

Interim Technical Report

**ANALYSIS OF A NAVIGATION CONCEPT EMPLOYING A
SPINNING RADIATING INTERFEROMETER IN
GEOSTATIONARY ORBIT**

by
George Ranjan Thomas

August 1971



UNIVERSITY of PENNSYLVANIA
The Moore School of Electrical Engineering
PHILADELPHIA, PENNSYLVANIA 19104

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gating craft as a double-sideband suppressed-carrier (DSB) waveform whose demodulation and subsequent processing form an important part of this study. The DSB signal is coherently demodulated to yield the intelligence. Two of the craft's position coordinates (the altitude being independently measured) are extracted optimally in the maximum a posteriori (MAP) sense from this intelligence. The MAP processor is designed to accept preliminary position information from on-board dead-reckoning equipment and stored a priori information, in addition to the incoming radio signals, to give improved estimates of position.

An error analysis reveals that the major contributors to position error are tropospheric phase fluctuations, multipath, uncertainty of user velocity introduces no error into this system. Systematic error due to satellite motion, multipath off the satellite surface and mismatch of interferometer antenna phase patterns are eliminated by the referenced radionavigation technique, a monitoring system using several ground stations. Bias error from user equipment is calibrated out by a technique which involves periodically transmitting a signal from a stationary antenna on the satellite.

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Estimation error analysis						
Orthogonal estimation errors						
Velocity errors						
Altitude errors						
Propagation errors						
Satellite-borne antenna error						
Satellite motion						

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ABSTRACT

Navigation satellite systems have been proposed in recent years as aids to long-distance navigation, particularly over large ocean areas. The lack of continuously operative systems that are both reliable and accurate over a wide coverage area is reflected in the stringent separation standards for air traffic. Since studies have shown that considerable savings result from the reduction of these separation standards, an economical satellite system is desired, preferably one that uses a single satellite to provide position information continuously to a field of users. Desirable attributes of such a system include unsaturability, which would require that the users be passive (nontransmitting), and maximum coverage area, requiring a synchronous (24-hour) satellite. A single satellite suitable for passive users necessitates interferometric (angle-measuring) methods and systems consisting of nonspinning synchronous satellites carrying crossed interferometer pairs have already been investigated. Since conventional spin-stabilization is employed by the majority of existing satellites, it is desired to apply interferometric techniques to spinning satellites.

This dissertation analyzes a navigation concept wherein a single radiating interferometer is placed on a spinning synchronous satellite. The L-band continuous-wave signals transmitted by the interferometer antennas are received by the navigating craft as a double-sideband suppressed-carrier (DSB) waveform whose demodulation and subsequent processing form an important part of this study. The DSB signal is coherently demodulated to yield the intelligence. Two of the craft's position coordinates (the altitude being independently measured) are extracted optimally in the maximum a posteriori (MAP) sense from this intelligence. The MAP processor is designed to accept preliminary position information from on-board dead-reckoning equipment and stored a priori information, in addition to the incoming radio signals, to give improved estimates of position.

An error analysis reveals that the major contributors to position error are tropospheric phase fluctuations, multipath, uncertainty in user altitude and additive receiver noise. The uncertainty of user velocity introduces no error into this system. Systematic errors due to satellite motion, multipath off the satellite surface and mismatch of interferometer antenna phase patterns are eliminated by the referenced radionavigation technique, a monitoring system using several ground stations. Bias error from user equipment is calibrated out by a technique which involves periodically transmitting a signal from a stationary antenna on the satellite.

The parameters of the system, such as the baselength of the interferometer, the skew-angle between the interferometer and the orbital plane, the gains of satellite and craft antennas, the transmitter power and the integration time-interval in the receiver, are traded off to optimize the system. It is concluded that position-fixes with 1σ accuracies on the order of 2 to 3 nmi can be provided in two orthogonal directions. Higher accuracies can be achieved in one direction but only at the cost of substantially reduced accuracy in the other. If the cross-track (north-south) accuracy of aircraft flying east-west routes over the North Atlantic is judged more important, then the system parameters can be appropriately selected, with the understanding that the along-track accuracy will deteriorate.

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PREFACE

The lack of a reliable and accurate navigation scheme operative over a large coverage area such as the North Atlantic Ocean which is heavily traveled is reflected in the stringent separation standards for aircraft. Since the reduction of these standards is expected to result in considerable savings, an economical radio-navigation system is desired, one that remains unsaturable while providing position information over a large geographic area. This work analyzes a concept wherein a conventionally spin-stabilized synchronous satellite carrying an interferometer transmits CW signals at L-band to a field of users. Suitable craft-borne receivers that extract position information from the received signal are designed, an analysis of the factors that introduce position error is performed, and the system is optimized by trading off various parameters. This navigation scheme may be used either as a pure radio aid for position determination or as a means for updating self-contained navigation equipment.

1.0 INTRODUCTION

Due to the rapid increase in air traffic densities over the last two decades, traditional radio systems for position determination, which involved a multiplicity of ground-based stations, have been judged increasingly inadequate [13, 44, 68]. This is particularly true for traffic over oceans and land masses that are either thinly-populated or economically developing. The lack of reliable and accurate systems has been reflected in the stringent separation standards for flight corridors over heavily traveled ocean routes such as those over the North Atlantic region. The lateral separation between lanes is 120 nmi, at present causing the diversion of air traffic from the optimum, minimum-time routes. One study claims an annual saving averaging one million dollars per airplane if the lateral separation standard is reduced to 60 nmi [47]. In view of the expected doubling by 1975 of transatlantic air traffic over the 1960 level, and the introduction of supersonic transports, the economic benefits of decreasing separation between lanes may be considerable.

The possibility of using earth satellites for navigation and air traffic control has been recognized by private as well as government interests [22, 54, 71]. Besides overcoming the basic disadvantage of existing terrestrial navigation schemes for long-range navigation, the limited geographical area covered, navigation satellites offer the prospects for combining, for the first time, a number of related functions such as communications, emergency operations, relaying of weather information, air traffic control and traffic surveillance.

There are many ways to classify navigation satellite systems. One is on the basis of satellite altitude, either synchronous or non-synchronous. Synchronous satellites are in 24-hour orbits which enable them to be stationary with respect to a point on earth, if the satellite orbit is in the earth's equatorial plane. Such satellites are 36,000 km (or 19,400 nmi) from the earth's surface. "Nonsynchronous" refers to low or medium-altitude satellites that are moving with respect to an observer on earth.

Nonsynchronous satellites are used at present in the Navy Navigation Satellite System [25, 40] wherein the user (ship or submarine) compares the frequency of the signal from a satellite with that of a highly stable local oscillator on board. The difference is a Doppler shift that is measured over a two-minute period. This measurement fixes the user on a hyperbola. A second two-minute interval provides another hyperbola and consequently the user's position coordinates. This system is capable of position-fix accuracies on the order of 1/4 mile [40], but since the total time required for the fix is several minutes, this system is not considered suitable for aircraft whose motion cannot be exactly accounted for during this period.

Besides the above disadvantage, there are many difficulties associated with systems employing nonsynchronous satellites. First, for continuous coverage of a given geographical area a higher number of satellites is required in nonsynchronous systems. Such satellites spend a higher percentage of orbital time in the earth's cold umbra, requiring active temperature control and extra rechargeable batteries,

resulting in a weight penalty. Furthermore, the earth's magnetic field affects low-altitude satellites in two ways: (a) the high-energy protons trapped in the Van Allen belt can affect solar cells and other solid-state components; this may be counteracted by protective coatings that impose a weight penalty; (b) the magnetic field causes precession of the satellite spin-axis and reduction of the spin-rate by interacting with power-supply currents and by inducing eddy-currents in the metallic body of the satellite. Finally, in the case of nonsynchronous satellites, ground station complexity is increased by the need for: (a) large tracking antennas that have a high performance over an entire hemisphere, (b) duplicate antennas, so that a rising satellite may be acquired while operation is being handed over by a setting satellite, (c) complex computing facilities for orbit determination and prediction, and (d) methods to counteract the increased radio-interference from other services, inevitable in a moving ground-based tracking antenna which must cover an entire hemisphere.

At synchronous altitude both single- and multiple-satellite systems are possible, depending on the spatial parameter (range, range-difference, range-rate, angle) that is being measured to generate lines of position (LOPs) or surfaces of position (SOPs). Each one of these techniques, with the sole exception of angle-measurement techniques, requires a multiplicity of satellites.

In a range-measurement navigation satellite scheme, two spherical SOPs are generated by a range-measurement from each of two satellites. A position-fix is obtained from the intersection

of these two SOPs with another spherical SOP (the earth itself in the case of ships, or a sphere concentric with the earth but with a radius increased by user altitude in the case of aircraft). Three satellites are required for range-difference measurement systems where two hyperboloidal SOPs are generated. If the user does not have an independent altitude measuring capability, an additional satellite will be required in each of the above systems.

Naturally, a reasonable objective in terms of minimizing the cost of the overall system would be to have as few satellites as possible, and single synchronous satellite systems are therefore attractive. Such systems always involve the measurement of spatial angles by interferometric techniques. Studies at Cubic Corp. [18,19], Westinghouse [71] and the University of Pennsylvania [41] have recommended that crossed radio-interferometer pairs be placed in synchronous orbit to provide measurements of the angles from the interferometers to the navigating user. The first two organizations suggested "active user" schemes where a signal would be transmitted by a user. The phase-differences between the signals arriving at each antenna of the satellite-borne interferometers are measured--each phase-difference measurement generating a conical SOP. The intersection of the two conical SOPs with a spherical SOP concentric with the earth fixes the user's position. Klein [41] suggested a "passive user" scheme where signals transmitted by the two interferometers are processed by the user to yield two angles of position, which together with an independently obtained estimate of his altitude gives complete position information.

All three of these interferometric systems require the emplacement of a crossed interferometer pair on a non-spinning synchronous satellite. The satellite is not permitted to spin since each interferometer must nominally be fixed in orientation relative to the earth. Stabilization of the satellite cannot therefore be accomplished by the conventional method of spinning the body about its axis of maximum moment of inertia: a passive stabilization scheme which relies primarily on natural laws to align itself in a stable attitude. Some other passive method, such as gravity-gradient techniques, or inherently more complex active stabilization methods must be employed. Gravity-gradient stabilization is the natural result of planetary attraction acting on an orbiting body. Widespread use has not been made of this technique although in 1967 successful gravity gradient experiments were conducted by General Electric Co. and the Department of Defense [31, 32, 41].

Conventional spin-stabilization has the advantage of simplicity and is a well-proven technique, having been used in many of the major operational satellites. This mode is highly resistant to disturbing torques, permits simpler thermal design (the rotation producing a uniform temperature environment within the satellite) and provides a centrifugal force field for the expulsion of liquid control propellants. It must be noted that "conventional" spin-stabilization requires the spin-axis to be perpendicular to the orbital plane, so that the axis direction is fixed relative to the earth.

Another desirable characteristic in a navigation system is "passive user" operation alluded to earlier, where the navigating user is not required to transmit. This permits continuous position-fix capability, reduces the possibility of interference (both deliberate and inadvertent) by not requiring an uplink from every craft to the satellite [39], and accommodates vehicles that are required to conceal their movements (e.g. military craft). "Passive user" systems, furthermore, are unsaturable since the traffic handling capability of a given route is limited only by the available position-fix accuracy and not by the craft-to-ground station link capacity. Existing links via the satellite, not being responsible for position-fixing, are freed for emergency operations, air traffic control, relaying of weather information, etc. The main disadvantage of such a system over an "active user" system is the need for measuring and computing facilities on board the navigating craft.

The system proposed here is an angle-measurement single interferometer scheme using a spin-stabilized synchronous satellite. The navigator is permitted to be passive for position-fixing purposes, utilizing uplinks to the satellite as stated above for communications, emergency operations, etc. Section 2.1 describes the system configuration, followed by the identification and derivation of the useful phase term containing user position information in Section 2.2.1. The signal-to-noise ratio at each stage of the demodulation process is derived in Section 2.2.2. Section 3 is concerned with the design of the receiving equipment on board the craft. The first part of the demodulation (after the RF-IF stages) is dealt with in Section 3.1,

where coherent methods are discussed and possible noncoherent techniques mentioned. Some intuitive schemes are outlined for the post-DSB-demodulation stage in Section 3.2.1, but the more interesting part is the systematic design of an optimum MAP^{*} processor in Section 3.2.2. An analysis of estimation errors appears in Section 4. Errors due to propagation effects (with emphasis on multipath) and a variety of random and systematic errors are discussed in Section 5. This work concludes with Section 6, where the major conclusions are summarized and some recommendations for future work are made.

* Maximum a posteriori.

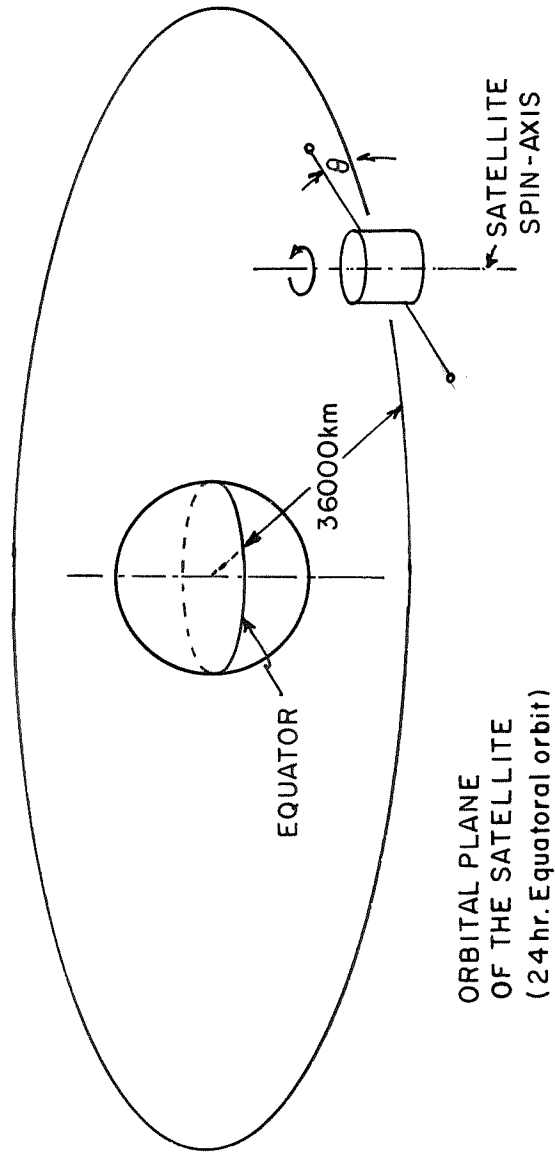


FIGURE 1 SYSTEM GEOMETRY OF THE SPINNING INTERFEROMETER NAVIGATION SATELLITE CONCEPT

2.0 SYSTEM DESCRIPTION AND ANALYSIS

2.1 System Description

The Spinning Interferometer Navigation Satellite concept consists of a spin-stabilized geostationary satellite carrying two co-linear booms. On the end of each boom is placed an antenna. These two antennas constitute the interferometer which rotates about the satellite's spin-axis, as shown in Fig. 1. The interferometer is "skewed" in the sense that its baseline is at an angle θ to the orbital plane (Fig. 2). The satellite's spin-axis, to permit conventional spin-stabilization, is perpendicular to the equatorial orbital plane.

For purposes of analysis, the antennas are assumed to be isotropic, but it is understood that they may be despun (either mechanically or electrically) to provide gain. The implementation of antenna despinning seems to be well within the capabilities of present-day technology [9], but is not part of this analysis. The signals transmitted by the two antennas are identical in phase, frequency and amplitude.

From the subsequent analysis it will be evident that the user position information is contained in the signal received by the craft even if only one rotating antenna at the satellite were transmitting, as in the configuration shown in Fig. 3(b). A combination of a rotating and a stationary antenna, as in Fig. 3(a), would also give rise to a received signal containing position information. The former is impractical to implement since the extraction of position information from the argument of the received signal is complicated

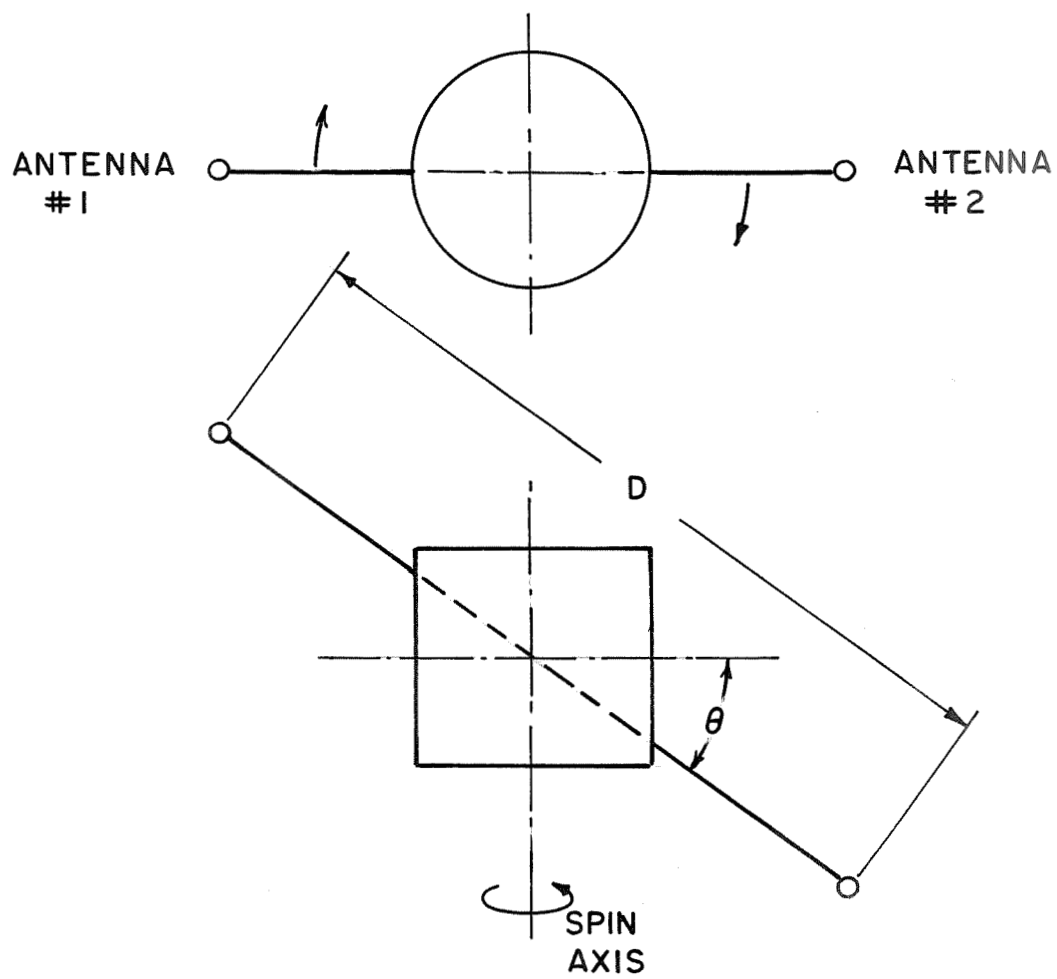
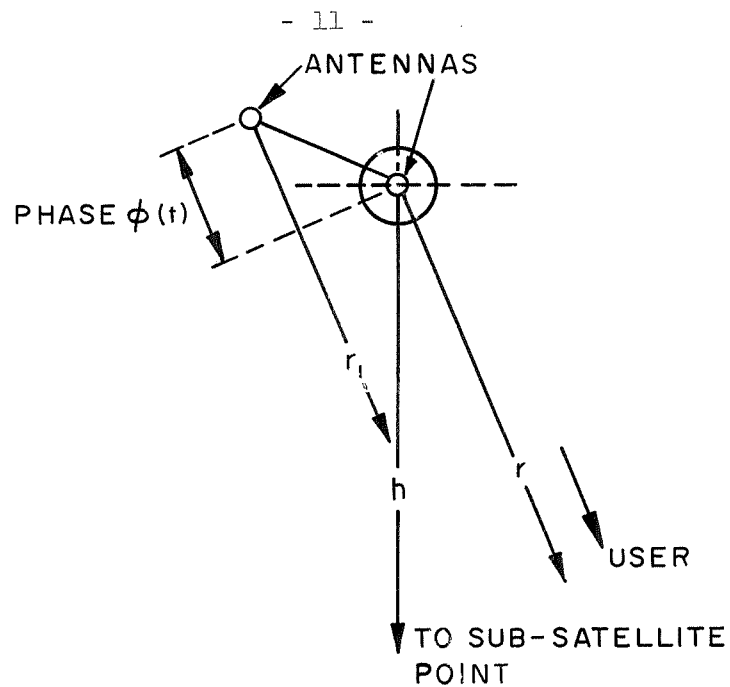
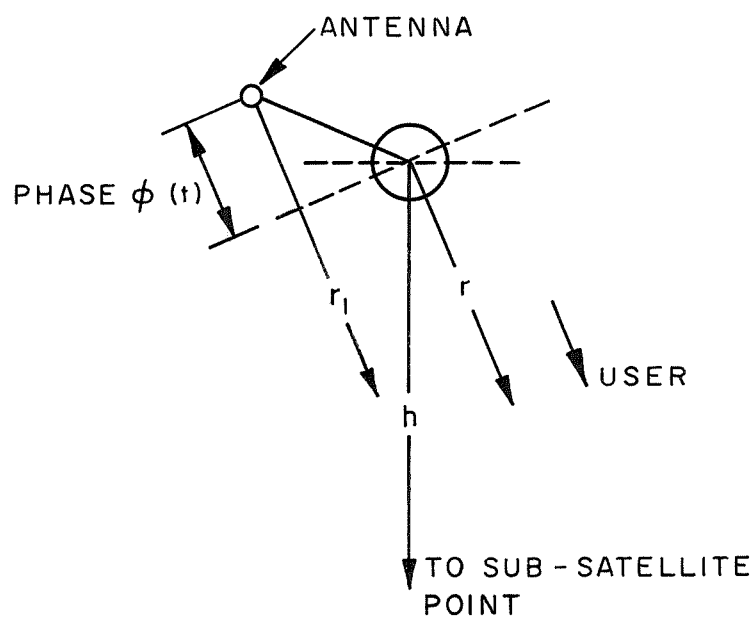


FIGURE 2 FRONT-VIEW AND PLAN-VIEW OF THE SATELLITE STRUCTURE



(a)



(b)

FIGURE 3 FRONT-VIEW OF ALTERNATE SATELLITE CONFIGURATIONS
 (a) ONE STATIONARY AND ONE MOVING ANTENNA
 (b) ONE MOVING ANTENNA

by the phase terms arising from craft motion. The latter, from preliminary investigation, does not seem to offer any advantages over the basic Spinning Interferometer concept. As analytical expressions are developed for the received signals in the above system configurations, it will become clear why the two alternative schemes mentioned in this paragraph are not considered further here.

2.2 System Analysis

2.2.1 Identification of the Information-Bearing Parameter

From Fig. 4 it is clear that the identical CW signals $A_T \cos \omega t$ transmitted by the two antennas of the interferometer follow different paths, r_1 and r_2 , to the user. The user receives these signals attenuated in amplitude and delayed in phase. The composite received signal $s_R(t)$ is given by

$$s_R(t) = A_R \cos \left(\omega t - \frac{\omega r_1}{c} \right) + A_R \cos \left(\omega t - \frac{\omega r_2}{c} \right) \quad (1)$$

assuming that the signals from both transmitting antennas are attenuated equally. If it is assumed further that the two rays at the satellite are essentially parallel, since the satellite's altitude is much greater than the interferometric baselength, r_1 and r_2 represent the phase delays

$$\frac{\omega r_1}{c} = \frac{\omega r}{c} + \varphi \left(t - \frac{r_1}{c} \right) \quad \frac{\omega r_2}{c} = \frac{\omega r}{c} - \varphi \left(t - \frac{r_2}{c} \right) \quad (2)$$

where r is the satellite-user range and φ is the phase corresponding to the distance of each antenna from the phase-front through the satellite center. In the subsequent analysis φ at the receiver is taken to be at $t - \frac{r}{c}$. The error introduced thereby is shown to be negligible in App. D.

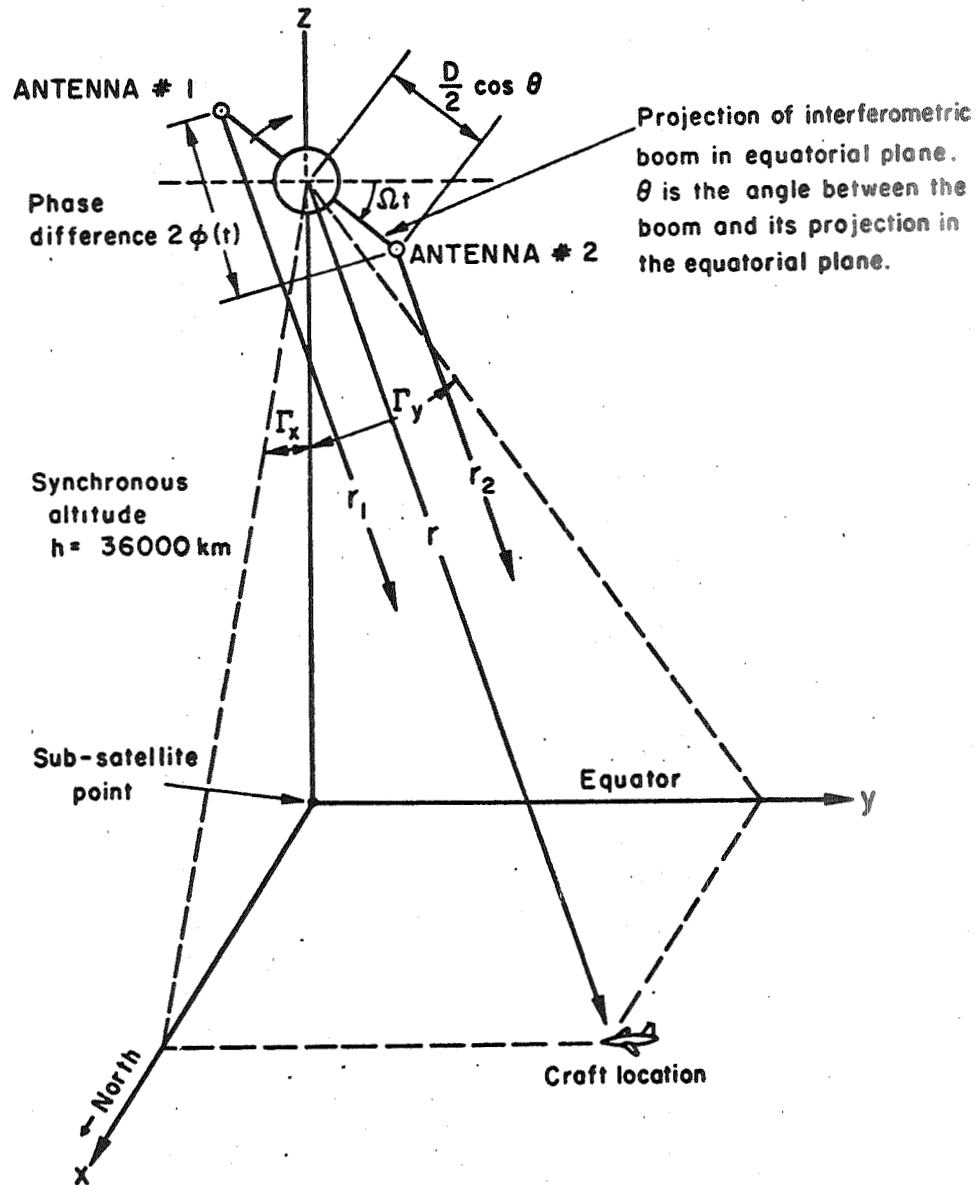


FIGURE 4 SYSTEM GEOMETRY IN RECTANGULAR COORDINATES

$\varphi(t)$ is the position-information-bearing term* and is derived from the system geometry in Appendix A:

$$\varphi(t) = C_1 + C_2 \cos (\Omega t - C_3) \quad (3)$$

where

$$\left. \begin{aligned} C_1 &\equiv \pi \frac{D}{\lambda} \frac{\Gamma_x}{\sqrt{1 + \Gamma_x^2 + \Gamma_y^2}} \sin \theta \\ C_2 &\equiv \pi \frac{D}{\lambda} \sqrt{\frac{1 + \Gamma_y^2}{1 + \Gamma_x^2 + \Gamma_y^2}} \cos \theta \\ C_3 &= \tan^{-1} \left[\frac{1}{\Gamma_y} \right] \end{aligned} \right\} \quad (4)$$

in which D/λ is the interferometric baselength-to-wavelength ratio, θ is the skew-angle between the interferometric baseline and the equatorial plane, Ω is the satellite spin-rate, Γ_x is the satellite-centered tilt angle corresponding to the craft's north-south position-coordinate and Γ_y corresponds to the east-west position coordinate (Fig. 4).

The isolation of the φ term and the ultimate extraction of (Γ_x, Γ_y) in some optimum fashion are two of the major objectives of this study.

* The fact that the craft position information is contained in C_1 , C_2 and C_3 is emphasized by writing φ as $\varphi(t, \underline{C})$ in subsequent sections. Both forms are used interchangeably and should cause no confusion. When $\varphi(t, \underline{C})$ is used as part of the signal received by the craft the propagation delay r/c is implicit.

At this point, it may be worthwhile inquiring into the nature of $\varphi(t)$ in order to discover its physical significance. From Eq. (2) it is clear that

$$\varphi(t) = \frac{1}{2} \cdot \frac{\omega}{c} (r_1 - r_2) \quad (5)$$

which becomes, if differentiated with respect to time:

$$\varphi'(t) = \frac{1}{2} \cdot \frac{\omega}{c} (r_1' - r_2') \quad (6)$$

where the primes indicate time-derivatives. Now the frequency ω_a received by an observer from antenna #1 that is moving at a velocity V_{r1} relative to the observer is

$$\omega_a = \omega \left(1 - \frac{V_{r1}}{c} \right) = \omega - \frac{\omega r_1'}{c} = \omega - \omega_{d1}$$

where ω_{d1} or $\frac{\omega r_1'}{c}$ is the Doppler shift in the frequency seen by the observer.

Similarly, the Doppler shifted frequency from antenna #2 is

$$\omega_b = \omega - \frac{\omega r_2'}{c} = \omega - \omega_{d2}$$

Rewriting Eq. (6) in terms of Doppler shifts

$$\varphi'(t) = \frac{1}{2} \cdot \left[\omega_{d1} - \omega_{d2} \right]$$

which means that the quantity of interest, $\varphi(t)$, is the time-integral of the difference in Doppler shifts due to the motion of the two antennas. Thus, the physical phenomenon operative in this navigation system is seen to be the Doppler Effect.

In Section 2.1 it was suggested that the position information contained in the argument of the signal received from one rotating antenna could possibly be extracted. Such a received signal can be written as the phase modulated waveform

$$s_R(t) = A_R \cos \left[\omega t - \frac{\omega r}{c} - \varphi \right] \quad (7)$$

from which φ is to be isolated. The propagation delay $\frac{\omega r}{c}$ in the argument could be of a form that makes the isolation of φ difficult, if not impossible. Therefore an inquiry into the nature of $\frac{\omega r}{c}$ is in order.

Defining $P(t)$ as the propagation delay

$$P(t) = \frac{\omega}{c} r(t)$$

from which it is clear that

$$P'(t) = \frac{\omega r'(t)}{c} = \frac{2\pi}{\lambda} r'(t).$$

Assuming that the craft flies at a constant speed V_u at a constant altitude as in Fig. 5,

$$r'(t) = - V_u \cos E(t) \quad (8)$$

where $E(t)$ is the time-varying elevation angle of the satellite as seen from the craft. Rewriting $P'(t)$,

$$P'(t) = - \frac{2\pi}{\lambda} V_u \cos E(t) \quad (9)$$

Figure 6 shows the existence of a relationship between elevation angle E , user longitude or latitude L , synchronous altitude h and earth radius R_e :

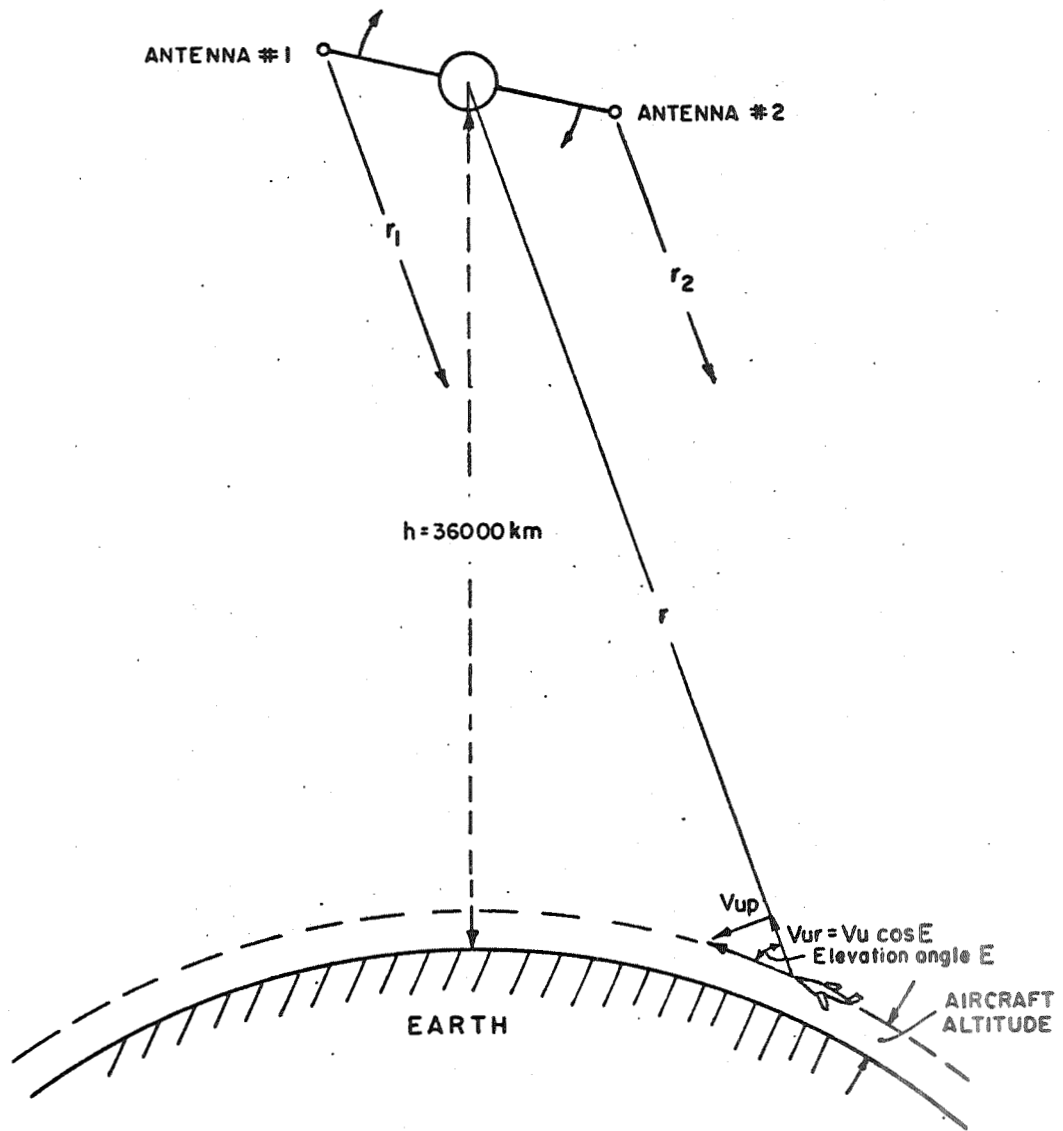


FIGURE 5 CRAFT VELOCITY EFFECTS

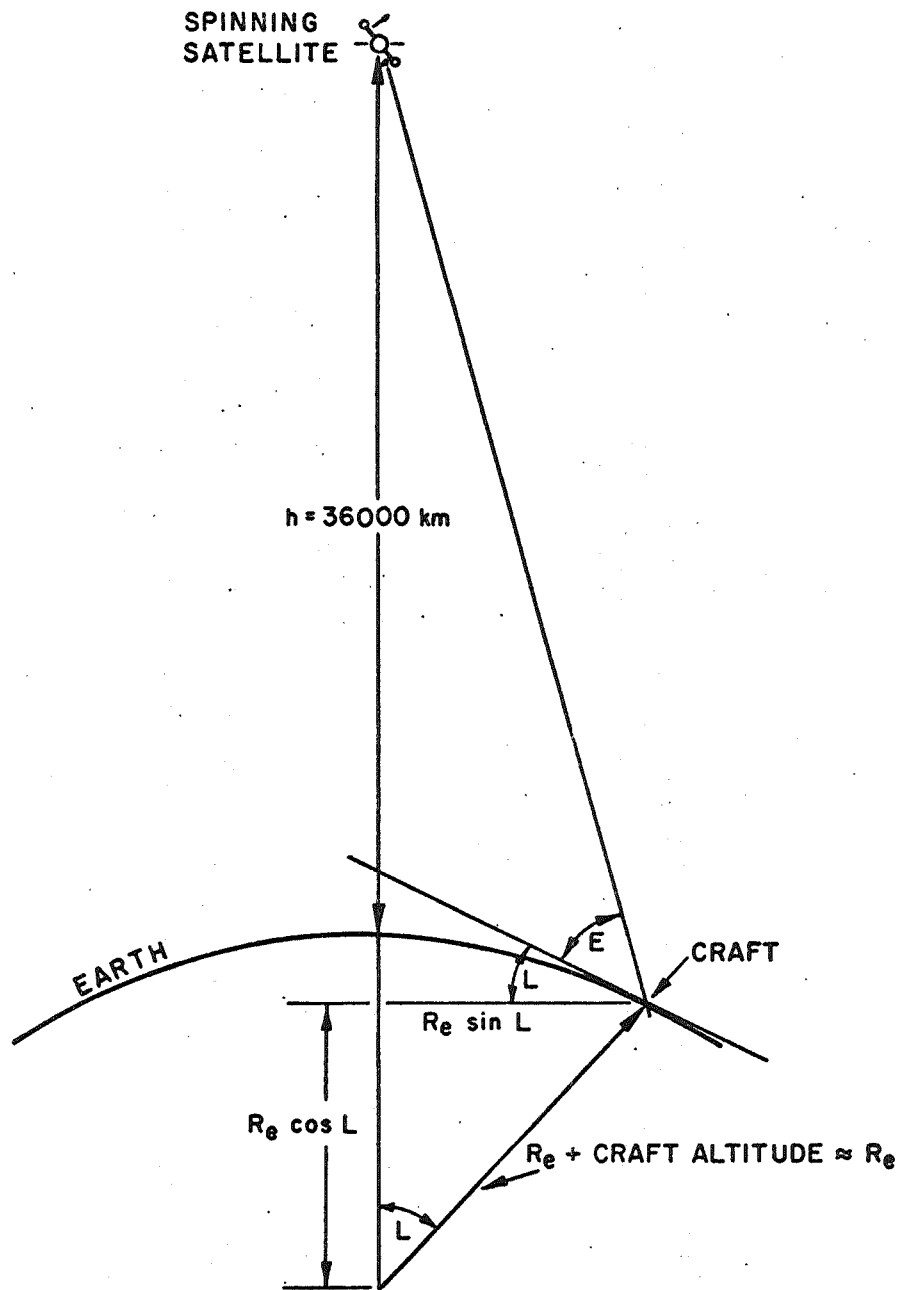


FIGURE 6 SATELLITE-CRAFT GEOMETRY SHOWING RELATIONSHIP BETWEEN ELEVATION ANGLE AND CRAFT LATITUDE (OR LONGITUDE)

$$\begin{aligned}
 \tan(L + E) &= \frac{h + R_e [1 - \cos L]}{R_e \sin L} \\
 &= \frac{\left(\frac{h}{R_e} + 1\right) - \cos L}{\sin L} \\
 &= \frac{H - \cos L}{\sin L}, \text{ replacing } \left[\frac{h}{R_e} + 1\right] \text{ by } H
 \end{aligned} \tag{10}$$

Therefore

$$E = \tan^{-1} \left[\frac{H - \cos L}{\sin L} \right] - L \tag{11}$$

Differentiating (11) with respect to time,

$$\begin{aligned}
 E'(t) &= \frac{H(\cos L - H)}{1 - 2H \cos L + H^2} \cdot L'(t) \\
 &= ML'(t)
 \end{aligned} \tag{12}$$

The limits on L are 0° and about 80° . When $L = 0^\circ$, $M = 1.18$; when $L = 40^\circ$, $M = -1.12$; when $L = 80^\circ$, $M = -0.98$. Clearly, M is of nearly unit magnitude for a wide range of craft locations. Equation (12) becomes

$$\begin{aligned}
 |E'(t)| &\approx |L'(t)| \\
 &= \frac{V_u}{R_e} \\
 &\approx 52.4 \times 10^{-6} \text{ rad/sec for } V_u = 1200 \text{ km/hr} \\
 &\approx 0.003 \text{ degrees/sec}
 \end{aligned}$$

Integrating $E'(t)$ for an aircraft that sees the satellite at an

elevation angle of E_o at time $t = 0$,

$$E(t) = \frac{V_u}{R_e} t + E_o$$

From Eq. (9),

$$\begin{aligned} P'(t) &= -\frac{2\pi}{\lambda} V_u \cos \left[\frac{V_u}{R_e} t + E_o \right] \\ &= -\frac{2\pi}{\lambda} V_u \left[\cos \frac{V_u}{R_e} t \cos E_o - \sin \frac{V_u}{R_e} t \sin E_o \right] \\ &\approx -\frac{2\pi}{\lambda} V_u \left[\cos E_o - \frac{V_u}{R_e} t \sin E_o \right] \end{aligned}$$

since $\frac{V_u}{R_e} t$ is very small for any time interval of interest, and the approximations $\cos \frac{V_u}{R_e} t \approx 1$ and $\sin \frac{V_u}{R_e} t \approx \frac{V_u}{R_e} t$ are valid.

Or,

$$P'(t) = P_2 t + P_1$$

where

$$P_2 = \frac{2\pi}{\lambda} \frac{(V_u)^2}{R_e} \sin E_o = 0.546 \sin E_o$$

$$P_1 = -\frac{2\pi}{\lambda} V_u \cos E_o = -10500 \cos E_o$$

the coefficients being evaluated for $\lambda = 20$ cm and $V_u = 1200$ km/hr.

For the geographical area of greatest interest the elevation angles lie between 20° and 55° corresponding to latitudes of $62^\circ N$ and $30^\circ N$.

P_2 ranges between 0.187 to 0.447 while $|P_1|$ ranges between 8600 and 9850. Since the observation interval is small (less than 1 second),

$P_2 t$ may be disregarded in comparison with P_1 . Integrating $P'(t)$

with respect to time and letting P_0 be the constant of integration,

$$P(t) \approx P_1 t + P_0$$

indicating that the propagation delay $\frac{\omega r}{c}$ is the sum of a constant and a linearly time-varying term. These terms are dependent on both craft velocity, not a very accurately known figure, and craft position. The extraction of position coordinates from Eq. (7) is impossible, since $P(t)$ cannot be isolated from φ . This may be seen more clearly by rewriting Eq. (7) in the following form:

$$s_R(t) = A_R \cos \left[(\omega - P_1) t - P_0 - \varphi \right] \quad (13)$$

which shows not only that φ is contaminated by P_0 , a term not precisely known, but also that the carrier frequency ω is changed by some unknown amount. Using a single moving antenna for position fixing purposes is therefore not considered a fruitful approach.

A further suggestion was made in Section 2.1 that a navigation scheme [Fig. 3(a)] employing an interferometer consisting of one stationary and one moving antenna be investigated. A preliminary study of the signal characteristics did not reveal any advantages. On the other hand, the structural imbalance caused by a single boom is likely to be disadvantageous. This concept has consequently been abandoned.

In the interferometer configuration adopted in this study the signal received by the craft is

$$\begin{aligned}
 s_R(t) &= A_R \left\{ \cos \left[\omega t - \frac{\omega r}{c} - \varphi \right] + \cos \left[\omega t - \frac{\omega r}{c} + \varphi \right] \right\} \\
 &= 2A_R \cos \varphi \cdot \cos \left(\omega t - \frac{\omega r}{c} \right)
 \end{aligned} \tag{14}$$

This is in the form of a double-sideband suppressed-carrier (DSB) signal with the "intelligence" part isolated from the contamination of the $\frac{\omega r}{c}$ term which constitutes the phase of the carrier term. The processing of $s_R(t)$ to yield useful information is the subject of Section 3.

In Section 3 the RF-IF stages are not discussed because they involve conventional heterodyning into the IF range of frequencies and some amplification but no change in signal-to-noise ratios. In order to indicate the existence of these stages and to provide some consistency in notation, Eq. (14) will be adjusted to read

$$s_{IF}(t) = 2A_O \cos \varphi \cos \left(\omega_{IF} t - \frac{\omega r}{c} \right) \tag{15}$$

in the analysis that follows in the next section which is concerned most with signal-to-noise ratios after the IF stage.

2.2.2 Signal-to-Noise Ratio Considerations

At the output of the IF stage, the total DSB signal $s_{IF}(t)$ may be written as the sum of two phase-modulated sinusoids and zero-mean white gaussian noise:

$$s_{IF}(t) = A_o \cos \left[\omega_{IF} t - \frac{\omega r}{c} - \varphi \right] + A_o \cos \left[\omega_{IF} t - \frac{\omega r}{c} + \varphi \right] + n(t) \quad (16)$$

where $\frac{\omega r}{c}$ is the propagation phase delay as encountered previously. The average power represented in $s_{IF}(t)$ may be obtained by squaring (16), averaging and excluding high frequency components:

$$\begin{aligned} \langle s_{IF}^2(t) \rangle &= \langle A_o^2 \cos^2 \left[\omega_{IF} t - \frac{\omega r}{c} - \varphi \right] \rangle \\ &\quad + \langle A_o^2 \cos^2 \left[\omega_{IF} t - \frac{\omega r}{c} + \varphi \right] \rangle \\ &\quad + 2 \langle A_o^2 \cos \left[\omega_{IF} t - \frac{\omega r}{c} - \varphi \right] \cos \left[\omega_{IF} t - \frac{\omega r}{c} + \varphi \right] \rangle \\ &\quad + \langle n^2(t) \rangle \\ &= \frac{1}{2} A_o^2 + \frac{1}{2} A_o^2 + A_o^2 \langle \cos 2\varphi \rangle + \langle n^2(t) \rangle \end{aligned}$$

The average signal power $P = A_o^2 \left[1 + \langle \cos 2\varphi \rangle \right]$

and the average noise power $N = \langle n^2(t) \rangle$.

As will be shown in Section 4.1, the spectrum of $\cos 2\varphi$ consists of a dc component $J_o(2C_2) \cos 2C_1$ and time-varying components at multiples of the satellite spin-rate Ω radians/sec. The averaging

operation removes the time-varying components, leaving

$$\langle \cos 2\varphi \rangle = J_0(2C_2) \cos 2C_1.$$

For a typical value of D/λ , say 100, C_2 is on the order of 314 radians and the argument of J_0 is on the order of 628. $J_0(628)$ being a very small number, the dc component of $\cos 2\varphi$ is also a very small number, i.e. $J_0(2C_2) \cos 2C_1 \ll 1$. The ratio $(P/N)_i$ of average signal power to average noise power at the input to the DSB-demodulator is

$$(P/N)_i = \frac{A_o^2}{\langle n^2(t) \rangle} \quad (17)$$

The process of coherent DSB-demodulation involves mixing the IF waveform $s_{IF}(t)$ with a locally generated signal

$$\cos \left(\omega_{IF} t - \frac{\omega r}{c} \right). \text{ The IF waveform can be rewritten as}$$

$$2 A_o \cos \varphi \cos \left(\omega_{IF} t - \frac{\omega r}{c} \right) + n_i \cos \left(\omega_{IF} t - \frac{\omega r}{c} \right) - n_q \sin \left(\omega_{IF} t - \frac{\omega r}{c} \right) \quad (18)$$

where n_i and n_q are the inphase and quadrature components of noise $n(t)$. Multiplying (18) by a local signal $\cos \left(\omega_{IF} t - \frac{\omega r}{c} \right)$ and excluding high frequency components, the demodulator output becomes

$$u(t) = A_o \cos \varphi + \frac{n_i(t)}{2} \quad \text{or}$$

$$= A_o \cos \varphi + w(t) \quad (19)$$

where $w(t)$ is the zero-mean additive gaussian noise assumed at the input to the MAP processor in Section 3.2.2.

The average power in $u(t)$ is obtained next:

$$\begin{aligned}\langle u^2(t) \rangle &= \langle A_o^2 \cos^2 \varphi \rangle + \langle w^2(t) \rangle \\ &= \frac{1}{2} A_o^2 \langle 1 + \cos 2\varphi \rangle + \frac{1}{4} \langle n_i^2(t) \rangle \\ &\simeq \frac{1}{2} A_o^2 + \frac{1}{4} \langle n_i^2(t) \rangle\end{aligned}$$

The ratio $(P/N)_d$ of average signal power to average noise power at the demodulator output is:

$$(P/N)_d = \frac{\frac{1}{2} A_o^2}{\langle w^2(t) \rangle} = \frac{2A_o^2}{\langle n^2(t) \rangle} \quad (20)$$

since the mean square noise $\langle n^2(t) \rangle$ is equal to the average power in the inphase or quadrature components of noise, i.e.

$$\langle n^2(t) \rangle = \langle n_i^2(t) \rangle = \langle n_q^2(t) \rangle$$

Equations (17) and (20) reveal the well-known characteristic of DSB-coherent demodulation of introducing a 3 dB gain in the signal-to-noise power ratio.

To see the difference in noise power spectral density (PSD) between output and input of the DSB-demodulator, express the average noise powers as the product of bandwidth and PSD at the demodulator output and input. The IF bandwidth of the received signal is, say B centered on the frequency ω_{IF} . The average noise power into

the demodulator is then

$$\langle n^2(t) \rangle = 2B \cdot N_{in} \text{ watts} \quad (21)$$

where N_{in} watts/Hz is the two-sided PSD of input noise.

After detection the average noise power over bandwidth B in $u(t)$ is

$$\begin{aligned} \langle w^2(t) \rangle &= \frac{1}{4} \langle n_i^2(t) \rangle = \frac{1}{4} \langle n^2(t) \rangle \\ &= B \cdot N_{out} \text{ watts} \end{aligned} \quad (22)$$

where N_{out} watts/Hz is the two-sided PSD of output noise.

From (21) and (22), $N_{in} = 2 N_{out}$, i.e. if the PSD of input noise $n(t)$ is N_o watts/Hz, then the PSD of output noise $w(t)$ is $\frac{N_o}{2}$ watts/Hz as will be assumed in Section 3.2.2.1.

The ratio of average signal power to noise power spectral density at the input to the DSB-demodulator is A_o^2/N_o . This ratio will be of importance in the definition of a system performance factor in Section 3.2.2.3 where the close correspondence between the receiver signal-to-noise ratio and observation time is discussed.

3.0 CRAFT-BORNE RECEIVER DESIGN

3.1 Introduction to Design

The received signal $s_R(t)$ may be viewed, as discussed in Section 2.2.1, either as the sum of two phase-modulated signals or as a double-sideband suppressed-carrier (DSB) waveform. Three stages may be distinguished in the signal processing to be done at the navigating craft: the RF-to-IF conversion, the isolation of the intelligence $A_o \cos \varphi(t, \underline{C})$ and finally the processing of this quantity to yield position information.

The first stage is accomplished in the conventional manner by providing a local oscillator that heterodynes the incoming signal down to some intermediate frequency range. The next stage involves the coherent demodulation of the DSB waveform. Possibilities for incoherent detection are also briefly discussed.

The last stage may be approached in basically two ways:

(a) by making measurements of the signal parameters C_i by phase-measuring devices and using these measured values in the navigation equations to solve for the position coordinates; (b) alternatively, it may be assumed that some a priori information on the user's position is available--a reasonable assumption since on-board dead-reckoning systems, inertial or otherwise, are expected to be carried by most large aircraft [57]. With such information assumed available, a processor optimum in the maximum a posteriori (MAP) sense can be designed. It may be noted that the satellite-aided radio navigation system could be viewed as a method to update or calibrate the self-contained on-board equipment.

3.1.1 DSB Demodulation by Coherent Methods

The demodulation of a DSB signal has been treated [65] as a special case of the general problem of estimating a continuous waveform which is modulated either linearly (AM, DSB, SSB) or nonlinearly (FM, PM). For the design of an optimum demodulator a suitable criterion of optimality must be selected and the estimator equations derived. Logical choices for the criterion are the minimum mean-square error (MMSE) and the maximum a posteriori probability (MAP) criteria. Both lead to identical answers for linear modulation schemes that are of interest here, but in nonlinear schemes both criteria have advantages and disadvantages depending on the properties of the intelligence to be extracted [65].

The line of attack followed in designing an optimum DSB demodulator is common to the construction of both realizable and unrealizable structures. An "unrealizable" filtering problem is simply one wherein an infinite delay is permitted. Although the problem of realizable demodulation in the presence of white noise is of interest here, it may be noted that the performance of these "practical" structures is quite close to the optimum unrealizable ones.

Van Trees [65] has shown that the optimum estimation for a DSB signal whose intelligence is disjoint from the carrier is demodulation by a coherent carrier waveform followed by a conventional optimum low-pass filter as shown in Fig. 7.

In a practical system the assumption that the carrier is known exactly at the receiver is not realistic, due to the variation

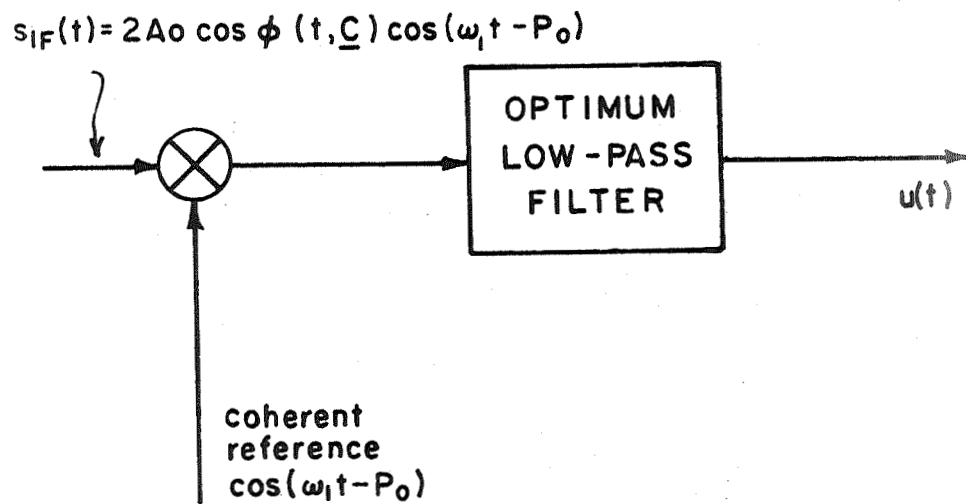


FIGURE 7 OPTIMUM DEMODULATOR FOR DOUBLE-SIDEBAND SUPPRESSED-CARRIER (DSB) SIGNAL

in phase and frequency introduced by the propagation delay, craft velocity and drifts in the oscillators at both transmitter and receiver. The actual carrier must be supplied in some manner for coherent demodulation.

Several ways of doing this have been discussed in the literature [56,65,60]. One method would be to transmit a low-power pilot carrier from a stationary antenna on the satellite. Since such a signal, when received by the user, will contain the effects of craft motion and propagation delay but not of the position-dependent phase quantity φ , it will be the appropriate coherent reference to use in the post-IF mixer. A phase-locked loop (PLL) can lock on the pilot and produce a reference signal at the output of the voltage-controlled oscillator (VCO) which may be used to coherently demodulate the DSB waveform.

Another method would be to transmit, not a low power "pilot" but a high power carrier from a satellite-borne stationary antenna. This would cause a conventional AM signal to be received:

$$s_R(t) = 2A_o [1 + \cos \varphi] \cos \left[\omega t - \frac{\omega r}{c} \right] + \text{noise}$$

Despite the obvious advantage of simplifying the user-borne equipment by requiring only an envelope detector to extract the intelligence, this method is unattractive because of the much higher transmitter power required at the satellite. Consequently, it is not further considered here. Instead, carrier synthesis methods are explored in the rest of this section.

The coherent methods to be discussed here involve the synthesis of the absent carrier from the sidebands of the received signal. Two methods are presented, one using a square-law device to deliver an appropriate frequency which is then tracked by a conventional phase-locked loop (PLL) whose output provides a coherent reference signal for demodulation purposes. The extraction of the reference depends on passing the received signal through a non-linearity.* The square-law device may be replaced by other nonlinearities such as multipliers as in the Costas loop [16] to achieve essentially the same performance as the squaring-loop method.

(1) Squaring-Loop Method (Fig. 8)

The input to the square-law device is $s_{IF}(t)$ as given in Eq. (15); replacing $\omega_{IF}t - P_1t - P_0$ by $\omega_1t - P_0$, this signal may be rewritten as:

$$s_{IF}(t) = 2A_0 \cos \varphi(t, \underline{C}) \cos (\omega_1t - P_0)$$

Squaring this function and replacing $\varphi(t, \underline{C})$ by φ produces

$$s_{IF}^2(t) = A_0^2 \left[1 + \cos 2\varphi + \cos 2(\omega_1t - P_0) + \frac{1}{2} \cos 2(\omega_1t - P_0 - \varphi) + \frac{1}{2} \cos 2(\omega_1t - P_0 + \varphi) \right] \quad (23)$$

which contains a double frequency component $\cos 2(\omega_1t - P_0)$ which is tracked by a PLL. Since a frequency doubler follows the VCO, the VCO-output signal is $\cos(\omega_1t - P_0 + \theta_e)$ where θ_e is the tracking

* Viterbi [66] presents an interesting argument to explain why a linear system cannot synthesize the synchronizing signal.

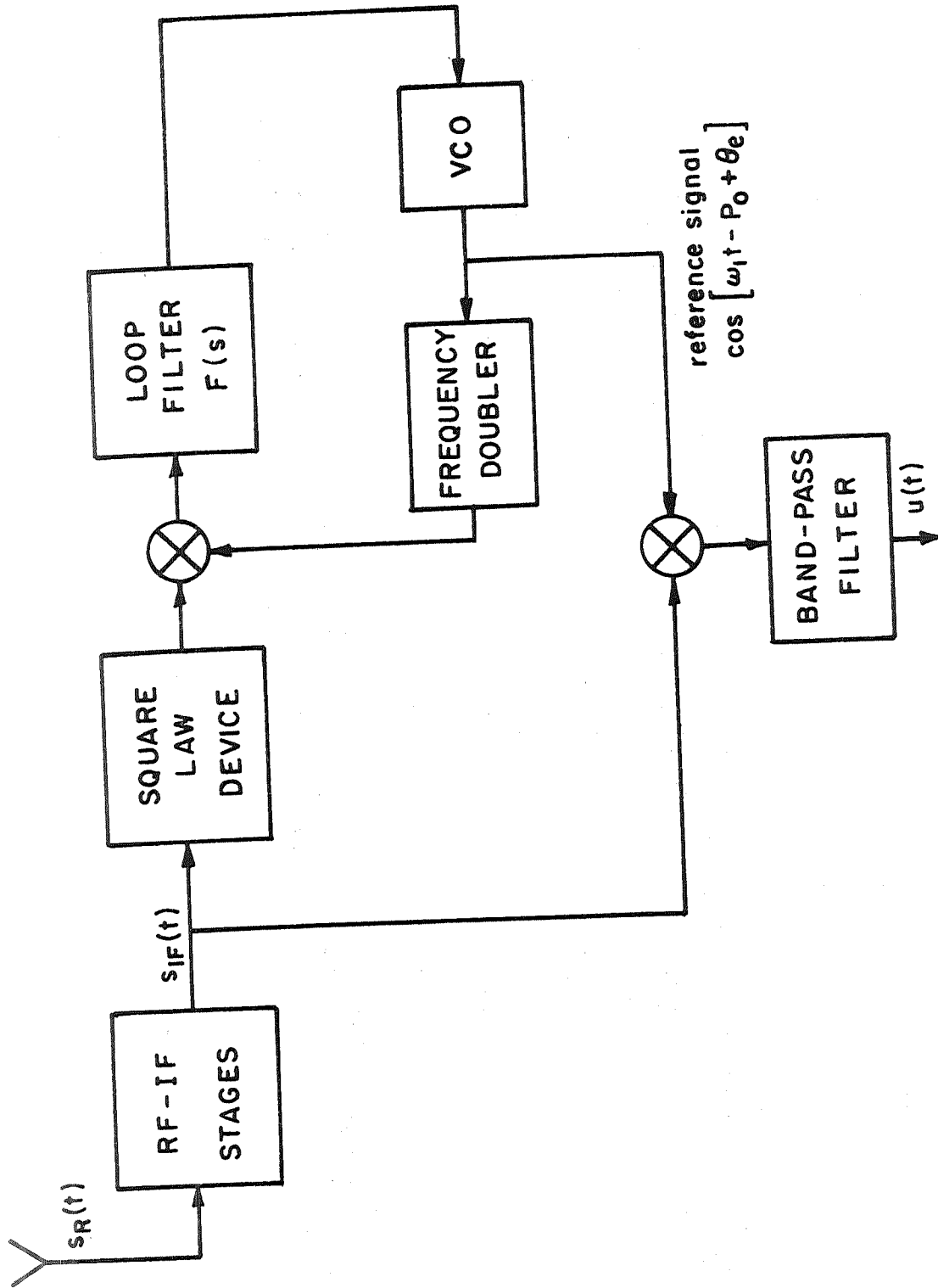


FIGURE 8 DEMODULATION BY SQUARING-LOOP

error. This constructed carrier is beaten against $s_{IF}(t)$, resulting in the low-pass waveform $u(t)$ that follows the removal of an accompanying double-frequency term by a low-pass filter:

$$u(t) = A_o \cos \varphi \cos \theta_e$$

indicating that the tracking error causes a power reduction by a factor of $\cos^2 \theta_e$ in the signal that has to be processed to yield position information. An estimate of typical θ_e is to be obtained and the performance of this system in the presence of additive noise is to be examined. Representing the entire IF signal as $s(t) + n(t)$, $n(t)$ being white gaussian noise of two-sided PSD N_o watts/Hz, the square-law device puts out

$$\begin{aligned} \left[s(t) + n(t) \right]^2 &= s^2(t) + 2s(t)n(t) + n^2(t) \\ &= s^2(t) + v(t) \end{aligned} \quad (24)$$

where $v(t)$ includes both the noise term due to the interaction of $n(t)$ with itself and that due to the interaction of $n(t)$ with $s(t)$. Since $s(t)$ and $n(t)$ are zero-mean independent processes assumed to be wide-sense stationary, the mean and auto-correlation functions of $v(t)$ are

$$\begin{aligned} \langle v(t) \rangle &= \langle n^2(t) \rangle = R_n(0) \\ R_v(\tau) &= 4 R_s(\tau) R_n(\tau) + 2 R_n^2(\tau) \end{aligned}$$

The above result [20,66] may be transformed to yield the PSD of $v(t)$:

$$S_v(\omega) = 4 \int_{-\infty}^{\infty} S_n(\omega - \zeta) S_s(\zeta) \frac{d\zeta}{2\pi} + 2 \int_{-\infty}^{\infty} S_n(\omega - \zeta) S_n(\zeta) \frac{d\zeta}{2\pi}$$

where S_n , S_s are PSD's of $n(t)$ and $s(t)$ respectively. The PLL may be assumed to have a very narrow loop bandwidth $B_L \ll B$ where B is the IF bandwidth. The PSD of $v(t)$ may therefore be approximated by its value at $\omega = 2\omega_1$. The one-sided PSD of $v(t)$, N_v , is necessary to obtain the variance of the tracking error.

$$N_v = 2S_v(2\omega_1) = 8 \int_{-\infty}^{\infty} S_n(2\omega_1 - \zeta) S_s(\zeta) \frac{d\zeta}{2\pi} \quad (25)$$

$$+ 4 \int_{-\infty}^{\infty} S_n(2\omega_1 - \zeta) S_n(\zeta) \frac{d\zeta}{2\pi}$$

The two-sided PSD of the input noise $n(t)$ has been taken to be N_o watts/Hz. This may be substituted in Eq. (25) for $S_n(2\omega_1 - \zeta)$, while $S_s(\zeta)$ is integrated over B Hz or in the range $2\omega_1 - \pi B \leq \omega \leq 2\omega_1 + \pi B$. The result is

$$N_v = 8N_o \int_B S_s(\zeta) \frac{d\zeta}{2\pi} + 4N_o^2 \int_B \frac{d\zeta}{2\pi}$$

The signal power is A_o^2 as shown in Section 2.2.2 but since the integration is over only the positive frequencies

$$\int_B S_s(\zeta) \frac{d\zeta}{2\pi} = \frac{1}{2} A_o^2$$

Therefore, the one-sided PSD of $v(t)$ may be written as

$$N_v = 4N_o A_o^2 + 4N_o^2 B \quad (26)$$

Assuming that the phase error θ_e is small enough for the linear model of the PLL to be valid, the variance of $2\theta_e$ may be found by using [66]

$$\begin{aligned}\sigma_{2\theta_e}^2 &= \frac{N_v (\text{Loop Bandwidth } B_L)}{\text{Signal Power}} \\ &= \frac{N_v B_L}{A_o^4/2}\end{aligned}\quad (27)$$

where $A_o^4/2$ is the power of the double frequency term in Eq. (23).

Using Eq. (26), the variance of $2\theta_e$ is seen to be

$$\begin{aligned}\sigma_{2\theta_e}^2 &= \frac{8 N_o B_L}{A_o^2} + \frac{8 N_o^2 B B_L}{A_o^4} \\ &= \frac{8}{A_o^2/N_o B_L} \left[1 + \frac{1}{A_o^2/N_o B} \right]\end{aligned}\quad (28)$$

from which the tracking error variance is

$$\sigma_{\theta_e}^2 = \frac{1}{4} \sigma_{2\theta_e}^2 = \frac{2}{A_o^2/N_o B_L} \left[1 + \frac{1}{A_o^2/N_o B} \right]\quad (29)$$

For a typical IF bandwidth of 2 kHz, loop bandwidth of 10 Hz and A_o^2/N_o of 1000 (30 dB), $\sigma_{\theta_e}^2 = 0.06$ or rms tracking error is on the order of 0.244 radian (14°). For higher SNR which is much more likely, the phase error due to noise is much less. Even at 14° , the power reduction factor $\cos^2 \theta_e$ is around 0.94 which does not affect $u(t)$ very much.

At this point it is necessary to consider the order of the PLL, i.e. the form of the loop filter transfer function $F(s)$. One reason is that the loop bandwidth B_L depends on the order of the PLL,

and therefore so does the loop signal-to-noise ratio $A_o^2/N_o B_L$. Furthermore, the PLL can successfully track the desired carrier waveform only if the loop filter transfer function is of a form suitable for the type of propagation delay $\frac{\omega r}{c}$ encountered.

In Section 2.2.1 it is seen that $\frac{\omega r}{c}$ is of the form $\frac{1}{2} P_2 t^2 + P_1 t + P_0$. Numerically evaluating the coefficients for the geographical region of interest, P_2 is found to be very small in comparison to P_1 . However, for purposes of design, the more complete form of $\frac{\omega r}{c}$ should be assumed. It is well known [63,66] that for a received signal whose phase is of the form $\frac{1}{2} P_2 t^2 + P_1 t + P_0$ an unbounded steady-state phase error results when either a first-order loop $[F(s) = 1]$ or an imperfect second-order loop $\left[F(s) = \frac{s+a}{s+\epsilon}\right]$ is used. The "imperfect" second-order loop is a consequence of not being able to mechanize the filter $F(s)$ with a perfect integrator. The quantities a and ϵ are functions of the resistors and capacitors used to construct $F(s)$. If however these circuit components are chosen so that $\epsilon \approx 0$, a perfect second-order loop results $\left[F(s) = 1 + a/s\right]$ and a bounded steady-state phase error is possible. The magnitude of this error is equal to $P_2/(a \cdot \text{loop gain} \cdot \text{signal amplitude})$.

For $P_2 \approx 0$ and the received signal having a propagation delay of $P_1 t + P_0$, the VCO quiescent frequency may be different from the incoming frequency ($\omega_{IF} - P_1$) or ω_1 by some fixed amount. No error due to this results if a perfect second-order loop or a third-order loop is used. A first-order loop introduces a steady-state error equal to

$$\frac{(\text{difference between VCO and incoming frequencies})}{(\text{loop gain} \cdot \text{signal amplitude})}$$

An imperfect second-order loop does not eliminate this error but reduces it by a factor of ϵ/a . For $P_2 \neq 0$, a third-order loop must be used to obtain an error-free output. In any case it may be assumed that a proper choice of loop filter transfer function is made so that the only phase tracking error is that due to additive noise as estimated in Eq. (29).

It is possible that due to some unpredictable malfunction of the equipment such as deterioration in the components of the VCO, the PLL may lock onto one of the sidebands $(A_o^2/2) \cos 2(\omega_1 t - P_o \pm \varphi)$ instead of the desired signal $A_o^2 \cos 2(\omega_1 t - P_o)$. In this event the output of the post-loop multiplier will be proportional to $\cos^2 \varphi$ instead of $\cos \varphi$. Fortunately the spectra of these two waveforms are disjoint in frequency, as seen in Appendix B, and a bandpass filter may be used to discriminate against the unwanted quantity. In the absence of noise, then, the output $u(t)$ to the post-demodulation processor will be zero when the PLL fails to properly lock rather than some nonzero quantity that will cause undetected and possibly large position errors.

(2) Costas Loop Method

Another coherent demodulation scheme first described by Costas [16] involves passing $s_{IF}(t)$ through essentially two basic receivers with local oscillator signals in phase quadrature to each other as shown in Fig. 9. If the local oscillator, a VCO, is perfectly synchronized to the constructed carrier, the output of the inphase

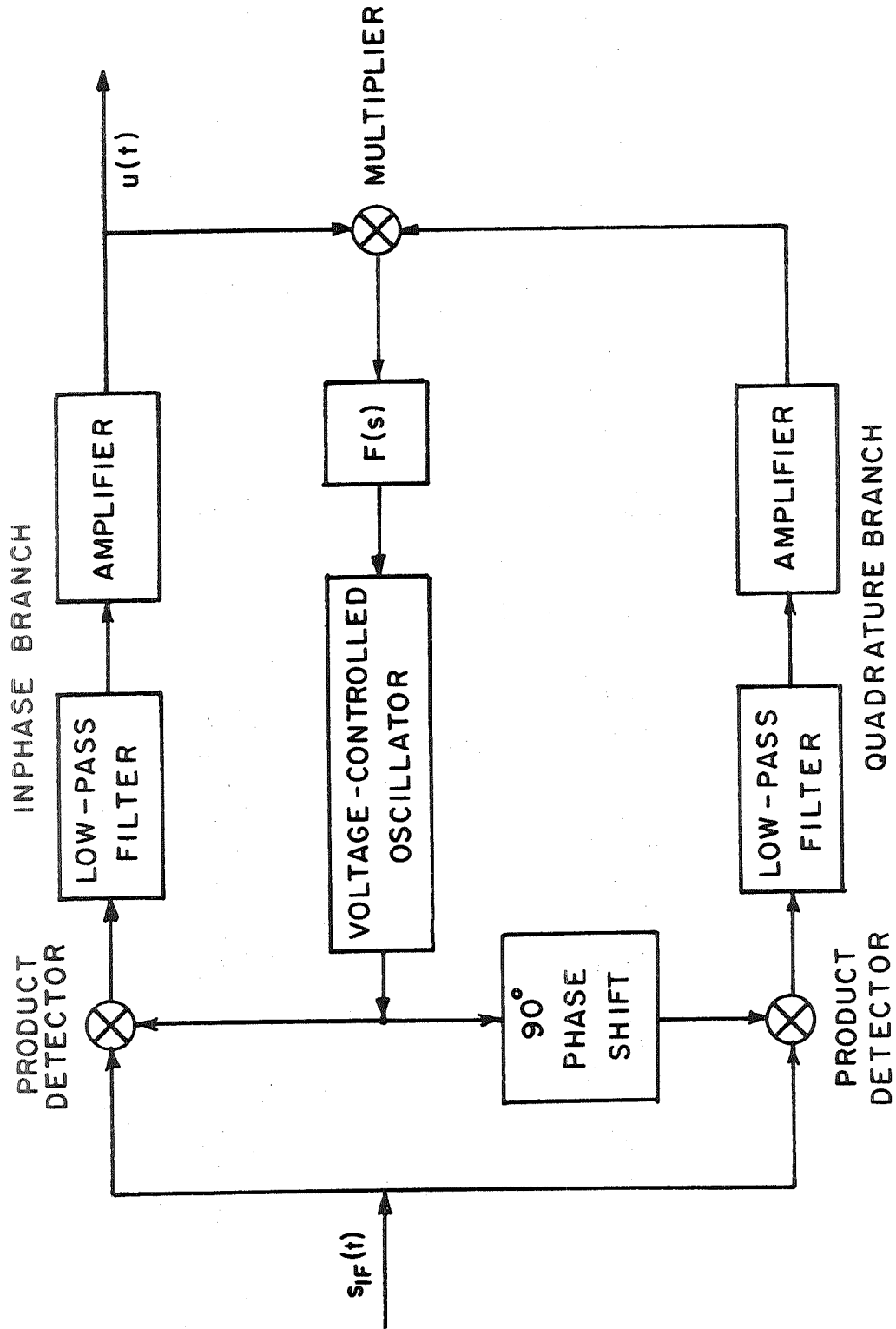


FIGURE 3 DEMODULATION BY COSTAS LOOP

amplifier will simply be the desired intelligence plus additive noise, $A_0 \cos \varphi(t, \underline{C}) + n_i/2$, and the output of the quadrature amplifier will be zero.

It is shown [46] that the differential equations that govern the behavior of the squaring-loop and the Costas loop are identical, leading to identical expressions for the probability density of the steady-state tracking error. The choice of which circuit to use therefore must be made on the basis of engineering hardware.

3.1.2 Possible Noncoherent Methods for DSB Demodulation

Although coherent methods are generally considered necessary for the demodulation of DSB signals, it must be noted that the present work is less concerned with faithfully reproducing the "intelligence" $A_0 \cos \varphi(t, \underline{C})$ than it is with extracting position-information. For this reason, incoherent methods may be used in the post-heterodyning stage.

In the coherent methods discussed in the preceding section, the removal of carrier results in simply $A_0 \cos \varphi(t, \underline{C})$, and the remaining problem is to optimally extract position-coordinates from this quantity. Incoherent methods do not result in $A_0 \cos \varphi(t, \underline{C})$ but rather some function of it.

A square-law detector, with an input of

$$s_{IF}(t) = 2A_0 \cos \varphi(t, \underline{C}) \cos \left(\omega_{IF} t - \frac{\omega r}{c} \right)$$

will have at its output, after filtering out the high-frequency terms

$$s_{IF}^2(t) = 2A_0^2 \cos^2 \varphi(t, \underline{C})$$

which must be processed to yield the desired information.

Another possibility would be a half-wave (piece-wise) linear detector whose output

$$s_o(t) = 2A_o \left| \cos \varphi(t, \underline{C}) \right|$$

would have to be processed. These possibilities are not explored further simply because each one would entail separate analyses of post-detection processing, and could be the subject of further work.

3.2 Post-Detection Processing Methods

3.2.1 Intuitive Schemes

The input $u(t)$ to the position-information extraction stage has been derived in Section 3.1.1 as

$$u(t) = A_o \cos \varphi(t, \underline{C}) + \frac{1}{2} n_i(t) \quad (30)$$

This representation, although suitable for Section 3.2.2, is not very convenient here as it is necessary to know "phase noise" as a function of the input signal-to-noise ratio when phase-measuring equipment is part of the processor. To get an alternate expression for $u(t)$, let the IF output be written as

$$\begin{aligned} s_{IF}(t) &= \left\{ A_o \cos \left[\omega_{IF} t - \frac{\omega_r}{c} - \varphi \right] + \frac{1}{2} n(t) \right\} + \\ &\quad \left\{ A_o \cos \left[\omega_{IF} t - \frac{\omega_r}{c} + \varphi \right] + \frac{1}{2} n(t) \right\} \\ &= A' \cos \left[\omega_{IF} t - \frac{\omega_r}{c} + \xi_1 \right] + A'' \cos \left[\omega_{IF} t - \frac{\omega_r}{c} + \xi_2 \right] \end{aligned} \quad (31)$$

where

$$\left. \begin{aligned} \tan \xi_1 &= \frac{-A_o \sin \varphi + \frac{1}{2} n_q}{A_o \cos \varphi + \frac{1}{2} n_i} \\ \tan \xi_2 &= \frac{A_o \sin \varphi + \frac{1}{2} n_q}{A_o \cos \varphi + \frac{1}{2} n_i} \\ A' &= A_o \left[1 + \frac{n_i^2 + n_q^2}{4A_o^2} + \frac{n_i}{A_o} \cos \varphi - \frac{n_q}{A_o} \sin \varphi \right]^{1/2} \\ A'' &= A_o \left[1 + \frac{n_i^2 + n_q^2}{4A_o^2} + \frac{n_i}{A_o} \cos \varphi + \frac{n_q}{A_o} \sin \varphi \right]^{1/2} \end{aligned} \right\} \quad (32)$$

ξ_1 and ξ_2 are the noisy phase terms; A' and A'' are the noisy amplitudes which may be seen to be equal to A_0 for a low-noise situation.

If $\xi_1 \equiv -\varphi + \alpha_1$ and $\xi_2 \equiv \varphi + \alpha_2$, where α_1 and α_2 are the phase noise terms to be investigated, then Eq. (31) may be rewritten as

$$\begin{aligned} s_{IF}(t) &= A_0 \left[\cos \left(\omega_{IF} t - \frac{\omega r}{c} - \varphi + \alpha_1 \right) + \cos \left(\omega_{IF} t - \frac{\omega r}{c} + \varphi + \alpha_2 \right) \right] \\ &= 2A_0 \cdot \cos \left(\varphi + \frac{\alpha_2 - \alpha_1}{2} \right) \cos \left(\omega_{IF} t - \frac{\omega r}{c} + \frac{\alpha_1 + \alpha_2}{2} \right) \end{aligned}$$

The noisy term $\frac{\alpha_1 + \alpha_2}{2}$ in the carrier part is of no interest since synchronous detection may be assumed to remove the entire carrier, leaving

$$u(t) = A_0 \cos \left(\varphi + \frac{\alpha_2 - \alpha_1}{2} \right) \quad (33)$$

The phase errors α_1 and α_2 may be written as

$$\begin{aligned} \alpha_1 &= \left(\frac{1}{2} n_i \right) \frac{\partial \xi_1}{\partial (\frac{1}{2} n_i)} + \left(\frac{1}{2} n_q \right) \frac{\partial \xi_1}{\partial (\frac{1}{2} n_q)} \\ \alpha_2 &= \left(\frac{1}{2} n_i \right) \frac{\partial \xi_2}{\partial (\frac{1}{2} n_i)} + \left(\frac{1}{2} n_q \right) \frac{\partial \xi_2}{\partial (\frac{1}{2} n_q)} \end{aligned}$$

Performing the indicated algebra and making the high signal-to-noise ratio assumption,

$$\alpha_1 = \frac{n_i}{2A_0} \sin \varphi + \frac{n_q}{2A_0} \cos \varphi$$

$$\alpha_2 = -\frac{n_i}{2A_o} \sin \varphi + \frac{n_q}{2A_o} \cos \varphi$$

and the phase noise in Eq. (33) becomes

$$\frac{\alpha_2 - \alpha_1}{2} = -\frac{n_i}{2A_o} \sin \varphi \equiv \varphi_n$$

The mean square phase noise $\langle \varphi_n^2 \rangle$ may be written as

$$\langle \varphi_n^2 \rangle = \frac{\langle n_i^2 \rangle}{4A_o^2} \langle \sin^2 \varphi \rangle \quad (34)$$

$\langle \sin^2 \varphi \rangle \approx \frac{1}{2}$ from the arguments in Sec. 2.2.2, $\langle n_i^2 \rangle = \langle n^2 \rangle \equiv N$ as is well known, and $A_o^2 \equiv P$, the signal power. Equation (34) may therefore be reduced to

$$\langle \varphi_n^2 \rangle = \frac{1}{8(P/N)_i} \quad (35)$$

where the subscript i stands for "input."

Now $N = N_o B$ where N_o is the noise PSD and B is the IF bandwidth.

Equation (35) becomes

$$\langle \varphi_n^2 \rangle = \frac{B}{8(A_o^2/N_o)} \quad (36)$$

Figure 10 shows the relation between the rms phase noise, $\sqrt{\langle \varphi_n^2 \rangle}$, and the signal power-to-noise PSD ratio, A_o^2/N_o , with D/λ as a parameter. D/λ is taken as a parameter, in lieu of the IF bandwidth only because they are closely related:

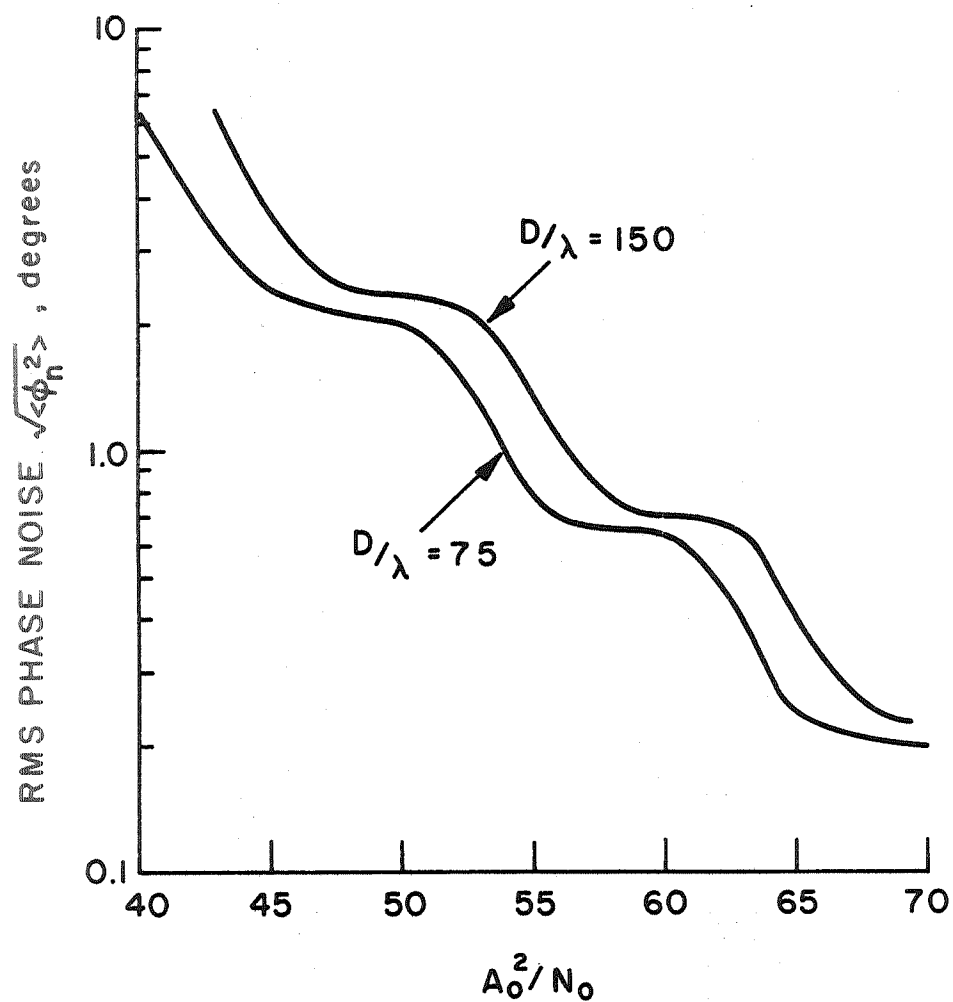


FIGURE 10 RMS PHASE NOISE PLOTTED AGAINST THE SIGNAL POWER-TO-NOISE POWER SPECTRAL DENSITY RATIO, WITH D/λ AS A PARAMETER

$$B = 2\pi \frac{D\Omega}{\lambda} \cos \theta \quad (37)$$

The bandwidth B may be obtained in two ways. The direct method would be to expand $\cos \phi$ into an infinite sum of terms that are products of trigonometric and Bessel functions and limit the bandwidth to include only the significant components. Rather than take this tedious approach, it is recognized here that the incoming waveform is the sum of two phase-modulated signals, each of which has an easily determined bandwidth. The two PM signals have identical spectra and their sum is likely to have the same bandwidth. Since the modulated phases are of opposite signs in the two waveforms, it is likely that some frequency components cancel and the resultant signal ends up with a somewhat smaller bandwidth. The assumption that the DSB signal has a bandwidth equal to that of either PM signal is thus seen to be the worst case.

Each PM signal has the form $\cos \left[\omega_{IF} t - \frac{\omega r}{c} \pm \phi \right]$ which may be written as

$$\cos \left[(\omega_{IF} - P_1)t - P_0 \pm C_1 \pm C_2 \cos \left(\Omega t - \frac{\Omega r}{c} - C_3 \right) \right]$$

when the expressions for the propagation delay and modulation ϕ are substituted for these quantities. The frequency deviation of this waveform has a magnitude of $C_2\Omega$ on either side of $(\omega_{IF} - P_1)$, the heterodyned carrier. Since C_2 , the phase-modulation index, is normally much larger than a radian, the bandwidth is equal to twice the maximum frequency deviation as in wideband FM. Hence the expression for B given in Eq. (37). It must be noted that B is centered on $(\omega_{IF} - P_1)$

not on ω_{IF} . This is possible since the coherent DSB demodulation techniques of Section 3.1.1 involve the synthesis of a carrier that includes the changing $\frac{\omega r}{c}$ term, thus permitting the extraction of the "intelligence" uncontaminated by $\frac{\omega r}{c}$. If this were not the case, in noncoherent demodulation techniques for example, the bandwidth would have to take the maximum change in carrier frequency into account. In this event B would be

$$B = 2\pi \frac{D}{\lambda} \Omega \cos \theta + (2P_1)_{\max}$$

a quantity larger by about 6 kHz for typical craft locations.

Two possible schemes to process $u(t)$ are outlined here. It must be noted that the extraction of user position requires the measurement of the phase quantity φ ; the instantaneous frequency φ' will not do because the time-constant part C_1 in φ which is indispensable to estimating Γ_x is not contained in φ' .

3.2.1.1 Method 1

The first approach is to extract φ' or the sinusoidal part of φ , mix it with $u(t)$ and follow this with low-pass filtering. Figure 11 indicates the process. The output of the discriminator, $u_d(t)$, assuming a noiseless situation for clarity, is proportional to the instantaneous frequency φ' ; with the delay $\frac{r}{c}$ implicit, $u_d(t)$ may be written as:

$$u_d(t) = -C_2 \Omega \sin(\Omega t - C_3)$$

where the discriminator constant of proportionality is ignored. A phase shift of 90° converts this into $C_2 \Omega \cos(\Omega t - C_3)$. On mixing

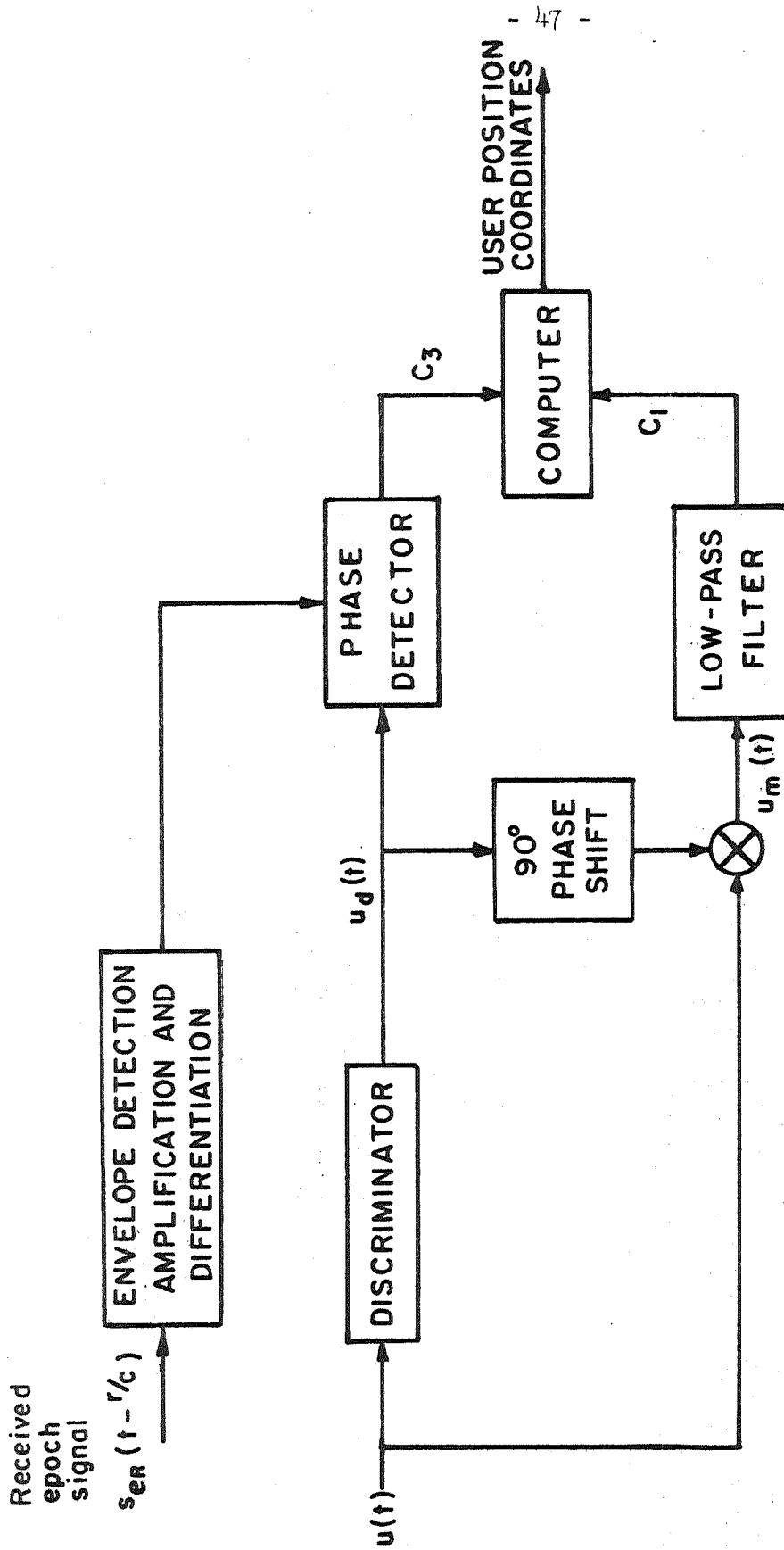


FIGURE 11 POSITION COORDINATE EXTRACTION BY INTUITIVE METHOD #1

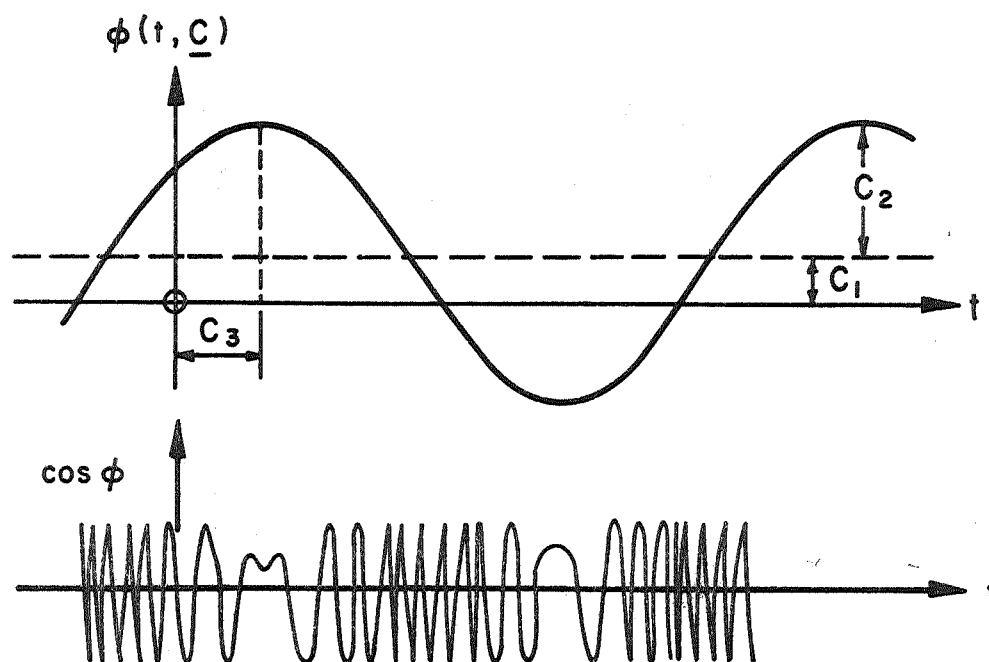


FIGURE 12 TIME DEPENDENCE OF $\phi(t, \underline{C})$ and $\cos \phi(t, \underline{C})$

this signal with $u(t)$, again assumed noiseless and ignoring the amplitude A_o , the result is $u_m(t)$:

$$\begin{aligned} u_m(t) &= C_2 \Omega \cos(\Omega t - C_3) \cos \varphi \\ &= C_2 \Omega \cos(\Omega t - C_3) \left[J_0(C_2) \cos C_1 - \right. \\ &\quad \left. 2J_1(C_2) \sin C_1 \cdot \cos(\Omega t - C_3) - \text{sinusoids at } 2\Omega, 3\Omega, \dots \right] \end{aligned}$$

$$u_m(t) \Big|_{\text{LPF}} = J_1(C_2) \sin C_1 \cdot C_2 \Omega$$

To a first approximation, C_2 is known since it is the least position-sensitive parameter as indicated by the elements of the sensitivity matrix in Section 4.1. Therefore the filtered signal above is proportional to $\sin C_1$. C_1 , being heavily dependent on Γ_x , may be used to evaluate this position-coordinate. The extraction of Γ_y requires the measurement of C_3 and to do this it is necessary to transmit an epoch signal to provide a time-(or phase-) reference. One possible way is shown in Fig.11 where a high-pass filter isolates this epoch signal and feeds it at a phase-detector whose output is C_3 . Essentially the detector measures the time-(or phase-) difference between the epoch signal and the point of maximum or minimum frequency of $\cos \varphi(t)$ as shown in Fig.12. The epoch signal is discussed in Appendix E.

3.2.1.2 Method 2

An alternative method (Fig.13) is to introduce a time delay δ and correlate this delayed signal with the undelayed $\cos \varphi(t)$. The mixer output $u_m(t)$ is

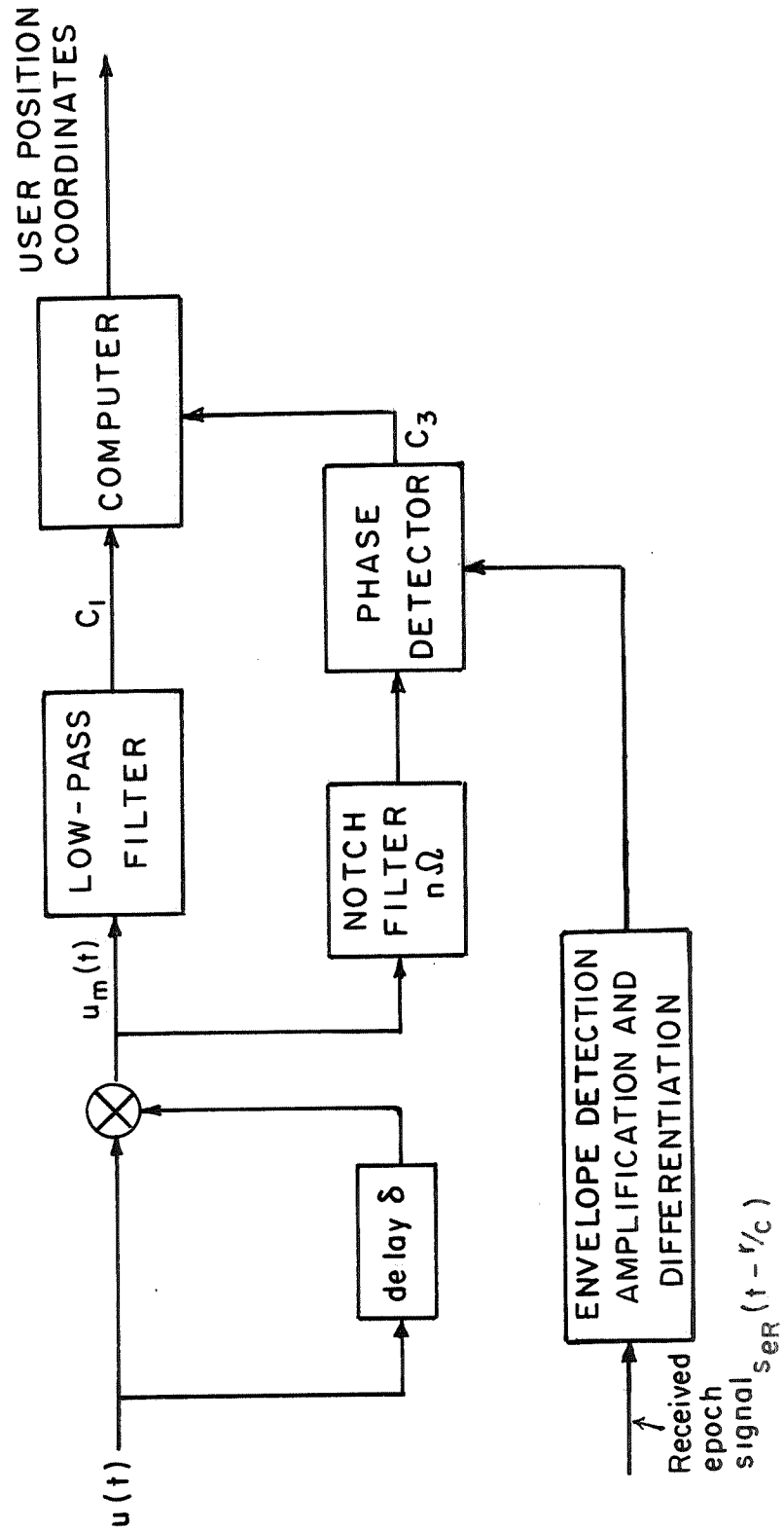


FIGURE 13 POSITION COORDINATE EXTRACTION BY INTUITIVE METHOD #2

$$\begin{aligned}
 u_m(t) &= \cos \varphi(t) \cos \varphi(t + \delta) \\
 &= \frac{1}{2} \cos[\varphi(t) + \varphi(t + \delta)] + \frac{1}{2} \cos[\varphi(t) - \varphi(t + \delta)] \\
 &= \frac{1}{2} \cos[2C_1 + C_2 \cos(\Omega t - C_3) + C_2 \cos(\Omega t + \Omega \delta - C_3)] \\
 &\quad + \frac{1}{2} \cos[C_2 \cos(\Omega t - C_3) - C_2 \cos(\Omega t + \Omega \delta - C_3)]
 \end{aligned}$$

If δ is chosen so that $\Omega \delta = \pi$ or an odd multiple of it,

$$\begin{aligned}
 u_m(t) &= \frac{1}{2} \cos 2C_1 + \frac{1}{2} \cos [2C_2 \cos(\Omega t - C_3)] \\
 &= \frac{1}{2} [\cos 2C_1 + J_0(2C_2)] + \text{sinusoids at } 2\Omega, 4\Omega, 6\Omega \dots
 \end{aligned}$$

Low-pass filtering will give the dc term. Assuming as before that C_2 is approximately known, the dc term is measured to yield a quantity proportional to C_1 . The extraction of C_3 , as in Method 1, requires an epoch signal to provide a time reference in the phase detector. The other signal into the phase-detector is picked off by a notch filter tuned to any convenient frequency $n\Omega$, n being chosen so that the frequency has an amplitude of significant size. The measured values of C_1 and C_3 are processed to yield the position-coordinates Γ_x and Γ_y .

3.2.1.3 Appraisal of the Intuitive Schemes

Both of the above methods have the obvious disadvantage of not using all the information available to the navigator concerning

the nature of the received signal as well as his prior knowledge of position. These methods share the disadvantage of requiring an epoch signal to provide a time-reference with the more sophisticated MAP processor to be discussed in Section 3.2.2.

Quite apart from this problem, Figs. 14 and 15 indicate that a 1 nmi resolution in the Γ_x coordinate will require C_1 to be measured to within 0.65° (at $D/\lambda = 100$), and a similar resolution of 1 nmi in the Γ_y coordinate will require C_3 to be measured to within 0.003° . Even assuming that phase detectors can be found to do the job, which is very unlikely for the C_3 measurement, the impracticality of doing this is evident from Fig. 10 where the rms phase noise is plotted against the signal power-to-noise PSD ratio. For $D/\lambda = 100$ and $A_O^2/N_O = 55$ dB, the phase noise is approximately 1.0° . Increasing A_O^2/N_O to 70 dB, the phase noise is still over 0.2° .

The phase detectors being able to read only the modulo 2π values of the phases C_1 and C_3 permit ambiguities in the determination of position. The earth-distance between ambiguous positions is of interest. From the expression for C_1 , a change ΔC_1 in electrical phase may be related to a change $\Delta \Gamma_x$ in spatial angle as follows:

$$\Delta C_1 = \pi \frac{D}{\lambda} \sin \theta \Delta \Gamma_x$$

Ambiguities occur when ΔC_1 exceeds 2π radians, or

$$\Delta \Gamma_x = \frac{2}{(D/\lambda) \sin \theta}$$

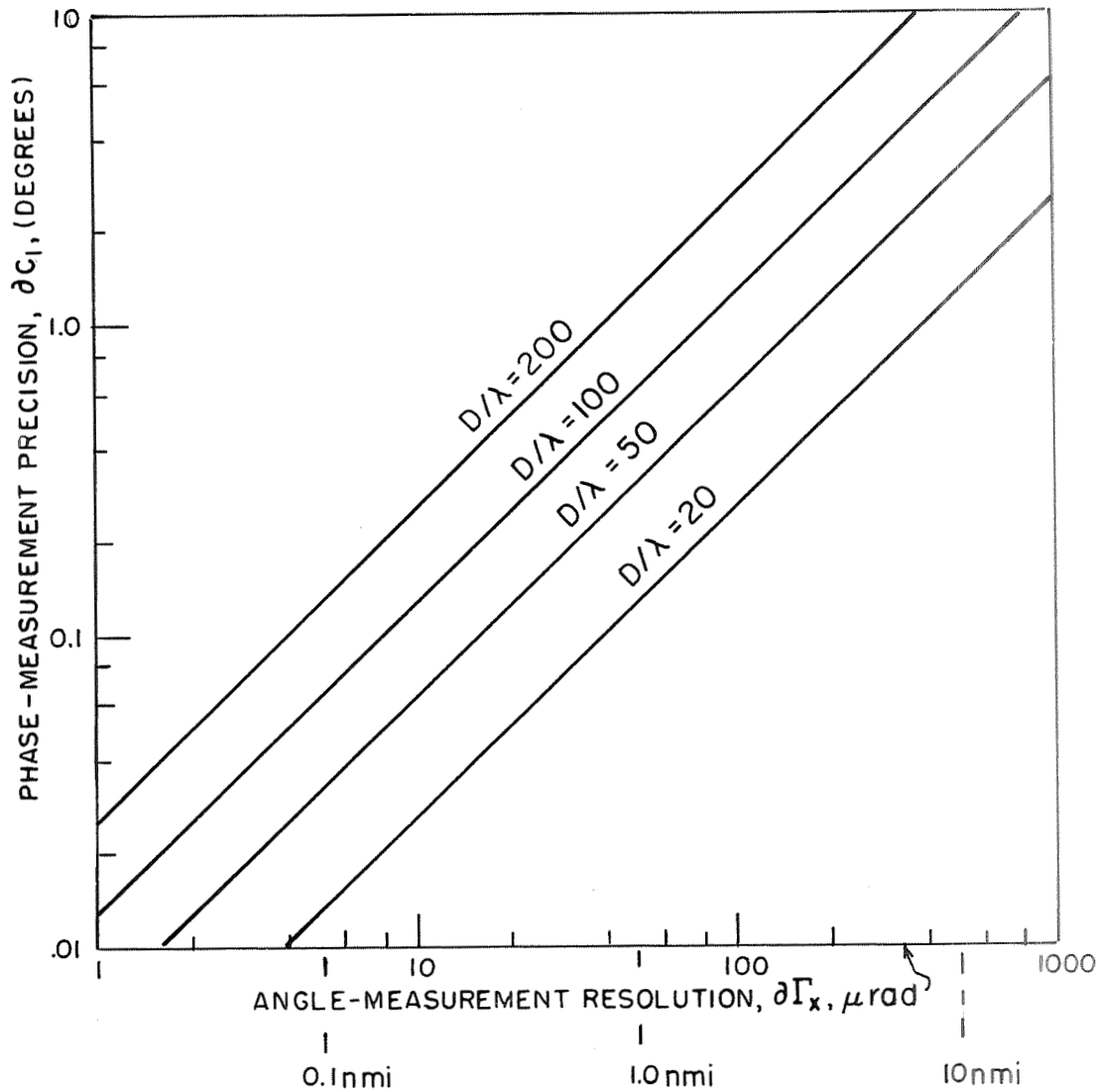


FIGURE 14 PHASE-MEASUREMENT PRECISION δC_1 (in degrees) PLOTTED AGAINST SPATIAL ANGLE-MEASUREMENT RESOLUTION $\delta \Gamma_x$ (IN MICRORADIANS AND NAUTICAL MILES), WITH D/λ AS A PARAMETER [$\theta = 45^\circ$]. GDOP NOT TAKEN INTO ACCOUNT.

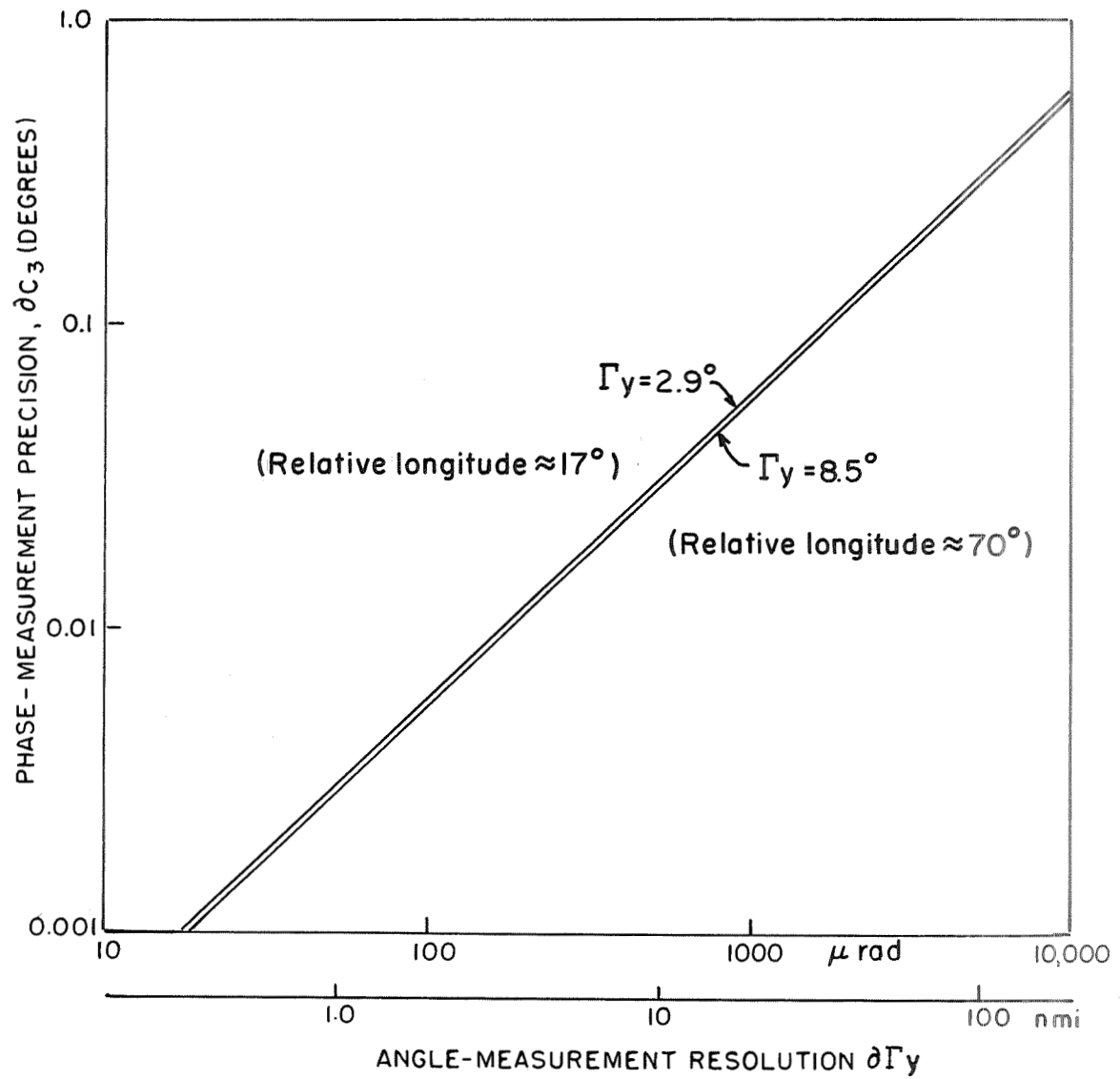


FIGURE 15 PHASE-MEASUREMENT PRECISION ∂C_3 PLOTTED AGAINST SPATIAL ANGLE-MEASUREMENT RESOLUTION $\partial \Gamma_y$. GDOP NOT TAKEN INTO ACCOUNT.

For $D/\lambda = 100$ and $\theta = 45^\circ$, $\Delta\Gamma_x$ corresponds to a distance of 550 nmi at very low latitudes increasing to 1760 nmi at 62°N latitude. At lower values of D/λ the spacing between ambiguities becomes much larger. At $D/\lambda = 20$, for instance, $\Delta\Gamma_x$ increases by a factor of five. The user's a priori knowledge of his general position will resolve this ambiguity.

The measurement of C_3 does not introduce ambiguities since the electrical angle ranges from 81.5° to 90° as may be seen by substituting 0° and 8.5° as the minimum and maximum values of Γ_y in the expression for C_3 .

The intuitive schemes are seen to be brute-force approaches to the navigation problem. For one thing, they involve measurements made of the entire (absolute) phase terms C_i , whereas it might be better to compare the incoming phase terms with phases corresponding to a preliminary estimate of craft location. This comparison procedure exploits the higher accuracy possible in a relative measurement. Furthermore, the intuitive methods do not provide the opportunity of using known characteristics of the signal, such as the rates of change of $\cos \varphi(t, \underline{C})$ with C_i , or the sensitivity relationships $\frac{\partial C_i}{\partial \Gamma_j}$, quantities that are known for a given craft location. A systematic design procedure for a processor that uses all this information is presented in the next section.

3.2.2 Maximum A Posteriori (MAP) Processor

The objective in this section is to design a processor that extracts craft position coordinates from the output $u(t)$ of the DSB demodulator in some optimal fashion, i.e. making use of as much information about the craft position and characteristics of the signal as possible. To do this a grid of "reference" points over the geographical field of users is postulated. These points may be fixed along a particular route, or they may be "floating", i.e. any point in the immediate neighborhood of the craft. In either case the values of C_1 , C_2 and C_3 and therefore the corresponding noiseless signal should be available for each grid-point. The on-board dead-reckoning equipment of the craft can be expected to give an estimate $\underline{\eta}$ of the position deviation from a particular grid-point. If $\underline{\eta}$ is known to be different from the actual position deviation $\underline{y}$ by an error $\underline{\epsilon}$ whose distribution has been determined from past flights, then an improved estimate $\hat{\underline{y}}$ is possible.

3.2.2.1 Derivation of Processor Equation

The output of the DSB demodulator is $A_o \cos \varphi$ mixed with noise. Defining $A_o \cos \varphi$ as the "intelligence" $s(t, \underline{C})$, the signal $u(t)$ at the input to the MAP processor may be written as

$$u(t) = s(t, \underline{C}) + w(t) \quad (38)$$

where the vector $^* \underline{C} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$, and $w(t)$ is stationary, white gaussian

* Underlined letters represent vector quantities.

noise with two-sided power spectral density $N_0/2$.

The signal $s(t, \underline{C})$ may be written as a truncated Taylor Series Expansion about its value at a grid-point:

$$s(t, \underline{C}) = s(t, \underline{C}_0) + \sum_{i=1}^3 \frac{\partial s}{\partial C_i} x_i \quad (39)$$

where the parameter deviations $x_i \equiv C_i - C_{i0}$. The subscript 0 refers to values of $\underline{C}$ at the grid-point near which the user is located. Also, the partial derivatives $\frac{\partial s}{\partial C_i}$ are evaluated at the grid-point.

Equation (39) may be rewritten as

$$s(t, \underline{C}) = s_0 + \underline{X}_t \underline{S} \quad (40)$$

$$\text{where } \underline{X} \equiv \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \underline{S} \equiv \begin{bmatrix} \frac{\partial s}{\partial C_1} \\ \frac{\partial s}{\partial C_2} \\ \frac{\partial s}{\partial C_3} \end{bmatrix}, \quad \text{and } s_0 \equiv s(t, \underline{C}_0)$$

Instead of Eq. (38), the received signal $u(t)$ may be rewritten as

$$u(t) = s_0 + u_0(t) \quad (41)$$

where $u_0(t)$ is defined as an observed signal which does not contain the known grid-point value of $s(t, \underline{C})$.

Rearranging the above equations, the observed signal $u_0(t)$ may be expressed as

$$\begin{aligned} u_0(t) &= \underline{X}_t \underline{S} + w(t) \text{ or} \\ &= \underline{S}_t \underline{X} + w(t) \end{aligned} \quad (42)$$

Now the C_i are functions of the craft position coordinates Γ_j . The Γ_j may be expressed as

$$\Gamma_j = \Gamma_{j0} + y_j \quad (43)$$

where Γ_{j0} are position coordinates of the grid-point and y_j are the position deviations from the grid-point.

The position deviation vector $\underline{y}$ is related to the parameter deviation vector $\underline{X}$ through

$$\underline{X} = A\underline{y} \quad (44)$$

where A is the 3×2 matrix formed by the partial derivatives of $\underline{C}$ taken with respect to Γ_j . That is, the elements A_{ij} of A are

$$A_{ij} = \left. \frac{\partial C_i(\Gamma_1, \Gamma_2)}{\partial \Gamma_j} \right| \begin{array}{l} i = 1, 2, 3 \\ \text{Evaluated at} \\ \text{the grid-point} \end{array} \quad j = 1, 2$$

To optimally estimate the position deviation vector $\underline{y}$, it is necessary to first obtain a likelihood function Λ . Such a function for nonlinear parameter estimation problems has been constructed in Appendix C in terms of an observed waveform, a signal containing the parameters to be estimated and white gaussian noise. The logarithm of the likelihood function, being easier to work with than Λ itself, is to be used in this development. It has the form given in Eq. (C.10). In terms of the noisy observed signal $u_o(t)$ of Eq. (42), the log likelihood function may be written as

$$\ln \Lambda [\underline{u}_o(t), \underline{y}] = \frac{2}{N_o} \int_0^T \underline{X}_t \underline{S} \underline{u}_o(t) dt - \frac{1}{N_o} \int_0^T \underline{X}_t \underline{S} \underline{S}_t \underline{X} dt$$

(45)

$$- \frac{1}{2} \underline{\epsilon}_t \mu^{-1} \underline{\epsilon}$$

From the discussion in Appendix C it is clear that the maximization of $\ln \Lambda$ consisting of the first two terms of Eq. (45) yields a maximum likelihood estimate of the parameter $\underline{y}$. The inclusion in Eq. (45) of a priori information in the form of the logarithm of the probability density of $\underline{\epsilon}$ and the subsequent maximization leads to a maximum a posteriori estimate. As mentioned previously, $\underline{\epsilon}$ is an error vector equal to $\underline{\eta} - \underline{y}$, the difference between the actual position deviation $\underline{y}$ from a grid-point and the preliminary estimate $\underline{\eta}$ available from the on-board dead-reckoning equipment. The covariance matrix μ of $\underline{\epsilon}$ or the corresponding probability distribution is obtained experimentally from past flights and assumed available to the processor. Note that the interval of observation $[0, T]$ is used for the limits of integration.

Equation (45) may be rewritten as the following with new quantities $\underline{V}$ and J properly defined:

$$\ln \Lambda = \underline{X}_t \underline{V} - \underline{X}_t \underline{J} \underline{X} - \frac{1}{2} \underline{\epsilon}_t \mu^{-1} \underline{\epsilon} \quad (46)$$

where the 3×1 vector $\underline{V} \equiv \frac{2}{N_o} \int_0^T \underline{S} \underline{u}_o(t) dt$ and the 3×3 matrix $J \equiv \frac{1}{N_o} \int_0^T \underline{S} \underline{S}_t dt$.

Substituting (44) in (46) and defining $A_t JA$ as K :

$$\ln \Lambda = \underline{y}_t \underline{A} \underline{V} - \underline{y}_t \underline{K} \underline{y} - \frac{1}{2} \underline{\epsilon}_t \mu^{-1} \underline{\epsilon} \quad (47)$$

Define a partial derivative operator $\nabla \equiv \begin{bmatrix} \frac{\partial}{\partial y_1} \\ \frac{\partial}{\partial y_2} \end{bmatrix}$. From the

definition of $\underline{\epsilon} = \underline{\eta} - \underline{y}$, another partial derivative operator which

can be defined as $\nabla_{\epsilon} \equiv \begin{bmatrix} \frac{\partial}{\partial \epsilon_1} \\ \frac{\partial}{\partial \epsilon_2} \end{bmatrix}$, is seen to be equal to $-\nabla$.

In order to maximize the log likelihood function, use these operators on Eq. (47). In general, the differential operator matrix has no meaning when applied to the individual matrices of the product UV [23], but in the special case where U is a $1 \times n$ matrix, V is an $n \times 1$ matrix and ∇ is an $n \times 1$ matrix

$$\nabla(UV) = (\nabla U)V + (\nabla V_t)U_t$$

Since this situation exists in the present problem,

$$\begin{aligned} \nabla \ln \Lambda &= \nabla \left[\underline{y}_t (\underline{A}_t \underline{V} - \underline{K} \underline{y}) \right] + \frac{1}{2} \nabla_{\epsilon} \left[\underline{\epsilon}_t (\mu^{-1} \underline{\epsilon}) \right] \\ &= (\nabla \underline{y}_t) (\underline{A}_t \underline{V} - \underline{K} \underline{y}) + \left[\nabla (\underline{V}_t \underline{A} - \underline{y}_t \underline{K}_t) \right] \underline{y} \\ &\quad + \frac{1}{2} \left[(\nabla_{\epsilon} \underline{\epsilon}_t) (\mu^{-1} \underline{\epsilon}) + \nabla_{\epsilon} (\underline{\epsilon}_t \mu^{-1}) \underline{\epsilon} \right] \\ &= (\nabla \underline{y}_t) (\underline{A}_t \underline{V} - \underline{K} \underline{y}) - \left[(\nabla \underline{y}_t) \underline{K}_t \underline{y} \right] \\ &\quad + \frac{1}{2} \left[(\nabla_{\epsilon} \underline{\epsilon}_t) (\mu^{-1} \underline{\epsilon}) + \nabla_{\epsilon} \underline{\epsilon}_t \mu_t^{-1} \underline{\epsilon} \right] \end{aligned} \quad (48)$$

μ and K are symmetric matrices since μ is a covariance matrix and K is equal to $A_t J A$ where J is symmetric. The substitutions $\mu_t = \mu$ and $K_t = K$ may be made in Eq. (48) which is rewritten as

$$\nabla \ln \Lambda = (\nabla \underline{y}_t) [A_t \underline{V} - 2K \underline{y}] + (\nabla_{\epsilon} \underline{\epsilon}_t) [\mu^{-1} \underline{\epsilon}] \quad (49)$$

$$\begin{aligned} \text{Now, } \nabla_{\epsilon} \underline{\epsilon}_t &= \nabla_{\epsilon} (\underline{\eta} - \underline{y})_t \\ &= \nabla \underline{y}_t \end{aligned}$$

Equation (49) becomes

$$\nabla \ln \Lambda = (\nabla \underline{y}_t) [A_t \underline{V} - 2K \underline{y} + \mu^{-1} \underline{\epsilon}] \quad (50)$$

Setting Eq. (50) equal to zero, and solving for the MAP estimate $\hat{\underline{y}}$ of the position deviation $\underline{y}$:

$$\begin{aligned} A_t \underline{V} - 2K \hat{\underline{y}} + \mu^{-1} \underline{\eta} - \mu^{-1} \hat{\underline{y}} &= 0 \\ \hat{\underline{y}} &= (2K + \mu^{-1})^{-1} (A_t \underline{V} + \mu^{-1} \underline{\eta}) \end{aligned} \quad (51)$$

The MAP processor of Fig. 16 is designed on the basis of this equation.* It may be noted that in the absence of a priori information in the form of a preliminary estimate $\underline{\eta}$ or a covariance matrix μ , the position coordinates obtained are optimum in the maximum likelihood sense rather than in the MAP sense.

That $\hat{\underline{y}}$ is an unbiased estimate may be seen by taking the expectation of $\hat{\underline{y}}$

$$E(\hat{\underline{y}}) = (2K + \mu^{-1})^{-1} [A_t E(\underline{V}) + \mu^{-1} E(\underline{\eta})] \quad (52)$$

* The role of the epoch signal in the Internal Signal Generating Mechanism is discussed in Appendix E.

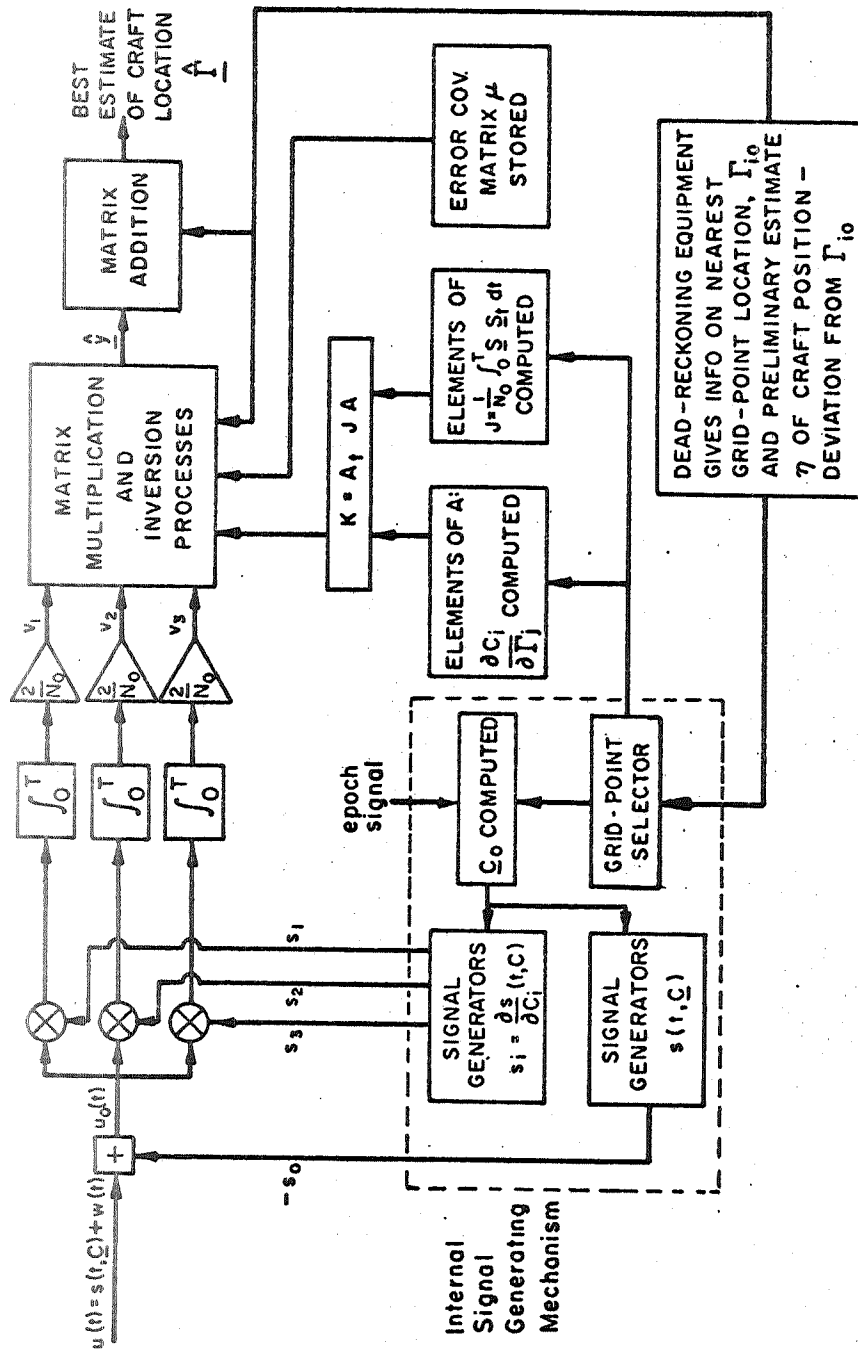


FIGURE 16 MAP Processor for optimally estimating craft position coordinates.

$$\begin{aligned}
 \text{But } E(\underline{V}) &= E \left[\frac{2}{N_0} \int_0^T \underline{S} u_o(t) dt \right] \\
 &= E \left(\frac{2}{N_0} \int_0^T \underline{S} [\underline{S}_t \underline{X} + w(t)] dt \right) \\
 &= \frac{2}{N_0} \int_0^T \underline{S} \underline{S}_t \underline{A} \underline{y} dt, \text{ since } E[w(t)] = 0 \\
 &= 2J \underline{A} \underline{y} \quad \text{from the definitions of } \underline{X} \text{ and } J. \\
 E(\underline{\eta}) &= E(\underline{y}) + E(\underline{\epsilon}) = \underline{y} \quad \text{since } E(\underline{\epsilon}) = 0
 \end{aligned}$$

Substituting these quantities into Eq. (52)

$$\begin{aligned}
 E(\hat{\underline{y}}) &= (2K + \mu^{-1})^{-1} (2A_t J A \underline{y} + \mu^{-1} \underline{y}) \\
 &= (2K + \mu^{-1})^{-1} (2K + \mu^{-1}) \underline{y} \\
 &= \underline{y}
 \end{aligned}$$

proving that $\hat{\underline{y}}$ is an unbiased estimate of the position deviation vector $\underline{y}$.

3.2.2.2 Derivation of the Estimation Error Covariance Matrix

The covariance matrix $\text{Cov}(\hat{\underline{y}})$ of the estimation error $\hat{\underline{y}} - \underline{y}$ may be derived in a conceptually straightforward but somewhat lengthy manner by expanding $E[(\hat{\underline{y}} - \underline{y})(\hat{\underline{y}} - \underline{y})_t^T]$ into the sum of four terms and finding the expectation of each. Alternately, the covariance matrices of errors in $\underline{V}$ and $\underline{\eta}$ may be used to get $\text{Cov}(\hat{\underline{y}})$ in a simpler way. From Eq. (51) $\text{Cov}(\hat{\underline{y}})$ may be written as

$$\text{Cov}(\hat{\underline{y}}) = (2K + \mu^{-1})^{-1} \left[A_t \text{Cov}(V) A + \mu^{-1} \text{Cov}(\underline{\eta}) \mu^{-1} \right] (2K + \mu^{-1})^{-1} \quad (53)$$

where $\text{Cov}(V)$ and $\text{Cov}(\underline{\eta})$ represent the error covariance matrices of $\underline{V}$ and $\underline{\eta}$. It is recognized here that $(2K + \mu^{-1})^{-1}$ and μ^{-1} , being symmetrical matrices, are identical to their transposed forms.

Now, from the definition of $\underline{V}$, and Eq. (42),

$$\begin{aligned} \underline{V} &= \frac{2}{N_o} \int_0^T \underline{S} u_o(t) dt \\ &= \frac{2}{N_o} \int_0^T \underline{S} \left[\underline{S}_t \underline{X} + w(t) \right] dt \\ &= \frac{2}{N_o} \int_0^T \underline{S} \underline{S}_t \underline{X} dt + \frac{2}{N_o} \int_0^T \underline{S} w(t) dt \end{aligned} \quad (54)$$

The error in $\underline{V}$ arises only from the noisy term in Eq. (54), and the error covariance matrix of $\underline{V}$ is therefore

$$\text{Cov}(V) = \left(\frac{2}{N_o} \right)^2 E \left\{ \int_0^T \underline{S}(t) w(t) dt \int_0^T \underline{S}_t(t') w(t') dt' \right\} \quad (55)$$

where the time dependence of $\underline{S}$ is explicit. Letting $t' = t + \tau$,

Eq. (55) may be rewritten as

$$\text{Cov}(V) = \left(\frac{2}{N_o} \right)^2 E \left\{ \int_0^T \int_0^T \underline{S}(t) \underline{S}_t(t+\tau) w(t) w(t+\tau) dt d\tau \right\}$$

Since $\underline{S}$ and w are independent and $\underline{S}$ is deterministic,

$$\text{Cov}(V) = \left(\frac{2}{N_o} \right)^2 \int_0^T \int_0^T \underline{S}(t) \underline{S}_t(t+\tau) R_w(\tau) d\tau dt \quad (56)$$

The autocorrelation function of the white noise process $w(t)$ is assumed to be $\frac{N_0}{2} \delta(\tau)$, which when substituted in Eq. (56) yields

$$\begin{aligned} \text{Cov}(V) &= \frac{2}{N_0} \int_0^T \underline{s}(t) \underline{s}_t(t) dt \\ &= 2J \end{aligned} \quad (57)$$

from the definition of J in Section 3.2.2.1.

The error covariance matrix of $\underline{\eta}$ may be obtained simply by recognizing that $\underline{\eta}$ is a "preliminary estimate" of the actual position deviation $\underline{y}$ which is in error by $\underline{\epsilon}$. So the probability distribution of the error in $\underline{\eta}$ is just the gaussian probability distribution of $\underline{\epsilon}$. Consequently, the error covariance matrix of $\underline{\eta}$ is

$$\text{Cov}(\underline{\eta}) = E(\underline{\epsilon} \underline{\epsilon}_t) = \mu \quad (58)$$

Substituting (57) and (58) into (53)

$$\begin{aligned} \text{Cov}(\hat{\underline{y}}) &= (2K + \mu^{-1})^{-1} (A_t 2JA + \mu^{-1} \mu \mu^{-1}) (2K + \mu^{-1})^{-1} \\ &= (2K + \mu^{-1})^{-1} (2K + \mu^{-1}) (2K + \mu^{-1})^{-1} \\ &= (2K + \mu^{-1})^{-1} \end{aligned} \quad (59)$$

In the absence of a priori information, μ^{-1} is equal to the null matrix and $\text{Cov}(\hat{\underline{y}})$ simply equals $(2K)^{-1}$. When a priori information is available, the elements of μ are finite valued, μ^{-1} is not a null matrix and its inclusion is seen to decrease the elements of $\text{Cov}(\hat{\underline{y}})$. The evaluation of the elements of $\text{Cov}(\hat{\underline{y}}) = (2K)^{-1}$ therefore provides an upper bound to the estimation errors.

4.0 ESTIMATION ERROR ANALYSIS

4.1 Evaluating the Estimation Errors

Assuming no a priori information on the craft location is available, i.e. μ^{-1} = null matrix, the estimation error covariance matrix as given by Eq. (59) is to be evaluated. The objective is to study the dependence of the elements of $\text{Cov}(\hat{\underline{y}})$ on the various system parameters. Define the 2×1 estimation error matrix $\underline{e}$ as

$$\underline{e} \equiv \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \hat{\underline{y}} - \underline{y} \quad (60)$$

where e_1 and e_2 are the errors in the north-south (x) and east-west (y) directions respectively. $\text{Cov}(\hat{\underline{y}})$ can be written as

$$\text{Cov}(\hat{\underline{y}}) = \begin{bmatrix} \langle e_1^2 \rangle & \langle e_1 e_2 \rangle \\ \langle e_2 e_1 \rangle & \langle e_2^2 \rangle \end{bmatrix} \quad (61)$$

The evaluation of the rms position errors $\sqrt{\langle e_1^2 \rangle}$, $\sqrt{\langle e_2^2 \rangle}$ corresponds to calculating the elements of the matrix

$$\begin{aligned} \text{Cov}(\hat{\underline{y}}) &= (2K)^{-1} \\ &= (2A_t J A)^{-1} \end{aligned} \quad (62)$$

The generation of numerical values for the above matrix can be mechanized via a computer program. The steps are as follows: the reading in of typical aircraft coordinates (Γ_x, Γ_y) enables the

evaluation of C_i , the phase terms in φ , and the elements A_{ij} of the matrix A defined in Eq. (44); the C_i enables the evaluation of the elements J_{ij} of the J matrix defined in Eq. (46); the matrix multiplication and inversion operations of Eq. (62) result in the desired values of Eq. (61).

The C_i terms have been defined exactly in Section 2.2, but the approximate expressions appearing below are considered adequate for the computer program:

$$C_1 \simeq \pi \frac{D}{\lambda} \Gamma_x \sin \theta \quad (63a)$$

$$C_2 \simeq \pi \frac{D}{\lambda} \cos \theta \quad (63b)$$

$$C_3 = \tan^{-1} \left(\frac{1}{\Gamma_y} \right) \quad (63c)$$

The elements of the sensitivity matrix A are listed below:

$$A_{11} = \frac{\partial C_1}{\partial \Gamma_x} \simeq \pi \frac{D}{\lambda} \sin \theta \quad (64a)$$

$$A_{12} = \frac{\partial C_1}{\partial \Gamma_y} \simeq - \pi \frac{D}{\lambda} \sin \theta \cdot \Gamma_x \Gamma_y \quad (64b)$$

$$A_{21} = \frac{\partial C_2}{\partial \Gamma_x} \simeq - \pi \frac{D}{\lambda} \cos \theta \cdot \Gamma_x \quad (64c)$$

$$A_{22} = \frac{\partial C_2}{\partial \Gamma_y} \simeq \pi \frac{D}{\lambda} \cos \theta \cdot \Gamma_x^2 \Gamma_y \quad (64d)$$

$$A_{31} = \frac{\partial C_3}{\partial \Gamma_x} = 0 \quad (64e)$$

$$A_{32} = \frac{\partial C_3}{\partial \Gamma_y} = - \frac{1}{1 + \Gamma_y^2} \quad (64f)$$

The approximations involved here are of the sort made in Eq. (63).

Quick calculations reveal virtually no difference in the final answers obtained by using the exact and the approximate expressions.

Equation (46) defines the elements of the 3×3 matrix J as

$$J_{ij} = \frac{1}{N_0} \int_0^T \frac{\partial s}{\partial C_i} \frac{\partial s}{\partial C_j} dt$$

where $(0, T)$ is the observation interval during which the integrators in Fig. 16 are operative.

The "intelligence" part of the received signal is*

$$s(t, \underline{C}) = A_0 \cos \left[C_1 + C_2 \cos(\Omega t - C_3) \right]$$

Differentiating $s(t, \underline{C})$ with respect to C_i :

$$\left. \begin{aligned} \frac{\partial s(t, \underline{C})}{\partial C_1} &= -A_0 \sin \left[C_1 + C_2 \cos(\Omega t - C_3) \right] \\ \frac{\partial s(t, \underline{C})}{\partial C_2} &= -A_0 \sin \left[C_1 + C_2 \cos(\Omega t - C_3) \right] \cdot \cos(\Omega t - C_3) \\ \frac{\partial s(t, \underline{C})}{\partial C_3} &= -A_0 C_2 \sin \left[C_1 + C_2 \cos(\Omega t - C_3) \right] \cdot \sin(\Omega t - C_3) \end{aligned} \right\} (65)$$

* The additional terms due to the arbitrary selection of time origin and the propagation path delay are obviated by the use of the epoch signal and hence are not included in this development.

The derivation of J_{11} proceeds as follows:

$$\begin{aligned}
 J_{11} &= \frac{1}{N_o} \int_0^T \left(\frac{\partial s}{\partial C_1} \right)^2 dt \\
 &= \frac{A_o^2}{N_o} \int_0^T \sin^2 [C_1 + C_2 \cos(\Omega t - C_3)] dt \\
 &= \frac{A_o^2}{2N_o} \int_0^T \left\{ 1 - \cos [2C_1 + 2C_2 \cos(\Omega t - C_3)] \right\} dt \quad (66)
 \end{aligned}$$

The second term in the above integrand may be expanded to read

$$- \cos 2C_1 \cos [2C_2 \cos(\Omega t - C_3)] + \sin 2C_1 \sin [2C_2 \cos(\Omega t - C_3)] \quad (67)$$

Using the Bessel function expansions^{*}

$$\cos[a \cos b] = J_0(a) - 2 [J_2(a) \cos 2b - J_4(a) \cos 4b + \dots]$$

$$\sin[a \cos b] = 2 [J_1(a) \cos b - J_3(a) \cos 3b + \dots]$$

the expression (67) may be written as a sum of terms at frequencies of zero and multiples of the satellite spin-rate Ω :

$$- J_0(2C_2) \cos 2C_1 + 2J_1(2C_2) \sin 2C_1 \cos(\Omega t - C_3) +$$

$$2J_2(2C_2) \cos 2C_1 \cos 2(\Omega t - C_3) - \dots$$

A careful choice of the interval of integration will result in all of the terms going to zero, except the dc term. Equation (66) may be rewritten therefore as

* The double subscripted J_{ij} terms are elements of the J matrix, whereas the single subscripted terms, e.g. J_0 , J_1 , J_2 are Bessel functions.

$$J_{11} = \frac{A_o^2 T}{2N_o} \left[1 - J_o(2C_2) \cos 2C_1 \right] \quad (68)$$

Similarly,

$$\left. \begin{aligned} J_{22} &= \frac{A_o^2 T}{4N_o} \left[1 - J_o(2C_2) \cos 2C_1 + J_2(2C_2) \cos 2C_1 \right] \\ J_{33} &= \frac{A_o^2 T C_2^2}{4N_o} \left[1 - J_o(2C_2) \cos 2C_1 - J_2(2C_2) \cos 2C_1 \right] \\ J_{12} = J_{21} &= \frac{A_o^2 T}{2N_o} \left[J_1(2C_2) \sin 2C_1 \right] \\ J_{13} = J_{31} &= 0 \\ J_{23} = J_{32} &= 0 \end{aligned} \right\} \quad (69)$$

Factoring $\frac{T A_o^2}{2N_o}$ out of the J_{ij} , a new matrix Q can be defined:

$$J = \frac{T A_o^2}{2N_o} Q \quad (70)$$

Equation (62) now becomes

$$\begin{aligned} \text{Cov}(\hat{\mathbf{y}}) &= (2 \cdot A_t J A)^{-1} \\ &= \left[\frac{T A_o^2}{N_o} (A_t Q A) \right]^{-1} \end{aligned} \quad (71)$$

The analysis in Section 2.2.2 showed that the ratio of the average signal power to the noise power spectral density at the input

to the DSB demodulator and hence at the receiver input is $\frac{A_o^2}{N_o}$. This quantity has the units of Hz. When multiplied by the integration time T seconds, a power ratio $\frac{TA_o^2}{N_o}$ results.

The evaluation of the estimation error covariance matrix, $Cov(\hat{y})$, involves computing the elements of A and Q, which depend on craft location and system parameters, then transposing A and multiplying the matrices A_t , Q and A. The factor $\frac{TA_o^2}{N_o}$ may be regarded as a performance figure that reveals the tradeoffs between A_o^2 , the signal power at the receiver (a quantity directly related to the transmitted power), N_o , the noise power spectral density which may be expressed also as the product of Boltzmann's constant k and the effective noise temperature T_e (in degrees Kelvin) of the craft receiver, and T, the integration time-interval in the MAP processor. Assigning a range of values to $\frac{TA_o^2}{N_o}$, multiplying these by A_tQA and inverting the resultant matrix provides $Cov(\hat{y})$. This operation was computerized and the results appear in Figs. 22 to 28.

It must be noted that although the initial system model assumed a flat earth, the position errors have been computed for a spherical earth. This was done by multiplying the error figures based on a flat earth by a Geometric Dilution of Precision (GDOP) Factor which depends on craft position, synchronous satellite altitude and the earth radius. Figure 18 gives this factor as a function of elevation angle and the following discussion clarifies it.

GDOP is an effect that "amplifies" the error that is geometrically projected onto the earth. The one-dimensional picture in Fig. 17 illustrates this effect for a synchronous satellite-aided navigation system. The error in tilt angle $\Delta\Gamma$ is related to craft position error of Δx by

$$\Delta x = \frac{r_s \Delta\Gamma}{\sin E} \quad (72)$$

where r_s is the satellite-craft slant range and E is the satellite elevation angle from the user.

The same tilt angle error produces a position error $(\Delta x)_s$ at the sub-satellite point of

$$(\Delta x)_s = h \Delta\Gamma \quad (73)$$

where h is the synchronous altitude of 36,000 km. From (72) and (73) a GDOP Factor may be defined as:

$$\text{GDOP Factor} = \frac{\Delta x}{(\Delta x)_s} = \frac{r_s}{h \sin E} \quad (74)$$

On multiplying this factor with the craft position error from the $\text{Cov}(\hat{y})$ matrix, the correct error for a spherical earth is obtained.

4.2 Correlation of Orthogonal Estimation Errors

If the vector $\underline{e}$ is assumed to have a bivariate gaussian distribution with a zero mean, the diagonal elements of $\text{Cov}(\hat{y})$ may be interpreted as error variances that are related to the major and minor axes of an ellipse of estimation error. Such an ellipse would be

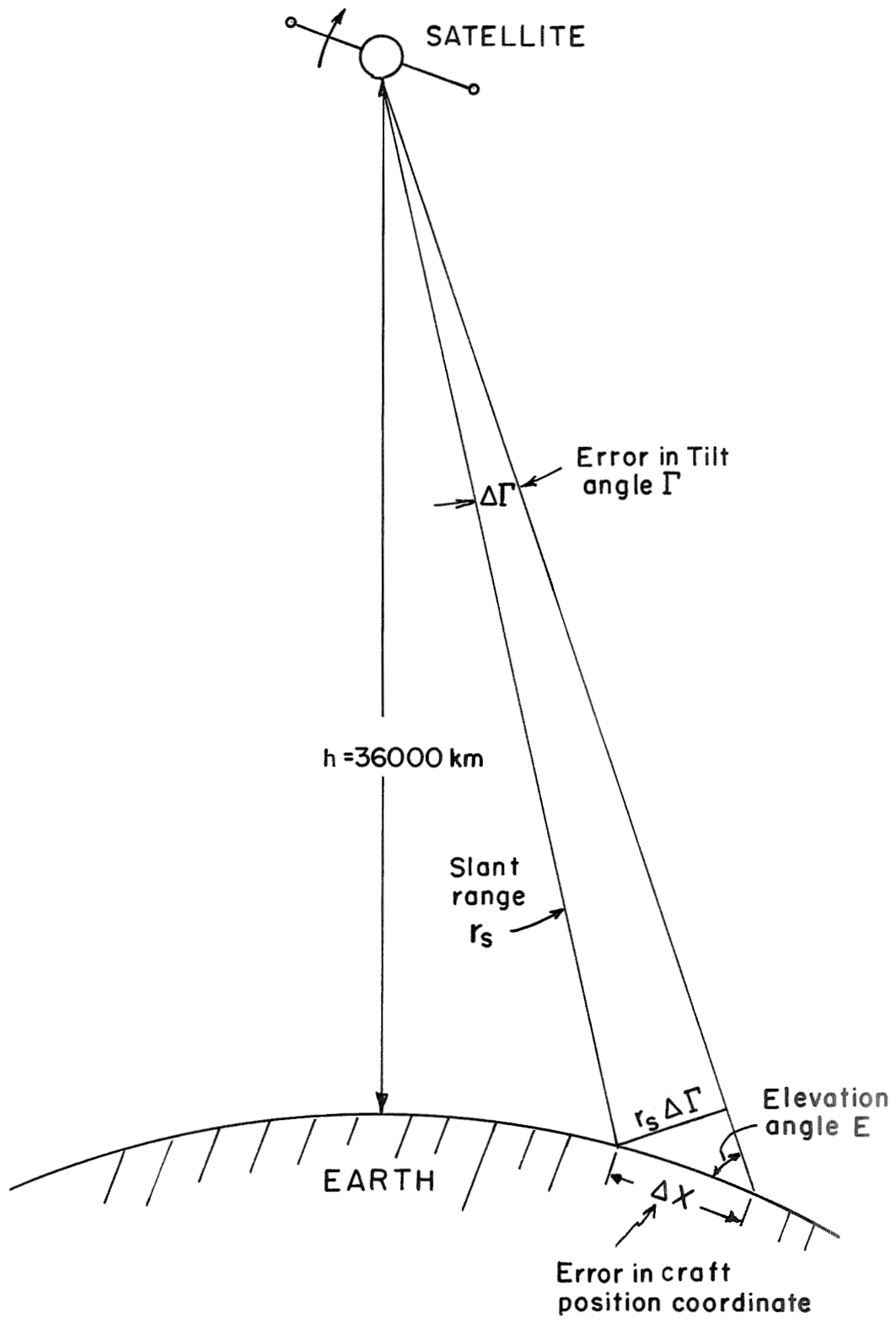


FIGURE 17 GEOMETRIC DILUTION OF PRECISION (GDOP) EFFECT

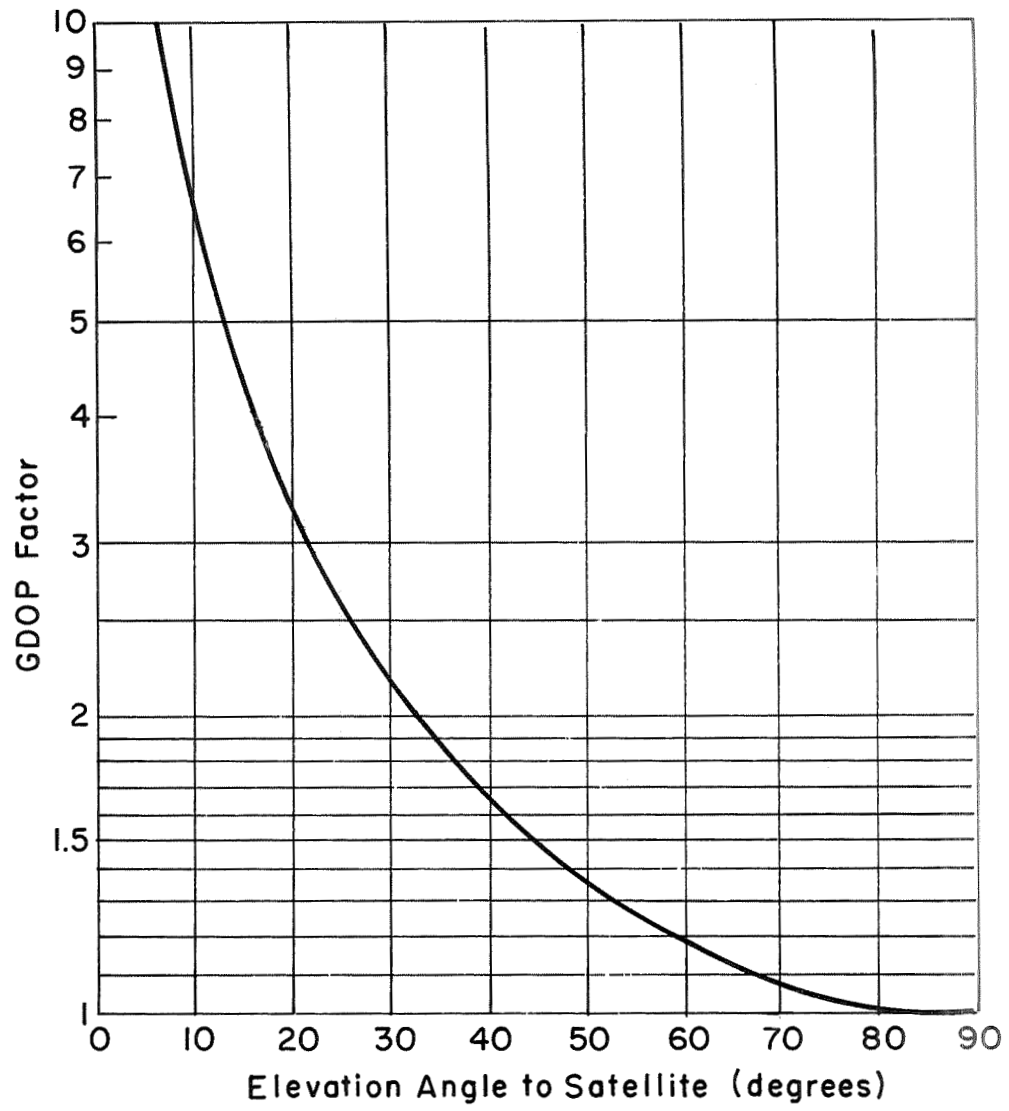


FIGURE 18 GDOP FACTOR AS A FUNCTION OF ELEVATION ANGLE

analogous to the concentration ellipse of Van Trees [65]. Analogously, therefore, the equation of the 1σ estimation error ellipse is

$$\underline{e}_t^T [\text{Cov}(\hat{\underline{y}})]^{-1} \underline{e} = 1 \quad (75)$$

or

$$(\underline{e}_1 \ \underline{e}_2) \begin{bmatrix} \langle (e_1^2) \rangle & \langle (e_1 e_2) \rangle \\ \langle (e_2 e_1) \rangle & \langle (e_2^2) \rangle \end{bmatrix}^{-1} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = 1 \quad (76)$$

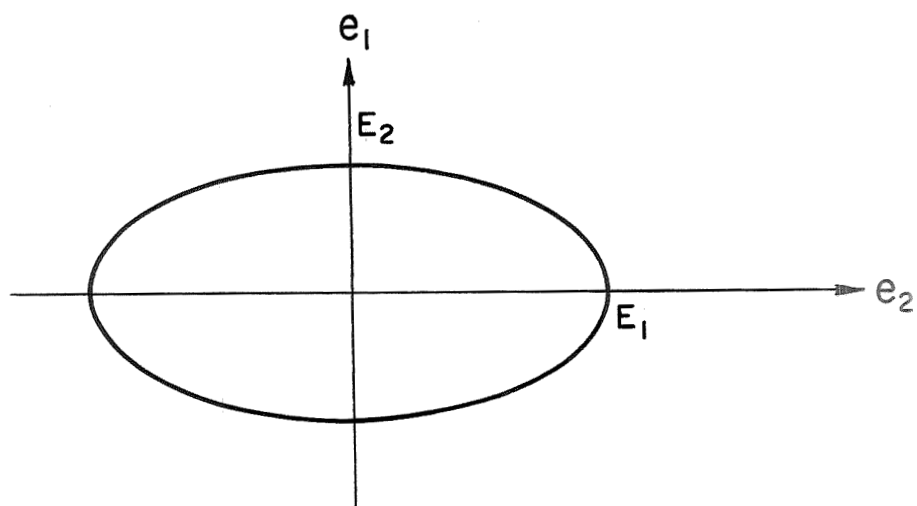
Redefining the diagonal elements of $\text{Cov}(\hat{\underline{y}})$ as E_1^2 and E_2^2 and the off-diagonal elements as $\rho E_1 E_2$, where ρ is a correlation coefficient, (76) may be rewritten as

$$(\underline{e}_1 \ \underline{e}_2) \begin{bmatrix} \frac{1}{E_1^2(1-\rho^2)} & \frac{\rho}{E_1 E_2(1-\rho^2)} \\ \frac{\rho}{E_1 E_2(1-\rho^2)} & \frac{1}{E_2^2(1-\rho^2)} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = 1 \quad (77)$$

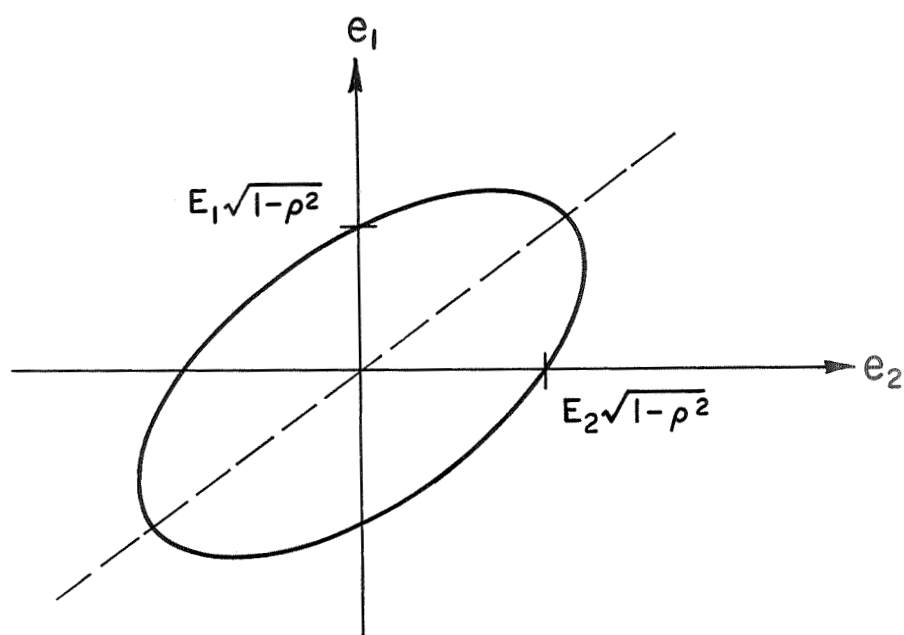
Carrying out the matrix multiplication, (77) becomes

$$\frac{e_1^2}{E_1^2(1-\rho^2)} + \frac{e_2^2}{E_2^2(1-\rho^2)} + \frac{2\rho e_1 e_2}{E_1 E_2(1-\rho^2)} = 1$$

Being a measure of correlation between the errors in the two orthogonal directions, ρ affects the error ellipse, thereby altering the error in each direction. For $\rho = 0$, the error ellipse is shown in Fig. 19a, but for a non-zero ρ , the ellipse is rotated as in Fig. 19b. To get some idea of its magnitude, ρ was calculated for several



(a)



(b)

FIGURE 19 ESTIMATION ERROR ELLIPSE: (a) for $\rho = 0$
(b) for $\rho \neq 0$

geographical locations with typical values of D/λ and θ . The results which appear in Figs. 20 and 21, may be summarized as follows: as θ is increased from 0° to 85° , ρ increases by two orders of magnitude; ρ increases in proportion to increases in latitude and relative longitude; it appears completely independent of the performance figure TA_0^2/N_0 and very weakly dependent on the baselength-to-wavelength ratio. For the areas of interest (below a relative longitude of 37.5° and $\theta = 45^\circ$), ρ is on the order of 10^{-2} and consequently has virtually no effect on the errors in the two directions.

4.3 Parameter Tradeoff for Minimum Estimation Errors

The 1 σ estimation errors in the north-south direction, e_1 , and the east-west direction, e_2 , show a marked dependence on the geographic location of the craft, on the physical parameters of the interferometer (θ , D/λ), on the transmitter power and on the integration time-interval in the craft receiver.

Figure 22 indicates the heavy dependence of the error on the interferometer skew-angle θ . For the interferometer entirely in the equatorial orbital plane, i.e. $\theta = 0^\circ$, the error e_2 is the smallest possible while e_1 is maximum. For $D/\lambda = 100$ and $TA_0^2/N_0 = 36.02$ dB at moderate latitudes (38°), $e_2 = 1.4$ nmi and $e_1 = 19$ nmi when $\theta = 0^\circ$. When the interferometer is skewed very heavily, say to $\theta = 85^\circ$, $e_2 = 16.4$ nmi and $e_1 = 1.5$ nmi. The errors are equal (≈ 2 nmi) for $\theta = 45^\circ$ at moderate latitudes. At very low latitudes, in the vicinity of the subsatellite point, the errors are equal (≈ 1.8 nmi) for $\theta \approx 37^\circ$.

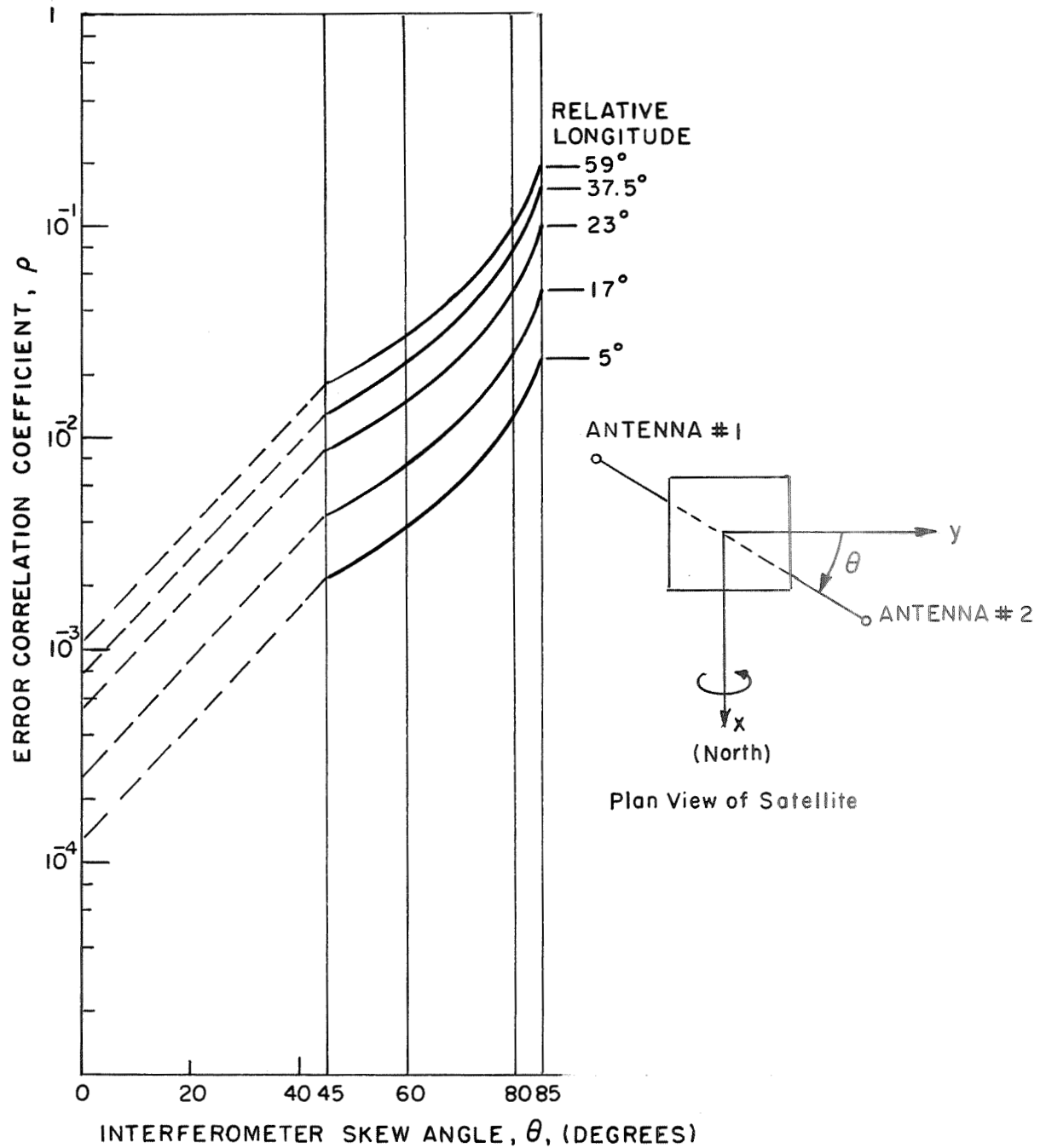


FIGURE 20 ESTIMATION ERROR CORRELATION COEFFICIENT ρ PLOTTED AGAINST INTERFEROMETER SKEW ANGLE θ , WITH RELATIVE LONGITUDE AS A PARAMETER [FOR $D/\lambda = 100$, $TA_0^2/N_0 = 36.02$ dB and $\Gamma_x = 5^\circ$ (30° LAT.)]

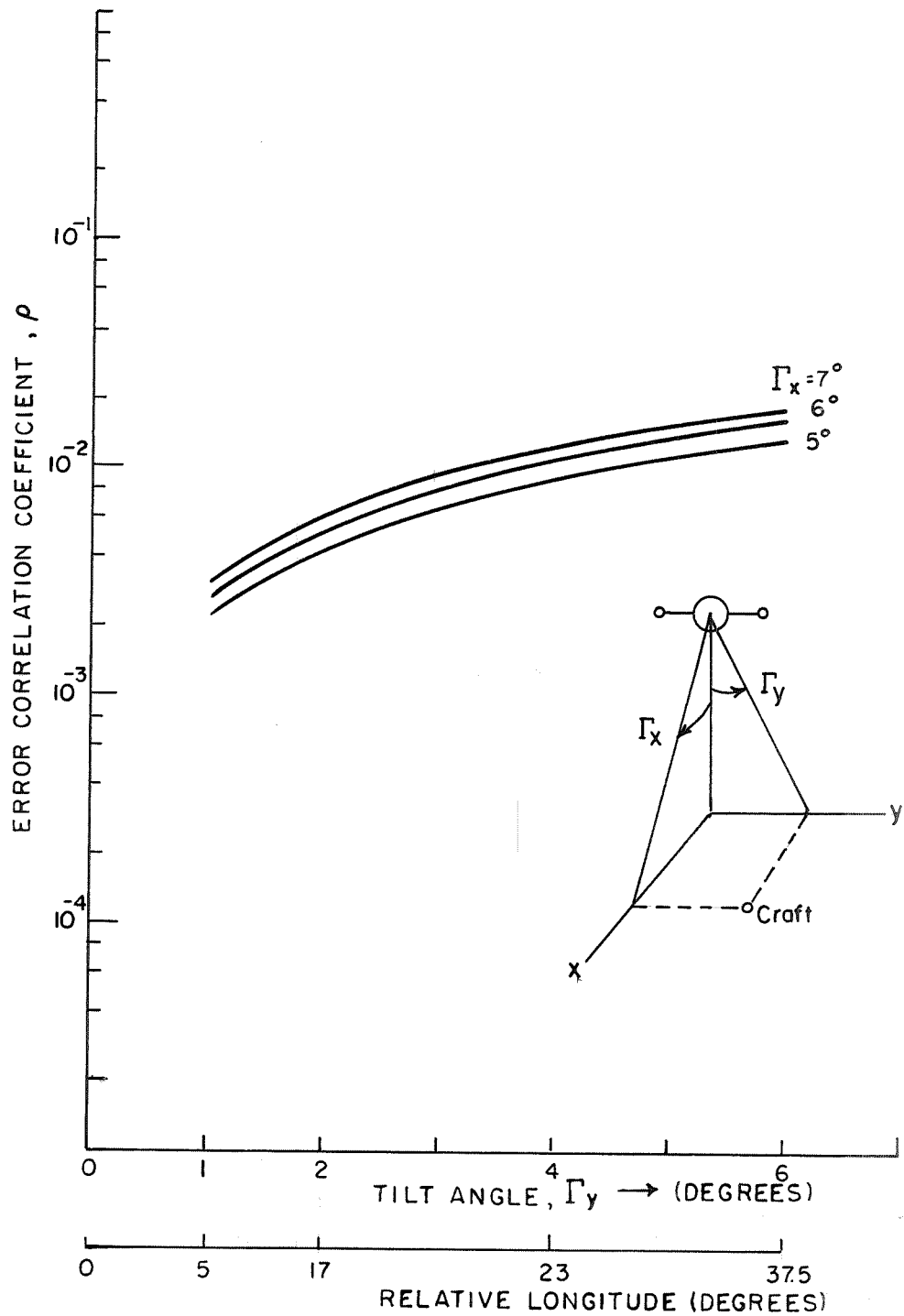


FIGURE 21 ESTIMATION ERROR CORRELATION COEFFICIENT ρ PLOTTED AGAINST TILT ANGLE Γ_y , WITH TILT ANGLE Γ_x AS A PARAMETER [FOR $\theta = 45^\circ$, $D/\lambda = 100$, $TA_0^2/N_0 = 36.02$ dB]

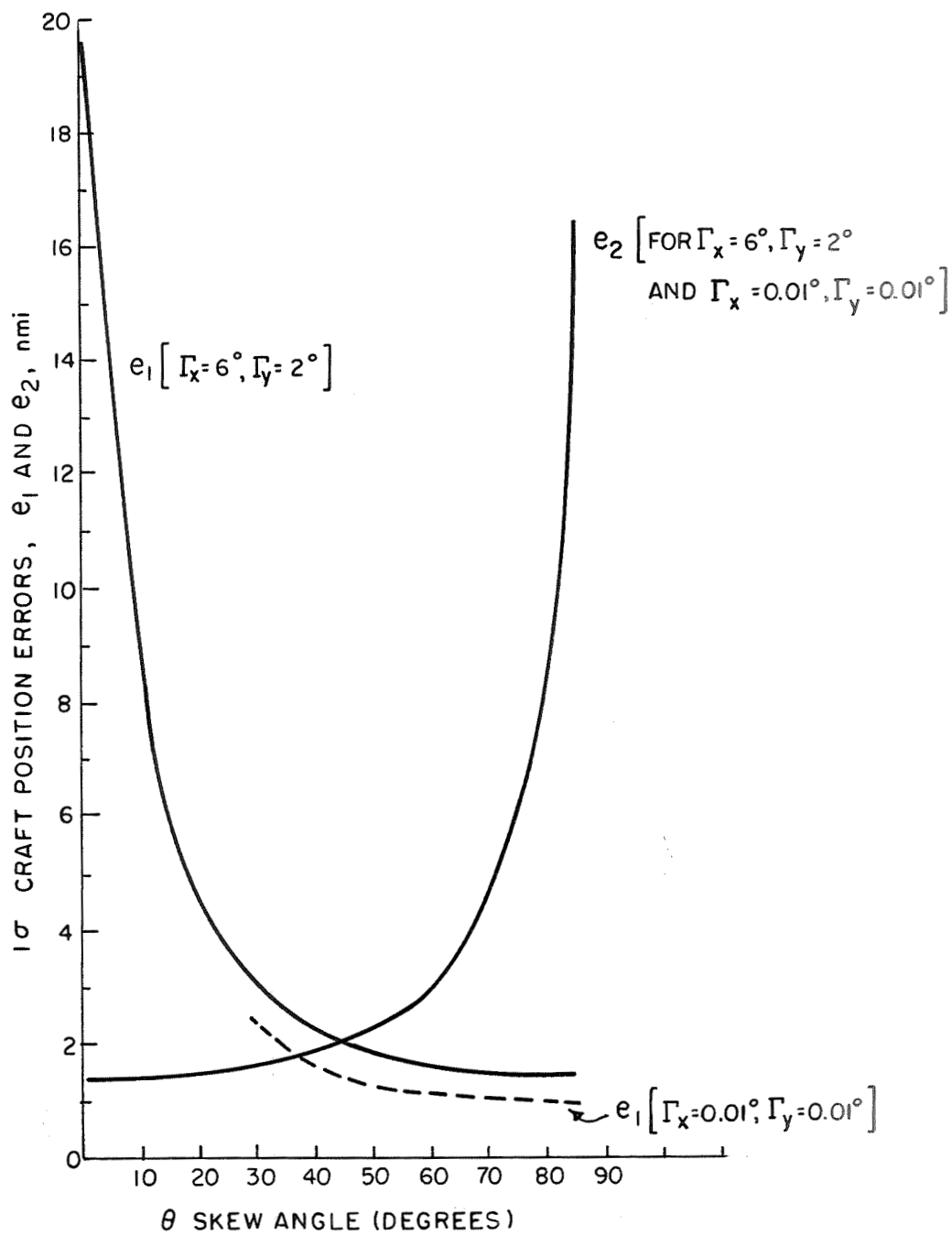


FIGURE 22 ESTIMATION ERRORS PLOTTED AGAINST INTERFEROMETER SKEW ANGLE [FOR $D/\lambda = 100$, $TA_0^2/N_0 = 36.02$ dB]

Clearly θ must be chosen with the requirements of the users of the navigation system in mind. If the users are mainly transatlantic aircraft which require high cross-track accuracy, then e_1 must be minimized at the expense of e_2 . θ may be increased to 60° to give $e_1 = 1.6$ nmi and $e_2 = 2.8$ nmi. Increasing it further deteriorates e_2 markedly while improving e_1 marginally. In Fig. 23 for $\theta = 0^\circ$, e_1 and e_2 are plotted against Γ_x . e_1 approaches a minimum around $\Gamma_x \approx 6^\circ$ while e_2 remains constant. For $\theta = 85^\circ$, e_2 remains constant whereas e_1 increases monotonically with increasing Γ_x . A similar behavior is noted for $\theta = 45^\circ$, although there is more error in the x-direction for $\theta = 45^\circ$ compared to when $\theta = 85^\circ$, but less error in the y-direction.

The dependence of errors on D/λ is illustrated in Fig. 24. Both e_1 and e_2 decrease at the same rate as D/λ is increased. It is clear that if estimation errors in the north-south direction are to be kept below 1 nmi, $D/\lambda > 100$ for low latitudes. If a 2 nmi error is tolerable, $D/\lambda > 50$ for low latitudes and $D/\lambda > 70$ for moderate latitudes.

Figure 25 shows the behavior of e_1 and e_2 with increasing Γ_y , at fixed values of θ , D/λ and TA_0^2/N_0 . The satellite, if located at 35° or 40° W. longitude could provide service for the entire east-west span of the North Atlantic, ensuring good coverage where it is needed most--in the central region (low Γ_y) far from the land masses--and poorer coverage near the American and European continents (high Γ_y), where land-based navigation systems are more readily available. At

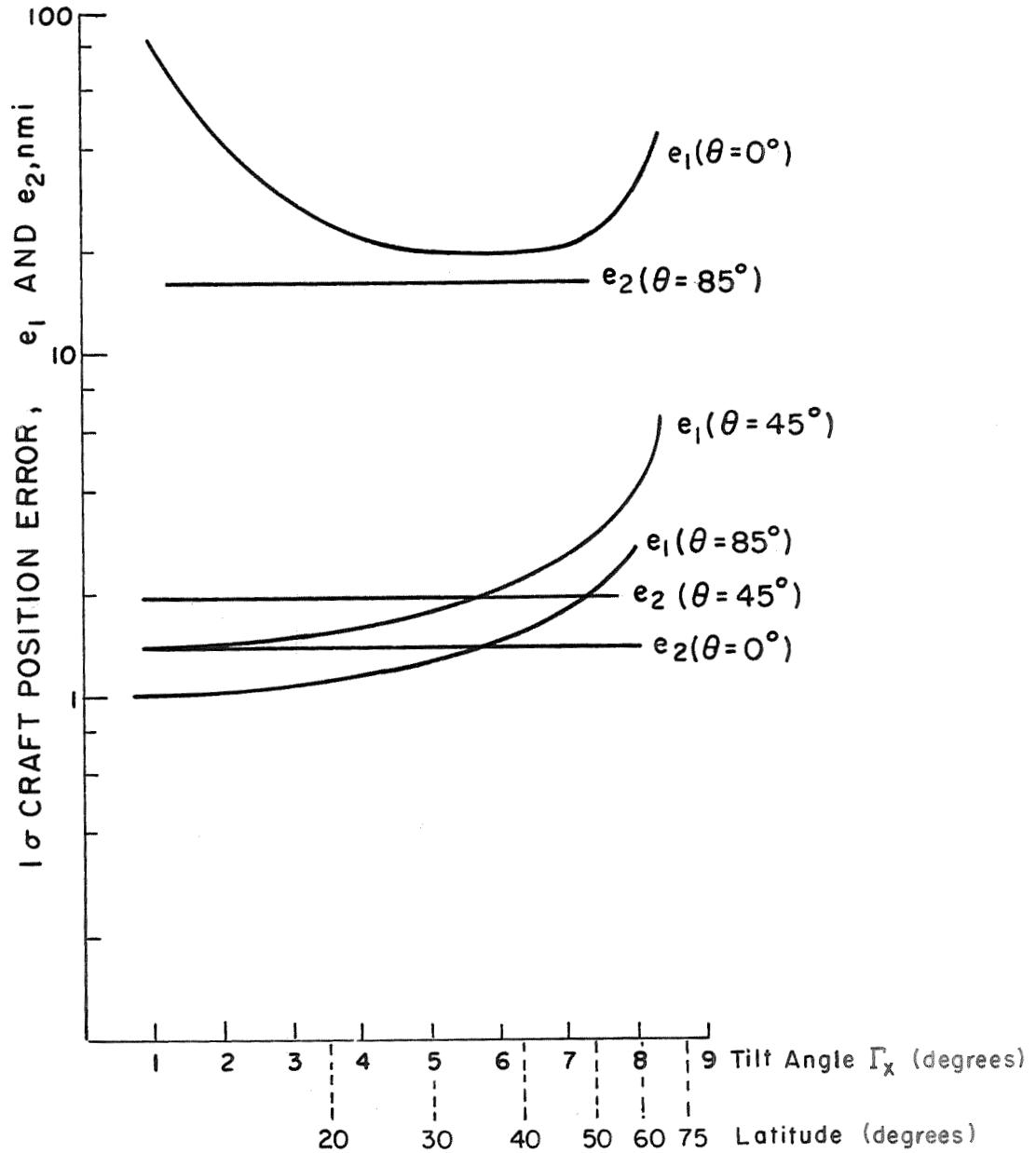


FIGURE 23 ESTIMATION ERRORS PLOTTED AGAINST TILT ANGLE Γ_x , WITH θ AS A PARAMETER $[TA_{\odot}^2/N_{\odot} = 36.02$ dB, $D/\lambda = 100$, $\Gamma_y = 1^\circ$]

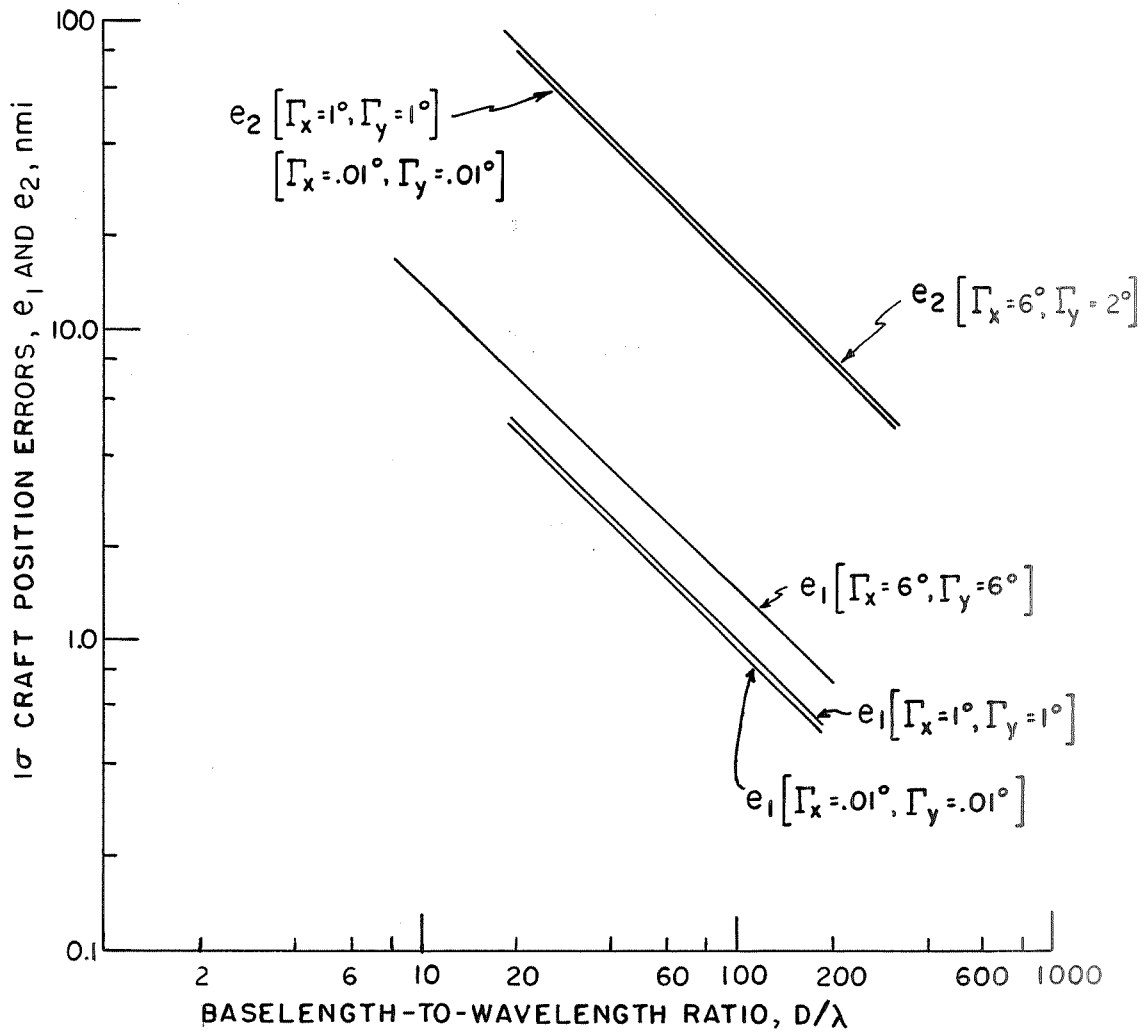


FIGURE 24 ESTIMATION ERRORS PLOTTED AGAINST THE BASELENGTH-TO-WAVELENGTH RATIO, WITH CRAFT LOCATION AS A PARAMETER [FOR $\theta = 85^\circ$, $TA_0^2/N_0 = 36.02$ dB]

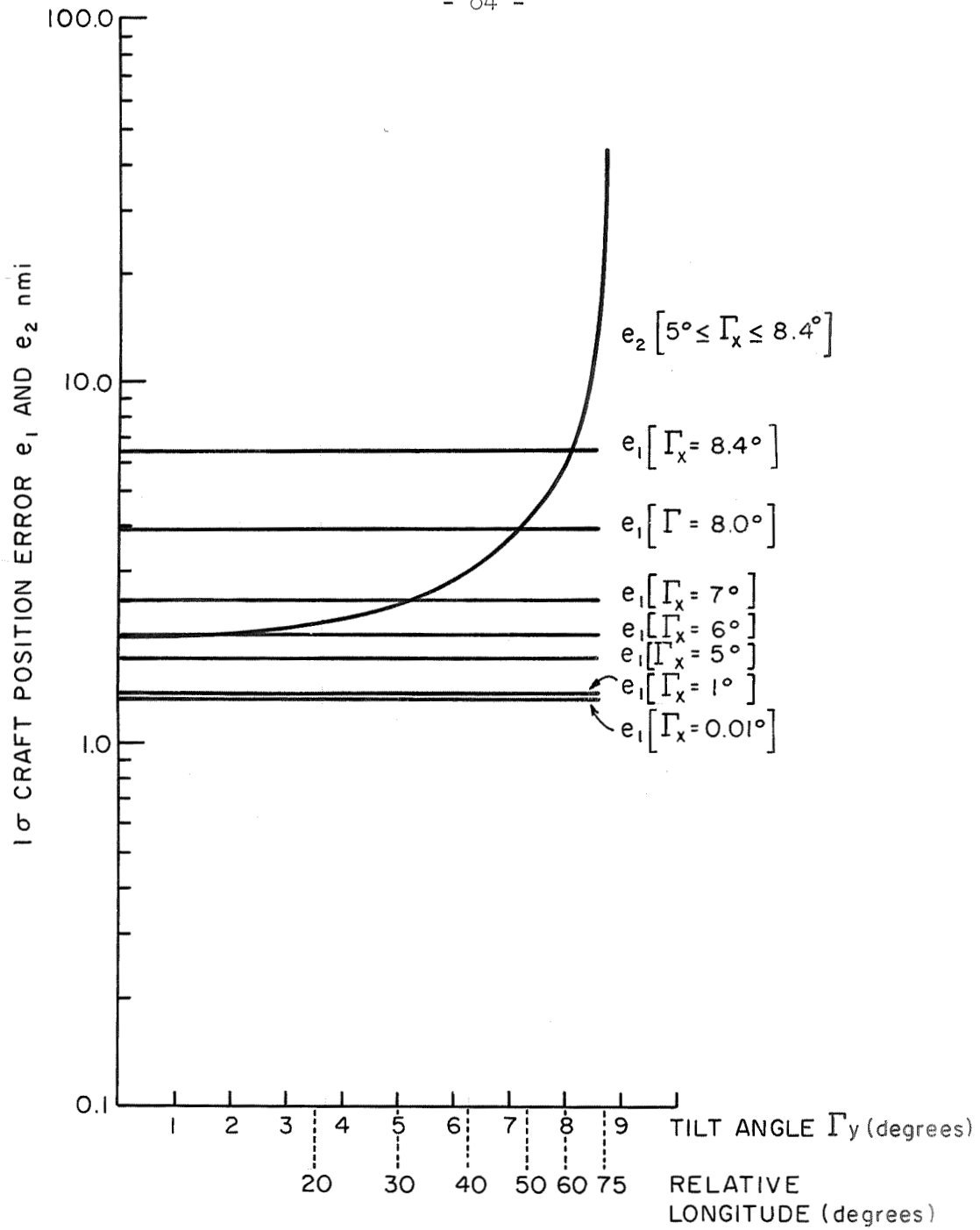


FIGURE 25 ESTIMATION ERRORS PLOTTED AGAINST TILT ANGLE Γ_y , with Γ_x
 [For $\theta = 45^\circ$, $D/\lambda = 100$, $TA_0^2/N_0 = 36.02$ dB]
 AS A PARAMETER

any rate Γ_y will not exceed 6° (corresponding to a relative longitude of 38°). The e_2 error locus is seen to increase monotonically with Γ_y reaching 2.8 nmi of error at $\Gamma_y = 6^\circ$. The e_1 loci are constant with Γ_y for a particular latitude, increasing with higher latitudes.

The performance factor TA_o^2/N_o illustrates the close interdependence between the signal power A_o^2 at the receiver input, the noise PSD N_o characteristic of the craft receiver used and the integration time-interval T of the MAP processor. A_o^2 is directly related to the satellite transmitter power through the free space path loss and the gains of both craft and satellite-borne antennas. N_o is commonly expressed in terms of the effective noise temperature T_e (in degrees Kelvin):

$$N_o = kT_e$$

where k , the Boltzmann's constant, is equal to 1.38×10^{-23} joules/ $^\circ K$. $T_e = 1000^\circ K$ is judged to be a reasonable value for an aircraft receiver. This includes losses in the transmission lines [41].

Figure 26 shows the improvement in e_1 possible by increasing TA_o^2/N_o . The curves are drawn with D/λ as a parameter. For a low latitude and low relative longitude, a 1 nmi accuracy is achieved with $TA_o^2/N_o = 38.5$ dB for $D/\lambda = 75$. A 2.5 dB saving is possible by increasing D/λ to 100, and a 4.5 dB saving is possible for $D/\lambda = 125$.

A tradeoff between integration time T and A_o^2/N_o (and hence transmitter power at the satellite) is shown in Fig. 29. The curves are plotted with the performance factor TA_o^2/N_o as a parameter. From

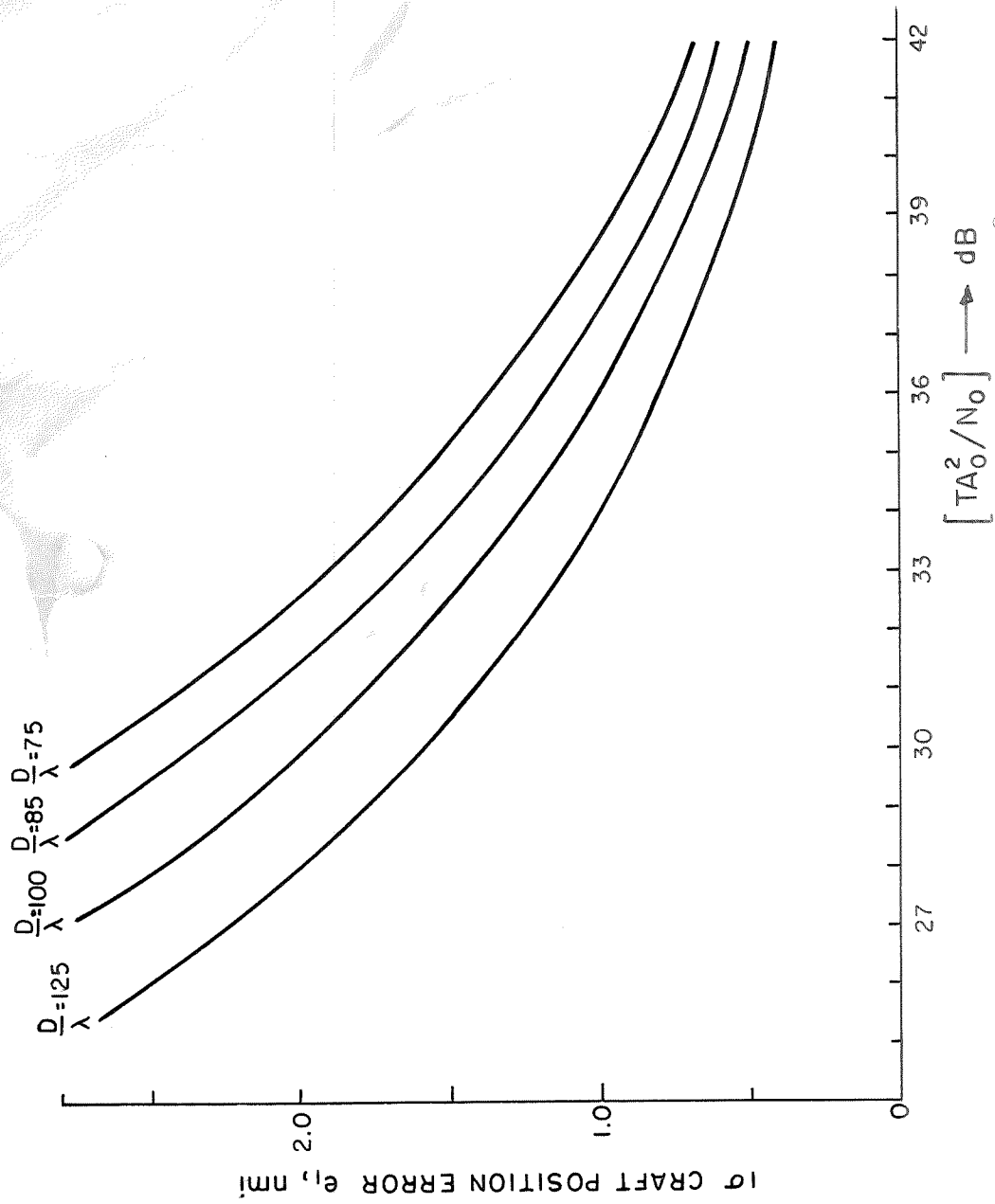


FIGURE 26 ESTIMATION ERROR e_1 PLOTTED AGAINST THE PERFORMANCE FACTOR TA_0^2/N_0 , WITH D/λ AS A PARAMETER [FOR $\Gamma_x = 1^\circ$, $\Gamma_y = 1^\circ$, $\theta = 85^\circ$]

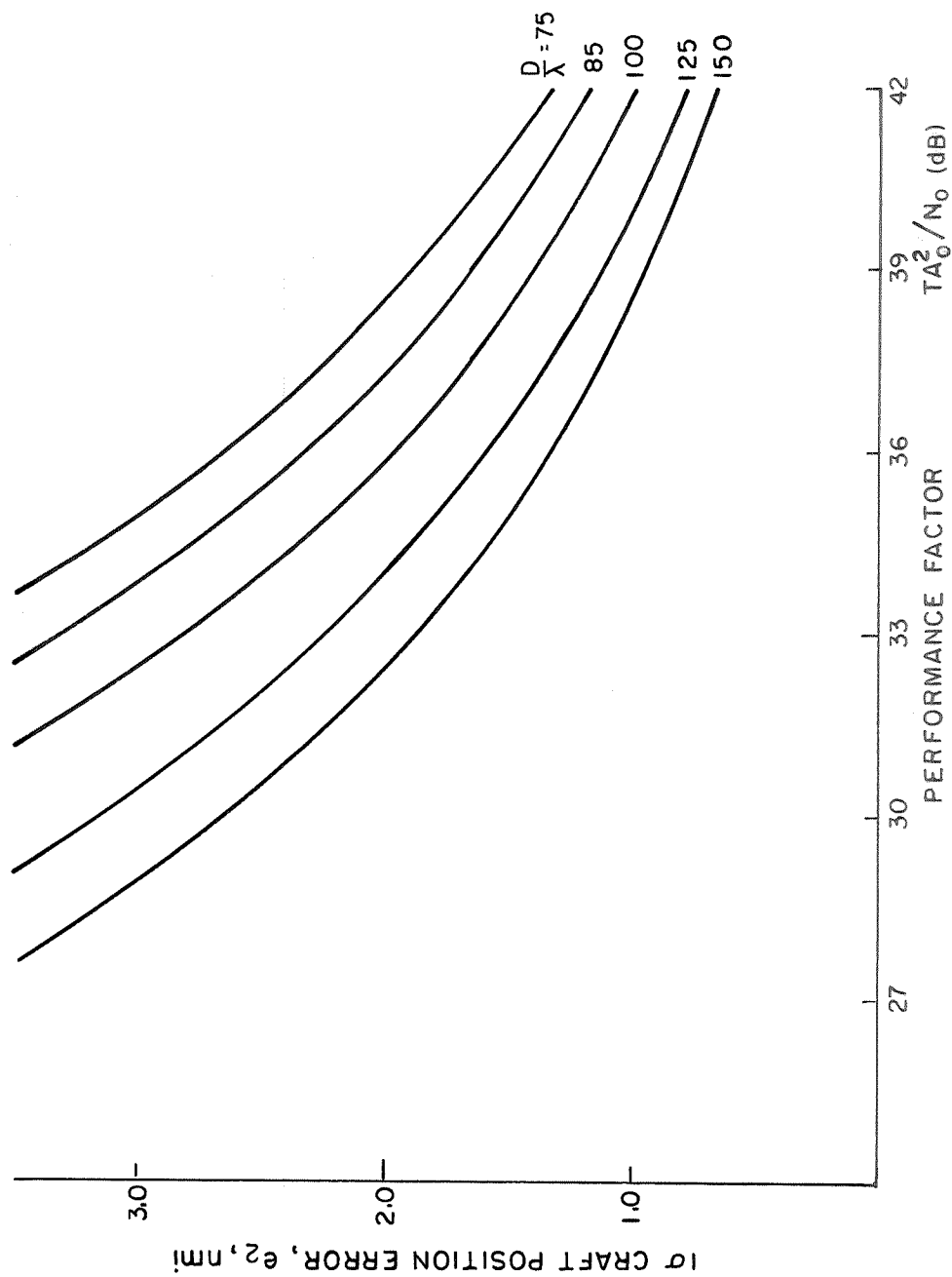


FIGURE 27 ESTIMATION ERROR e_2 PLOTTED AGAINST TA_0^2/N_0 , WITH D/λ AS A PARAMETER [FOR $\Gamma_x = 1^\circ$, $\Gamma_y = 1^\circ$, $\theta = 45^\circ$]

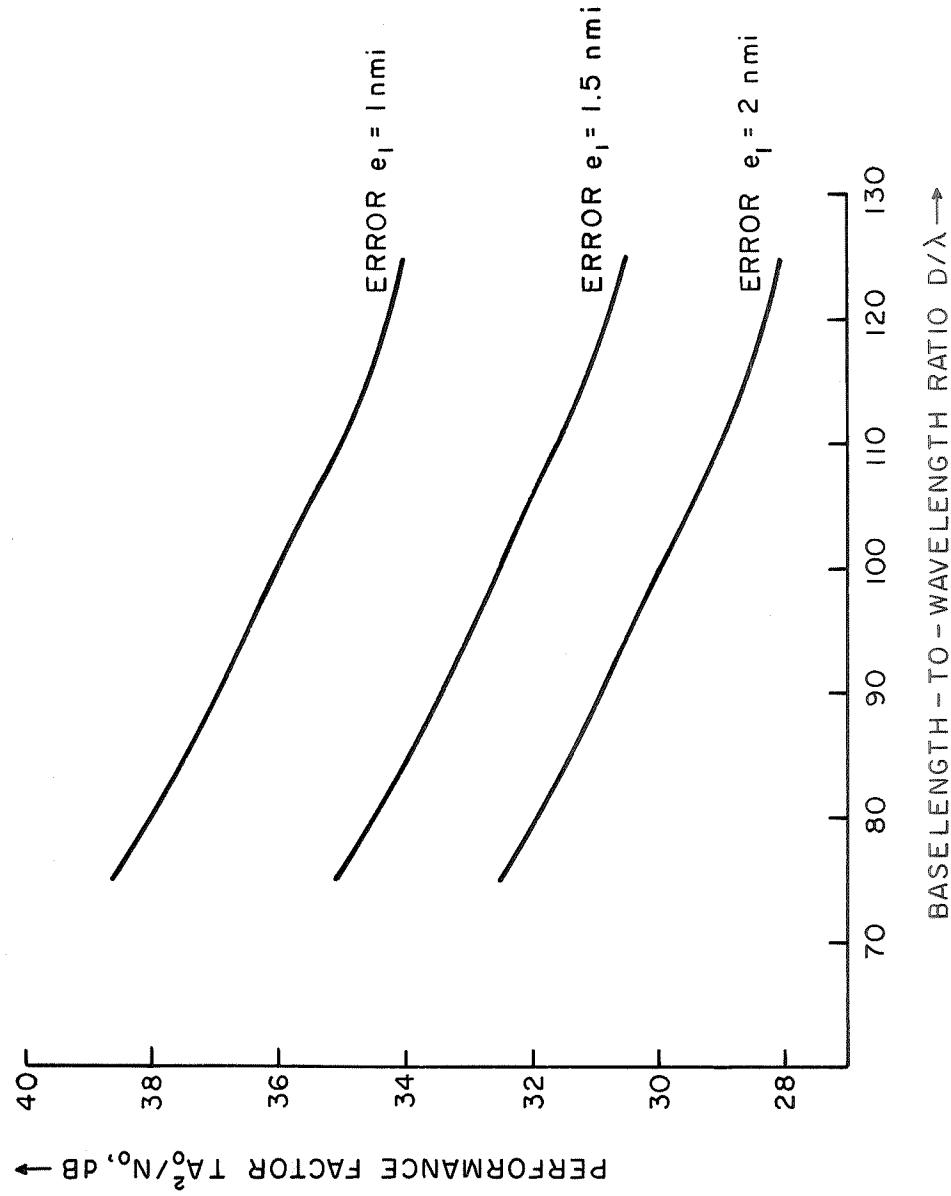


FIGURE 28 PERFORMANCE FACTOR TA_0^2/N_0 PLOTTED AGAINST D/λ , WITH e_1 AS A PARAMETER
[FOR $\Gamma_x = 10^\circ$, $\Gamma_y = 10^\circ$, $\theta = 85^\circ$]

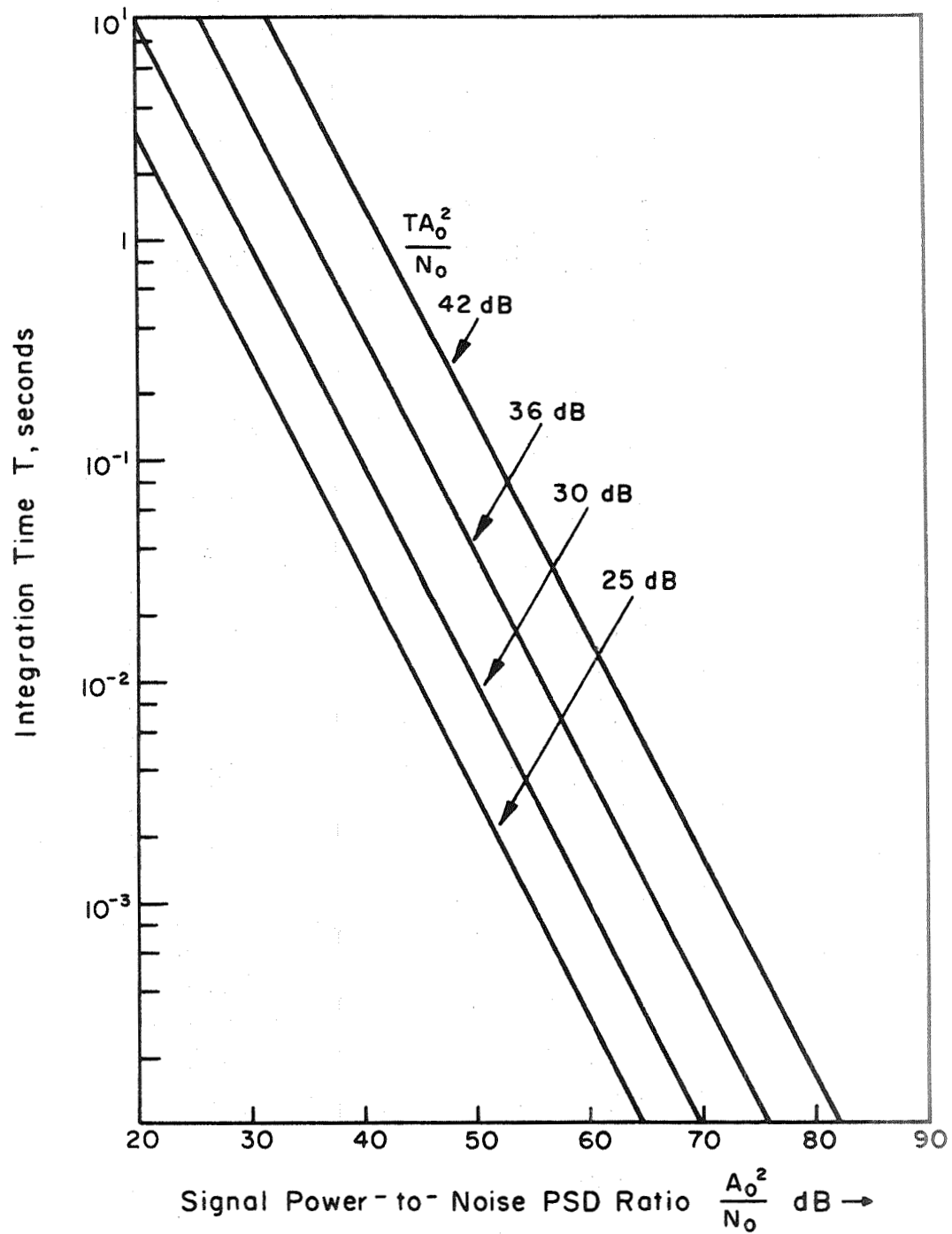


FIGURE 29 INTEGRATION TIME T PLOTTED AGAINST SIGNAL POWER-TO-NOISE PSD RATIO, $\frac{A_0^2}{N_0}$

Fig. 26 it is seen that TA_0^2/N_0 must exceed 36 dB for an error e_1 to be less than 1 nmi at $D/\lambda = 100$. Table I gives $A_0^2/N_0 = 35.5$ dB for a total satellite transmitter power of 50 watts (25 watts per transmitter) and no gain in the interferometer antennas. Figure 29 indicates that an integration time of approximately 1 second is required to give 1 nmi accuracy in the x-direction. For $e_1 = 2$ nmi, TA_0^2/N_0 is required to exceed 30 dB from Fig. 26. With the above constraint on satellite power and antenna gain, the integration time must exceed 0.27 second. If the satellite antennas are made directional by mechanically or electronically despinning them so that a gain of 18 dB (20° beamwidth) is introduced, a 1 nmi accuracy in the x-direction is possible with an integration time of only 16 msec.

Figure 30 indicates the tradeoffs possible between the gain G_s of the satellite interferometer antennas, the gain G_c of the navigating craft antenna and the power per satellite transmitter. Curves are plotted for $G_s = 0$ dB (isotropic), 10 dB (50° beamwidth) and 18 dB (20° beamwidth), the last two requiring some kind of despinning mechanism mentioned earlier; G_c is given values of 0 dB, 10 dB and 20 dB, the higher figure possible only if a steering arrangement is available to point the 16° beamwidth aircraft antenna toward the satellite; the range of satellite transmitter power considered is from 12.5 to 75 watts per transmitter. In arriving at the A_0^2/N_0 figures in Fig. 30 a free space loss of 190 dB and an aircraft receiver temperature of 1000°K have been assumed as in Table I. Figure 30 when used with the figures preceding it becomes a useful aid to the design of the entire navigation system.

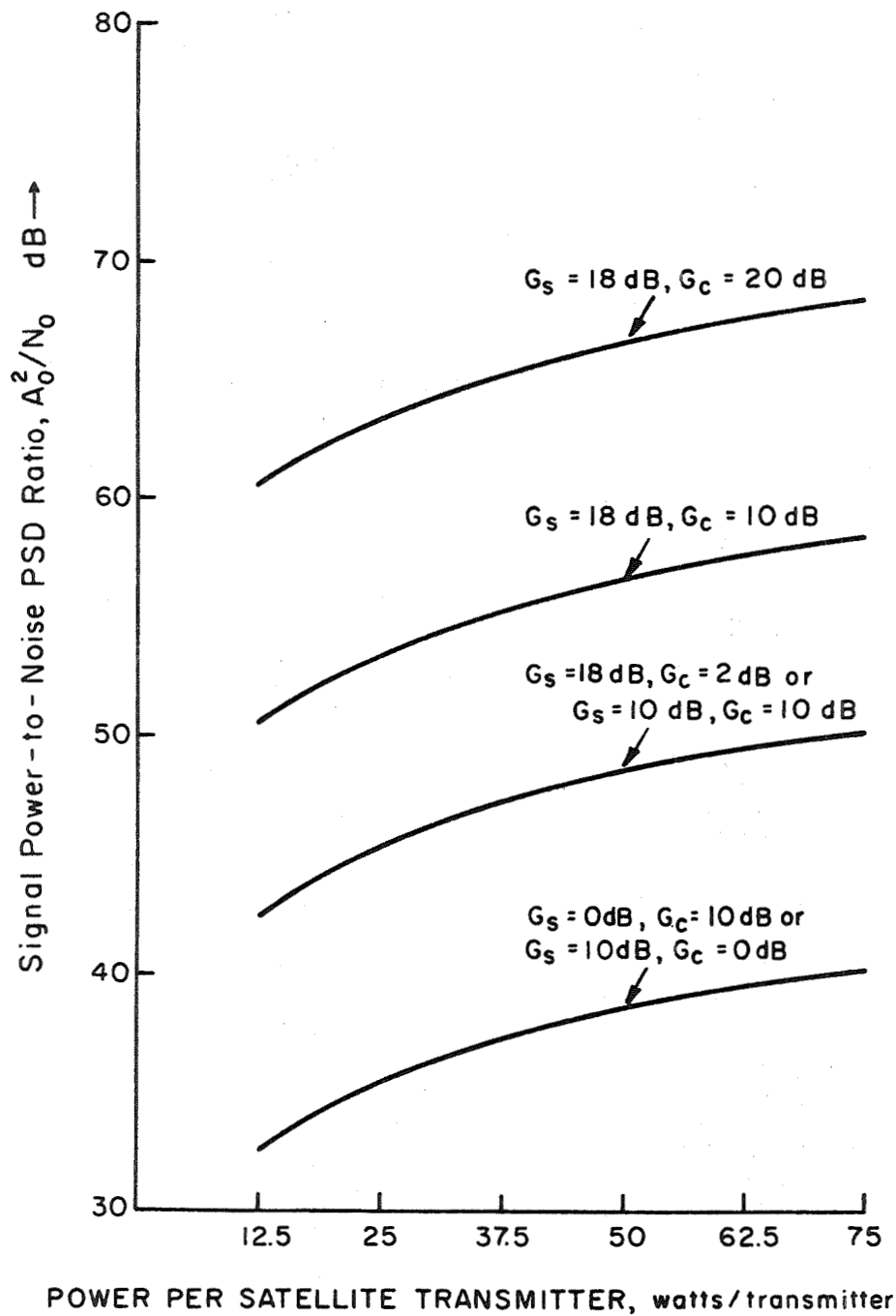


FIGURE 30 SIGNAL POWER-TO-NOISE PSD RATIO PLOTTED AGAINST POWER PER SATELLITE TRANSMITTER, WITH SATELLITE ANTENNA GAIN G_s AND CRAFT ANTENNA GAIN G_c AS PARAMETERS [AIRCRAFT RECEIVER TEMPERATURE 1000 K ASSUMED.]

Table I*

Power Budget for Satellite Signals Received by a Navigating Craft

Satellite transmitter power (25 watts per transmitter)	17 dBW
Satellite interferometer antenna gain G_s (isotropic)	0 dB
Free space loss at 1.6 GHz from synchronous altitude to horizon	-190 dB
Craft-borne receiving antenna gain G_c (50° beamwidth)	10 dB
Noise power spectral density N_o ($T_e = 1000^\circ\text{K}$)	-198.5 dB
Received signal power-to-noise PSD ratio, A_o^2/N_o	35.5 dB

* Sources of error other than receiver noise are discussed in Section 5 and are not included in this power budget.

5.0 ANALYSIS OF OTHER ERROR SOURCES

5.1 User Velocity and Altitude Errors

5.1.1 Error from Uncertainty in User Velocity

An important limitation in many proposed navigation satellite schemes including the only operational one, the Navy Navigation Satellite System, is that the velocity of the navigating craft must be known with precision (thereby making the system difficult to use with high-speed aircraft). This problem does not exist in the Spinning Interferometer technique as the following analysis will show.

The velocity of the user, V_u , may be resolved into two components, as shown in Fig. 31: V_{ur} along the user-antenna line-of-sight (LOS) and V_{up} perpendicular to this line. The tangential velocity v_a of the antenna can also be resolved into v_{ar} along the user antenna LOS and v_{ap} that is perpendicular to this line. The effect of V_u on the parameter of interest $\varphi(t)$ is to be analyzed.

By definition the phase quantity of interest is $\varphi(t) = \frac{\omega}{2c} [r_2 - r_1]$ and therefore $\varphi'(t) = \frac{\omega}{2c} [r_2' - r_1']$ where the primes indicate time-derivatives. Note that r_1' (or r_2') is the relative velocity between user and antenna #1 (or #2). In terms of the velocity components defined in Fig. 31, the relative velocity r_1' between user and antenna #1 is $V_{ur} - v_{ar}$, and r_2' is $V_{ur} + v_{ar}$.

Substituting these quantities in φ' , we get

$$\varphi' = \frac{\omega}{2c} [V_{ur} + v_{ar} - V_{ur} + v_{ar}] = \frac{\omega}{c} v_{ar}$$

which is independent of the radial contribution from the user's

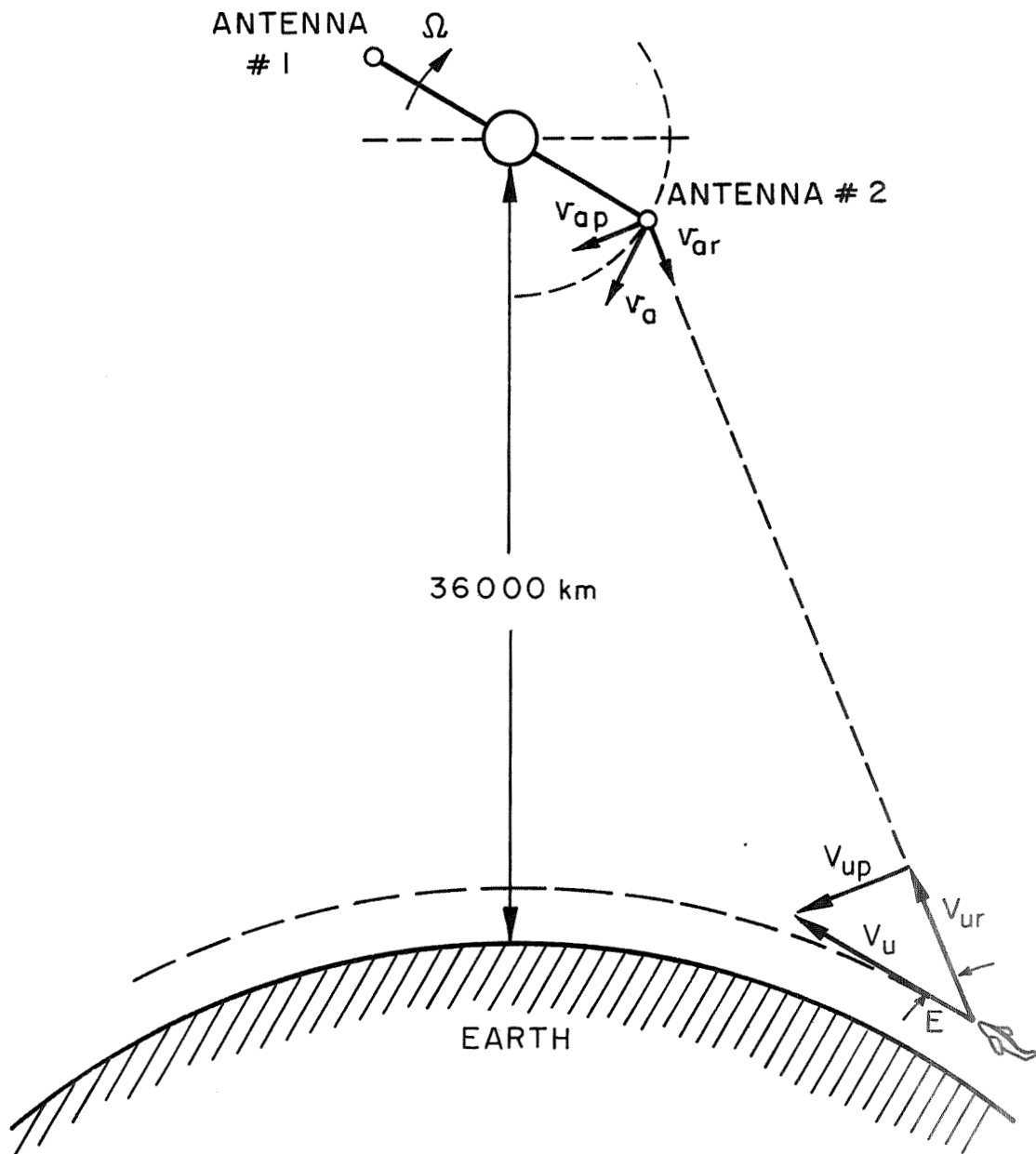


FIGURE 31 CRAFT VELOCITY COMPENSATION PROPERTY OF SPINNING INTERFEROMETER NAVIGATION SATELLITE CONCEPT

velocity. φ is therefore also unaffected by this.

As shown in Section 2.2.1 the velocity of the craft induces a shift in the "carrier" part of the DSB waveform. The post-IF signal, $s_{\text{IF}}(t)$, is written as $2A_o \cos \varphi(t, \underline{C}) \cos[(\omega_{\text{IF}} - P_1)t - P_o]$ where P_1 is dependent on user velocity. Figure 32 shows this dependence. The "intelligence" part of $s_{\text{IF}}(t)$, however, is unaffected by P_1 and the position accuracy therefore remains unaffected also. This happens because the VCO in the DSB demodulator is designed to synchronize its output with $\omega_{\text{IF}} - P_1$ and thus deliver the uncontaminated intelligence for further processing.

It is possible for V_{up} , the component of user velocity normal to the user-antenna LOS, to affect φ . This is because φ varies at a frequency equal to the satellite spin-rate Ω which is affected by V_{up} . The change $\Delta\Omega$ in Ω is, however, on the order of $\frac{V_{\text{up}}}{r}$. Considering the worst case, where the craft is at the subsatellite point, $V_{\text{up}} = V_u$, $r \cong$ synchronous altitude,

$$\Delta\Omega = \frac{V_{\text{up}}}{r} = 9.2 \times 10^{-6} \text{ radians/second}$$

where $V_u = 1200 \text{ km/hr}$ (near Mach 1).

The spin-rate of the satellite is on the order of 180 rpm (18.8 radians/second). The aircraft velocity changes the apparent spin-rate by $\frac{\Delta\Omega}{\Omega} \simeq 0.00005\%$, clearly an insignificant quantity. We are justified therefore in claiming that this navigation system entirely compensates for craft motion.

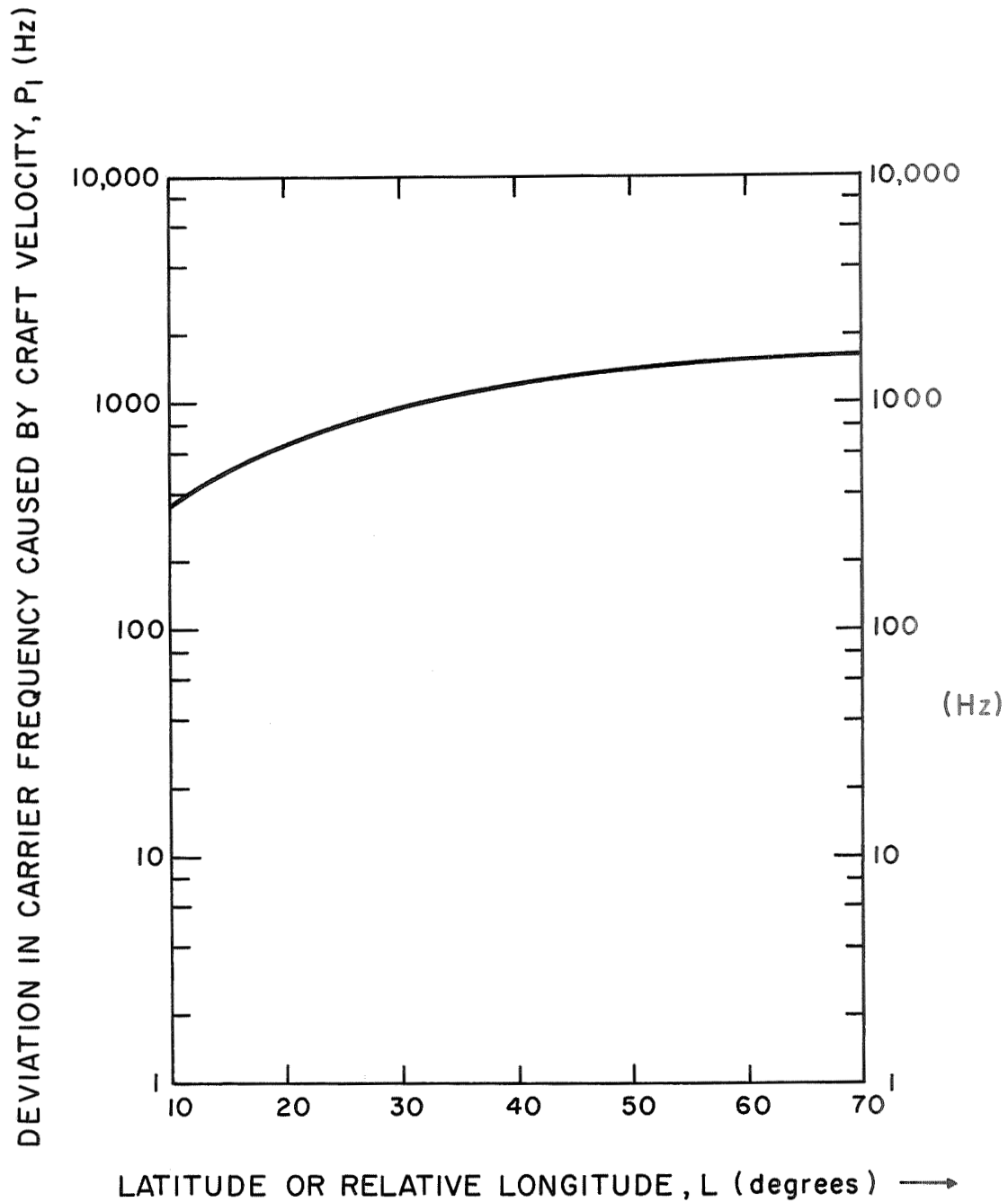


FIGURE 32 CARRIER FREQUENCY DEVIATION CAUSED BY CRAFT VELOCITY AS A FUNCTION OF LATITUDE (OR RELATIVE LONGITUDE) OF THE NAVIGATING CRAFT

5.1.2 Altitude Measurement Error

This navigation scheme provides only two out of the three surfaces of position (SOP) required to locate the navigating craft in three-dimensional space. It is assumed that the third SOP, a sphere concentric with the earth but with a radius equal to the earth radius augmented by the craft altitude, h_a , is available independent of the radionavigation system. Reasonably reliable altitude information is obtained from the barometric altimeter which is currently the basic instrument employed by most aircraft for altitude measurement.

Altimeter errors can normally be categorized as instrument (or mechanical) errors, aerodynamic (installation or flow field) errors and operational (flight technical) errors. An analysis of each group is important from the standpoint of component improvement. The emphasis in this work, however, is on the efficient and safe use of available airspace which is dependent on an overall "probable error" figure for the barometric altimeter system. Such a "probable error" in altitude is calculated for three nominal cruise altitudes of sea level, 6,000, 10,000 and 30,000 feet [61]. The factors that contribute to the probable error are based on experimental data compiled by several organizations [4, 5, 33, 52, 61]. In evaluating the probable error at the 0.003 level of significance (3σ) for a normal distribution, the convention used is [61]

$$P.E. = \sqrt{(e_1^2 + e_2^2 + \dots + e_n^2)}$$

where the e_i are the 3σ errors due to various sources of error. The

values for e_i are pessimistic but are taken to give an idea of the worst case performance of the navigation system. As tabulated in reference [61] the 3σ probable errors for sea-level, 6,000 feet, 10,000 feet and 30,000 feet are 664 feet, 583 feet, 582 feet and 980 feet respectively.

The interest in altitude error in the present work arises not only from the point of view of providing adequate altitude separation standards for the heavily traveled air corridors across the North Atlantic, but also because altitude errors introduce errors in the tilt angles Γ_x and Γ_y which the craft obtains by processing the navigation signals. The effect of altitude error on craft position error is to be considered next.

From Fig. 33 it is clear there is a relationship between the altimeter error Δh_a and tilt angle error $\Delta\Gamma$:

$$\Delta\Gamma = \frac{\Delta h_a}{r_s} \cos(90^\circ - \Gamma - L)$$

where the slant range r_s can be obtained from

$$r_s = \frac{(h + R_e) \sin L}{\cos E}$$

E being the satellite elevation angle, L the latitude or relative longitude, and $h + R_e = 42371$ km. $\Delta\Gamma$ expressed in nmi is plotted against L in Fig. 34. Note that $\Delta\Gamma$ has been corrected for GDOP.

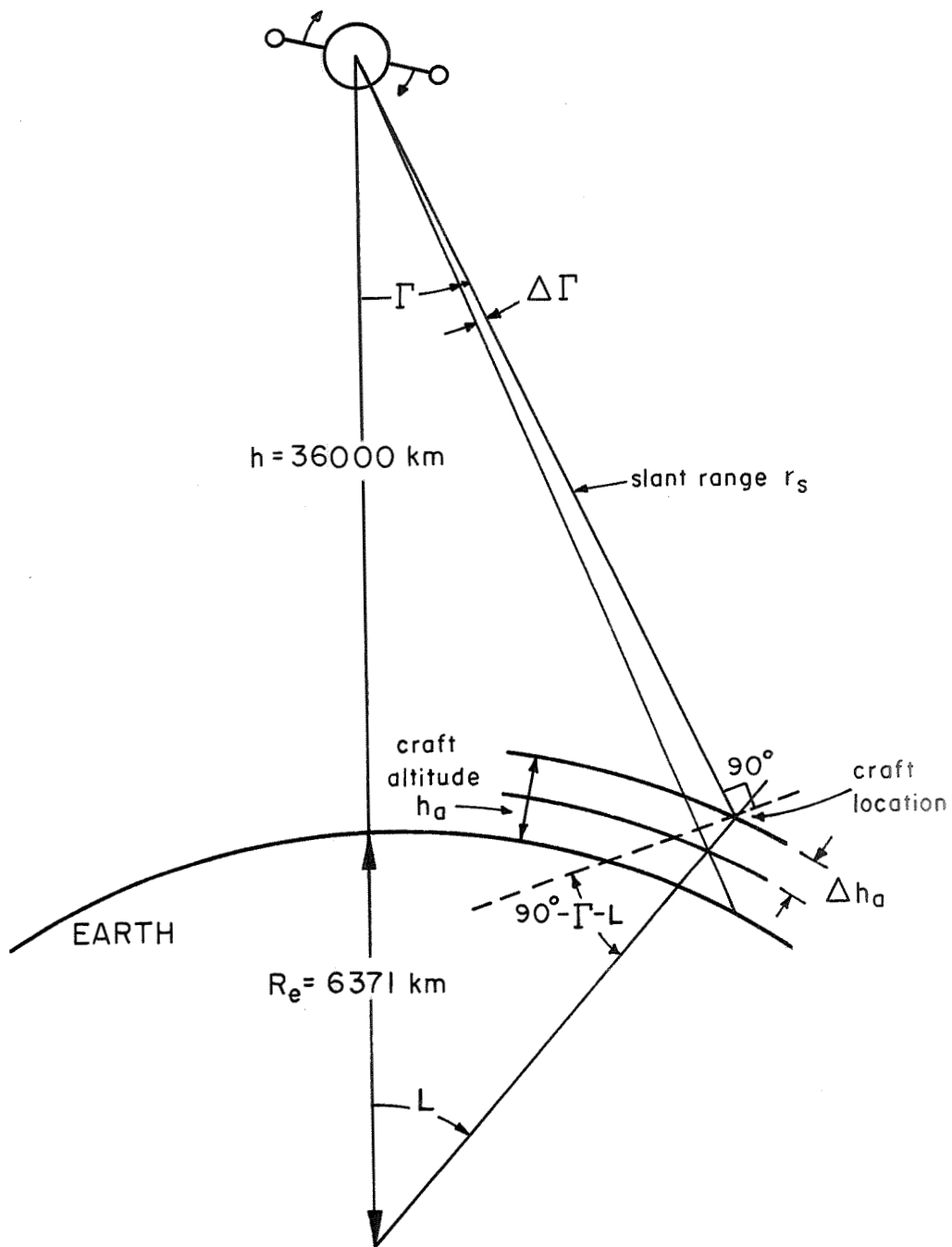


FIGURE 33 ALTIMETER ERROR GEOMETRY

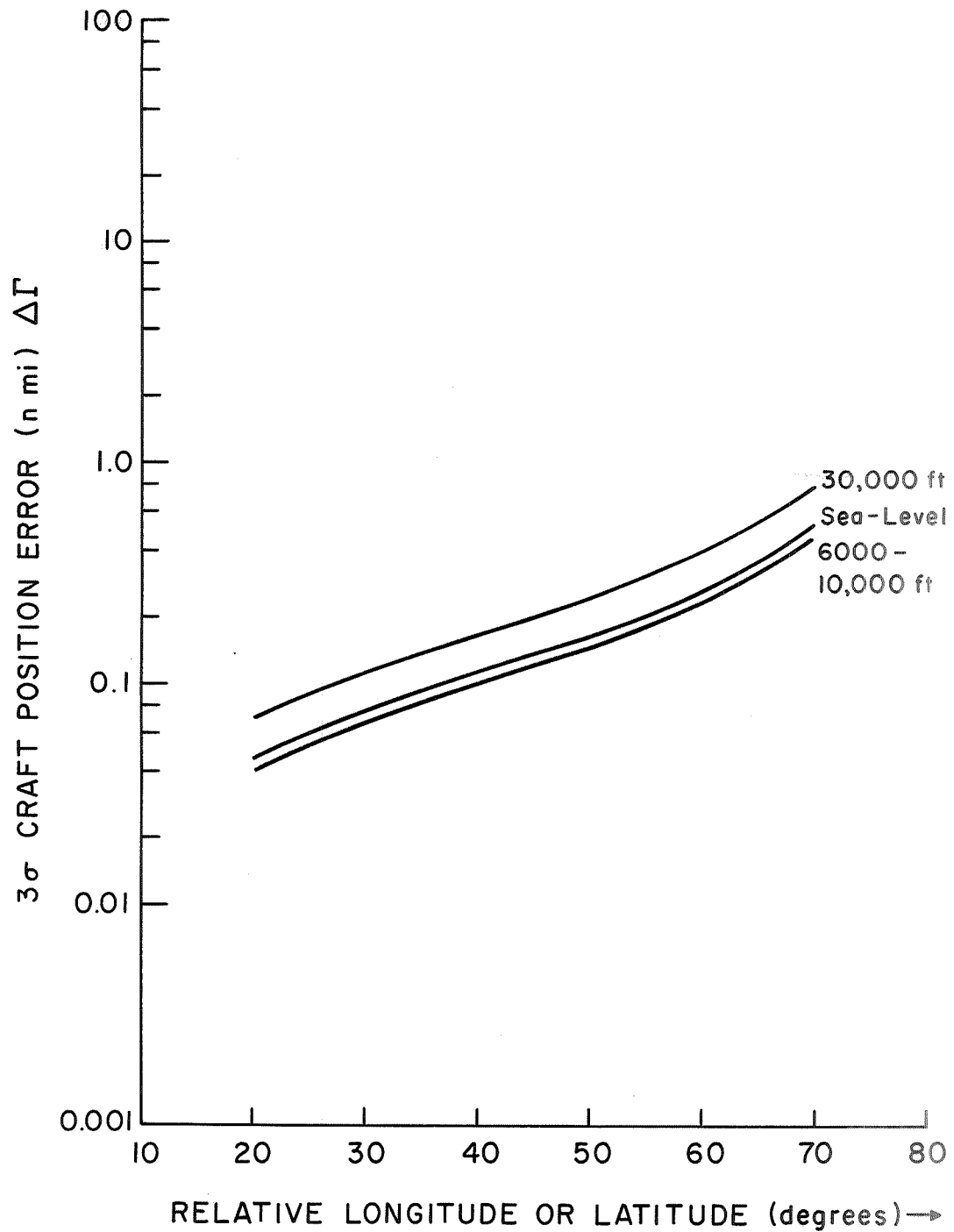


FIGURE 34 CRAFT POSITION ERROR (THREE STANDARD DEVIATIONS) PLOTTED AGAINST CRAFT LATITUDE OR RELATIVE LONGITUDE, WITH CRAFT ALTITUDE AS A PARAMETER

5.2 Propagation Errors

Errors are introduced into interferometric systems by differences in the signal propagation paths to (or from) two physically separated antennas, since the desired information is contained in the phase-difference quantity, and not in the total argument of either signal. Propagation error, therefore, will not exist if both paths are affected equally. In this section consideration is given mainly to those propagation effects that are likely to introduce error in the phase-difference. Other effects, such as absorption loss in the troposphere and polarization (Faraday) rotation exist. Tropospheric absorption loss at 1.5 GHz has been estimated to be 1.75 dB, 0.75 dB, 0.07 dB for elevation angles of 0° , 2° and 30° respectively [7] for a one-way transversal of the entire atmosphere. Ionospheric absorption has been calculated to be about 0.1 dB and 10^{-3} dB at 1 GHz and 10 GHz respectively [12]. Clearly, the signal-to-noise ratio at the craft receiver is not drastically affected by these small losses in power. The other phenomenon, Faraday rotation, is the rotation of the plane of polarization of a linearly polarized wave passing through the ionosphere which consists of ionized gas in a magnetic field [49,58]. This effect is inversely proportional to the square of the signal frequency and is considered insignificant above a few hundred megacycles [12]. If future L-band experiments show that this is not so, circularly polarized antennas should obviate the problem.

5.2.1 Refraction

Due to heterogeneities in the earth's ionosphere and troposphere radio waves are refracted, although for elevation angles above 15° no effect on system accuracy is expected [41]. Refractive effects for elevation angles above about 2° are considered negligible in reference [59].

These conclusions may be accepted intuitively with the aid of Fig. 35. The tilt angles Γ_x, Γ_y are calculated from the phase difference between the two signals following paths #1 and #2. An error in the tilt angles can result from the two paths changing in lengths by different amounts. If #1 and #2 undergo a direction change so that they are "as parallel" after refraction as before, then the user receives signals that are essentially unaffected by the refraction. But if #1 is refracted to a smaller extent than #2, then #2 travels an extra distance Δr as shown in Fig. 35. The question of whether Δr is significant compared to the phase-difference $\varphi(t)$ may be tentatively answered for the limiting cases of satellite elevation.

At high elevation angles, with the user directly below the satellite, both rays are practically normal to the ionospheric and tropospheric strata, and are therefore not refracted significantly, i.e. $\Delta r = 0$. At low elevation angles, however, the rays make large incident angles at the strata and consequently undergo some refraction. For an interferometer of any practical baselength located on a synchronous satellite, the two rays are virtually adjacent at

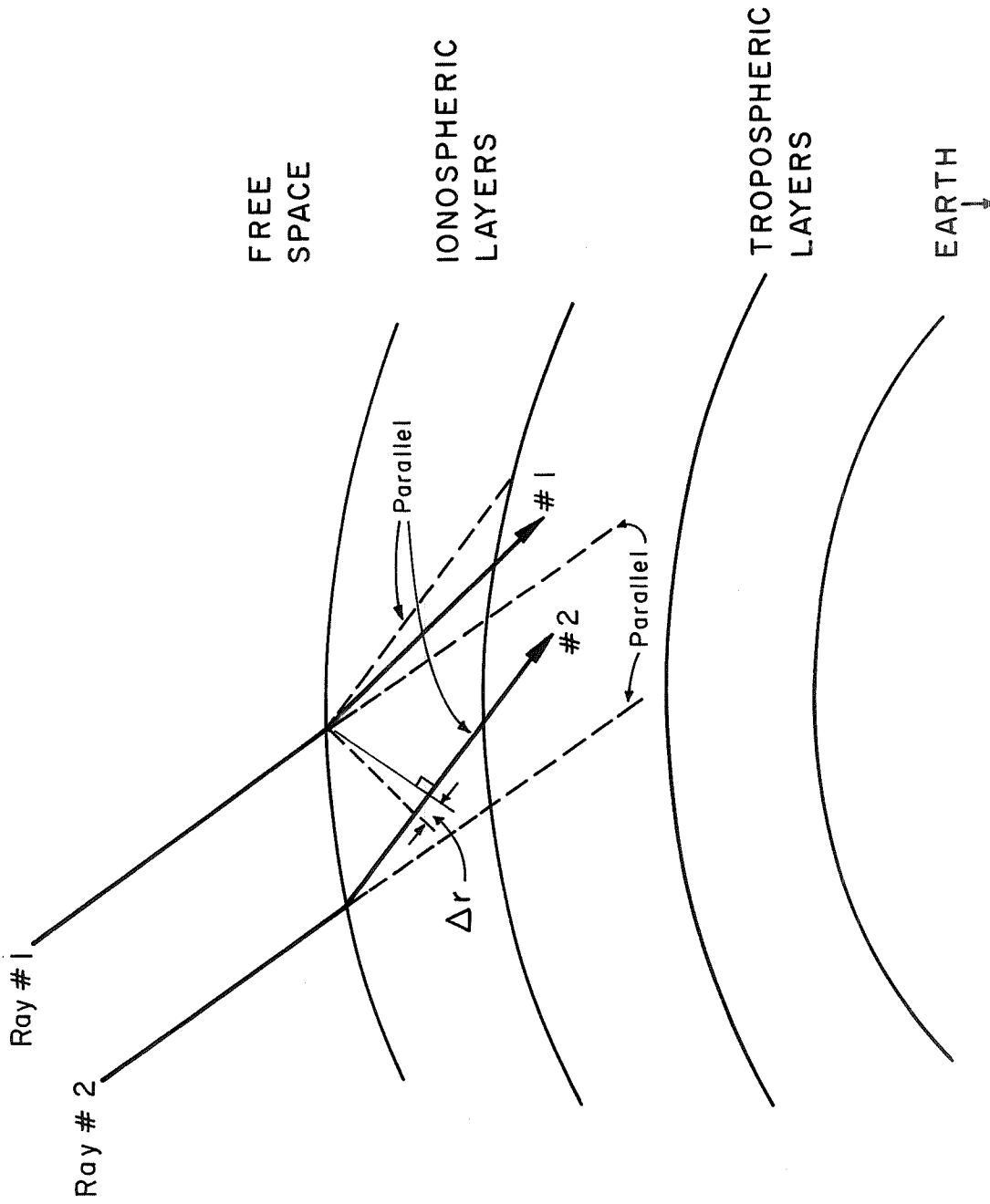


FIGURE 35 REFRACTIVE BENDING IN RAYS FROM SATELLITE TO EARTH

ionospheric altitude; the refraction undergone will be practically equal, leading Δr to equal zero again.

5.2.2 Dispersion

It is well known that the ionosphere is a dispersive medium for radio waves [21], which means that two signals differing slightly in frequency will travel at slightly different velocities. Error due to this effect has been considered negligible for frequencies above 1 GHz [41]. This view has been substantiated by radio astronomy measurements of the structure of Doppler-spread sources with a resolution of better than 0.5 microradians at 1400-1427 MHz (the Hydrogen line) and 1664-1668 MHz (the Hydroxide line) [55]. Furthermore, measurements at 2.7 GHz have indicated that even for radio sources spread over a bandwidth of 24 MHz the effect of dispersion is very small [43].

5.2.3 Tropospheric Phase Fluctuations

Tropospheric phase noise has been observed and measured in radio astronomy interferometer experiments [6, 11, 14]. With a ground-based interferometer 2 km long, it was found that phase noise at 2.7 GHz was on the order of $\pm 20^\circ$ rms [14]. With longer baselengths, 2.1 to 2.7 km, phase noise measured from 4 to 40 $^\circ$ rms at 2.7 GHz [6]. With a shorter baselength, 105 m, and a higher frequency, 3.075 GHz, phase fluctuations of 1.5 $^\circ$ have been measured [11]. It has been shown that phase noise increases linearly with operating frequency and with the 5/6 power of the interferometric baselength [6]. Using this relationship and the experimental evidence quoted above, the phase noise at

L-band with a 20-wavelength interferometer has been estimated to be on the order of $\pm 0.05^\circ$ or $\pm 0.06^\circ$ [41]. For a 100-wavelength interferometer, of interest in the present work, these figures may be extrapolated to give an estimate of tropospheric phase noise to be on the order of 0.19° to 0.23° .

The experiments cited above involved the observations of extraterrestrial radio sources from ground-based interferometers. These phase fluctuations, to some extent, seem to have been caused by atmospheric disturbances close to one of the two antennas [6]. For the satellite-borne interferometer situation, the phase fluctuations are likely to be much smaller since the two rays are very close to one another as they pass through the troposphere and therefore any tropospheric disturbances are likely to be perfectly correlated for the two signals. In contrast to this, radio astronomy experiments use very narrow beamwidth antennas, resulting in the two rays occupying separate columns as they pass through the troposphere. Furthermore, many of the users of this navigation scheme are high-altitude aircraft which on long-range flights are at altitudes where the tropospheric contribution to the error is minimal [41].

To arrive at orders of magnitude of position errors, differentiate the expression for $\varphi(t)$ with respect to the position coordinates Γ_x and Γ_y . From the expressions for C_i and A_{ij} in Section 4.1 it is clear that $\frac{\partial C_1}{\partial \Gamma_x} \gg \frac{\partial C_2}{\partial \Gamma_x}$, $\frac{\partial C_3}{\partial \Gamma_x} = 0$, $\frac{\partial C_3}{\partial \Gamma_y} \simeq -1$ and that $\frac{\partial C_1}{\partial \Gamma_y}$ and $\frac{\partial C_2}{\partial \Gamma_y}$ are very small. Making these approximations,

the following relations may be written:

$$\left| \frac{\partial \varphi}{\partial \Gamma_x} \right| \approx \pi \frac{D}{\lambda} \sin \theta$$

$$\left| \frac{\partial \varphi}{\partial \Gamma_y} \right|_{\max} \approx \pi \frac{D}{\lambda} \cos \theta$$

For $\theta = 85^\circ$ and $D/\lambda = 100$, the tropospheric phase noise of 0.19° and 0.23° correspond to position error in the x-direction of $10.5 \mu\text{rad}$ (0.20 nmi) and $10.8 \mu\text{rad}$ (0.21 nmi) respectively. These numbers may be considered to be worst case values as argued in the preceding paragraph. Position error in the y-direction, as expected, is much worse for $\theta = 85^\circ$. However, equal errors in the two directions result if $\theta = 45^\circ$, leading to $15 \mu\text{rad}$ (0.29 nmi) corresponding to phase noise of 0.19° .

5.2.4 Multipath

The multipath problem in satellite-aircraft communication links has been investigated both theoretically [10,35,51,67] and experimentally [38,45]. Measurements of the relative received multipath power have been made for grazing angles θ_g between 5° and 45° at a carrier frequency of 230 MHz. Frequency spreading of the multipath signal has also been measured as a function of grazing angle. Except at the lowest grazing angles, these results indicated that the multipath had a significant diffuse component for over-ocean flights [38].

From investigations of typical land and sea surfaces it is concluded on the basis of the criterion that for $\theta_g \geq 10^\circ$ diffuse conditions prevail whenever $\sigma_b > \lambda$ [24], that at L-band frequencies multipath reflection is predominantly diffuse [51]. σ_b is the standard deviation of the bump heights on the surface of reflection, estimated to be over 19 cm for 60.9 percent of all time. For a typical frequency of 1600 MHz, $\lambda = 18.75$ cm.

Having characterized the nature of the multipath signal, it is necessary first to determine the degradation in the performance of this navigation system due to multipath, and second, to explore possible techniques to improve the performance. Some results for a simplified geometry are obtained in this section followed by some multipath reduction techniques. Parenthetically, it may be noted that the following study is an attempt to obtain the order of magnitude of the multipath effect rather than an exhaustive treatment of the subject. Extensions are possible by assuming a detailed three-dimensional system geometry, assuming varying proportions of diffuse and specular components and investigating diversity and polarization techniques for the reduction of multipath.

5.2.4.1 Multipath Analysis

The objective here is to calculate the phase error $\Delta\varphi$ due to multipath. The signals s_1 , s_2 received by a craft may be written as

$$\begin{aligned} s_1 &= A_o \cos\left(\omega t - \frac{\omega r_1}{c}\right) + B_o \cos\left(\omega t - \frac{\omega r_1}{c} + \psi\right) \\ s_2 &= A_o \cos\left(\omega t - \frac{\omega r_1}{c} + 2\varphi\right) + B_o \cos\left(\omega t - \frac{\omega r_1}{c} + 2\varphi + \chi + \psi\right) \end{aligned} \quad (78)$$

where A_o is the direct signal amplitude, B_o is the (diffuse) multipath amplitude which is a Rayleigh distributed random variable, ψ is the multipath phase delay, a uniform random variable over $(0, 2\pi)$, 2φ is the true phase difference desired from the uncontaminated direct signals, r_1 is the craft range from antenna #1, and χ is the additional phase in the multipath in s_2 due to the difference in the directions of the multipath and direct phase fronts.

Equations (78) may be rewritten as

$$s_1 = \sqrt{A_o^2 + 2A_o B_o \cos \psi + B_o^2} \cos \left[\omega t - \frac{\omega r_1}{c} + \tan^{-1} \frac{B_o \sin \psi}{A_o + B_o \cos \psi} \right] \quad (79)$$

$$s_2 = \sqrt{A_o^2 + 2A_o B_o \cos(\psi + \chi) + B_o^2} \cos \left[\omega t - \frac{\omega r_1}{c} + 2\varphi + \tan^{-1} \frac{B_o \sin(\psi + \chi)}{A_o + B_o \cos(\psi + \chi)} \right]$$

The phase difference obtained by the navigator is then not 2φ as desired, but $2\varphi + \Delta\varphi$ where

$$\Delta\varphi = \tan^{-1} \frac{B_o \sin(\psi + \chi)}{A_o + B_o \cos(\psi + \chi)} - \tan^{-1} \frac{B_o \sin \psi}{A_o + B_o \cos \psi} \quad (80)$$

From the two-dimensional geometry in Fig. 36 the multipath and direct phasefronts are seen to be at an angle ϵ which depends on craft altitude and satellite elevation angle E . A plot of ϵ against elevation angle [51] for an aircraft flying at 9 km (29,500 feet)

FIGURE 36 MULTIPATH GEOMETRY: A TWO-DIMENSIONAL PICTURE

indicates that the maximum value of ϵ is around 0.024° for very low elevation angles. For $E = 90^\circ$, ϵ goes to zero. With this in mind χ is derived.

By definition and referring to Fig. 36,

$$\begin{aligned}\chi &= \frac{2\pi}{\lambda} \left[2\Delta_m - 2\Delta \right] \\ &= \frac{2\pi}{\lambda} \left[D \cos \theta \sin(\Gamma + \Omega t - \epsilon) - D \cos \theta \sin(\Gamma + \Omega t) \right] \quad (81)\end{aligned}$$

where $D \cos \theta$ is the projection of the interferometric baselength on the plane of the paper; Γ is the tilt angle of the craft in this plane and is equal to Γ_y for the satellite configuration assumed in this study. Rewriting Eq. (81),

$$\begin{aligned}\chi &= - 2\pi \frac{D}{\lambda} \cos \theta \left[2 \cos \left(\Gamma + \Omega t - \frac{\epsilon}{2} \right) \sin \left(\frac{\epsilon}{2} \right) \right] \\ &= - 2\pi \frac{D}{\lambda} \cos \theta \cdot \epsilon \cdot \cos \left(\Gamma + \Omega t - \frac{\epsilon}{2} \right) \quad (82)\end{aligned}$$

making the approximation $\sin \frac{\epsilon}{2} \simeq \frac{\epsilon}{2}$.

Since B_o and ψ are random variables, $\Delta\varphi$ is also a random variable. Its mean and variance can be obtained by averaging over B_o and ψ . Rather than tackle this mathematically tedious method, another approach [51] is taken.

Since $\Delta\varphi$ has a maximum with respect to ψ , while it monotonically increases with B_o , a value of ψ can be found for which $\Delta\varphi$ is a maximum. Differentiating $\Delta\varphi$ with respect to ψ , and setting the result equal to zero, gives

$$\psi = -\frac{\chi}{2}$$

which, when substituted for ψ in Eq. (80), gives

$$(\Delta\varphi)_{\max,\psi} = 2 \tan^{-1} \frac{\frac{B_o}{A_o} \sin \frac{\chi}{2}}{1 + \frac{B_o}{A_o} \cos \frac{\chi}{2}}$$

Simplifying, as in reference [51], and making the approximation that

$$B_o \ll A_o,$$

$$(\Delta\varphi)_{\max,\psi} \simeq 2 \frac{B_o}{A_o} \sin \frac{\chi}{2} \quad (83)$$

The Rayleigh distributed random variable B_o has a pdf

$$p(B_o) = \begin{cases} \frac{B_o}{\sigma_m^2} \exp \left[\frac{-B_o^2}{2\sigma_m^2} \right] & \text{for } B_o \geq 0 \\ 0 & \text{for } B_o < 0 \end{cases}$$

which indicates that the mean square value of B_o , $\langle B_o^2 \rangle = 2\sigma_m^2$.

The mean square value of $(\Delta\varphi)_{\max,\psi}$ may be written as

$$\sigma_\varphi^2 = \langle (\Delta\varphi)_{\max,\psi}^2 \rangle = (2)^2 \cdot \frac{2\sigma_m^2}{A_o^2} \cdot \langle \sin^2 \frac{\chi}{2} \rangle$$

If $\frac{2\sigma_m^2}{A_o^2}$ is considered the "multipath-to-signal power ratio" (MSR),

the rms value of $(\Delta\varphi)_{\max,\psi}$ becomes

$$\sigma_{\varphi} = 2 \sqrt{\text{MSR}} \sqrt{\left\langle \sin^2 \frac{\chi}{2} \right\rangle} \quad (84)$$

Now setting $\theta = 0$ in Eq. (82) to obtain the worst-case χ and substituting this in $\sin \frac{\chi}{2}$:

$$\begin{aligned} \sin \frac{\chi}{2} &= - \sin \left[\pi \frac{D}{\lambda} \epsilon \cos \left(\Gamma + \Omega t - \frac{\epsilon}{2} \right) \right] \\ &= - 2 \sum_k (-1)^k J_{2k+1} \left(\pi \frac{D}{\lambda} \epsilon \right) \cos(2k+1) \left(\Gamma + \Omega t - \frac{\epsilon}{2} \right) \end{aligned}$$

The baselength-to-wavelength ratio in the range of interest lies between $D/\lambda = 50$ and $D/\lambda = 200$. For the maximum value of ϵ , the values of the argument $\pi \frac{D}{\lambda} \epsilon$ for $D/\lambda = 50, 100$ and 200 are $0.066, 0.132$ and 0.264 . The first order Bessel function has the corresponding values $0.033, 0.066, 0.132$. The next Bessel function in the series is of order three. Since it is two orders of magnitude lower than J_1 , it is not considered significant in this analysis and may consequently be dropped. Therefore,

$$\begin{aligned} \sin \frac{\chi}{2} &\simeq - 2 J_1 \left(\pi \frac{D}{\lambda} \epsilon \right) \cos \left(\Gamma + \Omega t - \frac{\epsilon}{2} \right) \\ \sqrt{\sin^2 \frac{\chi}{2}} &= \sqrt{2} J_1 \left(\pi \frac{D}{\lambda} \epsilon \right) \end{aligned} \quad (85)$$

On substituting Eq. (85) in (84), the rms value of the phase error becomes

$$\sigma_{\varphi} = 2\sqrt{2} J_1 \left(\pi \frac{D}{\lambda} \epsilon \right) \sqrt{\text{MSR}} \quad (86)$$

From Section 4.1 the phase error and position error are related by the expressions

$$\left| \frac{\partial \phi}{\partial \Gamma_x} \right| = \pi \frac{D}{\lambda} \sin \theta$$

$$\left| \frac{\partial \phi}{\partial \Gamma_y} \right|_{\max} = \pi \frac{D}{\lambda} \cos \theta$$

Since a two-dimensional geometry has been assumed here and a skew angle θ of zero has been assumed to get a worst case value for χ , the second sensitivity expression is applicable here. An rms position error σ_Γ may be written in terms of σ_ϕ and the system parameters as

$$\sigma_\Gamma = \frac{\sigma_\phi}{\pi \frac{D}{\lambda}} \quad (87)$$

Inserting (87) into (86),

$$\sigma_\Gamma = 0.9 \frac{J_1 \left(\pi \frac{D}{\lambda} \epsilon \right)}{\frac{D}{\lambda}} \sqrt{\text{MSR}} \quad (88)$$

At low values of the Bessel function argument $\pi \frac{D}{\lambda} \epsilon$, $J_1 \left(\pi \frac{D}{\lambda} \epsilon \right)$ is approximately linear and so the coefficient of $\sqrt{\text{MSR}}$ in (88) is a constant. Equation (88) is plotted in Fig.37 which indicates the level of multipath suppression required for given values of craft position error. It is seen that the multipath power is not dependent on D/λ for baselength values of interest.

The approach taken in this section has been to assume a diffuse model for the received multipath, then expressing the phase error $\Delta\phi$

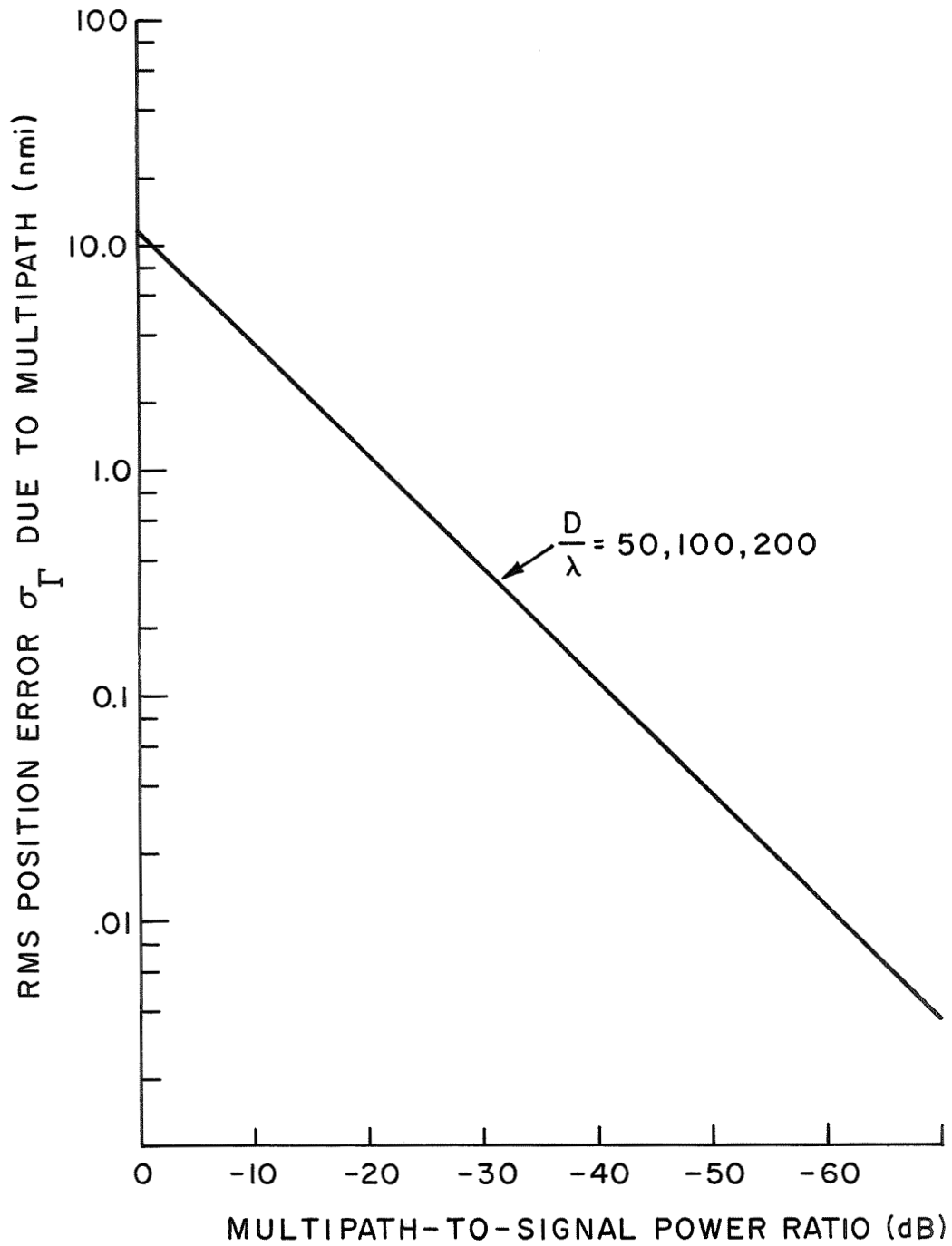


FIGURE 37 RMS POSITION ERROR DUE TO MULTIPATH PLOTTED AGAINST MULTIPATH-TO-SIGNAL POWER RATIO

as a function of the Rayleigh amplitude B_0 and uniform phase ψ of the multipath. $\Delta\varphi$ is then maximized with respect to ψ resulting in an upper bound obtained for the phase error. This maximum phase error is related to the multipath-to-signal power ratio and the system parameters through ϵ , the angle between the direct and multipath phase-fronts. Finally, the phase error is translated into position error. The next step is to explore ways to reject multipath and to estimate the resulting improvement in position accuracy.

It must be noted that some multipath may be caused by reflections from the surfaces of the satellite and the navigating craft. The former type is systematic over the field of users and is expected to be eliminated by the referenced radionavigation technique. The latter is not considered a serious problem if the craft antenna is suitably positioned on the craft structure.

5.2.4.2 Multipath Reduction Techniques

(1) Aircraft Antenna Pattern Discrimination

Since the type of antenna installed on a high-speed aircraft must be relatively compact and simple to avoid high installation costs and detrimental effects on aircraft dynamics, preference is given to top-mounted types with a low gain hemispheric coverage [17,45]. Past studies [41] of navigation satellite schemes have recommended a somewhat more directive antenna, one having gain on the order of 10 dB. Any further increase in gain, as noted earlier, will substantially increase the cost and complexity of the aircraft equipment since a steering mechanism will be required to keep the antenna pointed

toward the satellite. For a gain of 20 dB, for example, the beamwidth between the half-power points is about 16° and clearly would be useless even at moderate latitudes unless steering were employed.

Assuming a low-gain cardioid main-lobe pattern for the aircraft antenna, the geometry of Fig. 38 shows the earth-reflected multipath in a reduced gain position. For all intents and purposes the grazing angle, the direct signal arrival angle and the multipath signal arrival angle may be considered equal and approximately equal to the elevation angle E , for signals arriving from a synchronous satellite.

Most antenna patterns can be approximated over the $2E$ angle between the direct and multipath rays at the aircraft by a constant decrease in gain. Typical values for such a characterization are 0.1 dB/degree and 0.2 dB/degree [35]. For such typical antennas, a multipath voltage amplitude suppression factor ρ_m due to the antenna patterns may be defined:

$$\rho_m = \frac{\text{Antenna Gain at } \gamma + 2E}{\text{Antenna Gain at } \gamma} \quad (89)$$

which is plotted in Fig. 39 against the satellite elevation angle [35]. γ , the aircraft aspect angle, is shown in Fig. 38. This relation can be readily expressed as a direct signal-to-multipath power improvement which is sketched in Fig. 40. This plot shows a 10 dB improvement, solely due to lower hemisphere discrimination, at an elevation angle of 25° (57.2° latitude).

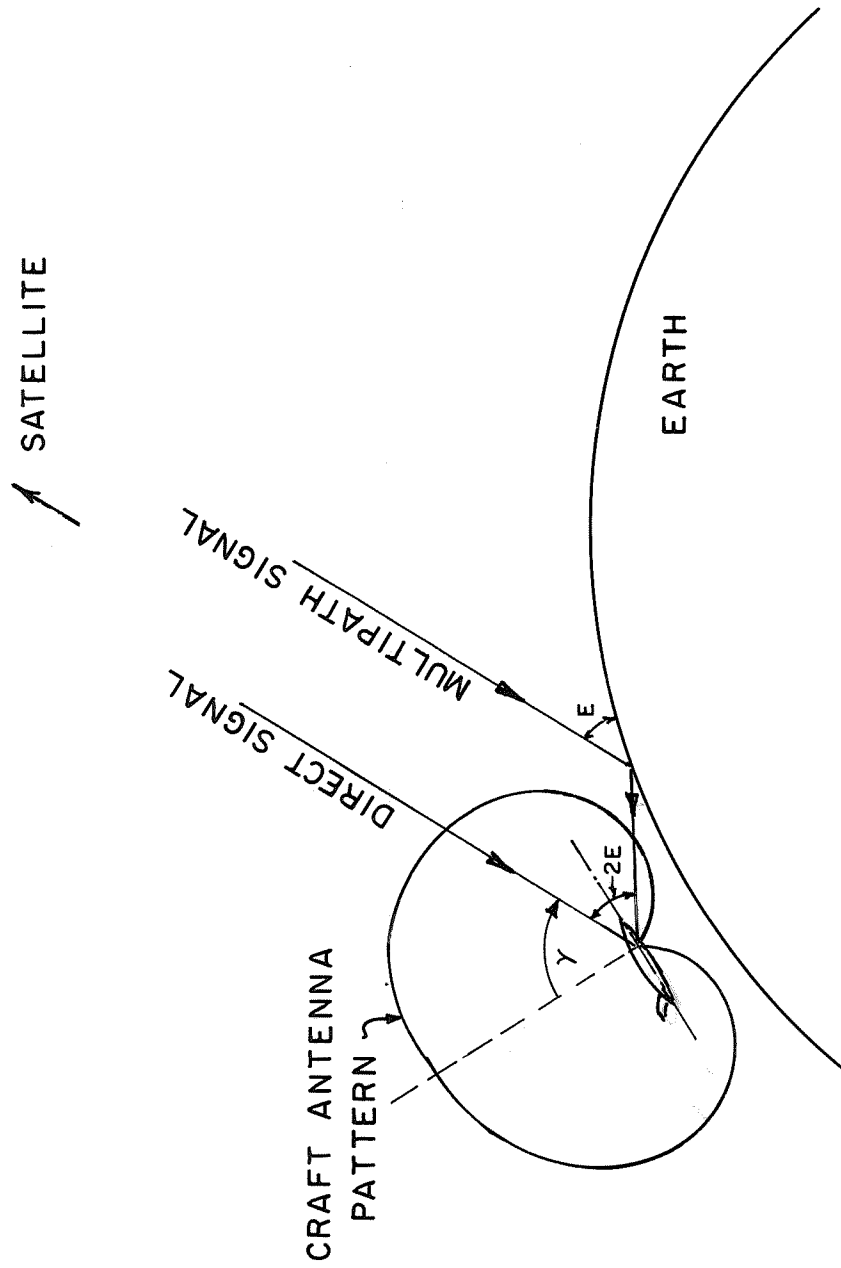


FIGURE 38 MULTIPATH REDUCTION BY AIRCRAFT ANTENNA PATTERN DISCRIMINATION

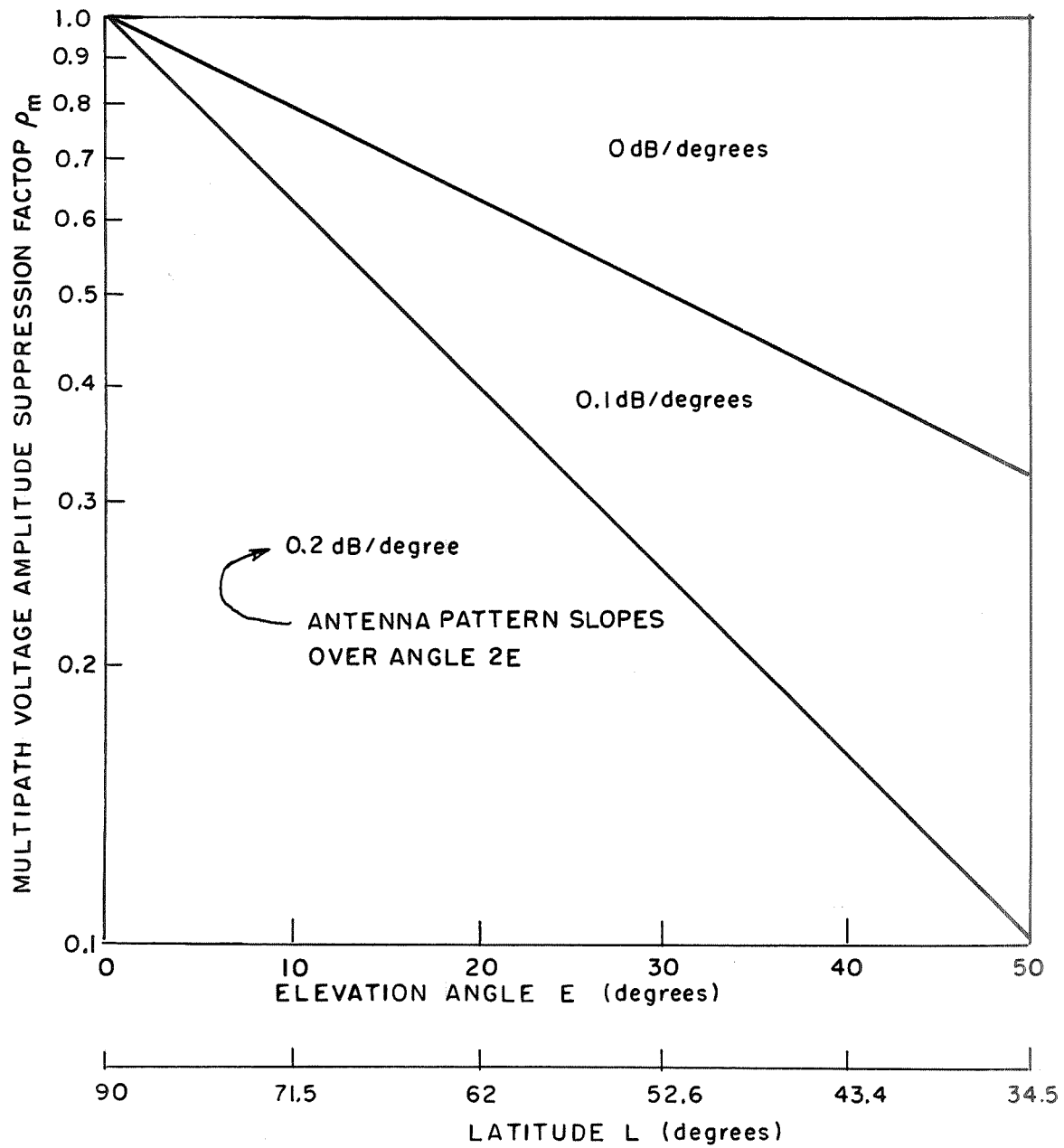


FIGURE 39 MULTIPATH VOLTAGE AMPLITUDE SUPPRESSION FACTOR PLOTTED AGAINST ELEVATION ANGLE AND LATITUDE

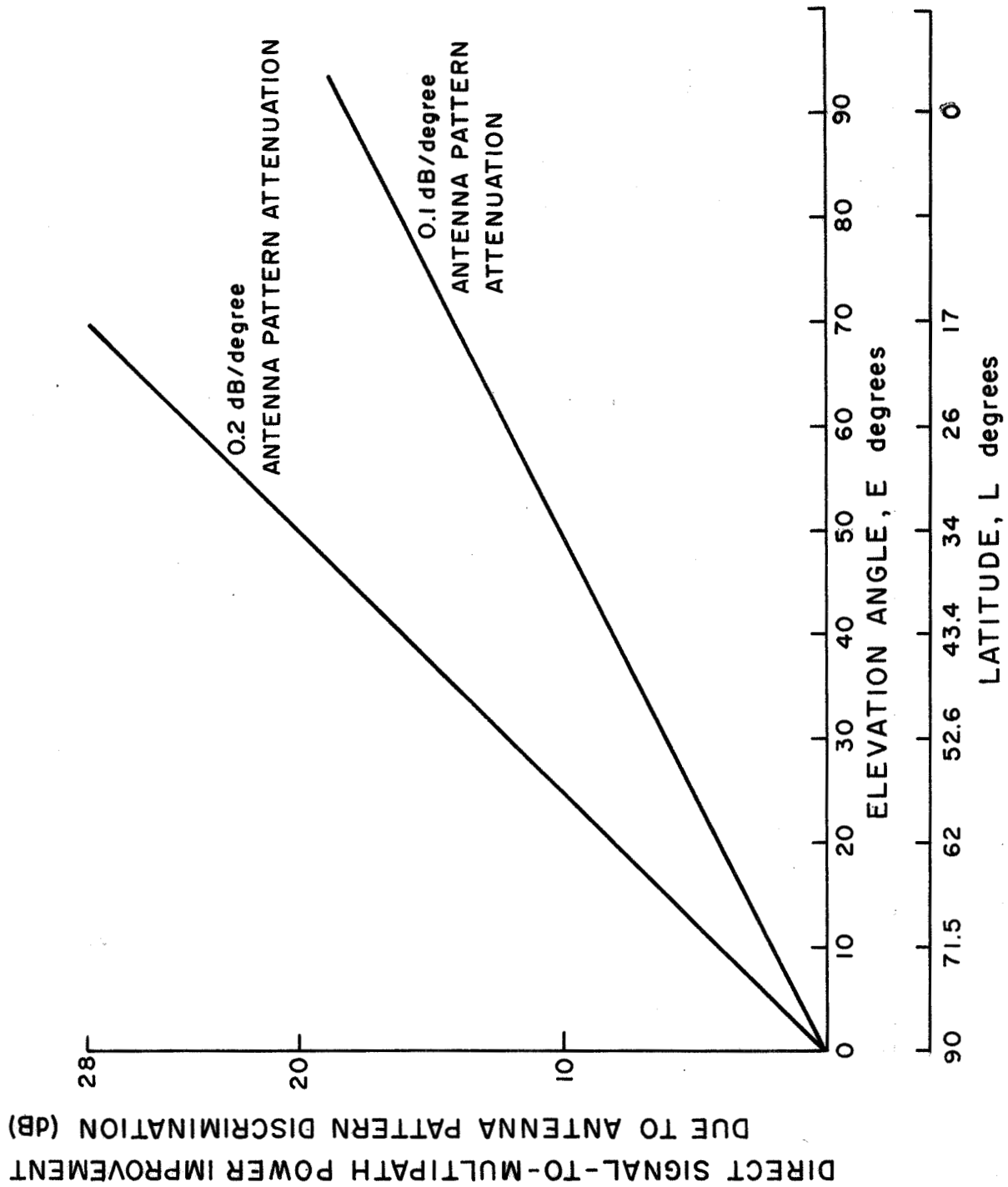


FIGURE 40 SIGNAL-TO-MULTIPATH POWER IMPROVEMENT BY ANTENNA PATTERN DISCRIMINATION

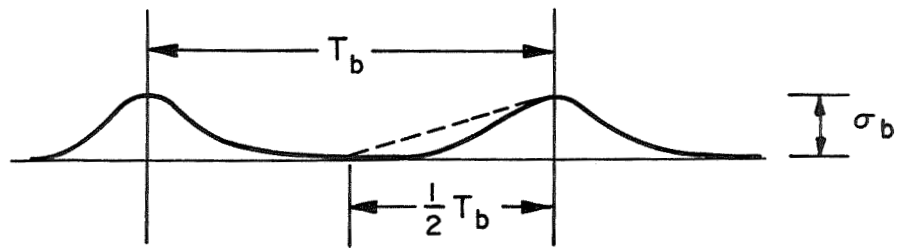
(2) Multipath Reduction by Narrowband Filtering

When the direct signal to the aircraft is of the form $A_o \cos(\omega_1 t + \varphi_1)$, the multipath signal from a rough earth is of the form $\sum_m B_m \cos(\omega_{2m} t + \varphi_{2m})$ where the amplitudes have a Rayleigh distribution and the phases have a uniform distribution. Therefore, the resultant signal has components at two types of beat frequencies: one type, between the direct signal and individual components of the scattered signal, and the second type, between different components of the scattered signal itself. The highest beat frequency then defines a "fading bandwidth." The highest beat frequency of the second type is found to be much greater than that of the first type and the "fading bandwidth" B_f consequently approximately equals the bandspread B_s of the scattered signal [26].

An expression for the bandspread of a diffuse multipath signal arriving at a craft moving at a velocity V_u is available from [51]:

$$B_f \cong B_s \cong 17.75 \frac{V_u}{\lambda} \left[\frac{2\sigma_b}{T_b} \right] \sin E \quad (90)$$

where λ , E , V_u have been defined previously, and $2\sigma_b/T_b$ is a surface characteristic parameter which may be interpreted as an average slope. Figure 41 makes this clear. The average slope is defined to be the ratio of the standard deviation of surface bump height σ_b and half the correlation interval T_b between bumps. The surface undulations have been described by a two-dimensional gaussian distribution.



$$\text{AVERAGE SLOPE} = \frac{\sigma_b}{\frac{1}{2} T_b}$$

FIGURE 41 SCATTERING SURFACE CHARACTERISTIC PARAMETER DEFINED
AS AN AVERAGE SLOPE

Equation (90) has been derived [51] on the assumptions of a stationary antenna on the satellite and a moving craft. The motion of the craft introduces a change in length of the reflection point-to-craft path. This path length change, a function of craft velocity and position, is used to obtain the correlation function of the scattered multipath power. Taking the Fourier transform of this correlation function yields the multipath power spectrum, which turns out to be a gaussian bell curve with a mean equal to the Doppler shift due to the craft velocity and a standard deviation which is a measure of the bandspread. B_s is defined to be twice this standard deviation.

In the present work, the reflection point-to-craft path length has an additional variation due to the spinning motion of the interferometer. From inspection of the power spectrum expression [51], this added variation seems likely to spread the multipath even more, so that the B_s of Eq. (90) may be taken as a lower limit to the bandspread occurring in the spinning interferometer system.

For $V_u = 1200$ km/hr, $\lambda = 20$ cm, $2\sigma_b/T_b = 0.1$ (implying a 3° average slope for the scattering surface), the bandspread may be written as

$$B_s = 472 \sin E \quad \text{Hz}$$

For $E = 20^\circ$ and 90° , B_s lies between 162 and 472 Hz. Since the bandwidth in the processor is $(T)^{-1}$ Hz, a "filtering factor," $(TB_s)^{-1}$ may be defined representing the improvement in the direct signal-to-multipath ratio by rejecting the multipath. Figure 42 shows this improvement possible as a function of T for the two limiting values

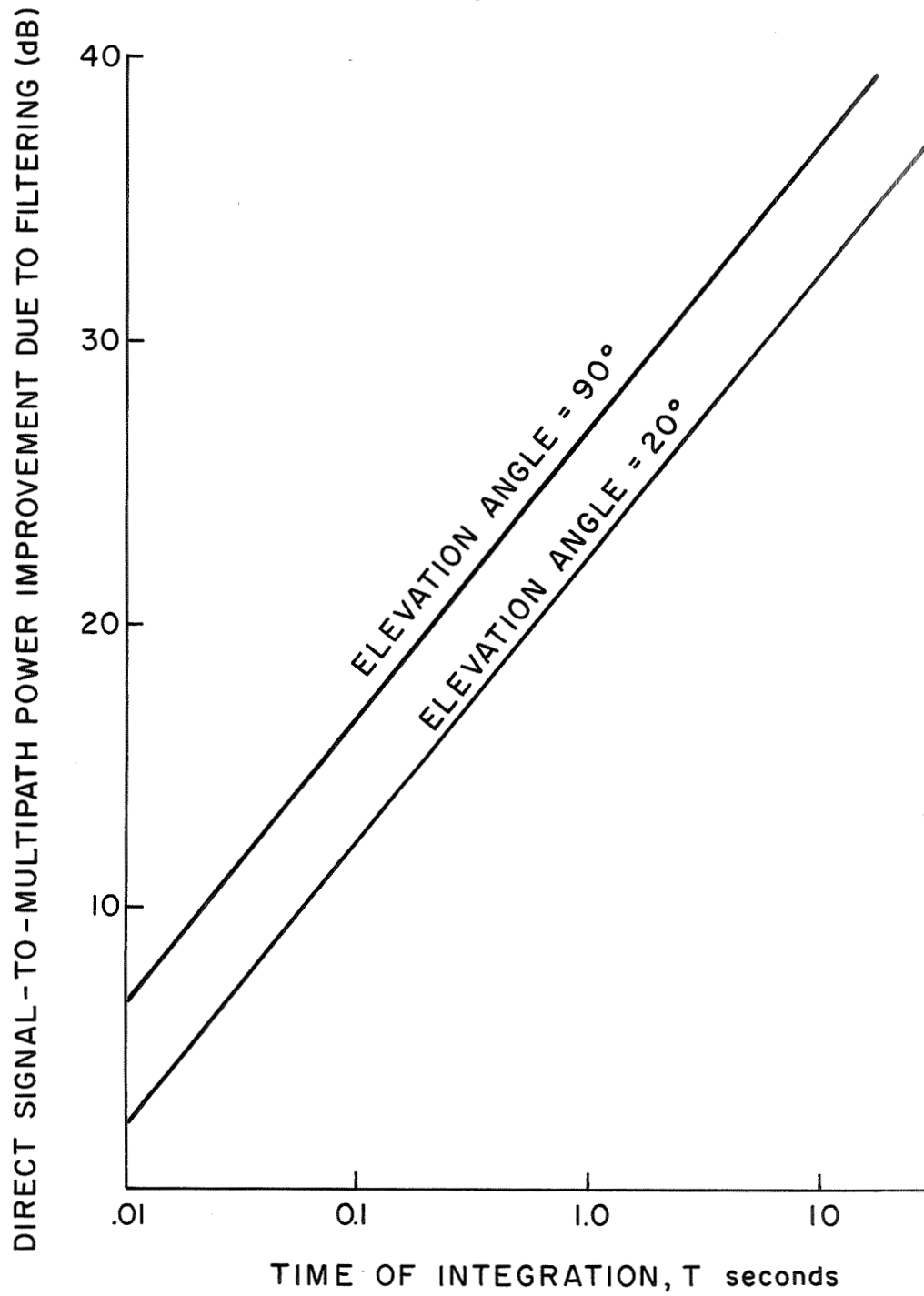


FIGURE 42 SIGNAL-TO-MULTIPATH POWER IMPROVEMENT BY NARROWBAND FILTERING (INCREASING T CORRESPONDS TO DECREASING PROCESSOR BANDWIDTH)

of elevation angle E . As mentioned earlier this is a lower bound for the improvement since the antenna motion will spread the multipath further, thereby concentrating less of it in the processor bandwidth $(T)^{-1}$.

5.2.4.3 Position Error Due to Multipath

An estimate of the position error due to multipath may be made from Figs. 40 and 42. At the lowest elevation angle of interest, 20° , Fig. 40 indicates a possible direct signal-to-multipath improvement of 8 dB while from Fig. 42 an improvement of 20 dB is possible when $T = 0.5$ second. The total improvement of 28 dB corresponds to an rms position error of 0.45 nmi, as seen in Fig. 37. This error can be reduced to 0.35 nmi if the integration time is increased to a second.

If errors on this order of magnitude are judged excessive, two other multipath reduction techniques may be incorporated, namely, space diversity and polarization. A space diversity technique, described in ref. [67] involves the use of two essentially omnidirectional antennas mounted symmetrically one above the other on the side of the aircraft and spaced by a distance roughly equal to one half of the carrier wavelength. The output of one of the antennas is electronically phase shifted and then added to the output of the other, leading to a cancellation of the multipath and a reinforcement of the direct signals.

Further multipath reduction is possible by circularly polarizing the signals transmitted by the satellite. For the

geometry of interest (grazing angles $> 3^{\circ}$) the sense of a circularly polarized wave reverses on reflection. The direct and multipath rays arrive at the craft with different senses of polarization, offering a means for further reduction of multipath [3,29,51]. Circular polarization might be deemed necessary for other reasons, as mentioned in Section 5.2.1, for instance, if future propagation experiments conducted at L-band show that Faraday rotation is not negligible at those frequencies.

Selecting a suitable circularly polarized aircraft antenna may be a problem in terms of cost and complexity. Top mounted circularly polarized antennas tend to be more cumbersome and expensive than vertically polarized blade antennas. However, a circularly polarized antenna installed in the craft-nose might be feasible [45].

5.3 Equipment Error and its Elimination

The errors introduced by equipment, both at the satellite and the navigating craft, may be serious unless corrected. The major sources of error are the interferometer antenna phase pattern mismatch and differential phase shifts in both satellite and user equipment. A description of these errors and possible methods to reduce or eliminate them are found in reference [41].

5.3.1 Satellite-Borne Antenna Error

The implicit assumptions were made while analyzing this navigation concept that the interferometer antennas were ideal point sources and that they could be electronically or mechanically despun if gain were required. The position of the phase center of a physical

antenna, however, is found to be a function of the tilt angles (Γ_x, Γ_y) even if a unique phase center can be defined. This can be done only for certain types of antennas, those that are symmetrical and which have either an even or odd, but smooth, distribution of phase across the antenna aperture [50]. Phase difference errors on the order of one electrical degree have been estimated as resulting from interferometer antenna phase pattern mismatch [18]. This mismatch can arise from deformities in the physical shape of the antenna reflectors, misalignments of the antenna feeds or from improper synchronization of the antenna despinning mechanism.

To correct all of these causes mechanically is very difficult, especially in the space environment where thermal expansion is a problem. Most of the mismatch error is likely to be systematic over the beamwidth of the antennas and is of the type that can be calibrated out through the application of the referenced radionavigation concept briefly discussed in Section 5.4 and described in detail in ref. [42].

5.3.2 Equipment Phase Shift Error

Differential phase shifts introduced by satellite and user equipment may be serious in a navigation scheme where a phase-difference quantity contains the desired position information. Error caused by satellite-borne equipment may be considered to be systematic over the field of users and will be assumed to be eliminated by the referenced radionavigation procedure. The effects of differential phase shifts in the craft-borne equipment result in errors that cannot be correlated with error information from ground-based reference

stations, requiring some other means of elimination as described in the next section. Such errors are those produced by the RF and IF amplifiers, the square-law detector, low-, high- and band-pass filters, the 90° phase-shift mechanism in the Costas circuit, the delay line in Section 3.2.1.2, the integrators and generators in the MAP processor of Section 3.2.2.1. These errors are peculiar to each craft navigation receiver and must be eliminated on board each craft during routine flights.

5.3.3 System Self-Calibration Technique

Reference [41] describes in detail a method to calibrate out equipment differential phase shifts and other bias errors. This system self-calibration technique, which involves the periodic transmission of a calibration signal, is adapted here for the Spinning Interferometer Navigation Satellite. During a fixed interval, say five seconds every minute, the signals that are normally emitted by the interferometric antennas are diverted to a body-fixed antenna on the satellite.

The signal received at the craft does not contain the position-information bearing term φ which in effect reduces the interferometer baselength to zero. After the IF stage the signal now looks like

$$s_{\text{IF}}(t) = 2A_o \cos\left(\omega_{\text{IF}}t - \frac{\omega r}{c}\right) + n(t)$$

since $\cos\left[C_1 + C_2 \cos(\Omega t - C_3)\right] = 1$. After the DSB demodulation stage, the signal becomes

$$u(t) = A_o + \frac{1}{2} n_i(t)$$

Now from the definition of the vector $\underline{V}$ in Sec. 3.2.2.1 the outputs of the integrator-amplifiers in Fig. 16 collectively may be written as

$$\underline{V} = \frac{2}{N_o} \int_0^T \underline{S} u_o(t) dt$$

where the partial derivative vector $\underline{S}$ is internally generated for a particular neighboring grid-point and the observed signal $u_o(t)$ contains both the received signal $u(t)$ and the internally generated signal $s(t, \underline{C}_o)$:

$$u_o(t) = u(t) - s(t, \underline{C}_o)$$

In the navigation mode of operation

$$\begin{aligned} u_o(t) &= A_o \cos \varphi(t, \underline{C}) + \frac{1}{2} n_i(t) - A_o \cos \varphi(t, \underline{C}_o) \\ &\equiv A_o \cos \varphi + \frac{1}{2} n_i - A_o \cos \varphi_o \end{aligned}$$

$A_o \cos \varphi_o$ is substituted for $s(t, \underline{C}_o)$ for clarity in subsequent operations. In the calibration mode, letting $\varphi = 0$

$$u_o(t) = A_o + \frac{1}{2} n_i - A_o \cos \varphi_o$$

The partial derivative vector $\underline{S}$ is composed of

$$\frac{\partial s}{\partial C_1} = - A_o \sin \varphi_o$$

$$\frac{\partial s}{\partial C_2} = - A_o \sin \varphi_o \cos(\Omega t - C_3)$$

$$\frac{\partial s}{\partial C_3} = - A_o C_2 \sin \varphi_o \sin(\Omega t - C_3)$$

all of which have significant frequency components in the region around $C_2 \Omega$ Hz. Using $\underline{S}$ in the expression for $\underline{V}$ for the navigation mode of operation

$$V_1 = - \frac{2A_o^2}{N_o} \int \sin \varphi_o \cos \varphi dt + \frac{A_o^2}{N_o} \int \sin 2\varphi_o dt - \frac{A_o}{N_o} \int \sin \varphi_o n_i dt$$

$$\Delta = V_{1s} + V_{1n}$$

$$V_2 = - \frac{2A_o^2}{N_o} \int \sin \varphi_o \cos \varphi \cos(\Omega t - C_3) dt + \frac{A_o^2}{N_o} \int \sin 2\varphi_o \cos(\Omega t - C_3) dt$$

$$- \frac{A_o}{N_o} \int \sin \varphi_o n_i \cos(\Omega t - C_3) dt$$

$$\Delta = V_{2s} + V_{2n}$$

$$V_3 = - \frac{2A_o^2 C_2}{N_o} \int \sin \varphi_o \cos \varphi \sin(\Omega t - C_3) dt + \frac{A_o^2 C_2}{N_o} \int \sin 2\varphi_o \cos(\Omega t - C_3) dt$$

$$- \frac{A_o C_2}{N_o} \int \sin \varphi_o n_i \sin(\Omega t - C_3) dt$$

$$\Delta = V_{3s} + V_{3n}$$

The integrations are performed over $(0, T)$, the observation time-interval defined previously. In the navigation mode, for the user not exactly at the grid-point, $\underline{V}$ consists of the elements

$$V_1 = V_{1s} + V_{1n}$$

$$V_2 = V_{2s} + V_{2n} \quad \text{or } \underline{V} = \underline{V}_s + \underline{V}_n$$

$$V_3 = V_{3s} + V_{3n}$$

where the subscript "s" represents the part of $\underline{V}$ uncontaminated by the noisy terms $\underline{V}_n$. If the craft coincides with the grid-point, $\underline{V}_s = 0$.

In the calibration mode, $\varphi = 0$ leads to the first terms in V_i to integrate out to zero leaving $\underline{V} = \underline{V}_n$ when no bias errors are present, so that the output of the MAP processor during the calibration mode must register a position deviation vector $\hat{\underline{y}}$ of zero, within the limits of the position uncertainty due to noise. A nonzero value for $\hat{\underline{y}}$ indicates a bias error that may be entered into the craft's computer memory to correct the position-coordinates estimated during the navigation mode of operation. The position uncertainty due to noise is reduced as T increases.

It was mentioned at the beginning of this section that the calibration mode interval (CMI) be five seconds during every minute. The considerations in choosing suitable values of CMI and the navigation mode interval (NMI) are briefly discussed here. The CMI must be sufficiently long to permit the PLL's in the craft receiver to track the rapid phase changes that occur when the calibration mode is initiated. Furthermore, the CMI must exceed the integration time T . A few seconds are judged sufficient to meet both these requirements.

The rate of change in bias error will determine the NMI. For very slow variation in bias error, calibration might be required only every one or two hours or at the beginning of a trip. Relatively rapid changes will dictate a smaller NMI. Experimentation will be required to select suitable intervals.

5.4 Satellite Motion and its Compensation

The information-bearing phase difference term $\varphi(t, \underline{C})$ is used in this analysis under the assumption that the spinning satellite is located at its nominal synchronous position. The effect on $\varphi(t, \underline{C})$ of the satellite's motion away from its nominal position is explored in Appendix A where a general expression for $\varphi(t, \underline{C})$ is derived in terms of the coordinates of translation (a, b, c) and rotation (α, β, γ). A dynamic study of the boom-bearing satellite could use this general $\varphi(t, \underline{C})$ to estimate craft position errors that result from satellite motion. The present work however assumes that the referenced radio-navigation concept [41] corrects for such errors.

5.4.1 Referenced Radionavigation

Referenced radionavigation is a method for reducing bias error that is correlated over the entire field of users, such as errors due to imperfect matching of the interferometer antenna phase patterns, differential phase shifts introduced by satellite equipment, and changes in satellite position and altitude. This technique uses several ground-based reference stations at well-defined locations in the satellite's field of view. Each reference station is equipped with navigation equipment similar to that carried by the craft, but

superior in terms of higher antenna gain and lower receiver temperature. Since the true position of each station is known, the received signal can be processed to yield error correction information which is transmitted to the users via a data link. These navigating craft utilize this information to calibrate out external bias errors. Details of this technique appear in [41, 42].

5.4.2 Satellite Motion

The signals received from the spinning interferometer are affected by the orbital, attitude and configuration perturbations of the satellite. The phenomena that will disturb the nominal synchronous equatorial circular orbit include the asphericity of the earth's gravitational field, solar and lunar effects and solar radiation pressure. Air drag is small enough at synchronous altitude to be disregarded [48]. Solar radiation pressure has no net effect on the orbit, since it increases the speed of the satellite during one half of the orbit and decreases it during the other. This change in speed results in a change in longitude of the subsatellite point, on the order of 0.03° for SYNCOM [73]. Solar radiation pressure also increases the inclination of the orbit during one half of the year and reduces it during the other. Both of these effects were judged negligible. The major solar and lunar effect is to cause an inclination of the satellite orbit on the order of $0.958^\circ/\text{year}$ for SYNCOM.

The orbital perturbations due to the asphericity of the earth's gravitational field has been analyzed in detail [30]. The results indicate that only two truly stable positions exist for a synchronous

satellite, namely $255 \pm 1/2^\circ$ E and $75 \pm 3/4^\circ$ E longitude.* To establish a synchronous satellite at any other longitude will require station-keeping propulsion of 6.36 feet/sec./year. Without this, the satellite will oscillate about the closest of the two stable points with periods in excess of 2.2 years. Associated with these large-angle oscillations are variations of the orbital radius of the same period but of amplitudes less than 37 miles. All these effects that disturb the satellite's nominal orbit seem to be very slow and hence the errors that they produce are capable of being eliminated by the referenced radionavigation technique.

Attitude and configuration perturbations deserve a thorough dynamic analysis for the case of a boom-bearing spinning satellite. Several forcing functions may be significant among which are solar radiation pressure, thermal cycling, magnetic torques and gravity gradient torques. Slow variations in spin-rate, introduction of a slow precession and periodic but slow bending of the booms are all presumably correctable by referenced radionavigation. The possibilities of vibrations of a relatively high frequency in the booms and its compensation by proper choice of material and design must be studied. A rapid change in the positions of the interferometer antennas, if it is large enough to cause a significant craft position error, will require that the integration time T be very small, thus increasing the power requirements on the satellite transmitter. A mathematical treatment on solar induced bending

* These figures are different from those in [30], being based on more recent data. From Dr. V. J. Slabinski of Communications Satellite Corp.

vibrations of a flexible member given in [34] could be a starting point for such an investigation.

5.5 Error Budget Considerations

From the investigation in Section 5.0 it may be concluded that the contributions to craft position error from the uncertainty in craft velocity and from the refraction and dispersion of radio waves are negligible. Errors introduced by satellite equipment, certain types of multipath and by the attitude and position variations of the satellite are expected to be systematic over the field of users, and hence amenable to correction by the referenced radio-navigation technique. Errors from user equipment are peculiar to each craft, but are assumed to be eliminated by the system self-calibration technique.

The major contributors to position error are tropospheric phase fluctuations, multipath, uncertainty in user altitude and additive receiver noise. This last source of error appears in the estimation error analyzed in Section 4.0 where a tradeoff analysis is presented for various system parameters such as the baselength-to-wavelength ratio D/λ , interferometric skew-angle θ , satellite transmitter power, integration time-interval in the processor, antenna gain at the interferometer and the user, all at various geographical locations of the navigating craft. The other three error sources are treated in Sections 5.1 to 5.4 and the worst-case values of position error are included in the error budget of Table II.

The total error figure in Table II is obtained by assuming that the errors due to each source are independent and hence add in the root-sum-square sense. This total position error can be kept to within 1.58 nmi, depending on the choice of system parameters, in either the north-south direction or the east-west direction. By letting $\theta = 45^\circ$, equal errors in both directions are possible. A higher value for the skew-angle will permit a somewhat higher accuracy in the north-south direction at the expense of substantial reduction in the east-west accuracy. In any case the total error figure resulting from this analysis is judged to be an adequate goal for a navigation system serving the North Atlantic air traffic^{*} in view of the opinion of the International Civil Aeronautics Organization (ICAO) that a 10 navigation accuracy of 10 nmi would be sufficient to support a 60 nmi lateral separation standard [57,68].

The present trend is toward self-contained systems, particularly inertial navigation systems (INS) because inertial platforms provide attitude and heading information besides position-fix capability [57]. Maximum inertial drifts of 3.2 nmi/hour have been observed on North Atlantic flights resulting in a maximum terminal radial error of 20.6 nmi for the New York-London route.^{**}

* This area of interest is shown in Fig. 43 with typical air traffic routes [22].

** These figures are in agreement with INS accuracy information in Advisory Circular No. 25-4 of the Federal Aviation Administration [57].

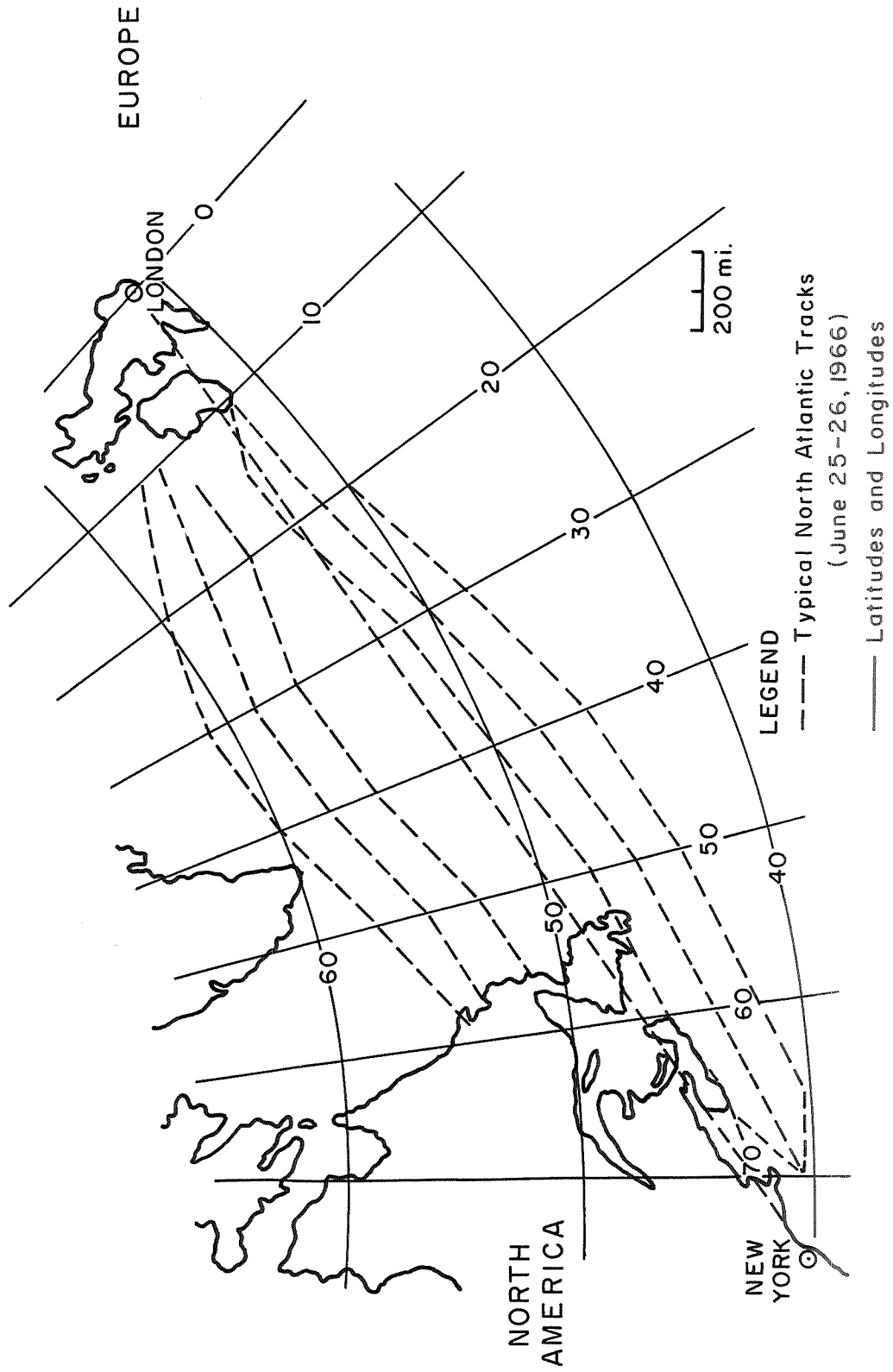


FIGURE 43 THE NORTH ATLANTIC OCEAN AREA OF INTEREST WITH TYPICAL AIR TRAFFIC ROUTES

External aids independent of the self-contained systems, such as the satellite-aided navigation scheme analyzed here, could periodically update the on-board system thereby providing a significantly higher position accuracy. Moreover, external aids would protect craft against gross navigation or "blunder" errors, i.e. those erratic errors without predictable distributions. Such errors can be disastrous particularly in the cross-track direction.

Table II

1σ Craft Position Error Budget

Estimation error e_1 (with $D/\lambda = 100$, $\theta = 85^\circ$, $TA_0^2/N_0 = 36.02$ dB)*	1.50 nmi
Error due to tropospheric phase noise	0.21 nmi
Error due to multipath (with $T = 0.5$ second)	0.45 nmi
Error due to uncertainty in aircraft altitude ($h_a = 30,000$ feet, $L = 60^\circ$)**	0.13 nmi
Total craft position error (by combining the above in a root-sum-square sense)***	1.58 nmi

* This figure for e_1 corresponds to a reduced accuracy figure for e_2 since $\theta = 85^\circ$. If accuracy in both x- and y-directions is desired, θ can be reduced to 45° , for which $e_1 = e_2 \simeq 2.0$ nmi.

The tropospheric phase noise contribution will be somewhat higher in this event, around 0.29 nmi, resulting in a total error of 2.07 nmi. It must be noted that these values for e_1 and e_2 do not take into account the error reduction possible by introducing a priori information into the MAP processor.

** 1σ error extracted from Fig. 34 which gives 3σ position errors.

*** Errors introduced by epoch signal considerations and interferometric motion are the subject of Appendices D and E and do not appear in this table.

Table III
A Comparison of the Spinning Interferometer Navigation
Satellite System with Some Operational Schemes

System	Accuracy	Max. Range	Operating Frequency	Measured Parameters	Remarks
VOR	1-2 degrees	200 nmi	108-118 MHz	Bearings	Colocating VOR with DME permits position fixing by measuring one bearing and one range
DME	0.2 mi or 5% of measured distance	300	1025-1150 MHz	Range	
ATC Radar	2% of range	200	1030-1090 MHz	Range and Bearing	Used to locate aircraft independently of pilot's on-board navigation equipment.
Consol	6-24 mi day 10-40 mi night	1000 day 1200 night	190-194 kHz United States 257-263 kHz Europe	Bearings	
Loran-A	2-10 mi	700 day 1400 night	1750-2000 kHz	Range-difference	
Loran-C	1/4 nmi (for range < 1200 mi)	550 day 1600 night	100 kHz	Range-difference	Accuracy is an order of magnitude lower for ranges > 1200 mi
Decca	0.05-1.1 mi	300	70-130 kHz	Range-difference	

continued

continued

Table III

System	Accuracy	Max. Range	Operating Frequency	Measured Parameters	Remarks
Omega	1 mi day 2 mi night	Thousands of miles	10.2-13.6 kHz	Range-difference	Partially operational at present
Navy Nav. Sat. Syst. (Transit)	1/4 mi (ships)	Thousands of miles	150, 400 MHz	Range-difference	3 satellites in 600 mi polar orbit; user velocity must be known accurately. Not suitable for aircraft.
Spinning Interferometer Navigation Satellite	3 mi	Thousands of miles	1540-1660 MHz	Angle	Complements on-board dead-reckoning system. Provides rapid position fix. Suitable for high speed aircraft.

The following were used as references:

1. P. I. Klein: Satellite Based Navigation, Traffic Control and Communications to Mobile Terminals, UCLA Course Notes, 1969.
2. R. B. Kershner: Status of the Navy Navigation Satellite System, presented at American Astronautical Society, National Meeting on Practical Space Applications, Feb. 21, 1966, San Diego, Calif.
3. W. Bauss (technical editor): Radio Navigation Systems, AGARDograph 63, 1963.

6.0 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A navigation concept consisting of a spin-stabilized synchronous satellite bearing a single radiating interferometer has been investigated in this work. It is concluded that aircraft and ships in a large area such as the Atlantic Ocean could be provided with position fixes with 1 σ accuracies on the order of 2 to 3 nmi in two orthogonal directions. A higher accuracy in one direction is possible but only at the cost of a marked deterioration in accuracy in the other direction. For aircraft flying east-west routes over the North Atlantic as shown in Fig. 43 the cross-track accuracy (north-south) might be judged more important, and the navigation satellite parameters can be selected appropriately, with the expectation of a corresponding decrease in along-track accuracy.

This study assumed a CW signal transmitted by each antenna of the spinning interferometer resulting in a double-sideband suppressed-carrier waveform received at the navigating craft. Coherent demodulation yielded the intelligence which was then processed to give the craft position coordinates. The design of this processor was approached in two ways: first, from inspection of the form of the intelligence, two possible designs from an intuitive viewpoint were suggested involving the measurement of the information-bearing phase terms and the subsequent solution of the navigation equations; second, a systematic approach to the design of a processor optimum in the maximum a posteriori sense was presented, where as much as possible of the available position

information and the received signal characteristics was used to extract the user's position coordinates.

Rather than demodulate the DSB signal coherently, it was noted that noncoherent techniques (e.g. square-law detector and half-wave piecewise linear detector) could be used to extract quantities proportional to the intelligence waveform. This is possible since the important thing is not to construct the intelligence faithfully, for which coherent techniques would be necessary, but rather to extract position information. Entirely different receivers could be designed on this basis. The receiver study could be profitably extended to estimate the navigator's computer capacity that will be required to store a priori information and the complexity required to perform the necessary mathematical manipulations.

Another extension to the present work is possible in the area of multipath. The degradation in system performance could be studied in more detail by means of a three-dimensional geometry resulting in more exact values for the multipath-to-signal power ratio. The investigation of multipath reduction techniques could be extended to the use of circular polarization and space diversity.

A detailed study of the dynamic behavior of a spinning satellite bearing booms would be desirable from the point of view of investigating rapid changes in satellite attitude or vibrations set up in the booms.

Experimental data on the effects of propagation anomalies on signals from satellite-borne interferometers are desirable particularly at L-band, one of the aeronautical mobile bands allocated by the ITU Extraordinary Administrative Radio Conference [36]. Finally, it will be necessary to demonstrate that the components of the receiver, voltage-controlled oscillators and signal generators in particular remain stable in an operational environment.

Appendix A

DERIVATION OF THE PHASE DIFFERENCE TERM $\varphi(t, \underline{C})$

A general expression for $\varphi(t)$ when the satellite is experiencing drift and attitude change from its nominal position is derived here. A rigid baselength is assumed for the interferometer. As shown in Fig. A1 a displacement of the satellite may be seen as a combination of the translation of its center-of-mass away from the nominal position and a rotation about the new (shifted) center-of-mass. Figure A2 describes the positions of the interferometric antennas in the body-fixed xyz coordinate system. These antenna coordinates have to be transferred into the inertial (XYZ) system wherein the navigating craft is located, to obtain $\varphi(t)$. This operation involves first, the transformation of the xyz coordinates into the "intermediate" X'Y'Z' system by means of the well-known Eulerian Transformation [64], and then a linear transformation into the XYZ system.

The Euler angles as shown in Fig. A1 are α , the angle of nutation, β , the angle of precession, and γ , the angle of pure rotation. For a reasonably well-stabilized spacecraft which is not tumbling or executing any other unusual motions, it is assumed here that the nutation angle is small, i.e. $\sin \alpha \doteq \alpha$ and $\cos \alpha \doteq 1$. With this restriction the transformation becomes

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} 1 & \alpha \sin \gamma & \alpha \cos \gamma \\ \alpha \sin \beta & \cos(\beta + \gamma) & -\sin(\beta + \gamma) \\ -\alpha \cos \beta & \sin(\beta + \gamma) & \cos(\beta + \gamma) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (\text{A.1})$$

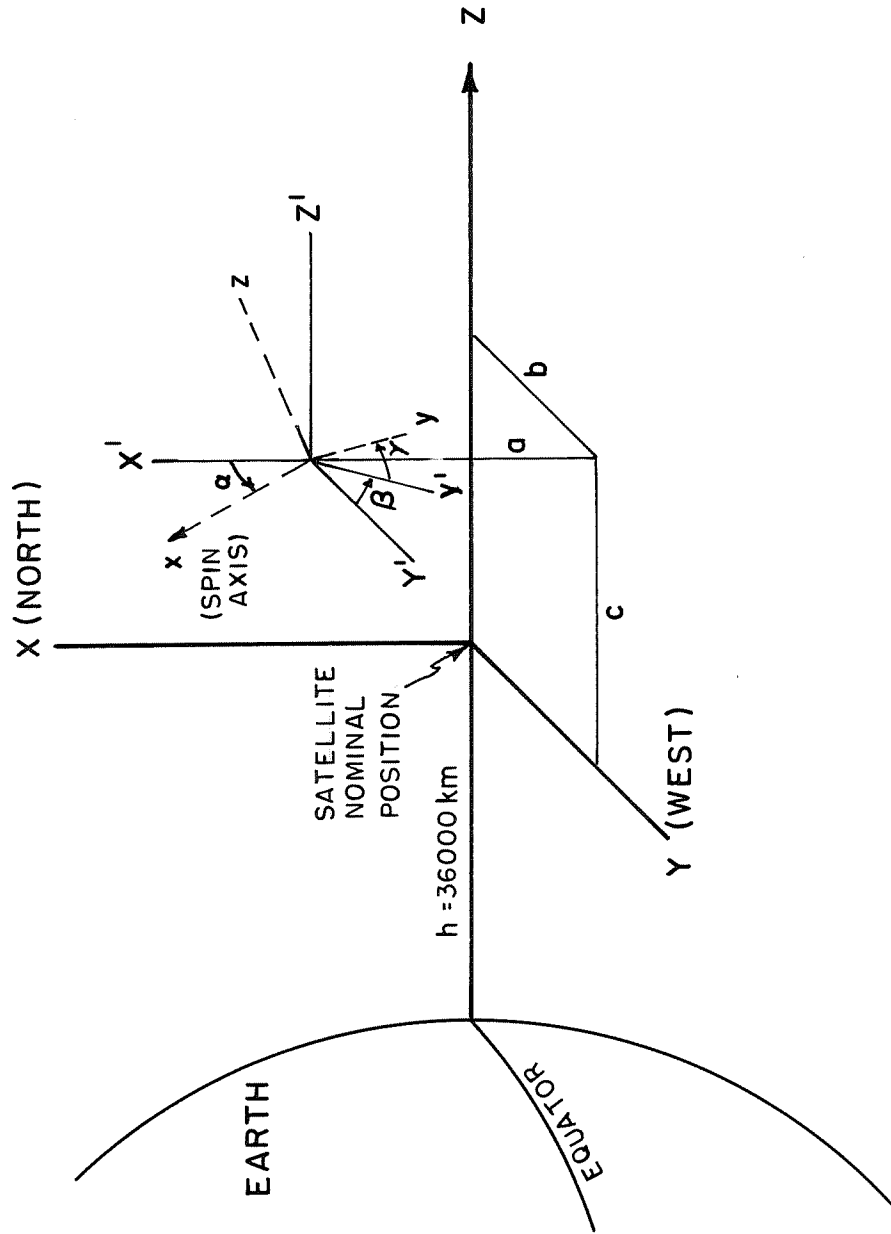


FIGURE A1 RELATIVE POSITIONS OF THE INERTIAL COORDINATE AXES (XYZ), THE TRANSLATED AXES ($x'y'z'$) AND THE BODY-FIXED AXES ($x'y'z'$). (NOT DRAWN TO SCALE)

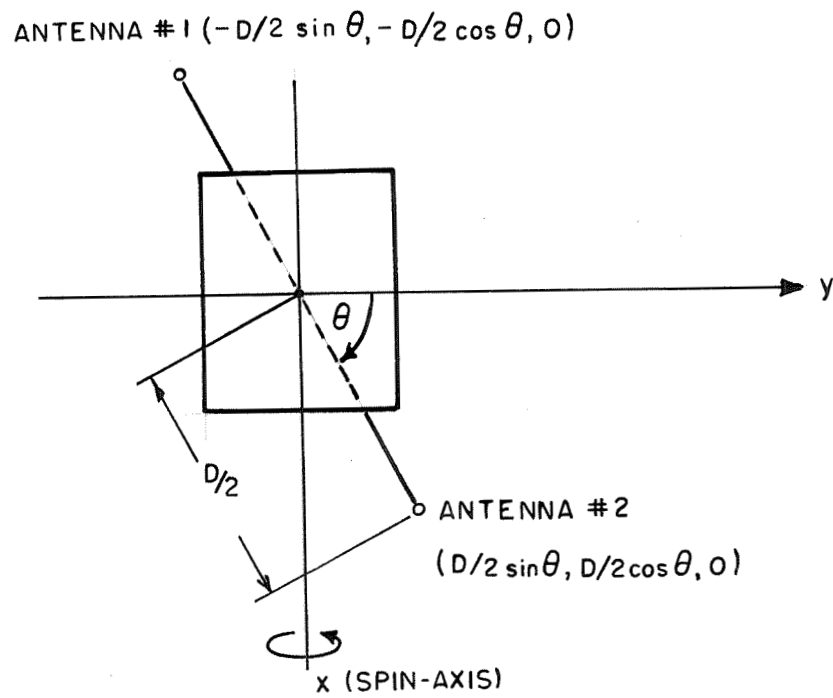


FIGURE A2 PLAN-VIEW OF SATELLITE IN THE BODY-FIXED (xyz) COORDINATE SYSTEM

The coordinates of antennas #1 and #2 in the body-fixed system are

$$\begin{aligned} x_1 &= -\frac{D}{2} \sin \theta & x_2 &= \frac{D}{2} \sin \theta \\ y_1 &= -\frac{D}{2} \cos \theta & y_2 &= \frac{D}{2} \cos \theta \\ z_1 &= 0 & z_2 &= 0 \end{aligned} \quad (A.2)$$

These correspond to the following in the XYZ system:

$$\begin{aligned} X_1 &= -\frac{D}{2} [\sin \theta + \alpha \sin \gamma \cos \theta] - a & &= -X - a \\ Y_1 &= -\frac{D}{2} [\alpha \sin \beta \sin \theta + \cos(\beta + \gamma) \cos \theta] - b & &= -Y - b \\ Z_1 &= -\frac{D}{2} [-\alpha \cos \beta \sin \theta + \sin(\beta + \gamma) \cos \theta] - c & &= -Z - c \\ \\ X_2 &= \frac{D}{2} [\sin \theta + \alpha \sin \gamma \cos \theta] - a & &= X - a \\ Y_2 &= \frac{D}{2} [\alpha \sin \beta \sin \theta + \cos(\beta + \gamma) \cos \theta] - b & &= Y - b \\ Z_2 &= \frac{D}{2} [-\alpha \cos \beta \sin \theta + \sin(\beta + \gamma) \cos \theta] - c & &= Z - c \end{aligned} \quad (A.3)$$

where X, Y, Z simply stand for the complicated expressions in the brackets and should not be confused with the XYZ-coordinate system.

Let the craft coordinates be $(X_u, Y_u, -h)$ in the inertial system.

The range from each antenna to the user becomes r_1 and r_2 given by

$$\begin{aligned} r_1^2 &= (X_1 - X_u)^2 + (Y_1 - Y_u)^2 + (Z_1 + h)^2 \\ &= (-X - a - X_u)^2 + (-Y - b - Y_u)^2 + (-Z - c + h)^2 \\ r_2^2 &= (X - a - X_u)^2 + (Y - b - Y_u)^2 + (Z - c + h)^2 \end{aligned} \quad (A.4)$$

Expanding and subtracting the above,

$$r_1^2 - r_2^2 = 4X(a + X_u) + 4Y(b + Y_u) + 4Z(c - h) \quad (A.5)$$

$\varphi(t)$ is related to $r_1 - r_2$ by definition as

$$\begin{aligned}\phi(t) &\triangleq \frac{2\pi}{\lambda} \frac{r_1 - r_2}{2} \\ &= \frac{\pi}{\lambda} \frac{r_1^2 - r_2^2}{r_1 + r_2}\end{aligned}\tag{A.6}$$

Now $r_1 + r_2 \approx 2r$ for the geometry under consideration

$$\begin{aligned}r^2 &= (a - X_u)^2 + (b - Y_u)^2 + (c + h)^2 \\ r &= h \left[1 + \Gamma_x^2 + \Gamma_y^2 + f_a + f_b + f_c \right]^{1/2}\end{aligned}\tag{A.7}$$

where $\tan \Gamma_x = X_u/h$, $\tan \Gamma_y = Y_u/h$, or

$$\Gamma_x \cong X_u/h, \quad \Gamma_y \cong Y_u/h$$

$$\begin{aligned}f_a &= \frac{a^2 - 2aX_u}{h^2} \\ f_b &= \frac{b^2 - 2bY_u}{h^2} \\ f_c &= \frac{c^2 + 2ch}{h^2}\end{aligned}\tag{A.8}$$

Equation (A.5) can be rewritten as

$$\begin{aligned}r_1^2 - r_2^2 &= 2Dh \left[\left\{ \sin \theta + \alpha \sin \gamma \cos \theta \right\} \left(\Gamma_x + \frac{a}{h} \right) + \left\{ \alpha \sin \beta \right. \right. \\ &\quad \left. \left. \sin \theta + \cos (\beta + \gamma) \cos \theta \right\} \left(\Gamma_y + \frac{b}{h} \right) + \left\{ -\alpha \cos \beta \sin \theta \right. \right. \\ &\quad \left. \left. + \sin (\beta + \gamma) \cos \theta \right\} \left(\frac{c}{h} - 1 \right) \right]\end{aligned}$$

Since the rate of spin $\Omega = -(\dot{\beta} + \dot{\gamma})$, it follows that

$$\beta + \gamma = -\Omega t \quad \text{or} \quad \gamma = -(\Omega t + \beta)$$

assuming that the interferometer is horizontal at $t = 0$.

Therefore

$$\begin{aligned} \kappa_1^2 - \kappa_2^2 = 2Dh \left[\sin \theta \left\{ \Gamma_x + \frac{a}{h} + \alpha \sin \beta \left(\Gamma_y + \frac{b}{h} \right) - \alpha \cos \beta \left(\frac{c}{h} - 1 \right) \right\} + \cos \theta \left\{ -\alpha \sin (\Omega t + \beta) \left[\Gamma_x + \frac{a}{h} \right] + \cos \Omega t \left[\Gamma_y + \frac{b}{h} \right] - \sin \Omega t \left[\frac{c}{h} - 1 \right] \right\} \right] \end{aligned}$$

The coefficient of $\cos \theta$ above may be expressed as

$$\begin{aligned} & \left[-\alpha \sin \beta \left(\Gamma_x + \frac{a}{h} \right) + \left(\Gamma_y + \frac{b}{h} \right) \right] \cos \Omega t + \\ & \left[-\alpha \cos \beta \left(\Gamma_x + \frac{a}{h} \right) - \left(\frac{c}{h} - 1 \right) \right] \sin \Omega t \end{aligned}$$

or as

$$F_2 \cos \Omega t + F_3 \sin \Omega t$$

From Eq. (A.6)

$$\begin{aligned} \phi(t) &= \frac{\pi}{\lambda} \frac{\kappa_1^2 - \kappa_2^2}{2h} \\ &= \pi \frac{D}{\lambda} \frac{F_1 \sin \theta + \cos \theta (F_2 \cos \Omega t + F_3 \sin \Omega t)}{\sqrt{1 + \Gamma_x^2 + \Gamma_y^2 + f_a + f_b + f_c}} \\ &= C_1' + C_2' \cos(\Omega t - C_3') \end{aligned} \tag{A.9}$$

$$\begin{aligned} \text{where } C_2' &= \pi \frac{D}{\lambda} \left[\frac{\left(\Gamma_y + \frac{b}{h} \right)^2 + \left(\frac{c}{h} - 1 \right)^2 + \alpha^2 \left(\Gamma_x + \frac{a}{h} \right)^2 + 2\alpha \left(\Gamma_x + \frac{a}{h} \right) \left[\cos \beta \left(\frac{c}{h} - 1 \right) - \sin \beta \left(\Gamma_y + \frac{b}{h} \right) \right]}{1 + \Gamma_x^2 + \Gamma_y^2 + f_a + f_b + f_c} \right]^{\frac{1}{2}} \cos \theta \end{aligned} \tag{A.10}$$

$$C_1' = \pi \frac{D}{\lambda} \frac{\left[\Gamma_x + \frac{a}{h} + \alpha \sin \beta \left(\Gamma_y + \frac{b}{h} \right) - \alpha \cos \beta \left(\frac{c}{h} - 1 \right) \right]}{\sqrt{1 + \Gamma_x^2 + \Gamma_y^2 + f_a + f_b + f_c}} \sin \theta \tag{A.11}$$

$$C_3' = \tan^{-1} \left[\frac{1 - \frac{\epsilon}{h} - \alpha \cos \beta \left(\Gamma_x + \frac{a}{h} \right)}{\Gamma_y + \frac{b}{h} - \alpha \sin \beta \left(\Gamma_x + \frac{a}{h} \right)} \right] \quad (\text{A.12})$$

For the nominal position of the satellite, the quantities $a, b, c, \alpha, \beta, \gamma$ are set equal to zero. Equation (A.9) reduces to*

$$\phi(t) = C_1 + C_2 \cos(\Omega t - C_3) \quad (\text{A.13})$$

where

$$\left. \begin{aligned} C_1 &= \pi \frac{D}{\lambda} \frac{\Gamma_x}{\sqrt{1 + \Gamma_x^2 + \Gamma_y^2}} \cdot \sin \theta \\ C_2 &= \pi \frac{D}{\lambda} \sqrt{\frac{1 + \Gamma_y^2}{1 + \Gamma_x^2 + \Gamma_y^2}} \cdot \cos \theta \\ C_3 &= \tan^{-1} \left[\frac{1}{\Gamma_y} \right] \end{aligned} \right\} \quad (\text{A.14})$$

* An additional term will appear in the argument $\Omega t - C_3$ if the horizontal position of the interferometer is not taken as the time origin. $\phi(t)$ as written, however, may be used for analytical purposes without any loss in generality.

Appendix B

POWER SPECTRUM OF THE INTELLIGENCE

The spectrum of $\cos \phi(t, \underline{C})$ is of interest since filtering operations depend on some idea of what the important components of the signal are.

$$\begin{aligned} \cos \phi(t, \underline{C}) &\equiv \cos [C_1 + C_2 \cos(\Omega t - C_3)] \\ &= \cos [C_2 \cos(\Omega t - C_3)] \cos C_1 \\ &\quad - \sin [C_2 \cos(\Omega t - C_3)] \sin C_1 \end{aligned} \quad (B.1)$$

The Bessel function expansions of the quantities in brackets follow the rules:

$$\begin{aligned} \cos(a \cos b) &= J_0(a) - 2[J_2(a) \cos 2b - J_4(a) \cos 4b + \dots] \\ \sin(a \cos b) &= 2[J_1(a) \cos b - J_3(a) \cos 3b + \dots] \end{aligned}$$

Rewriting $\cos \phi(t, \underline{C})$

$$\begin{aligned} \cos \phi(t, \underline{C}) &= J_0(C_2) \cos C_1 - 2J_1(C_2) \cos(\Omega t - C_3) \cdot \sin C_1 \\ &\quad - 2J_2(C_2) \cos 2(\Omega t - C_3) \cdot \cos C_1 \\ &\quad + 2J_3(C_2) \cos 3(\Omega t - C_3) \cdot \sin C_1 + \dots \end{aligned}$$

Now

$$\begin{aligned} C_2 &\cong \pi \frac{D}{\lambda} \cos \theta \\ &= 222 \text{ for } D/\lambda = 100 \text{ and } \theta = 45^\circ \end{aligned}$$

which implies that the important Bessel functions are of the order around 222 and that the others are essentially zero in magnitude.

The important terms have frequencies around 222 Ω or 666 Hz.

For $\cos 2\varphi(t, \underline{C})$ the spectrum is similar except for a doubling of the significant frequencies to 1332 Hz for the example chosen. The spectra of both $\cos \varphi(t, \underline{C})$ and $\cos 2\varphi(t, \underline{C})$ are shown in Fig. B1.

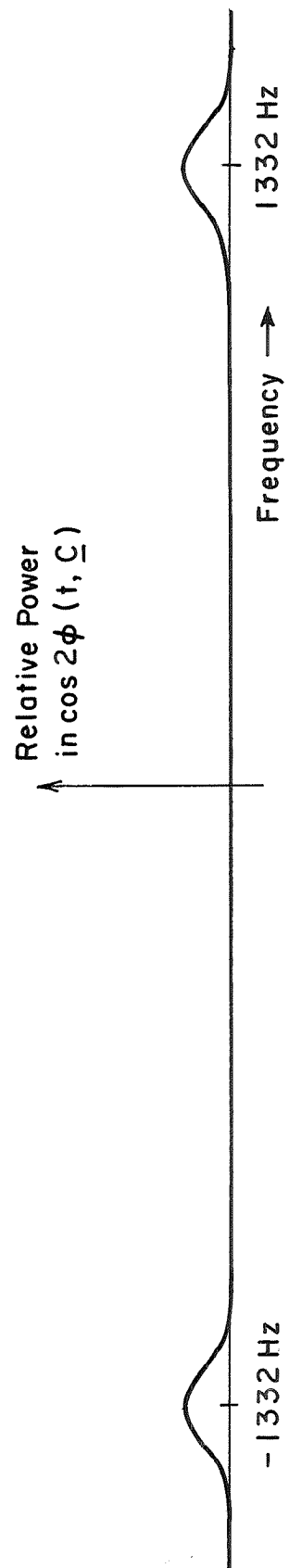
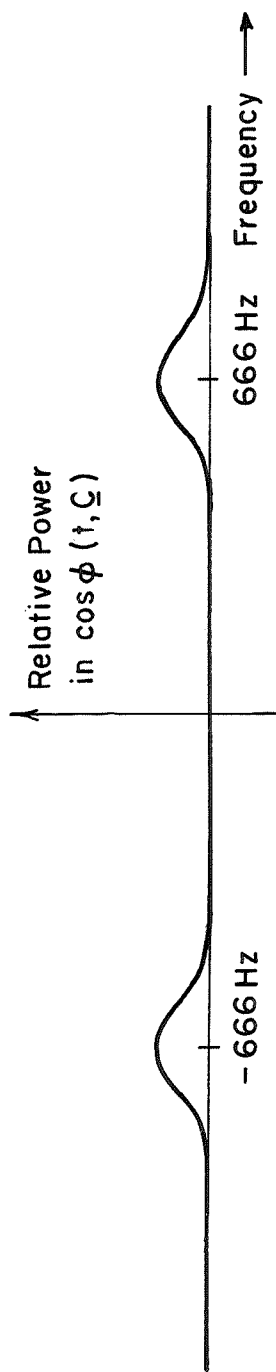


FIGURE B1 RELATIVE POWER SPECTRA OF $\cos \phi(t, \underline{C})$ AND $\cos 2\phi(t, \underline{C})$

Appendix C

THE DERIVATION OF A LIKELIHOOD FUNCTION FOR THE NONLINEAR
PARAMETER ESTIMATION PROBLEM

The derivation of the MAP processor equation in Section 3.2.2.1 begins with a log likelihood function $\ln \Lambda [r(t), A]$. This Appendix explores the reasons for this choice. The results obtained here and the notation followed are largely due to Van Trees [65], Chapters 2 and 4.

The basic problem is to "optimally" estimate the parameters A of a signal $s(t, A)$ that is contaminated by additive white gaussian noise $w(t)$. The parameters A may be either random or nonrandom. For the estimation of random parameters it is necessary to have a probabilistic mapping $p_{r/a}(R/A)$ to the observation space containing the vector R , an a priori pdf^{*} $p_a(A)$, and a cost function $C(A_e)$ which is a measure of the cost incurred by a particular decision made at the receiver. A_e is the estimation error. It is desired that the expected value of the cost, or risk, be minimum--a condition satisfied by a Bayes estimate. The choice of a parabolic cost function, $C(A_e) = A_e^2$, leads to a minimum mean square error estimate (MMSE) which can be shown to be the mean of the a posteriori pdf $p_{a/r}(A/R)$. A uniform cost criterion results in another Bayes estimate, the maximum a posteriori (MAP) estimate which may be shown to be the mode of the a posteriori pdf.

* pdf stands for the probability density function.

For nonrandom parameters a different approach must be used. A transition pdf, $p_{r/a}(R/A)$, known as a likelihood function is constructed, and that value of A is chosen which most likely caused a given value of R to occur. That is, the maximum likelihood (ML) estimate of the parameter A is that value at which the likelihood function (or its logarithm) is a maximum. Symbolically, $A = \hat{a}_{ML}$ when

$$\frac{\partial}{\partial A} \ln p_{r/a}(R/A) = 0 \quad (C.1)$$

Maximizing the a posteriori pdf or its logarithm gives the MAP equation

$$\frac{\partial}{\partial A} \ln p_{a/r}(A/R) = 0 \quad (C.2)$$

In order to separate the roles of the observed vector R and the a priori knowledge, the a posteriori pdf may be written as

$$p_{a/r}(A/R) = \frac{p_{r/a}(R/A) \cdot p_a(A)}{p_r(R)} \quad (C.3)$$

Taking logarithms,

$$\ln p_{a/r}(A/R) = \ln p_{r/a}(R/A) + \ln p_a(A) - \ln p_r(R) \quad (C.4)$$

The first two terms on the right-hand side represent a sufficient statistic, since the third term is not a function of A and will not enter the maximization of $\ln p_{a/r}(A/R)$. Equation (C.2) now becomes

$$\frac{\partial}{\partial A} \ln p_{n/a}(R/A) + \frac{\partial}{\partial A} \ln p_a(A) = 0 \quad (C.5)$$

when $A = \hat{a}_{\text{MAP}}$. Comparing Eqs. (C.1) and (C.5) it is seen that the ML estimate corresponds mathematically to the limiting case of a MAP estimate when the a priori knowledge approaches zero.

The general problem may be restated as the estimation of the parameters A when the noisy received signal $r(t)$ is

$$r(t) = s(t, A) + w(t) \quad (C.6)$$

The function $s(t, A)$ is a deterministic mapping of A into a time function. If $s(t, A)$ can be written as $A s(t)$, the problem is one of linear estimation. A likelihood function is needed but much simplification is possible by finding a sufficient statistic and maximizing it to find the ML estimate. For the nonlinear estimation case encountered in this work a sufficient statistic does not exist in general and a likelihood function has to be constructed directly.

The approach is to approximate $r(t)$ in terms of a finite set $\underline{r}$ of numbers $r_1, r_2, \dots, r_K$ choosing this set so that

$$p_{r_1, r_2, \dots, r_K / A}(R_1, R_2, \dots, R_K / A) = \prod_{i=1}^K p_{r_i / A}(R_i / A) \quad (C.7)$$

which says that the observations are conditionally independent. Since $r(t)$ can be represented completely only by an infinite set of numbers r_i , the solution must be in a form which permits letting $K \rightarrow \infty$. The

K-coefficient approximation to $r(t)$ is accomplished by using $r_K(t)$ defined as

$$r_K(t) = \sum_{i=1}^K r_i \Phi_i(t) \quad 0 \leq t \leq T$$

where

$$r_i = \int_0^T r(t) \Phi_i(t) dt \quad i = 1, 2, \dots, K$$

and the $\Phi_i(t)$ belong to an arbitrary complete orthonormal set of functions. The transition probability $p_{\underline{r}/\underline{a}}(\underline{R}/\underline{A})$ may be written as follows, assuming gaussian noise and hence gaussian distribution for r_i :

$$\Lambda_1[r_K(t), A] = \prod_{i=1}^K \frac{1}{\sqrt{\pi N_0}} \exp \left[-\frac{1}{2} \frac{(R_i - S_i(A))^2}{N_0/2} \right] \quad (C.8)$$

where the conditional variances

$$\text{var}(r_i/A) = N_0/2$$

and

$$S_i(A) = \int_0^T S(t, A) \Phi_i(t) dt$$

If $K \rightarrow \infty$ in Eq. (C.8), $\Lambda_1(r_K(t), A)$ is not well defined. On the principle that a likelihood function may be divided by anything that does not depend on A and still remain a likelihood function, divide

Eq. (C.8) by

$$\prod_{i=1}^K \frac{1}{\sqrt{\pi N_0}} \exp \left[-\frac{1}{2} \frac{R_i^2}{N_0/2} \right]$$

obtaining a new likelihood function $\Lambda[r_K(t), A]$. Taking the logarithm and canceling the common terms

$$\ln \Lambda [r_k(t), A] = \sum_{i=1}^K \frac{R_i s_i}{N_0/2} - \frac{1}{2} \sum_{i=1}^K \frac{s_i^2(A)}{N_0/2} \quad (C.9)$$

The two sums may be expressed as integrals using Parseval's Theorem,

$$\sum_{i=1}^K R_i s_i = \int_0^T h_k(t) s_k(t) dt$$

and

$$\sum_{i=1}^K s_i^2 = \int_0^T s_k^2(t) dt$$

Letting $K \rightarrow \infty$, the log likelihood function becomes

$$\ln \Lambda [h(t), A] = \frac{2}{N_0} \int_0^T h(t) s(t, A) dt - \frac{1}{N_0} \int_0^T s^2(t, A) dt \quad (C.10)$$

To find the ML estimate of A , $\hat{a}_{ML}$, the absolute maximum of Eq. (C.10) must be found. The MAP estimate $\hat{a}_{MAP}$ is found by adding $\ln p_a(A)$ in Eq. (C.10) and maximizing this new function.

In the Spinning Interferometer Navigation scheme the initial estimate of position $\underline{\eta}$ is made by on-board dead-reckoning equipment. The radio system is expected to provide an improved estimate $\hat{\underline{y}}$ that differs from $\underline{\eta}$ by an error $\underline{\epsilon}$. The distribution of this error may be experimentally determined for a particular aircraft flying a specific route, and this a priori information used to improve position determination in subsequent flights. If such information is not fed into the estimator structure, the craft obtains a maximum likelihood estimate corresponding to a MAP estimate with a uniform a priori distribution.

Appendix D

NATURE OF ERROR INTRODUCED BY INTERFEROMETRIC MOTION

The spinning interferometer consists of two antennas transmitting identical CW signals. The waveform $s_R(t)$ received by the craft at time t may be written as

$$s_R(t) = A_0 \cos \left[\omega t - \frac{\omega r_1}{c} \left(t - \frac{r_1}{c} \right) \right] + A_0 \cos \left[\omega t - \frac{\omega r_2}{c} \left(t - \frac{r_2}{c} \right) \right] \quad (D1)$$

where the craft-to-antenna #1 range r_1 is taken at time $t - r_1/c$, and r_2 is taken at time $t - r_2/c$. These two ranges can be expressed in terms of the craft-to-satellite center range r and differential distances ρ_1 and ρ_2 from each antenna to the phase-front through the satellite center, as shown in Fig. D1:

$$r_1(t - r_1/c) = r + \rho_1(t - r/c - \rho_1/c) \quad (D2)$$

$$r_2(t - r_2/c) = r - \rho_2(t - r/c + \rho_2/c) \quad (D3)$$

The time-varying differential distance from an antenna to the phase front may be written in terms of the baselength D , skew angle θ , spin-rate Ω as

$$\rho(t) = \frac{D}{2} \sin \theta \cdot T_x + \frac{D}{2} \cos \theta \cdot \cos(\Omega t - C_3) \quad (D4)$$

$$\equiv \varepsilon_1 + \varepsilon_2 \cos(\Omega t - C_3)$$

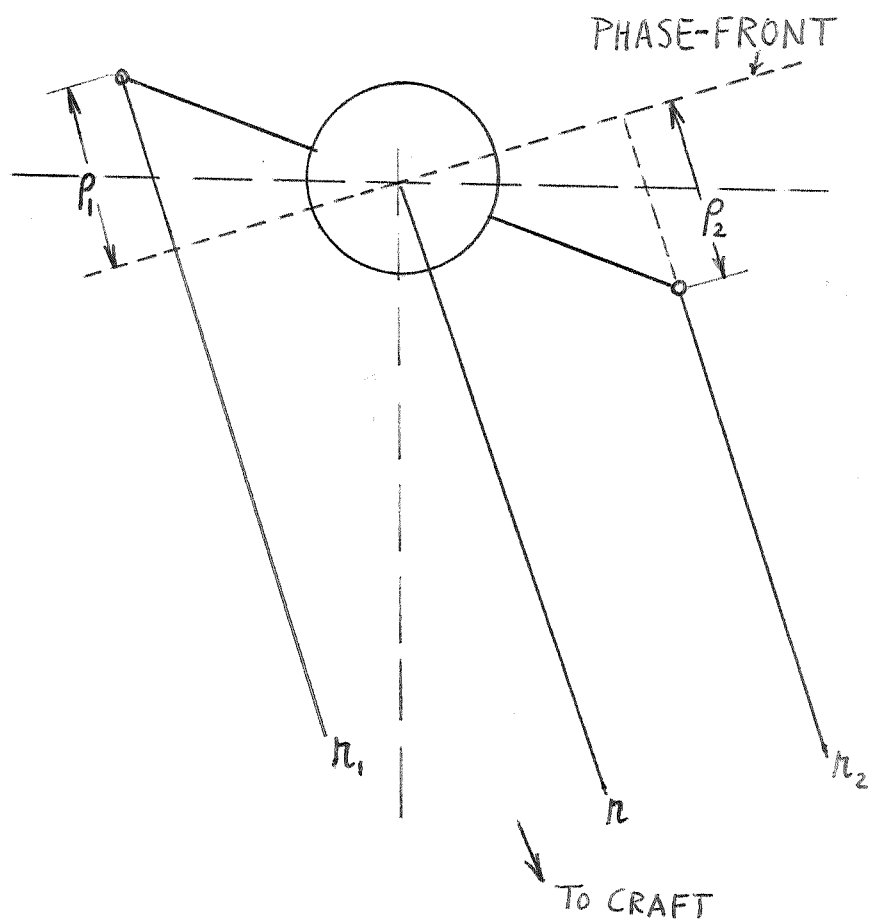


FIGURE D1 ANTENNA-CRAFT RANGES RELATIVE TO SATELLITE-CRAFT RANGE

This expression has been derived as $\frac{r_1 - r_2}{2}$ in Appendix A, under the assumption that the interferometer is horizontal at $t = 0$.

Using this relation, (D2) and (D3) may be written as

$$\begin{aligned} \rho_1(t - r/c - \rho_1/c) &= \varepsilon_1 + \varepsilon_2 \cos \left[\Omega t - \frac{\Omega r}{c} - \frac{\Omega \rho_1}{c} - C_3 \right] \\ \rho_2(t - r/c + \rho_2/c) &= \varepsilon_1 + \varepsilon_2 \cos \left[\Omega t - \frac{\Omega r}{c} + \frac{\Omega \rho_2}{c} - C_3 \right] \end{aligned}$$

or, if $\Omega t - \frac{\Omega r}{c} - C_3 \triangleq Q$,

$$\rho_1 = \varepsilon_1 + \varepsilon_2 \cos(Q - \Omega \rho_1/c)$$

$$\rho_2 = \varepsilon_1 + \varepsilon_2 \cos(Q + \Omega \rho_2/c)$$

With $\cos \frac{\Omega \rho_1}{c} \cong 1$ and $\sin \frac{\Omega \rho_1}{c} \cong \frac{\Omega \rho_1}{c}$ since $\frac{\Omega \rho_1}{c}$ is on the order of tenths of microradians,

$$\rho_1 = \varepsilon_1 + \varepsilon_2 \cos Q + \frac{\varepsilon_2 \Omega \rho_1}{c} \sin Q \quad (D5)$$

Similarly,

$$\rho_2 = \varepsilon_1 + \varepsilon_2 \cos Q - \frac{\varepsilon_2 \Omega \rho_2}{c} \sin Q \quad (D6)$$

Solving (D5) and (D6) for ρ_1 and ρ_2

$$\rho_1 = \frac{\epsilon_1 + \epsilon_2 \cos Q}{1 - \frac{\epsilon_2 \Omega}{c} \sin Q} \quad (D7)$$

$$\rho_2 = \frac{\epsilon_1 + \epsilon_2 \cos Q}{1 + \frac{\epsilon_2 \Omega}{c} \sin Q} \quad (D8)$$

Since $\frac{\epsilon_2 \Omega}{c}$ is on the order of tenths of microradians (D7) and (D8) may be rewritten as

$$\rho_1 = [\epsilon_1 + \epsilon_2 \cos Q] \left[1 + \frac{\epsilon_2 \Omega}{c} \sin Q + \left(\frac{\epsilon_2 \Omega}{c} \sin Q \right)^2 \right]$$

$$\rho_2 = [\epsilon_1 + \epsilon_2 \cos Q] \left[1 - \frac{\epsilon_2 \Omega}{c} \sin Q + \left(\frac{\epsilon_2 \Omega}{c} \sin Q \right)^2 \right]$$

or, using (D4),

$$\left. \begin{aligned} \rho_1 &= \rho(t - r/c) + \delta_1 + \delta_2 \\ \rho_2 &= \rho(t - r/c) - \delta_1 + \delta_2 \end{aligned} \right\} \quad (D9)$$

where the error terms δ_1 and δ_2 are

$$\left. \begin{aligned} \delta_1 &= \frac{\epsilon_1 \epsilon_2 \Omega}{c} \sin Q + \frac{\epsilon_2^2 \Omega}{2c} \sin 2Q \\ \delta_2 &= \frac{\epsilon_1}{2} \left(\frac{\epsilon_2 \Omega}{c} \right)^2 [1 - \cos 2Q] \\ &\quad + \frac{\epsilon_2}{4} \left(\frac{\epsilon_2 \Omega}{c} \right)^2 [\cos Q - \cos 3Q] \end{aligned} \right\} \quad (D10)$$

Using (D2), (D3) and (D9) in (D1):

$$\begin{aligned} s_R(t) = A_0 &\left[\cos \left(\omega t - \frac{\omega r}{c} - \frac{\omega \rho}{c} \left(t - \frac{r}{c} \right) - \frac{\omega \delta_1}{c} - \frac{\omega \delta_2}{c} \right) \right. \\ &\left. + \cos \left(\omega t - \frac{\omega r}{c} + \frac{\omega \rho}{c} \left(t - \frac{r}{c} \right) - \frac{\omega \delta_1}{c} + \frac{\omega \delta_2}{c} \right) \right] \quad (D11) \end{aligned}$$

Expanding (D11) and combining terms,

$$\begin{aligned} s_R(t) &= 2 A_0 \cos \left[\frac{\omega \rho}{c} \left(t - \frac{r}{c} \right) + \frac{\omega \delta_2}{c} \right] \cos \left(\omega t - \frac{\omega r}{c} - \frac{\omega \delta_1}{c} \right) \\ &= 2 A_0 \cos \left[\varphi \left(t - \frac{r}{c} \right) + \frac{\omega \delta_2}{c} \right] \cos \left(\omega t - \frac{\omega r}{c} - \frac{\omega \delta_1}{c} \right) \end{aligned}$$

The first-order error term $\omega \delta_1 / c$ appearing in the carrier part of the DSB waveform $s_R(t)$ is seen to consist of two slowly varying sinusoids of very small amplitude. The amplitude of $\sin Q$ is on the order of $14.85 \mu\text{rad}$ and that of $\sin 2Q$ is about $49.3 \mu\text{rad}$.

The system parameters used to obtain these figures are $D/\lambda = 100$, $\theta = 45^\circ$, $\Omega = 6\pi \text{ rad/sec}$, $\omega/2\pi = 1.5 \text{ GHz}$. Coherent detection which

is expected to remove the carrier part of the DSB signal is not likely to be affected by the slowly varying $\omega\delta_1/c$.

The information-bearing term $\phi(t - r/c)$ is contaminated by $\omega\delta_2/c$. This second-order error consists of terms at dc, Ω , 2Ω and 3Ω radians/sec. A worst case figure on the magnitude of this error can be obtained by adding the amplitudes of all four terms. The first two terms have amplitudes

$$\frac{\omega}{c} \frac{\epsilon_1}{2} \left(\frac{\epsilon_2 \Omega}{c} \right)^2 \cong 3.33 \times 10^{-12} \text{ radians}$$

The last two terms have amplitudes

$$\frac{\omega}{c} \frac{\epsilon_2}{4} \left(\frac{\epsilon_2 \Omega}{c} \right)^2 \cong 11.1 \times 10^{-12} \text{ radians}$$

The worst case, then, is an error in $\phi(t - r/c)$ of around 30×10^{-12} radians. For $D/\lambda = 100$, $\theta = 45^\circ$, the corresponding craft position error is given by

$$\begin{aligned} \delta T_x &= \frac{\partial \phi}{\pi D/\lambda \sin \theta} = \frac{\partial \phi}{222} \\ \delta T_y &= \frac{\partial \phi}{\pi D/\lambda \cos \theta} = \frac{\partial \phi}{222} \end{aligned}$$

which result in errors in the x- and y-directions of about 2.6×10^{-9} nmi, clearly an insignificant quantity. It may be concluded that the use of $\phi(t - r/c)$ as the sole information-bearing term is justified.

Appendix E

EPOCH SIGNAL CONSIDERATIONS

The internal signal generating mechanism which is part of the MAP processor is shown in some detail in Fig. E1. Its function is to generate the s_0 , s_1 , s_2 and s_3 waveforms corresponding to the grid-point in the vicinity of which the craft is located. These waveforms are of amplitude A_0 and contain phase quantities C in their arguments. Being dependent on the grid-point coordinates and system parameters (D/λ , θ , transmitted power), A_0 , C_1 and C_2 may be considered known. A_0 could also be obtained from the received signal envelope and fed into this generating mechanism. C_3 , however, depends on the time origin selected at the navigating craft. For coherent processing to be accomplished in the MAP structure, the receiver time origin must either be identical to that at the transmitter, or different by a known amount.

The incoming phase quantity $\varphi(t-r/c)$ and the internally generated $\varphi_0(t)$ may be rewritten as

$$\varphi(t-r/c) = C_1 + C_2 \cos\left(\Omega t - \frac{\Omega r}{c} - \Omega t_0 - C_3\right)$$

$$\varphi_0(t-t_1) = C_1 + C_2 \cos(\Omega t - \Omega t_1)$$

where t_0 is the time origin at the transmitter, t_1 is the time origin at the receiver, and $\Omega r/c$ is the propagation phase delay. If t_1 is not equal to $t_0 + r/c$ an error in the estimated craft location will be introduced. To ensure that $t_1 = t_0 + r/c$, a time reference is provided at the craft.

Such a time reference may be established by transmitting an epoch signal (e.g. a sinusoid synchronous with the satellite spin-rate, an RF

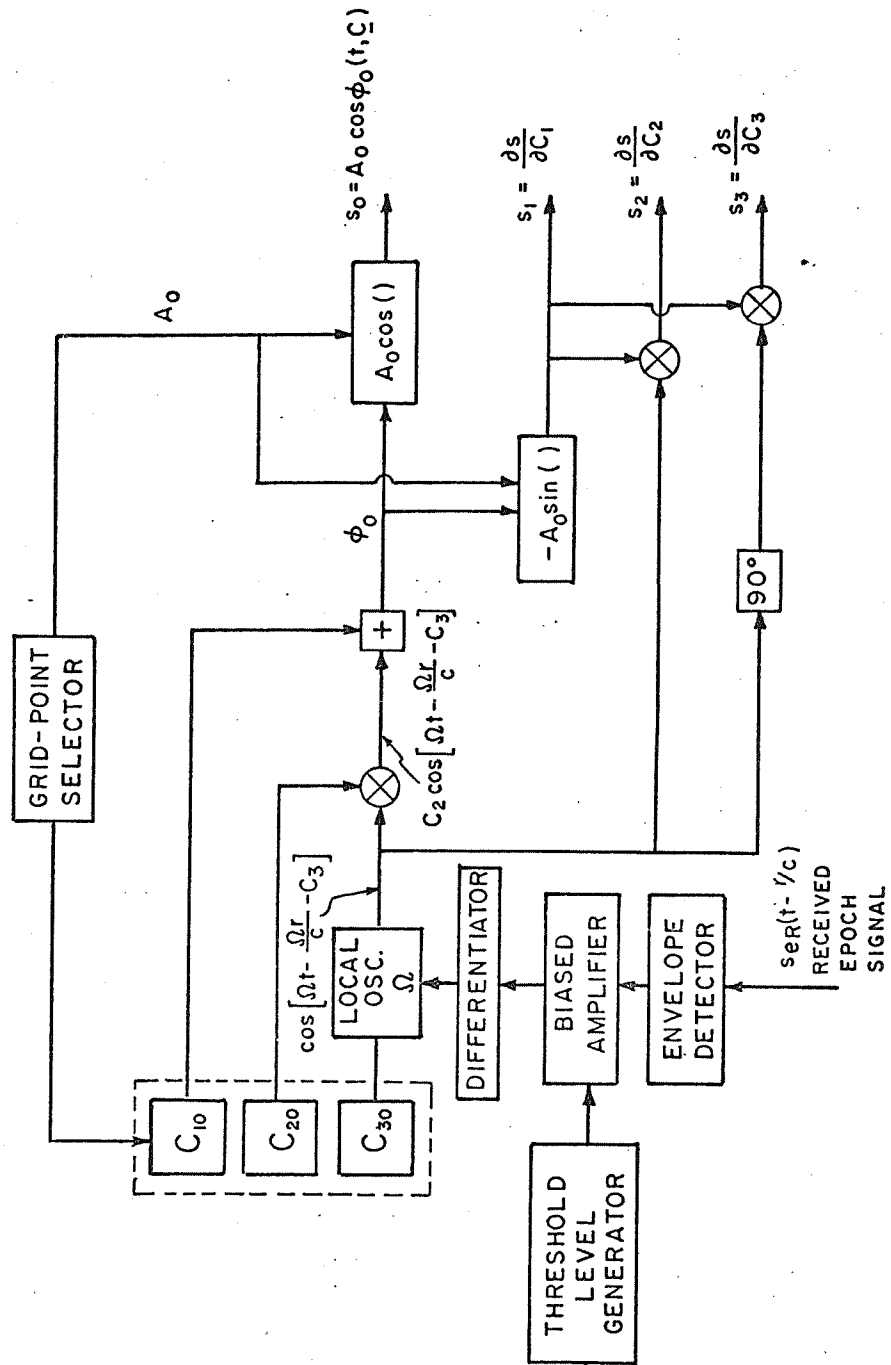


FIGURE E1 INTERNAL SIGNAL GENERATING MECHANISM FOR THE MAP PROCESSOR, INCLUDING A SIMPLIFIED BLOCK DIAGRAM FOR EPOCH SIGNAL RECEPTION

pulse train where the pulses may be rectangular, triangular, trapezoidal etc.) and allowing this signal to reference an internal oscillator. The instant of transmission fixes the time origin $t=0$, thus establishing the magnitude of C_3 . This time instant could be at the horizontal position of the interferometer as has been assumed in the derivation of $\varphi(t, \underline{C})$. Any other orientation could be assumed provided it is consistent.

Two methods of detecting the interferometer position are visualized. The first one is based on the spatial coherence property of laser light and star light.* Assume that the satellite is illuminated by either a laser or a star. The radiation is sampled at two points on the satellite and the phase difference between the planes of constant phase is measured thus fixing the satellite attitude. The same basic technique has been proposed using radio waves and a radio interferometer. Due to the longer wavelengths at radio frequencies, the accuracies are much less than those from the optical systems. Attitude and hence interferometric position may be determined with an accuracy of between ± 0.1 and ± 1.0 seconds of arc with star trackers and an order of magnitude higher, ± 0.01 seconds of arc, with laser radiation. The uncertainty in interferometric position introduces an equal spatial angle uncertainty in $\mathbf{r}_x$ and $\mathbf{r}_y$. A second of arc corresponds to about $5 \mu\text{rad}$ which is about 0.1 nmi in earth distance.

The second method is to observe the incoming $\varphi(t - r/c)$ waveform at a ground station, to detect the instants of maxima or the instant of zero crossing, and, knowing the ground location to estimate the time origin. The estimated time origin could be used by the ground station to activate

*A. B. DeCou: Spacecraft Attitude Measurement Using Spatial Coherence of Laser or Star Light, IEEE Trans. AES-7, No. 2, March 1971, p. 288-298.

a satellite transponder to transmit an epoch signal to the field of users. This method effects a simplification of satellite equipment by not requiring satellite-borne detection and processing equipment as in the optical techniques, but their relative merits bear further study.

In any case, it may be assumed that an RF pulse train is transmitted from a body-fixed satellite antenna. It is received with an amplitude A by the navigating craft where it is used to time reference the Ω oscillator as shown in Fig. E1. The time origin may be fixed when the received pulse crosses a threshold, say $A/2$, where the slope of the pulse is often greatest though not so for all filters.

For analytical purposes, then, assume that the received pulse has an amplitude A and a rise time t_r . The useful information is contained in the time location of the pulse edge. If the pulse were perfectly rectangular, additive noise would have no effect on its edges since the noise is a vertical perturbation. However, infinite bandwidth is required for such a pulse. In practice, bandwidth is always limited since the received pulse has to go through band limited filters and amplifiers before it can activate the Ω oscillator. Furthermore, an unlimited increase in bandwidth permits a corresponding increase in the rms noise voltage, causing occasional noise peaks to be mistaken for signal peaks.

With a finite system bandwidth B , the received pulses have a non-zero rise time $t_r = 1/B$, and the noise will alter the edge position by an error t_e . Fig. E2 shows the relationship between t_r , A , t_e and the noise $n(t)$:

$$\frac{t_e}{n} = \frac{t_r}{A} \quad (\text{E1})$$

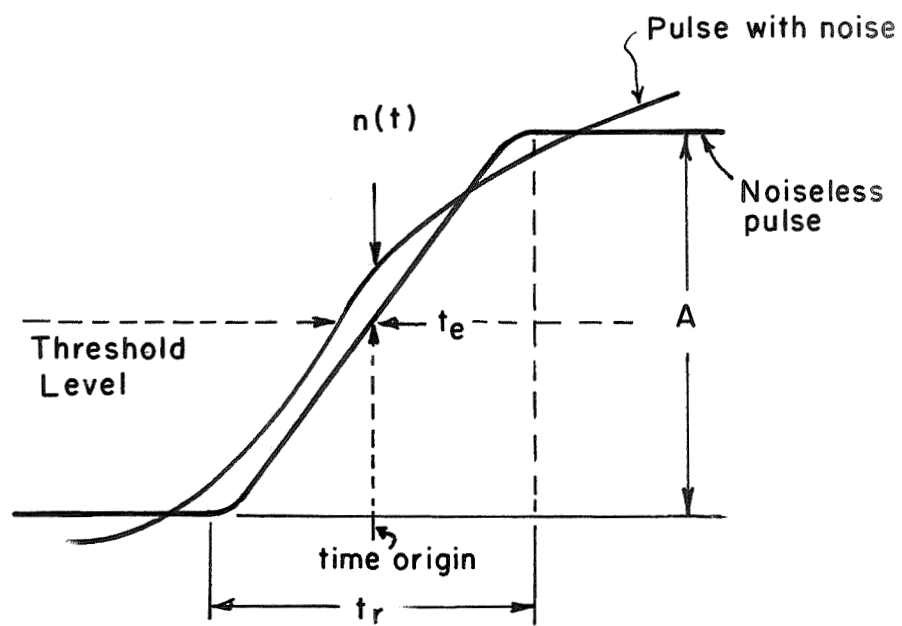


FIGURE E2 PULSE EDGE POSITION PERTURBED BY NOISE

which may be rewritten in terms of the mean square time error

$$\langle t_e^2 \rangle = \left(\frac{t_n}{A} \right)^2 \langle n^2 \rangle$$

Assuming $n(t)$ to be bandlimited white noise of two-sided spectral density N_0 watts/Hz over the system bandwidth B , the above equation may be written as

$$\langle t_e^2 \rangle = \frac{2 N_0 B}{B^2 A^2} = \frac{2 N_0}{B A^2} \quad (E2)$$

If the pulse is τ seconds and the time between pulses T_0 , a duty cycle τ/T_0 may be defined, to express the average power P_r arriving at the receiver as

$$P_r = (\tau/T_0)(A^2/2) \quad (E3)$$

Since the time information is entirely in the pulse edges, the pulse duration τ could be taken to be simply twice the rise time, or $\tau = 2/B$, which makes the pulses triangular. The two threshold crossing instants available with each pulse can be averaged to give a better estimate of time origin. Equation (E3) may be written as

$$P_r = A^2/BT_0 \quad (E4)$$

In terms of the above quantities an rms error in time origin may be expressed as

$$\Delta t \equiv \sqrt{\langle t_e^2 \rangle} = \sqrt{\frac{2}{B^2 T_0 (P_r/N_0)}} \quad (E5)$$

This time error appears as an error in C_3 and hence Γ_y . From the expression for C_3

$$C_3 = \tan^{-1} [1/T_y]$$

$$\frac{\partial C_3}{\partial T_y} = \frac{-1}{1 + T_y^2} \simeq -1 \quad \text{since } T_y^2 \leq 0.02$$

$$\therefore |\partial T_y| \cong \partial C_3$$

At a spin-rate Ω ,

$$\partial C_3 = \Omega \Delta t$$

and therefore

$$|\partial T_y| = \Omega \Delta t$$

which may be used to convert time error into location error.

A power budget based on an assumed average transmitted power of 25 watts is presented in Table E1. Fig. E3 shows a plot of Δt and the corresponding position error vs. average transmitted power for several combinations of B and T_0 . The required peak transmitted power may be obtained immediately from (E4). The power requirements could be eased by allowing gain in the satellite antenna. Fig. E3 and Table E1 assume a zero dB antenna at the satellite.

The interval T_0 determines how frequently position fixes are available. An epoch signal every 2 seconds or once in 6 revolutions for a satellite spinning at 180 rpm allows virtually continuous operation since even at 1200 km/hr the aircraft has moved only 2/3 km during this interval. The permissible error in the east-west coordinate T_y if judged to be 1 nmi, assuming cross-track accuracy is of more importance, will allow the average transmitted power to be less than 19 watts for B=10 KHz and $T_0 = 2$ sec. If the bandwidth is reduced to 1 KHz the power requirements is seen to rise rapidly to 119 watts for T_0 of 20 sec, and to 1.19 Kw for T_0 of 2 sec.

Table EI

Epoch Signal Transmission Power Budget

Average satellite transmitter power (25 W)	14	dBW
Satellite antenna gain*	0	dB
Free space loss (at 1.6 GHz)	-190	dB
Craft-borne antenna gain	10	dB
Received power at craft, P_r	-166	dB
Noise PSD N_o ($T_e = 1000^\circ\text{K}$)	<u>-198.5</u>	<u>dB</u>
P_r/N_o	32.5	dB

* A higher gain will reduce the transmitter power requirements.

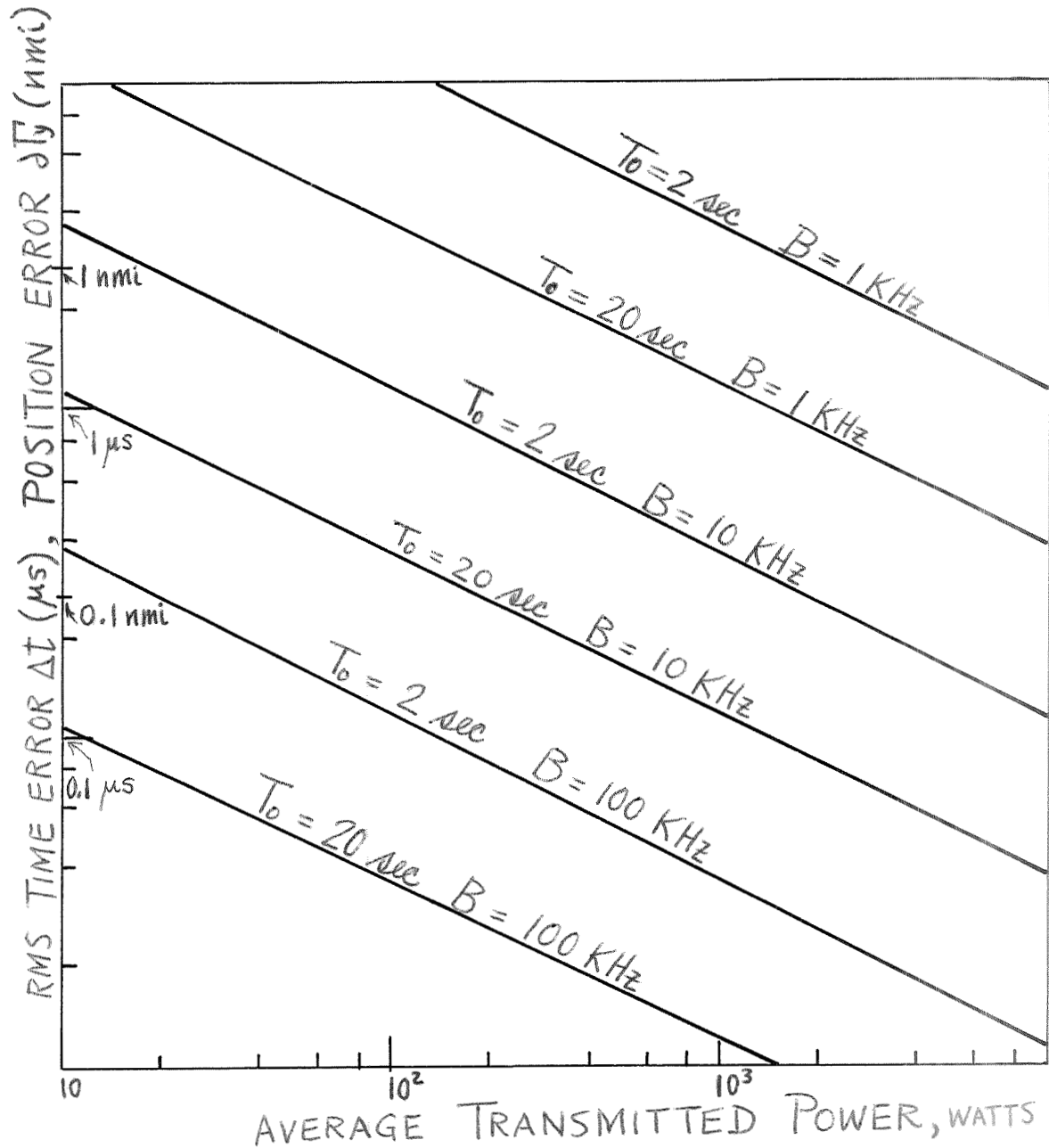


FIGURE E3 RMS TIME ERROR AND THE CORRESPONDING POSITION ERROR
VS AVERAGE TRANSMITTED POWER WITH B AND T_0 AS PARAMETERS.
[$\tau = 2/B$, RECEIVER TEMPERATURE = 1000 $^{\circ}K$, $\Omega/2\pi = 3 \text{ Hz}$,
SATELLITE ANTENNA GAIN = 0 dB]

These values of power are excessive, and it may be concluded that this method of implementing an epoch signal requires considerable additional spectrum, nearly five times the intelligence bandwidth. From Fig. E3 it appears that a decrease in T_0 at a constant value of average transmitted power, corresponding to a lower peak power, results in a higher Δt or poorer position accuracy. It must be noted, however, that a lower T_0 means a larger number of pulses in a given time interval. This means that a larger number of threshold crossing instants are available to be averaged to improve the estimated time origin.

Mention must be made of the possibility of operation without an epoch signal, that is, by designing the MAP processor to estimate the time origin difference. This could be subject for further work directed toward simplifying satellite equipment.

Appendix F

COMPUTER PROGRAM TO EVALUATE ESTIMATION ERRORS

The steps necessary to generate numerical values for the elements of the estimation error covariance matrix are discussed in section 4.0. The computer program presented on the next page reads in typical values for craft coordinates, interferometric skew-angle, baselength-to-wavelength ratio and performance factor, and computes the GDOP factors, error correlation coefficients and the 1σ estimation errors in the x- and y-directions.

The craft coordinates, Γ_x and Γ_y , and skew-angle are expressed in degrees, the program converting them to radians. The performance factor is expressed as an integer to be used in the calculations, but is printed out in decibels. It must be noted that Γ_x , Γ_y , θ , D/λ and TA_o^2/N_o in the text are named X, Y, TH, D and TSNR respectively in the program. Other symbols should be clear from the context.

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C NAVSAT RECEIVER OPTIMIZATION
0001 C DIMENSION X(6),Y(6),TH(5),D(5),TSNR(6),Q(3,3),A(3,2),B(2,3),
      1 P(2,3), R(2,2), KL(2), KM(2)
C
C A CONVERSION FACTOR WILL BE DEFINED HERE CON(2)
0002 CON(2) = 3.141593 * 2 / 180.0
0003 CG = 1755.216 E+06
C
C READ IN THE POSITION COORDS, THETA, D/L, TIME-SNR
0004 READ (5,100) X
0005 READ (5,100) Y
0006 READ (5,200) TH
0007 READ (5,200) D
0008 READ (5,300) TSNR
C
C
0009 100 FORMAT (6F10.0)
0010 200 FORMAT (5F10.0)
0011 300 FORMAT (6F10.0)
0012 66 FORMAT ('0',5F10.2,10X,5E12.4)
C
C DO LOOPS START AT THIS POINT
C
0013 DO 10 I = 1,6
0014 XX = TAN(CON(X(I)))
0015 XP = X(I)
C
C CALCULATE GOOP FACTOR FOR THE X COORDINATE
0016 XS = SIN ( CON (X(I)))
0017 XC = COS ( CON ( X(I)))
0018 BX = 42378. * XC
0019 RTX = SQRT ((BX*BX) - CG)
0020 SLANTX = BX - RTX
0021 SAX = XS * SLANTX / 6378.
0022 ANGX = ARSIN(SAX)
0023 ELX = 1.570797 - CON (X(I)) - ANGX
0024 SELX = SIN(ELX)
0025 GOOPX = SLANTX / (36000. * SELX)
C
0026 DO 10 J = 1,6
0027 YY = TAN(CON(Y(J)))
0028 YP = Y(J)
C
C CALCULATE THE GOOP FACTOR FOR THE Y COORDINATE.
0029 YS = SIN ( CON (Y(J)))
0030 YC = COS ( CON ( Y (J)))
0031 BY = 42378. * YC
0032 RTY = SQRT((BY*BY) - CG)
0033 SLANTY = BY - RTY
0034 SAY = YS * SLANTY / 6378.
0035 ANGY = ARSIN (SAY)
0036 ELY = 1.570797 - CON (Y(J)) - ANGY
0037 SELY = SIN (ELY)
0038 GOOPY = SLANTY / (36000. * SELY)
C
0039 DO 10 K = 1,5
      THH = CON(TH(K))
      THP = TH(K)
      DO 10 L = 1,5
        DD = D(L)
        E = DD*3.141593
        DO 10 M = 1,6
          TS = TSNR (M)
          DR = 10.0 * ALOG10(TSNR(M))
C
C COMPUTE SINES AND COSINES OF SKEW ANGLE THETA
0048 ST = SIN(THH)
0049 CT = COS(THH)
C
C COMPUTE THE C PARAMETERS
0050 C1 = E*XX*ST
0051 C2 = E*CT
0052 CC = COS(2.0*CT)
0053 SC = SIN(2.0*CT)
C
C COMPUTE SECOND ORDER BESSEL FUNCTION...RECURRENCE RELATION
0054 F0 = FJ0(2.0 * C2)
0055 F1 = FJ1(2.0 * C2)
0056 F2 = (F1 / C2) - F0
C
C NOW GET THE BESSEL SINUSOID PRODUCTS
0057 BS1 = F0 * CC
0058 BS2 = F2 * CC
C
C COMPUTE THE ELEMENTS OF THE A MATRIX
0059 A(1,1) = E*ST
0060 A(1,2) = -E*ST*XX*YY
0061 A(2,1) = -E*CT*XX
0062 A(2,2) = E*CT*XX*XX*YY
0063 A(3,1) = 0.0
0064 A(3,2) = -1.0 / (1.0 + YY*YY)
C
C COMPUTE THE ELEMENTS OF THE Q MATRIX
0065 Q(1,1) = 1.0 - BS1
0066 Q(1,2) = F1 * SC
0067 Q(1,3) = 0.0
0068 Q(2,1) = Q(1,2)
0069 Q(2,2) = 0.5 * (1.0 - BS1 + BS2)
0070 Q(2,3) = 0.0
0071 Q(3,1) = 0.0
0072 Q(3,2) = 0.0
0073 Q(3,3) = 0.5 * C2 * C2 * (1.0 - BS1 - BS2)
C
C SSP SUBROUTINES ARE CALLED IN AT THIS POINT
C
C TRANSPOSE A TO GET B
0074 CALL GMTRA (A,B,3,2)
C
C MULTIPLY B AND Q TO GIVE P
0075 CALL GMPRO (B,Q,P,2,3,3)
C
C MULTIPLY P AND A TO GIVE R
0076 CALL GMPRO (P,A,R,2,3,2)
C
C INVERT THE MATRIX R
0077 CALL MINV (R,2,DET,KL,KM)
0078 DO 13 II = 1,2
0079 DO 13 JJ = 1,2
0080 R(II,JJ) = R(II,JJ) / TS
13
C
0081 FAC = 51.479E-06
0082 SOX = SORT(R(1,1))
0083 SOY = SORT(R(2,2))
0084 RHU1 = R(1,2) / (SOX * SOY)
0085 RHO2 = R(2,1) / (SOX * SOY)
0086 ERX = GOOPX * SOX / FAC
0087 ERY = GOOPY * SOY / FAC
C
0088 WRITE(6,66) XP,YP,THP,DD,DB,ERX,ERY,GOOPX,GOOPY,DET
C
0089 10 CONTINUE
0090 END

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