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STATUS REPORTS ON CONTROL OPTIMIZATION, STABILIZATION AND COMPUTER ALGORITHMS FOR SPACE APPLICATIONS

Report ESL-SR-446
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Ninth and Tenth Status Reports

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Ninth Status Report for Period 1 March to 1 September 1970
Tenth Status Report for Period 1 September 1970 to 1 March 1971

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TABLE OF CONTENTS

Page

Ninth Status Report

1. General 1
2. Technical Accomplishment 1
3. Project Documentation 2
4. Budget Status 2

Tenth Status Report

1. General 3
2. Technical Accomplishment 4
   - 2.1 Estimation and Control for Uncertain Systems 4
   - 2.2 Stochastic Stability with Applications to the Study of the Human Operator 13
   - 2.3 Controllability of Infinite Dimensional Systems 16
   - 2.4 Deterministic Compensator Design 17
   - 2.5 Air Traffic Control 19
3. Project Documentation 24
4. Budget Status 24
5. Bibliography 25

NINTH STATUS REPORT

1. GENERAL

This status report covers research done during the period 1 March to 1 September 1970 under Research Grant No. NGL-22-009-124 by faculty members, research assistants, and graduate students associated with the Decision and Control Sciences Group of the Electronic Systems Laboratory.

During the above period the following people contributed to the project: Professors M. Athans, I. B. Rhodes, and J. C. Williams (all part time); Research Assistants (full time) J. H. Davis and H. P. Geering. In addition, the following graduate students, without salary support from this project, contributed to it: G. Blankenship, T. L. Johnson, L. Kramer, and L. W. Porter.

2. TECHNICAL ACCOMPLISHMENT

The research accomplished during this reporting period is adequately described in the "Technical Proposal for Continuation of Research on Control Optimization, Stabilization and Computer Algorithms for Space Applications," June 1970, and other documents listed in the next section.

3. PROJECT DOCUMENTATION

The following documents have been submitted as part of the technical reporting under this grant:

Joseph, Keith M. "On the Design of Optimal and Suboptimal Feedback Systems." M.I.T. Electronic Systems Laboratory Report ESL-R-421, March 1970.

Mitter, S. K. and R. Foulkes. "Controllability and Pole Assignment for Discrete Time Linear Systems Defined Over Arbitrary Fields." M.I.T. Electronic Systems Laboratory Report ESL-P-424, May 1970.

Athans, M., S. K. Mitter, and G. C. Newton, principal investigators. "Technical Proposal for Continuation of Research on Control Optimization, Stabilization and Computer Algorithms for Space Applications," June 1970.

Athans, Michael. "On the Design of P-I-D Controllers Using Optimal Linear Regulator Theory." M.I.T. Electronic Systems Laboratory Report ESL-P-432, July 1970.

All of the above research has been supported wholly or in part by this grant.

4. BUDGET STATUS

As of 30 August 1970, approximately 104.5 per cent of the grant funds have been expended in 100 per cent of the grant period. No effect has been given to step funding in these figures.

TENTH STATUS REPORT

1. GENERAL

This status report pertains to research progress made under Research Grant No. NGL-22-009-124 during the period 1 September 1970 to 1 March 1971 by faculty members, research assistants, and graduate students comprising the research staff assigned to this project. Progress was made in the five areas reported on in the next section.

During the report period the following people contributed to this research: Professors M. Athans and S. K. Mitter, both part time (G. C. Newton was on leave). In addition, the following people contributed to this research without requiring salary support from this grant: instructor L. Kramer; teaching assistants H. P. Geering and A. H. Sarris; graduate students D. P. Bertsekas, G. L. Blankenship, C. Chong, T. L. Johnson, S. H. Lin, and H. F. Vandevenne. The reduced salary support level during this period was necessitated by the need to bring the expenditures for the project into line with its funding.

2. TECHNICAL ACCOMPLISHMENT

2.1 Estimation and Control for Uncertain Systems

A. Estimation Techniques

A.1 Background

Techniques for estimating state variables and parameters of linear and non-linear dynamical systems on the basis of uncertain or noisy observations are of fundamental importance to a whole variety of engineering applications. We have commented on the application of these ideas when we considered radar tracking and resource allocation problems.

In this section we outline our research in this area.

A.2 Observer-Estimator Theory

There are many problems in which some measurements are quite accurate, while others are relatively inaccurate. For example, in advanced phased-array radars the range measurement is much more accurate than the corresponding azimuth and elevation measurements.

Such problems provided the motivation for reexamining the issue of stochastic estimation when some of the measurements are noisy and when some of the measurements are modeled as noiseless. It is well known that when all measurements are noise-free, one can use a Luenberger-type observer. On the other hand, when all measurements are noisy, then one can use a Kalman filter.

The problem of dealing with a mixture of noisy and noise-free measurements was initially considered in the Ph.D. dissertation of Tse Subsequently, the results were refined. For discrete time systems, the results obtained are summarized in the paper by Tse and Athans [T2]. The equivalent results for the continuous time case can be found in another paper by Tse and Athans [T3].

The basic results obtained in this study can be briefly summarized as follows: given an nth order linear dynamical system with α noisy measurements and β noiseless measurements, then one can construct an optimal state estimator whose order is $n-\beta$. The complete mathematical characterization of such an observer-estimator can be found in the above-cited references.

A.3 Set Theoretic Description of the Uncertainty

The usual assumption about the uncertainty associated with inaccuracies in measurements is that it can be modeled by a probabilistic noise process (white or colored). On the other hand, certain types of radar and sonar problems give rise to a different characterization of the measurement errors. One can assume that the measurement errors belong to some set and that no probabilistic information is available.

Such problems have been examined in some detail. The technical results can be found in the paper by Bertsekas and Rhodes [B2] and in the forthcoming (in June 1971) doctoral thesis of Bertsekas. In the above research effort emphasis has been placed on problems in which the uncertain quantities are unknown except for their membership in given sets. The two types of sets that have received most attention are an energy-type constraint on the uncertain quantities and instantaneous constraints on the uncertain quantities. Within this framework, attention has been directed principally toward the estimation problem in which one wishes to characterize the set of possible system states consistent with the observed output. In the case of energy constraints, this problem has been solved by reducing it to a standard tracking (or servomechanism) problem of optimal control theory, and recursive estimators that are similar in form to the corresponding stochastic Kalman filters have been found for the filtering problem, the prediction problem, and the smoothing problem. These results have then been used to derive ellipsoidal bounds to the set of possible states consistent with the observed output in the case of instantaneous constraints on the uncertain quantities. The resulting recursive estimator is again formally identical to a stochastic Kalman filter. It is also similar to that proposed by Schweppe but has a major practical advantage that the estimator gains do not depend on the entities generated in any particular sample run and may be precomputed. In addition, it permits a steady-state solution. In all other respects this estimator and that due to Schweppe are comparable.

B. Stochastic Control and Compensation Methods

B.1 Background

The control of linear and non-linear dynamical systems on the

basis of noisy measurements and subject to unknown or stochastic disturbances is one of the most fundamental problems in the analysis and design of aerospace systems. The problem in particular appears in

a) autopilot design

b) aircraft autopilot design

c) helicopter autopilot design

d) space shuttle control system design

as well as for most other servomechanism oriented applications.

Modern control and estimation theory has had a major impact upon design techniques for such problems. The tutorial paper by Athans [X] contains an extensive discussion of the philosophy, techniques, theory, and trends in this exciting area of research and applications. Our research in the area of stochastic control has proceeded simultaneously along several directions.

In any control system uncertainty is always more or less present in different forms. For example, the input signals may be different from the precomputed ones, the measurements performed on the output of the system may be inaccurate, the parameters of the system may be imperfectly known, etc. In many cases this uncertainty cannot be neglected and must be explicitly taken into account in the design of the system.

The most often used approach in describing the uncertainty in a control problem is a probabilistic one; i.e., all uncertain quantities are modeled as random variables with known statistics, and one seeks the control for which the expected value of a suitable cost functional is minimized. This formulation yields the control law which is best "on the average" with respect to the cost functional under consideration. This approach has yielded some elegant and useful results, particularly for linear systems, notable cases being the Kalman filter and the separation theorem for linear systems and quadratic criteria. The disadvantage of this approach is that it is necessary for the designer to know the a priori statistics of all the quantities involved. This is quite often the source of arbitrary assumptions, since the uncertain quantities may not have a stochastic nature at all, or, if they do, their statistics may be unknown to the designer. In such cases a different model for uncertainty may be

more appropriate. Such a model is the case where the uncertain quantities are considered unknown, except for the fact that they belong to given sets. This "unknown but bounded" model has been studied, and practical schemes for estimation and control of uncertain systems have been developed for the case of linear systems.

Still, however, the subject of the control of uncertain systems remains largely unexplored. Only a small number of problems, particularly those involving linear systems, has been sufficiently investigated. On the other hand, it is necessary to evaluate the merits and drawbacks of the two approaches, "stochastic" and "unknown but bounded", as regards practical design problems and to delineate their similarities. Another area involves the simultaneous online identification and control of linear and non-linear systems with unknown structure and/or parameters. This is often called adaptive stochastic control.

B.2 Stochastic Control for Bounded, but Unknown, Uncertainty

The followup to the estimation problem using a bounded but unknown description of the uncertainty (see Section A.3) has been pursued by Bertsekas, Rhodes, and Mitter. The control problem consists in driving the state of the system to a prescribed target set or keeping the state trajectory within a "tube" for all time and in the presence of disturbances. The paper by Bertsekas and Rhodes[B.1] contains the technical details on this class of problems. The forthcoming doctoral thesis of Bertsekas (June 1971) will contain more details.

B.3 Stochastic Control for Mixed Noisy and Noise-free Measurements

This problem occurs when (see also Section A.2) some measurements are exact while others are noisy. The stochastic control problem under the linear-quadratic-gaussian assumptions has been formulated. It was found that the separation theorem still holds. The details can be found in the doctoral thesis by Tse

B.4 Adaptive Control

By the term "adaptive control" we refer to the problem of controlling a dynamical system, subject to stochastic disturbances, with noisy measurements, and which, in addition, has a number of unknown parameters which may also vary in a stochastic manner.

The ability to control a partially unknown system subject to stochastic disturbances and errors is a fundamental problem that arises in a multitude of aerospace applications. Indeed, the major impetus for research on adaptive systems has been provided by problems encountered in autopilot design for high performance aircraft. The fact that such aircraft undergo rapid changes in flight conditions, results in changes in the values of their aerodynamic derivatives which are by no means small. Hence, a standard autopilot, i.e., one that has been designed to operate only for one set of flight conditions, cannot be expected to yield satisfactory performance. Therefore, adaptive autopilots are of primary importance.

The current state of stochastic estimation, non-linear filtering, and stochastic control has matured enough so that advances in realistic adaptive control system design are possible. Our research efforts have been focused on understanding the implications of using online stochastic adaptive optimal and suboptimal control algorithms.

The initial class of problems which was considered dealt with the control of linear discrete time stochastic systems with unknown and stochastically varying control gains. Roughly speaking, such problems arise when one considers the sampled data version of linear systems with known poles but with

a) unknown d-c gain,

b) unknown number of zeros,

c) unknown zero location.

This class of problems has been investigated in detail, both from a theoretical and a simulation point of view. The technical details can be found in Tse's thesis [T1] the two papers by Tse and Athans [T4] [T5] and the abbreviated version [T6] of the latter references. The basic results of these investigations were to show that the gains of the adaptive control system were modulated by the current and projected uncertainty (covariance matrices) of the parameter estimation. Thus, the usual separation theorem did not hold. It was also found that the behavior of the adaptive control system was drastically different for stable and unstable systems. Furthermore, it was found that excellent control can result even if the identification algorithm did not identify the parameters accurately.

At the present time we are actively pursuing the extension of the above results to a broader class of more realistic problems from an applications viewpoint.

B.5 Autopilot Design for Large Flexible Aircraft

A specific application of the theory of optimal stochastic control that was carried out was concerned with the design of an autopilot for the C-5A aircraft. In this study, in addition to the rigid dynamics, one must pay special attention to the wing flexure modes. The main criterion of performance was the minimization of stresses in critical points along the aircraft fuselage and wings. Flexure control was accomplished by pairs of ailerons; the optimal location of the ailerons along the wing was also determined. The technical details of this study can be found in the paper by Johnson, Athans, and Skelton [J1].

The above study is typical of a variety of recent applications of modern control theory to compensator design. As explained in more detail in reference [A1] the major advantage of modern control theory is that once the modeling and performance index are specified by the control system designer, the calculations can be readily carried out by a digital computer. In fact, the report by Athans [A2] shows how an automated algorithm can compute the transfer function of the required compensator starting from the transfer function of the plant. Hence such a digital computer program accepts "inputs" and yields "outputs" in the frequency domain, although all internal calculations are carried out using state variable representations.

C. Differential Games and Vector-Valued Criteria

C.1 Background

One of the most exciting areas of research at the present time is focused upon understanding the behavior of systems with multiple controllers and with, perhaps, conflicting performance criteria. This is an attempt to enlarge the available theoretical and algorithmic techniques which have been developed for systems with a single controller and with a scalar index of performance.

There exists a large class of engineering and non-engineering systems for which one must develop techniques for overall coordination of individual objectives. Some remarks on this class of problems were pre-

sented in Reference [R1]. However, there are many appropriate formulations of problems of this type. In this section we present two distinct approaches that we have examined.

C.2 Non-Scalar-Valued Performance Criteria in Optimal Control

One of our continuing projects deals with the theory of optimal control theory for non-scalar-valued cost functionals. We have been able to find an infimum principle, which is analogous to the well known Pontryagin minimum principle in optimal control theory for scalar-valued cost functionals. As an interesting application, these results were used to rederive the Kalman-Bucy filter with infimal error covariance matrix at the final time. These results can be found in the recent paper by Geering and Athans [G1].

Optimality usually refers to a performance criterion, which is a scalar-valued function of the parameters (controls, states, gains, etc.) of the optimal control or estimation problem, and which has to be minimized. It is often difficult to choose a suitable scalar-valued optimality criterion, because several things may simultaneously be of interest, such as energy consumption, transfer time, and integral-squared error in optimal control problems, or the mean error vector and the whole covariance matrix in optimal estimation problems.

It then is natural to consider non-scalar-valued performance criteria, e.g., vectors, matrices, sets, etc. The meaning of "better than" has to be defined, which mathematically speaking is done by introducing a partial order relation in the space of the cost function (see B.Z. Vulikh, "Introduction to the Theory of Partially Ordered Spaces," 1967, or G. Birkhoff, "Lattice Theory," 3rd ed., 1967). For non-scalar-valued performance criteria, the notion of optimality splits up into superiority and non-inferiority, because "better than" is not the complement of "worse than." Thus, a superior solution (or infimum) is "better than" all other solutions, whereas a non-inferior solution (or minimum) is not "worse than" any other solution.

L. A. Zadeh was the first researcher considering vector-valued cost criteria (IEEE Trans. Automatic Control AC-8 (1963), 59-60). In this case, a superior solution minimizes all of the cost components, whereas for non-inferior solutions, no other solution can be found which

simultaneously improves all of the cost components. Based on Zadeh's correspondence item, S. S. L. Chang (SIAM J. Control 4 (1966), 46-55), D. H. Chyung (SIAM J. Control 5 (1967), 345-351, and N. O. Da Cunha and E. Polak (J. Math. Anal. Appl. 19 (1967), 103-124) have given necessary conditions in the form of a minimum.

The costate now is a linear map from the (extended) state space into the cost space. The Hamiltonian attains its values in the cost space and has to be infimized by the superior control for all times along the optimal state trajectory and all costate maps whose component of the final time boundary condition mapping the cost space into itself is a positive map.

At the present time the research efforts are concentrated on the problems of existence of superior solutions and sufficiency of these conditions. It is expected that the investigation of these problems will parallel those for scalar-valued cost functionals to a high degree, except that now the notion of convexity has to be suitably redefined.

Another topic of further research will be the investigation of the cross-connections between this optimization theory for non-scalar-valued performance criteria and the theory of differential games.

These extensions will be the major topic of Geering's Ph.D. dissertation (September 1971).

C.3 Differential Games

Another class of problems that involve multiple cost criteria is that of differential games. The paper by Rhodes [R 1] summarizes the major part of our research in this area. What was considered was a class of deterministic and stochastic non-zero sum games (two-person) with linear dynamics and quadratic cost functionals.

This research was concerned with two-person non-zero-sum linear differential games with quadratic cost functionals. First, we investigated deterministic non-zero-sum games and provided some sufficient conditions for the existence of a solution. Second, we examined the corresponding stochastic problem where the initial state is a Gaussian random vector and the output measurements are corrupted by white Gaussian noises. Attention was directed primarily to the special case in which one controller

has no output measurements and is therefore restricted to using only the a priori information available to him. It was shown that, in contrast to the corresponding zero-sum situation, the optimum controls are not simply related to those given by a separation theorem. This fact was discussed in terms of the difference between the optimum closed-loop solutions and the optimum open-loop feedback solutions to the deterministic non-zero-sum game.

In addition, we reviewed the formulation and solution of a two-person non-zero-sum deterministic linear differential game with quadratic cost functionals. In particular, we emphasized the essential distinction between the optimum closed-loop solution and the optimum open-loop feedback solution, a difference that is of interest in our later interpretation of the solution to the stochastic game. Then we reviewed the linear system-quadratic cost deterministic zero-sum game and discussed sufficient conditions for the existence of a solution. These conditions were used to provide sufficient conditions for the existence of a solution to the original non-zero-sum game by relating its solution to those of certain zero-sum games. Finally we formulated a general stochastic differential game involving a linear system, quadratic cost functionals, and independent white Gaussian noises additively corrupting the output measurements. This was followed by an examination and discussion of the special case where one controller has no output measurements and is therefore restricted to using only the a priori information available to it. Included is a comparison of the solution to that given by a separation theorem and an interpretation of the optimum controls in terms of the optimum closed-loop and open-loop feedback strategies for the corresponding deterministic game.

D. Numerical Solution of Optimal Control Problems

The main investigation has been concerned with the stability and convergence of discretization schemes for optimal control problems for systems described by ordinary and partial differential equations. Preliminary reports of this work have been presented in the JACC Conference last year and in the Hawaii International Conference in January of this year.

For linear dynamical systems (finite or infinite dimensional) with a quadratic or convex cost function and with inequality constraints on the control variable, it can now be proved that certain discretization

schemes for the control problem itself are both stable and convergent (to the solution of the continuous optimal control problem). A detailed report on this work will be available in the near future. Work on extending this analysis to certain classes of non-linear systems and to problems involving state variable constraints is now in progress and we expect to perform numerical calculations for certain VSTOL aircraft problems.

It is worth pointing out that these results have the following practical significance: when sophisticated minimization techniques, such as the conjugate gradient and variable metric methods, are used to solve optimal control problems, it is necessary to pay some attention to the question of discretization. If this is not done, the integration of the adjoint equation will be inaccurate, resulting in inaccurate gradient computations and poor convergence [M1].

E. Minimization Algorithms for Non-differentiable Functions

In problems involving worst-case design (min-max problems) one is faced with the problem of minimizing a function which is not differentiable. A convergent algorithm for the minimization of such functions was presented [M2]. We hope to do some numerical computations in the near future.

2.2 Stochastic Stability with Applications to the Study of the Human Operator

A. Work on Stability of Uncertain Systems

This research has been concerned with the analysis of the asymptotic properties of Feedback systems subjected to stochastic perturbations. The possibility of random parameters in the system itself was not excluded. Moreover, the avoidance of Lyapunov arguments (used exclusively in the past) was taken as an a priori condition. Thus, the natural vehicle for the analysis became equations like

$$u(t,\omega) = e(t,\omega) + (Ge)\,(t,\omega)$$

where u represents the input (stochastic) process, e the output process, t time, ω the random event, and G the system operator (possibly random).

The stability properties of deterministic equations of the above species are well understood. By "stability" we mean that bounded inputs (u) produce bounded outputs (e), and "bounded" is to be understood in a functional sense. An equivalent statement is that I + G as an operator have a bounded inverse on the function space of interest; criteria for the existence of such an inverse then become stability criteria.

In the stochastic setting, stability is a somewhat less intuitive concept, as may be surmised from the numerous definitions in terms of sample paths and moments offered in the past. Though it is relatively easy to give moment bounds, using the analog of the deterministic input-output stability theory, in this research the issue of which stochastic property of the system signals to consider has been overcome by considering the probability distributions of the random processes involved directly. Using some recent work on the convergence of probability distributions, it is possible to prove the following general result:

Theorem: Assume the following:

(i) For every ω $e(\cdot, \omega)$, $u(\cdot, \omega)$ are continuous functions

(ii) $E\|u(t) - u(s)\|^2 \leq \gamma |t-s|^2$ For some $\gamma > 0$

(iii) $E\|(Ge)(t) - (Ge)(s)\|^2 \leq \alpha E\|e(t) - e(s)\|^2$
For some $\alpha > 0$

(iv) u is a stationary process (or more generally the distributions of u form a compact set in some well-defined sense).

Then a sufficient condition for e to be asymptotically stationary (or more generally for the distributions of e to form a compact set) is that $\alpha < 1$.

This result is thus the complete analog of the "Small Gain Theorem" of deterministic input-output stability theory. That is, $u = (I + G)e$ is stable as discussed above if $\|Ge\| / \|e\| < 1$ for every e.

The constant α is relatively easy to evaluate; consider the

linear equation

$$u(t,\omega) = e(t,\omega) + \int_0^t g(t-\tau)\, e(\tau,\omega)\, dK(\tau,\omega)$$

representing a linear system with white noise ($\dot{K}$) multiplication feedback. In this case

$$\alpha = \frac{\sigma^2}{2\pi} \int_{-\infty}^{\infty} |G(j\nu)|^2 \, d\nu$$

when

$$G(j\nu) = \int_0^{\infty} g(t) e^{-j\nu t} \, dt$$

and σ^2 is the variance constant of $\dot{K}$.

Moreover, the condition is necessary as well in this case, and for this reason it may be regarded as an analogy of the Nyquist criterion in a stochastic setting.

By replacing $e(\tau)$ by $f(e(\tau))$ in the integrand above, it is possible to prove a result akin to the deterministic "Circle Theorem". That is, the distribution of e is ultimately stationary if the Nyquist locus of

$$W(j\nu) = \int_{-\infty}^{\infty} G(j\nu - j\nu_0)\, G(j\nu_0)\, d\nu_0$$

lies outside a certain disk in the complex plane determined by f. Moreover, it is possible to consider noise processes more general than $\dot{K}$ and thereby account for random phenomena discontinuous in nature.

In summary, in this research we have analyzed the asymptotic properties of stochastic processes determined by feedback systems driven by noise. By using input-output techniques, we are able to avoid Lyapunov arguments and treat non-Markovian signals. In the specific cases examined, the criteria are explicit conditions in the frequency domain.

B. The Human Operator as a Stochastic Operator

One of the most intriguing and important examples of a system that may be modeled as a random operator in a feedback system is the human as a feedback controller. In a control task the human may be modeled as a cascade of a random sampler, a random observation gain, an estimator, a delay, and a final gain. The sampler accounts for the tendency of the human to observe at random instants of time. The random gain, which may be replaced by additive observation noise, reflects the human's tendency

to make observation errors. The estimator accounts for the human's judgment of the variable to be controlled and his tendency to optimize his assessment. The delay is a lumped quantity accounting for neuromotor delays and other effects. The final gain accounts for errors in execution of the control action.

Thus, in first order analysis the human may be represented as a random sample and hold, a linear deterministic element, a delay, a stochastic gain, and, in the crudest approximation, simply as a white noise multiplicative gain. The reasons for assuming stochastic variations in the human control actions are to account for the essentially random features of human behavior, like the observation sampling, and to escape the impossibility of any one model accounting for the individual nature of different human operators.

It is easy then, with models incorporating these simple elements, to approximate the human behavior in a control loop. In particular, stability of a steady-state control task is particularly easy to handle, because the magnitude of perturbations may be assumed small and the model determined accurately about this equilibrium.

Details of this work will appear in a forthcoming technical report.

2.3 Controllability of Infinite Dimensional Systems

The objective of this research is to develop a theory of controllability for parabolic and second-order hyperbolic systems and for hereditary differential systems. An attempt is made to use abstract semigroup theory and spectral theory to develop the necessary results. It is proposed to apply these ideas to the stabilization problem of high speed aircraft in the presence of severe gust load disturbances.

In contrast to finite dimensional systems, autonomous infinite dimensional systems exhibit some fundamental differences in its controllability aspects. These are:

(i) the set of states attainable from the origin in time $[0,T]$ need not coincide with the set of states that can be driven to zero in $[0,T]$;

(ii) the range of the solution operator need not be closed;

(iii) the notion of analyticity, which says that if an initial state can be driven to the origin in time $0,T$ then it can be driven to the

origin in time $[0,\bar{t}]$, $\forall\, t > 0$, may not hold.

Moreover in a distributed parameter system, control may be exercised in a distributed manner, pointwise or through the boundary, and one has to take into account explicitly these various possibilities.

Various explicit results on necessary and sufficient conditions for approximate controllability of diffusive and delay systems have been obtained. These results make use of the spectral characteristics of the unbounded system operator. Similar results are now being sought for wave-type systems and neutral functional differential equations. The relationship of these results to the problem of stabilizability is being investigated. Details of these results will be presented in the future.

2.4 Deterministic Compensator Design

A. Background

In spite of many advances in the design of compensators through the use of Kalman filters and Luenberger observers in conjunction with the separation theorem, there is still a host of interesting problems that remain in the area of time-invariant compensator design for linear deterministic systems. As we have mentioned before, techniques for compensator design are of paramount importance in a large number of aerospace applications in which simple analog-type compensators are required for designing aircraft and missile autopilots.

B. Fixed-Order Compensator Design

The primary motivation for research in this area arises from the fact that the minimal dimensionality of the compensators obtained from the use of Kalman filters or Luenberger observers is fixed as a function of the system order and the number of noiseless measurements. Since dimensionality is often directly proportional to complexity, the control designer may naturally want compensators of as low an order as possible, perhaps at the expense of optimality.

It is well known from classical servomechanism design theory that relatively high-order single-input, single-output plants can be stabilized and yield satisfactory performance with a relatively low-order compensator. Such design techniques, however, are not available for multi-input, multi-output plants.

Our research in this area was directed toward developing a set of design techniques which can be used to obtain a compensator whose order can be specified a priori by the designer. What we sought is an algebraic characterization of the parameters of a dynamic compensator, which is optimal in a certain well-defined sense, and which can be used for the design of feedback control systems for multi-input, multi-output linear time-invariant systems.

The technical details for such a design technique can be found in the paper by Johnson and Athans [J2]. It turned out that the parameter matrices of the fixed-order feedback compensator must satisfy a set of three simultaneous non-linear algebraic matrix equations (similar in structure to the algebraic Riccati equation). Furthermore, it was shown that the structure of the algebraic equations is similar to those found for the so-called output-feedback or limited-state variable feedback problem; these results can be found in the paper by Levine, Johnson, and Athans [L1].

C. Design of Proportional-Integral-Derivative Controllers

This research effort was motivated by the fact that there is a lack of well-understood techniques for the design of optimal linear-quadratic-type control systems when the desired input to be tracked is a step, ramp, or polynomial-type time function. For single-input, single-output systems, it is well known from classical servomechanism theory that in order to avoid steady-state errors, one may have to introduce one or more integrators in the forward-loop plant.

Surprisingly enough, this type of problem has not been considered in detail in the modern control literature. Notable exception to this is the work of C. D. Johnson, which, however, deals with disturbance nulling rather than input tracking per se.

A brief report by Athans [A3] considers this class of problems in a modern control theoretic context. It points out that one must be extremely careful in defining the appropriate optimal control problem. Furthermore, it points out by a simple example that the actual implementation (structurewise) of a mathematical control law may lead to sensitive designs or insensitive designs.

This work is currently being pursued in more detail by Sandell and Athans. Hopefully, insensitive design techniques for this class of problems

will be developed in Sandell's S.M. Thesis (September 1971).

2.5 Air Traffic Control

Significant progress has been made in the area of air traffic control. Our research results up to now are summarized in the papers by Athans and Porter [A5] and Athans [A4].

Briefly we considered the problem of coordinating the traffic flow and holding patternsof N aircraft which desire to land in a single runway. A distance separation is to be enforced over the outer marker. It was shown that this problem can be attacked as a variation of a linear-quadratic optimal control problem. The solution of this optimization problem can be used to indicate which aircraft can accomplish headway corrections by velocity control, and which require to undergo path stretching or holding maneuvers. The gradual implementation of these strategies in current and evolutionary ATC systems was also discussed.

The motivation for this research was provided by system delays encountered in the current operation of the ATC systems during heavy traffic conditions. At present, there are small time delays which are associated with "unstacking" the aircraft, since the aircraft at the bottom of the stack may have the wrong heading at the time of the command to "leave the stack." These small time delays can, however, become significant when many aircraft are holding; hence, this contributes to an effective reduction in the landing rate. These accumulated delays cannot be blamed upon the air traffic controllers, since in heavy traffic conditions they cannot monitor the detailed trajectory of each aircraft that is holding. Rather, it is the result of overall system uncoordination due to the many decisions that must be carried out continuously by the human controllers.

This lack of system coordination is also exhibited when a particular aircraft in a queue develops an emergency. In such cases, it is often desirable to have this aircraft land first, and this necessitates the reordering of the aircraft which were supposed to land before the troubled one. The determination of the proper trajectories for all aircraft concerned is a complex task which, in heavy traffic conditions, can tax the ability of the most capable human.

Similar situations arise when a sudden wind shift requires a change in the landing runway. Once more, the rescheduling of the aircraft is a complex decision task.

From a conceptual viewpoint, there is no reason why the ATC system cannot be completely automated. One can envision an automated set of command and control system in which digital computers obtain all the information from the ground and airborne sensors, smooth the data, compute trajectories for all aircraft, and transmit the appropriate commands to the autopilot of each and every aircraft.

However, such a completely automated system neglects the economic and political constraints that exist. It takes many years to develop and test any new equipment, and even more time to negotiate and change all international agreements that exist. Hence, it is safe to assume that such a completely automated system, even if developed now, will not take effect for at least 20-30 years. Thus, any proposed innovations must take into account the need for gradual evolution and the fact that man-machine interaction will be an essential part of the ATC system for many years to come.

It is the contention of our research that the digital computer can be used as an effective tool that carries out routine decisions and advises the human of suggested courses of action via a display. It is most important, however, that the basis for the automated suggested decisions is such that as technological advances in displays, instrumentation, and autopilots are gradually incorporated into the ATC system, that these changes do not necessitate drastic changes in the way that the decisions are being made. For example, if improved accuracies in locating the aircraft become possible by advanced sensors and can be used to reduce the safe separation distance, this advance should not require a completely new conceptual and algorithmic package for system coordination.

There are many system studies for improved scheduling and trajectory control. However, most of the suggested approached basically attempt to <u>have the digital computer execute decisions in essentially the same manner as a human does</u>. This introduces an artificial element of complexity in the algorithms because the human basic decision rules

cannot be defined precisely and are modulated to a large extent by past rules, regulations, constraints, and "we always did it this way" doctrines. Even if such computational algorithms could be developed, then they would not necessarily be in tune with the future automatic control capabilities that can be realized by hardware advances.

The approach taken was to attack the problem of automated decision making by the digital computer essentially as though all decisions could be implemented completely automatically. This would define the "best possible" way that the aircraft could be controlled. However, these automated decisions are only presented as suggestions, via displays, to the human controller, and eventually the pilot. The human essentially can follow the suggested actions or he can ignore them. The decision-making algorith should be a general one; yet it should have free parameters that can be adjusted by having realistic man-machine simulation experiments. In this manner, at any phase of implementation, one would be able to "tune" the algorithm parameters to be in phase with the human constraints. On the other hand, as more and more true automation is introduced in the system, this would only necessitate changes in certain algorithm parameters rather than a complete overhaul of the decision-making algorithm.

We are continuing our research efforts on this important class of problems, following the above philosophy, and attempting to make our approach as practical as possible.

Currently our research efforts are directed toward solving a more realistic problem. Our basic assumption will be that aircraft landing speeds are mixed. The basic problem is to channel the aircraft to the runway without any violations of separation standards, but also spaced as closely as possible so that the landing rate will approach the capacity of the runway. The design of a regulator, as the algorithm that does the optimal spacing and sequencing in the area beyond the outer marker is called, has not been attacked analytically before because of its complexity. We believe that the mathematical tools required to solve the problem exist.

The system model we will assume at first will be simple and devoid of all uncertainties. Our assumptions will be:

(i) Position and ground velocities of all aircraft are known accurately in the NTA.

(ii) The air routes the aircraft fly are fixed.

(iii) The landing speeds of aircraft are uniformly distributed between 90 and 170 mph.

(iv) All the landings and possibly take-offs are done on the same runway.

(v) The aircraft must be separated by at least D units of distance for safety.

(vi) There are no wind disturbances.

(vii) The aircraft will land on a first-come/first-served basis.

The way we shall attack the problem is as follows:

(i) Divide aircraft into four classes according to their landing speeds. A class will be characterized landing speeds in the region $100 + 10k \pm 10$ knots ($k = 0, 1, 2, 3$).

(ii) Specify four parallel planes, each to be used by aircraft of the same speed class.

(iii) Define boundaries or points on each of the four planes, such that for any aircraft on any of the four planes, the time to travel the distance between the boundary defined on the aircraft's particular plane and the OM is constant. This scheme will solve the merging problem.

(iv) The first-come/first-served scheme establishes the landing order and the specific point in time each aircraft should cross the OM.

(v) Once the priorities and the times the aircraft will land are known, the speed profile of every aircraft is known from the time it enters the NTA until it crosses the OM at which point no more control can be applied. Linearization of the aircraft dynamics around this speed profile will leave a linear time varying system whose states will be the position and velocity deviations from the nominal. Application of quadratic criteria and solution of the resulting optimal control problem will result in the correction maneuvers that every aircraft will have to perform to maintain its nominal speed profile and land at the specified time.

(vi) Subsequently, uncertainties will be introduced, such as wind disturbances and position- and velocity-measurement uncertainties, and the same problem with uncertainties will be attacked.

(vii) Finally, other than first-come/first-served landing schemes will be investigated.

3. PROJECT DOCUMENTATION

All of the papers listed in Section 5, Bibliography, are the result of research supported wholly or in part by this research grant. Copies of those documents which have already appeared have been forwarded to the Grant Office.

4. BUDGET STATUS

As of 28 February 1971, approximately 92.0 per cent of the grant funds have been expended in 91.0 per cent of the grant period. No effect has been given to step funding in these figures.

5. BIBLIOGRAPHY

A1 Athans, M. "The Role of the Linear-Quadratic-Gaussian Problem in Control System Design." M.I.T. Electronic Systems Laboratory Report ESL-P-438, Cambridge, Mass., February 1971; also to appear in IEEE Trans. on Automatic Control (Special Issue), Vol. AC-16, December 1971.

A2 Athans, M. "On the Design of a Digital Computer Program for the Design of Feedback Compensators in Transfer Function Form." M.I.T. Electronic Systems Laboratory Report ESL-R-409, Cambridge, Mass., December 1969 (DDC-AD700431).

A3 Athans, M. "On the Design of P-I-D Controllers Using Optimal Linear Regulator Theory." M.I.T. Electronic Systems Laboratory Report ESL-P-432, Cambridge, Mass., July 1970 (DDC-AD710300).

A4 Athans, M. "System Aspects of Air Traffic Control," Proc. 4th Hawaii International Conference on Systems Sciences, Honolulu, Hawaii, January 1971.

A5 Athans, M. and L. W. Porter. An Approach to Semiautomated Optimal Scheduling and Holding Strategies for Air Traffic Control." M.I.T. Electronic Systems Laboratory Report ESL-P-437, Cambridge, Mass., December 1970; also to appear in Proc. 1971 Joint Automatic Control Conference, St. Louis, Missouri, August 1971.

B1 Bertsekas, D. P. and I. B. Rhodes. "On the Minimax Reachability of Target Sets and Target Tubes," Automatica (accepted) (DDC-AD706079).

B2 Bertsekas, D. P. and I. B. Rhodes. "Recursive State Estimation for a Set-Membership Description of Uncertainty," IEEE Trans. on Automatic Control (accepted) (DDC-AD706114).

G1 Geering, H. P. and M. Athans. "Optimal Control Theory for Non-Scalar Valued Criteria," Proc. Fifth Annual Princeton Conference on Information Sciences and Systems, Princeton, New Jersey, March 1971.

J1 Johnson, T. L., M. Athans, and G. B. Skelton. "Optimal Control Surface Locations for Aircraft Flexure Controllers," Proc. 1970 Joint Automatic Control Conference, Atlanta, Georgia, June 1970; also IEEE Trans. on Automatic Control, Vol. AC-16, August 1971 (accepted).

J2 Johnson, T. L. and M. Athans. "On the Design of Optimal Constrained Dynamic Compensators for Linear Constant Systems," IEEE Trans. on Automatic Control, Vol. AC-15, December 1970 (DDC-AD706115).

L1 Levine, W. S., T. L. Johnson, and M. Athans. "Optimal Limited State Variable Feedback Controllers for Linear Systems," IEEE Trans. on Automatic Control (Special Issue), Vol. AC-16, December 1971 (accepted).

M1 Mitter, S. K. "Stability and Convergence of Discretization Schemes for Optimal Control Problems." Invited Paper, 4th Hawaii International Conference on Systems Sciences, Honolulu, Hawaii, January 1971.

M2 Mitter, S. K. (with D. P. Bertsekas). "Steepest Descent for Optimization Problems with Non-differentiable Cost Functions," Proc. Fifth Annual Princeton Conference on Information Sciences and Systems, Princeton, New Jersey, March 1971.

R1 Rhodes, I. B. "On Nonzero Sum Differential Games with Quadratic Cost Functionals," Proc. First International Conference on Differential Games, Amherst, Mass., September 1969 (DDC-AD691147).

T1 Tse, E. "On the Optimal Control of Linear Systems with Incomplete Information." Ph. D. Thesis, M.I.T. Department of Electrical Engineering; also M.I.T. Electronic Systems Laboratory Report ESL-R-412, January 1970 (DDC-AD701797).

T2 Tse, E. and M. Athans. "Optimal Minimal Order Observers-Filters for Discrete Time Linear Time-Varying Systems," IEEE Trans. on Automatic Control, Vol. AC-15, pp. 416-426, August 1970 (DDC-AD690573).

T3 Tse, E and M. Athans. "Observer Theory for Continuous-Time Linear Systems." (submitted to Information and Control) (DDC-AD706939).

T4 Tse, E. and M. Athans. "Adaptive Stochastic Control for Linear Systems: Part I: Solution Methods," Proc. 9th IEEE Symposium on Adaptive Processes, Decision and Control, Austin, Texas, December 1970.

T5 Tse, E. and M. Athans. "Adaptive Stochastic Control for Linear Systems: Part II: Asymptotic Properties and Simulation Results," Proc. 9th IEEE Symposium on Adaptive Processes, Decision and Control, Austin, Texas, December 1970 (DDC-AD706939).

T6 Tse, E. and M. Athans. "Adaptive Stochastic Control for a Class of Linear Systems," IEEE Trans. on Automatic Control (submitted).