBRARY
KIRTLAND AFB NM 87117

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

— NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546