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HEAT TRANSFER OPTIMIZATION
AND REACTIVITY OF
NUCLEAR ROCKET REACTORS

by

Terry R. Simpson

Edward A. Mason

CSR T 70-3

CENTER FOR SPACE RESEARCH
MASSACHUSETTS INSTITUTE OF TECHNOLOGY



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Terry Regg Simpson
B.S., California Institute of Technology
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Heat Transfer Optimization and Reactivity
of Nuclear Rocket Reactors
by
Terry Regg Simpson

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Abstract

A method of optimizing the axial power profile of a solid-core nuclear rocket reactor has been developed. It is shown that the power profile is very dependent upon the heat transfer correlation used to predict channel wall temperatures, especially in the reactor core inlet region. It is demonstrated that the optimum reactor core dimensions and I_{sp} are insensitive to the form of the heat transfer correlation used.

Reactivity characteristics for such compact, high performance reactor cores have been calculated. It is shown that the hydrogen in the reflector can contribute several times more reactivity worth than the hydrogen in the reactor core. The critical uranium concentration, total and specific hydrogen reactivity worths, and mean fissioning energy are all shown to be strongly correlated to the geometrical buckling. Several equations are derived relating hydrogen worth to geometrical buckling. The total reactivity worth of hydrogen in the reflector is shown to be linearly proportional to the average hydrogen concentration in the reflector. Power tilting and loss of reactivity by core thermal expansion and corrosion loss of carbon are calculated. These reactivity characteristics are shown to agree favorably with values in the literature cited for the Rover nuclear rocket reactors.

Thesis Supervisor: Edward A. Mason
Professor of Nuclear Engineering

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1. Summary

1.1 Introduction

A review of the available literature (6 - 14, 17 - 23) on solid-core nuclear rocket propulsion indicates that there are important heat transfer and nuclear considerations that should be investigated and defined for an optimum nuclear reactor system. Radiation shielding for the nuclear reactor imposes large weight penalties upon those nuclear propulsion reactors which are not as compact as possible, especially radially compact. Compactness for the nuclear reactor must not decrease the achievable specific impulse of the rocket system. This specific impulse, I_{sp} (sec), is directly proportional to the square root of the rocket-nozzle chamber temperature. I_{sp} is the single most important parameter for the rocket system and is a measure of the efficiency with which the rocket engine utilizes propellant. Since the hydrogen propellant may comprise 90% of the entire system weight, small improvements in I_{sp} mean large savings in total weight. Compactness of reactor core means a savings in the large shielding weight penalty. In terms of total weight in earth orbit (WIEO) for a manned mission for Mars, to be assembled in orbit, these typical mission weight sensitivity relations are found (36): one second I_{sp} = 3000 lbs WIEO and one lb. engine weight = 16 lbs. WIEO. This means that an improvement in I_{sp} by one second will reduce WIEO by 3000 lbs; one lb. increase in engine weight will increase WIEO by 16 lbs.

A very compact reactor can be designed such that all portions of a representative core coolant channel operate at the maximum temperatures that materials constraints will allow. Therefore compactness and maximum Isp can result. Such a compact reactor should have a very fast spectrum and the reactivity worth of the hydrogen in both reflector and reactor core needs to be defined.

1.2 Heat Transfer Considerations

1.2.1 Temperature Anomaly

Previous heat transfer studies (6, 8, 11, 17, 19) for the nuclear rocket have used a variety of heat transfer correlations for the turbulent flow of hydrogen in the reactor core coolant channels. A careful study of these reported results shows that the correlations all predict a wall temperature anomaly that could seriously limit full-power, steady-state design performance. For combinations of high power density and low inlet bulk coolant temperatures, a wall temperature distribution may be predicted that exhibits an unusual behavior in the core inlet region with respect to bulk coolant temperature: the wall temperature decreases as the bulk coolant temperature increases. It can be shown that correlations which show this effect will predict very large increases in inlet wall temperature as the bulk temperature is decreased at constant power. This means, if the anomaly actually exists and is not a correlational defect, that operation at 100% of design power would be impossible because reductions in inlet bulk temperature, always possible at full power operation, could cause

fuel element melting or failure. Some experimental evidence tentatively supports the existence of this temperature effect (4) although this experimental evidence was not taken at the conditions of high power density, high wall temperature, and very low bulk coolant temperatures predicted for nuclear rocket channel conditions.

1.2.2 Heat Transfer Optimization

Compact nuclear rocket reactors require that the core power profile be optimized such that all portions of the core operate at the highest temperature that materials constraints will allow. A computer program, MAXPOW, was written that would calculate the optimum axial power profile by maximizing the power density subject to three constraints: (1.) maximum stress, (2.) maximum fuel centerline temperature, and (3.) maximum wall temperature. The computer program proceeds along a representative coolant channel and at each channel position calculates three power densities: (1.) P_s = power density that will give the maximum stress, (2.) P_c = power density that assumes that the fuel centerline is at its maximum temperature, and (3.) P_w = power density that assumes that the channel wall is at its maximum temperature. The smallest of P_s , P_c , P_w is selected as the maximum power density, P , at each channel position before proceeding further along the channel.

The reactor core with the highest allowable average power density has the highest exit temperature and the shortest channel length. The exit temperature plus the given flow rate per channel determine thrust per channel so that for a given, fixed total thrust, the highest average

power density requires the least number of channels. The optimum power profile is radially uniform and the core radius is directly proportional to the square root of the total number of channels. Therefore, MAXPOW defines the most compact reactor core for a given total thrust and given flow rate per channel.

Calculations of the stress-limited power density, P_s , requires an inversion of the plane-strain relationships for cylindrical geometry. Calculation of the fuel centerline-limited power density, P_c , requires an inversion of the steady-state heat conduction equation. The calculation of the wall temperature-limited power density, P_w , and calculation of all the wall temperatures by MAXPOW requires that the turbulent hydrogen heat transfer correlation be known. A comparison can thus be made of the effect that correlational form has upon predicted optimum axial power profiles.

1.2.3 Effect of Correlational Form Upon Predicted Optimum Power Shapes

Four heat transfer correlations for turbulent hydrogen flow were examined. These four were selected primarily because of their association with either previous nuclear rocket studies or with the present NERVA nuclear rocket development program. The four correlations are:

$$(10) \quad Nu = (0.023) Re_f^{0.8} Pr_f^{0.4} (T/T_f)^{0.8} \quad (1.1)$$

$$(11) \quad Nu = (0.045) Re^{0.8} Pr^{0.4} (T/T_w)^{0.55} \left(1 + \frac{x}{D}\right)^{-0.15} \quad (1.2)$$

$$(4) \quad Nu = (0.021) Re^{0.8} Pr^{0.4} \left[\frac{T_w}{T} \right]^{-(0.29 + 0.0019 \frac{x}{D})} \quad (1.3)$$

$$(1) \quad Nu = (0.023) Re^{0.8} Pr^{0.4} \left[\frac{T_w}{T} \right] \exp - (0.57 - 1.59D/x) \quad (1.4)$$

where Nu = Nusselt number = $\frac{hD}{K}$,

h = heat transfer coefficient, $\frac{(\text{ft-lbf})}{\text{ft}^2\text{-sec-}^\circ\text{R}}$

D = channel hydraulic diameter, (ft)

K = thermal conductivity of the propellant, $\frac{(\text{ft-lbf})}{\text{ft-}^\circ\text{R}}$

Re = Reynolds number = $\frac{GD}{\mu}$,

G = flow rate per channel per unit flow area = $\frac{w}{(\pi D^2/4)}$, $\frac{\text{lbm/s/channel}}{\text{ft}^2}$

w = flow rate per channel, (lbm/s/channel)

μ = thermal viscosity of the propellant, $\frac{(\text{lbm})}{\text{sec-ft}}$

Pr = Prandtl number = $\frac{\mu c_p}{K}$

c_p = specific heat of the propellant, $\frac{(\text{ft-lbf})}{\text{lbm-}^\circ\text{R}}$

T_f = film temperature = $1/2 (T + T_w)$, ($^\circ\text{R}$)

T = bulk coolant temperature, ($^\circ\text{R}$)

T_w = channel wall temperature, ($^\circ\text{R}$)

x = distance from core inlet, (ft)

subscript f = evaluation at the film temperature, and all unsubscripted variables are evaluated at the bulk coolant temperature.

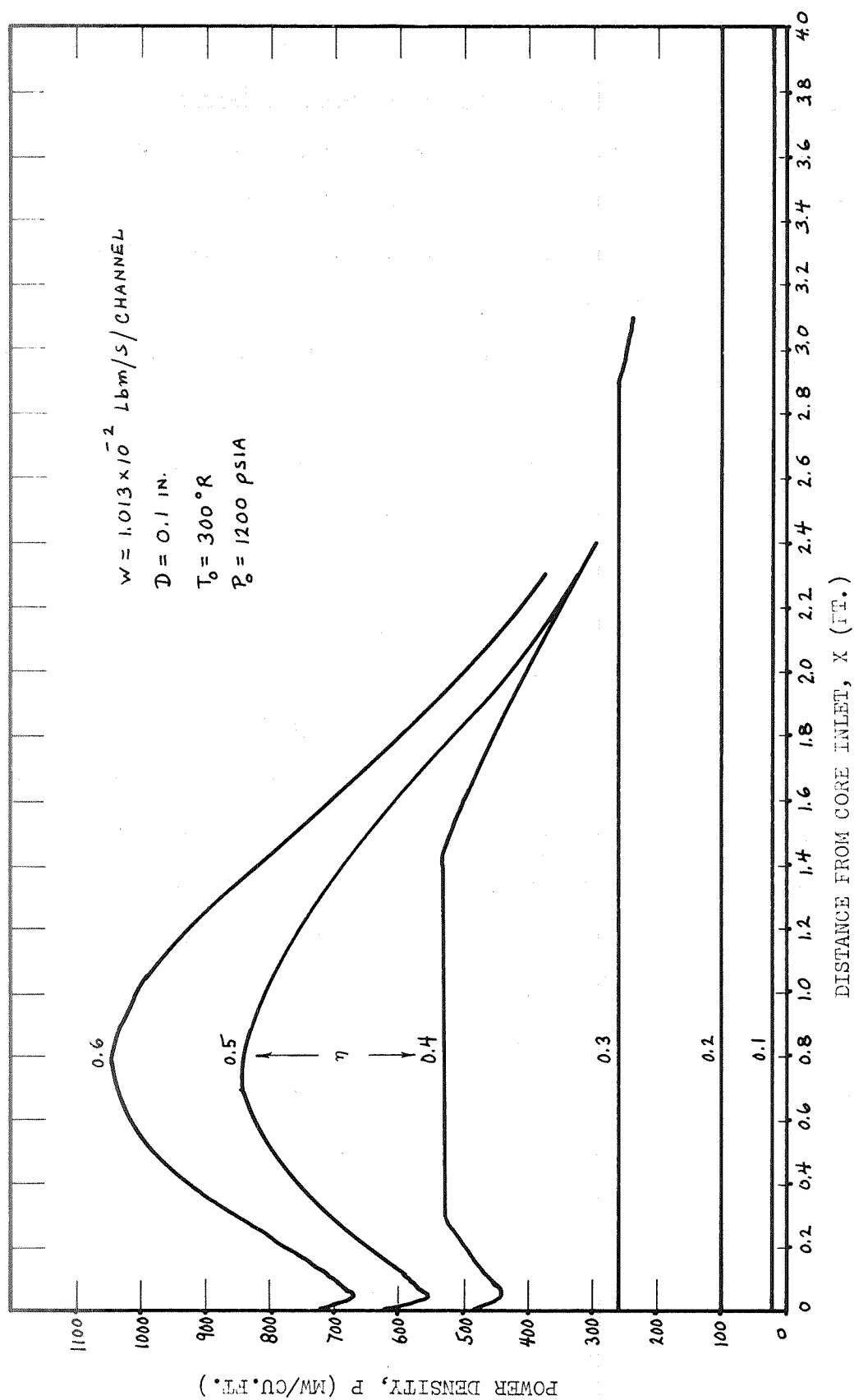
All hydrogen properties: μ , c_p , and K , are temperature dependent and are described by curve-fits of NASA data (9).

1.2.4 Heat Transfer Optimization Parametric Survey

Correlations (1.1), (1.2), and (1.3) all predict the anomalous wall temperature behavior while Correlation (1.4) does not. Extensive parametric surveys of Correlation (1.2), representative of the "anomaly" class and Correlation (1.4), the only representative of the "non-anomaly" class, were performed. The following nominal parameters were used:

- (a) $w = 1.013 \times 10^{-2}$ lbm/s/channel; reported as optimum for minimum reactor weight per unit flow area (10).
- (b) $F = \text{total thrust} = 250,000$ lbf; reported in an extensive nuclear mission optimization study (12, 13, 14).
- (c) $D = 0.1$ inches; also from (12, 13, 14).
- (d) $S = \text{maximum stress} = 5000$ psia (10).
- (e) $TFMAX = \text{maximum fuel centerline temperature} = 5500$ °R (10).
- and (f) $TWMAX = \text{maximum channel wall temperature} = 5000$ °R (10).

Figure 1-1 shows the optimum power profiles predicted by Correlation (1.2) as a function of the core void fraction, Figures 1-2 and 1-3 show the wall and fuel centerline temperature distributions associated with these optimum power profiles. The flat sections in the power shapes in Figure 1-1 are strictly stress-limited regions. Figures 1-2 and 1-3 must be compared to determine where $TFMAX$ and $TWMAX$ are dominant. The wall temperature anomaly in Figure 1-2 is reflected in the fuel centerline temperature distributions of Figure 1-3.


 FIGURE 1-1 OPTIMUM POWER PROFILES PREDICTED BY CORRELATION (1.2) AS A FUNCTION OF η

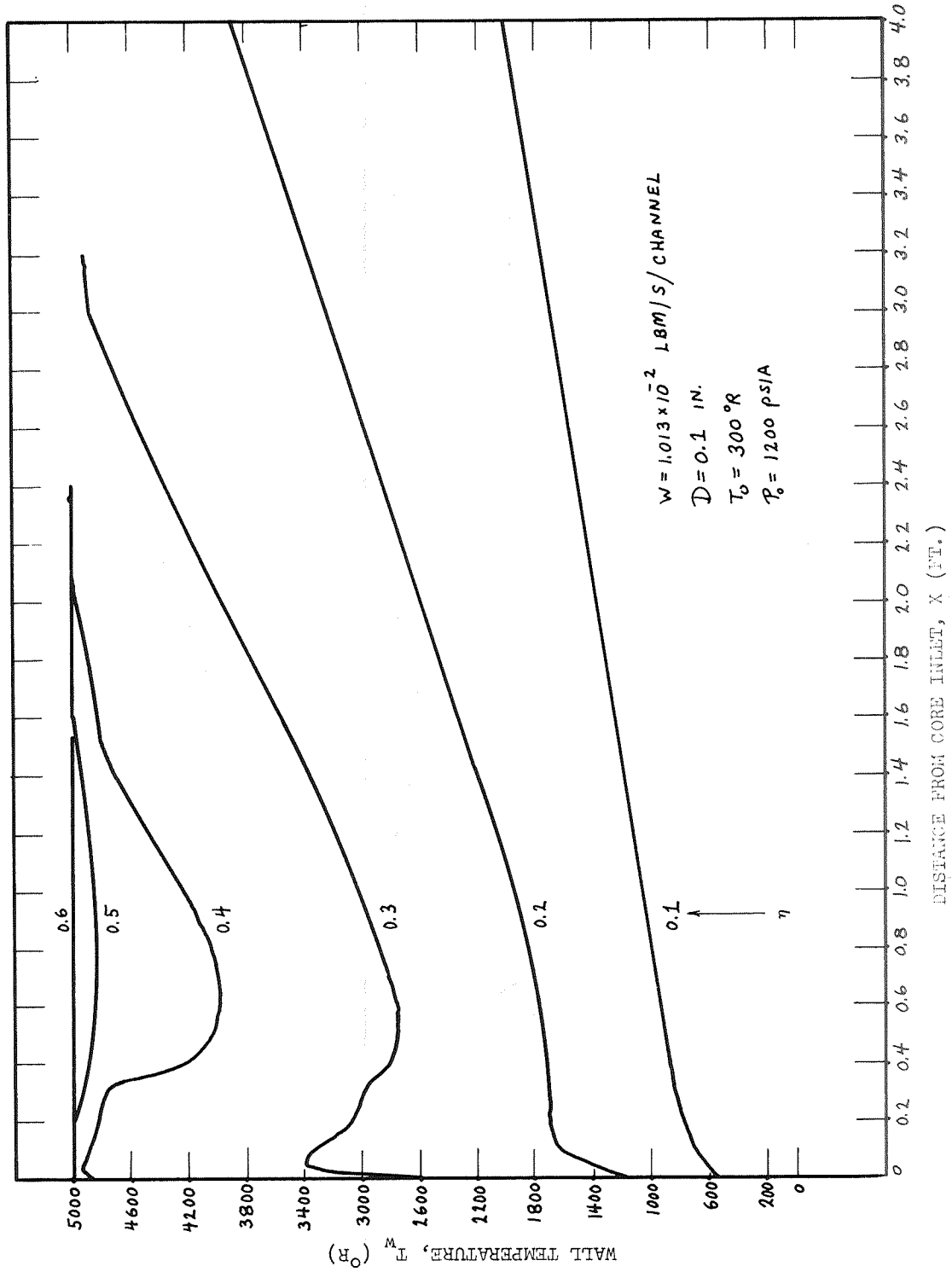


FIGURE 1-2 WALL TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (1.2)

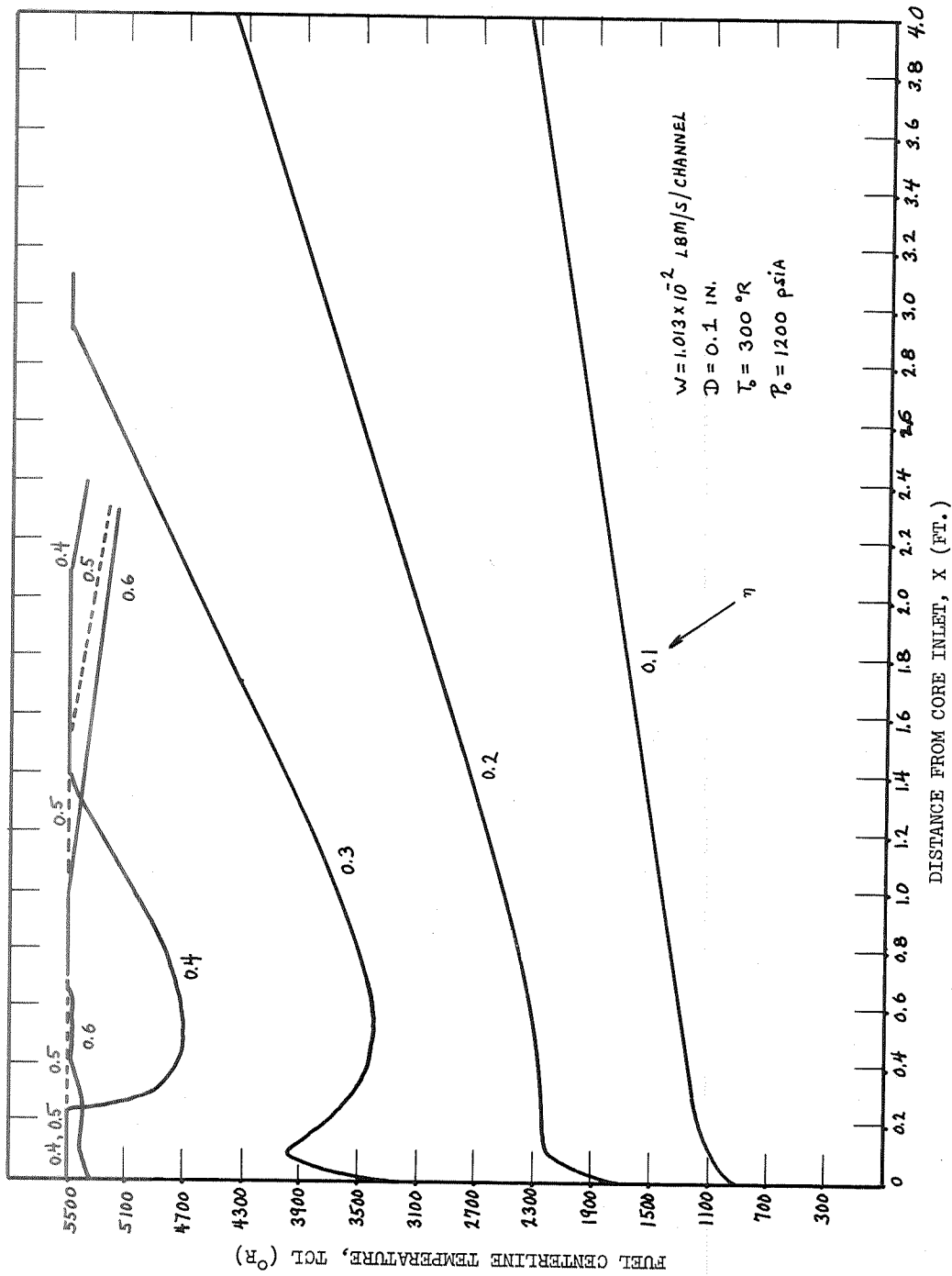


FIGURE 1-3 FUEL CENTERLINE TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (1.2)

Figure 1-4 shows the optimum power profiles predicted by Correlation (1.4). As indicated on this figure, the three principal limits always follow each other in the sequence: S, TFMAX, TWMAX, along each core coolant channel for this correlation only. Figure 1-5 shows the wall temperature distributions associated with these optimum power profiles. No wall temperature anomaly exists for this correlation.

Table 1-1 summarizes representative results of the parametric survey on Correlations (1.2) and (1.4), for one set of nominal parameters. In this table the following symbols are used:

D = channel hydraulic diameter, (in.)

P_o = core entrance hydrogen pressure, (psia)

w = channel flow rate, (lbm/s/channel)

T_o = core entrance hydrogen temperature, (°R)

F = total thrust level, (lbf)

η = void fraction = hydrogen free flow fraction

L_c = channel total length, (ft)

R_c = core radius, (ft)

I_{sp} = specific impulse, (sec.)

$\langle w \rangle$ = total flow rate, (lbm/s)

N = total number of core channels

P_{tot} = total core power, (MW)

δP_* = total core pressure drop, (psia)

and Anoz. = throat area of the converging-diverging rocket nozzle, (ft²).

Table 1-1

Parametric Survey, Optimized Reactor Characteristics*

*See preceding discussion for explanation of symbols.

$D = 0.1$ in., $P_o = 1200$ psia, $w = 1.013 \times 10^{-2}$ lbm/s/channel,

$T_o = 300^\circ\text{R}$, $F = 250,000$ lbf

Correlation (1.2):

η	Lc (ft)	Rc (ft)	Isp (sec)	$\langle w \rangle$ (lbm/s)	N	Ptot (MW)	δP^* (psia)	Anoz. (ft ²)
0.1	4.0	2.77	556	449	44,351	2209	187	0.91
0.2	4.0	1.66	773	323	31,928	3570	452	1.24
0.3	3.1	1.29	862	290	28,635	4204	502	1.32
0.4	2.4	1.10	887	282	27,828	4391	447	1.23
0.5	2.3	0.98	899	278	27,467	4480	463	1.26
0.6	2.3	0.89	902	277	27,363	4507	474	1.27

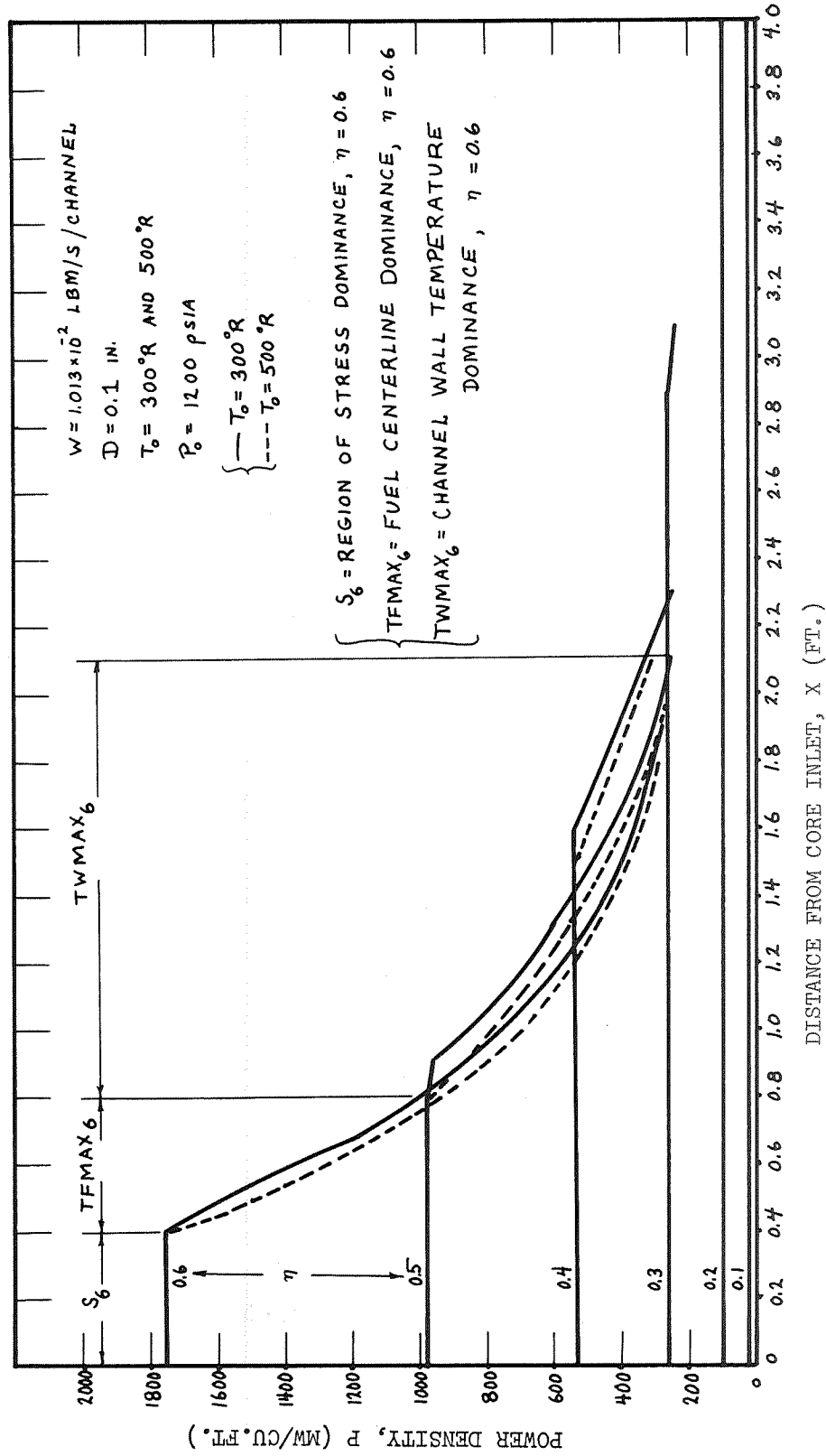
Correlation (1.4):

η	Lc (ft)	Rc (ft)	Isp (sec)	$\langle w \rangle$ (lbm/s)	N	Ptot (MW)	δP^* (psia)	Anoz. (ft ²)
0.1	4.0	2.77	556	449	44,350	2209	187	0.91
0.2	4.0	1.66	773	323	31,928	3570	453	1.24
0.3	3.1	1.29	862	290	28,638	4203	501	1.32
0.4	2.4	1.10	889	281	27,774	4403	456	1.24
0.5	2.1	0.97	903	277	27,323	4513	467	1.26
0.6	1.9	0.89	909	275	27,164	4547	460	1.25

1.2.5 Heat-Transfer Optimization Conclusions

The major conclusions of the heat transfer parametric surveys are:

- (1) The detailed power and temperature distributions for an optimum nuclear rocket reactor core are very dependent upon the particular mathematical form of the heat transfer correlation used.


 FIGURE 1-4 OPTIMUM POWER PROFILES PREDICTED BY CORRELATION (1.4) AS A FUNCTION OF η

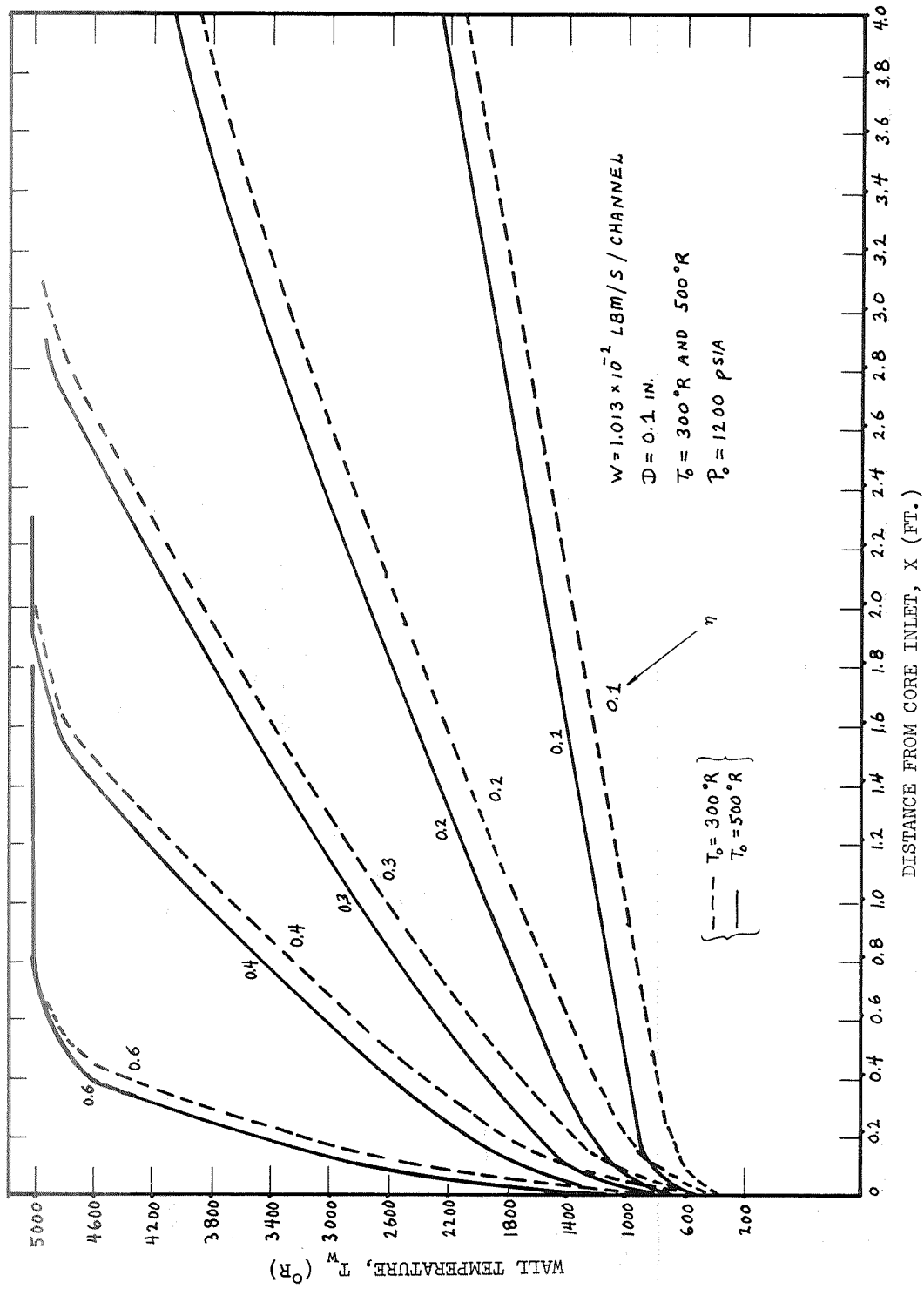


FIGURE 1-5 WALL TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (1.4)

This is especially true in the core inlet region where large power densities and low bulk coolant temperatures are predicted for nuclear rocket operation.

(2) Despite the sensitivity of local optimum reactor channel characteristics to the particular correlation used, the total length, radius, and specific impulse for an optimum nuclear rocket reactor core apparently depend only upon four gross engineering parameters:

(a) w , flow rate per channel; (b) F , total thrust; (c) D , channel hydraulic diameter; and (d) η , reactor core void fraction.

(3) For the nominal parameters cited in Section 1.2.4, the "best" case was found to be for $\eta = 0.6$. The following optimum characteristics for cases with $\eta = 0.6$ were derived and are shown in Table 1-2:

- (a) I_{sp} = ideal, vacuum specific impulse = 843-939 sec.
- (b) L_c = total reactor core channel length = 1.10-2.80 ft.
- (c) D_c = reactor core diameter = 1.78-1.84 feet.

(4) If a lower flow rate, $w = 1.013 \times 10^{-3}$ lbm/s/channel, or a larger hydraulic channel diameter, $D=0.2$ inches, are used, then the I_{sp} did increase to 954 sec., but "pancake" core configurations result. The "pancake" will have $L_c/D_c = 0.38$, typically.

(5) The selection of reactor cores with $\eta = 0.6$ appear to be favored by the results of this heat transfer analysis. The suitability of these "best" optimum power profiles depends heavily upon the actual magnitude of the temperature anomaly, if it does exist and

Table 1-2

"Best" Cases for the Parametric Heat-Transfer Optimization Survey

$\eta = 0.6$ for all the cases below and $F = 250,000$ lbf

Correlation (1.2):

$\frac{w}{(lbm/s \text{ channel})}$	$\frac{D}{(in.)}$	$\frac{Po}{(psia)}$	$\frac{To}{(^{\circ}R)}$	$\frac{Lc}{(ft)}$	$\frac{Rc}{(ft)}$	$\frac{\langle w \rangle}{(lbm/s)}$	$\frac{N}{}$	$\frac{P_{tot}}{(MW)}$	$\frac{\delta P^*}{(psia)}$	$\frac{Isp}{(sec)}$
1.013×10^{-2}	0.1	1200	500	2.1	0.89	277	27,368	4310	453	902
1.013×10^{-2}	0.1	1200	300	2.3	0.89	277	27,363	4507	474	902
1.013×10^{-3}	0.1	1200	400	2.2	2.74	262	258,824	4854	7.	954

Correlation (1.4):

1.013×10^{-3}	0.1	1400	400	2.1	2.74	262	258,739	4716	6.	954
1.013×10^{-2}	0.1	1400	400	2.8	0.87	266	26,277	4714	633	939
1.013×10^{-2}	0.2	1400	400	4.0	1.75	268	26,484	4661	24	932
1.013×10^{-2}	0.1	1200	400	1.9	0.89	275	27,128	4460	468	910
1.013×10^{-2}	0.1	1200	300	1.9	0.89	275	27,164	4547	460	909
1.013×10^{-2}	0.1	1200	500	1.8	0.89	276	27,248	4329	434	906
1.013×10^{-2}	0.1	1000	400	1.1	0.92	297	29,271	3929	283	843

if it is not due to a correlational defect. While a temperature anomaly has been found (4), the extrapolation of existing correlations must be regarded as tentative and unreliable until a turbulent hydrogen heat transfer correlation is reported which can experimentally confirm or deny the anomaly effect. Some of the correlations cover the range of wall and bulk coolant temperatures predicted for nuclear rocket operation, but not all combinations of these temperatures have been verified.

1.3 Nuclear Considerations

1.3.1 The Nuclear Reactor Model

Previous nuclear rocket reactor studies (9, 10) have demonstrated the reactivity importance of hydrogen in such reactor cores, but none have reported the reactivity effect of hydrogen being also present in the reflector. The compact nuclear reactors studied in this thesis have very fast spectra and hydrogen in the reflector of such reactors can have a significant reactivity contribution because of the importance of the relatively few neutrons which are moderated to thermal energies.

In order to calculate the hydrogen reactivity worth and neutron physics characteristics of a series of the optimum reactors studied in this thesis, the following nuclear reactor model was used:

(1) The reactor is reflected radially and on the reactor core inlet end by a beryllium reflector whose density was adjusted to allow the presence of hydrogen at steady-state conditions. The reflector thickness is chosen to be 6 inches (15.25 cm.), reported as optimum for minimum reactor core weight per unit flow area (33).

(2) The reactor core dimensions are determined by the heat-transfer optimization discussed above. These dimensions are based upon the nominal parameters cited in Section 1.2.4 and are in the ranges reported in conclusion (3) of Section 1.2.5.

(3) The reactors are uniformly-loaded with uranium -235. There is a uniform hydrogen distribution in the reflector and a non-uniform hydrogen distribution in the reactor core as calculated by MAXPOW for the heat-transfer optimization, discussed above.

(4) Criticality is assumed to occur with all hydrogen being present in the reactor. Therefore no extra uranium has been allowed for control and these calculated critical masses are lower than if the hydrogen moderation had not been utilized.

(5) All nuclear calculations are performed with the EXTERMINATOR-2 (38) multigroup, diffusion-theory code. The nuclear cross sections are taken from the 16-group LASL set (37).

1.3.2 Correlations of the Heat-Transfer Optimum Nuclear Reactor Characteristics

The very high leakage for these compact nuclear reactors makes it reasonable that all of the important characteristics can be correlated against geometrical buckling, B^2 , defined by Equation (1.5):

$$B^2 = \left(\frac{\pi}{L_c} \right)^2 + \left(\frac{2.405}{R_c} \right)^2, \quad [cm]^{-2} \quad (1.5)$$

Here L_c = channel length [cm] and R_c = core radius [cm] as calculated for the compact, heat-transfer optimum, nuclear rocket reactors.

Table 1-3 summarizes the cases investigated for the nuclear calculations. The only previously undefined symbol in this table is T_m = mean temperature $[\text{°R}]$ of the hydrogen in the reflector. The sequence A-D will illustrate the trade-off in I_{sp} and compactness (the reactor core size increases and I_{sp} decreases from A to D) for decreased critical mass requirement and increased total hydrogen reactivity worth. In particular the change in hydrogen specific worth ($\$/K_g H_2$) with B^2 is very well illustrated by the A-D sequence (shown in a following figure). Cases A, E and F are "best" cases with high I_{sp} . Case E will illustrate the effects of a decreased hydrogen density in the reflector in comparison to Case A. Case F is a "pancake" core resulting from a "low" flow rate one-tenth that of the other cases.

Table 1-4 summarizes the critical core characteristics for the nine reactor cases studied. The square cylindrical reactors, $L_c/2R_c$ approximately unity, have critical masses in the range 66 - 223 K_g . A decreased reflector density, reflector void fraction $\phi = 0.30$, incurs a +30% critical mass penalty with respect to the $\phi = +0.15$ reflectors, of approximately 50 - 63 K_g . The pancake core, Case F, has the largest critical mass calculated, 737 K_g . This pancakeness means increased fast leakage from the unreflected rocket-nozzle end and a large critical mass penalty. The majority of these critical masses are lower than that reported for the Phoebus - 2A reactor (46), which is 300 K_g . This Phoebus critical mass, however, is reported to have a large amount of reactivity tied up in niobium carbide which was not studied in this thesis.

Table 1-3

Reactor Cases Considered for Hydrogen Reactivity Worth Computations*

Case	A	B	C	D	E	F
w	1.013×10^{-2}	1.013×10^{-2}	1.013×10^{-2}	1.013×10^{-2}	1.013×10^{-2}	1.013×10^{-3}
(lbm/s/channel)						
D	0.1	0.1	0.1	0.1	0.1	0.1
(in.)						
η	0.6	0.5	0.4	0.3	0.6	0.6
P _o	1200	1200	1200	1200	1200	1400
(psia)						
T _o	300	300	300	300	500	400
(°R)						
T _m	170	170	170	170	387	220
(°R)						
L _c	1.9	2.1	2.4	3.1	1.8	2.2
(ft.)						
R _c	0.89	0.97	1.1	1.3	0.89	2.74
(ft.)						
Isp	909	903	889	862	906	954
(sec.)						
$\langle w \rangle$	275	277	281	290	276	262
(lbm/s.)						
P _{tot.}	4547	4513	4403	4203	4329	4854
(MW)						

34.

*Note: 1. Cases considered which have a beryllium reflector with 85% dense beryllium and 15% void fraction (hydrogen fraction) are designated A.15, B.15, etc.

2. Cases considered which have a beryllium reflector with 70% dense beryllium and 30% void fraction are designated A. 30, B.30, C.30.

Table 1-4
Critical Core Characteristics with All Hydrogen Present

Case	B^2	$[u]$	x_u	x_c	c/u	$L_c/2R_c$	M235	η
A.15	105×10^{-4}	3.70×10^{21}	0.076	0.324	7.5	1.07	197.1	0.6
B.15	87×10^{-4}	2.29×10^{21}	0.048	0.452	16.8	1.05	170.2	0.5
C.15	70×10^{-4}	1.27×10^{21}	0.026	0.574	38.7	1.09	129.7	0.4
D.15	47×10^{-4}	3.75×10^{21}	0.008	0.692	15.8	1.19	65.6	0.3
E.15	109×10^{-4}	4.36×10^{21}	0.091	0.309	5.9	1.00	223.3	0.6
F.15	32×10^{-4}	1.27×10^{21}	0.026	0.374	25.2	0.38	736.9	0.6
A.30	105×10^{-4}	4.72×10^{21}	0.097	0.303	5.5	1.07	253.5	0.6
B.30	87×10^{-4}	3.12×10^{21}	0.064	0.436	11.9	1.05	233.0	0.5
C.30	70×10^{-4}	1.80×10^{21}	0.037	0.563	26.7	1.09	180.0	0.4

B^2 = Geometrical buckling, (cm^{-2})

$[u]$ = Critical Uranium concentration, (atoms $\text{U-235}/\text{cm}^3$)

x_u = Volume fraction of uranium

x_c = Volume fraction of carbon

c/u = Critical carbon to uranium ratio

$L_c/2R_c$ = Length to diameter ratio of core

M235 = Critical mass of uranium, (Kg)

η = Core void fraction

Figure 1-6 shows critical uranium concentrations plotted against B^2 . As expected, critical uranium concentration increases with B^2 because the relative leakage increases as the reactor volume decreases. It is interesting to note that for B^2 greater than $70 \times 10^{-4} \text{ cm}^{-2}$ both critical uranium trends, characterized by the value of the reflector void fraction, exhibit an almost linear dependence upon B^2 . A probable explanation for this behavior is that the high energy neutrons, which are responsible for the majority of fissions and leakage, do not "see" the beryllium reflector in the sense that significant moderation of these high energy neutrons does not take place in the reflector. The core flux distributions for these high energy groups therefore have similar spatial shapes, or bucklings, and the linear behavior of critical concentrations with B^2 results just as for unreflected, one-group reactors (39).

Figure 1-7 shows the total reactivity worth for the reflector hydrogen, Δk_{tr} , and core hydrogen Δk_{tc} , correlated against B^2 . This figure demonstrates that the total reflector hydrogen worth can be more than three times the core hydrogen contribution. For example, case A.30 has $\Delta k_{tr} = 7.70\%$ and $\Delta k_{tc} = 2.53\%$, one $\% = 0.0064 \frac{\Delta k}{k}$. This occurs because of the greater average density of hydrogen in the reflector. Also for the reflector thickness assumed, the reflector volume is a sizeable fraction of the core volume and the steady-state number of hydrogen atoms in the reflector is greater than the total number in the core. The total Δk contributed by

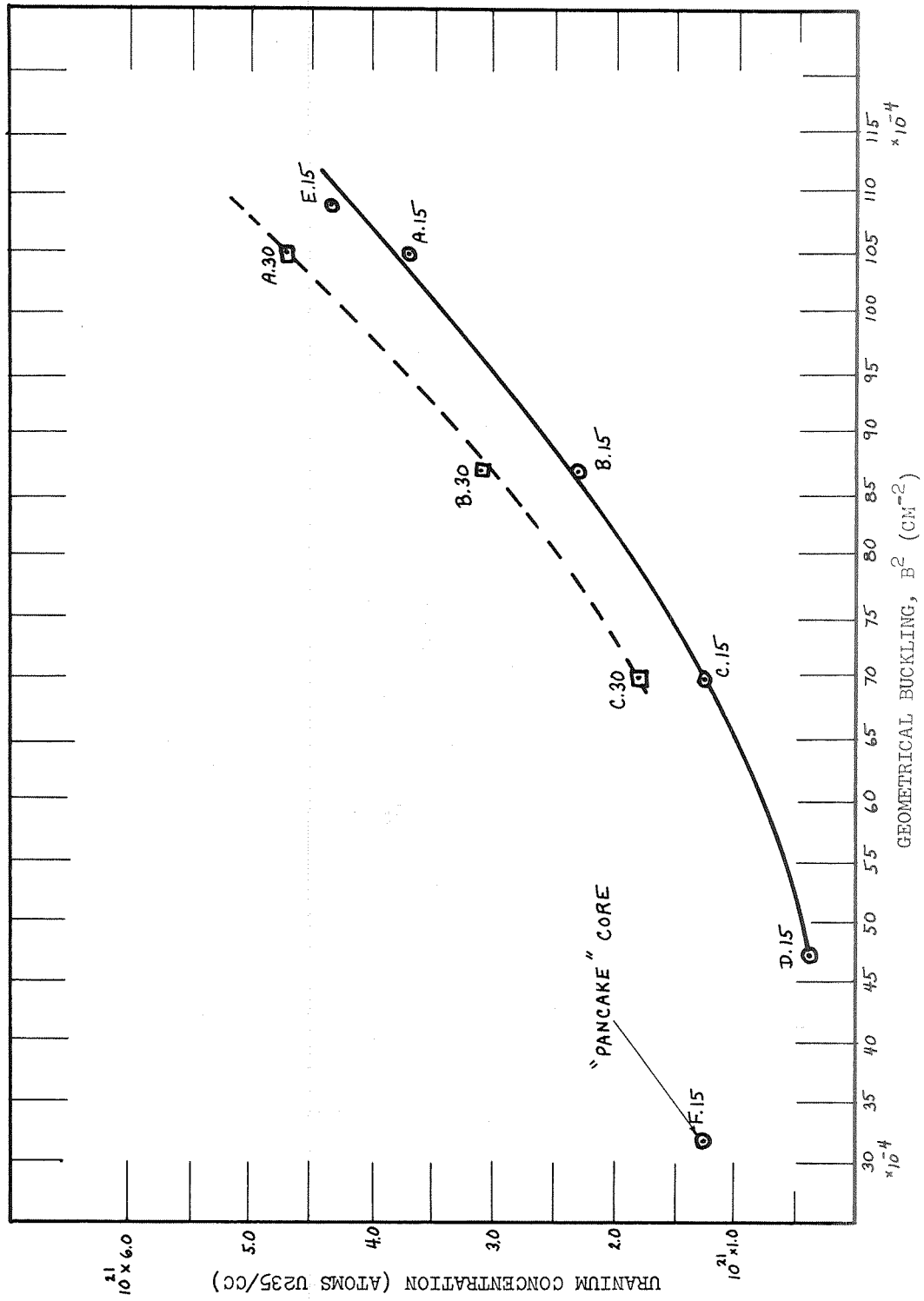


FIGURE 1-6 CRITICAL URANIUM CONCENTRATIONS, ALL HYDROGEN PRESENT, VERSUS GEOMETRICAL BUCKLING

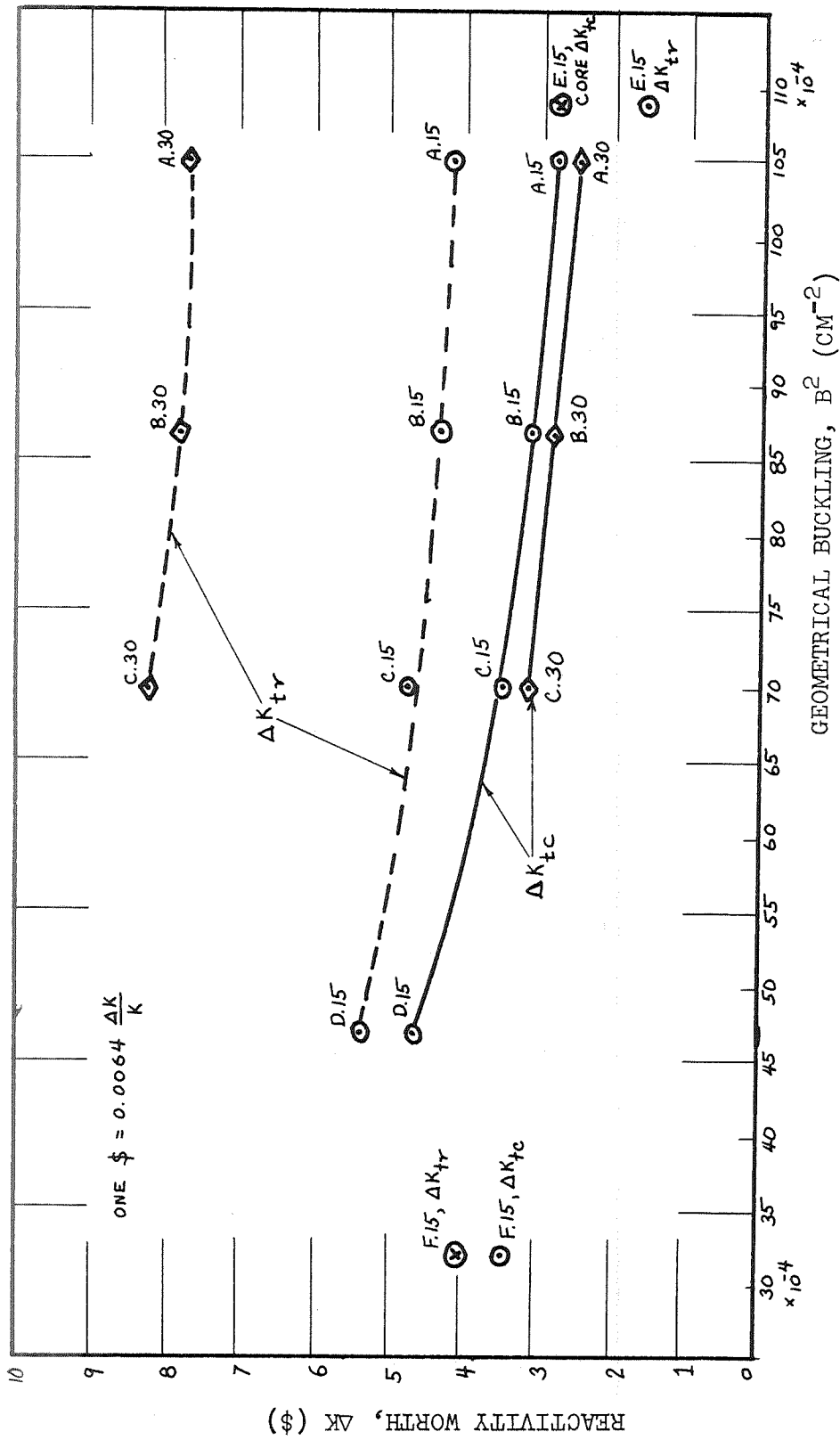
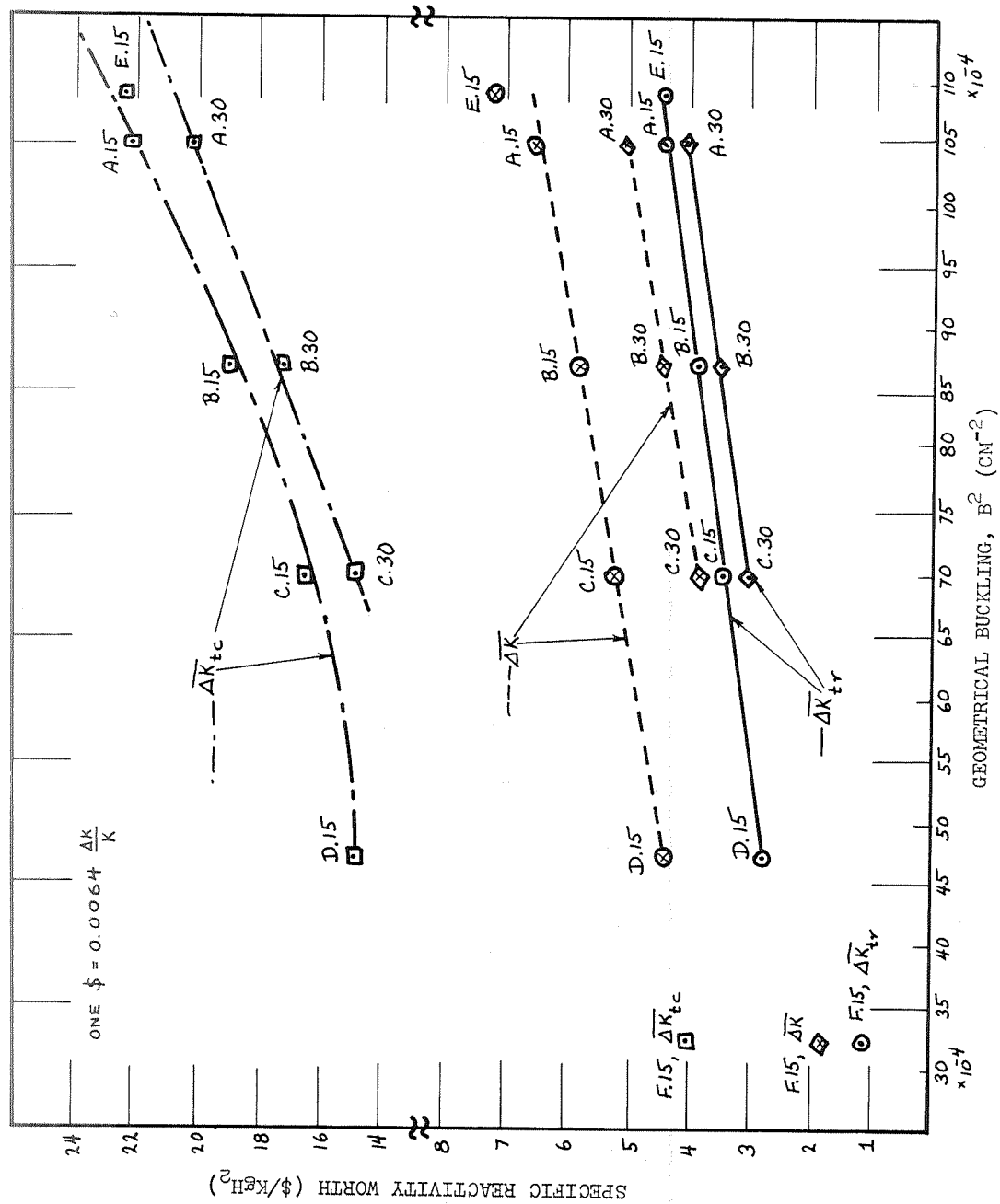


FIGURE 1-7 CORE, ΔK_{tc} , AND REFLECTOR ΔK_{tr} , HYDROGEN REACTIVITY WORTHS VERSUS GEOMETRICAL BUCKLING

hydrogen for the $\phi = 0.15$ reactors ranges from 6.95\$ for A.15 to 9.95\$ for D.15. Case E.15 has $\Delta k = 4.15\$$ because of the decreased reflector hydrogen density. The $\phi = 0.30$ reactor cases have Δk ranging from 10.23\$ for A.30 to 11.26\$ for C.30. All of the core values for Δk_{tc} for all nine cases are in the range 2.36\$ to 4.63\$. The reported value of the core hydrogen reactivity worth for Phoebus-2A is 3.30\$ (46), for Kiwi is 2\$ (48), and for the NRX-A2 (1120 MW, 71 lbm/s) (47) is 1.73\$. However, the value of delayed neutron fraction for these reactors is reported by Mills (48) to be computed as one \$ = 0.0074 $\frac{\Delta k}{k}$, so that the values reported in this thesis should be lowered by a factor of 0.865 for comparison. No specific value for reflector hydrogen worth has been reported in the literature.

Figure 1-7 indicates that there is a definite trend of decreasing total hydrogen reactivity contribution as B^2 increases. However, there are actually different amounts of hydrogen present from case to case and the trend in Figure 1-7 only shows the effect of a decreasing amount of total hydrogen as B^2 increases. The smaller cores with large B^2 have smaller total hydrogen masses.

Figure 1-8 shows the normalized reactivity contributions for reflector hydrogen, $\overline{\Delta k_{tr}}$, and core hydrogen $\overline{\Delta k_{tc}}$. The specific hydrogen worth, as expressed in terms of (\$/KgH₂), exhibits the opposite trends shown in Figure 1-7. From Figure 1-8 one can see that the effectiveness of a unit mass of hydrogen increases as B^2 increases. The relationship displayed by $\overline{\Delta k_{tr}}$ and $\overline{\Delta k}$ enabled the following straight-line curve-fits to be calculated:

FIGURE 1-8 SPECIFIC HYDROGEN WORTH, ΔK , ΔK_{tc} , AND ΔK_{tr} , (\$/kgH₂) VERSUS GEOMETRICAL BUCKLING

$$\overline{\Delta k_{tr}} = 1.413 + 2.845 \times 10^2 B^2 \quad ; \quad \phi = 0.15 \quad (1.6)$$

$$\overline{\Delta k} = 2.758 + 3.545 \times 10^2 B^2 \quad ; \quad \phi = 0.15 \quad (1.7)$$

$$\overline{\Delta k_{tr}} = 0.900 + 3.000 \times 10^2 B^2 \quad ; \quad \phi = 0.30 \quad (1.8)$$

$$\overline{\Delta k} = 1.370 + 3.514 \times 10^2 B^2 \quad ; \quad \phi = 0.30 \quad (1.9)$$

where $\overline{\Delta k_{tr}} = \frac{\Delta k \text{ (reflector hydrogen)}}{\text{Total reflector hydrogen}}, (\$/\text{KgH}_2)$

$$\overline{\Delta k} = \frac{\Delta k \text{ (total hydrogen)}}{\text{Total hydrogen}}, (\$/\text{KgH}_2)$$

and ϕ = reflector void fraction. Mills (48) reports that the average worth of hydrogen in the Kiwi nuclear reactor is worth approximately $5\$/\text{KgH}_2$ (one $\$ = 0.0074 \frac{\Delta k}{k}$) which compares favorably with the values of $\frac{\Delta k}{k}$ shown in Figure 1-8 which are in the range 3.83 ($\$/\text{KgH}_2$) to 7.16 ($\$/\text{KgH}_2$) (one $\$ = 0.0064 \frac{\Delta k}{k}$).

Figure 1-9 shows that the mean leakage energies, $\overline{E_L}$, and mean energy of neutrons causing fission, $\overline{E_p}$, defined below, increase as B^2 increases so that Figure 1-8 apparently shows that hydrogen increases in reactivity worth as the average energy of the system increases. This is puzzling at first sight because the total scattering cross section for hydrogen decreases as the average neutron energy increases (37). However, Figure 1-6 shows that the uranium concentration increases with B^2 and comparison with the curves for $\overline{\Delta k_{tc}}$ in

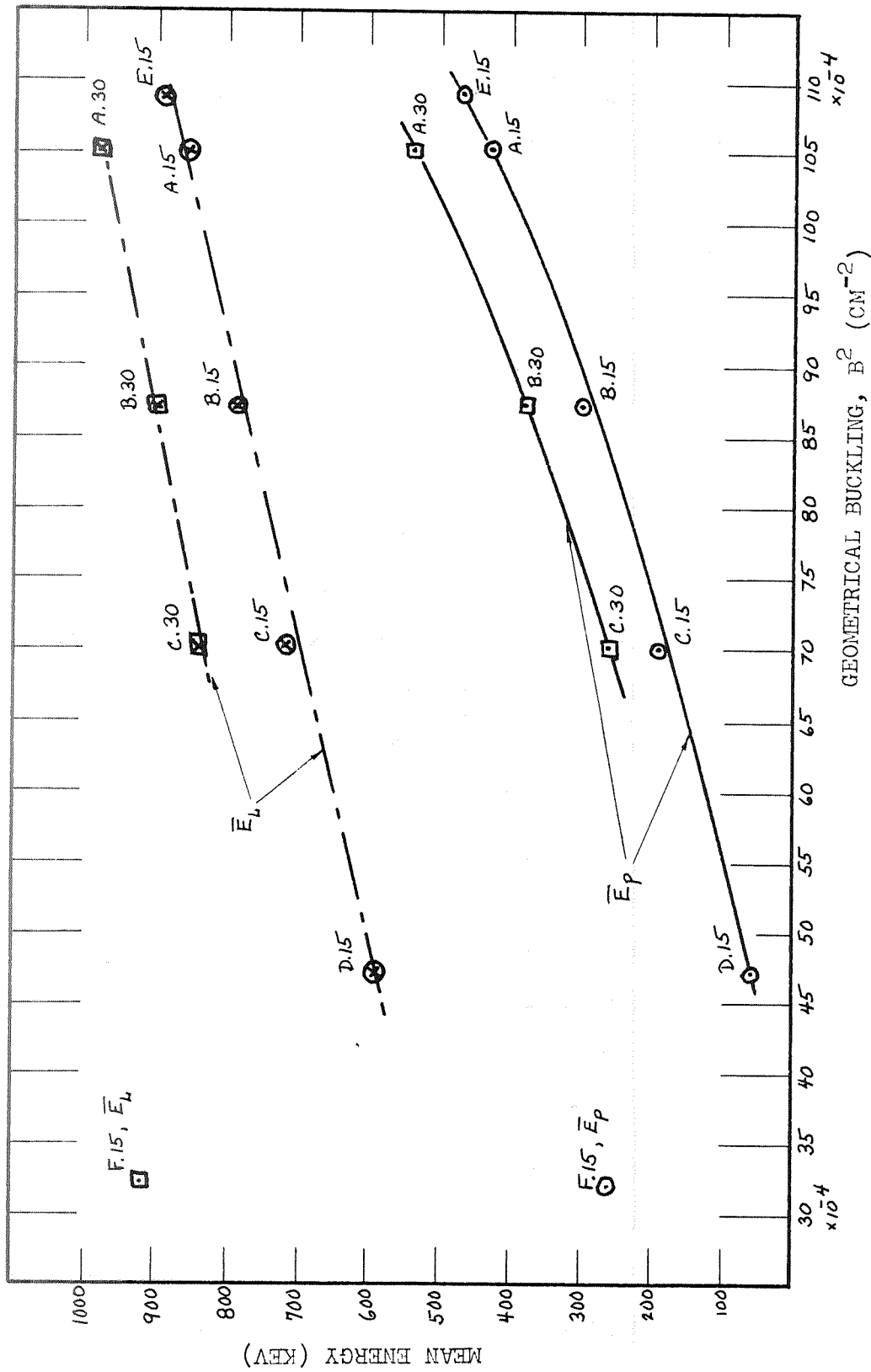


FIGURE 1-9 MEAN LEAKAGE AND NEUTRON PRODUCTION ENERGIES VERSUS GEOMETRICAL BUCKLING

Figure 1-8 shows a definite similarity in the shapes of critical uranium versus B^2 and $\overline{\Delta k_{tc}}$ versus B^2 . This is probably not a coincidence. Lamarsh (39) reports that hydrogen increases in moderating effectiveness in the presence of uranium. Uranium has inelastic scattering which allows hydrogen to "see" an increased available flux due to fast neutrons which are inelastically downscattered into an energy range for which hydrogen is very effective for further moderation. It is therefore concluded that the trends in Figure 1-6 are the result of uranium inelastic scattering and the increase in critical uranium concentration with increasing B^2 .

The definitions of $\overline{E}_L$ and $\overline{E}_p$ in Figure 1-9 are:

$$\overline{E}_L = \frac{\sum_i \overline{E}_i L_i}{\sum_i L_i}, \text{ (Kev)} \quad (1.10)$$

$$\overline{E}_p = \frac{\sum_i \overline{E}_i P_i}{\sum_i P_i}, \text{ (Kev)} \quad (1.11)$$

where $\overline{E}_i$ = average energy of neutrons in group i, taken from the LASL cross section set (37),

L_i = total number of neutrons leaking out in energy group i, $\left[\frac{(\text{neutrons})}{\text{sec}} \right]$ and P_i = total number of neutrons produced by fission in energy group i, $\left[\frac{(\text{neutrons})}{\text{sec}} \right]$.

Figure 1-10 shows the results of correlation of total reflector hydrogen density versus total reflector hydrogen reactivity worth for cases A.15 and A.30. Over the range from 25% to 100% of steady-state hydrogen concentration, one can conclude that the total hydrogen worth in the reflector is directly proportional to the average hydrogen concentration in the reflector. Little (9) found that the total reactivity contribution for average amounts of hydrogen in the core of nuclear rocket reactors, of the same composition as the ones studied in this thesis, also has a linear relationship to total reactivity worth. This would suggest that similar physical processes are producing analogous results. The physical processes in this case are due to fast neutron down-scattering by hydrogen in the presence of uranium.

In addition to hydrogen reactivity characteristics for these compact cores, several calculations were made to estimate the reactivity losses caused by core thermal expansion and loss of carbon by hydrogen corrosion. For a thermal expansion of the reactor core corresponding to $\Delta T = 5000^\circ R$ a total $\Delta k = -0.78\%$ to -1.45% was calculated.

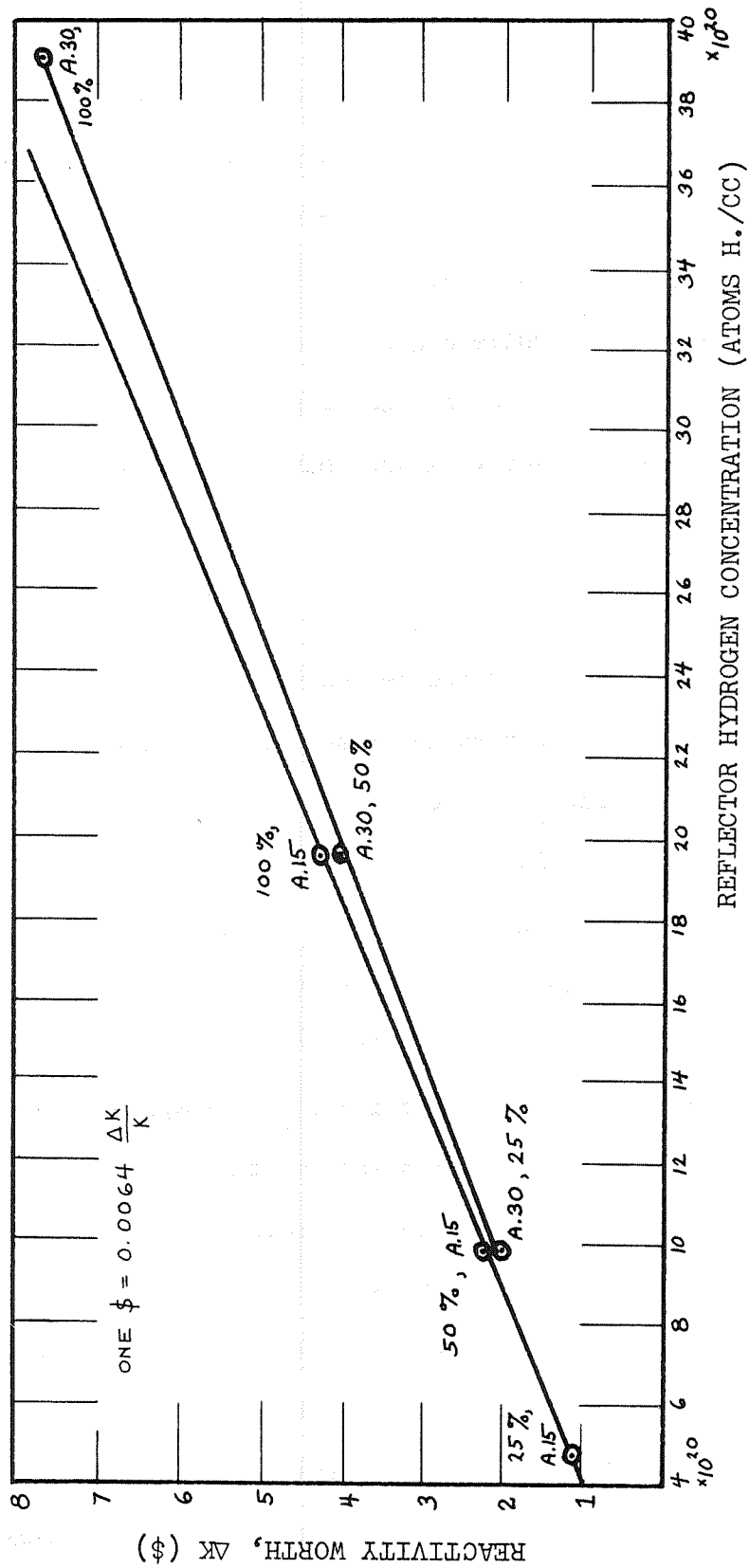


FIGURE 1-10 REFLECTOR HYDROGEN REACTIVITY WORTH VERSUS CONCENTRATION OF HYDROGEN IN THE REFLECTOR

This value compares very well to values cited in the literature for the Rover nuclear rocket reactors (46, 47, 48). Carbon deletion from the reactor core channels was calculated to be worth -0.2161 ($\$/K_gC$) to -0.3150 ($\$/K_gC$). If the hydrogen corrosion rate for unprotected graphite (49) is assumed, unacceptably high rates of reactivity loss will occur, typically a rate of $-70\$/min$ would result. An estimate for acceptable corrosion rates showed that coating materials must reduce hydrogen-caused carbon losses to approximately 0.10% of the unprotected rate.

1.4 Conclusions

From the nuclear calculations it can be concluded that:

- (1) For such compact nuclear rocket reactors, the reactivity worth of hydrogen in the reflector can be significantly greater than the reactivity worth of hydrogen in the core.
- (2) The total worth of reflector hydrogen is directly proportional to the average hydrogen concentration in the reflector.
- (3) The very definite correlations with B^2 that have been demonstrated for critical uranium concentration and hydrogen reactivity worths could be very useful in future optimization studies where an accurate estimate of critical mass and total hydrogen worth could be had without extensive multigroup calculations.
- (4) For the reactors in this thesis, the smallest reactor cores have the highest I_{sp} , highest critical mass, and lowest total hydrogen worth. Pancake cores suffer from two serious disadvantages: (a) greatly increased core radius and (b) greatly increased critical mass.

Increasing the reflector void fraction for a given size reactor core increases (a) critical mass, and (b) total hydrogen worth.

The overall conclusions for this study can be summarized as:

(1) Compact nuclear rocket reactors can significantly reduce shielding weight requirements while yielding very high performance (Isp). However, better heat-transfer correlations are needed for an accurate determination of non-uniform uranium distributions needed to match desired power shapes.

(2) The optimum nuclear reactor cores should have a length-to-diameter ratio near unity and should have core void fractions near 0.6 because of the Isp advantage. Large reflector void fractions should be avoided.

(3) Further work should better define the reflector hydrogen distribution since a major share of the total hydrogen reactivity worth can be due to hydrogen there. Further studies should be made to see if the trends demonstrated for this simplified nuclear model will still hold for a more practical reactor that has rotating poison plates in the reflector (27) for control and which has strongly absorbing structural material in the core; for instance, niobium (47) as in the Phoebus reactor core.

2. Introduction and Problem Statement.

2.1 Nuclear Propulsion.

The United States is currently developing a hydrogen-propelled, solid-core, graphite-moderated nuclear rocket for such intermediate energy missions as a lunar transfer shuttle in the late 1970's and for high energy, manned space missions such as a Mars stopover or fly-by in the middle 1980's (16, 24). The nuclear rocket is potentially several times more efficient than present hydrogen-oxygen or projected hydrogen-fluorine chemical systems for these space missions. Propulsion efficiency is customarily expressed in terms of the specific impulse, I_{sp} , which is simply the rocket exit velocity, v_e [ft/sec], divided by the acceleration of gravity at the earth's surface, $g_0 = 32.2$ ft/sec:

$$I_{sp} = \frac{v_e}{g_0} \quad [\text{seconds}] \quad (2.1)$$

A momentum balance on any rocket will yield a statement about the total masses before and after an incremental change in velocity:

$$\frac{m_b}{m_0} = - \left(\frac{\Delta v_c}{v_e} \right). \quad (2.2)$$

where m_b = total mass [lbm] remaining after the rocket changes its velocity by thrusting propellant; this includes nuclear reactor engine, thrust structure, empty propellant tanks, and any residual propellant plus scientific payload;

m_0 = total mass [lbm] of entire rocket system before changing velocity and is equal to m_b plus the propellant expended to achieve the required velocity change;

and Δv_c = "characteristic" velocity increment [ft/sec] which is the sum of the actual velocity increment achieved, Δv [ft/sec], plus the loss in potentially achievable velocity increment due to thrusting in a gravitational field, defined in Equation (2.4) below.

This "rocket equation", as it is called, sometimes is written in the following manner to emphasize the important dependence of the characteristic velocity upon exit velocity and burn-out ratio:

$$\Delta v_c = v_e \ln \left[m_0 / m_B \right] \quad (2.3)$$

This characteristic velocity is the sum of the actual velocity increment and the gravity loss incurred for a finite burn time in a gravitational field (11):

$$\Delta v = v_e \ln \left[m_0 / m_B \right] - \overline{g \sin \theta} t_B \quad (2.4)$$

where $\overline{g \sin \theta}$ = average value of the gravitational acceleration [ft/sec] opposing the motion of a rocket thrusting at an angle to the direction of the normal from the surface of the Earth,

and t_B = total time of thrust or "burn time" [seconds]. Since the nuclear rocket, as presently conceived, will be assembled in a 100 nautical mile earth orbit or will be the upper stage of a Saturn V - type rocket (16), a nominal value of $\overline{g \sin \theta}$ is 5 ft/sec² for these situations (11, p. 104) and using a maximum burn time of 1800 seconds for the nuclear engine (13, 14) one can see that gravity losses can be very important for high energy missions; for instance the minimum required Δv for a Mars soft-landing is 31,000 ft/sec (11, p. 49) and the direct-injection gravity loss 9,000 ft/sec would substantially increase the characteristic velocity. For mission planners, the gravity loss is a penalty incurred for direct-injection and the required Δv increment is a variable determined by the selection of a transfer free-flight trajectory considering the time of arrival at the planet or target and the inbound-and outbound-leg trip times (15). The 31,000 ft/sec cited

above is for a minimum-energy transfer ellipse called a Hohmann transfer, (15, p. 100) which requires a transfer time of about 9 months (15, p. 137). Faster transfers would require larger ΔV 's. Because the characteristic ΔV_e is large for manned planetary missions, the mission planner needs a rocket system that can deliver as high an exit velocity as possible so as to decrease the amount of propellant needed and therefore increase the weight of scientific and support payload that can be carried. The larger the exit velocity, the more efficiently the system utilizes propellant mass.

An energy balance across the converging-diverging nozzle of any rocket, assuming adiabatic and isentropic flow, yields the dependence of I_{sp} ($g_0^{-1} v_e$) on chamber temperature, T_c [°Rankine] and propellant molecular weight, M [lbm - mole] (25):

$$I_{sp} = \sqrt{\frac{2\gamma R_u}{(\gamma-1)g_0} \left(\frac{T_c}{M} \right)} \eta_c \quad (2.5)$$

where γ = ratio of specific heats $c_p/c_v = 1.35$ for hydrogen,

R_u = universal gas constant = $1545.33 \frac{\text{ft-lbf}}{\text{lbm-mole-}^\circ\text{R}}$,

and η_c = ideal nozzle efficiency.

The ideal nozzle efficiency, η_c , will be taken to be unity in this thesis so that the specific impulse should be referred to as the vacuum specific impulse. Explicitly, $\eta_c = 1 - (P_e / P_o)^{\frac{\gamma-1}{\gamma}}$ and a current practical value quoted for the inverse pressure ratio is 140:1 (12,32) which would yield $\eta_c = 0.7223$ and $\sqrt{\eta_c} = 0.85$. The chemical constituents which in chemically-powered rockets react to generate the heat also form the reaction products, which then becomes the propellant. If hydrogen and oxygen burn to release energy, they form H_2O with molecular weight 18. The nuclear rocket's advantage is that the nuclear reactor provides the

heat while the propellant may be selected independently to be hydrogen with molecular weight 2. If chamber temperatures are equal, this gives a factor of $\sqrt{\frac{18}{2}} = 3$ improvement in Isp. The solid-core nuclear rocket is limited by material temperatures to potential chamber temperatures lower than those achievable by chemical rockets so that the advantage in Isp is not in the exact ratio of propellant molecular weights; relatively the attainable Isp's are 800 - 900 seconds for solid-core nuclear, versus 400 - 500 seconds for chemical (2). The effect of this Isp improvement upon deliverable payload is very mission dependent. However, for most missions presently conceived this means a doubling of effective payload. In terms of a lunar landing for which a Saturn V permits a stay time of one day, the nuclear rocket replacing the third stage of the Saturn V would allow a stay time of 100 days. (2,16). For a manned Mars landing, the use of all-nuclear versus all-chemical propulsion can result in the saving of millions of pounds for a mission initially assembled in Earth orbit (12).

2.2 The Nuclear Rocket Program

Since 1950 the United States has been conducting a research and development program to utilize nuclear power for space propulsion, called Project Rover (24). The primary system concept is to use the nuclear reactor core as a straight, one-pass (open cycle) heat exchanger to heat hydrogen propellant to very high temperatures. Both metallic and graphite reactor cores were considered and graphite fuel elements containing uranium - 235 were decided upon as being most favorable for early development because graphite had long been used as a high temperature material. The strong, unfavorable reaction of graphite with hot

hydrogen has been reduced to tolerable corrosion levels by the development of refractory coating materials such as niobium carbide (26,27). Special hydrogen turbo-pumps have been developed and optimized for the nuclear rocket (36). A schematic picture of the nuclear rocket system is presented in Figure 2-1. Hydrogen leaves the pump and enters the rocket engine system via the regeneratively cooled nozzle, cools the beryllium reflector and internal shield, enters the reactor core where it is heated, and then is accelerated through the converging-diverging nozzle. Power for the turbine to drive the turbo-pump comes from hot hydrogen at the nozzle inlet chamber in a hot-bleed-cycle system. Thermal energy stored in the system starts the pump initially in a bootstrap operation (26).

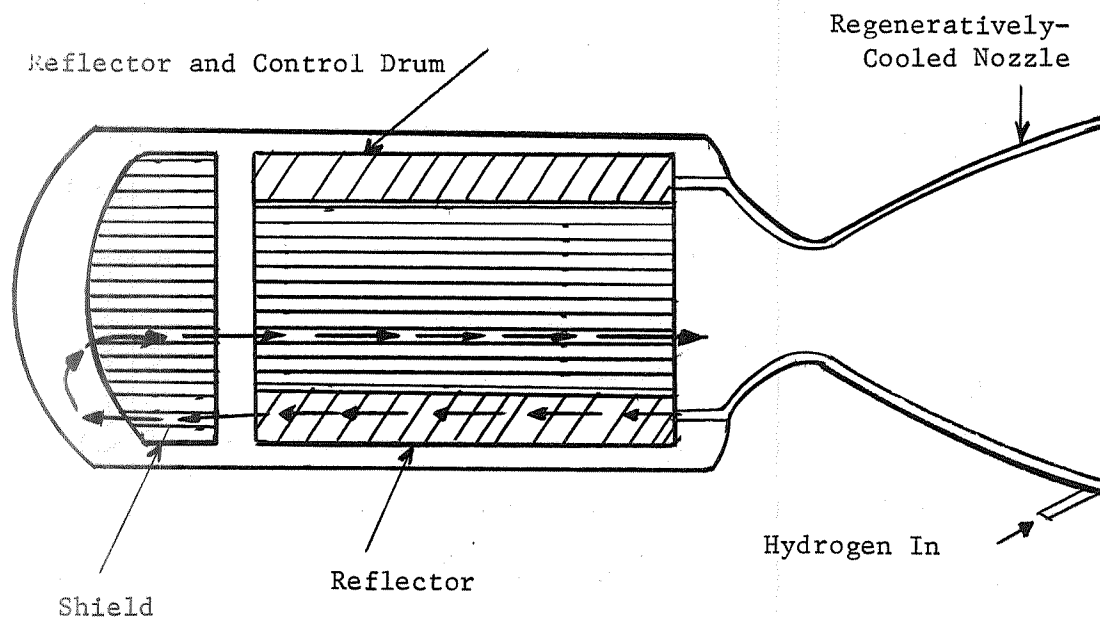


Figure 2-1 NERVA Rocket Engine
from Reference (26).

Hydrogen which had been used to cool the core support structure formerly was exhausted into the nozzle inlet chamber where its cooler temperature diluted the chamber temperature and reduced the specific impulse. Now a separate coolant circuit for the core support system allows an increase in the Isp of about 25 seconds at any chamber temperature (27).

Project Rover has successively designed and developed three series of nuclear reactors: Kiwi, NERVA, and Phoebus. Kiwi is the name of a flightless Australian bird and correspondingly designates a series of prototype reactors never to be flown, and designed to prove the feasibility of a hydrogen-cooled, solid-core, graphite-moderated nuclear reactor. Testing began in 1959 and ended in 1964 with the Kiwi B4D reactor as the successful prototype for the succeeding NERVA and Phoebus reactor series. NERVA, an acronym for Nuclear Engine for Rocket Vehicle Applications, designates an 1100 megawatt pioneer reactor series whose chief purpose is to advance nuclear propulsion technology. Phoebus designates the 5,000 megawatt, higher power density reactor series intended originally to be developed into a 200,000 lbf-thrust nuclear rocket. Variable uranium content and inlet hydrogen jets to vary the flow rate from fuel element to fuel element were to have been tested and used in Phoebus. Details of these innovations as well as the flow pattern of fuel element geometry are classified (24). Because plans for planetary manned missions have been postponed indefinitely, emphasis has shifted to developing a 75,000 lbf-thrust lunar shuttle rocket resulting in a nuclear flight model by 1975 (26,16). In line with this shift in emphasis, a 400 MW scaled-down reactor named Pewee-I has been designed to test the advanced fuel elements in simulated NERVA and Phoebus reactor environments and became operational at the end of 1968 (26,34).

Before itemizing the progress that has been made to date in the nuclear rocket program, one should be cautioned that many of the following accomplishments are often reported simply as technical achievements with little supplementary data appearing in the open literature. This seems to be especially true of technical features in the nuclear core design such as fuel element innovations and methods of protective coating to reduce hydrogen corrosion.

With the completion of the reactor portion of the NERVA technology program at the end of 1967, the development of a reactor with a sufficient lifetime for extended missions has been accomplished. These following key feasibility unknowns have been resolved:

(1.) reactor structural integrity and reliability, (2.) stable control, (3.) capability of multiple full-power restart, (4.) mapping of hot-bleed-cycle engine performance characteristics, (5.) full power operation at 1500 MW for 30 minutes and with exit temperatures above 4,100°R, (6.) development of alternative bootstrap startup methods, and (7.) analytical predictability of reactor performance (29).

Continued testing of NERVA and Phoebus reactors have yielded these design achievements: (1.) separate coolant circuit for the core support system, (2.) reflector system in which only poison plates rotate for neutron control, (3.) high power density fuel elements, (4.) new high performance Hastelloy-X nozzle, (5.) dual turbopump feed system capable of delivering hydrogen at 285 lbm/sec. and 1,300 psia, and (6.) significant progress in zirconium-uranium-carbide/carbon composite fuels which should have longer operating lifetimes than graphite-based fuels (27).

Materials research for the NERVA program has been concerned with the special environments uniquely associated with a nuclear rocket; i.e., (a.) nuclear and solar radiation, (b.) deep space vacuum, (c.) corrosion, (d.) very high and very low (cryogenic) temperatures, and (e.) intense thermal gradients. This research has led to the announcement of the publication of a NERVA Materials Properties Data Book in order to standardize properties of materials used in the nuclear rocket program (35).

2.3 Previous Analytical Heat Transfer and Nuclear Studies.

Because of the very large improvement in specific impulse to be gained by using nuclear propulsion, there is a need to determine the possible maximum performance levels for nuclear rocket systems in order that nuclear mission capabilities be clearly defined and to enable the recognition of optimum rocket system characteristics which should be developed. Historically most of the available studies analyzing nuclear rocket systems have appeared somewhat independently of the technical development of the Rover program as outlined in Section 2.2 because few details of the physics and important materials limitations have been available as they pertain to the actual Kiwi, NERVA, or Phoebus reactors. There appears to be a serious unresolved limitation in heat transfer for optimum nuclear rocket reactors that previous authors have not examined in any detail. This heat transfer limitation is defined in Section 2.4 after this following review which emphasizes work pertaining to NERVA-type graphite-moderated nuclear rockets.

Stenning (17), 1960, presented a simplified method of analyzing nuclear rocket performance which was used in a slightly modified form for several parametric surveys of nuclear rocket reactors. Briefly, one assumes a linear shear stress, or equivalently a linear hydrogen velocity profile along a representative coolant channel, and the validity of the

Reynolds analogy (41, p. 243) between friction and heat transfer. A convenient parameter $y = 4fL_c/D$ is derived with which different nuclear reactor cores may be quickly compared for effectiveness as efficient heat exchangers; for this parameter f = Fanning friction factor, L_c = channel length [ft]. This simplification is very useful because effects of channel length, channel diameter, flow rate per channel, and axial power profile upon exit temperature and pressure drop can easily be calculated.

Yasui (8), 1961, did a moderately detailed analysis of convective heat transfer in the channels of a typical nuclear rocket using Stennings' approach. He estimated core sizes by restricting $L_c = D_c$ (core diameter) for "best" neutronic efficiency and no attempt was made to optimize any configuration. While the parametric calculations covered a wide range of reactor cores, most of his detailed work applies to a channel diameter of 0.2 inches, channel lengths of 3 and 5 feet, and an exit temperature of 4,000°R. His major conclusion was that with several limitations simultaneously operating along a coolant channel, namely: (1.) thermal stress, (2.) maximum wall temperature, and (3.) maximum fuel centerline temperature, a best combination of power density shapes might mean selecting a uniform axial power profile (flat shape) in the inlet section which would be followed by a power shape that would give constant wall temperatures in the final sections of the channel. No neutronic calculations were done.

Cooper (6, 7), 1962, conducted both heat transfer and criticality calculations for fast reactor cores. Again, no attempt was made to optimize since the purpose was to determine feasible power densities, exit gas temperatures, critical masses, and critical dimensions as functions of the reactor design parameters such as core void fraction and nuclear fuel composition. From an estimated desirable pressure drop of 200 psia and Stennings's method, a maximum flow rate of 1 lbm/sec/sq. in.

of core flow area was calculated. Simple axial sinusoidal power density variation was used as being adequate for uniformly loaded and unreflected nuclear reactors. Cooper calculated that a graphite reactor with channel flow rate per channel flow area, $G = 1.0 \text{ lbm/sec/in}^2$ flow area, $L_c = 4 \text{ ft.}$, $D = 0.1 \text{ in.}$, and maximum wall temperature $TW_{MAX} = 5,000^\circ \text{ R}$ was capable of $ISP = 830 \text{ sec.}$ (ideal vacuum conditions with no pressure ratio considered across the rocket nozzle). Cooper's heat transfer conclusions were that: (a.) a flow rate of $G = 1.0 \text{ lbm/sec/in}^2$ flow area was feasible for typical axial sinusoidal power profiles, (b.) bulk power densities on the order of 500 to 1000 MW/cu.ft. of core were required and feasible, and (c.) typical temperature differences across the fuel would be 200 to 500° R . No stress limitations were evaluated and the criticality calculations did not consider hydrogen.

Both of these surveys noted that ideally it is very efficient to add as much heat as possible when the propellant is the coldest; that is, at the reactor core inlet. In Appendix A, the simplified Stenning analysis is used to compare uniform, cosine, and linear (maximum at the core entrance) axial power profiles. The linear profile is clearly superior with respect to achievable I_{sp} and the comparison also demonstrates a trade-off between I_{sp} and channel length. Where the channel length is fixed, a high I_{sp} results from the linear profile and where the I_{sp} (or outlet temperature) is fixed, a shorter channel length results from the more efficient linear power profile. The detailed numerical results in Chapter 4 verify that power shaping can simultaneously improve outlet temperatures and allow a compact nuclear core. If a maximum wall temperature were the only limit and if the Reynolds analogy were strictly true, then a steeper-than-linear power profile would yield the highest I_{sp} at the shortest channel length;

that is, the reactor core with the largest power density will have the highest outlet temperature.

The present conception of the nuclear rocket puts a premium upon compactness for shielding reasons. A thick axial shadow shield is needed to protect electronic instrumentation and this puts a very restrictive weight disadvantage on increases in core radius. For this reason a uniform heat release in the radial direction is very desirable. Changes in channel length bring weight penalties from the pressure shell, beryllium reflector, and overall graphite reactor core volume. However an extension of channel length may increase the propellant exit temperature which increases the specific impulse. Because the specific impulse is related to the total propellant weight in an exponential manner through the rocket Equation (2.2) and since the propellant may typically be 90% of the total mission system weight, an increase in specific impulse is generally much more beneficial than a decrease in channel length. The actual sensitivity to weight change is very mission dependent since changes in propellant weight naturally are less for low energy missions than for high energy missions. For a 1982 manned Mars landing with an initial weight in earth orbit (WIEO) of 2×10^6 lbs and a nuclear Isp of 825 seconds, these mission performance influence coefficients result: 1 lb engine weight = 16 lbs WIEO and 1 second Isp = 3,000 lbs WIEO (36). This means that every extra pound of engine weight above that assumed for this mission will incur an increase of 16 pounds for the entire weight in earth orbit and that every second of specific impulse above the 825 seconds assumed will decrease the WIEO weight by 3,000 lbs. These numbers are a result of an extensive nuclear mission parametric survey (12,13,14). Therefore, from the simplified Stenning approach one can

see a means of simultaneously increasing specific impulse and decreasing the nuclear reactor core weight penalty by power profile optimization.

The implicit assumption of a constant channel friction factor requires a constant channel heat transfer coefficient, $h \left[\frac{\text{ft lbf}}{\text{ft}^2 \cdot ^\circ\text{R}} \right]$, because of the Reynolds analogy (41):

$$\frac{f}{2} = \frac{h}{G c_p} \quad (2.6)$$

where c_p is the specific heat capacity of the propellant $\left[\frac{\text{ft-lbf}}{\text{lbm} \cdot ^\circ\text{R}} \right]$. For the local channel temperatures, a local heat transfer coefficient should be used, and especially for detailed heat transfer optimization this coefficient cannot be taken as constant. Yasui used the following heat transfer correlation to calculate channel conditions in selected reactor cores:

$$\text{Nu}_f = (0.036) \text{Re}_f^{0.8} \text{Pr}_f^{0.4} (T/T_f)^{0.8} (1 + \frac{x}{D})^{-0.1} \quad (2.7)$$

and Cooper used:

$$\text{Nu} = (0.036) \text{Re}^{0.8} \text{Pr}^{0.4} (T/T_w)^{0.8} (1 + \frac{x}{D})^{-0.1} \quad (2.8)$$

where $\text{Nu} = \text{Nusselt number} = \frac{hD}{k}$,

k = thermal conductivity of the propellant $\left[\frac{\text{ft-lbf}}{\text{ft} \cdot ^\circ\text{R}} \right]$

$\text{Re} = \text{Reynolds number} = \frac{GD}{\mu}$,

μ = thermal viscosity of the propellant $\left[\frac{\text{lbm}}{\text{sec-ft}} \right]$,

T = local bulk coolant temperature [$^\circ\text{Rankine}$],

T_f = film temperature = $1/2 (T + T_w)$ [$^\circ\text{R}$],

T_w = local wall temperature [$^\circ\text{R}$],

$\text{Pr} = \text{Prandtl number} = \frac{\mu c_p}{k}$,

and x = distance from entrance of channel [ft],

and where the only difference is the choice of bulk (no subscript) or film temperatures (f subscript) for evaluation of the hydrogen properties. However, the $(T/T_w)^{0.8}$ correction to these correlations admittedly had doubtful validity for the large temperature gradients expected between wall and bulk coolant for the channel inlet sections of rocket nuclear reactor cores (6, p. 43). It is shown in Chapter 4 that the mathematical form of the local heat transfer coefficient has a very important influence upon possible dynamic operation of the most efficient rocket nuclear reactor cores.

Plebuch (10), 1963, assumed certain nominal materials properties for a solid-core, hydrogen-cooled, graphite-moderated nuclear rocket and calculated optimum axial power density profiles in a detailed numerical manner simultaneously taking into account the three thermal limitations of maximum thermal stress, maximum wall temperature, and maximum fuel centerline temperature. These derived power density profiles conformed to Yasui's preliminary conclusions and confirmed Cooper's calculations about required power densities. However, Plebuch's choice of the local heat transfer correlation was:

$$Nu = (0.023) Re_f^{0.8} Pr_f^{0.4} (T/T_f)^{0.8} \quad (2.9)$$

With an emphasis upon variables most effective in shaping the power generation for improved heat transfer, Plebuch then undertook a study of the reactor physics of non-symmetrically reflected, fully enriched, hydrogen-cooled, graphite-moderated nuclear rocket reactors. His principal reactor physics model was a "modified" analytical 3 group diffusion theory using one fast group and two overlapping thermal groups to account for the large temperature discontinuity between the reactor core, which may

be at 5,500°R, and the reflector, which may be at 400°R. Calculations that had to account for a non-uniform hydrogen distribution in the core used a numerical two-dimensional diffusion theory code with the appropriate 3-group constants. With no hydrogen present and a uniformly-loaded nuclear reactor, these variables: (1.) moderator-to-fuel ratio, (2.) core temperature, (3.) core solid fraction, (4.) reflector material, (5.) reflector thickness, (6.) reflector solid fraction, and (7.) reflector temperature were investigated for their effects upon (a.) critical dimensions and (b.) axial and radial power peaking at the interface between the nuclear reactor core and the reactor reflector. Hydrogen, uniformly distributed throughout a nuclear reactor core having a solid fraction of 0.7, was evaluated for its effects upon (1.) neutron slowing-down spectrum, (2.) fast group reactor physics constants, (3.) critical core dimensions, and (4.) power peaking. Non-uniform hydrogen reactivity insertion for this solid fraction for two carbon-to-U235, C/U, values, and for two core temperatures was calculated by the numerical diffusion code. Temperature reactivity coefficients due to the shift of the core thermal spectrum as a function of the moderator-to-fuel ratio were calculated for uranium -235, uranium -233, and plutonium -239. Finally, the use of a non-uniform uranium distribution and a suitable reflector thickness duplicated the calculated "desirable" optimum axial power profile and flat radial profile for a nuclear reactor core having the solid fraction =0.7.

Plebuch thus demonstrated that desirable axial power profiles and flat radial power profiles could be achieved by the judicious use of non-uniform uranium distribution and choice of reflector materials and thick-

ness. Large negative reactivity defects due to the shift in reactor core thermal neutron spectrum indicated that large thermal nuclear rocket reactors would be impractical. Hydrogen uniformly distributed throughout the core could appreciably decrease the critical dimensions, change the neutron slowing down spectrum, and alter the power peaking.

Due to the large number of variables concerned, not all of the hydrogen effects influencing the neutron physics of a nuclear rocket reactor could be investigated. A reactor core with a solid fraction of 0.7 was the only case examined for hydrogen effects and only a few non-uniform-hydrogen, whole-core reactivity calculations were made. In particular no hydrogen reactivity calculations were performed concerning any situation having non-uniform-hydrogen at a steady-state distribution in the core with hydrogen also present in the reflector or with only hydrogen in the reflector such as might occur prior to reactor start-up. While the choice of one fast and two overlapping thermal energy groups might reasonably account as a physics model for the large core-reflector temperature discontinuity in thermal or epithermal nuclear reactors, small and therefore fast systems that are dominated by fast neutron leakage require analysis by many more fast energy groups. This fast leakage means that the hydrogen in both core and reflector should have an important reactivity effect, because relatively few neutrons reach the thermal energy group so that any extra neutrons which may be down-scattered by an excellent moderator such as hydrogen will be very important. Also because of these relatively few neutrons reaching the thermal energy, Plebuch demonstrated that shifts in the thermal neutron spectrum due to relatively large changes in core

or reflector temperatures have small reactivity effects for these fast systems.

Little (9), 1964, investigated non-uniform-hydrogen reactivity insertions for large, thermal and epithermal, nuclear rocket reactor cores having a solid fraction = 0.7 and just one reference power distribution. His reactor physics model treated the fast energy dependence in great detail with 55 groups using the Goertzel-Selengut equations with no fast fission sources, but the spatial dependence was only approximated by assuming a constant buckling for all the fast groups and the leakage neutrons returning to the core were approximated as a constant fraction of those calculated to be leaving the core. No calculations treating the presence of hydrogen in the reflector were performed, and his choice of the local heat transfer correlation was:

$$Nu = 0.045 Re^{0.8} Pr^{0.4} \left(\frac{T}{T_w} \right)^{0.55} \left(1 + \frac{x}{D} \right)^{-0.15}. \quad (2.10)$$

Little's major conclusions were that large thermal nuclear rockets have large reactivity coefficients due to changes in hydrogen density and changes in core temperature during rapid startups, and that small nuclear rockets (C/U = 250) are much less sensitive to these changes.

2.4 The Problem Statement.

From the preceding analytical work there is doubt as to the correct heat transfer relation to use for turbulent hydrogen flow in the channels of a nuclear rocket reactor core, and there has been no determination of the effect and importance of hydrogen present in the reflector of optimum reactor cores. Concerning turbulent heat transfer where the wall is at a high temperature and the coolant is at a low bulk temperature,

and with a coolant such as hydrogen whose properties are quite temperature dependent, some (empirical) factor is needed to adjust the usual constant-properties correlations. Generally this corrected correlation takes the form (4) :

$$Nu = K Re^A Pr^B (T/T_w)^C \quad (2.11)$$

where the constants A and B are usually taken to be 0.8 and 0.4 respectively and the functions K and C are adjusted to suitable constants, depending upon whether the Reynolds and Prandtl numbers are evaluated at film or bulk temperatures, except when the physical situation requires attention to entrance effects, in which case either K or C is described as a function of (x/D). There are many combinations of K and C which could fit the same experimental heat transfer data to the same degree of accuracy. For the reactor designer the consequences of using any particular form of the heat transfer correlation should be examined, especially for heat transfer optimization where the correlations are extrapolated to extreme conditions not verified by experiment.

Yasui (8) and Cooper (6) both noted that the tentative (T/T_w)^{0.8} correction for large temperature-gradient heat transfer, which amounts to the use of a modified Reynolds number, led to the prediction of relatively high entrance wall temperatures when low inlet bulk temperatures were used in conjunction with high bulk power densities. A careful study of their results indicates the appearance of a temperature effect hereafter referred to as the "temperature anomaly." This is an anomaly not in the sense that it is physically impossible, but in the sense that it is contrary to common experience with constant-property coolants. This

effect is that with the modified Reynolds number, a decrease in the predicted wall temperature occurs where the bulk coolant temperature increases, but only for that inlet region where the (T/T_w) ratio is quite low. That the wall temperature should increase as the bulk coolant temperature increases is strictly true only if the heat transfer coefficient remains relatively constant, since by Fourier's law:

$$\frac{Q}{A} = h (T_w - T) , \quad (2.12)$$

where Q/A = power transferred to coolant per sq. ft. of wall surface area. Thus, the heat transferred per unit area per unit time is equal to the product of the heat transfer coefficient and the temperature difference. For a constant Q/A the mathematical form of h could allow T_w to increase while T decreased, and vice versa. Experimentally such an effect can be produced (4, p. 3) so that if Correlations (2.7), (2.8), and (2.9) were true over the entire range of temperatures and temperature ratios to be found in a nuclear rocket reactor, then it may be impossible to design an efficient core system to 100% capability since a reduction in inlet bulk temperature at any time during full power operation could cause the wall temperature to greatly exceed its limit with resultant melting or fuel element failure through corrosion. An examination of the effect of the form of the hydrogen heat transfer correlations upon nuclear rocket reactor heat transfer optimization is reported in Chapter 4. The sense of the optimization used in this thesis is aimed at maximizing the power density axially such that all positions along the reactor core channel operate at some thermal limit. As indicated by the analysis in Appendix A with Stenning's simplified method, the reactor core with the maximum out-

put power per unit volume gives the highest Isp. Having the highest Isp at the shortest channel length is extremely important for compactness related to the shielding requirements for any given nuclear rocket mission. A high Isp means a high thrust per channel and for any total thrust level, the core radius is thus made as compact as possible since the core radius is proportional to the square root of the total number of channels. Although no nuclear engine weight or mission WIEO calculations are performed in this thesis, previous authors have demonstrated that the nuclear shielding is the major weight component of a nuclear engine; for example, Cooper (7) reports that a 1,000 MW graphite reactor with a core diameter of 36 in. and a 12 in. beryllium side reflector has an engine weight (core plus reflector) of 5,500 lbs while the 10 inch lead shadow shield (similar to that depicted in Figure 2-1) weighs 9,000 lbs. and a 5 inch circumferential lead shield would weigh an additional 16,000 lbs. Sams (33) also confirms the dominance of the shield weight for nuclear rockets. The compactness, especially radially, of heat-transfer optimized nuclear rocket reactors is the primary motivation for this optimization investigation.

Consideration of all of the nuclear aspects of a given nuclear rocket reactor core can be conceptually complicated because of the manner in which significant core and reflector variables can change with respect to one another. Plebuch (10) has shown that at least 8 significant variables must be considered for the criticality and power peaking of uniformly-loaded nuclear reactor cores - that is, (1.) moderator-to-fuel ratio, (2.) core temperature, (3.) core void fraction, (4.) hydrogen distribution in the core, (5.) reflector material, (6.) reflector thick-

ness, (7.) reflector solid fraction, and (8.) reflector temperature. Because of the large number of variables and because of his few-group analytical model, Plebuch did not attempt to optimize the reactor configuration completely although he did show that a desirable axial and radial power distribution could be accomplished by non-uniform uranium distribution and proper selection of reflector thicknesses.

Heat-transfer optimization along a nuclear rocket reactor channel is limited by the available pressure drop to a maximum channel length as discussed in Section 3.7, and the outlet temperature defines an I_{sp} and a thrust per channel. Selection of the total thrust for the rocket system then determines the number of channels and thus the reactor core radius, assuming that the optimum power profile is radially uniform. Therefore, heat transfer-optimization will yield definite physical dimensions for nuclear rocket reactor cores. For the thrust level selected in this thesis, which is in the manned-Mars-landing range (250,000 lbf) and which is a representative value taken from an extensive nuclear mission optimization study (12,13,14), these dimensions are on the order of a few feet with the length-to-diameter ratio of the core tending toward unity as the core void fraction increases. The compactness of these optimum cores suggest that a high loading of uranium will be required and that the neutron spectra will be very fast and whose physics will be dominated by fast neutron leakage.

The nuclear reactor core which has been sized by the heat-transfer optimization method described in Chapter 3 will greatly reduce the number of independently-varying factors that must be considered. The following complete problem is thus analyzed. First in Chapter 4 the mathematical forms of turbulent hydrogen heat transfer correlations are compared and

the effect that the correlational form has upon the optimum power density profile and the magnitude of the predicted temperature "anomaly", are reported and discussed. The correlation used by Plebuch (10) is compared with the correlation given in Bussard and DeLauer's book on nuclear propulsion (11), with a 1964 correlation by Taylor (4), and with a 1968 correlation again by Taylor (1). These last three correlations are used because of their association with NERVA-type nuclear rockets. The effect of (1.) flow rate per channel, (2.) channel diameter, (3.) core void fraction, and (4.) the entrance hydrogen pressure and temperature upon (a.) optimum reactor channel characteristics, (b.) achievable ideal specific impulse, and (c.) heat-transfer optimized reactor core dimensions is reported. The computer method for the heat-transfer optimization is discussed in Chapter 3.

With the dimensions of the reactor core determined by heat transfer considerations then the reactivity effects of uniform hydrogen in the reflector and a non-uniform hydrogen distribution in the reactor core which produces the optimum axial power profile for that particular reactor, as well as a study of the critical concentration of uranium 235 uniformly distributed in a graphite matrix in the presence of this hydrogen, is reported and discussed in Chapter 5. This approach assumes that for these small, fast reactor cores the reactivity effects will be dominated by neutron leakage and the hydrogen reactivity values are insensitive to the local distribution of uranium. In this manner the criticality calculation determines the minimum moderator-to-fuel ratio and minimum critical mass associated with an optimum core that fully utilizes the reactivity insertions due to hydrogen. The approach followed here is

therefore different than in previous studies in that the physical size of the reactor core is first set by heat transfer considerations to be the most compact possible and then for this size the required uniform carbon/uranium ratio and the total hydrogen reactivity effect are derived.

In addition to the above (1.) hydrogen reactivity due to core and reflector, the following aspects will also be reported: (2.) spatial perturbation in power distributions caused by hydrogen insertion, (3.) effect of variable radial reflector thickness as an aid in axial power shaping, (4.) core reactivity loss caused by thermal expansion of the core radius, and (5.) core reactivity loss caused by carbon deletion.

Plebuch (10) and Little (9) have both shown that temperature effects such as the core-reflector temperature discontinuity and the shift in thermal neutron spectrum are small for low carbon/uranium, fast reactor systems. For instance, Little cites a total reactivity change of -0.20% ($\Delta k/k = 0.0064 \frac{\Delta k}{k}$) for a temperature shift from 300°K to 2,400°K in a reflected core having C/U = 250. Therefore, in the nuclear calculations in Chapter 5 no adjustments are made for temperature discontinuities or shifts in thermal neutron spectrum. The 16-group LASL cross-section set (37) is used with the two-dimensional, multigroup, diffusion-theory code, EXTERMINATOR - 2 (38). The cases considered in this thesis are limited to one total thrust level and one reflector thickness. However, these nominal values are taken from an extensive nuclear mission-optimization study (12,13,14) so that the results of this investigation are consistent with future nuclear missions under active consideration.

3. Heat Transfer Optimization - Computer Model

3.1 Introduction

Because the heat transfer optimization of a nuclear rocket reactor core must consider the local heat transfer coefficients for turbulent hydrogen flow as well as the point values of the channel wall temperature, fuel temperature, hydrogen pressure, and hydrogen temperature, it is necessary to develop a numerical method of simultaneously solving for these point channel characteristics and selecting the maximum power density at each point. The optimum power density profile is uniform in the radial direction because all reactor core channels are assumed to have the same flow rate so that the model will only need to calculate the axial power profile.

3.2 Flow Channel Equivalent Cell

The cylindrical reactor core is assumed to consist of many "unit flow channels" of constant flow diameter aligned parallel to the major axis, each equivalent to a hollow rod cooled at the inner wall (or radius) by a gas at the axially variable pressure $P_*(x)$, and each thermally isolated from neighboring cells by requiring $\frac{dT}{dr} = 0$ at the outer radius of the cell. The cell outer radius is at the centerline dividing two adjacent cells and since the fuel temperature is a maximum there, this position will always be referred to as the "centerline fuel temperature." The cell outer radius, r_o , is determined by the core void fraction, ϵ , or hydrogen free flow fraction, since $r_o = D/2\sqrt{\epsilon}$. The channel length is determined in the course of the optimization.

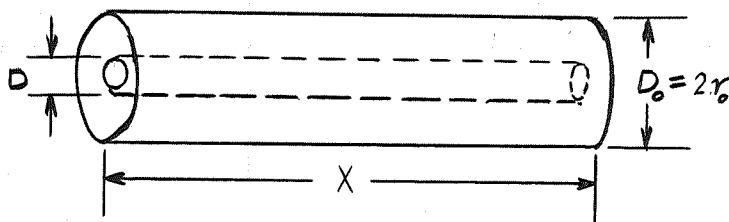


Figure 3-1 Flow Channel Equivalent Cell

3.3 The Model for Calculating Hydrogen Pressures

The momentum equation describes the change in hydrogen pressure along a channel due to friction and density changes:

$$\frac{\delta P_*}{\delta x} = \frac{-G^2}{g_c} \left[\frac{2f}{pD} + \frac{\delta}{\delta x} \left(\frac{1}{p} \right) \right] \quad (3.1)$$

where P_* = local coolant pressure $\left[\frac{\text{lb}_f}{\text{ft}^2} \right]$,

x = distance from channel entrance to local position $[\text{ft}]$,

G = flow rate per channel divided by channel flow area

$$= w / \left(\pi \frac{D^2}{4} \right) \left[\frac{\text{lb}_m}{\text{sec-ft}^2\text{-channel}} \right],$$

f = Fanning friction factor,

p = local coolant density $\left[\frac{\text{lb}_m}{\text{ft}^3} \right]$,

D = channel hydraulic diameter $[\text{ft}]$.

This is integrated using numerical difference techniques as:

$$[P_*(x + \Delta x) - P_*(x)] = \frac{G^2}{g_c p(x)} \left(-2f(x) \frac{\Delta x}{D} - \frac{[p(x) - p(x + \Delta x)]}{p(x)} \right) \quad (3.1.1)$$

where the density contribution has been treated exactly since $\delta / \delta x \left(\frac{1}{p} \right)$ is a perfect differential in this one-dimensional model, and the friction factor is evaluated at x and taken as constant over the following interval Δx . Since, as will be shown, the friction factor increases along the channel, this integration underestimates any pressure drops strongly influenced by friction. For the reactor cores in this study, the friction term, $\frac{2f\Delta x}{D}$, over most of the channel length is greater than the density term, $\frac{\Delta p}{p}$, because of the rapid heating of the gas. However, the friction factor is not a strong function of temperature and, for large increases in channel temperature, changes relatively slowly. The correlation for the turbulent friction factor used in this study is (5) :

$$\frac{f}{2} = \left(0.0007 + \frac{0.0625}{Re_s^{0.32}} \right) \left(\frac{T}{T_w} \right)^{0.5} \quad (3.2)$$

where Re_s = modified surface Reynolds number = $(Re_w) (T/T_w)$ (3.2.1) and the w subscript always means evaluation at the wall temperature. Tables 3-1 and 3-2 illustrate actual temperatures, friction factors and pressure drops for two optimization calculations. Values from an older turbulent friction factor correlation (6) :

$$\frac{f}{2} = 0.023 / Re_f^{0.2} \quad (3.3)$$

are included for comparison. For several optimization runs, the exact form of the friction factor, either Equation (3.2) or Equation (3.3), made

Table 3-1

Channel Conditions and Friction Factorsfor an Optimized Channel, High Flow Rate

D=0.1 in., $w=1 \times 10^{-2}$ lbm/s/channel, P =1,200 psia,
 $T_o=300^\circ\text{R}$, core void fraction $\eta=0.4$, $\rho_{sp}=880$ sec.

<u>X, ft.</u>	<u>T, °R</u>	<u>T_w, °R</u>	<u>P_o, psia</u>	<u>f (5)</u>	<u>f (6)</u>	<u>p $\frac{\text{lbm}}{\text{ft}^3} \frac{(\text{flow})}{\text{volume}}$</u>
0	300	648	1200	0.00313	0.00384	0.74547
.1	503	1485	1195	0.00322	0.00420	0.44320
.2	705	1822	1189	0.00344	0.00432	0.31451
.3	906	2109	1182	0.00361	0.00441	0.24319
.4	1105	2370	1173	0.00375	0.00455	0.19782
.5	1304	2615	1164	0.00387	0.00461	0.16640
.6	1500	2848	1154	0.00397	0.00466	0.14332
.7	1695	3071	1142	0.00406	0.00471	0.12562
.8	1887	3285	1130	0.00415	0.00475	0.11160
.9	2077	3491	1116	0.00422	0.00479	0.10019
1.0	2264	3692	1102	0.00429	0.00482	0.09071
1.1	2449	3885	1086	0.00435	0.00485	0.08268
1.2	2630	4074	1069	0.00441	0.00488	0.07577
1.3	2809	4257	1051	0.00446	0.00491	0.06973
1.4	2985	4436	1032	0.00451	0.00494	0.06440
1.5	3158	4610	1010	0.00456	0.00497	0.05963
1.6	3328	4778	988	0.00460	0.00498	0.05532
1.7	3488	4839	964	0.00465	0.00500	0.05149
1.8	3632	4893	938	0.00469	0.00501	0.04814
1.9	3762	4940	911	0.00473	0.00502	0.04513
2.0	3880	4982	882	0.00477	0.00503	0.04237
2.1	3988	5000	851	0.00480	0.00503	0.03980
2.2	4083	5000	819	0.00483	0.00504	0.03736
2.3	4168	5000	783	0.00485	0.00505	0.03501
2.4	4243	5000	744	0.00488	0.00505	0.03268

Table 3-2

Channel Conditions and Friction Factorsfor an Optimized Channel, Low Flow Rate

D=0.1 in., $w=1 \times 10^{-3}$ lbm/s/channel, $P_o=1400$ psia,
 $T_o=400^\circ\text{R}$, core void fraction $\gamma=0.4$, $I_{sp}=955$ sec.

<u>X, ft.</u>	<u>T, °R</u>	<u>Tw, °R</u>	<u>P_x, psia</u>	<u>f (5)</u>	<u>f (6)</u>	<u>$\frac{\text{lbm}}{\text{p ft}^3} / (\text{flow volume})$</u>
0	400	1545	1400	0.00554	0.00663	0.65228
0.1	1976	5004	1400	0.00741	0.00773	0.13204
0.2	2794	5012	1400	0.00800	0.00783	0.09326
0.3	3249	5000	1399	0.00827	0.00789	0.08026
0.4	3564	5010	1399	0.00844	0.00792	0.07317
0.5	3800	5001	1399	0.00857	0.00795	0.06860
0.6	3985	5005	1399	0.00866	0.00797	0.06540
0.7	4135	5007	1398	0.00873	0.00798	0.06302
0.8	4258	5002	1398	0.00879	0.00800	0.06119
0.9	4359	5001	1398	0.00884	0.00801	0.05975
1.0	4445	5000	1397	0.00888	0.00802	0.05859
1.1	4517	5000	1397	0.00891	0.00802	0.05764
1.2	4579	5000	1397	0.00894	0.00802	0.05685
1.3	4632	5000	1397	0.00896	0.00803	0.05618
1.4	4678	5000	1396	0.00898	0.00803	0.05562
1.5	4718	5002	1396	0.00900	0.00804	0.05514
1.6	4753	5000	1396	0.00901	0.00804	0.05473
1.7	4783	5002	1395	0.00903	0.00804	0.05437
1.8	4809	5000	1395	0.00904	0.00805	0.05406
1.9	4832	5001	1395	0.00905	0.00805	0.05379
2.0	4852	5001	1394	0.00906	0.00805	0.05356
2.1	4870	5000	1394	0.00906	0.00805	0.05336
2.2	4885	5000	1394	0.00907	0.00806	0.05318
2.3	4898	5000	1393	0.00908	0.00806	0.05302

no difference at all in the resultant optimum channel characteristics which is a consequence of the insensitivity of the optimum power density to pressure. The numerical value of f is of interest because Stenning (17) used a value of 0.005 for his rapid parametric method. A more elaborate or accurate determination of the friction factor as calculated in this thesis would be necessary if either the optimum power density profile or the hydrogen distribution along the channel were greatly sensitive to it. The stress-limited power density as described in Section 3.5 is the only limitation on maximum power density which has a dependence upon pressure and will be shown in Section 3.6 to be nearly independent of hydrogen conditions. As for the frictional effect upon hydrogen density, the two Tables 3-1 and 3-2 show that at a low flow rate $w = 1 \times 10^{-3}$ lbm/s/channel, frictional and density changes have negligible effect upon pressure because G^2 appears in the pressure drop calculation, Equation (3.1.1) and therefore rapid heating of the gas essentially determines the density changes; at a higher flow rate $w = 1 \times 10^{-2}$ lbm/s/channel the friction factor changes slowly from increment to increment. The friction factor therefore does not significantly affect the power distribution and the hydrogen distribution is predominantly determined by the rapid heating of the gas.

3.4 The Hydrogen and Nominal Core Materials Properties.

Hydrogen is assumed to be a semi-perfect gas which obeys the perfect gas relation:

$$\frac{P_*}{P} = \left(\frac{R_u}{M} \right)^T \quad (3.4)$$

and whose physical properties are calculated from the following curve-

fit relations of NASA hydrogen data (9) :

$$\text{Specific Heat} = c_p = 2601. + 0.5353 \times 10^{-4} T^2 \left[\frac{\text{ft-lbf}}{\text{lbm-}^\circ\text{R}} \right] \quad (3.5)$$

$$\text{Thermal Conductivity} = K_H = 0.218 \times 10^{-3} T^{0.74} \left[\frac{\text{ft-lbf}}{\text{sec-ft-}^\circ\text{R}} \right] \quad (3.6)$$

$$\text{Viscosity} = \mu = 1.553 \times 10^{-7} T^{0.6} \left[\frac{\text{lbm}}{\text{sec-ft}} \right] \quad (3.7)$$

An average $\bar{c}_p$ is used in the energy balance along the channel:

$$\bar{c}_p = \int_{T(x)}^{T(x+\Delta x)} c_p(T) dT / [T(x+\Delta x) - T(x)] \quad (3.5.1)$$

$$= 2601. + 0.5353 \times 10^{-4} \left[\frac{T(x+\Delta x)^2 + T(x+\Delta x) T(x) + T(x)^2}{3} \right] \quad (3.5.2)$$

The core nominal materials properties in the following Table 3-3 are those of graphite and are consistent with those values used in previous studies (9,10).

Table 3-3

Nominal Reactor Core Materials Properties

Young's Modulus = $E = 1.5 \times 10^6$ [psia]

Coefficient of Thermal Expansion = $\alpha = 2.75 \times 10^{-6}/^\circ\text{R}$

Poisson's Ratio = $\nu = 0.25$

Thermal Conductivity of Graphite = $K_g = 3.136$ [ft-lbf/ft-sec- $^\circ\text{R}$]

Maximum Thermal Stress = $S = 5000$ [psia]

Maximum Wall Temperature = $TW_{MAX} = 5000$ $^\circ\text{R}$

Maximum Fuel Centerline Temperature = $TF_{MAX} = 5500$ $^\circ\text{R}$

3.5 The Maximum Axial Power Densities $P_w(x)$, $P_c(x)$, $P_s(x)$, and $P(x)$

A computer program, MAXPOW, was written that would begin with the entrance hydrogen pressure, entrance hydrogen bulk temperature, flow rate per channel, channel diameter, core void fraction, materials thermal limitations, and materials properties, and then proceed in finite steps, of length Δx , along the channel determining the maximum specific power density, $P(x)$ [$\frac{\text{MW}}{\text{Cu. Ft.}}$] by solving and inverting the following point heat transfer equations. The method used is not sensitive to the axial step size; that is, although $\Delta x = 0.1$ ft. is generally used, much larger Δx were tested and the resultant channel power, pressure, etc. distributions agreed with those calculated using the smaller Δx .

Fourier's law with uniform heat generation (no radial dependence on the specific power density) at each position x gives:

$$[T_w(x) - T(x)] h = D (1 - \eta) P(x) / 4 \eta , \quad (3.8)$$

where the heat transfer coefficient is generally a function of the Reynolds number, Prandtl number, x position, and wall-to-bulk temperature ratio. Setting $T_w(x) = \text{TWMAX}$ and inverting Equation (3.8) gives:

$$P_w(x) = (\text{TWMAX} - T(x)) \frac{4 h \eta}{(1 - \eta) D} , \quad (3.8.1)$$

which is the wall temperature - limited power density at x .

Fourier's law and steady-state heat conduction within the cylindrical channel wall at the cell fuel centerline yield:

$$[T_{CL}(x) - T(x)] = \left[\frac{(1 - \eta) D}{4 h \eta} - \frac{D^2}{16 K_g \eta} \left(\ln \eta + \frac{(1 - \eta)}{\eta} \right) \right] P(x) \quad (3.9)$$

Setting the centerline temperature $TCL(x) = TFMAX$ and inverting Equation (3.9) gives:

$$P_c(x) = \frac{[TFMAX - T(x)]}{\left[\frac{(1-\eta)D}{4h\eta} - \frac{D^2}{16K_g\eta} \left(\ln\eta + \frac{(1-\eta)}{\eta} \right) \right]}, \quad (3.9.1)$$

which is the centerline temperature-limited power density at x .

With the hydrogen gas pressure $P_*(x)$, steady-state heat conduction, requiring $\frac{dT}{dr} = 0$ at the outer cell radius, and assuming that the channel length is long enough such that conditions of plane strain apply, then the three components of the stress, $\sigma_x(x, r)$, $\sigma_r(x, r)$, and $\sigma_\theta(x, r)$ can be determined and are derived in Appendix B. The root-mean-squared stress, $\sigma^2(x)$, satisfies the following equation:

$$2\sigma^2(x) = (\sigma_x - \sigma_r)^2 + (\sigma_x - \sigma_\theta)^2 + (\sigma_\theta - \sigma_r)^2. \quad (3.10)$$

The stress has its maximum value at the inner radius, so that:

$$\text{MAX } \sigma^2(x) = P_*(x)^2 + P_*(x) [2F_0 - F_1] P(x) + P(x)^2 [F_1^2 - F_0 F_1 + F_0^2], \quad (3.10.1)$$

$$\text{where } F_0 = \left[\frac{\alpha_{ED}^2}{16K_g(1-\nu)} \right] \left[\frac{-\ln\eta}{\eta(1-\eta)} - \frac{3(1-\eta)}{2\eta} \right]$$

$$\text{and } F_1 = \left[\frac{\alpha_{ED}^2}{32K_g(1-\nu)} \right] \left(\frac{1-\eta}{\eta} \right)^2$$

as derived in Appendix B. Setting $\text{MAX } \sigma^2(x) = S^2$, then:

$$P_s(x) = \frac{-P_*(x)(2F_1 - F_0)}{2(F_0^2 - F_0 F_1 + F_1^2)} + \sqrt{\left[\frac{P_*(x)(2F_1 - F_0)}{2(F_0^2 - F_0 F_1 + F_1^2)} \right]^2 - \left[\frac{P_*(x)^2 - S^2}{F_0^2 - F_0 F_1 + F_1^2} \right]}, \quad (3.10.2)$$

which is the stress-limited power density at x .

The hydrogen coolant temperature is found by an energy balance along the channel:

$$\bar{w}c_p [T(x+\Delta x) - T(x)] = \int_x^{x+\Delta x} P(x) \frac{\pi D^2}{4} \left(\frac{1-\eta}{\eta} \right) dx, \quad (3.11)$$

where $\bar{c}_p$ is defined in Section 3.4. The following implicit approximation to the integral insures that the power densities as determined are continuous along the channel:

$$\bar{w}c_p [T(x+\Delta x) - T(x)] = \left(\frac{1-\eta}{\eta} \right) \left(\frac{\pi D^2 \Delta x}{4} \right) \left[\frac{P(x+\Delta x) + P(x)}{2} \right]. \quad (3.11.1)$$

This form is the same as requiring that the power density be linear with x across any segment.

With these equations one can simultaneously solve for the hydrogen bulk temperature and anyone of the power densities. Only for P_s is the bulk coolant temperature indirectly involved through the determination of the point hydrogen pressure. After calculating all three limiting power densities, the smallest of P_w , P_c , and P_s is then finally selected as the optimum power density at each position x . These power densities should be called "specific" $\left[\frac{\text{MW}}{\text{cu.ft. of channel wall}} \right]$ as opposed to "bulk" $\left[\frac{\text{MW}}{\text{cu.ft. of core}} \right]$ because the heat transfer relations must consider the power actually produced in the equivalent flow channel walls. However, calculations with nuclear multigroup diffusion-theory computer codes, which must typically homogenize regions of the reactor, i.e., lump together fuel, moderator, and voids, find that the use of bulk power density is more useful and realistic. In this thesis the bulk power density is always reported and graphed except for two figures (Figures 4-6

and 4-7) related to heat transfer correlations which should use specific power densities. The relation between bulk and specific power densities is:

$$\frac{\text{"bulk" } P}{(1-\eta)} = \text{"specific" } P \quad (3.12)$$

In the following chapter the optimum power density is not designated as either specific or bulk because all of the results are for bulk power densities. The total power reported for each reactor core, of course, is the same regardless of which term is used, and this reported power is the actual integrated power for each channel, assuming a linear-with-x dependence between channel positions, times the number of channels.

3.6 The Stress-limited Power Density as a Nearly Constant Baseline

In describing the numerical procedure for optimizing the power density profile it was noted that the stress-limited power density P_s depended in a second-order manner upon the hydrogen gas temperature through the hydrogen pressure. The exact relation:

$$P_s = \frac{-P_*[2F_0 - F_1]}{2(F_0^2 - F_0F_1 + F_1^2)} + \sqrt{\left[\frac{P_*[2F_0 - F_1]^2}{2(F_0^2 - F_0F_1 + F_1^2)} \right]^2 - \frac{(P_*^2 - S^2)}{(F_0^2 - F_0F_1 + F_1^2)}}, \quad (3.10.2)$$

and the zero-pressure stress-limited specific power

$$P_s^0 = \frac{S}{\sqrt{F_0^2 - F_0F_1 + F_1^2}} \quad (3.13)$$

are compared at $P_* = 1200$ psia in Table 3-4.

Table 3-4

Comparison of Exact and Zero-Pressure Stress-Limited
Specific Power Densities for Nominal Material Properties

at $P = 1200$ psia and $D = 0.1$ in., $D = 0.2$ in.

η	$P_s^\circ, \frac{MW}{cu.ft.}, D = 0.1 \text{ in.}$	$P_s, \frac{MW}{cu.ft.}, D = 0.1 \text{ in.}$	$P_s^\circ, \frac{MW}{cu.ft.}, D = 0.2 \text{ in.}$	$P_s, \frac{MW}{Cu.Ft.}, D = 0.2 \text{ in.}$
0.1	24.6	25.4	6.1	6.3
0.2	127.4	127.8	31.8	32.0
0.3	377.2	371.1	94.3	92.8
0.4	914.	886.1	228.5	221.5
0.5	2054.	1968.	513.5	492.1
0.6	4609.	4376.	1152.	1094.
0.7	1.111×10^4	1.047×10^4	277.9	2618.
0.8	3.255×10^4	3.047×10^4	8137	7617.
0.9	1.642×10^5	1.528×10^5	41040.	38210.

For a pressure of 1200 psia, the exact stress-limited power is affected by at most 5% by including the effect of hydrogen pressure. Exceeding a stress limit for any proposed rocket reactor channel then is practically independent of the hydrogen conditions and the stress-limited power density depends primarily upon the materials properties and channel dimensions in this manner:

$$P_s^\circ = \frac{S}{\frac{\alpha_{ED}^2}{32Kg(1-\nu)} q(\eta)} \quad (3.13.1)$$

with

$$q(\eta) = \sqrt{\left(\frac{1-\eta}{\eta}\right)^4 - \left(\frac{1-\eta}{\eta}\right)^2 \left[\frac{-2\ln\eta}{\eta(1-\eta)} - \frac{(3-\eta)}{\eta} \right] + \left[\frac{-2\ln\eta}{\eta(1-\eta)} - \frac{(3-\eta)}{\eta} \right]^2} \quad (3.13.2)$$

Aside from reconfirming the friction factor independence noted in

Section 3.3, this clearly exhibits the stress-limited power density as a useful power density baseline. Any core, optimized or not, cannot contain power densities above this limit regardless of the hydrogen conditions. Also for those channel regions where P_c and P_w are calculated to be greater than P_s , the selection of P_s will give a nearly constant value that flattens or "tops" the power profile. This feature is very prominent in stress-limited, high flow rate reactor cores.

3.7 Computational Limitations Upon the Optimization.

Because h , the heat transfer coefficient is generally a complicated function of $T(x)$ and $T_w(x)$, quantities which themselves depend upon the determination of $P(x)$, an iteration at each point is required until consistent power densities and temperatures are found. As a rule, the stress-limited maximum power density requires the fewest iterations, being almost entirely determined by material properties and channel dimensions.

As will be fully explained, the program stops the computation when any of 4 conditions are encountered:

- (1) The hydrogen temperature is in the range $T_{MAX} > T(x) > T_{MAX} - 100$,
- (2) The channel length would exceed 4 feet,
- (3) The calculated pressure drop is greater than the available pressure, and
- (4) The Mach number would exceed 0.5 at the exit of the flow channel.

Since the coolant temperature cannot physically exceed the wall temperature, one has to consider just how closely one wishes to approach

the maximum wall temperature and the computational time involved in getting very near this limit. Situations did occur in which the optimized heating of the gas produced hydrogen temperatures quite close to TWMAX in a channel of 4 feet or less, but which optimization calculation of $P(x)$, etc. would have continued on for several more feet with calculated power densities becoming very small and the increments of rising gas temperature contributing smaller and smaller changes to the specific impulse. The first computational stop was then found to eliminate consideration of these very low power "tails". Computational stop number (2) was decided upon to limit computer time spent upon those core situations which were hopelessly stress-limited to very low power densities and which would have had lengths several times those of the other optimized cores. These were clearly not competitive. The stop (3) is a computational necessity recognizing that heat addition and frictional losses may exhaust the available pressure. Before proceeding from x to $x+\Delta x$, the computer program tests the available pressure drop by assuming a linear extrapolation of the present power $P(x)$ to zero in the next increment Δx and then calculates the predicted pressure drop. The friction factor, although underestimated, is at nearly a constant value when significant pressure drops occur. Extrapolation of the power to zero is necessary to find the minimum coolant temperature for the density change. The equations for heat transfer are not accurate at high Mach numbers since the adiabatic wall temperature should be used in place of the wall temperature T_w .

$$T'_w = \left(1 + \frac{\gamma-1}{2} M^2 \right) T_w, \quad (3.14)$$

where T_w = adiabatic wall temperature,

γ = ratio of propellant specific heats,

and M = Mach number = velocity of propellant / velocity of sound.

At Mach 0.5 the use of wall temperature introduces an error of 4%.

4. Heat Transfer Optimization - Results.

4.1 Introduction and Summary.

Several position dependent (i.e., local and not average) heat transfer correlations have been used or proposed for turbulent hydrogen heat transfer in nuclear reactor cores (1, 4, 6, 8, 9, 11). No survey of the consequences of using one particular correlation over any other has been reported to date, specifically for optimizing the nuclear rocket performance. These correlations must be extrapolated to extreme conditions, and it is important to understand the dependence of an optimum power shape and the attendant wall temperature distribution upon the particular mathematical form of the heat transfer correlation. These extreme conditions are a result of the low entrance hydrogen temperatures and high power densities which exist in nuclear rockets. The bulk power densities are generally on the order of hundreds of megawatts per cubic foot (6, 8).

The hydrogen gas enters the core at fairly low temperatures because prior to insertion into the regeneratively cooled rocket nozzle, it is most compactly stored as a liquid. This is a necessity because the tank mass required to store any propellant is related to the propellant mass, M_p , and density, ρ_p , by (11, p. 73):

$$M_t = \frac{C_{st} M_p}{\rho_p^{2/3}} \quad (4.1)$$

where C_{st} is typically $0.25 \text{ (lbm / cu. ft.)}^{2/3}$. Tank mass competes with useful payload and hydrogen has a low density, 4 lbm/cu.ft., even as a liquid. Depending upon the kind of single or multiple pass arrangement in the reflector, the hydrogen removes the heat acquired by the beryllium through absorption of core gamma rays. The temperature rise should be several hundred degrees Rankine and the hydrogen must be a gas upon entering the core or an unacceptable reactivity insertion results (3,9).

At the high power densities and low inlet temperatures considered for nuclear rocket operation, large ratios of wall-to-bulk temperatures could result in the core inlet sections. For an optimized nuclear rocket reactor core, the wall temperature may be 5000°R while the inlet bulk coolant is 200°R , producing a T_w/T ratio of 25 which is significantly greater than the ratio of 8.0, which is a typical experimental limit assigned to one of the NERVA-related heat transfer correlations discussed in Section 4.2.

The primary motivation for studying the dependence of optimum power profiles upon the particular heat transfer correlation used was that the turbulent hydrogen correlation assumed by several previous (6,8,10) nuclear rocket studies predicts an unusual phenomenon : any decrease in the inlet hydrogen bulk temperatures could cause a large increase in predicted channel wall temperatures. Heat-transfer optimization would naturally accentuate this feature, which will be referred to here as a wall temperature anomaly because this relationship between coolant and wall temperature is intuitively anomalous, although it may be real. This anomaly characteristically is found in a wall tempera-

ture distribution which begins with an initially high T_w at the nuclear reactor core entrance, decreases for a short distance along the channel despite the fact that the bulk hydrogen temperature increases meanwhile, and then reaches a point beyond which the T_w does increase. The disturbing aspect about this is that any decrease in entrance hydrogen bulk temperature at full power operation could increase fuel element wall and fuel centerline temperatures beyond the material limits with subsequent melting or unacceptable hydrogen surface corrosion of the fuel elements. If this anomaly were real, the attainment of compact and highly efficient nuclear rocket reactor cores would be seriously limited. Several turbulent hydrogen heat transfer correlations more recent than the ones used by Yasui (8), Cooper (6), and Plebuch (10) still predict this behavior as is shown in Sections 4.3 and 4.4. Moreover, Taylor (4), 1964, experimentally produced such an effect although this result was tempered by cautionary remarks about the experimental accuracy and the fact that the experimental effect was of smaller magnitude than would be expected by strictly extrapolating the many correlations he surveyed. Regardless of this, the improved (1964) correlation Taylor devised still predicts this behavior as is seen in Section 4.3.

The most recent correlation by Taylor (1), 1968, is simpler in form, comprehends a very large number of data points, and when applied in Section 4.5 predicted no such anomaly. For this reason the correlational effects surveyed in this Chapter are deliberately separated into three sections : Sections 4.3 and 4.4 will compare correlations exhibiting the temperature anomaly and show the dependence of the optimum axial

power profile upon those "anomalous" correlations, and Section 4.5 will examine the axial power profile consequences of a "non-anomaly" correlation. This is done for reasons of clarity and because this seems to be a natural classification. Comparison between these two correlation classifications is shown at the beginning of Section 4.5 to emphasize the contrasting wall temperature behavior in the inlet section.

Therefore, in this Chapter will be demonstrated that: (1.) the older heat transfer correlations for turbulent hydrogen flow are inconsistent in predictions of entrance wall temperatures and predict questionable (i.e., anomalous) wall temperatures for large wall-to-bulk temperature ratios, (2.) that the detailed optimum axial power profile is dependent upon the particular correlation used, but (3.) that the physical dimensions of the optimum reactor core and its specific impulse are insensitive to the particular heat transfer correlation used and depend primarily upon (a.) flow rate per channel, $\dot{W}$ (b.) channel diameter, D , and (c.) core void fraction, η . Although only further experimental work could resolve whether a temperature anomaly really exists, the inter-comparisons of "anomalous" correlations in Section 4.3 plus the lack of an anomaly as shown in Section 4.5 seem to indicate that such an effect could be due merely to correlational form and experimental inaccuracy.

4.2 The Heat Transfer Correlations.

The turbulent heat transfer correlation used by Plebuch, 1963, (10) for calculating his optimum power profiles is:

$$Nu = (0.023) Re_f^{0.8} Pr_f^{0.4} (T/T_f)^{0.8} , \quad (4.2)$$

which is similar to the correlations used by Yasui and Cooper, except that their constant factor is different from 0.023 as noted in Section 2.3. There is no explanation as to why the constant should differ.

Bussard and DeLauer's book on nuclear propulsion (1965) (11, p.130) gives this correlation:

$$Nu = (0.045) Re^{0.8} Pr^{0.4} (T/T_w)^{0.55} (1 + \frac{x}{D})^{-0.15} , \quad (4.3)$$

which is said to agree with experiment with an average deviation less than 10 per cent. Taylor (4, p.7) also gives a similar correlation:

$$Nu = (0.025) Re^{0.8} Pr^{0.4} (\frac{T}{T_w})^{0.55} , \quad (4.4)$$

for which the deviation is within 10% and $1.44 < T_w/T < 9.2$ and which Taylor says to be an equation of the type used for some NERVA heat transfer calculations. In this thesis only Equation (4.3) rather than Equation (4.4) was extensively investigated because the similarity in (T_w/T) ratio in Equation (4.4) would probably produce similar results.

An improved correlation, in the sense that recent (up to 1964) hydrogen experimental data had been incorporated into the correlation, is given by Taylor (4) as:

$$Nu = (0.021) Re^{0.8} Pr^{0.4} (\frac{T_w}{T})^{- [0.29 + 0.0019 (\frac{x}{D})]} , \quad (4.5)$$

which deviates from experiment by $\pm 10\%$ and covers experimental data for $10 < \frac{x}{D} < 240$, $200^\circ R < T < 2800^\circ R$, $1.1 < T_w/T < 8.0$.

The most recent hydrogen heat transfer correlation successfully used for NERVA and Phoebus regeneratively-cooled nozzles is given by

Taylor (1) as:

$$Nu = (0.023) Re^{0.8} Pr^{0.4} \left(\frac{T_w}{T} \right) \exp - (0.57 - 1.59D/x), (4.6)$$

and is the result of an extensive review of over 4600 data points. The range of experimental conditions were : $2 < \frac{x}{D} < 252$, $1.1 < T_w/T < 27.6$, $4.5^\circ R < T_o < 573^\circ R$, $18 \text{ psia} < P_o < 2500 \text{ psia}$, $0.036 \frac{\text{Btu}}{\text{s-in}^2} < \frac{Q}{A} < 27.6 \frac{\text{Btu}}{\text{s-in}^2}$ and $53^\circ R < T_w < 5600^\circ R$, where T_o and P_o are the inlet temperature and pressure. Notice that this correlation is much simpler in form than any of the others, there being no exponent for the wall-to-bulk temperature ratio, and the range of experimental conditions satisfactorily covers the range of channel conditions expected in nuclear rocket reactor cores. The Q/A values comfortably bracket the conditions actually found in the optimized reactor cores in Sections 4.4 and 4.5; for instance, a rough calculation for the $D = 0.1 \text{ in.}$ and $\eta = 0.3$ "high-flow-rate" optimum channel yields a Q/A value of approximately $12 \text{ Btu/sec-sq. in.}$ However, the reported accuracy is such that 87% of the heat transfer coefficients predicted by Equation (4.6) deviate less than $\pm 25\%$ from the measured values. This is a much larger deviation than the deviations claimed for Correlations (4.3) and (4.5).

4.3 A Wall Temperature "Anomaly" and Inlet Temperature Inconsistencies.

4.3.1 The Unusual Behavior Predicted by Equation (4.2)

Correlation (4.2) has the modifying coefficient $(T/T_f)^{0.8}$ which in effect represents the introduction of a "modified" Reynolds number:

$$\text{Re}_f^* = \left(\frac{p_v D}{\mu_f} \right) \left(\frac{T}{T_f} \right), \quad (4.7)$$

and is of historical importance for comparative purposes because it has been used by several authors as discussed in Section 2.3. Yasui and Cooper both commented upon the high wall temperature predicted at the channel entrance for high power densities and Plebuch displays without comment, the large wall temperature anomaly associated with this correlation in several of his optimum power shapes-optimum in the sense used throughout this thesis; i.e., the nuclear reactor core has every point along a channel operating at the steady-state condition under one of the three thermal limits of maximum stress, S , maximum wall temperature, T_{WMAX} , or maximum fuel centerline temperature, T_{FMAX} .

Figure 4-1 and Table 4-1 illustrate the peculiar wall temperatures predicted by Equation (4.2) for an optimum power shape that happens to be constant because nearly the entire channel length is stress-limited (see Section 3.6). This figure was generated by the computer program, MAXPOW, discussed in Chapter 3 and is essentially Figure I-12 on page 30 of Plebuch's Sc.D. thesis. Incidentally, this modifying temperature ratio, $(T/T_f)^{0.8}$, is extremely important for duplicating his power density figures because when the heat transfer coefficient as reported in Plebuch's reference 26 is used, without assuming the use of a modified Reynolds number, only monotonically-decreasing power profiles result and, for example, this particular figure cannot be derived.

For this core void fraction of 0.3 and the 400°R inlet gas temperature, Correlation (4.2) predicts a 3272°R ΔT at the entrance.

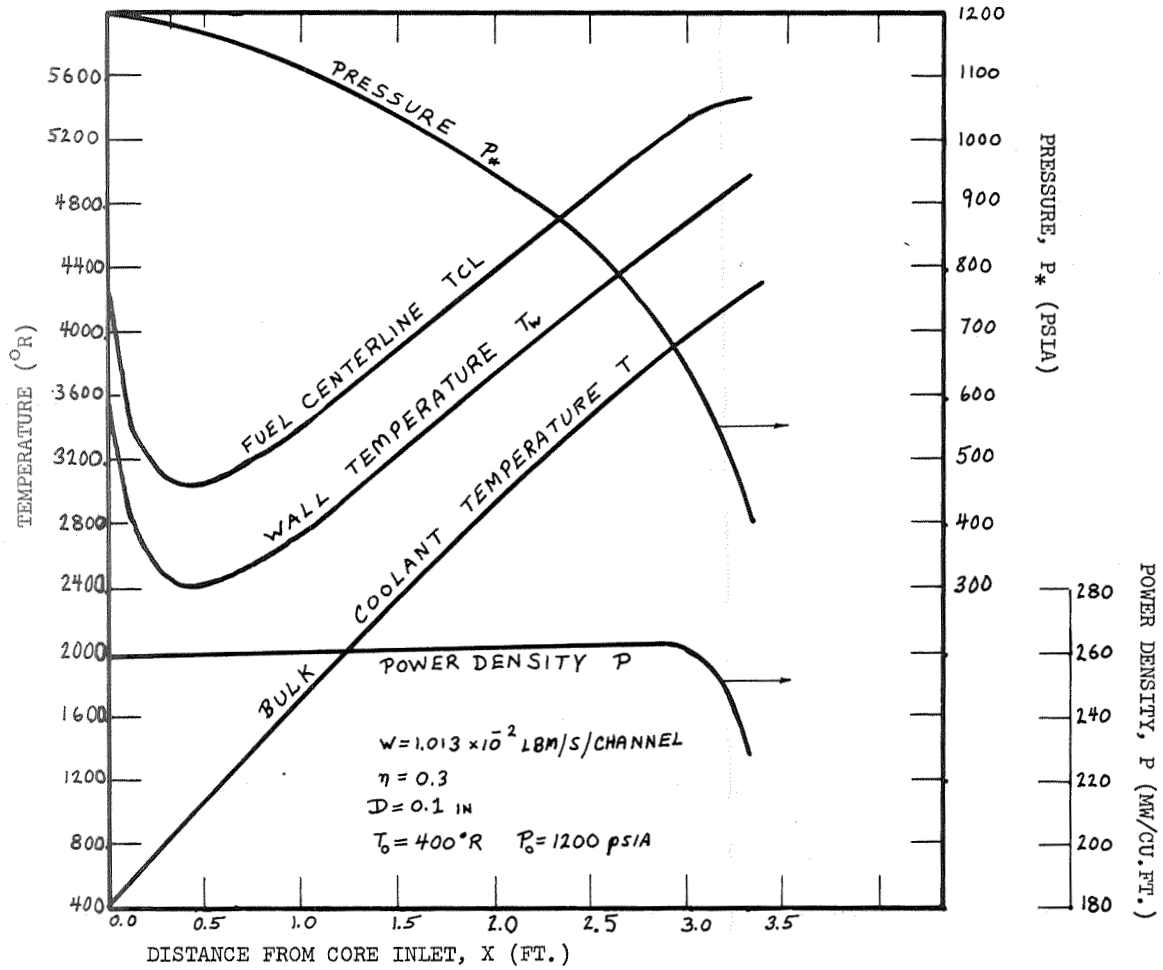


FIGURE 4-1 OPTIMUM REACTOR CORE CHANNEL DISTRIBUTIONS,
CORRELATION (4.2)

Table 4-1

Reactor Core Channel Characteristics for Figure 4-1

<u>X, ft</u>	<u>T, ° R</u>	<u>Tw, ° R</u>	<u>TCL, ° R</u>	<u>Bulk P, Cu.Ft. ^{MW}</u>	<u>P_o, psia</u>
0	400	3672	4308	260	1200
0.167	619	2675	3312	260	1190
0.333	838	2452	3089	260	1179
0.500	1055	2444	3081	260	1166
0.667	1270	2522	3160	260	1151
0.833	1483	2640	3278	260	1135
1.000	1694	2781	3419	261	1116
1.167	1902	2934	3573	261	1096
1.333	2107	3094	3733	261	1073
1.500	2309	3258	3898	261	1049
1.667	2508	3425	4065	261	1021
1.833	2703	3592	4233	262	991
2.000	2895	3759	4400	262	957
2.167	3085	3926	4568	262	922
2.333	3271	4091	4734	262	882
2.500	3453	4255	4898	262	837
2.667	3632	4416	5061	263	786
2.833	3808	4576	5221	263	727
3.000	3980	4733	5379	264	656
3.167	4147	4868	5500	258	564
3.333	4300	4934	5500	231	423

The wall temperature decreases by approximately 1230 ° R while the bulk coolant temperature rises by 650 ° R! This represents a serious problem because at full power operation, a reduction in entrance coolant temperature, which is always possible, would further increase the entrance wall temperature and possibly exceed the materials limit for either fuel centerline melting or excessive wall corrosion by hydrogen.

"Logically" a warmer coolant should be accompanied by a warmer wall at constant power, but this is strictly mathematically true only if the heat transfer coefficient remains constant. Since the same tendency

in the inlet region was also noted with the chopped cosine shape, same entrance hydrogen conditions, and same 0.3 core void fraction in Cooper's survey, this trend is not a result of the optimization and must be a consequence of the high power density, low inlet temperature, or the particular mathematical form of the heat transfer coefficient. Certainly any such trend will be accentuated by an optimization that tries to push all materials to their temperature limits. Yasui, 1961, (8) acknowledged that this $(T/T_f)^{0.8}$ correction might not be adequate for the large temperature gradients considered for nuclear operation so the question as to the actual validity of results such as Figure 4-1 remained until further experimental work could confirm the correlational correction or produce more accurate correlations. Since Equations (4.3) and (4.5) became available later than Correlation (4.2), presumably these are on firmer experimental footing and would supersede Correlation (4.2), but no intercomparison with respect to optimum power profiles has been reported.

4.3.2 The Temperature "Anomaly" Predicted by Equations (4.3) and (4.5).

Correlations (4.3) and (4.5) also predict wall temperature anomalies as seen in Figures 4-2 and 4-3 which assume the same reactor core parameters and identical initial hydrogen conditions as Figure 4-1. Tables 4-2 and 4-3 list the channel temperatures for these figures. These tables exhibit a slightly longer channel than Table 4-1 only because the pressure drop limitation (see Section 3.7) is not reached as quickly with the coolant bulk heating allowed by Correlations (4.3) and (4.5), as by Correlation (4.2). Notice that for this core void

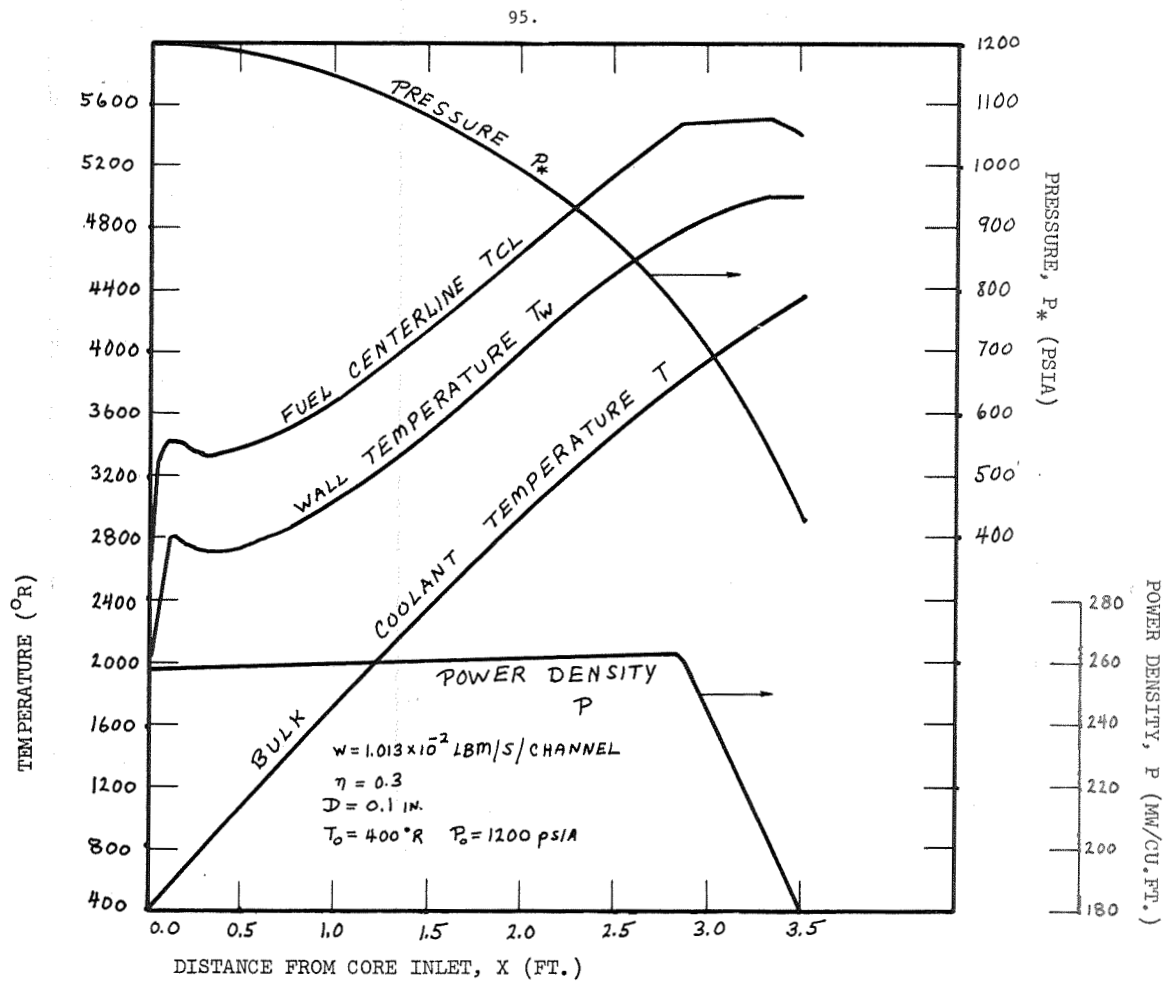


FIGURE 4-2 OPTIMUM REACTOR CORE CHANNEL DISTRIBUTIONS,
CORRELATION (4.3)

Table 4-2

Channel Temperatures for Figure 4-2

<u>X, ft.</u>	<u>T, °R</u>	<u>Tw, °R</u>	<u>TCL, °R</u>
0	400	1974	2611
0.167	619	2769	3406
0.333	838	2700	3337
0.500	1055	2718	3355
0.667	1270	2797	3434
0.833	1483	2913	3551
1.000	1693	3051	3689
1.167	1901	3202	3840
1.333	2106	3360	3999
1.500	2308	3524	4163
1.667	2507	3689	4329
1.833	2702	3856	4497
2.000	2894	4023	4664
2.167	3084	4189	4831
2.333	3269	4354	4997
2.500	3452	4518	5161
2.667	3631	4680	5323
2.833	3806	4839	5483
3.000	3971	4904	5500
3.167	4121	4955	5500
3.333	4254	5000	5495
3.500	4370	5000	5426

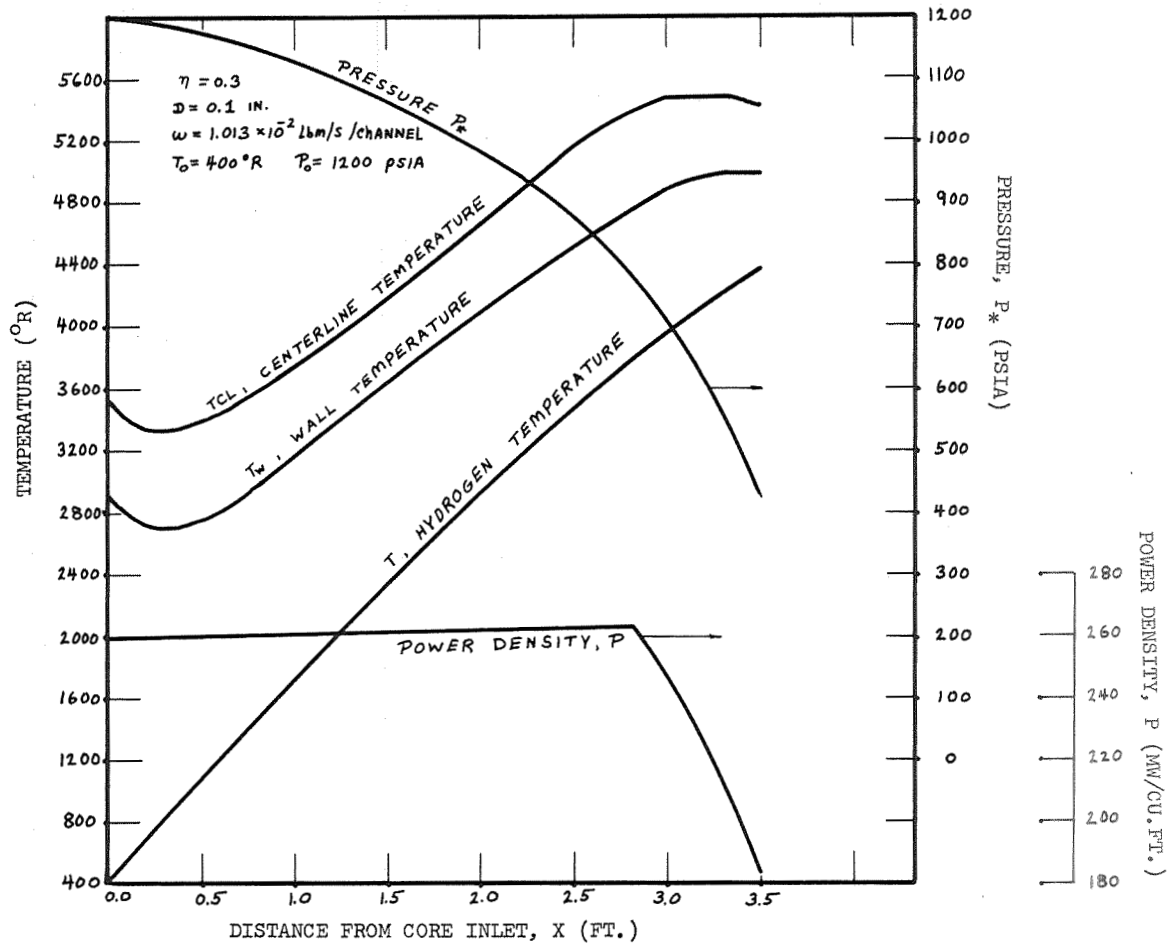


FIGURE 4-3 OPTIMUM REACTOR CORE CHANNEL DISTRIBUTIONS, CORRELATION (4.5)

Table 4-3

Channel Temperatures for Figure 4-3

<u>X, ft.</u>	<u>T, °R</u>	<u>T_w, °R</u>	<u>TCL, °R</u>
0	400	2887	3524
0.167	619	2693	3329
0.333	838	2675	3312
0.500	1055	2734	3371
0.667	1270	2833	3470
0.833	1483	2956	3594
1.000	1693	3094	3732
1.167	1901	3241	3879
1.333	2106	3394	4033
1.500	2308	3551	4190
1.667	2507	3709	4349
1.833	2702	3869	4509
2.000	2894	4028	4669
2.167	3084	4188	4830
2.333	3269	4347	4989
2.500	3452	4504	5147
2.667	3631	4660	5303
2.833	3806	4814	5458
3.000	3973	4892	5500
3.167	4125	4941	5500
3.333	4263	4987	5500
3.500	4384	5000	5450

fraction neither correlation predicts as large an anomaly as Equation (4.2) although the power shapes are practically identical because the nearly constant stress-limited power density (see Section 3.6) dominates. In other words, Equations (4.3) and (4.5) may predict wall-temperature-limited maximum power densities, P_w , and fuel centerline-temperature-limited maximum power densities, P_c , which are quite different, but regardless of this the magnitudes of each particular P_w or P_c are greater than the nearly constant stress-limited maximum power density, P_s , which is thus selected as the optimum power density in accordance with the procedure explained in Section 3.5.

However, as expected because of the tentative nature of Equation (4.2), the channel temperature distributions predicted by Correlations (4.3) and (4.5) are significantly different from the temperature distributions predicted by Correlation (4.2). The composite plot of wall temperature distributions in Figure 4-4 and fuel centerline temperature distributions in Figure 4-5 show that Equations (4.3) and (4.5) agree closely for predicted temperatures nearly everywhere except at the channel entrance where the predicted temperatures disagree by 913°R for both wall and fuel centerline. Inlet temperature predictions are very inconsistent for the first 0.5 feet of channel until the bulk coolant temperature reaches 1055°R and $T_w \approx 2700^{\circ}\text{R}$, a (T_w/T) ratio of ≈ 2.6 .

These three correlations predict a stress-limited situation so that the wall and centerline temperatures do not approach their respective limits until near the channel exit. If there is a reduction in the hydrogen entrance bulk temperature then the inlet region wall tempera-

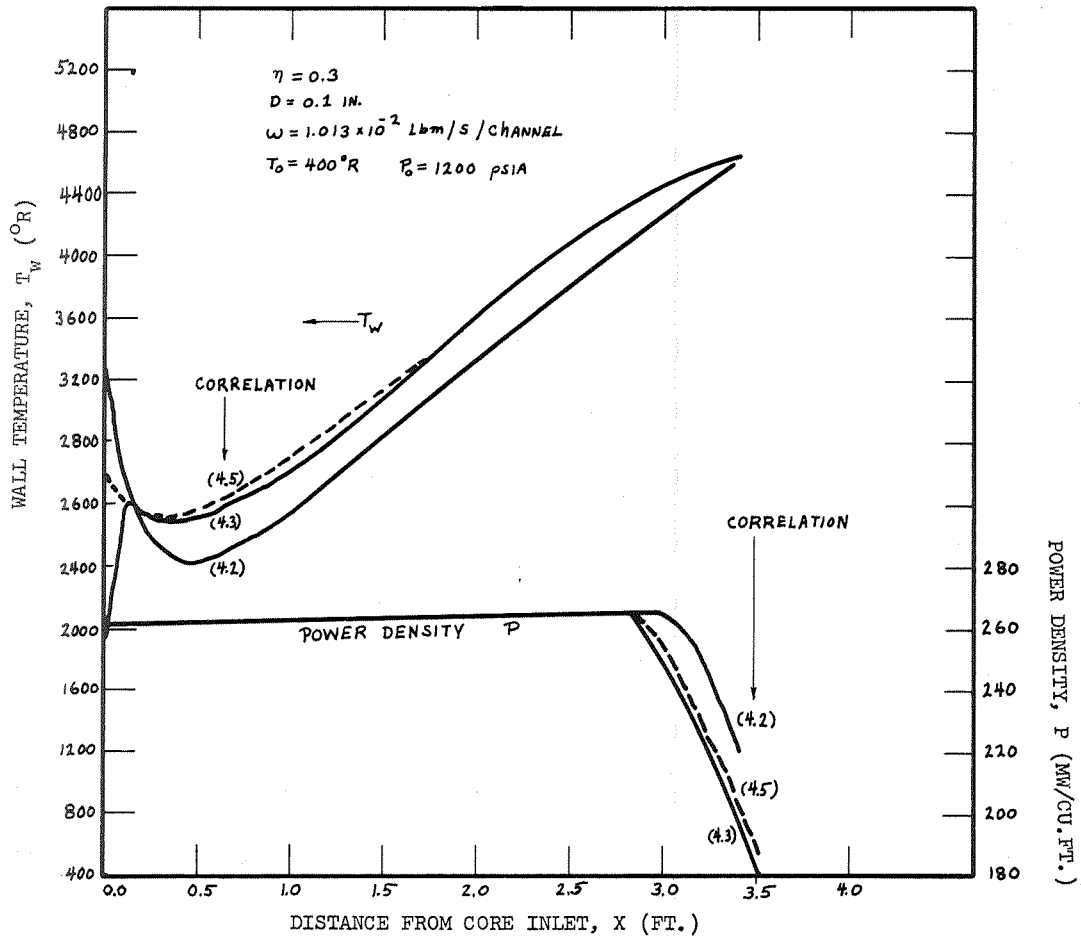


FIGURE 4-4 WALL TEMPERATURE DISTRIBUTIONS AND OPTIMUM POWER PROFILES FOR THREE CORRELATIONS

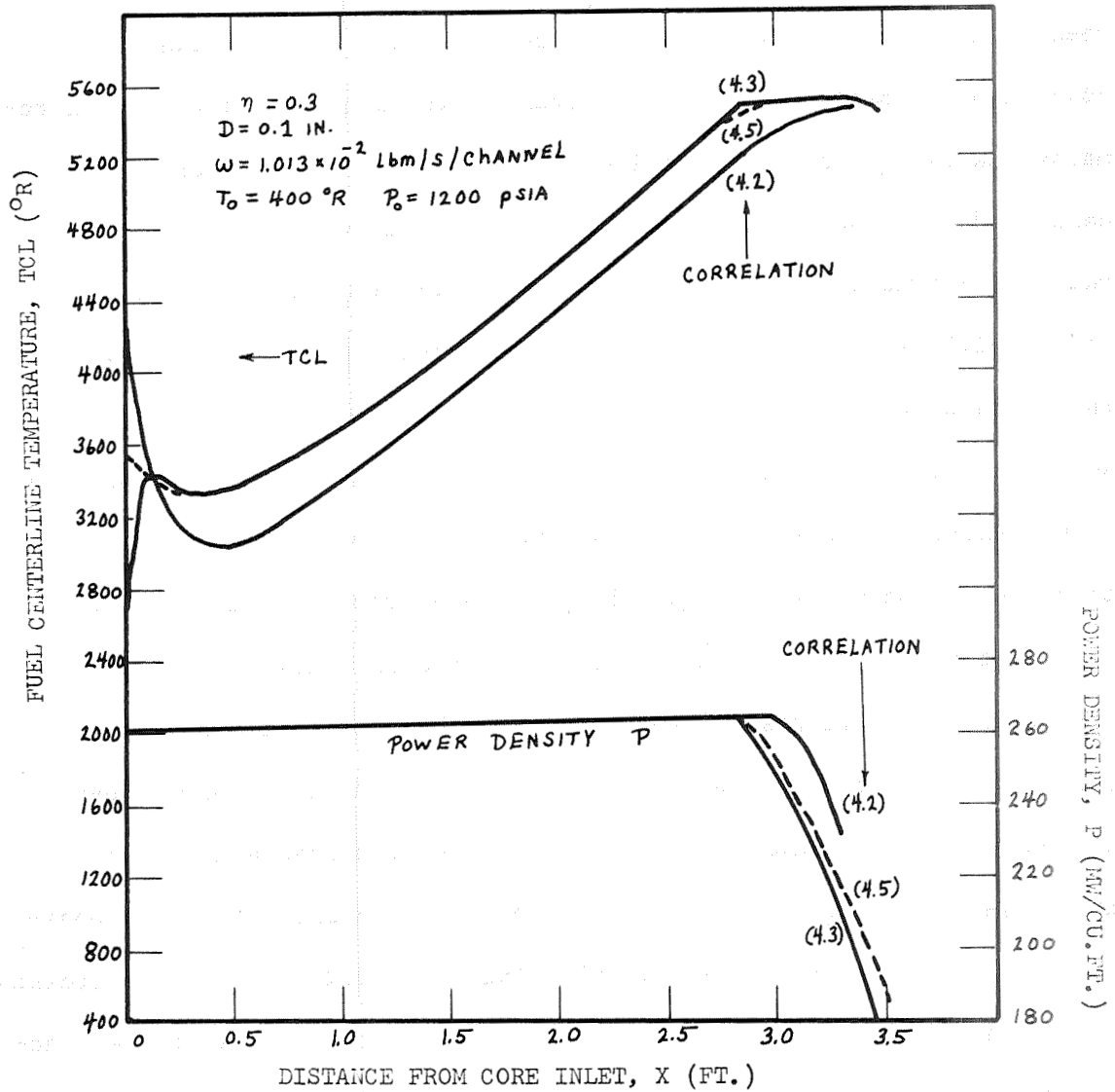


FIGURE 4-5 FUEL CENTERLINE TEMPERATURE DISTRIBUTIONS AND OPTIMUM POWER
 PROFILES FOR THREE CORRELATIONS

tures will increase at constant power, but not all wall temperatures everywhere along the channel will increase. That this is so will be seen by the following argument. Assume that at constant power the power density profile after the entrance bulk temperature reduction remains exactly the same. This ignores the perturbation in fission density due to the introduction of colder and more dense hydrogen which will actually occur in a nuclear rocket reactor. However, this effect will be largely restricted to a short entrance region because the hydrogen will rapidly become heated and the perturbation will be shown in Chapter 5 to increase the local power density by a maximum of 10-60% there. With the same power profile and momentarily colder entrance hydrogen, the hydrogen bulk temperatures along the channel will just be uniformly reduced. From the present wall temperature distribution in Figures 4-2 and 4-3 it can be seen that there is a bulk coolant temperature above which the wall and bulk coolant obey a "normal" relationship, i.e., they increase or decrease together. Therefore those exit portions of the reactor channel, which were operating near the material limits, will find the local T_w and TCL decreased after the entrance T_o reduction. For those inlet sections of the reactor channel which after the T_o reduction have bulk coolant temperatures below this critical temperature, the local T_w and TCL will have increased according to the "anomalous" behavior of the correlation prediction at low, bulk temperatures and high power densities.

However, the dynamic situation - reduction of inlet bulk temperature at full power - is qualitatively different for each particular correlation. Notice that Correlation (4.3) displays a "normal" tend-

ency for the wall temperature to increase with an increase in bulk coolant temperature at least for the first 1/3 foot, and then follows the anomaly depression for another short distance until the "normal" trend is resumed. Calculations with Equation (4.3) assuming an entrance bulk reduction from 400°R to 300°R show that the entrance wall temperature, $T_w(0)$, jumps from 1970°R to 2540°R and entrance fuel centerline temperature, $T_{CL}(0)$, jumps from 2610°R to 3180°R and that, moreover, the local "peak" in wall temperature distribution persists. Therefore this local peak would be the location at which fuel centerline melting or excessive wall corrosion by hydrogen would occur. On the other hand, Correlation (4.5) would predict trouble at the entrance $x=0$ only. Both Equations (4.3) and (4.5) therefore "allow" a certain amount of inlet bulk temperature reduction for stress-limited channels before material limits are exceeded. The temperature gradient across the channel wall in the example given was not increased so that the stress limit is not exceeded. If the channel situation were all wall-or fuel centerline - temperature limited, the reactor core could not "tolerate" the entrance bulk temperature reduction at constant power because one of the material temperature limits would be immediately exceeded by the increase in inlet T_w . Therefore, if the predicted optimum power density using Correlations (4.3) and (4.5) is stress-limited in the inlet region, which is the region immediately affected by bulk coolant temperature reductions, some entrant bulk temperature reduction can be tolerated, the amount of reduction depending in each situation upon how near to the materials limits the inlet region wall or fuel centerline is at steady-state operation.

The assertion that the anomalous behavior of T_w with respect to bulk temperature T does in fact depend upon some critical bulk temperature is seen in Figures 4-6 and 4-7 where the wall temperatures predicted by Correlations (4.3) and (4.5), respectively, are plotted against the bulk coolant temperatures for one set of reactor core parameters and one position, $x=0$ at the channel entrance. The critical temperature occurs at the point of inflection and is dependent upon specific power density - the larger that the power density is, then the higher that the critical temperature will be and consequently, the longer the channel region affected by the "anomalous" behavior of wall temperature. These figures show that many combinations of low initial bulk coolant temperature with high specific power density can greatly magnify the tendency to predict very high wall temperatures as result of reduction in bulk coolant temperature.

Figures 4-6 and 4-7 can be used for other void fractions at the same channel position, $x=0$, by scaling the specific power density curve parameter in this manner:

$$\frac{(1-\eta')}{\eta'} P' = \frac{(1-\eta)}{\eta} P, \quad (4.8)$$

where the quantities with a prime are the reference values for Figures 4-6 and 4-7. Since $\eta' = 0.5$ this scaling simplifies to :

$$P' = \frac{(1-\eta)}{\eta} P \quad (4.8.1)$$

as an example, Figures 4-1 through 4-5 are based upon $\eta=0.3$ and a situation where the bulk power density is stress-limited to 260 MW/Cu.Ft. By Equation (3.12):

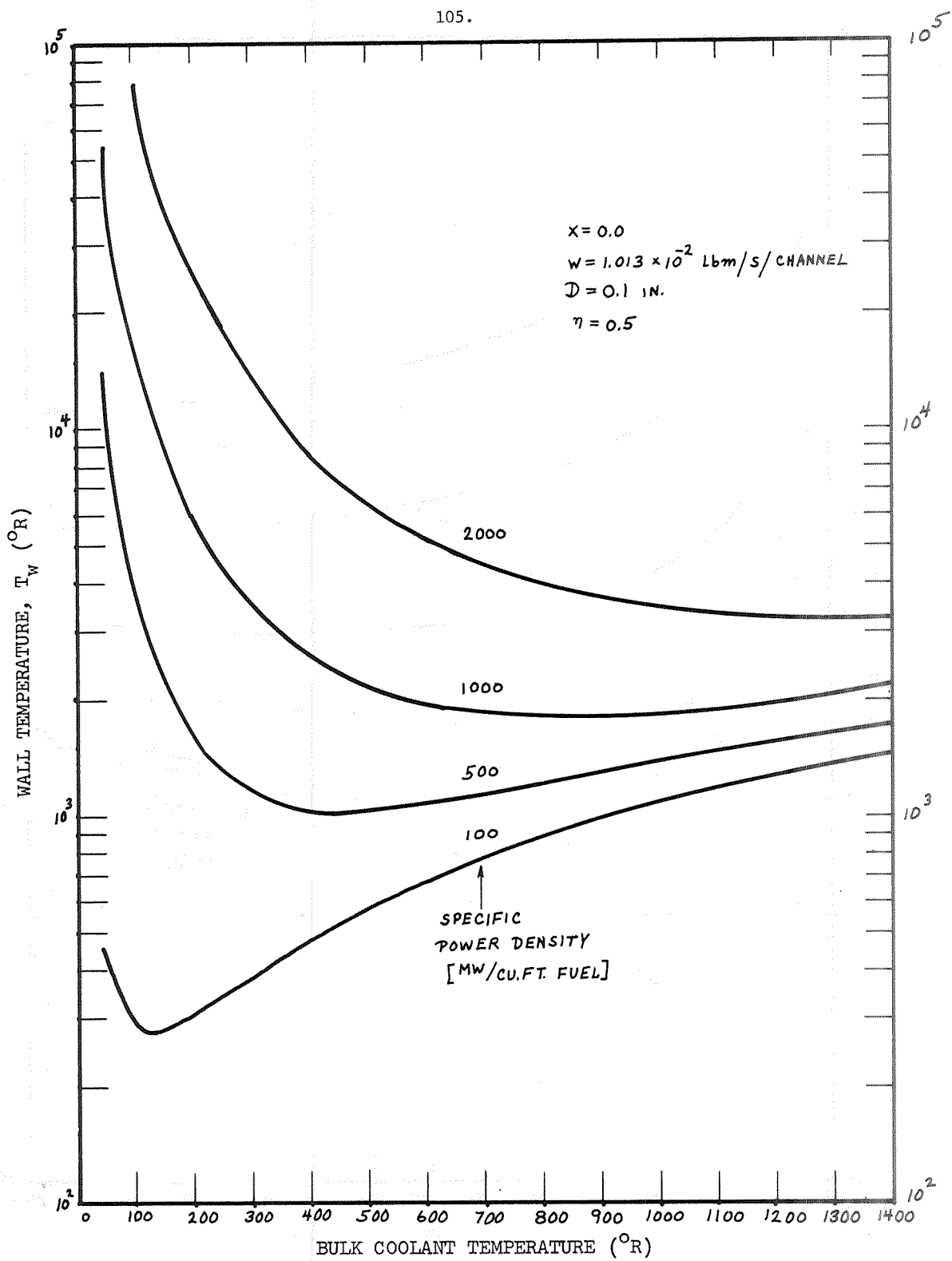


FIGURE 4-6 WALL TEMPERATURE VERSUS BULK COOLANT TEMPERATURE,
PREDICTED BY CORRELATION (4.3)

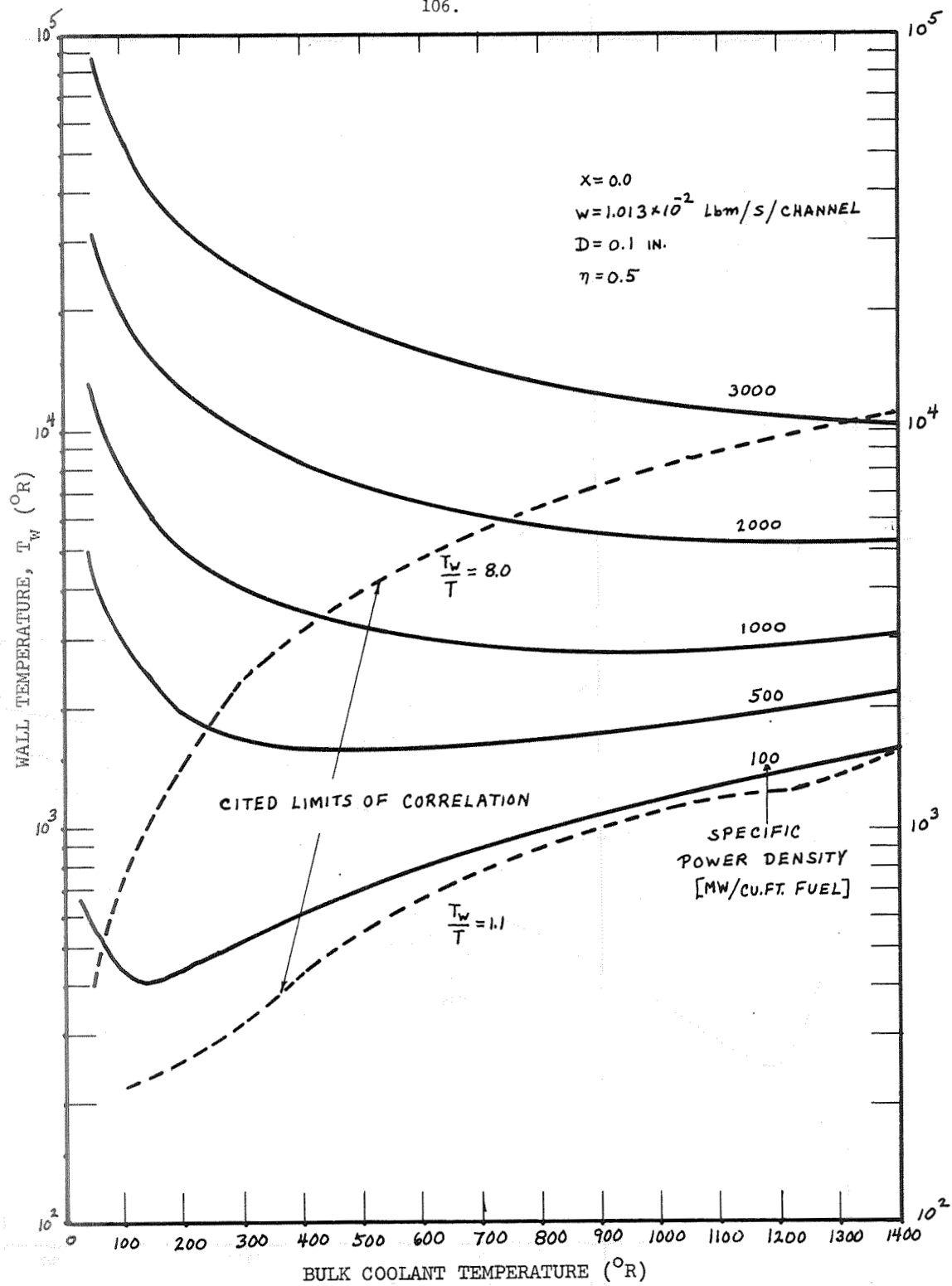


FIGURE 4-7 WALL TEMPERATURE VERSUS BULK COOLANT TEMPERATURE,
 PREDICTED BY CORRELATION (4.5)

$$\frac{\text{"Bulk" } P}{(1-\eta)} = \text{"specific" } P \quad (3.12)$$

(see Section 3.6) one finds $P' = 260/0.3 = 867 \text{ MW/Cu.Ft.}$ as the specific power density parameter to use in either Figure 4-6 or Figure 4-7. Thus, following an estimated curve labeled 867 MW/Cu.Ft. would enable one to predict the increase in T_w for a decrease in bulk T at $x=0$ for $\eta = 0.3$.

This scaling for void fractions is due to Equation (3.8):

$$[T_w(x) - T(x)] h = D (1-\eta) P(x)/4\eta . \quad (3.8)$$

All one can say from this equation is that a particular T_w and T are consistent with the quantity $(1-\eta) P/\eta$. If one wished to use Figures 4-6 and 4-7 to predict T_w versus T for a position different from $x=0$, then the explicit form for the heat transfer coefficient, h , would have to be considered. Equation (4.3) very easily lends itself to position scaling because of the separation of spatial and temperature ratio corrections:

$$\frac{hD}{k} = N_u = (0.045) \text{Re}^{0.8} \text{Pr}^{0.4} \left(\frac{T}{T_w} \right)^{0.55} \left(1 + \frac{x}{D} \right)^{-0.15} \quad (4.3)$$

so that T_w versus T could be found at any position x by using the curves of Figure 4-6 with the specific power density parameter P' changed according to:

$$P' = \left(\frac{1-\eta}{\eta} \right) P \left(1 + \frac{x}{D} \right)^{+0.15} . \quad (4.8.2)$$

Equation (4.5) does not lend itself to position scaling.

4.3.3. The Power Density "Anomaly"

The results of Sections 4.3.1 and 4.3.2 have been based upon one flow rate per channel, $w=1.013 \times 10^{-2}$ lbm/s/channel. For the same void fraction, but a lower flow rate, the optimum power density profile should be expected to become less stress-limited and even more influenced by the mathematical form of the heat transfer correlations. At a particular position, x , where the bulk coolant temperature is fixed at $T(x)$, the heat transfer coefficient goes approximately as $\sim w^{0.8} \left(\frac{T}{T_w} \right)^c$ or $\sim w^{0.8} \left(\frac{T}{T_{wMAX}} \right)^c$ for calculating the wall-temperature-limited power density, P_w where c = proper exponent at x and therefore a constant. Then by Equation (3.8.1) :

$$P_w(x) = [T_{wMAX} - T(x)] \frac{4 h \eta}{(1-\eta) D} , \quad (3.8.1)$$

and P_w goes approximately as $\sim \left[T_{wMAX} - T(x) \right] w^{0.8} \left(\frac{T}{T_{wMAX}} \right)^c$. Since the stress-limited maximum power density, P_s , is essentially constant, at some w , P_w must become smaller than P_s at the lower flow rate. The same is obviously true at a constant flow rate as the increase in bulk temperature decreases the $(T_{wMAX} - T)$ temperature difference. When the leveling or flattening of the stress limit is not found at x , then the mathematical form of the heat transfer function determines the predicted power shape to follow. In Figure 4-8 a flow rate $w = 0.44 \times 10^{-2}$ lbm / s/ channel, about one-half that used for Figures 4-1 through 4-7, results in partially-stress-limited optimum power profiles. Once again Equation (4.2) magnifies the wall temperature anomaly, being almost 600°R deeper than predictions by Equations (4.3) and (4.5) which agree well over the entire channel length.

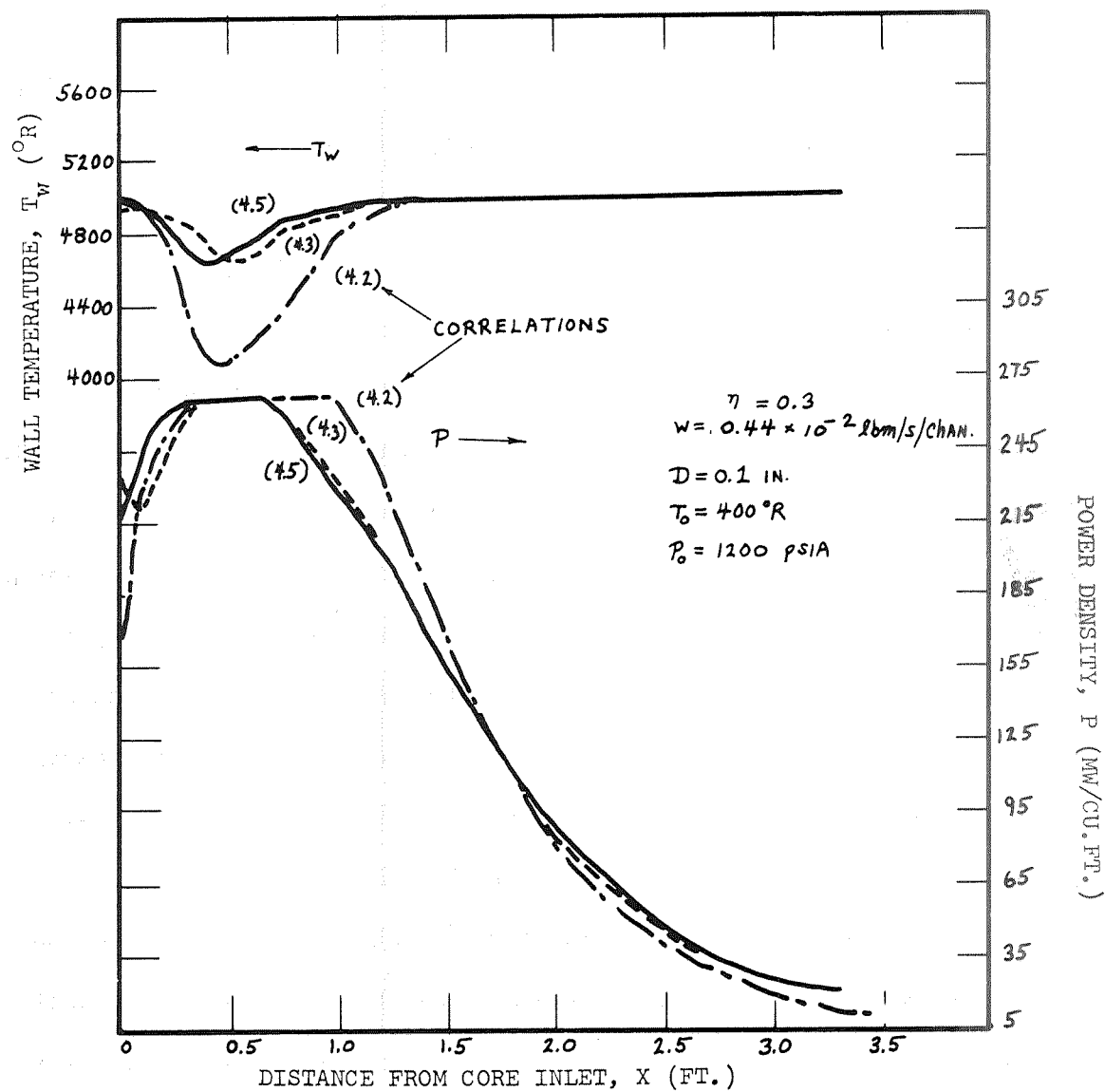


FIGURE 4-8 WALL TEMPERATURE DISTRIBUTIONS AND OPTIMUM POWER PROFILES FOR THREE CORRELATIONS, MEDIUM FLOW RATE PER CHANNEL

Relaxing the stress limit at $w = 1.013 \times 10^{-2}$ lbm/s/channel can also be accomplished by going to a reactor core with larger core void fraction. Physically this stress limit relaxation happens because the actual channel wall thickness becomes relatively thinner :

$$\frac{1}{\eta} - 1 = (D_o - D)/D \quad (4.9)$$

In terms of allowable maximum stress-limited power density, Table 3-4 shows that P_s^o increases from 377.2 MW/Cu.Ft. at core void fraction 0.3 to 2054 MW/Cu.Ft. at core void fraction 0.5. The allowable P_w and P_c also increase with void fraction, but not as rapidly as P_s so that at the higher void fraction they may be the limiting predicted power densities rather than P_s . Figure 4-9 confirms that for void fraction = 0.5, the stress power limitation is above any of the wall and fuel center-line temperature - limited power densities for the entire channel length, and the predicted optimum power profiles are solely influenced by the form of the heat transfer coefficient. The more recent Correlations (4.3) and (4.5) agree in predicted optimum power densities, except for those points very close to the entrance, while Correlation (4.2) peaks almost 140 MW/Cu.Ft. greater than the others.

The non-stress-limited optimum power profiles in Figures 4-8 and 4-9 are a result of variation in the calculated heat transfer coefficient. Because the increasing bulk temperature would always tend to decrease the temperature difference ($T_w - T$), then a constant h would imply an optimum power profile decreasing with x , by Fourier's law, Equation (3.8). So for wall-and centerline - limited optimum reactor core channels the so-called wall temperature "anomaly" converts to a "power density anomaly". At the partially-stress-limited situation of Figure

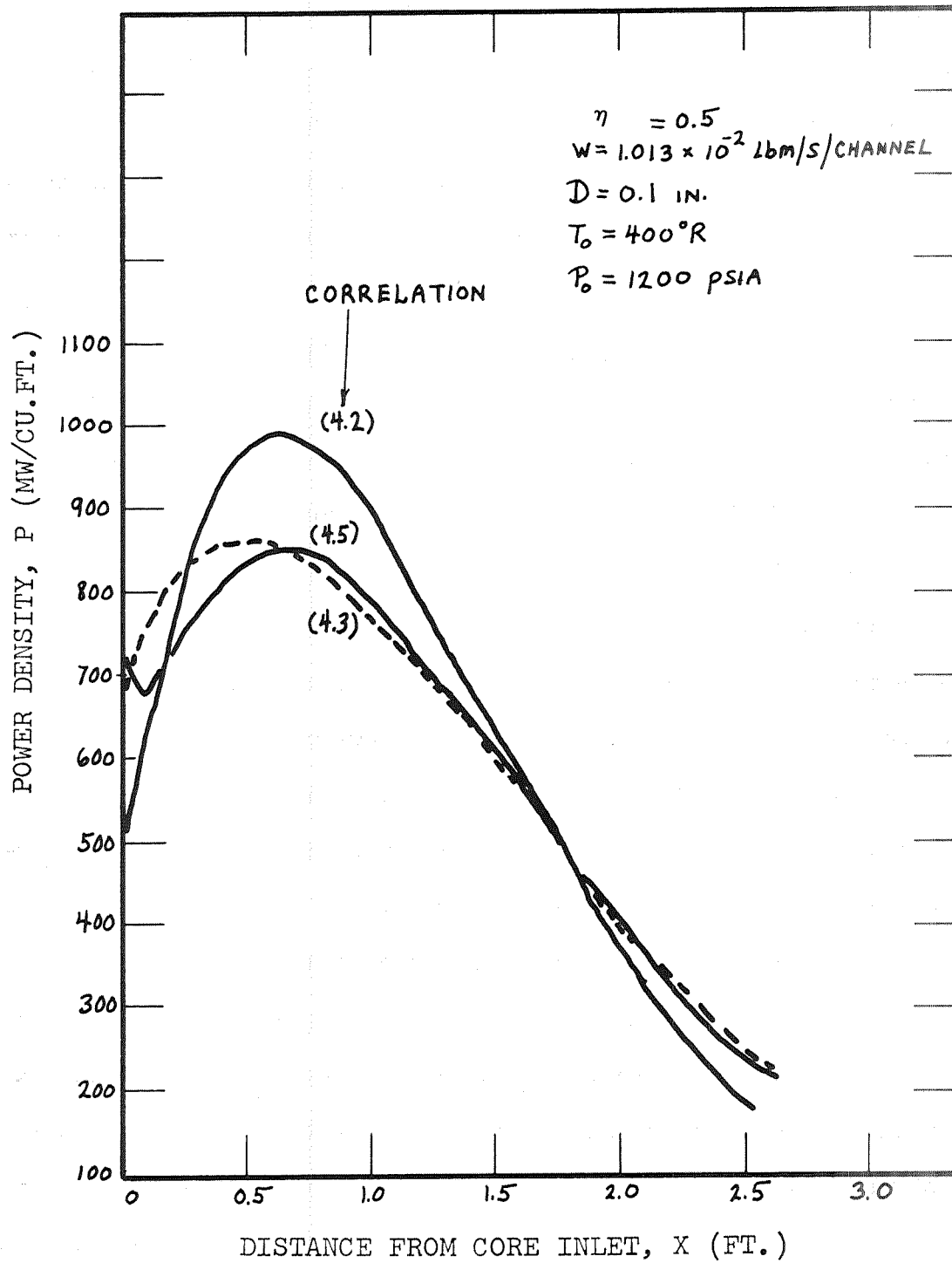


FIGURE 4-9 OPTIMUM POWER PROFILES FOR THREE CORRELATIONS,
HIGH CORE VOID FRACTION

4-8, we find a mixture: a power "anomaly" preceding in channel position x, a wall temperature "anomaly". The power density "anomaly" situation can tolerate, in the physical sense, no reduction in entrance hydrogen bulk temperature at constant power because this would immediately cause either the predicted wall or centerline temperature to rise above the respective limit.

4.3.4 Comparison of Equation (4.3) with Equation (4.6)

In Sections 4.3.1 through 4.3.3 it has been emphasized that Correlations (4.2), (4.3), and (4.5) all predict temperature anomalies for channel conditions associated with high power densities and low bulk coolant temperatures. Even though Correlation (4.5) is a systematic attempt to more accurately represent the hydrogen data available up to 1964, results using Equation (4.5) still agree closely with Equation (4.3). Such a wall temperature behavior could be experimentally produced (4), but most of the data had been taken at wall-to-bulk temperature ratios less than 4.0 and not at the high wall temperatures desired for nuclear rocket operation.

Unfortunately, the reactor designer must assume that correlations which accurately represent heat transfer data at one range of hydrogen conditions may remain approximately valid for extrapolation to more extreme conditions. Continuing experimentation and a comprehensive survey of all available turbulent single phase hydrogen heat transfer data resulted in Equation (4.6) (1968). Figure 4-10 very clearly shows the significant change that has occurred in the channel inlet section because of Equation (4.6). For these channel parameters and entrance hydrogen conditions, the power "anomaly", seen in Figure 4-9 and re-

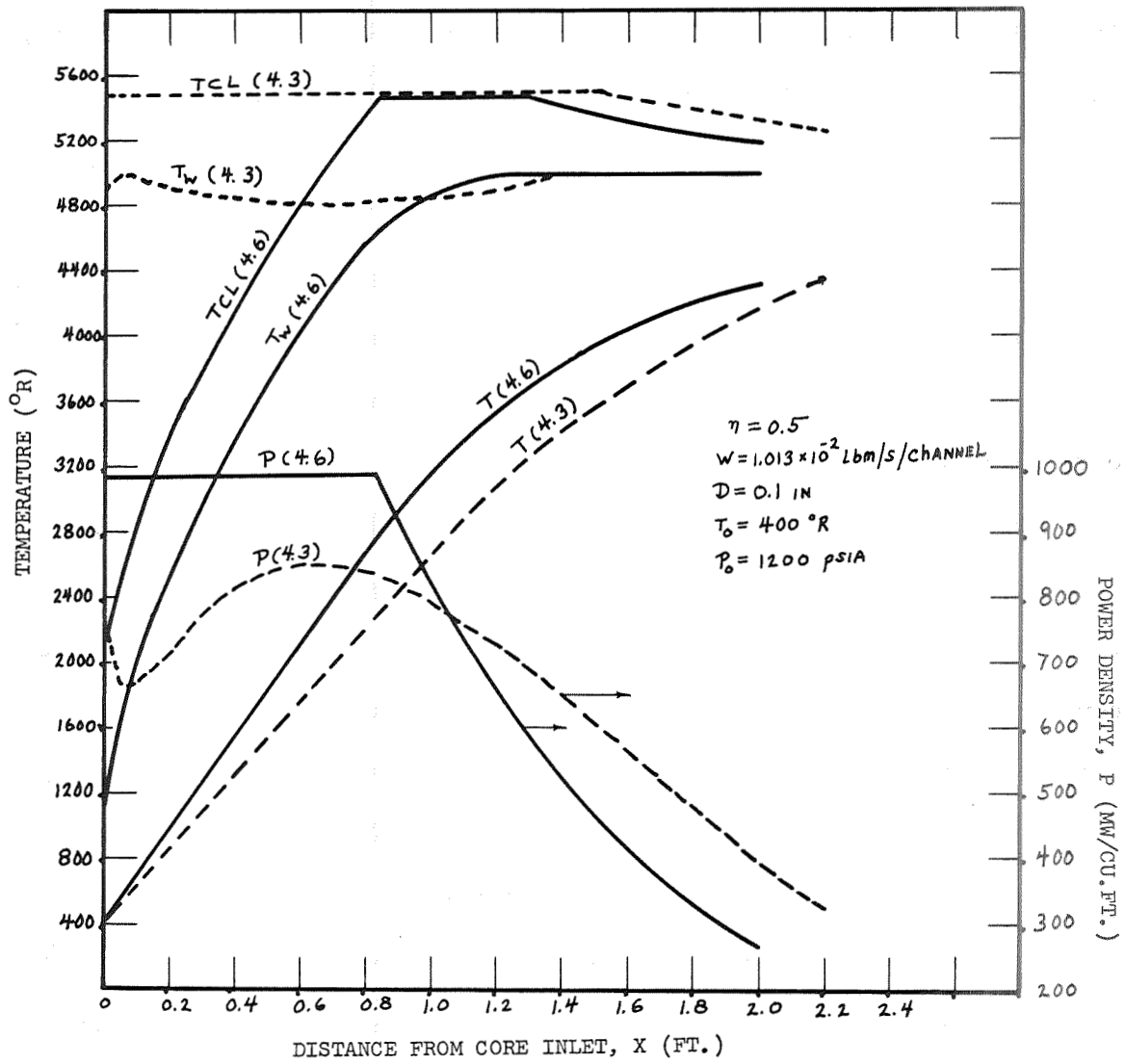


FIGURE 4-10 COMPARISON OF OPTIMUM REACTOR CHANNEL DISTRIBUTIONS AS PREDICTED BY CORRELATION (4.3) AND CORRELATION (4.6)

plotted in this figure, has yielded to the flat stress-limited power profile. Because P_s is essentially constant and independent of the correlational form as demonstrated in Section 3.6 then Correlation (4.6) must predict P_c 's and P_w 's which are larger than P_s ; hence, P_s becomes the limiting power density. Therefore Figure 4-10 indirectly graphically demonstrates an inequality for predicted maximum power densities :

$$P_w [(4.6)] > P_s > P_w [(4.3)]$$

and Equation (4.6) must represent a lesser resistance to heat flow into the hydrogen coolant at low bulk temperatures. That is, a given $(T_w - T)$ allows a greater power density with Correlation (4.6) than with Correlation (4.3).

Because of this lower resistance to heat flow into the hydrogen coolant (as indicated by lower wall temperatures), it is interesting to note that with Correlation (4.6) the wall-to-bulk temperature ratio never goes very much above 2, even though the experimental range of (T_w/T) covered by Equation (4.6) extends to $(T_w/T) = 27.6$, while Equation (4.3) which is presumably valid for all $(T_w/T) < 9.2$, consistently predicts $(T_w/T) > 12$ in the inlet channel regions. It cannot be definitely proven that Correlation (4.6) represents an improvement over Correlations (4.3) and (4.5), although the channel conditions surveyed for Correlation (4.6) would indicate this, because the stated accuracy of Equation (4.6) is much less than the stated accuracy of the other correlations. There is no explanation why this correlation should not predict the same "anomaly" as predicted by the others.

4.4 Parametric Survey of Optimum Reactor Channel Characteristics Associated with Correlation (4.3) Which Exhibits a Wall Temperature Anomaly.

4.4.1 Selection of w , T_o , P_o and D .

Experimental evidence indicates that a wall temperature drop can occur as the bulk coolant temperature rises (4), although to a smaller extent than predicted by either Equations (4.2) and (4.3). It is necessary to determine the effect that such a wall temperature anomaly has upon the predicted optimum power profiles. Since both Correlations (4.3) and (4.5) generally agree as to predicted power shapes and channel temperatures for many cases calculated, only Correlation (4.3) was selected for a parametric study. Both correlations are sensitive to the low inlet bulk temperatures probably required for the nuclear rocket hydrogen coolant so two core entrance temperatures of $T_o=300^\circ \text{R}$ and $T_o=500^\circ \text{R}$ were chosen for the calculations in this Section. The selection of the core inlet hydrogen temperature for steady-state operation can be changed arbitrarily by the reactor designer by adjusting the temperature of the hydrogen inserted into the regeneratively-cooled nozzle. Earlier studies (6,10) using the correlational $(T/T_f)^{0.8}$ form of Equation (4.2) indicated that a flow rate near $w = 1.013 \times 10^{-2} \text{ lbm/s/channel}$ (4.6 grams/s/channel) is optimum for producing minimum weight nuclear rocket systems for a flow diameter (or hydraulic diameter) of $D = 0.1$ inches. This flow rate is tentatively adopted in this parametric survey as the "high flow rate", but a lower flow rate might be necessary if the large pressure drops along the channel which result as reported in this Section are unaccept-

from more detailed considerations of the mechanical support of the reactor core.

Selection of a lower steady-state flow rate is complicated because the temperature dependence of hydrogen viscosity,

$$\mu = 1.553 \times 10^{-7} T^{0.6} \left[\frac{\text{lbm}}{\text{sec-ft}} \right] \quad (3.7)$$

means that, in terms of Reynolds number, hydrogen can enter a channel safely in the turbulent flow condition and leave the channel safely in the laminar flow regime because the Reynolds number has decreased as the bulk temperature has increased. Heat transfer is much more efficient for turbulent flow than for laminar flow meaning that the laminar flow portions of the channel are much more restricted as to allowable power densities. For all-laminar flow, the possibility of a "laminar flow instability" exists in that for two identical channels having the same rate of heat addition and with the total pressure drop held constant there can be two flow rates. The lower flow rate is unstable and channel burn-out can result. Reshotko (20) has analytically verified this behavior in terms of the amplifications of infinitesimal disturbances and estimated that the characteristic times of this instability are on the order of seconds and minutes. In other words, a design based upon steady-state laminar flow at a flow rate per channel W_1 , might find that laminar flow operation could actually establish flow rates of W_1 and W_2 in adjacent channels having identical heat addition rates and a common δP_* imposed upon both. The "laminar flow instability" then would drive the lower flow rate W_2 to zero and burn-out would result. To avoid channel burn-out the cool-down of a nuclear rocket reactor would require pulsed

cooling to insure turbulent flow, rather than a steady low flow of propellant to remove the fission product decay heat and recently reported NERVA test data (42) show that cool-down can take days for an hour's steady operation.

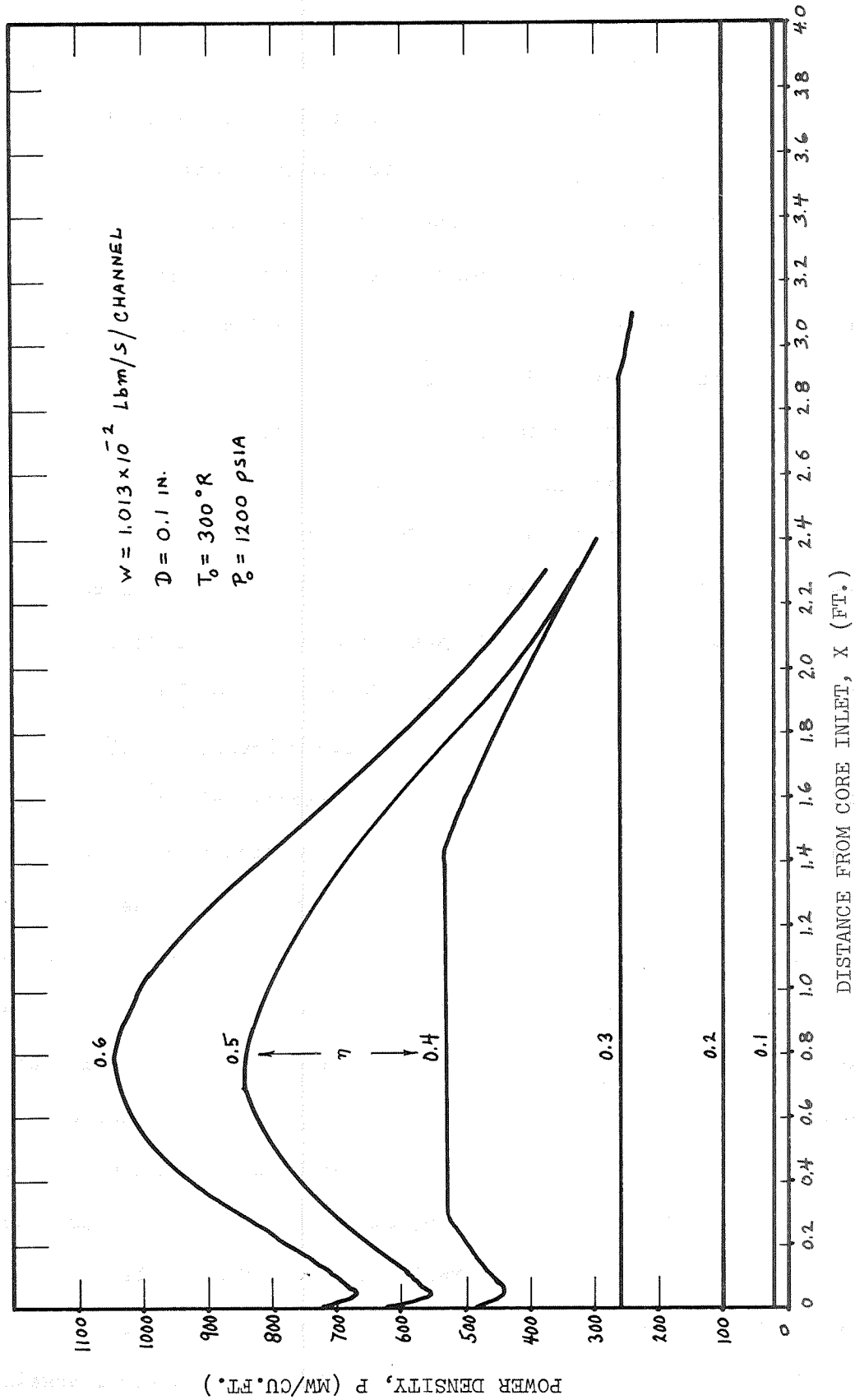
Steady-state operation must also insure turbulent flow. The proper Reynolds number for transition from turbulent to laminar flow is taken to be 2100 by Bussard (11, p. 128), 2200 by Ellerbrock (19), and 5000 by Yasui (8), while Harry (21) and Turney (22) use the criteria that laminar flow will remain laminar up to $Re = 2100$ and turbulent flow will remain turbulent down to $Re = 2100$. A flow rate of $w = 1.013 \times 10^{-3}$ lbm/s/channel is adopted for convenience in this study as the "low flow rate" and insures that turbulent flow conditions prevail along any entire channel length; that is, the Reynolds number corresponding to the above "low flow rate" at $T = 5000^\circ R$ is $Re = 6.014 \times 10^3$.

The above calculations are based upon a flow (or hydraulic) diameter $D = 0.1$ inches. Preliminary calculations (6,8) have shown that a very small flow diameter (as small as possible consistent with fabrication capabilities) is desirable and the recent extensive nuclear mission optimization study (13, Figure 7) indicates that this flow diameter is a practicality for the Rover nuclear rocket engines. Calculations with several entrance hydrogen pressures, P_o , have shown this P_o parameter to be not critical in determining any optimum power profile - which is consistent with the demonstration in Section 3.6 that the stress-limited maximum power density, P_s , is practically independent of the channel hydrogen conditions. The value 1200 psia is presented in the following figures and is a familiar pressure level for nuclear rocket studies.

4.4.2 Optimum Reactor Channel Characteristics as a Function of Void Fraction.

Figures 4-11 through 4-15 show the optimum reactor characteristics as predicted by Correlation (4.3) with the core void fraction as the curve parameter and $T_o = 300^\circ \text{R}$. Figure 4-11 presents the optimum power profiles for the steady state "high flow rate", $w = 1.013 \times 10^{-2} \text{ lbm/s/channel}$. The two most prominent features for these power profiles are the increase in peak maximum power density and the relaxation of the stress limitation as the core void fraction is increased. For a constant channel flow diameter, larger void fractions mean relatively thinner channel wall thicknesses and less limitation from thermal stresses. Those portions of the channel which have the flat power shape are stress limited as explained in Section 3.6, and therefore the wall and fuel centerline temperatures there are below TWMAX and TFMAX, respectively, because only one thermal limitation can apply at each channel position.

Without knowing the internal channel temperature distributions, one can see that an unusual situation must occur at the inlet sections of these optimized channels. For $\eta = 0.4$ the power profile goes through a minimum between $x=0$ and $x = 0.3 \text{ ft.}$, followed by the flatness indicating stress limitation. Because this is a maximum power density profile any increase in optimum power density along the channel is highly unusual and contrary to common experience with constant-property coolants. This is the power density "anomaly" explained in Section 4.3.3 and can only occur for non-constant heat transfer coefficients. With Correlation (4.3) the onset of the power density anomaly occurs at $x = 0.05 \text{ feet}$ ($x/D = 6$). Interestingly, the optimum power shapes for $\eta = 0.5$ and $\eta = 0.6$ are very similar to those found in typical end-reflected, uniformly loaded nuclear reactor cores, whereas the optimum power shapes


 FIGURE 4-11. OPTIMUM POWER PROFILES PREDICTED BY CORRELATION (4.3) AS A FUNCTION OF η

for $\eta = 0.4$ would certainly require a non-uniform uranium distribution to form the proper steady-state power distribution. If Correlation (4.3) is valid for the large wall-to-bulk temperature gradients expected in nuclear rocket reactor steady-state operations, then having a core $\eta \geq 0.5$ would be very desirable from the standpoint of uranium fuel rearrangement since the power shape for uniformly loaded nuclear reactors would closely match the desired profile.

The reactor cores having the low void fractions $\eta = 0.1$ and $\eta = 0.2$ consistently display unfavorably long channel lengths as compared to the higher η 's, whether optimized or not. For these cases, the optimum power densities are stress-limited to such low values that the gas cannot be heated very rapidly. Figure 4-12 shows that at $x = 3.1$ ft., $\eta = 0.1$ achieves $T = 1360^\circ \text{R}$, $\eta = 0.2$ achieves $T = 2680^\circ \text{R}$, while $\eta = 0.3$ achieves $T = 4000^\circ \text{R}$. Even at $x = 4$ ft., $\eta = 0.1$ achieves $T_{\text{exit}} = 1650^\circ \text{R}$ as compared to $T_{\text{exit}} = 3200^\circ \text{R}$ for $\eta = 0.2$. Figure 4-12 shows the definite advantage in choosing a high core void fraction to both increase outlet temperature of the propellant hydrogen and decrease the required channel length. For $\eta = 0.4$ an outlet temperature of 4000°R is reached at a foot shorter distance than for $\eta = 0.3$, and $\eta = 0.4$ can continue to heat the coolant to $T = 4200^\circ \text{R}$ for 0.3 extra feet as explained in the next paragraph. There is a diminished incentive for much higher core void fractions because the hydrogen coolant temperature distribution changes very slightly from $\eta = 0.4$ to $\eta = 0.6$, although the peak optimum power density required increases from 530 MW/Cu.Ft. to 1040 MW/Cu.Ft.

The decreased channel length that accompanies the more efficient heating profile of the higher η 's is determined by the available pressure

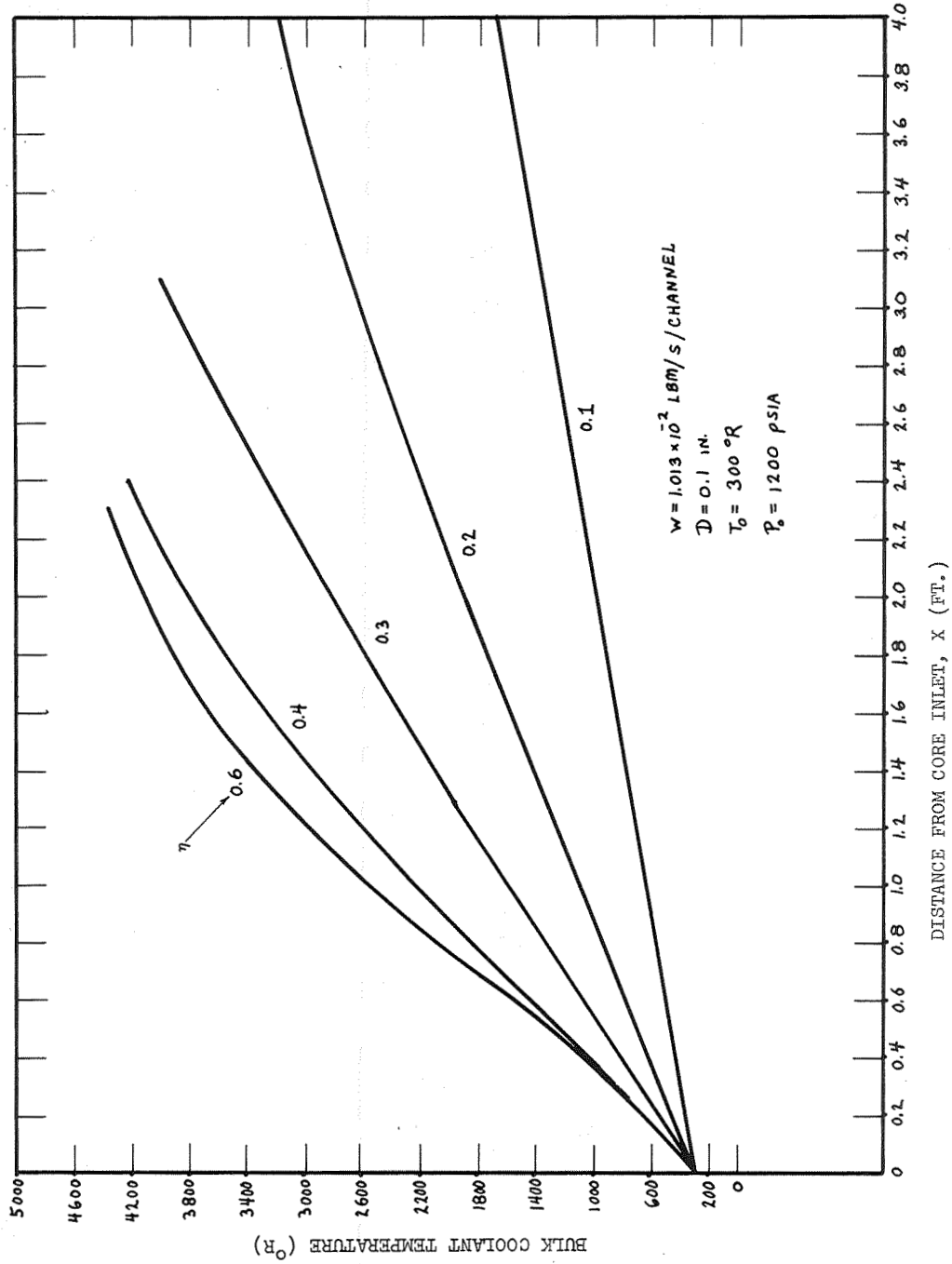


FIGURE 4-12 BULK COOLANT TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (4.3)

drop. Figure 4-13 displays the pressure and the density distributions associated with the optimum power and temperature profiles of Figures 4-11 and 4-12. At any given channel position, x , the pressure drop increases as η increases, although between $\eta = 0.4$ and $\eta = 0.6$ the pressure distribution shifts relatively little. The hydrogen density, p , also follows this trend. As discussed in Section 3.7, the computer program "tests" the available pressure drop at position x to see if there is enough pressure to allow the minimum extrapolation of the present power density, $P(x)$, to zero at $x + \Delta x$. In other words, since the implicit integration of Equation (3.11.1) :

$$w \bar{c}_p [T (x + \Delta x) - T (x)] = \left(\frac{1 - \eta}{\eta} \right) \left(\frac{\pi D^2 \Delta x}{4} \right) \left[\frac{P (x + \Delta x) + P (x)}{2} \right] \quad (3.11.1)$$

assumes a linear-with- x behavior of the calculated power density, the bulk coolant temperature $T (x + \Delta x) > T (x)$ even with $P (x + \Delta x) = 0$ and this, in turn, implies a hydrogen density change and a minimum pressure drop from x to $x + \Delta x$. If the minimum pressure drop is greater than the available pressure $P_*(x)$ then the calculation is terminated at x and no attempt is made to achieve $P_{*exit} = 0$. In a sense, the channel lengths calculated are not really the maximum allowable, but because $x \leq 0.1$ ft. the difference is very slight. Figure 4-13 shows that δP_* increases from 190 psia at $\eta = 0.1$ to 470 psia at $\eta = 0.6$. Due to the increased force imposed on the core, these higher pressure losses increase the structural weight of the base core inlet support; however, all reactor cores with $\eta \geq 0.3$ bear the same weight penalty since the δP_* is roughly the same for all η above this core void fraction. Increasing P_o would increase the allowable channel length because there is an inverse relationship be-

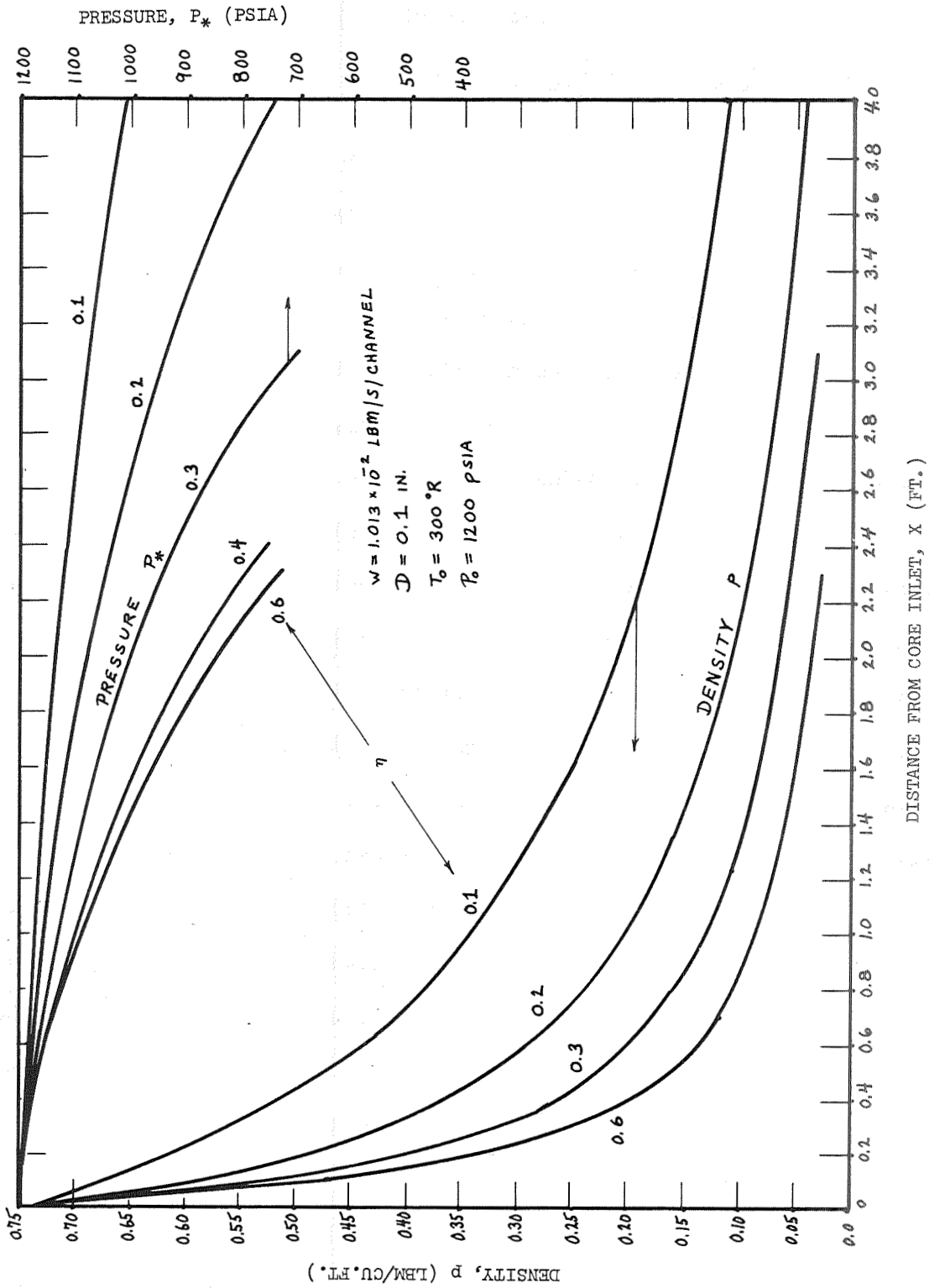


FIGURE 4-13 PRESSURE AND DENSITY DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (4.3)

tween entrance pressure and total pressure drop. The momentum Equation (3.1) can be approximately integrated to give (11, p. 146):

$$\Delta P_* = \frac{R u G^2 T_m^1}{g_c P_m^M} \left[\ln \frac{P_{in}}{P_{out}} + 2f \frac{L_c}{D} \right], \quad (4.10)$$

where T_m^1 = mean bulk coolant temperature = $1/2 (T_o + T\text{-exit})$,

P_m = mean coolant pressure = $1/2 (P_o + P_{*-exit})$,

P_{in} = coolant density at channel entrance,

P_{out} = coolant density at channel exit,

and the remaining symbols have been defined in Equation (3.4), Section 3.4, which shows that a higher mean pressure P_m , and thus higher P_o , yields a lower ΔP_* . Decreasing the flow rate would also decrease ΔP_* .

From Figure 4-11 one can only tell where the wall or fuel centerline temperature restrictions are not limiting because of the flatness of the optimum power profile, otherwise one must look at the detailed temperature distributions to see which restriction limits the power density at position x . In Figure 4-14 one sees that for $\eta = 0.6$ the wall temperature T_{WMAX} dominates the entire channel length. Proper wall coating materials to reduce carbon corrosion by the hot hydrogen coolant would be extremely important for steady-state operation of any optimum reactor core with $\eta \geq 0.6$. Reactor cores with the lower void fractions $\eta = 0.4$ and $\eta = 0.5$ are more fuel-centerline temperature limited because of the relatively thicker channel walls, and reactor cores with the lowest void fractions $\eta = 0.1$ to 0.3 and also the thickest channel walls are almost entirely stress-limited.

Figure 4-14 shows that the wall temperature anomaly first appears for $\eta = 0.2$ where a slight dip is discernable at $x = 0.3$ feet. The anomaly temperature depression reaches its maximum for $\eta = 0.4$ where T_w de-

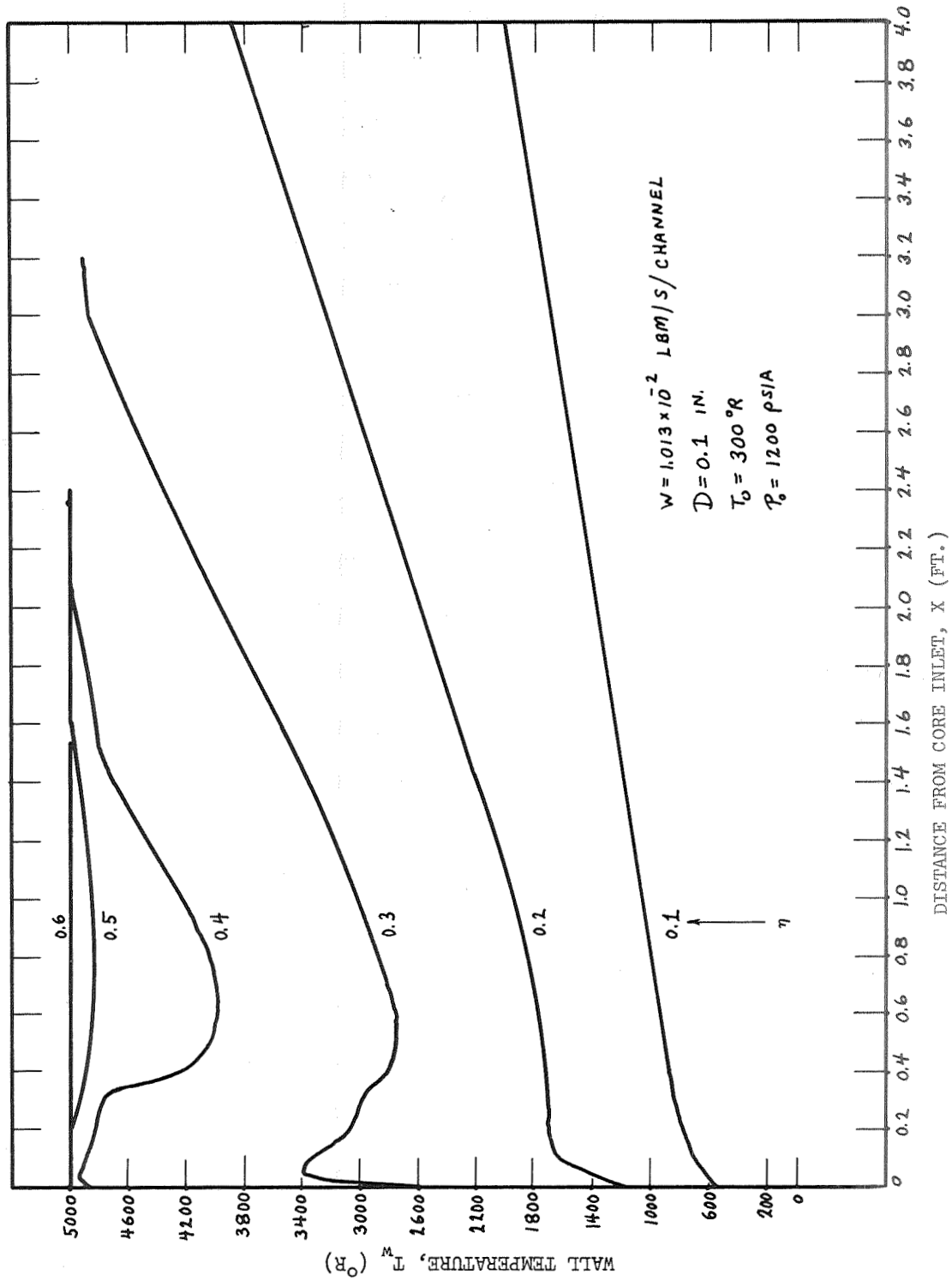


FIGURE 4-14 WALL TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (4.3)

creases by 960°R over the same 0.6 ft. interval for which T increases by 1140°R . The increasing magnitude of the wall temperature depression in going from $\eta = 0.2$ to $\eta = 0.4$ can be easily understood with referral to Figure 4-6 where T_w is plotted versus T on lines of constant specific power density. At any fixed bulk coolant temperature, T , one sees that increases in specific power density (or bulk power density since there is the simple relationship of Equation (3.12) between these two power densities) predict larger T_w 's. The maximum allowable specific power density increases with core void fraction (refer to Table 3-4) so that the inlet section wall temperatures should increase with η , as Figure 4-14 shows. The point of inflection for any curve in Figure 4-6, which marks the "critical" bulk coolant or wall temperature beyond which T_w and T increase or decrease together (at constant power density), also increases as the specific power density increases, but more slowly than the "peak inlet T_w ". Therefore the magnitude of "peak inlet" T_w minus "critical" T_w , which is the size of the wall anomaly depression, also increases with core void fraction. Figure 4-15 shows that it is the fuel-centerline temperature limitation which restricts both wall temperatures and maximum power densities at the inlet sections for $\eta = 0.4$ and $\eta = 0.5$ and the curves of Figure 4-6 cannot be so straight-forwardly applied because the power density is no longer constant along the channel. However, because the inlet T_w 's can no longer increase due to the fuel-centerline temperature limitation, the temperature depression reaches a maximum at $\eta = 0.4$, decreases for $\eta = 0.5$, and then vanishes entirely at $\eta \geq 0.6$ because the wall temperature limitation dominates the entire channel, forcing all $T_w = T_{w\text{MAX}}$.

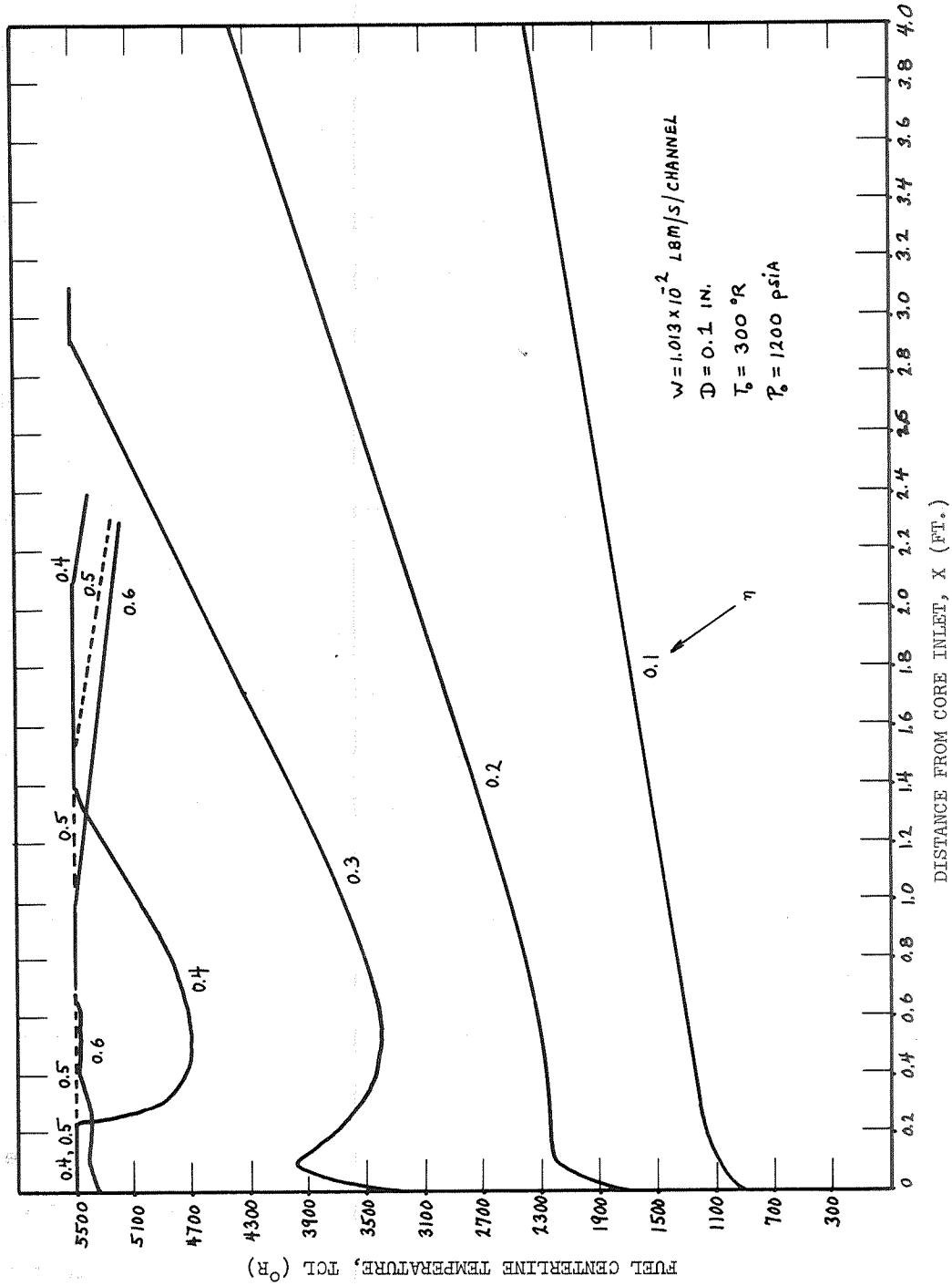


FIGURE 4-15 FUEL CENTERLINE TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (4.3)

When using a chopped-cosine-axial power shape, Cooper (6) noted a wall temperature depression (predicted by use of the modified Reynolds number) and suggested that at least the steady-state appearance of this anomaly could be made to "go away" by pre-heating the entrance coolant, although he expressed a warning about the possibilities of any inlet bulk temperature reductions occurring at constant power. Figures 4-6 and 4-7 show that, depending upon the operating power density, a considerable increase in inlet gas temperature above 300°R could be required to place steady-state operation at $x = 0$ safely above the anomaly-producing "knee" of the constant-power-density curves. In any case, preheating the entrance hydrogen while maintaining the same steady-state axial power profile does not eliminate the anomalous behavior of the wall temperature if entrance bulk coolant temperature is reduced to a very low value, and pre-heating could incur a weight penalty if this meant that hydrogen gas had to be introduced into the regeneratively-cooled nozzle at a higher temperature. By Equation (4.1) a decrease in hydrogen density means that more propellant tank mass is required.

However, preheating the incoming hydrogen gas would be advantageous when combined with heat-transfer optimization because of the peculiar behavior of Correlation (4.3). Because T_w and T behave inversely for bulk temperatures below a "critical" value, increasing the entrance T will predict a larger allowable maximum power density for the same value of T_w - this behavior manifests itself as the power density anomaly for those optimum power profiles in Figure 4-11 having $\eta \geq 0.4$. Beginning the axial power distribution with higher power densities should increase the achievable outlet temperature and I_{sp} which would offset the disadvan-

tage of increased propellant tank mass, remembering that typically one second $I_{sp} = 3000$ lbs WIEO (36).

Figures 4-16 through 4-20 show the optimum reactor channel characteristics as predicted by Correlation (4.3) with $T_o = 500^\circ R$. Figure 4-16 also displays the power density anomaly for $\eta = 0.5$ and $\eta = 0.6$. In comparison with these same core void fractions at $T_o = 300^\circ R$ (see Figure 4-11), the entrance and peak optimum power densities are roughly 30 MW / Cu.Ft. higher, confirming that an increase in T_o allows higher power densities along the channel. At $T_o = 500^\circ R$, $\eta = 0.4$ has no power density anomaly in the inlet section. Notice by comparison of Figure 4-16 with Figure 4-11 that if steady-state operation were designed around $T_o = 500^\circ R$, a reduction to $T_o = 300^\circ R$ at constant power would cause the inlet section of $\eta = 0.4$ to operate 90 MW / Cu. Ft. in excess (at the worst point) of the maximum power density determined for $T_o = 300^\circ R$. For $\eta = 0.1$ to 0.3 and $T_o = 500^\circ R$ the optimum power profiles are still all stress - limited and identical to the $T_o = 300^\circ R$ shapes so that one can directly compare the coolant, wall, and fuel-center-line temperature distributions for the effects of increasing T_o .

Figure 4-17 in comparison with Figure 4-12 shows the effect of increased T_o : $\eta = 0.6$ has $T_{\text{exit}} = 4380^\circ R$ at $x = 2.1$ ft. for $T_o = 500^\circ R$ while $T_{\text{exit}} = 4360^\circ R$ at $x = 2.3$ ft. for $T_o = 300^\circ R$, $\eta = 0.4$ has $T_{\text{exit}} = 4260^\circ R$ at $x = 2.3$ ft. for $T_o = 500^\circ R$ while $T_{\text{exit}} = 4240^\circ R$ at $x = 2.4$ ft. for $T_o = 300^\circ R$. Therefore, increasing T_o has resulted in shorter channel lengths and higher outlet temperatures for those core void fractions ($\eta = 0.4$ to $\eta = 0.6$) which do not have stress-limitations.

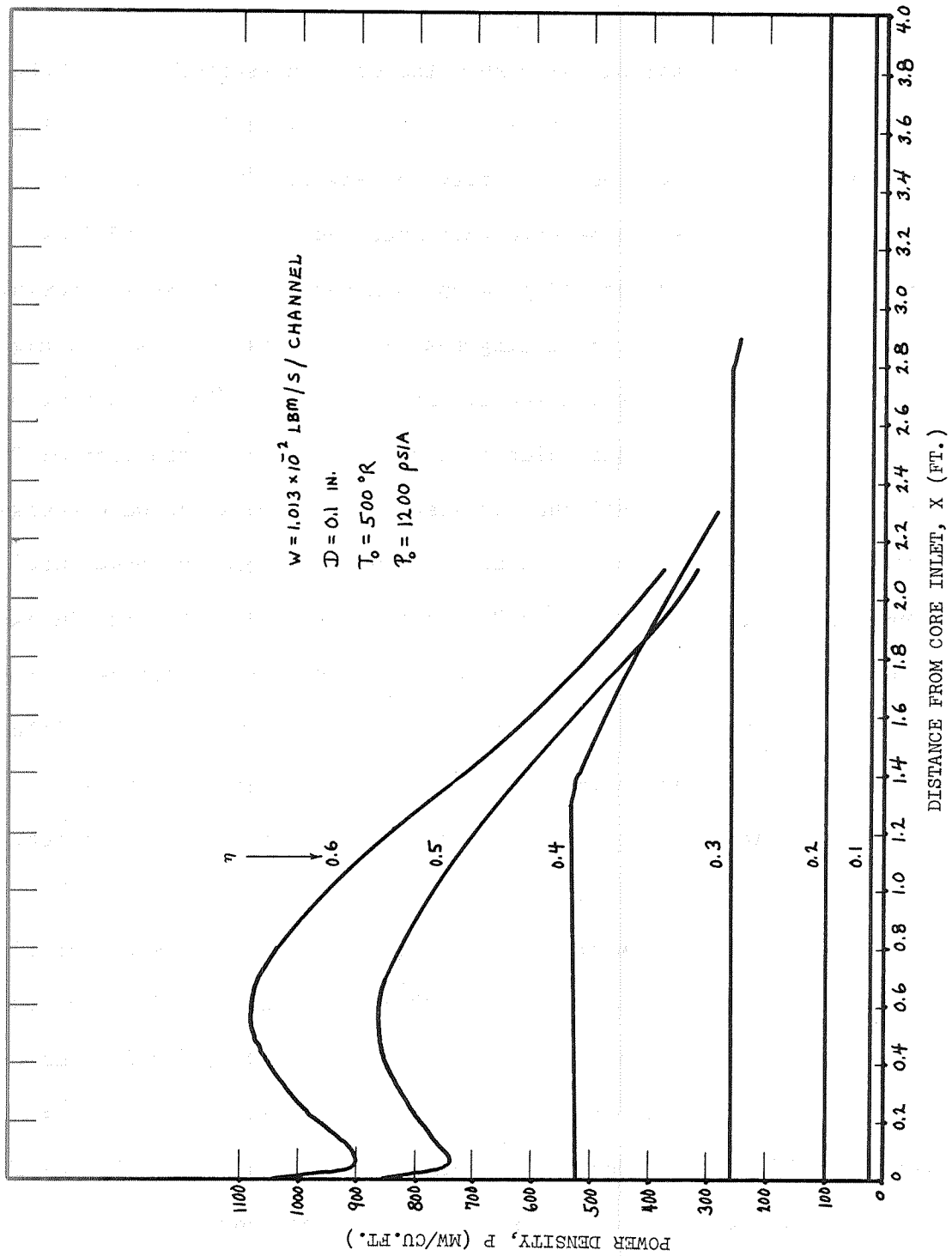


FIGURE 4-16 OPTIMUM POWER PROFILES PREDICTED BY CORRELATION (4.3) AS A FUNCTION OF η , INCREASED T_0

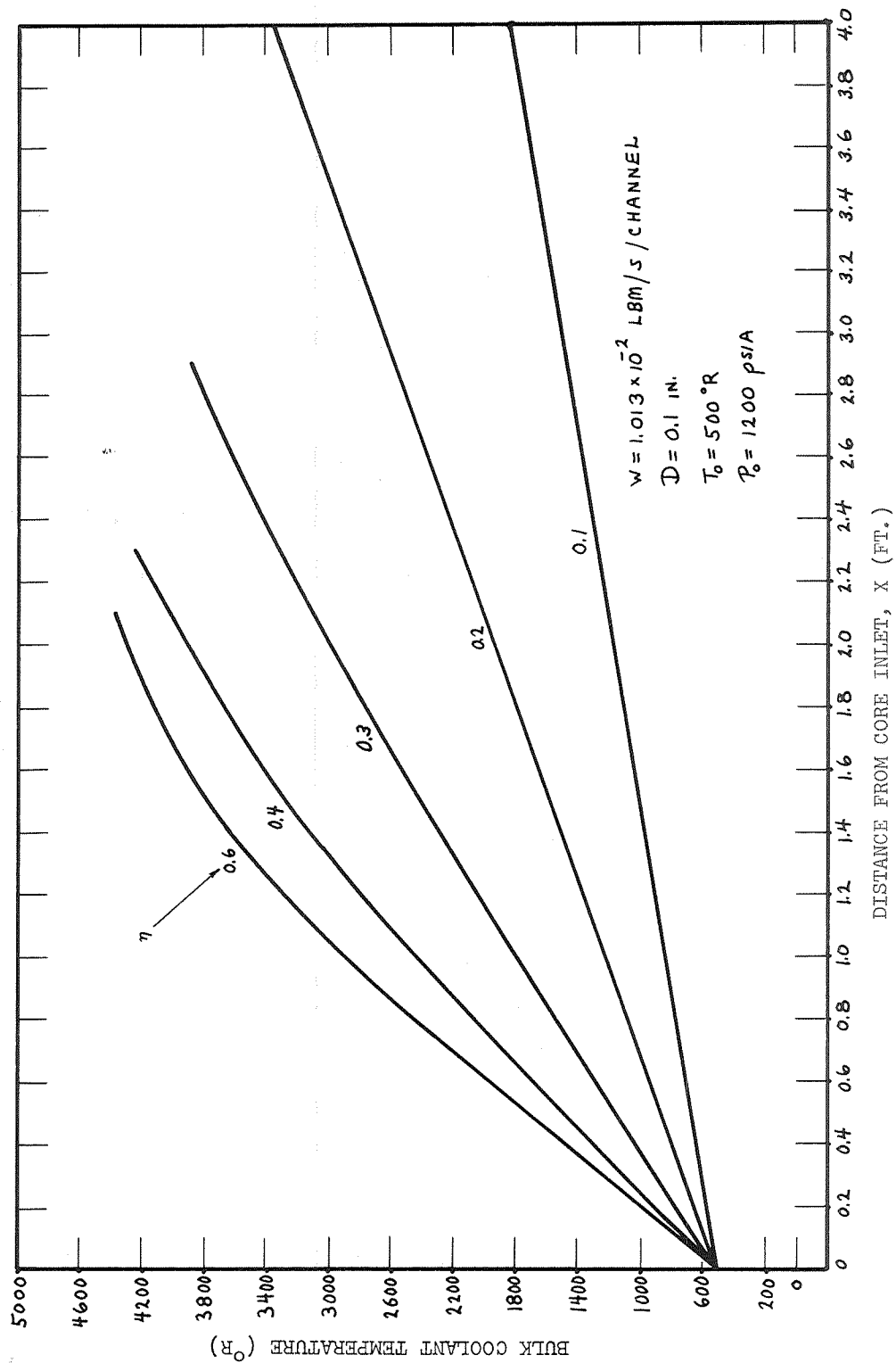


FIGURE 4-17 BULK COOLANT TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (4.3), INCREASED T_0

Therefore, increasing T_o has resulted in shorter channel lengths and higher outlet temperatures for those core void fractions ($\eta=0.4$ to $\eta=0.6$) which do not have stress-limitations.

Figure 4-18 exhibits the same trends as Figure 4-13 and the only significant change is that the hydrogen densities are lower for $T_o = 500^\circ\text{R}$ than for $T_o = 300^\circ\text{R}$. Figure 4-19 shows that the wall temperature anomaly is just discernable for $\eta=0.3$, whereas at the lower T_o (see Figure 4-14) the wall anomaly is just discernable for $\eta=0.2$. The magnitude of the anomaly temperature depression also passes through a maximum for $\eta=0.4$. The wall temperature limitation TWMAX dominates $\eta=0.6$, and Figure 4-20 also reveals $\eta=0.4$ and 0.5 to be partially fuel-centerline limited. (see Figure 4-15).

The completely stress-dominated void fractions $\eta=0.1$ to 0.3 reveal in Table 4-4 the effects of increasing the inlet gas temperature T_o at constant power.

Table 4-4

Effects of Increasing T_o at Constant Power (Stress-Limited)

All comparisons are at $x = 0$				
η	$T_o = 300^\circ\text{R}$ $T_w(0)$	$T_o = 500^\circ\text{R}$ $T_w(0)$	$T_o = 300^\circ\text{R}$ $T_{CL}(0)$	$T_o = 500^\circ\text{R}$ $T_{CL}(0)$
0.1	558	698	921	1062
0.2	1164	1057	1693	1585
0.3	2556	1741	3192	2622

For $\eta=0.3$, Figures 4-20 and 4-15 also show that the fuel-centerline temperature jumps by 900°R at $x = 0.1$ ft. as T_o is decreased by 200°R . Further decreases in T_o would cause the fuel-centerline temperature limit

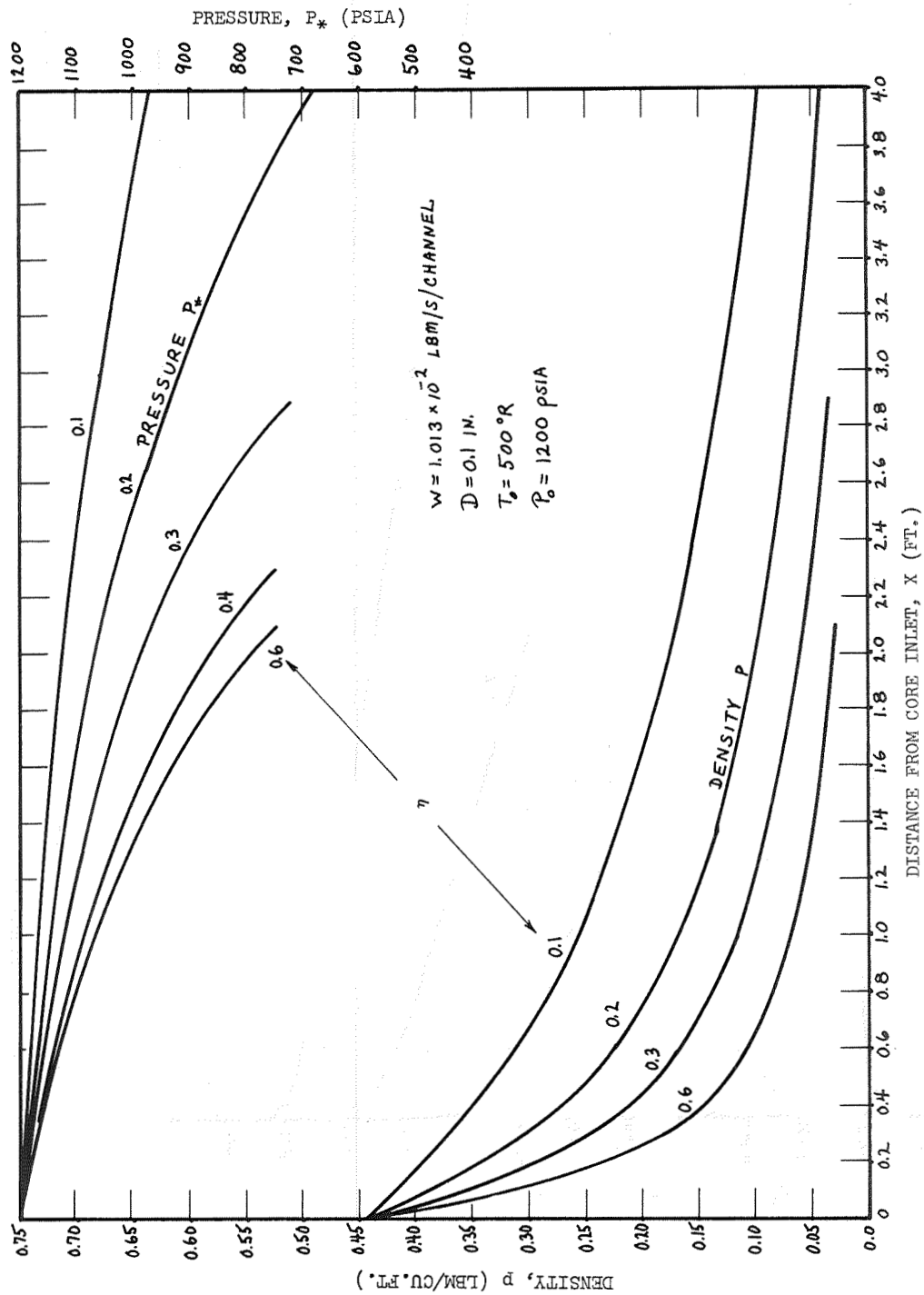


FIGURE 4-18 PRESSURE AND DENSITY DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (4.3), INCREASED T_0

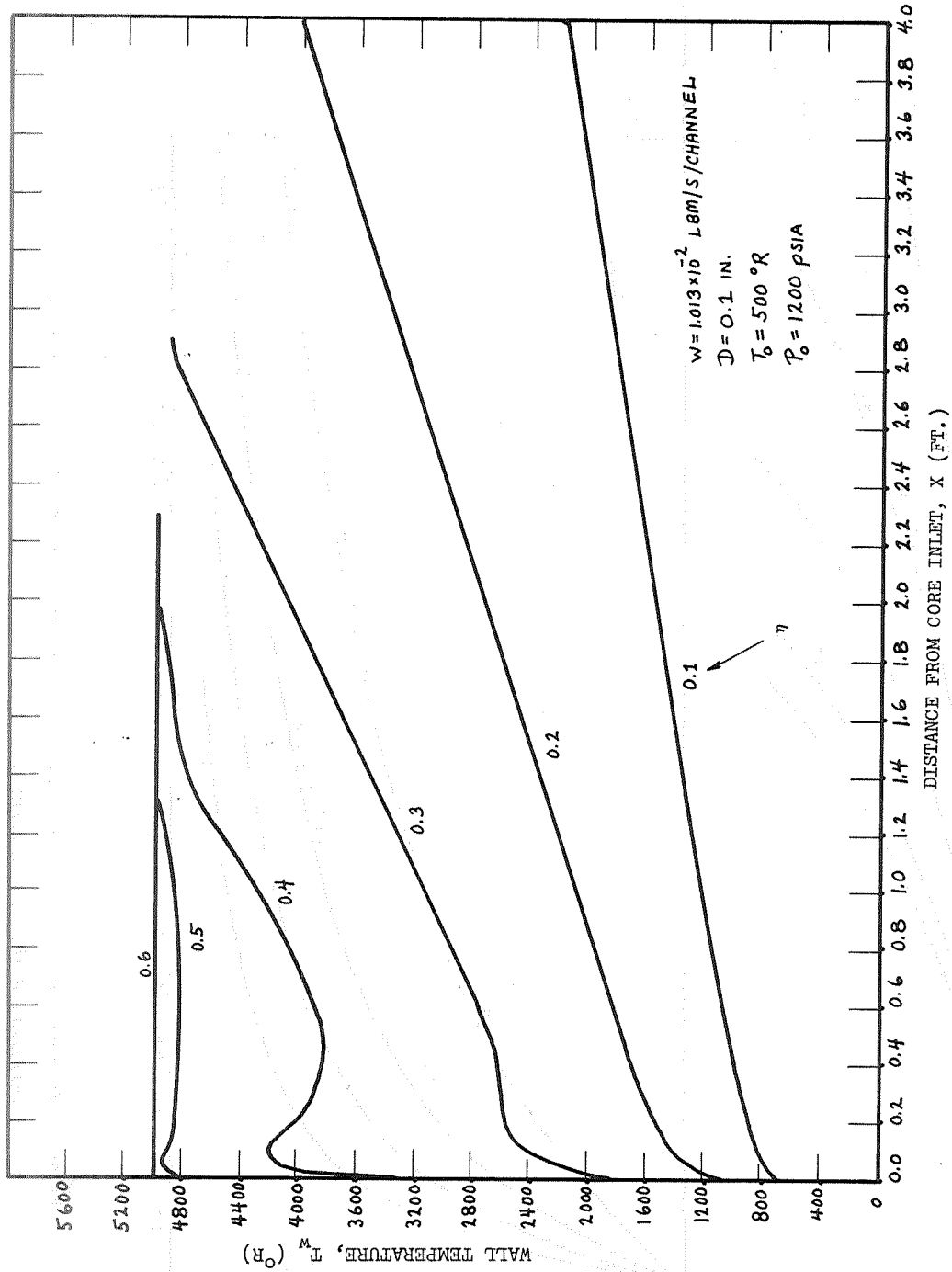


FIGURE 4-19 WALL TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (4.3), INCREASED T_0

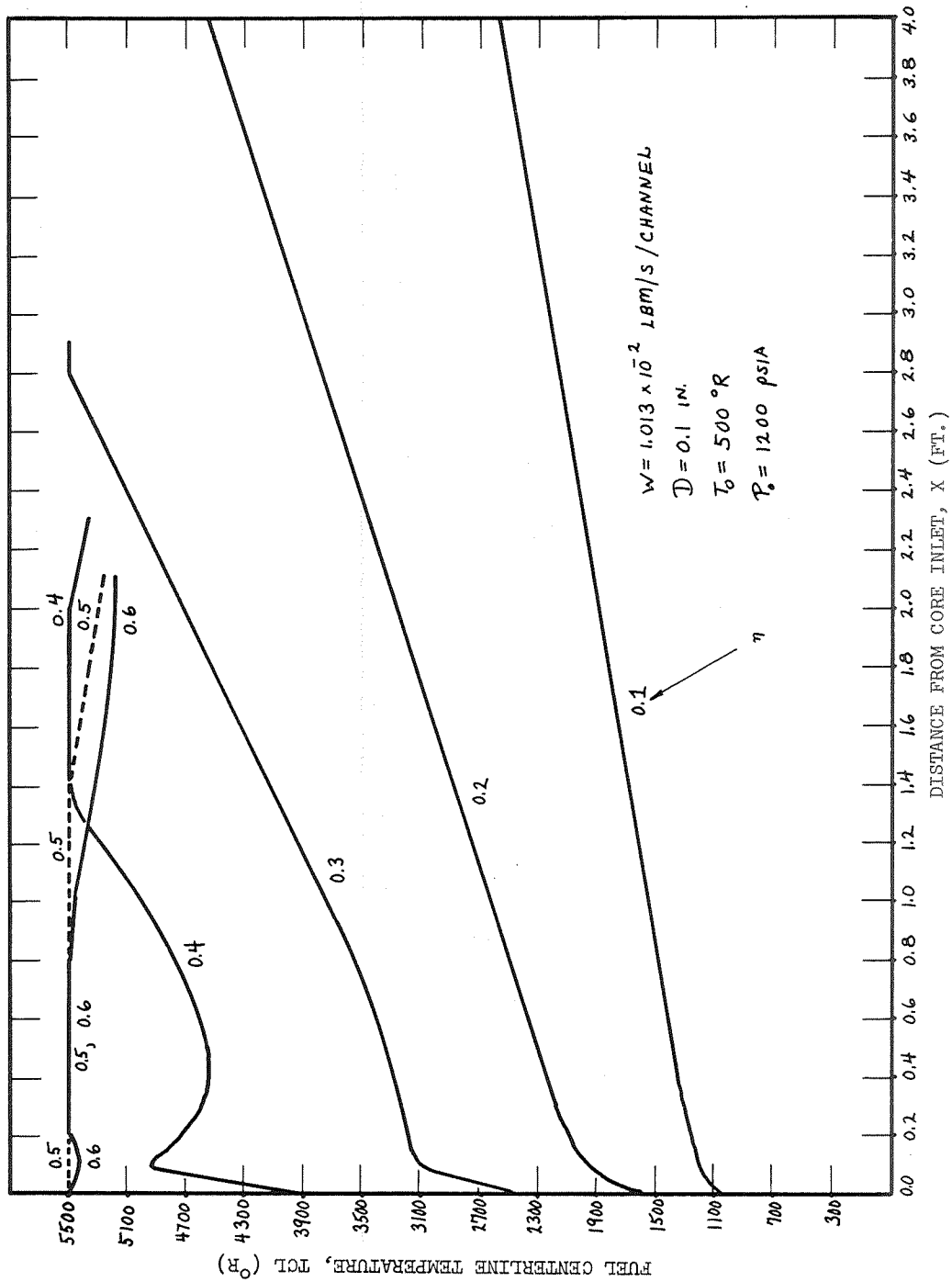


FIGURE 4-20 FUEL CENTERLINE TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (4.3), INCREASED T_0

to be exceeded a short distance from the entrance. Similarly, the trouble position for the wall temperature is at $x = 0.05$ ft. where Figure 4-19 and 4-14 show that T_w jumps by 1200°R for the 200°R drop in T_o .

In addition to the parametric survey at the high flow rate, $w = 1.013 \times 10^{-2}$ lbm/s/channel, a survey was undertaken at the low flow rate, $w = 1.013 \times 10^{-3}$ lbm/s/channel. For the low flow rate one would expect a much lower pressure drop because of the G^2 dependence in the momentum Equation (3.1.1). The pressure drop for all the cases at this low flow rate was 7 psia ΔP_* . Figure 4-21 shows that the power density anomaly predominates for all η except $\eta = 0.1$. The peak optimum power densities are considerably lower than those of the high flow rate. The peak power density is 180 MW/ Cu. Ft. for $\eta = 0.6$ at $w = 1.013 \times 10^{-3}$ lbm/s/channel while the peak power density is 1040 MW/Cu.Ft. for $\eta = 0.6$ at $w = 1.013 \times 10^{-2}$ lbm/s/channel. This is a consequence of the $w^{0.8}$ dependence of the maximum wall temperature - limited and centerline - limited power densities. Figure 4-22 shows that very short channel lengths are a result of the rapid coolant heating. The maximum channel length is not determined by the available pressure drop, but by the computational limit number (1) in Section 3.7 : whenever the bulk coolant temperature is in the range $T_{WMAX} > T(x) > T_{WMAX} - 100$, computation is terminated. This computational limit eliminates consideration of very low power density "tails".

Except for $\eta = 0.1$ these low flow rate cases are entirely wall temperature limited. Exit temperatures are greater than those of the high flow rate and the vacuum specific impulse, I_{sp} , attainable is 50 seconds greater than the high flow rate I_{sp} 's. Unfortunately, for any

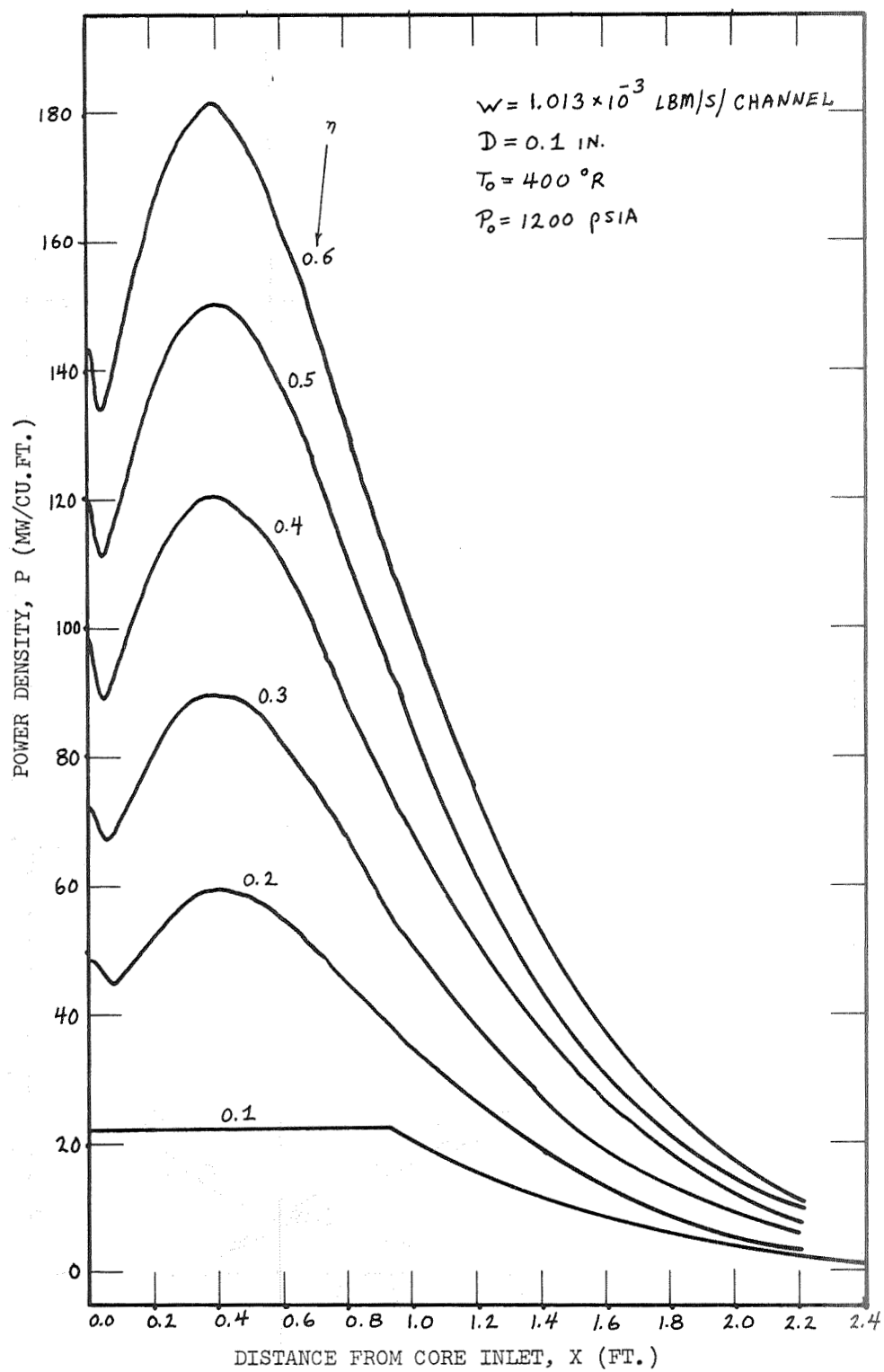


FIGURE 4-21 OPTIMUM POWER PROFILES PREDICTED BY CORRELATION (4.3)

AS A FUNCTION OF η , LOW FLOW RATE

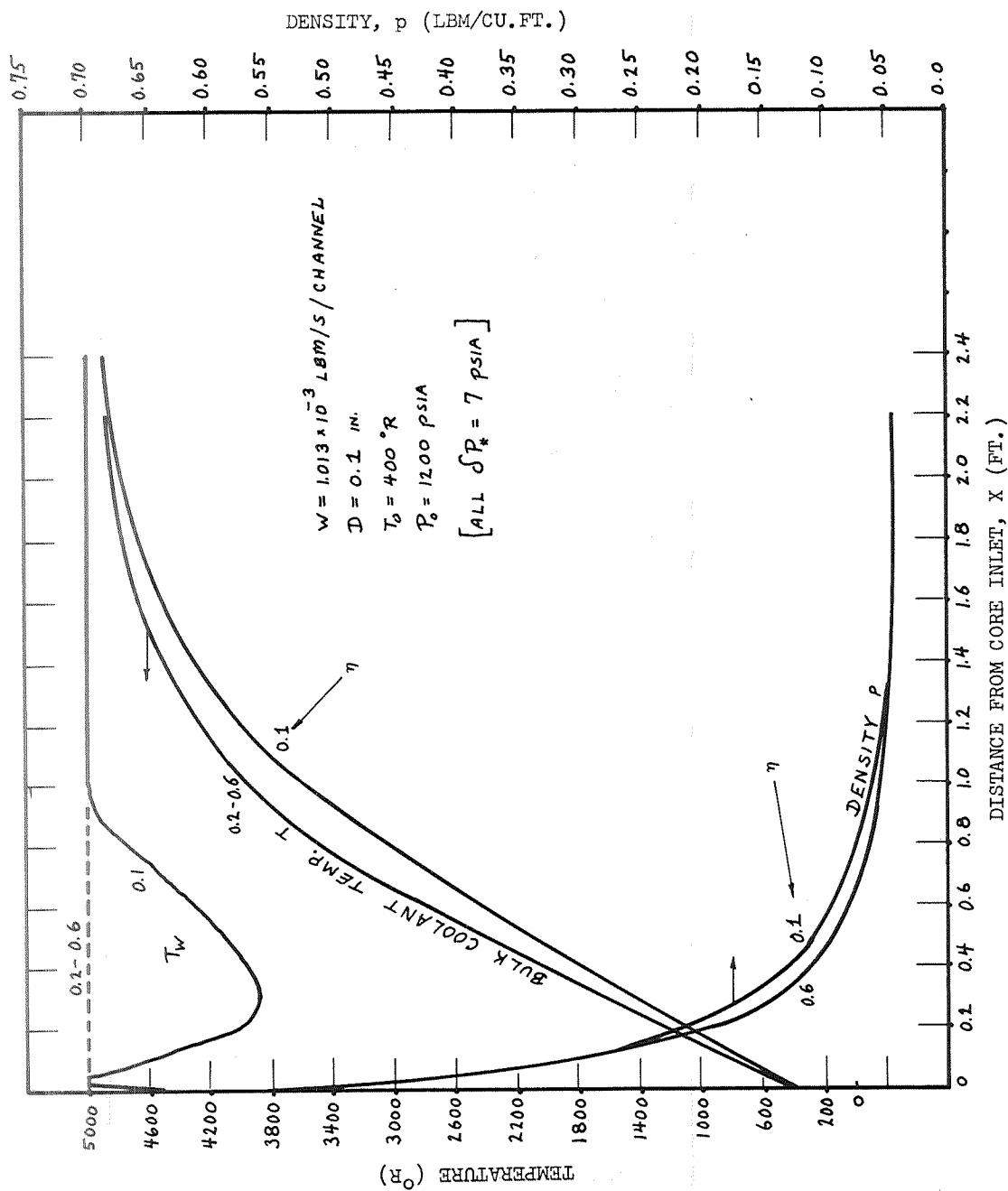


FIGURE 4-22 OPTIMUM REACTOR CORE CHANNEL DISTRIBUTIONS PREDICTED BY CORRELATION (4.3), LOW FLOW RATE

desired total thrust this lower flow rate will have approximately $\sqrt{10}$ times the core radius of the high flow rate. The total thrust $F = \langle w \rangle I_{sp}$ where $\langle w \rangle$ = total flow rate = number of channels (N) x w. Because the optimum power density is radially uniform, each channel has the same I_{sp} and the core radius is then calculated from

$$R_c = \frac{D}{2} \sqrt{N \frac{1-\eta}{\eta}} = \frac{D}{2} \sqrt{\frac{F}{w I_{sp}}} \sqrt{\frac{1-\eta}{\eta}} \quad (4.11)$$

The low flow rate is one-tenth the high flow rate and although the I_{sp} is 50 seconds greater for low w than for high w, the core radius ratio is still about $\sqrt{10}$.

4.4.3 Major Results for Parametric Survey Using Equation (4.3)

The major conclusions of the parametric survey using Correlation (4.3) and high flow rate $w = 1.013 \times 10^{-2}$ lbm/s/channel are:

(1.) The inlet pressure P_o has no effect upon determining any optimum power profile and affects the allowable channel length by the available pressure drop limitation. For $P_o = 1200$ psia, the computed pressure drop ΔP_* for all core void fractions $\eta \geq 0.3$ is roughly 450 psia.

(2.) Increasing the entrance bulk temperature T_o from 300°R to 500°R does not eliminate the wall temperature anomaly or power density anomaly for heat-transfer optimization with core void fractions greater than 0.3. Where stress-limited power densities are not controlling, increasing the entrance T_o increases performance by allowing greater power densities because of the unusual behavior of Correlation (4.3). Larger power densities simultaneously increase the achievable exit temperature and shorten the total channel length.

(3.) The low core void fractions $\eta = 0.1$ and 0.2 are entirely stress-limited to such low optimum power densities that their exit

temperatures and channel lengths cannot compete with optimized cores with higher η 's.

(4.) The intermediate core void fractions $\eta = 0.3$ to 0.5 exhibit a mixture of all three limitations. Void fraction $\eta = 0.3$ is predominantly stress-limited while $\eta = 0.4$ is stress and fuel-centerline temperature-limited and $\eta = 0.5$ is predominantly fuel-centerline temperature-limited. The void fraction $\eta = 0.4$ has a significantly greater exit temperature and a significantly shorter channel length than $\eta = 0.3$, while cores having void fractions greater than 0.4 are roughly comparable in both exit gas temperature and channel length.

(5.) All cores with void fractions $\eta \geq 0.6$ are entirely wall temperature limited.

(6.) Only those optimum cores having stress-limitations in the inlet section ($\eta \leq 0.4$) can allow any decreases in the entrance bulk temperature T_0 at constant power, but the anomalous behavior of Correlation (4.3) will predict large wall temperature increases in the inlet sections for any reactor core channel if the bulk temperature is reduced to very low values.

(7.) For reactor cores with void fractions $\eta \geq 0.5$ the optimum power profiles approximately match those power profiles typically found in reactor cores having a uniform loading of uranium. Therefore, there is an incentive to use reactor cores with large void fractions since little uranium redistribution would be needed for forming the desired power profile. However, the optimum power profile in the inlet for these high η reactor cores is an apparent anomaly associated with Correlation (4.3).

Using Correlation (4.3) and a low flow rate $w = 1.013 \times 10^{-3}$

lbm/s/channel:

(1.) All reactor cores with $\eta > 0.1$ are entirely wall temperatures-limited, the power density anomaly dominates, and the peak optimum power densities are approximately one-tenth those required at the high flow rate.

(2.) As shown in the following Table 4-5, the pressure drop is constant at 7 psia and all reactor core voids $\eta > 0.1$ have channel lengths equal to 2.2 feet and exit gas temperatures equal to 4900°R .

(3.) However, for any desired total thrust these low flow rate reactor cores will have core radii roughly $\sqrt{10}$ times the core radii of the high flow rate optimum cores and will tend to be "pancake" in appearance and thus suffer a weight penalty due to increased radius.

Table 4-5 summarizes the physical and performance parameters of these optimum cores associated with Correlation (4.3). Symbols not previously defined are: L_c = core length, $P_{tot.}$ = integrated total core power, δP_* = core pressure drop, $A_{noz.}$ = throat area of the converging - diverging nozzle (derived in Appendix C), and V_c = core volume. The "best" case, $\eta = 0.6$, is separately compared in Table 4-6.

Table 4-5

Parametric Survey, Optimized Reactor Characteristics
Using Correlation (4.3)

D = 0.1 in., P_o = 1200 psia, w = 1.013 x 10⁻² lbm/s/channel,
 total F = 250,000 lbf. T_o = 500 R

η	L_c feet	Isp sec	R_c feet	$\langle w \rangle$ lbm/s	N	Ptot. MW	ΔP_* psia	Anoz. ft ²	V _c cu. ft.
0.1	4.0	587	2.70	426	42,033	2094	225	0.95	91.7
0.2	4.0	793	1.64	315	31,138	3483	517	1.35	34.0
0.3	2.9	857	1.29	292	28,800	3960	470	1.27	15.2
0.4	2.3	890	1.10	281	27,740	4214	449	1.23	8.7
0.5	2.1	898	0.98	278	27,483	4279	441	1.22	6.3
0.6	2.1	902	0.89	277	27,368	4310	453	1.24	5.2

T_o = 300° R, other parameters same as above

η	L_c feet	Isp sec	R_c feet	$\langle w \rangle$ lbm/s	N	Ptot. MW	ΔP_* psia	Anoz. ft ²	V _c cu. ft.
0.1	4.0	556	2.77	449	44,351	2209	187	0.91	96.8
0.2	4.0	773	1.66	323	31,928	3570	452	1.24	34.8
0.3	3.1	862	1.29	290	28,635	4204	502	1.32	16.1
0.4	2.4	887	1.10	282	27,828	4391	447	1.23	9.1
0.5	2.3	899	0.98	278	27,467	4480	463	1.26	6.9
0.6	2.3	902	0.89	277	27,363	4507	474	1.27	5.7

T_o = 400° R, w = 1.013 x 10⁻³ lbm/s/channel, all else the same.

η	L_c feet	Isp sec	R_c feet	$\langle w \rangle$ lbm/s	N	Ptot. MW	ΔP_* psia	Anoz. ft ²	V _c cu. ft.
0.1	2.4	954	6.70	262	258,728	4858	7	0.77	338.7
0.2	2.2	954	4.74	262	258,824	4854	7	0.77	155.3
0.3	2.2	954	3.87	262	258,824	4854	7	0.77	103.5
0.4	2.2	954	3.35	262	258,824	4854	7	0.77	77.6
0.5	2.2	954	3.00	262	258,824	4854	7	0.77	62.1
0.6	2.2	954	2.74	262	258,824	4854	7	0.77	51.8

Table 4-6

"Best" Case for Correlation (4.3)

$P_o = 1200$ psia
 $D = 0.1$ inch
 $F = 250,000$ lbf
 $\eta = 0.6$

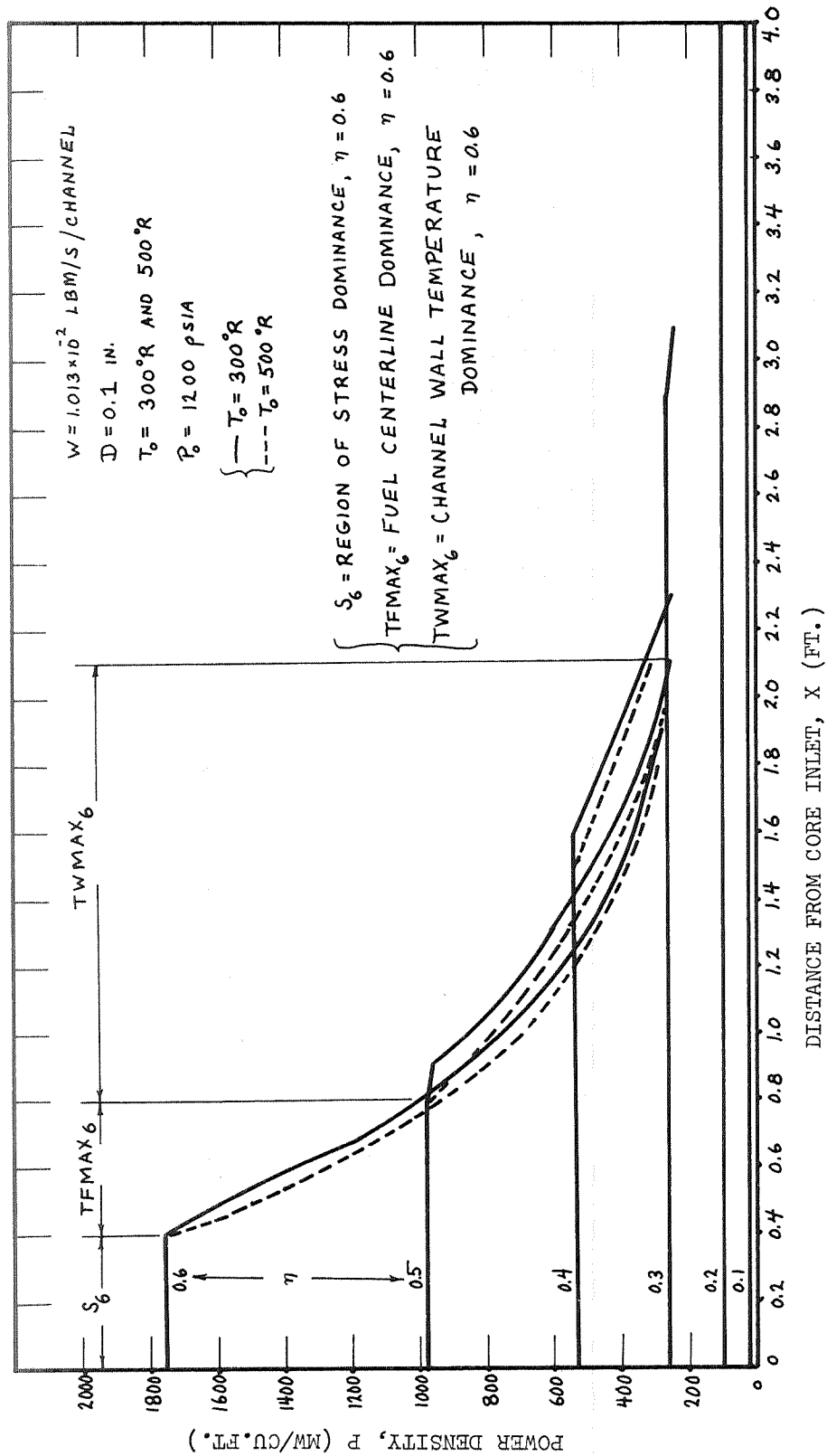
w lbm/s/channel	T_o [°R]	L_c [ft]	R_c [ft]	$\langle w \rangle$ lbm/s	N	Ptot. MW	P psia	Isp sec	Angz. ft ²
1.013×10^{-2}	500	2.1	0.89	277	27,368	4310	453.	902	1.24
1.013×10^{-2}	300	2.3	0.89	277	27,363	4507	474.	902	1.27
1.013×10^{-3}	400	2.2	2.74	262	258,824	4854	7.	954	0.77

4.5 Parametric Survey of Optimum Reactor Channel Characteristics
Associated with NERVA - Phoebus Nozzle Correlation (4.6)

4.5.1 Optimum Reactor Channel Characteristics as a Function
of Core Void Fraction.

The values of P_o , T_o , w and D as selected in Section 4.4.1 are used for the parametric survey of Equation (4.6) in this section and additional results for $P_o = 1400$ psia, $P_o = 1000$ psia, $T_o = 400^\circ$ R, and $D = 0.2$ in. are tabulated. As Figure 4-10 (Section 4.3) illustrated, the optimum channel characteristics using Correlation (4.6) exhibit none of the temperature or power density anomalies consistently predicted by Correlations (4.2), (4.3), and this is the principal reason for isolating the results of this correlation from the results illustrated in Section 4.4.

Figure 4-23 shows the optimum axial power profiles as a function of core void fraction, η . Comparing these power profiles with those in Figures 4-11 and 4-16 there are several immediate differences: (1.) No power density anomalies; (2.) All power profiles in the range of η examined have a "topped" or flattened appearance in the inlet regions of the reactor core, meaning that the stress limitation dominates; and


 FIGURE 4-23 OPTIMUM POWER PROFILES PREDICTED BY CORRELATION (4.6) AS A FUNCTION OF η

(3.) for $\eta \geq 0.4$ an increase in T_o from 300°R to 500°R always results in a decrease in the allowable optimum power density in the reactor channel sections not dominated by the stress limitation. Notice that this is directly opposite to the results using Correlation (4.3) where the anomalous behavior allowed higher power densities for lower T_o 's.

Figure 4-24 shows the significant advantage of going to reactor cores having $\eta \geq 0.4$. For example $\eta = 0.6$ has $L_c = 1.9$ ft. and $T - \text{exit} = 4440^\circ\text{R}$ while $\eta = 0.3$ has $L_c = 3.1$ ft. and $T - \text{exit only} = 4000^\circ\text{R}$. At about $\eta = 0.3$ the allowed power densities can begin to achieve vacuum Isp's of over 850 seconds with interestingly short channel lengths - interesting because for the total thrust level of $F = 250,000$ lbf and the thrust per channel at the nominal high flow rate $w = 1.013 \times 10^{-2}$ lbm/s/channel, these reactor cores have an $L_c/2R_c$ close to unity which implies a reasonable neutronic "compactness" to be kept in mind. The critical mass calculations reported in Chapter 5 demonstrate the large increase in uranium needed to compensate for a low $L_c/2R_c$ ratio. For the end-reflected nuclear reactor model in this thesis, flattening or "pancaking" the reactor greatly increases the unreflected fast neutron leakage from the reactor bottom.

Figure 4-25 shows the completely anomaly-free wall temperature behavior of this correlation and demonstrates that an increase in T_o to 500°R causes the channel wall temperatures to reach TWMAX at shorter channel positions than found for $T_o = 300^\circ\text{R}$. A comparison of Figures 4-23 and 4-25 easily shows the dominant region for each of the thermal limitations used in this thesis. The reactor core having $\eta = 0.6$ is illustrated as an example: (a) Figure 4-23 shows by the power flatness

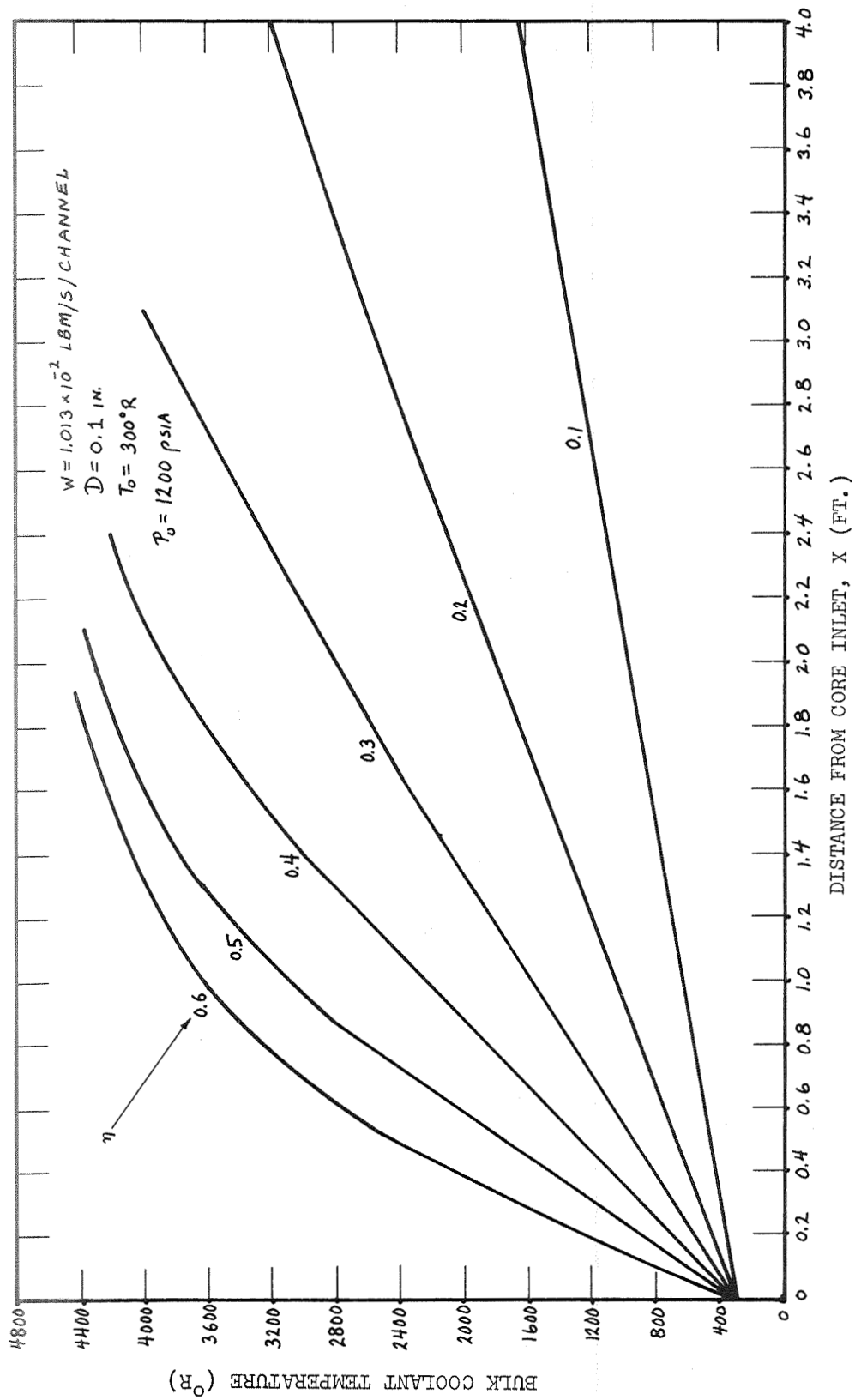


FIGURE 4-24 BULK COOLANT TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES,

CORRELATION (4.6)

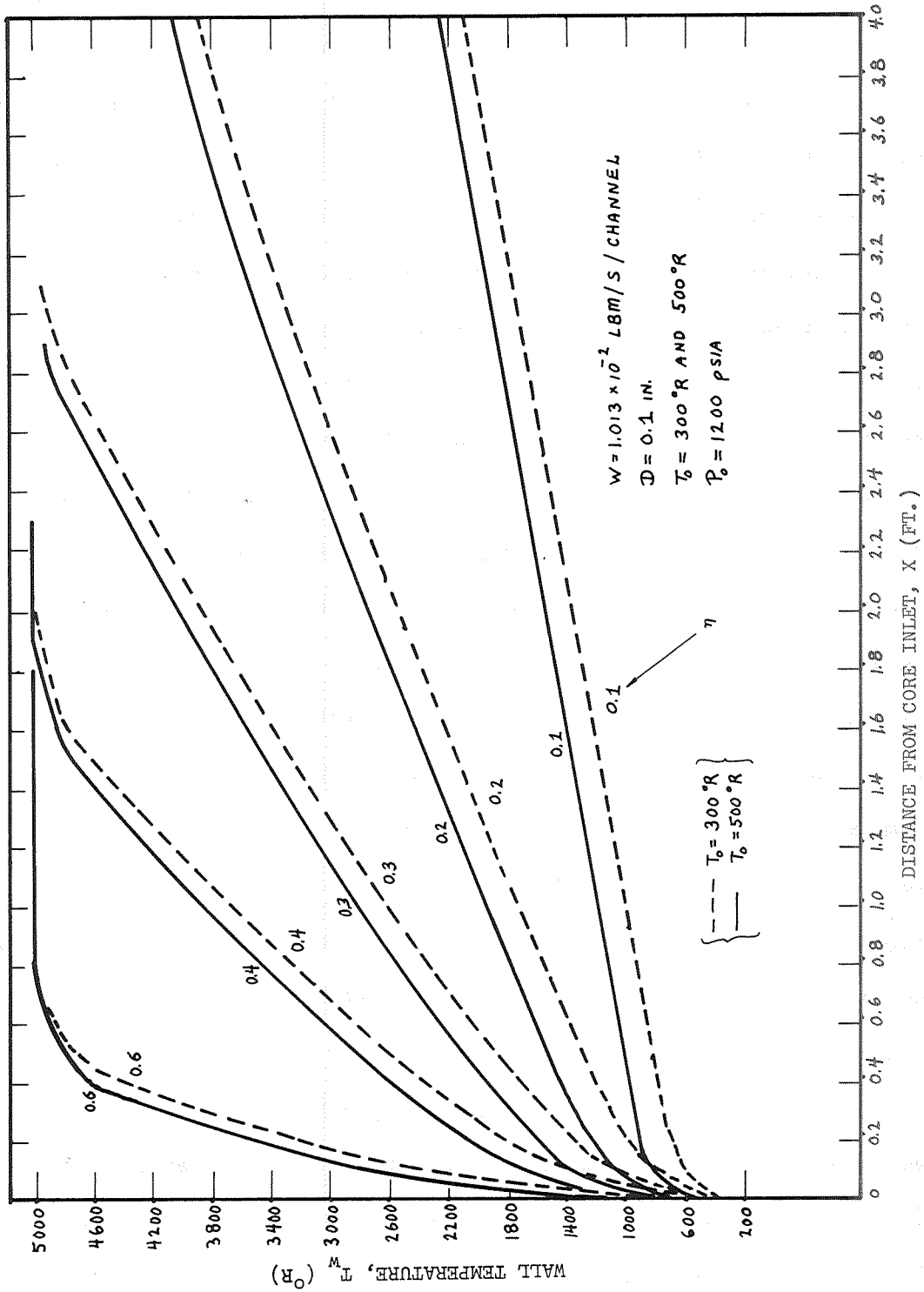


FIGURE 4-25 WALL TEMPERATURE DISTRIBUTIONS RESULTING FROM OPTIMUM POWER PROFILES, CORRELATION (4.6)

or level characteristic that the maximum stress limit, S , dominates up to $x = 0.4$ ft.; (b) Figure 4-25 shows that TW_{MAX} dominates from $x = 0.8$ ft. to the channel exit; (c) therefore, TF_{MAX} must dominate in the region $x = 0.4$ ft. to $x = 0.8$ ft. For graphical clarity, the wall temperature distribution for $\eta = 0.5$ was not plotted in Figure 4-25, but a comparison of the figures for $\eta \leq 0.4$ shows that reactor cores having $\eta \geq 0.4$ operate at steady-state under all three thermal limitations. In contrast to the reactor core channel characteristics predicted by Correlation (4.3), Correlation (4.6) predicts this orderly succession of regions of dominance along the channel length: stress > fuel centerline temperature > wall temperature. In other words, starting from the core entrance the S limit will always precede in channel position the TF_{MAX} limit which will always precede the TW_{MAX} limit. For a given η not all of the thermal limitations need be present as shown for $\eta \leq 0.3$. Based upon these facts, the computer calculations described in Chapter 3, MAXPOW, could have been simplified in that all three maximum power densities for a given x need not be simultaneously compared.

Figure 4-26 demonstrates a radical change in optimum power profiles for the "low flow rate" $w = 1.013 \times 10^{-3}$ lbm/s/channel. In marked contrast to Correlation (4.3)'s power density distributions in Figure 4-21, the power density anomaly is non-existent and stress-limited flattening always appears because of the more efficient transfer of heat to the coolant as predicted by Equation (4.6); i.e., for a given power density and bulk coolant T , Equation (4.3) predicts a much higher T_w than Equation (4.6) and therefore, optimum power densities

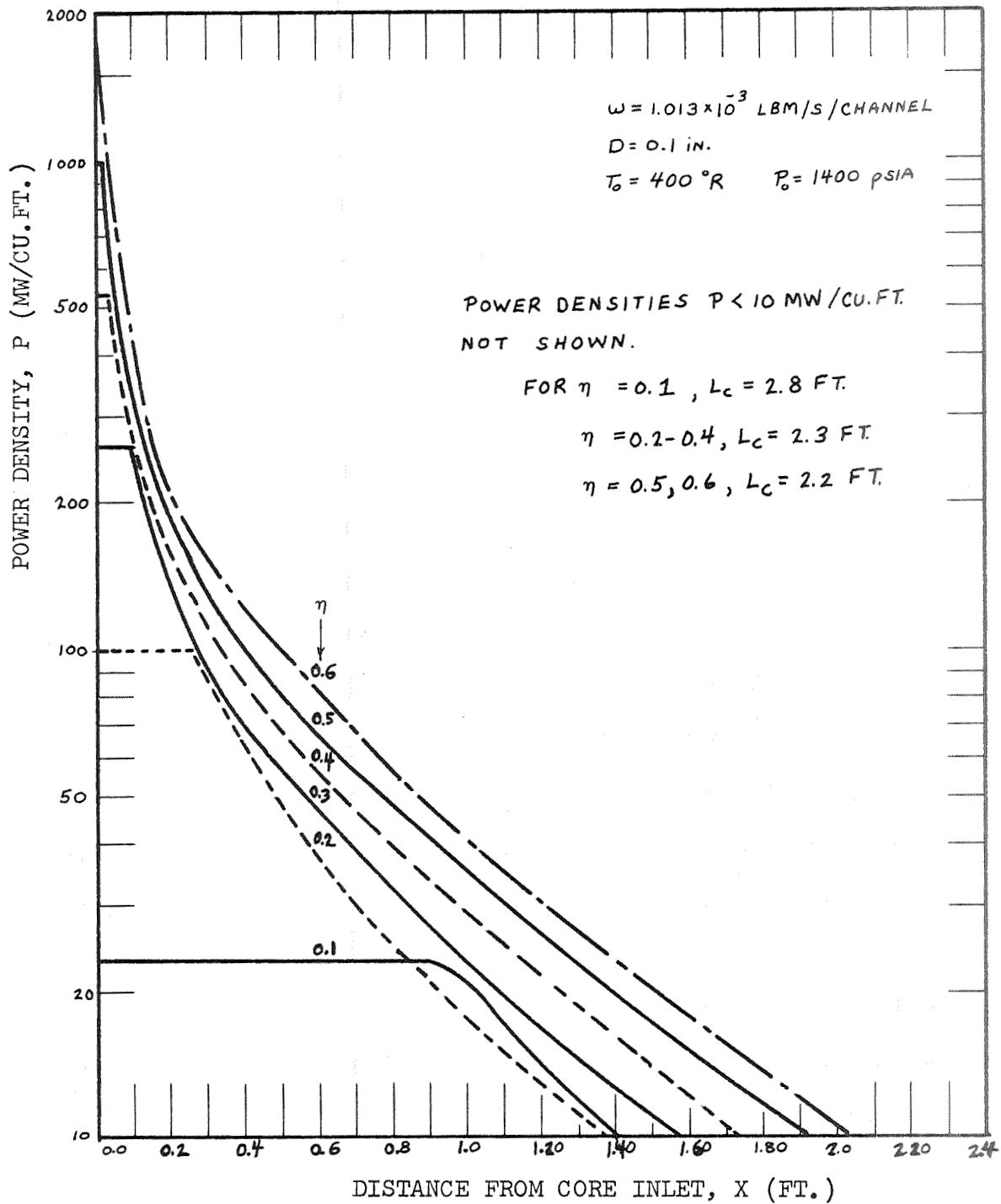


FIGURE 4-26 OPTIMUM POWER PROFILES AS PREDICTED BY
CORRELATION (4.6) AS A FUNCTION OF η , LOW FLOW RATE

associated with Equation (4.3) are wall and centerline temperature-constrained in regions where those power profiles associated with Equation (4.6) are stress - constrained. The low flow rate peak power density for Equation (4.6) for each η is equal to the peak power densities at the high flow rate because the stress limit is essentially independent of flow rate per channel. Although the allowable maximum power density using Correlation (4.6) is greater than that allowed by Correlation (4.3), there is no noticeable performance difference as evidenced by outlet temperature or channel length because it is the integrated power that primarily determines the performance characteristics. With $\eta = 0.6$, for example, correlation (4.3) allows a peak power density of ≈ 180 MW/cu. ft. while Correlation (4.6) allows ≈ 1750 Mw/cu. ft. The low flow rate causes the bulk temperature to quickly increase along the channel and approximately at $x \approx 0$ for Equation (4.6) and $x \approx 0.4$ ft. for Equation (4.3) the TFMAX and TWMAX limitations force a "tail-off" in the optimum power density. Thus, the power distribution which begins at 1750 Mw/cu. ft. tails-off more rapidly than the distribution which peaks at 180 Mw/cu. ft. further into the channel, with the result that when the computational limitation number (1.), Section 3.7 : $TWMAX > T(x) > TWMAX - 100$, is reached the integrated power is identical for both correlations and the engine performance characteristics are identical.

These computations for $P_o = 1200$ psia, $T_o = 300^\circ R$, $T_o = 500^\circ R$, high and low flow rates are given in Table 4-7. In addition, calculations with $P_o = 1000$ psia, $P_o = 1400$ psia, $T_o = 400^\circ R$, and $D = 0.2$ inch are tabulated.

Table 4-7

Optimum Reactor Channel Characteristics Using Correlation (4.6)

$w = 1.013 \times 10^{-2}$ lbm/s/channel, $D = 0.1$ in., $P_o = 1200$ psia

$T_o = 500^\circ R$

η	Lc feet	Isp Sec.	Rc feet	(w) lbm/s	N	Ptot MW	ΔP^* psia	A_{noz} ft ²	V_c ft ³
0.1	4.0	587	2.70	426	42,032	2094	225	0.95	91.7
0.2	4.0	793	1.64	3.5	31,138	3483	517	1.35	34.0
0.3	2.8	857	1.29	292	28,801	3960	470	1.27	15.2
0.4	2.3	888	1.10	281	27,778	4203	447	1.23	8.7
0.5	2.0	901	0.98	277	27,379	4302	447	1.23	6.0
0.6	1.8	906	0.89	276	27,248	4329	434	1.21	4.5

$w=1.013 \times 10^{-2}$ lbm/s/channel, $D=0.1$ in, $P_o=1200$ psia, $T_o=300^\circ R$

η	Lc feet	Isp Sec.	Rc feet	(w) lbm/s	N	Ptot MW	ΔP^* psia	A_{noz} ft ²	VC ft ³
0.1	4.0	556	2.77	449	44,350	2209	187	0.91	96.8
0.2	4.0	773	1.66	323	31,928	3570	453	1.24	34.8
0.3	3.1	861	1.29	290	28,638	4202	501	1.32	16.1
0.4	2.4	889	1.10	281	27,774	4403	456	1.24	9.1
0.5	2.1	903	0.97	277	27,323	4513	467	1.26	6.3
0.6	1.9	909	0.89	275	27,164	4547	460	1.25	4.7

$w=1.013 \times 10^{-2}$ lbm/s/channel, $D=0.1$ in., $P_o=1400$ psia, $T_o=400^\circ R$

η	Lc feet	Isp Sec.	Rc feet	(w) lbm/s	N	Ptot MW	ΔP^* psia	A_{noz} ft ²	VC ft ³
0.1	4.0	572	2.74	437	43,131	2150	169	0.75	94.1
0.2	4.0	782	1.66	320	31,578	3516	363	0.89	34.7
0.3	3.9	918	1.25	272	26,872	4560	635	1.21	19.1
0.4	3.3	932	1.07	268	26,491	4665	631	1.20	11.9
0.5	3.0	937	0.96	267	26,334	4705	641	1.22	8.6
0.6	2.8	939	0.89	266	26,277	4714	633	1.21	6.7

$w=1.013 \times 10^{-2}$ lbm/s/channel, $D=0.1$ in., $P_o=1000$ psia, $T_o=400^\circ R$

η	Lc feet	Isp Sec.	Rc feet	(w) lbm/s	N	Ptot MW	ΔP^* psia	Anoz ft ²	Vc ft ³
0.1	4.0	571	2.74	437	43,184	2146	266	1.26	94.2
0.2	3.2	718	1.73	348	34,376	3082	420	1.59	30.0
0.3	2.2	764	1.37	327	32,294	3390	348	1.42	12.9
0.4	1.7	816	1.15	306	30,237	3749	341	1.40	7.0
0.5	1.3	831	1.02	301	29,698	3849	296	1.31	4.2
0.6	1.1	843	0.92	297	29,271	3929	283	1.29	2.9

$w=1.013 \times 10^{-2}$ lbm/s/channel, $D=0.1$ in., $P_o=1200$ psia, $T_o=400^\circ R$

η	Lc feet	Isp Sec.	Rc feet	(w) lbm/s	N	Ptot MW	ΔP^* psia	Anoz ft ²	Vc ft ³
0.1	4.0	572	2.75	437	43,142	2149	205	0.93	94.1
0.2	4.0	783	1.66	319	31,524	3526	484	1.29	34.4
0.3	3.0	859	1.30	291	28,719	4082	486	1.29	15.7
0.4	2.4	892	1.10	280	27,670	4331	472	1.27	9.1
0.5	2.1	905	0.98	276	27,268	4430	479	1.28	6.2
0.6	1.9	910	0.89	275	27,128	4460	468	1.26	4.7

$w=1.013 \times 10^{-3}$ lbm/s/channel, $T_o=400^\circ R$, $P_o=1400$ psia, $D=0.1$ in.

η	Lc feet	Isp Sec.	Rc feet	(w) lbm/s	N	Ptot MW	ΔP^* psia	Anoz ft ²	Vc ft ³
0.1	4.0	572	5.47	437	43,132	2150	7	0.66	376.4
0.2	4.0	781	3.31	320	31,618	3509	13	0.67	138.0
0.3	4.0	898	2.52	278	27,486	4390	19	0.67	80.0
0.4	4.0	921	2.16	272	26,810	4574	22	0.67	58.5
0.5	4.0	928	1.92	269	26,584	4636	24	0.67	46.4
0.6	4.0	932	1.75	268	26,484	4661	24	0.67	38.5

4.5.2 Major Results for Parametric Survey Using Equation (4.6)

While Table 4-7 reports the total results of the parametric survey using Equation (4.6), it is obvious that all cases achieve their maximum specific impulse at $\eta = 0.6$ and therefore these "best" cases are separately tabulated in Table 4-8, where the cases are organized according to the achievable Isp. Table 4-8 illustrates the many trade-offs possible in order to achieve a maximum Isp for the most compact and least weight nuclear engine. Since no shielding or extensive weight calculations were carried out in this thesis, only these tentative observations can be made:

(1) The highest Isp has the lowest (w) and the largest R_c . Because of the large weight penalty of increasing the end-shield radius, this advantage in Isp is partially offset. Using the mission weight sensitivity values cited in Section 2.3, where 1 lb. engine weight = 16 lbs WIEO (Weight In Earth Orbital) and 1 sec Isp = 3000 lbs WIEO (36), then a one second Isp advantage could be nullified relative to the entire mission system weight if the nuclear engine weight increases as a result of the one-second Isp increase by 187.5 lbs. Because the shield weight is the major nuclear engine weight component, a carefully evaluation of the mission shielding requirements is needed for this low flow rate.

(2) Increasing P_o to 1400 psia relative to the $P_o = 1200$ psia nominal pressure increases the Isp from 910 seconds to 939 seconds while L_c increases from 1.9 ft. to 2.8 ft. (2nd and 4th rows in Table 4-8). In other words, the increase in P_o allows a longer channel

length to be optimized before the available pressure drop limitation terminates computation. The $\Delta P_o = +200$ psia results in $\Delta ISP = +29$ seconds, $\Delta L_c = +0.9$ ft., $\Delta(\delta P_x) = +165$ psia, $\Delta P_{tot} = +254$ Mw, and $\Delta R_c = -0.02$ ft.

(3) Increasing the channel diameter D to 0.2 inches (3rd row of Table 4-8) results in an optimum core which is much larger than would result with $D = 0.1$ inches, all other parameters constant (2nd row). The reactor core radius scales in direct proportion to the ratio of channel diameters.

(4) Variation of T_o (rows 4, 5 and 6), all other parameters constant, does not significantly change the performance characteristics of these optimum nuclear rocket reactor cores.

(5) Reduction of P_o by 200 psia below the nominal $P_o = 1200$ psia (last row to be compared with the 4th row) will result in a "pancake" configuration with $L_c/2R_c \approx 0.6$ and a reduction in I_{sp} to 843 seconds. The lower P_o simultaneously restricts the channel length that may be optimized before the available pressure drop limitation terminates computation and restricts the achievable exit temperature. A lower T_o exit means a low thrust per channel, more channels than must be required to produce the 250,000 lbf thrust, and hence a "pancake" result because the reactor core radius scales as $R_c \sim \sqrt{N}$, where N = total number of parallel channels.

Table 4-8

"Best" Case for Correlation (4.6)

$$\eta = 0.6$$

$\frac{w}{\text{lbm/s/Channel}}$	D Inch	P _o Psia	T _o R	L _c ft	R _c ft	(w) lbm/s	N	P _{tot} MW	ΔP_* Psia	Isp Sec	Anoz ft ²
1.013x10 ⁻³	0.1	1400	400	2.1	2.74	262	258,739	4716	6.	954	0.66
1.013x10 ⁻²	0.1	1400	400	2.8	0.87	266	26,277	4714	633.	939	1.21
1.013x10 ⁻²	0.2	1400	400	4.0	1.75	268	26,484	4661	24.	932	0.67
1.013x10 ⁻²	0.1	1200	400	1.9	0.89	275	27,128	4460	468.	910	1.26
1.013x10 ⁻²	0.1	1200	300	1.9	0.89	275	27,164	4547	460.	909	1.25
1.013x10 ⁻²	0.1	1200	500	1.8	0.89	276	27,248	4329	434.	906	1.21
1.013x10 ⁻²	0.1	1000	400	1.1	0.92	297	29,271	3929	283	843	1.29

4.6 Heat-Transfer Optimum Nuclear Rocket Reactor Core Characteristics

A comparison of Tables 4-5 through 4-8 demonstrates that while Correlations (4.3) and (4.6) have been shown to predict substantially different optimum reactor core power distributions under identical core situations, the overall reactor core dimensions and the mission capability in terms of Isp are primarily determined by the physical parameters w, D, and F and the size and Isp are insensitive to either the particular turbulent hydrogen heat transfer correlation used or the entrant channel hydrogen conditions (with the exception of P_o's gross influence upon allowable channel length). Therefore, Chapter 4 provides a realistic estimate of mission capability - Isp - and a realistic reactor core size as a model for investigating the reactivity characteristics of such heat-transfer optimum reactors. However, the suitability of the best

axial power distributions for non steady-state operation is unresolvable strictly from heat transfer considerations because of the temperature anomaly. If the anomaly really exists, then the reactor system cannot be designed for steady state operation at 100% full power. The proper non-uniform uranium distribution is also dependent upon an accurate prediction of the desired power distribution. The optimum power profiles calculated in this thesis are significantly different from those previously reported (10) and any calculated non-uniform uranium distributions to match the power profiles in Sections 4.4 and 4.5 would probably differ significantly from that previously reported also. However no calculations were performed in this thesis to accurately match specific uranium distributions to desired power profiles under the assumption that such compact cores will be fast, dominated by neutron leakage, and therefore the hydrogen reactivity worths, both uniform and non-uniform, should be largely determined by physical core size and average uranium concentration rather than local details of non-uniform uranium concentrations.

The results of this heat transfer analysis may be somewhat dependent upon the thermal constraints used, although the basic discrepancy between the "anomalous" and "nonanomalous" correlations will still persist for different S, TFMAX, TWMAX. A sensitivity analysis using Correlation (4.6) and $\eta = 0.3 - 0.6$ is reported in Appendix F. In terms of Isp, an improvement in TWMAX is most influential. Increasing TWMAX by 100° R increases Isp by 7 seconds in the $\eta = 0.6$, "best" case studied.

5. Reactivity and Neutron Physics Characteristics of Heat-Transfer Optimum Nuclear Rocket Reactors.

5.1 The Nuclear Reactor Model

The comparison of turbulent hydrogen heat transfer correlations in Chapter 4 has demonstrated that the physical dimensions of the optimum nuclear reactor core are determined principally by the core parameters η and D and the rocket system parameters w and F . The dimensions of these compact reactor cores for the nominal 250,000 lbf thrust (12,13,14) are on the order of a few feet and as shown in this chapter, they have fast neutron spectra. This agrees with the findings of Cooper (7) who has shown that tungsten and carbide reactor cores of comparable dimensions, when no hydrogen is present, have fast spectra and that it is justifiable to assume that the reactor core is effectively homogeneous because the void spaces and fuel channel thicknesses are much smaller than the neutron mean free path. Therefore, uniformly-loaded uranium-235 reactor cores are used in this thesis to determine criticality and hydrogen reactivity worths on the assumption that the value of any hydrogen reactivity calculated would not be greatly changed by the actual non-uniform uranium distribution that would be required to match the desired power profile for each optimized case. This assumption was tested for one relatively non-uniform uranium distribution and good agreement for the hydrogen reactivity was obtained, as reported in Section 5.4.

For the numerical calculations using the two-dimensional diffusion theory computer code, EXTERMINATOR - 2 (38), the nuclear reactor is assumed to be homogeneous by regions - uranium and carbon uniformly distributed in the reactor core, beryllium and hydrogen uniformly distributed in the reflector - except for the steady state, non-uniform hydrogen distribution in the reactor

core. Reactor core dimensions and hydrogen distributions associated with Correlation (4.6) are used because of the wide experimental range covered by this correlation. The existence of the anomalous temperature behavior as discussed in Chapter 4 might actually be present and the hydrogen distributions associated with Correlations (4.3) and (4.5) could as easily have been used. The reactor core is divided into many axial (x) regions and the non-uniform core hydrogen density distribution across each region is averaged as shown in Figure 5-1 for the case A (defined in the following tables). The hydrogen concentration is taken to be radially uniform for each axial region. The region hydrogen density, $\bar{p}'$, is found from the average bulk density, $\bar{p}$, by Equation (5.1) :

$$\bar{p}' = \bar{p} \eta \quad (5.1)$$

where η = core void fraction,

$$\begin{aligned} \bar{p} &= \text{average bulk coolant density} \left[\text{atoms H/cm}^3 \text{ of hydrogen flow volume} \right], \\ \bar{p}' &= \text{hydrogen region density} \left[\text{atoms H/cm}^3 \text{ of entire region volume} \right]. \end{aligned}$$

Table 5-1 lists the characteristics of the reactor cases investigated in this chapter and Table 5-2 lists their associated non-uniform core hydrogen density distributions as used in the computer code. Cases A, B, C, D form a sequence of descending core void fraction (or increasing reactor volume) and have identical $P_0 = 1200$ psia, $T_0 = 300$ R, $w = 1.013 \times 10^{-2}$ lbm/s/channel, and $D = 0.1$ inch. Case E is identical to Case A except that $T = 500^\circ\text{R}$ and this changes the calculated core dimensions slightly while case F is a "pancake" reactor configuration that results from the "low" flow rate. The high flow rate reactor cores are emphasized because their achievable Isp's are relatively high while the calculated core radius yields an $L_c/2R_c$ near unity; i.e., "square" cylinders. Although optimum nuclear rocket reactor cores have their

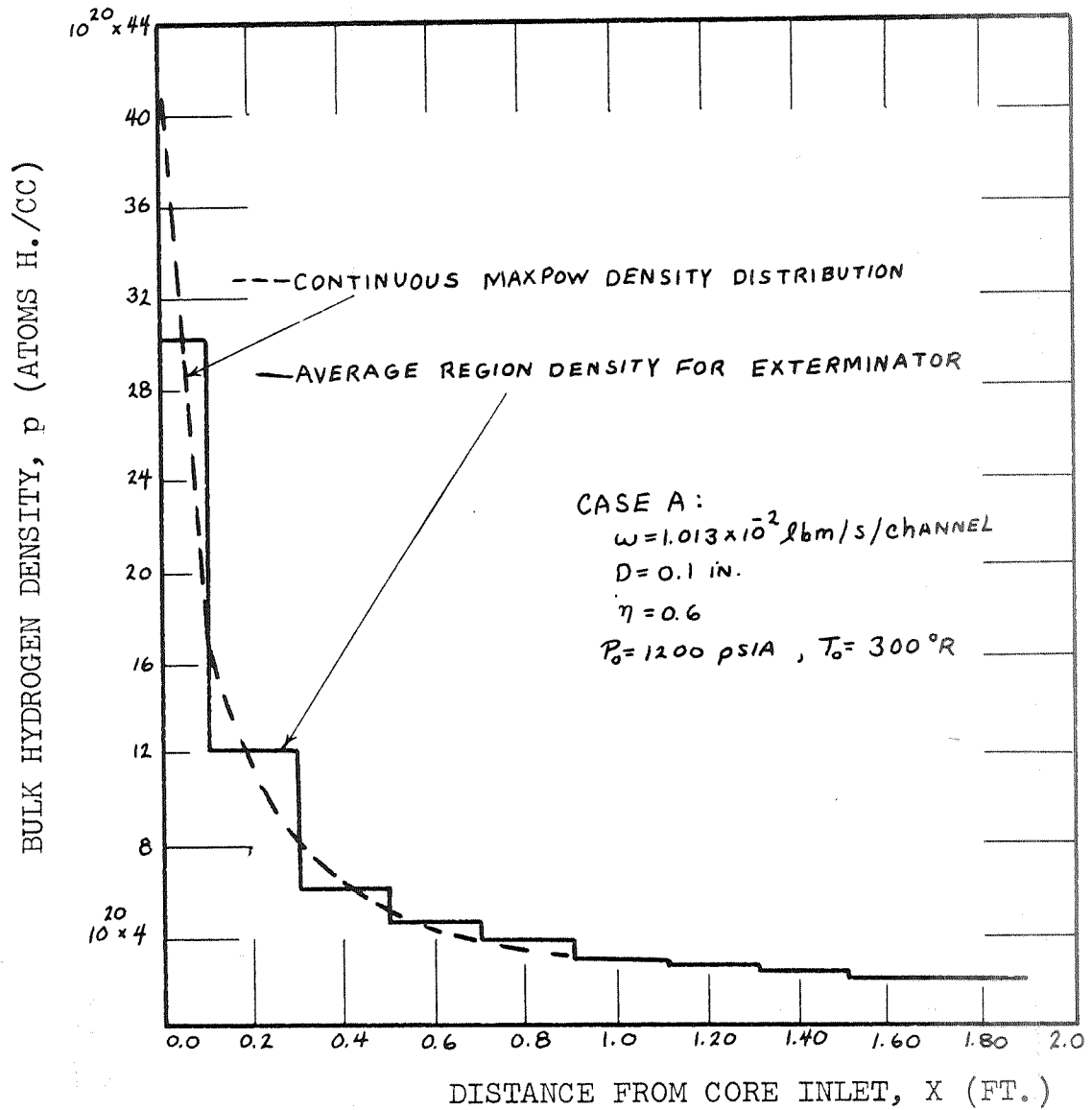


FIGURE 5-1 AVERAGE HYDROGEN REGIONS
 IN THE NUCLEAR ROCKET REACTOR CORE

Table 5-1

Reactor Cases Considered for Hydrogen Reactivity Worth Computations*

Case	A	B	C	D	E	F
w [lbm/s/channel]	1.013×10^{-2}	1.013×10^{-2}	1.013×10^{-2}	1.013×10^{-2}	1.013×10^{-2}	1.013×10^{-3}
D [in.]	0.1	0.1	0.1	0.1	0.1	0.1
η	0.6	0.5	0.4	0.3	0.6	0.6
P _o [psia]	1200	1200	1200	1200	1200	1400
T _o [°R]	300	300	300	300	500	400
T _m [°R]	170	170	170	170	387	220
L _c [ft.]	1.9	2.1	2.4	3.1	1.8	2.2
R _c [ft.]	0.89	0.97	1.1	1.3	0.89	2.74
I _{sp} [sec.]	909	903	889	862	906	954
$\langle w \rangle$ [lbm/s.]	275	277	281	290	276	262
P _{tot.} [MW]	4547	4513	4403	4203	4329	4854

*Note: 1. Cases considered which have a beryllium reflector with 85% dense beryllium and 15% void fraction (hydrogen fraction) are designated A.15, B.15, etc.

2. Cases considered which have a beryllium reflector with 70% dense beryllium and 30% void fraction are designated A.30, B.30, C.30.

Table 5-2

Reactor Core Region Hydrogen Distributions

$[H]$ = Concentration of hydrogen atoms, $\left[\text{atoms/cm}^3 \right]$

X_0 = Distance from core inlet, start of core hydrogen region $\left[\text{cm} \right]$

X_1 = Distance from core inlet at end of core hydrogen region $\left[\text{cm} \right]$

$\Delta X = X_1 - X_0$ $\left[\text{cm} \right]$

<u>Case A</u>	$[H]$	X_0	X_1	ΔX
	3.0190×10^{21}	0.0	3.05	3.05
	1.1633×10^{21}	3.05	9.15	6.10
	6.2250×10^{20}	9.15	15.25	6.10
	4.3700×10^{20}	15.25	21.35	6.10
	3.5215×10^{20}	21.35	27.45	6.10
	3.0265×10^{20}	27.45	33.55	6.10
	2.6805×10^{20}	33.55	39.65	6.10
	2.4030×10^{20}	39.65	45.75	6.10
	2.1565×10^{20}	45.75	51.85	6.10
	1.9190×10^{20}	51.85	57.95	6.10
<u>Case B</u>	$[H]$	X_0	X_1	ΔX
	2.6900×10^{21}	0.0	3.05	3.05
	1.2605×10^{21}	3.05	9.15	6.10
	7.1780×10^{20}	9.15	15.25	6.10
	5.0050×10^{20}	15.25	21.35	6.10
	3.8135×10^{20}	21.35	27.45	6.10
	3.0790×10^{20}	27.45	33.55	6.10
	2.6140×10^{20}	33.55	39.65	6.10
	2.2865×10^{20}	39.65	45.75	6.10
	2.0320×10^{20}	45.75	51.85	6.10
	1.8130×10^{20}	51.85	57.95	6.10
	1.6060×10^{20}	57.95	64.05	6.10
<u>Case C</u>	$[H]$	X_0	X_1	ΔX
	1.8755×10^{21}	0.0	6.10	6.10
	9.6250×10^{20}	6.10	12.20	6.10
	6.4945×10^{20}	12.20	18.30	6.10
	4.8775×10^{20}	18.30	24.40	6.10
	3.8805×10^{20}	24.40	30.50	6.10
	3.1980×10^{20}	30.50	36.60	6.10
	2.6945×10^{20}	36.60	42.70	6.10
	2.3030×10^{20}	42.70	48.80	6.10
	1.9895×10^{20}	48.80	54.90	6.10
	1.6897×10^{20}	54.90	64.05	9.15
	1.3957×10^{20}	64.05	73.20	9.15

Case D

<u>H</u>	<u>X₀</u>	<u>X₁</u>	<u>ΔX</u>
1.8236x10 ²¹	0.0	3.05	3.05
1.3155x10 ²¹	3.05	6.10	3.05
9.3711x10 ²⁰	6.10	12.20	6.10
6.6798x10 ²⁰	12.20	18.30	6.10
5.1715x10 ²⁰	18.30	24.40	6.10
3.7129x10 ²⁰	24.40	39.65	12.20
2.4717x10 ²⁰	39.65	58.15	18.50
1.4465x10 ²⁰	58.15	88.65	30.50
1.4465x10 ²⁰	88.65	94.75	6.10

Case E

1.5565x10 ²¹	0.0	6.10	6.10
7.1375x10 ²⁰	6.10	12.20	6.10
4.7295x10 ²⁰	12.20	18.30	6.10
3.7055x10 ²⁰	18.30	24.40	6.10
3.1440x10 ²⁰	24.40	30.50	6.10
2.7799x10 ²⁰	30.50	36.60	6.10
2.4795x10 ²⁰	36.60	42.70	6.10
2.2285x10 ²⁰	42.70	48.80	6.10
1.9915x10 ²⁰	48.80	54.90	6.10

Case F

2.1465x10 ²¹	0.0	3.05	3.05
4.8702x10 ²⁰	3.05	6.10	3.05
4.3399x10 ²⁰	6.10	9.15	3.05
4.0326x10 ²⁰	9.15	12.20	3.05
3.8259x10 ²⁰	12.20	15.25	3.05
3.5294x10 ²⁰	15.25	27.45	12.20
3.2478x10 ²⁰	27.45	45.75	18.30
3.1097x10 ²⁰	45.75	64.05	18.30

"best" cases for $\eta = 0.6$, such small cores with high leakage require a very high uranium loading to maintain criticality. The sequence of cases A-D is selected to demonstrate the trade-off in Isp and relatively low total hydrogen reactivity worths at high η , for lower Isp, increased hydrogen reactivity worths and increased core volume, but lower critical masses at low η . Also the A-D sequence very effectively demonstrates several important regularities in relative hydrogen reactivity worth with respect to the reactor core size which should be very important to know if a further detailed optimization of a complete nuclear rocket engine is to be performed. Cases A, E, and F are "best" cases having high Isp at $\eta = 0.6$. As demonstrated in Chapter 4, the variation in T_o had a very small effect upon the calculated optimum reactor core dimensions, but case E has a mean temperature, T_m , substantially different from the T_m of the A-D sequence so case E is used to show the effect that a decrease in reflector hydrogen density has upon the calculated uranium critical mass. Case F is explicitly selected to demonstrate the changes in reactivity characteristics due to "pancakeness". The entrance pressure, P_o , and temperature, T_o , for case F are not consistent with those for A-E in order that a reasonable channel length for this "pancake" configuration be used.

Figure 2-1 shows that the present NERVA concept has all of the hydrogen passing through the regeneratively-cooled rocket nozzle and beryllium reflector before entering the reactor core. Figure 5-2 is the simplified concept used for the EXTERMINATOR computations. This model is asymmetrically reflected, radially and at the core entrance end, by a beryllium reflector uniformly 6 inches (15.25 cm) thick. This beryllium thickness is reported to be optimum for minimum reactor core weight per unit flow area (33).

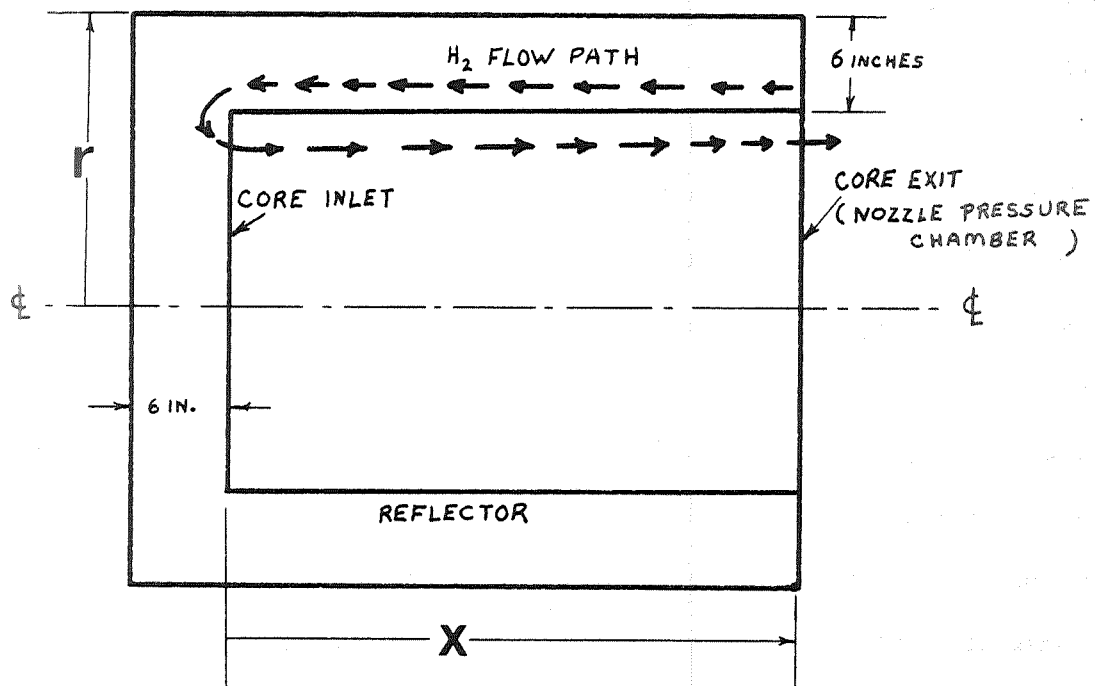


FIGURE 5-2 NUCLEAR ROCKET REACTOR MODEL

The hydrogen temperature at the reactor core entrance, T_o , will depend upon both the initial hydrogen temperature entering the regeneratively-cooled rocket nozzle and upon the fraction of core thermal power that is gained by the coolant in flowing through the nozzle and reflector coolant passages. In this thesis a mean temperature is assigned to the hydrogen in the reflector, assuming that hydrogen enters the nozzle coolant circuit at 40 R:

$$T_m = \frac{1}{2} (T_o + 40) \quad (5.2)$$

As explained in Section 4.1, this implies that all of these reactor cases, with one exception as explained below, have no relative disadvantage in hydrogen propellant tank mass. Equation (4.1):

$$M_t = \frac{C_{st} M_p}{p_p^{2/3}} \quad (4.1)$$

shows that the mass of the propellant tankage, M_t , is inversely related to the rocket nozzle inlet hydrogen density, p_p . Consequently, an inlet temperature greater than 40 °R (critical temperature for hydrogen; $p_p = 4$ lbm/cu.ft.) incurs a weight penalty due to increased propellant tankage. For cases A through F, the fraction of the reactor core power that the reflector and rocket nozzle must acquire in order that the 40 °R coolant be heated to the core entrance temperature, T_o , is presented in Table 5-3.

Table 5-3 *
Fraction of Core Power Absorbed
by the Reflector - Nozzle Coolant Circuit

Case	$\langle w \rangle$	Ptot.	$\left(\frac{\bar{c}_p \langle w \rangle \Delta T}{P_{tot.}} \right)$
A	275	4547	0.056
B	277	4513	0.056
C	281	4403	0.059
D	290	4203	0.063
E	276	4329	0.104
F	262	4716	0.055

*

$$\begin{aligned}
 \langle w \rangle &= \text{Total flow rate, } \left[\frac{\text{lbm}}{\text{sec}} \right] \\
 P_{\text{tot.}} &= \text{Total core power } \left[\text{MW} \right] \\
 \bar{c}_p &= 0.353 \times 10^4 \frac{\text{watt-sec}}{\text{lbm-R}} \text{ (from Equation (3.5))} \\
 \frac{\Delta T}{T_0} &= T_0 - 40 \left[^\circ \text{R} \right] \\
 \frac{\bar{c}_p \langle w \rangle \Delta T}{P_{\text{tot}}} &= \text{Fraction of core power absorbed in the reflector-rocket nozzle.}
 \end{aligned}$$

With the exception of Case E, all cases have roughly the same requirement, so that the required core-power fraction for Case E was arbitrarily set equal to 0.06. This means that its $T_m = 387 \left[^\circ \text{R} \right]$ rather than $270 \left[^\circ \text{R} \right]$ and the nozzle inlet temperature is $234 \left[^\circ \text{R} \right]$ instead of $40 \left[^\circ \text{R} \right]$. By Equation (4.1), case E thereby has a propellant tankage mass roughly 3.35 times that of the other cases. Because a particular mission has not been selected, the mass of propellant, M_p , and therefore, the actual amount of this weight penalty, is unknown. The fraction of core power calculated in Table 5-3 is close to the nominal value of 5% cited by Bussard and DeLauer (11, p. 183). According to them, nominally 3% of the core thermal power is retained by the reflector through gamma photon absorptions, neutron heating, and conductive heat transfer, while 2% of the core thermal power is added to the coolant through the converging-diverging rocket nozzle.

The mean pressure in the reflector is taken to be equal to P_0 for all cases. Calculations were performed with the integrated momentum Equation (4.10)

$$\delta P_* = \frac{R u G^2 T_m}{g c P_m M} \left[\ln \left\{ \frac{P_{\text{in}}}{P_{\text{out}}} \right\} + \frac{2 f L}{D} \right] \quad (4.10)$$

and the results showed that pressure losses across the reflector are quite negligible. The mean reflector channel hydrogen density is calculated from the perfect gas law, Equation (3.4). For the EXTERMINATOR calculations this mean channel density must be converted to a reflector region density, $\bar{p}_r$:

$$\bar{p}_r = \phi P_m \left[\frac{H \text{ atoms}}{\text{cm}^3 \text{ reflector}} \right] \quad (5.1.1)$$

where ϕ = the reflector void fraction.

There are several reflector void fraction values reported in the literature. Bussard and DeLauer (11) cite a reflector void fraction of 15% as representative for nuclear rocket reactors; Plebuch (10) reported on beryllium reflectors with $\phi = 0.10$; and Cooper (7) used $\phi = 0.25$ in his fast reactor calculations. Three important considerations might govern the selection of an ideal nuclear rocket reflector void fraction : (1.) having a large reflector void fraction will certainly increase the required uranium critical mass because of the increased neutron leakage; (2.) on the other hand, Plebuch has shown that the power peaking at the radial core-reflector interface for uniformly loaded reactors decreases as the solid fraction of beryllium is decreased. Less redistribution of uranium in the radial direction would be required to shape an optimum flat radial power profile, assuming that the thick radial reflector always produces a power peak otherwise. This does not represent the situation if an increased reflector void fraction allows more hydrogen to be introduced into the reflector. In this case the power peaking may increase because of increased moderation by the hydrogen; (3.) the reflector may be used for control. A decreased beryllium density will make the reflector less effective - if the decreased beryllium density does not allow additional hydrogen into the reflector. The total reactivity contribution represented by a reflector having an increased void fraction filled by hydrogen may be larger than the lower void fraction also filled with hydrogen.

No reflector void fraction has been specifically reported for the NERVA or Phoebus nuclear rocket reactors. For comparative purposes, the two nominal values of 15 and 30% reflector void fractions are used. In the following sections, the short-hand notation, A.15, means that the reactor core of case A is reflected by a beryllium reflector containing 15 volume % hydrogen and 85 volume % beryllium. A.30 has a beryllium reflector containing 30 volume % hydrogen and 70 volume %

beryllium.

5.2 Hydrogen Reactivity Worth

5.2.1 Introduction

Criticality for a nuclear reactor system is simply the precise balance between neutrons being produced in the reactor system and neutrons being lost from the system through absorptions and leakage across the physical boundaries. This is expressed in the effective multiplication constant, Equation (5.3).

$$k = \frac{\text{Total Productions}}{\text{Total Absorptions} + \text{Leakage}} \quad (5.3)$$

The reactor model used in this thesis, shown in Figure 5-2, is an extremely complicated system to attempt to predict in any detailed manner because the three components of Equation (5.3) are not conceptually easy to separate. The reactor is non-symmetrically reflected which means that neutron leakage is non-symmetrical. Hydrogen is present in both reflector and reactor core which will perturb the neutron energy spectrum. The non-uniform distribution of hydrogen in the reactor means that the neutron energy spectrum is spatially dependent. Because of this complexity, the interpretation of multigroup calculations will be greatly aided by correlating the results to show obvious trends.

Primarily because neutron leakage is so prominent in the reactors investigated in this thesis, all of the numerical results of this chapter are plotted against the geometrical buckling for each particular reactor. In cylindrical geometry, the buckling is defined by Equation (5.4) :

$$B^2 = \left(\frac{\pi}{L_c} \right)^2 + \left(\frac{2.405}{R_c} \right)^2 \left[\text{cm}^{-2} \right] . \quad (5.4)$$

as used in this thesis, the L_c and R_c in Equation (5.4) are the actual physical reactor core length and radius previously determined in Chapter 4. The buckling is a convenient parameter which has been found to be very important in correlating

the characteristics of these compact reactor cores.

The theoretical motivation behind using B^2 comes from analyzing very simple reactors - one group, one region, and unreflected. For these simple systems, Equation (5.4) is the first eigenvalue of the critical diffusion equation and the total leakage from a critical reactor is directly proportional to B^2 (39). It is natural to attempt to use a parameter that describes leakage for a simple system in order to understand and interpret a complicated system that is also very leakage dependent. From an engineering viewpoint, the critical core dimensions in Equation (5.4) are present by the MAXPOW heat transfer optimization before any other parameter of interest, such as the C/U ratio, can be calculated. The clear trends in hydrogen reactivity worth as correlated with B^2 , shown in Section 5.2.3, could eliminate some future extensive multigroup computations.

5.2.2 Critical Uranium Concentrations with Hydrogen

During full power operation of a nuclear rocket there is hydrogen flowing through the reflector and core, contributing reactivity because of hydrogen's excellent moderating properties. The minimum critical mass required for full power operation will occur for the steady-state situation in which there are no control rods or extra neutron absorbing materials present in the reactor system and therefore full use is made of the increase in reactivity contributed by all of the hydrogen. Table 5-4 and Figure 5-3 show the critical uranium concentrations and critical masses for the nine reactor cases examined in this thesis. Appendix D lists the material densities and nuclear multigroup constants used in these calculations.

The sequence A-D exhibits decreasing critical masses as the physical size of the reactor cores increases and relative neutron leakage decreases.

Table 5-4

Critical Core Characteristics with All Hydrogen Present

Case	B^2	$[u]$	x_u	x_c	c/u	$L_c/2R_c$	M235	η
A.15	105×10^{-4}	3.70×10^{21}	0.076	0.324	7.5	1.07	197.1	0.6
B.15	87×10^{-4}	2.29×10^{21}	0.048	0.452	16.8	1.05	170.2	0.5
C.15	70×10^{-4}	1.27×10^{21}	0.026	0.574	38.7	1.09	129.7	0.4
D.15	47×10^{-4}	3.75×10^{21}	0.008	0.692	15.8	1.19	65.6	0.3
E.15	109×10^{-4}	4.36×10^{21}	0.091	0.309	5.9	1.00	223.3	0.6
F.15	32×10^{-4}	1.27×10^{21}	0.026	0.374	25.2	0.38	736.9	0.6
A.30	105×10^{-4}	4.72×10^{21}	0.097	0.303	5.5	1.07	253.5	0.6
B.30	87×10^{-4}	3.12×10^{21}	0.064	0.436	11.9	1.05	233.0	0.5
C.30	70×10^{-4}	1.80×10^{21}	0.037	0.563	26.7	1.09	180.0	0.4

 B^2 = Geometrical buckling, (cm^{-2}) $[u]$ = Critical Uranium concentration, (atoms U-235/ cm^3) x_u = Volume fraction of uranium x_c = Volume fraction of carbon c/u = Critical carbon to Uranium ratio $L_c/2R_c$ = Length to diameter ratio of core

M235 = Critical mass of uranium, (Kg)

= Core void fraction

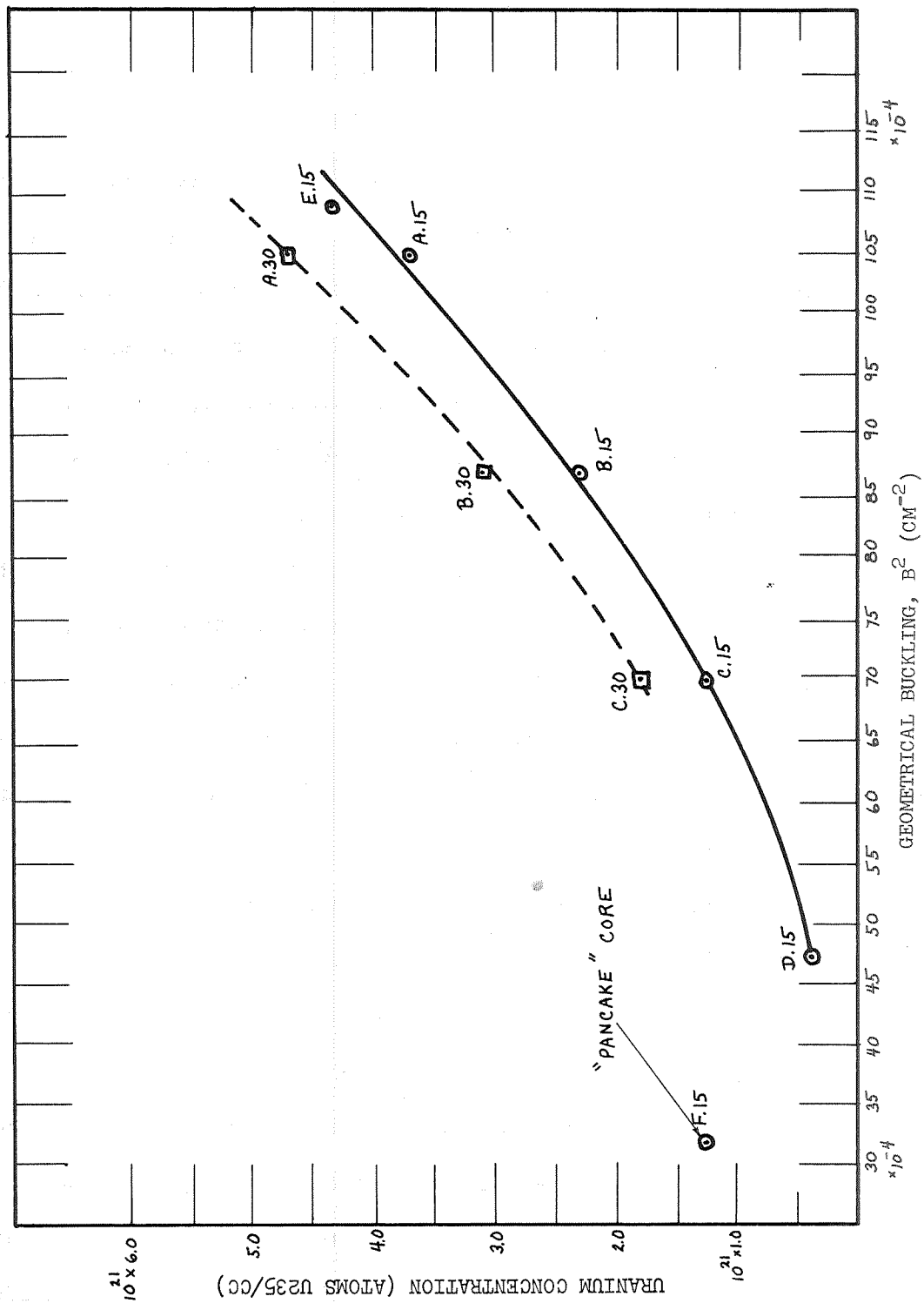


FIGURE 5-3 CRITICAL URANIUM CONCENTRATIONS, ALL HYDROGEN PRESENT, VERSUS GEOMETRICAL BUCKLING

Case E.15 has a lower reflector hydrogen density as compared to A.15. From Table 5-4, E.15 has a 26.2 Kg critical mass disadvantage. Figure 5-3, however, shows that the critical uranium concentrations that result from a simultaneous change in physical size and changes in core and reflector hydrogen densities are very strongly correlated by the geometrical buckling.

Since leakage is so dominant (as shown by the B^2 dependence) then it is probable that most of this 26 Kg mass disadvantage is due to the small change in physical size between case A and case E. In Section 5.2.3 it is shown that the total worth of hydrogen in the reflector of a fixed core size is directly proportional to the hydrogen density. If the mean temperature, T_m , for E.15 had been $170^\circ R = T_m (A.15)$ then E.15 would have had $\frac{387}{170} \times (1.79\$) = 4.08 \$$ reactivity contribution from its reflector hydrogen. (The 1.79 \$ value is reported in Table 5-5 in the next section.) A criticality search for E.15 showed that uranium is worth approximately 0.228 \$/KgU (this value determined when all hydrogen present) so that $4.08 \$ - 1.79 \$ = 2.29 \$ = 10.0 \text{ Kg uranium}$ is the true penalty case E.15 incurs because of decreased reflector hydrogen density. The remaining 16. Kg disadvantage is due to increased leakage which is the dominant characteristic of Figure 5-3.

The reactor cases group themselves according to the reflector void fraction. Those reactors with $\phi = 0.30$ have larger critical uranium concentrations and critical masses because the less dense beryllium reflector allows more leakage. It is interesting to note that for B^2 greater than $70 \times 10^{-4} \text{ cm}^{-2}$, the two groups show a parallel, almost linear trend over a fairly wide range of critical uranium concentrations. Although these reactors contain hydrogen, the addition of all of the hydrogen that this thesis assumes for steady-state conditions does not appreciably soften the neutron energy spectrum. The hydrogen addition significantly reduces the critical mass, but in other respects the energy spectrum is

so fast that most of the neutron productions and leakage are found in the high energy groups, as will be shown in Section 5.3. These high energy neutrons do not "see" the reflector in the sense that the reflector "bump" does not appear. Figure 5-4 shows a representative axial flux plot for case A.15. The reflector "bump", or increase in a group flux in the reflector, represents significant moderation by the reflector. As seen in the figure, the high energy groups tend to have identical spatial shapes, or constant group bucklings, until energy group 9 (100 - 550 eV) must alter its shape to adjust to the reflector's presence. The groups higher than group 9 behave in essentially the same manner as unreflected neutron fluxes. Since the majority of neutron productions and leakages occur in these high energy groups, Figure 5-3 shows an almost linear dependence of critical uranium concentration upon B^2 , just as in the simple one-group, one-region, bare reactor (39). Case D.15 has its mean neutron production energy (defined and tabulated in Section 5.3) in the intermediate range and therefore deviates from the smooth line predicted by cases A.15 - C.15.

The notable exception to the $\phi = 0.15$ line is case F.15. This is the "pancake" core resulting from a low flow rate per channel. Because the reactor model, shown in Figure 5-2, has no reflector on the rocket nozzle end, pancakeness greatly increases the fast leakage from this end. Critical uranium concentration relative to a square cylinder, sequence A-E, must increase to maintain criticality. Table 5-4 shows that F.15 has the largest critical mass, 737 Kg. The weight of the uranium critical mass is small compared to the entire rocket system, especially the shielding. However, the pancake core now has two inherent disadvantages:

- (1.) an increased end shield mass because of its large radius, and
- (2.) an increased critical mass because of the increased bottom leakage.

These disadvantages must be offset by the increase in Isp when this type of core is evaluated for any planned mission.

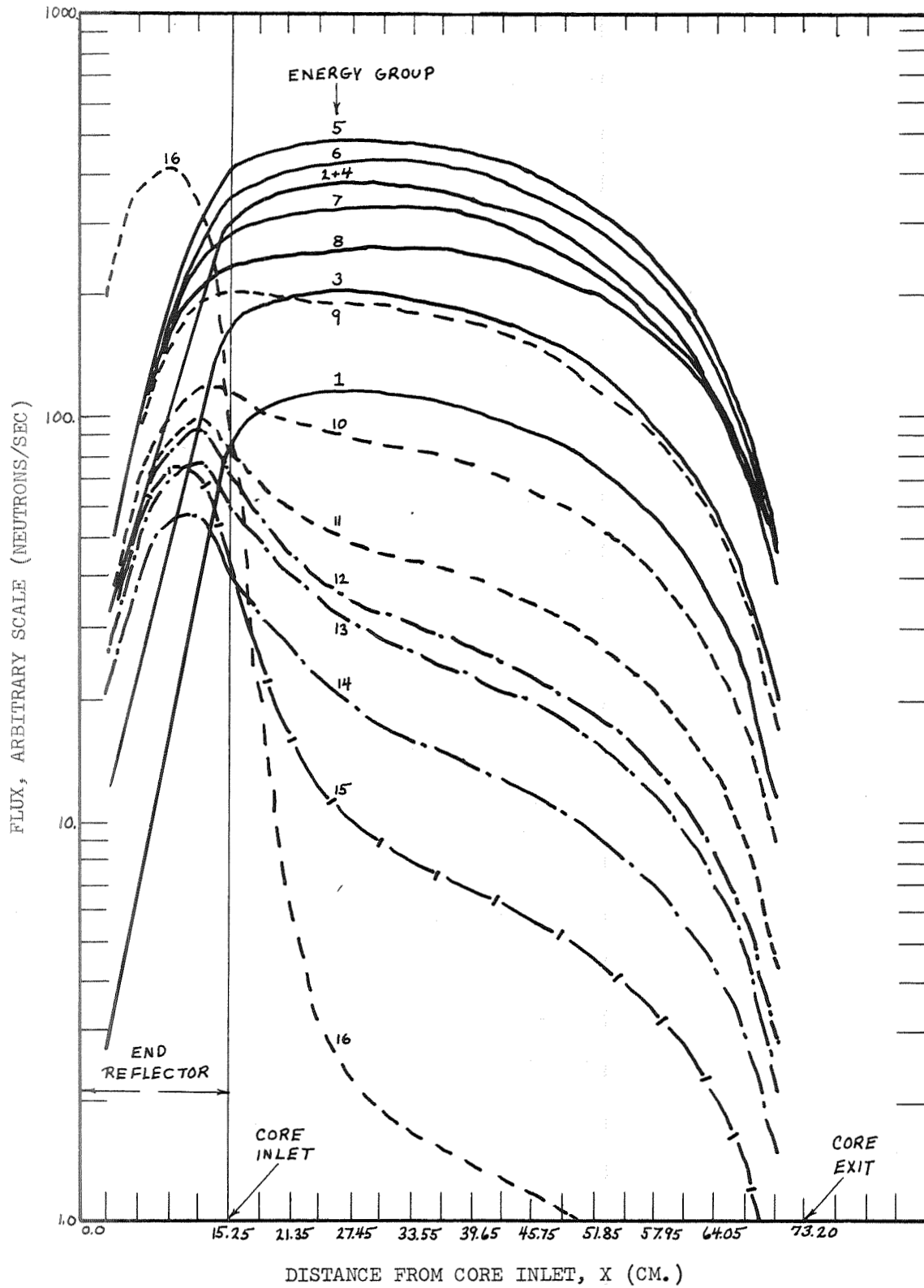


FIGURE 5-4 REPRESENTATIVE AXIAL NEUTRON GROUP FLUX DISTRIBUTIONS, CASE A.15

5.2.3 Reactivity Worths of Core and Reflector Hydrogen as Correlated by the Geometrical Buckling.

The results of Section 5.2.2 define critical, steady-state reactor configurations for which the total and relative hydrogen reactivity contributions in the reactor core and reflector can be calculated. Reflector hydrogen worth is always evaluated in this thesis with the reactor core empty of hydrogen and reactor core hydrogen worth is always evaluated with the reflector filled uniformly with hydrogen at P_0 and T_m . This corresponds to the physical situation shown in Figure 2-1 : hydrogen enters the nuclear reactor reflector before entering the reactor core.

To avoid the overuse of the written phrase "hydrogen reactivity worth", the following symbols are defined:

$$\Delta k_{tr} = \text{reactivity worth due to all of the hydrogen in a reflector } [\$, (5.5)$$

$$\Delta k_{tc} = \text{reactivity worth due to all of the non-uniform hydrogen in a reactor core } [\$, (5.6)$$

$$\overline{\Delta k}_{tr} = \frac{\Delta k_{tr}}{\text{total reflector hydrogen atoms}} \left[\$/\text{kg H}_2 \right], (5.7)$$

$$\overline{\Delta k}_{tc} = \frac{\Delta k_{tc}}{\text{total core hydrogen atoms}} \left[\$/\text{kg H}_2 \right], (5.8)$$

$$\overline{\Delta k} = \frac{\Delta k_{tr} + \Delta k_{tc}}{\text{total reactor hydrogen atoms}} \left[\$/\text{kg H}_2 \right], (5.9)$$

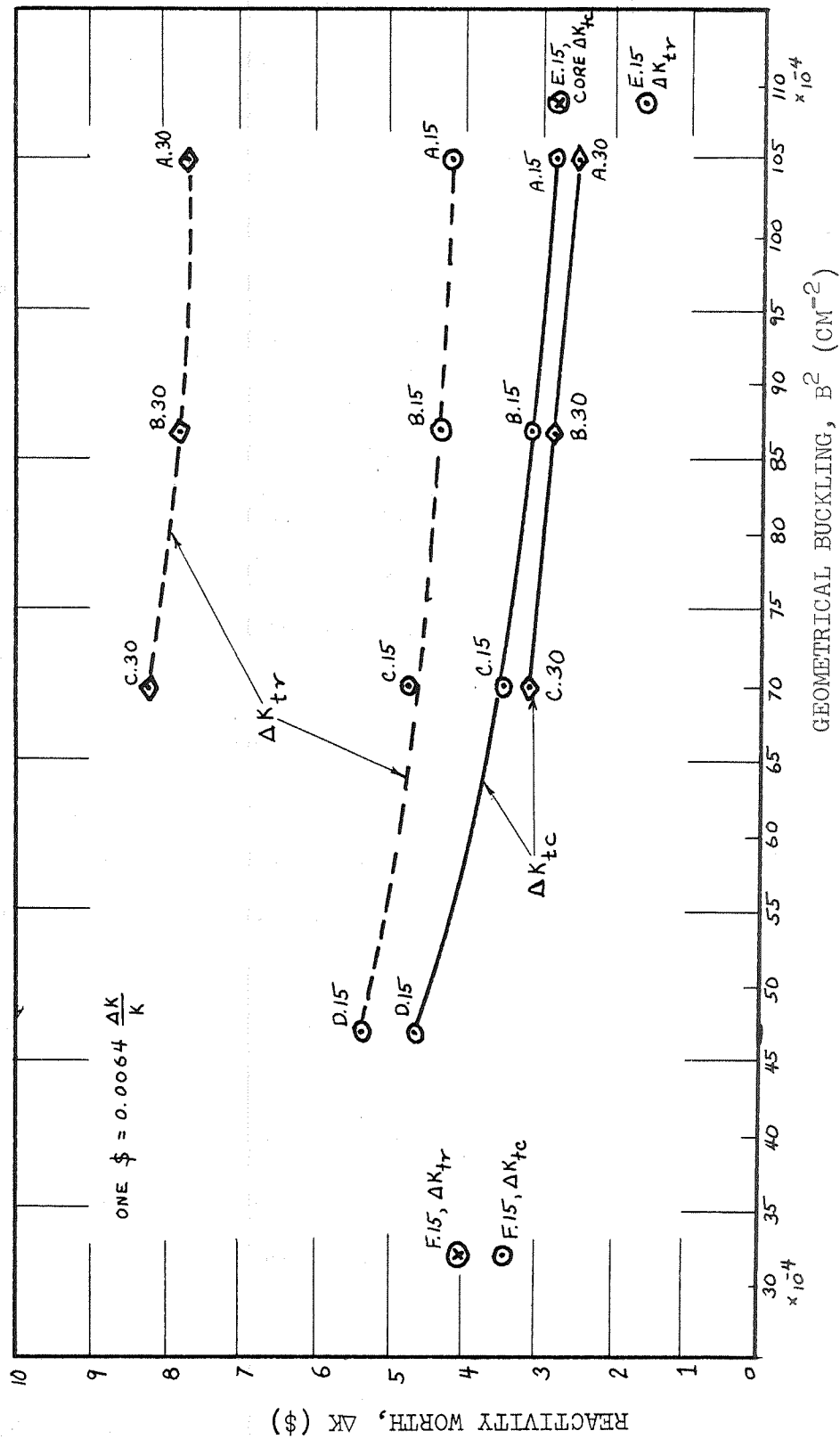
$$\text{and } 1.0\$ = 0.0064 \frac{\Delta k}{k} \quad (9).$$

Thus Δk_{tr} = whole-reflector hydrogen reactivity worth and Δk_{tc} = whole-core hydrogen reactivity worth. The results of the multigroup calculations are presented in Table 5-5 and Figure 5-5. The data form very regular groupings characterized by the reflector void fraction, ϕ .

Table 5-5
Reactivity Worth of Uniform Hydrogen in the Reflector and
Non-Uniform Hydrogen Distribution in the Reactor Core

Case	η	P_o	T_o	w	T_m	B^2	Δk Reflector	Δk Core	Δk Total
A.15	0.6	1200	300	1.013×10^{-2}	170	105×10^{-4}	4.18	2.77	6.95
B.15	0.5	1200	300	1.013×10^{-2}	170	87×10^{-4}	4.33	3.12	7.45
C.15	0.4	1200	300	1.013×10^{-2}	170	70×10^{-4}	4.66	3.46	8.12
D.15	0.3	1200	300	1.013×10^{-2}	170	47×10^{-4}	5.32	4.63	9.95
E.15	0.6	1200	500	1.013×10^{-2}	387	109×10^{-4}	1.79	2.36	4.15
F.15	0.6	1400	400	1.013×10^{-3}	220	32×10^{-4}	3.98	3.43	7.41
A.30	0.6	1200	300	1.013×10^{-2}	170	105×10^{-4}	7.70	2.53	10.23
B.30	0.5	1200	300	1.013×10^{-2}	170	87×10^{-4}	7.82	2.83	10.65
C.30	0.4	1200	300	1.013×10^{-2}	170	70×10^{-4}	8.18	3.08	11.26

P_o = Core entrance pressure, (psia)
 T_o = Core entrance temperature, ($^{\circ}$ R)
 w = Flow rate per channel, (lbm/s/channel)
 T_m = Mean hydrogen temperature in reflector, ($^{\circ}$ R)
 B^2 = Geometrical buckling of the core, (cm^{-2})
 ΔK = Reactivity change, (β) ($1 \beta = 0.0064 \Delta k/k$)

FIGURE 5-5 CORE, ΔK_{tc} , AND REFLECTOR ΔK_{tr} , HYDROGEN REACTIVITY WORTHS VERSUS GEOMETRICAL BUCKLING

Several obvious conclusions can be drawn from Figure 5-5:

- (1.) There is a very definite almost-linear dependence upon B^2 within each grouping. As the physical size of the reactor cores increase (B^2 decreases) both Δk_{tr} and Δk_{tc} increase;
- (2.) The total reflector hydrogen reactivity contribution can be significantly greater than the total reactivity contributed by the core hydrogen distribution. Table 5-5 shows that A.15 has $\Delta k_{tr} = 4.18 \$$ versus $\Delta k_{tc} = 2.77 \$$, while A.30 has $\Delta k_{tr} = 7.70 \$$ versus $\Delta k_{tc} = 2.53 \$$. Case E.15, with a $T_m = 387 R$ that is substantially higher than the uniform $T_m = 170 R$ for the A-D sequence, has a lower Δk for the reflector hydrogen than for the core hydrogen. This would indicate that it is the quantity of hydrogen in the reflector relative to the core that accounts for the greater reactivity for the reflector hydrogen and this is shown later;
- (3.) When the Δk_{tr} for the three corresponding sets of cases, A(.15,.30), B(.15,.30), and C(.15,.30), are compared, it is found that the ratio of the reflector hydrogen worths for each set are all approximately in the ratio 1 : 1.80 which is in nearly the same ratio as the reflector hydrogen densities (exactly 1:2);
- (4.) The pancake core, case F.15, does not fall into the $\phi = 0.15$ grouping as correlated by B^2 , although the total core plus reflector hydrogen worth, 7.41 \$, is in the same range as the total Δk 's of the $\phi = 0.15$ A-D sequence.

Reasonable care was taken to closely approximate the continuous core hydrogen distributions calculated by MAXPOW when converting to the lumped regional hydrogen densities for analysis by EXTERMINATOR, illustrated in Figure 5-1, so that the regularity of the trends shown by Figure 5-5 are thought not to be the result of any systematic error in data preparation. However, interpretation of Figure 5-5 is somewhat obscured by the unequal amounts of hydrogen being compared

from case to case. As B^2 decreases, not only do the reactor core volumes increase, but the reflector volume also increases. Therefore equal densities of hydrogen mean different masses of hydrogen from case to case.

The whole-core and whole-reflector hydrogen worths are divided by the total number of hydrogen atoms in each section of the reactor and these results are reported in Table 5-6 and Figure 5-6 as specific reactivity worth, dollars per $\text{kg} \cdot \text{H}_2$, where one \$ = $0.0064 \frac{\Delta k}{k}$.

Table 5-6

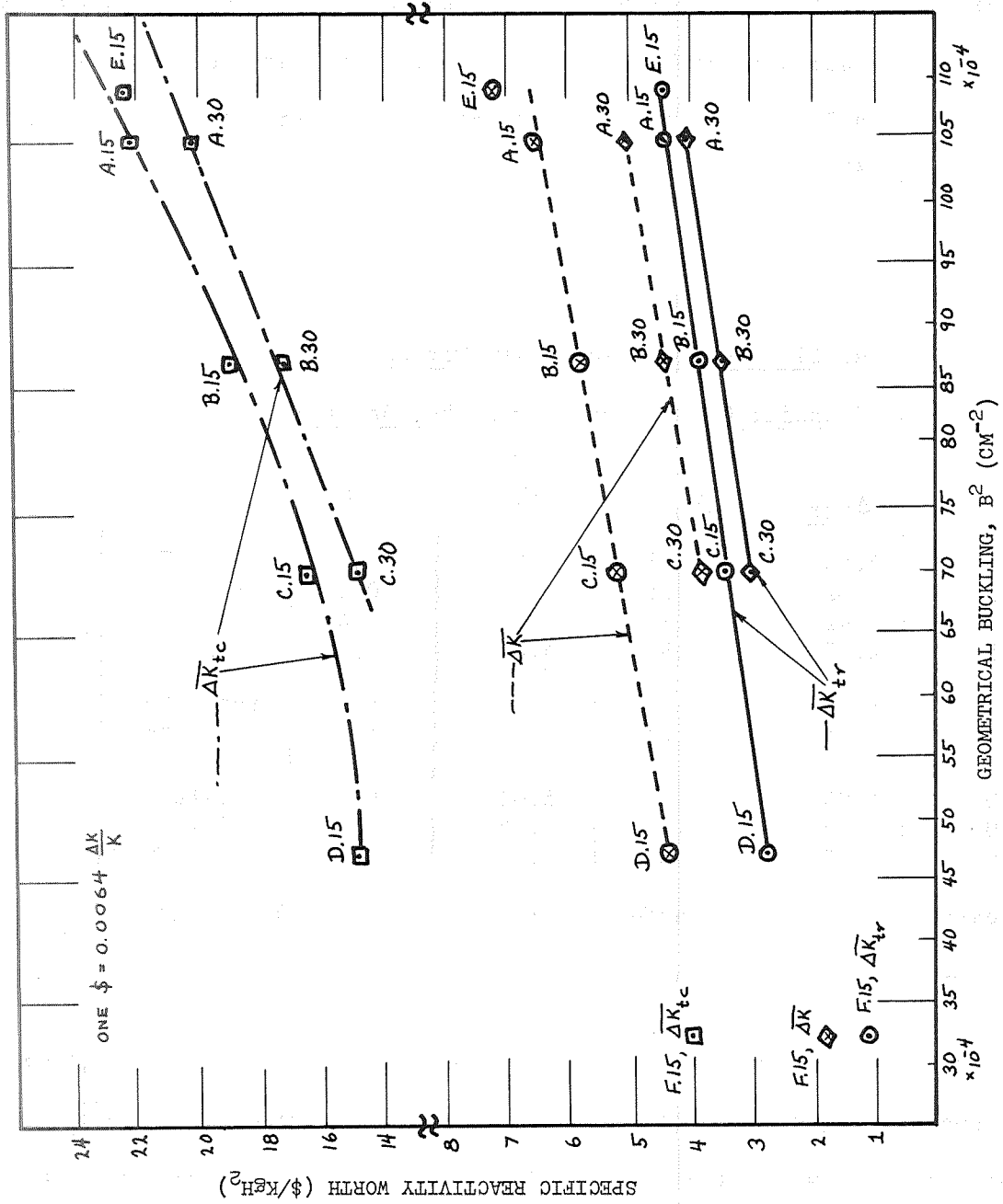
Specific Hydrogen Reactivity Worths*

For Heat-Transfer Optimum Nuclear Rocket Reactors

Case	$\overline{\Delta k}_{tr}$	$\overline{\Delta k}_{tc}$	$\overline{\Delta k}$	η	B^2
A.15	4.40	22.28	6.48	0.6	105×10^{-4}
B.15	3.82	18.95	5.74	0.5	87×10^{-4}
C.15	3.42	16.49	5.16	0.4	70×10^{-4}
D.15	2.75	14.84	4.43	0.3	47×10^{-4}
E.15	4.40	22.41	7.16	0.6	109×10^{-4}
F.15	1.10	3.92	1.79	0.6	32×10^{-4}
A.30	4.05	20.29	5.06	0.6	105×10^{-4}
B.30	3.45	17.18	4.38	0.5	87×10^{-4}
C.30	3.00	14.68	3.83	0.4	70×10^{-4}

* Reactivity worths expressed in $\left[\$/\text{kg H}_2 \right]$, 1 \$ = $0.0064 \frac{\Delta k}{k}$
 B^2 = Geometrical buckling $\left[\text{cm}^{-2} \right]$

From Figure 5-6 one can see that the data again form definite groupings based upon reflector void fraction, . $\overline{\Delta k}_{tc}$ is much greater than $\overline{\Delta k}_{tr}$ because the fast neutrons which are down-scattered in the core are more important to the criticality of the system than fast neutrons down-scattered in the reflector.

FIGURE 5-6 SPECIFIC HYDROGEN WORTH, ΔK , ΔK_{tc} , AND ΔK_{tr} , ($\$/kH_2) VERSUS GEOMETRICAL BUCKLING$

Thus, hydrogen is more effective in the core. As the magnitude of $\overline{\Delta k}$ shows, the greater effectiveness of core hydrogen is outweighed by the greater number of reflector hydrogen atoms and the $\overline{\Delta k}$ value closely matches $\overline{\Delta k}_{tr}$ rather than $\overline{\Delta k}_{tc}$.

The linearity demonstrated in Figure 5-6 for the $\phi = 0.15$ group of $\overline{\Delta k}_{tr}$ is more apparent than that previously demonstrated for $\overline{\Delta k}_{tr}$ in Figure 5-5 because the normalization places cases D.15 and E.15 definitely in line with A-C. The obvious conclusion to be drawn from the B^2 trends for all of the ϕ groupings is that a unit mass of hydrogen in either core or reflector is more effective as the average energy of the system increases since both $\overline{\Delta k}_{tc}$ and $\overline{\Delta k}_{tr}$ increase with increasing B^2 . By Equation (5.4) an increasing B^2 means a decreasing core size, more relative neutron leakage, and a faster neutron spectrum, as is shown in the following Section 5.3. Notice that the normalized trends in Figure 5-6 are opposite to the trends shown in Figure 5-5 which show an increase in total worth as B^2 decreases. The increase in $\overline{\Delta k}$ with B^2 as shown in Figure 5-6 is a change in the reactivity effectiveness of a unit mass of hydrogen which is compensated for, as in Figure 5-5, by an increase in the amount of hydrogen.

When cases A.15 - D.15 and A.30 - C.30 are approximately fit by a straight line, these equations result:

$$\phi = 0.15; \quad \overline{\Delta k}_{tr} = 1.413 + 2.845 \times 10^{-2} B^2 \quad (5.10)$$

$$\phi = 0.15; \quad \overline{\Delta k} = 2.758 + 3.545 \times 10^{-2} B^2 \quad (5.11)$$

$$\phi = 0.30; \quad \overline{\Delta k}_{tr} = 0.900 + 3.000 \times 10^{-2} B^2 \quad (5.12)$$

$$\phi = 0.30; \quad \overline{\Delta k} = 1.370 + 3.514 \times 10^{-2} B^2 \quad (5.13)$$

A correlation of the reflector hydrogen density versus total reflector hydrogen worth for cases A.15 and A.30 is plotted in Figure 5-7. Three values of hydrogen density are correlated for each case, with the critical uranium con-

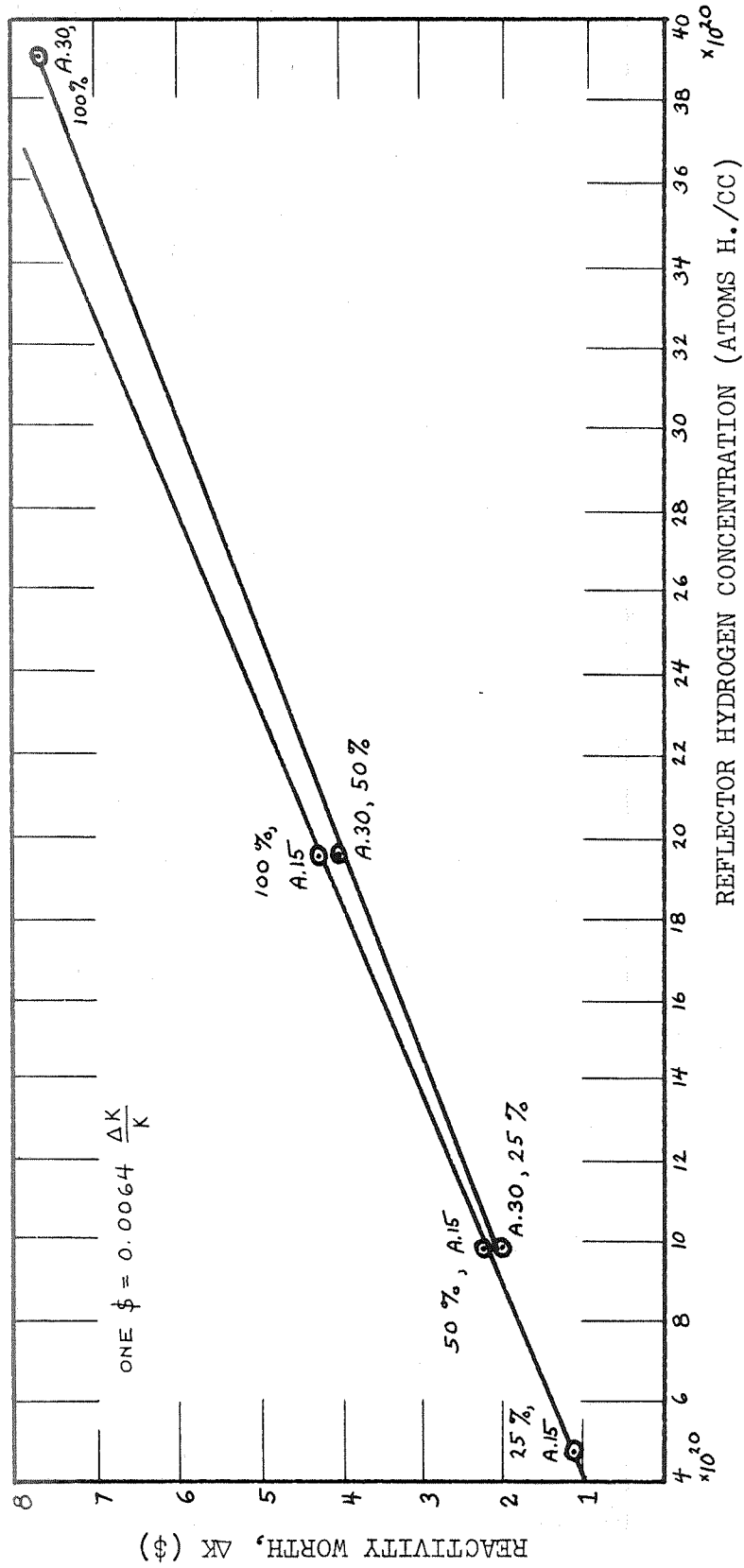


FIGURE 5-7 REFLECTOR HYDROGEN REACTIVITY WORTH VERSUS CONCENTRATION OF HYDROGEN IN THE REFLECTOR

centrations previously determined for A.15 and A.30 remaining unchanged. The lowest hydrogen density for each ϕ grouping in Figure 5-7 is at 25% of full steady-state density and the middle value is at 50% of full density. Therefore, equal quantities of hydrogen are directly compared for the 50%, A.30 point, and the 100%, A.15 point. Figure 5-7 shows: (1.) For the density range, 25% - 100% of full steady-state hydrogen density, the Δk_{tr} contribution is directly proportional to the hydrogen density; a similar conclusion has been reported by Little (9) who found that the total reactivity worth of uniform hydrogen in a small core is directly proportional to density; (2.) an equal amount of hydrogen is worth more in the $\phi = 0.15$ reflector than in the $\phi = 0.30$ reflector. This difference in reactivity worth per unit mass of hydrogen cannot be due to the increase in the average energy of the neutron spectrum for the A.30 reactor system, but is probably due to the decreased beryllium density for $\phi = 0.30$.

Corresponding to the decrease in specific worth for hydrogen in the $\phi = 0.30$ reflector, Figure 5-6 shows that the same effect is true for hydrogen in the reactor core. Table 5-7 shows that $\bar{\Delta k}_{tr}(.15) - \bar{\Delta k}_{tr}(.30) = 0.37$ [$\$/\text{KgH}_2$] and $\bar{\Delta k}_{tc}(.15) - \bar{\Delta k}_{tc}(.30) = 1.80$ [$\$/\text{KgH}_2$], independent of B^2 . Apparently

Table 5-7

The Effect of an Increased Reflector ϕ
Upon Specific Hydrogen Worth*

Case	$\bar{\Delta k}_{tr}(.15)$	$\bar{\Delta k}_{tr}(.30)$	.15 - .30	$\bar{\Delta k}_{tc}(.15)$	$\bar{\Delta k}_{tc}(.30)$	.15 - .30
A	4.40	4.05	0.35	22.28	20.29	1.99
B	3.82	3.45	0.37	18.95	17.18	1.77
C	3.42	3.00	0.42	16.49	14.68	1.81

* [$\$/\text{KgH}_2$]

the decreased diffusion coefficient in the reflector has altered the relative flux distributions in both core and reflector such that hydrogen is not as effective in reducing leakage.

The following tentative explanation for the behavior of hydrogen effectiveness for these very fast systems is prompted by a discussion by Lamarsh on neutron moderation (39, Chapter 6). Lamarsh observes that oxygen in water acts like a "uniformly distributed reflector" with respect to hydrogen in that: (a) oxygen does not appreciably moderate neutrons by itself, but (b) neutron collisions with oxygen tend to prevent the neutrons from traveling very far from a source, in which case the presence of oxygen will increase the "available" neutron flux in the vicinity of hydrogen. Increased moderation results and the neutron age, τ , is decreased significantly. Fermi age theory gives $e^{-B^2\tau}$ as the fast non-leakage probability (40) and one can see that the presence of oxygen makes hydrogen more effective in reducing leakage. This same effect may be responsible for the decrease in hydrogen effectiveness say, from A.15 to A.30. Figure 5-4 shows that thermal neutrons have a very high peak in the reflector. Figure 5-8 shows that it is primarily the beryllium that provides the majority of the thermal flux even though the hydrogen significantly increases the flux. Therefore, it is reasonable to conclude that hydrogen's effectiveness in both core and reflector may depend upon the reflector density.

Lamarsh observes that a decrease in the so-called neutron age, τ , also occurs for metal-water mixtures, uranium specifically. Lamarsh concludes that since these metals do not moderate neutrons appreciably by elastic scattering, then an increase in moderation must occur by inelastic scattering. Inelastic scattering for uranium increases in effectiveness with neutron energy as Table 5-8 shows (summarized from Table D-2).

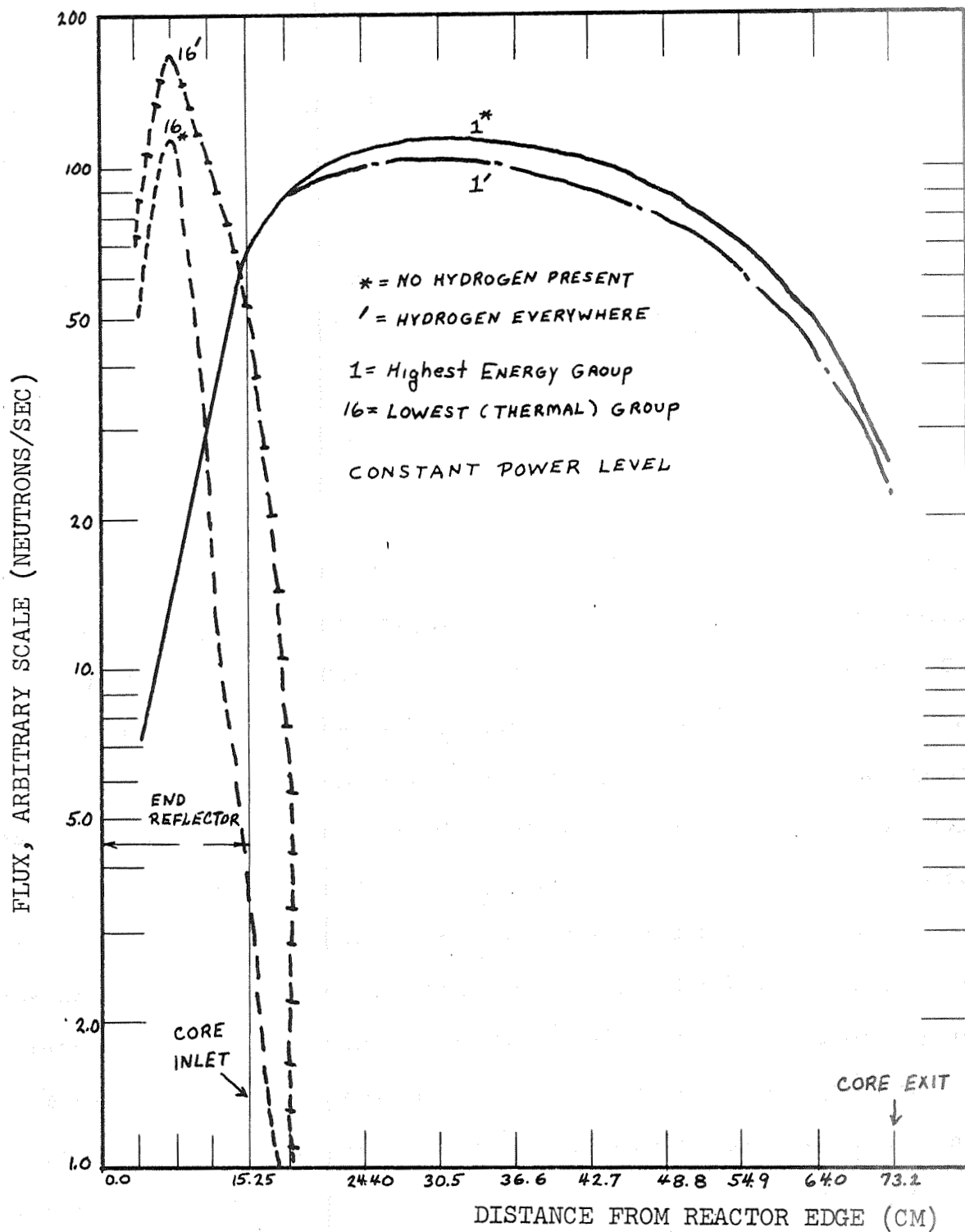


FIGURE 5-8 REPRESENTATIVE HIGHEST AND LOWEST ENERGY GROUP FLUX DISTRIBUTIONS, WITH AND WITHOUT HYDROGEN, CASE A.15

Table 5-8

Total Microscopic Scattering Cross Section of UraniumTop 4 Groups

<u>Energy Group</u>	<u>σ_s [Barns]*</u>	<u>Energy Range</u>
1	1.79	3 - MeV
2	1.43	1.4 - 3 MeV
3	1.02	0.9 - 1.4 MeV
4	0.43	0.4 - 0.9 MeV

*1 Barn = 10^{-24} cm²

Hydrogen therefore appears to increase in effectiveness as the reactor cores in this thesis decrease in size because the total number of uranium atoms is increasing. The higher energies of these reactor cores allows very effective use of the increased uranium to inelastically down-scatter relatively more fast neutrons to the energy range for which hydrogen is very effective. Thus the same mass of hydrogen "sees" an increased neutron flux. Indirect confirmation of the correctness of this concept may be seen by comparing Figure 5-3, which shows critical uranium concentration versus B^2 , and Figure 5-6. The $\phi = 0.15$ curve in Figure 5-3 is very similar to the $\phi = 0.15, \bar{\Delta k}_{tc}$ curve, in Figure 5-6, which is no coincidence if hydrogen specific worth is directly related to the total number of uranium atoms in the reactor system. However, there is no constant relationship between $\bar{\Delta k}_{tc}$ and the critical uranium concentration which one would expect if uranium were the only inelastic scatterer. The actual situation is complicated by the presence of carbon which also has inelastic scattering in the highest energy group. As B^2 increases, the concentration of carbon must decrease because the core void fraction increases. The specific worth of hydrogen should be affected by the carbon con-

centration although the dominant effect appears to be due to uranium.

It can be concluded from the above arguments that the trends discovered in this section are based upon a combination of: (a) "distributed reflector" effect and (b) inelastic uranium scattering. Trends such as Equations (5.10), (5.11), (5.12), and (5.13) can be very useful in estimating reactivity worth for very compact nuclear rocket reactors.

Because the non-uniform hydrogen distributions used in this reactivity survey are a result of using Correlation (4.6), rather than Correlation (4.3), the positional dependence of the core hydrogen was investigated. Because of the very rapid heating of the hydrogen coolant that occurs for these optimum reactor cores, it was expected that most of the non-uniform reactivity would be found to be due to the first few centimeters of the channel length. Table 5-9 shows that over half of the Δk_{tc} is due to that front portion of the channel that contains the majority of hydrogen. Referral to Figure 4-13, which shows the density distributions for Correlation (4.3), and Table 5-2, which shows the region density distributions actually used, in EXTERMINATOR, would enable one to estimate the change in Δk_{tc} due to use of Correlation (4.3) by a simple linear scaling of the first few channel sections. In general this will amount to no more than a few cents of reactivity.

Table 5-9

Position Dependence of Core Hydrogen WorthFor Several Cases

x_0 = Distance from core inlet, start of core hydrogen region

x_1 = Distance from core inlet, end of core hydrogen region

$\Delta k'$ = Incremental reactivity change

Δk_{tc} = Total reactivity change for entire core [\$]

Case	x_0 [cm]	x_1 [cm]	$\Delta k' [\$]$	$\overline{\Delta k'} [\$ / \text{KgH}_2]$	$\frac{\Delta k'}{\Delta k_{tc}}$
A.15	0.0	3.05	0.84	23.82	0.302
	3.05	9.15	0.72	26.34	0.561
	9.15	57.95	1.22	19.64	1.000
A.30	0.0	3.05	0.70	19.86	0.277
	3.05	9.15	0.64	23.48	0.530
	9.15	57.95	1.19	19.13	1.000
E.15	0.0	6.10	0.93	24.91	0.394
	6.10	12.20	0.48	28.04	0.597
	12.20	18.30	0.32	28.21	0.733
	18.30	24.40	0.23	25.88	0.830
	24.40	54.90	0.40	13.05	1.000
F.15	0.0	3.05	1.26	5.28	0.317
	3.05	6.10	0.30	5.54	0.392
	6.10	9.15	0.25	4.62	0.455
	9.15	12.20	0.26	4.80	0.520
	12.20	15.25	0.20	4.70	0.570
	15.25	27.45	0.73	4.65	0.754
	27.45	45.75	0.69	3.18	0.927
	45.75	64.05	0.29	1.31	1.000

The hydrogen distribution in the reflector is treated in an approximate manner in this thesis because the specific hydrogen worth of the reflector is much lower than the specific hydrogen worth of the core. However, since it has been shown that the total reactivity contribution from the reflector can be several times the reactor core contribution, the positional dependence of hydrogen in the reflector

was examined. Figure 5-9 shows the model used to investigate the sequential insertion of hydrogen at full P_0 , T_m , in cases A.15 and A.30. The hydrogen is assumed to flow by stages (conceptually only) first into reflector section a, b, c, and then d. The results of these calculations are shown in Table 5-10. Approximately 65% of the total reflector reactivity is introduced in a and b. The Δk 's for section b are quite a bit higher than the uniform Δk_{tr} calculated at the beginning of this Section. The reflector section at the core entrance is about 72% higher than the uniform estimate also. Reflector sections b, c, and d would be expected to be at higher temperatures than section a. What appears to be an underestimate at b and d because the lower Δk_{tr} is assumed, will tend to be compensated by an increase in actual temperature above T_m . A detailed hydrogen distribution in the reflector (and an accurate determination of the gamma heating fraction in the reflector) would be required to completely determine the error involved for a uniform assumption. However, it is felt that the model used is accurate enough for the survey purpose of this thesis.

Table 5-10

Position Dependence of Reflector Hydrogen Worth, Cases A.15 and A.30

Case	Section	$\Delta k''$ [\$]	$\Delta k''$ [\$/KgH ₂]	$\frac{\Delta k''}{\Delta k_{tr}}$
A.15	a	1.83	3.78	0.438
	b	0.89	6.53	0.651
	c	0.47	2.23	0.763
	d	0.99	7.59	1.000
A.30	a	3.69	3.82	0.480
	b	1.53	5.61	0.679
	c	0.78	1.85	0.793
	d	1.69	7.16	1.000

a - d = Sections of reflector, keyed to Figure 5-9

$\Delta k''$ = Total reactivity increment

Δk_{tr} = Total reactivity worth of reflector

$$\left(\begin{array}{l} \Delta k_{tr}(.15) = 4.40 \text{ } \$/\text{KgH}_2 \\ \Delta k_{tr}(.30) = 4.05 \text{ } \$/\text{KgH}_2 \end{array} \right)$$

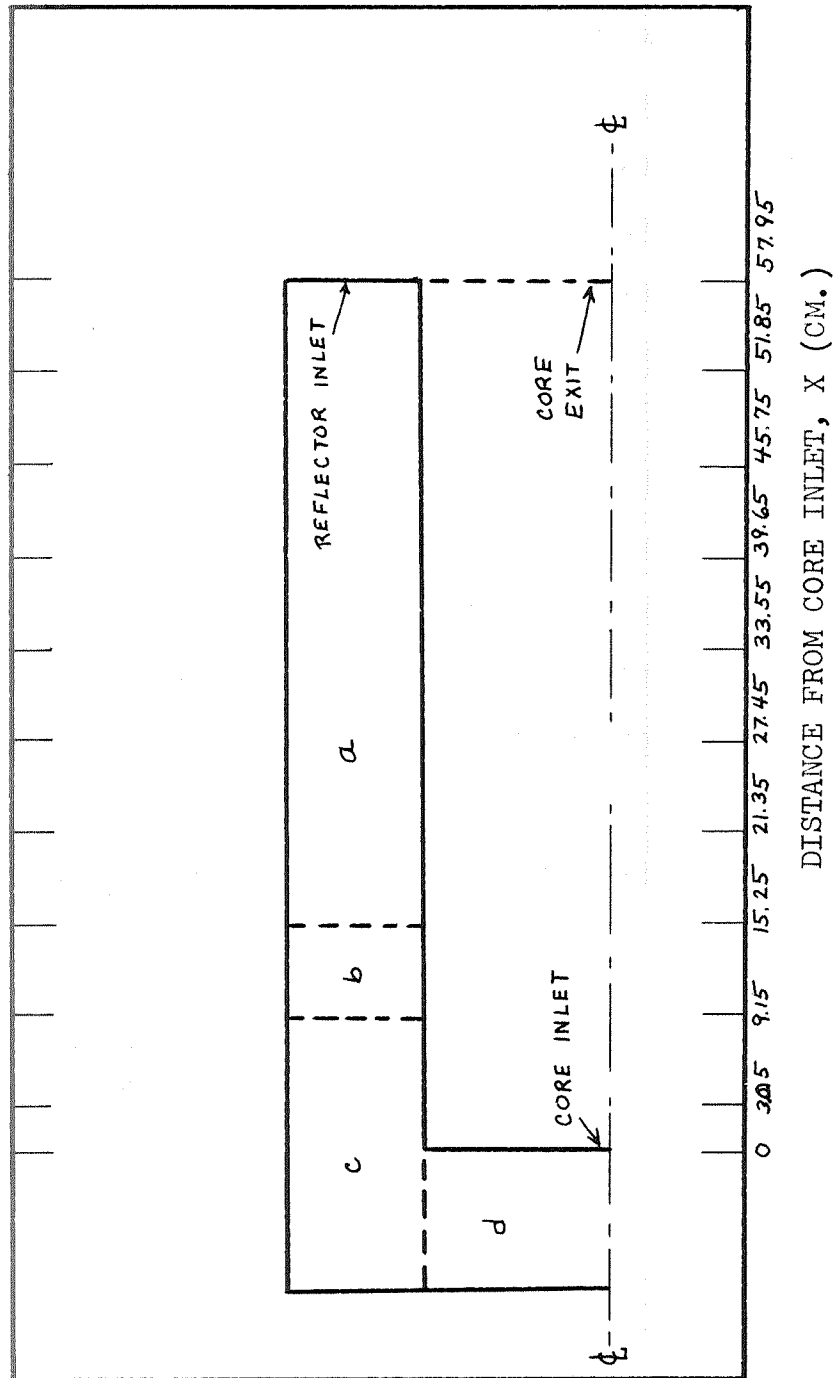


FIGURE 5-9 MODEL FOR REFLECTOR POSITION WORTH OF HYDROGEN, CASES A.15 AND A.30

5.3 Changes in Neutron Production and Leakage Spectra Due to the Presence of Hydrogen

The mean neutron production energy and neutron leakage energy were calculated for each case investigated in this thesis. The definition of these mean energies are:

$$\bar{E}_p = \text{mean energy of neutrons causing productions} = \frac{\sum_i \bar{E}_i (\nu \Sigma_f \phi)_i}{\sum_i (\nu \Sigma_f \phi)_i} \quad [\text{KeV}] \quad (5.14)$$

= "mean production energy"

where $(\nu \Sigma_f \phi)_i$ = total neutrons produced by fissions in group i, $\left[\frac{\text{neutrons}}{\text{sec}} \right]$

and $\sum_i (\nu \Sigma_f \phi)_i$ = total neutrons produced by fissions in all groups, $\left[\frac{\text{neutrons}}{\text{sec}} \right]$,

and $\bar{E}_i$ = mean energy of group i, [KeV].

The group energies are listed in Table 5-11.

Table 5-11

Average Group Energies

<u>Energy Group</u>	<u>$\bar{E}$ [MeV]</u>
1	4.23
2	2.20
3	1.15
4	0.65
5	0.25
6	5.85×10^{-2}
7	1.00×10^{-2}
8	1.775×10^{-3}
9	3.250×10^{-4}
10	6.5×10^{-5}
11	2.0×10^{-5}
12	6.5×10^{-6}
13	2.0×10^{-6}
14	7.0×10^{-7}
15	2.5×10^{-7}
16	2.5×10^{-8}

$$\bar{E}_L = \text{mean leakage energy} = \frac{\sum_i \bar{E}_i (L_i)}{\sum_i (L_i)} \quad [\text{KeV}] \quad (5.15)$$

where L_i = total leakage neutrons in group i , $\left[\frac{\text{neutrons}}{\text{sec}} \right]$,
 and $\sum_i (L_i)$ = total leakage neutrons for all groups, $\left[\frac{\text{neutrons}}{\text{sec}} \right]$.

The results of these calculations are shown in Table 5-12 and Figure 5-10. As Figure 5-10 shows, the mean production and leakage energies group themselves according to reflector ϕ and correlate almost linearly with the geometrical buckling, B^2 . Again, case F.15 exhibits different characteristics than the other cases because of its increased relative leakage.

Figures 5-11 and 5-12 show the effect that hydrogen addition has upon the neutron production spectrum for case B.15. With the addition of hydrogen uniformly to the reflector in Figure 5-11, the major effect is to remove the neutron productions at fast and intermediate energies (lethargy = 6 is the lower limit for "fast" energies ≥ 100 KeV) and increase the thermal productions significantly. For only an 18 KeV reduction in the $\bar{E}_p$ the result is $\Delta k = 4.33\%$. Actually the calculation of a mean production energy does not really reflect the large increase

Table 5-12

Mean Neutron Leakage Energies and Mean Neutron Production Energies

Case	B^2 [cm^{-2}]	$\bar{E}_p$ [KeV]	$\bar{E}_L$ [KeV]
A.15	105×10^{-4}	426	868
B.15	87×10^{-4}	296	785
C.15	70×10^{-4}	187	716
D.15	47×10^{-4}	62	591
E.15	109×10^{-4}	470	890
F.15	32×10^{-4}	261	920
A.30	105×10^{-4}	540	979
B.30	87×10^{-4}	369	900
C.30	70×10^{-4}	256	837

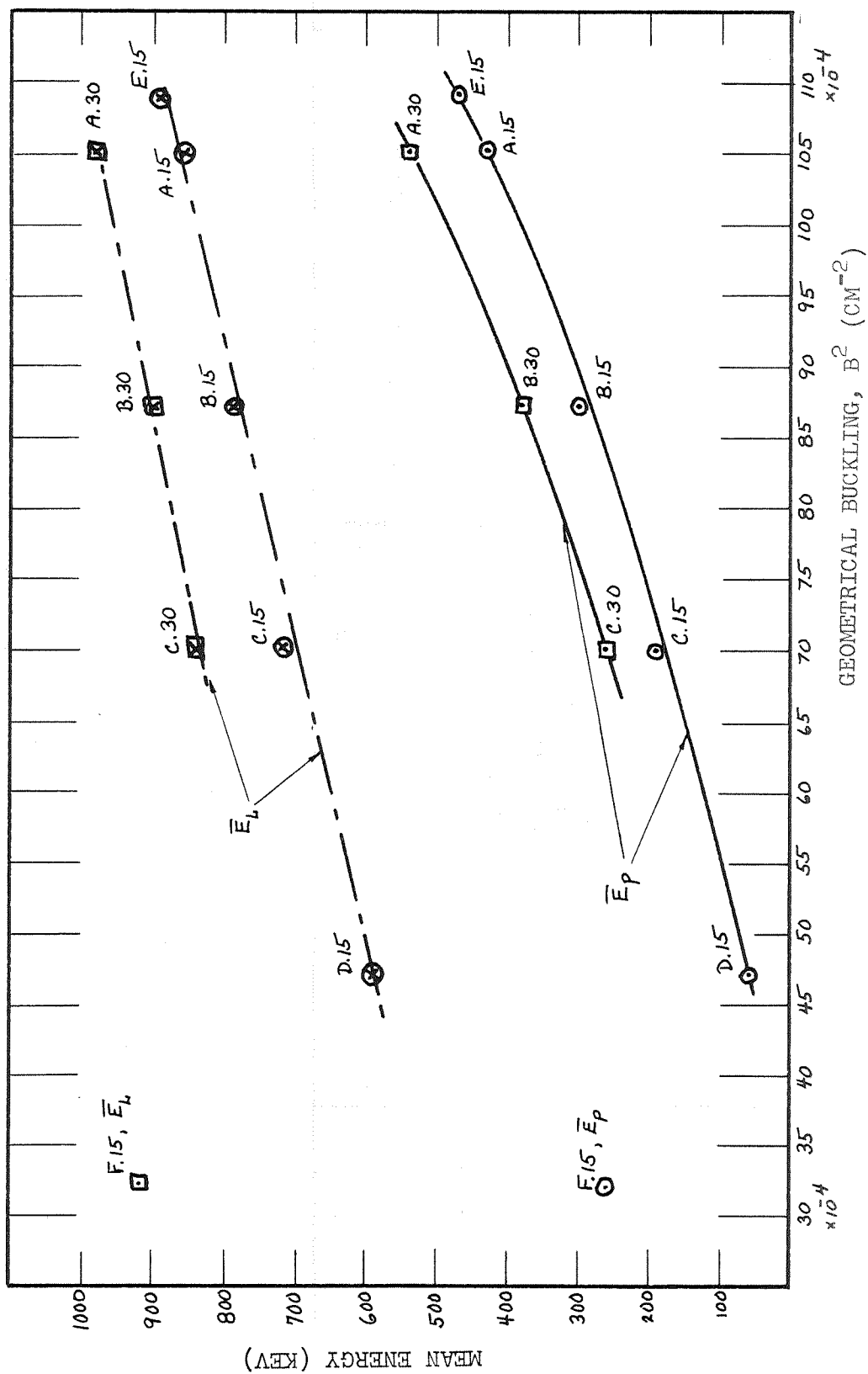


FIGURE 5-10 MEAN LEAKAGE AND NEUTRON PRODUCTION ENERGIES VERSUS GEOMETRICAL BUCKLING

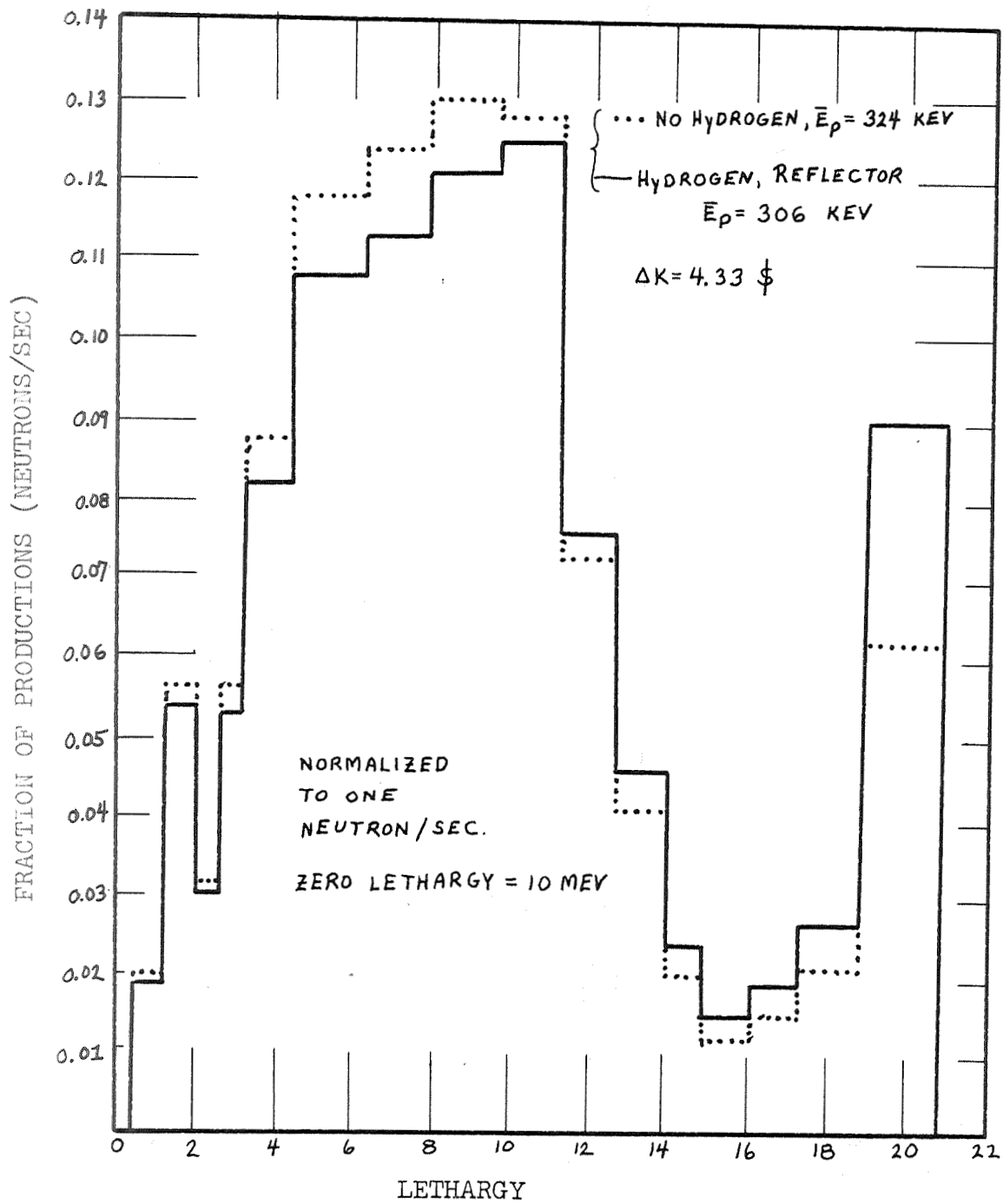


FIGURE 5-11 EFFECT OF REFLECTOR HYDROGEN UPON NEUTRON
 PRODUCTION SPECTRUM, CASE B.15

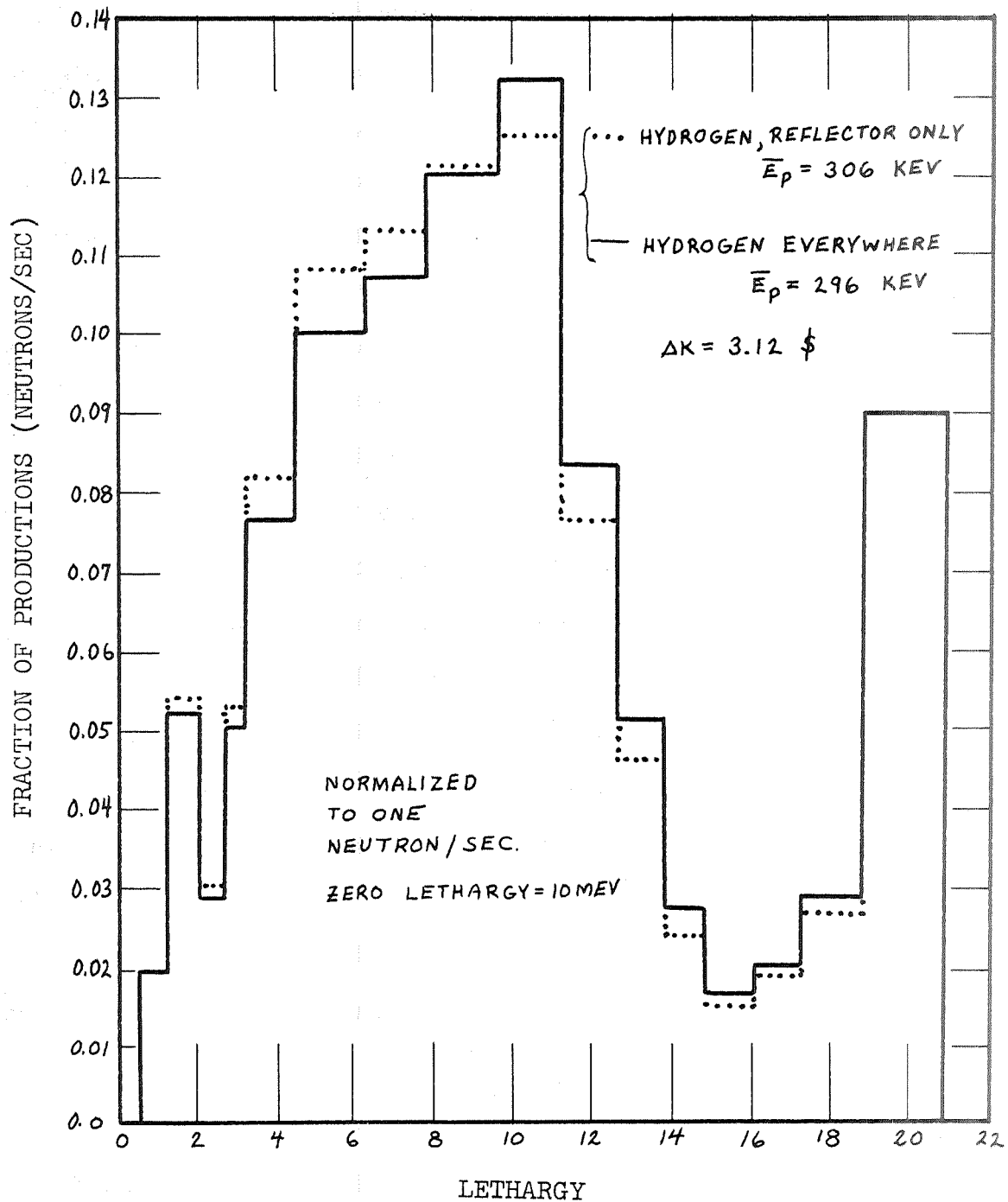


FIGURE 5-12 EFFECT OF CORE HYDROGEN UPON NEUTRON
PRODUCTION SPECTRUM, CASE B.15

in thermal productions. The average energies of groups 1-8 are so much larger than the 0.025eV of the 16th group, that changes in $\bar{E}_p$ only show shifts among these higher groups. With the addition of the non-uniform hydrogen distribution to the reactor core in Figure 5-12, no detectable relative change occurs in the thermal group when compared with Figure 5-11 and the only change in the production spectrum is a further shift from fast to intermediate energies. For this hydrogen addition the change in $\bar{E}_p$ is 10 KeV and $\Delta k = 3.12\%$. For the entire reactor, hydrogen has softened the spectrum only 28 KeV and added 7.45% in reactivity. This is a representative result for all of these reactors. The majority of moderation in these fast, compact cores is due to inelastic scattering by uranium and elastic scattering by carbon. Hydrogen is primarily effective in utilizing the presence of uranium inelastic scattering to significantly increase the thermal group productions and in that manner reduce leakage.

Similarly, the addition of hydrogen does not greatly affect the neutron leakage spectrum in terms of $\bar{E}_L$. Figure 5-13 shows the typical prominence of the second energy group in the leakage spectra. Although A.30 has twice the number of hydrogen atoms in the reflector, A.15 still has relatively more thermal neutron leakage. This points out the effectiveness of denser beryllium in combination with hydrogen in moderating neutrons.

5.4 Non-Uniform Effects

5.4.1 Perturbations in Power Distributions Due to the Presence of Hydrogen in Uniformly-Loaded Reactors

The results reported in Section 5.2.2 demonstrate that the critical reactors with the dimensions predicted by MAXPOW have very high uniform

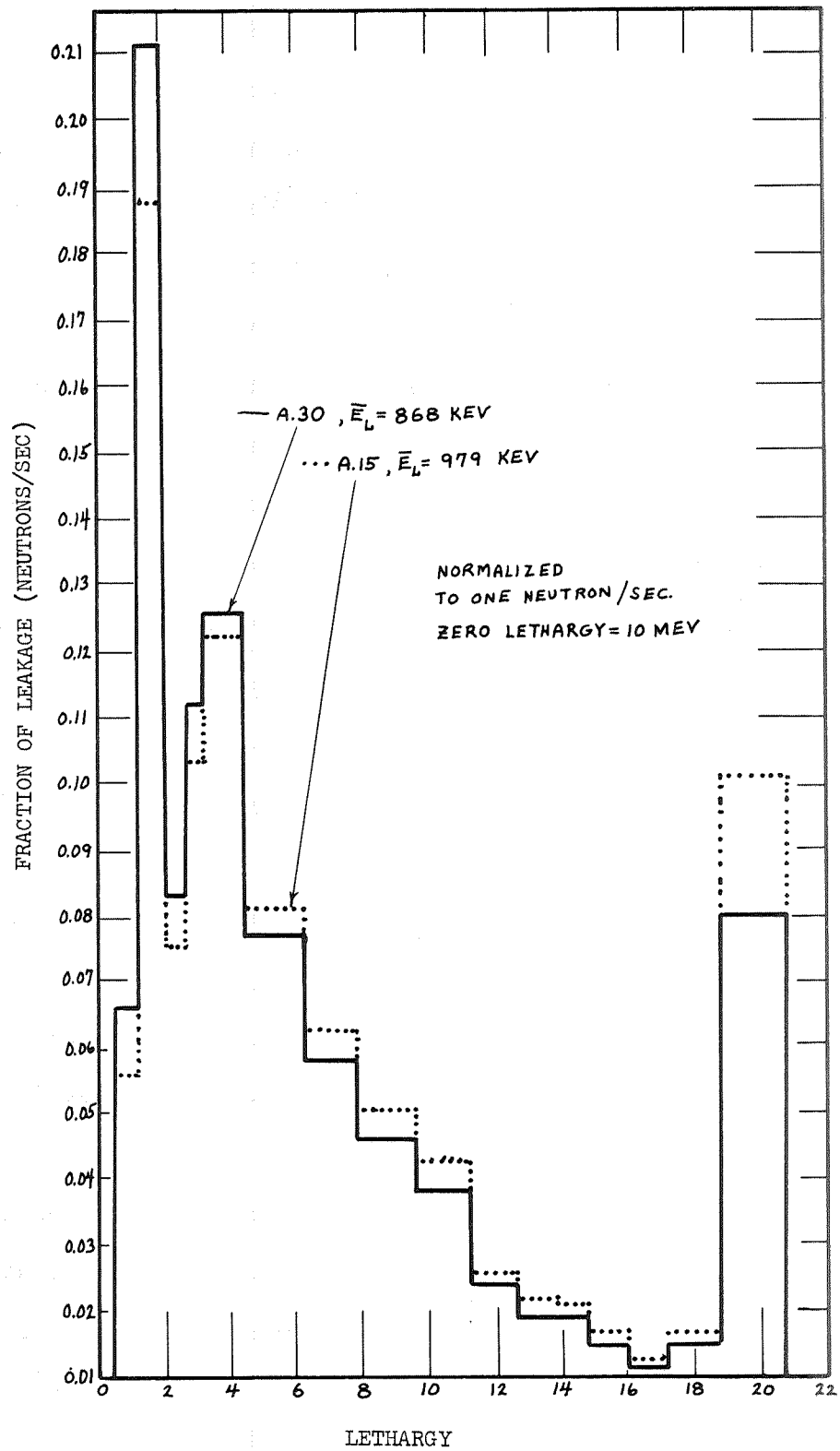


FIGURE 5-13 TOTAL NEUTRON LEAKAGE SPECTRA, CASES A.15 AND A.30

uranium concentrations. When hydrogen is introduced into the reflector there is an increase in the number of thermal neutrons going from the reflector into the core. Because of the high concentration of uranium, these thermal neutrons are very quickly absorbed and the change in power density will show the largest increase at the core-reflector interface. When hydrogen is introduced into the reactor core the hydrogen will very rapidly decrease in density along the channel if the reactor core is already heated up. Little (9) has shown that unacceptably large reactivity insertions can result if cold, dense hydrogen is directly injected into the reactor core. From the results reported in Section 5.2.3 this danger is confirmed by noting the relatively much greater specific reactivity worth of core hydrogen with respect to reflector hydrogen. Therefore it is assumed that the scheme of reactor startup will be such that spatial shifts in power density in the reactor core need not consider the insertion of cold, dense hydrogen. If a comparison is made of power distributions in the reactor core with no hydrogen present anywhere and then with all of the hydrogen present, distributed as previously assumed for the critical mass and reactivity calculations in Section 5.2, then a reasonable idea can be had of the amount of power tilting that must occur during startup.

Figure 5-14 shows the relative changes in axial power shape as hydrogen is first fully inserted into the reflector and then subsequently fully inserted as a non-uniform distribution into the reactor core. This is only a convenient calculational technique to show the maximum variation in power shape. For an actual reactor system, the hydrogen would be introduced in a much more continuous manner, starting with low pressure hydrogen

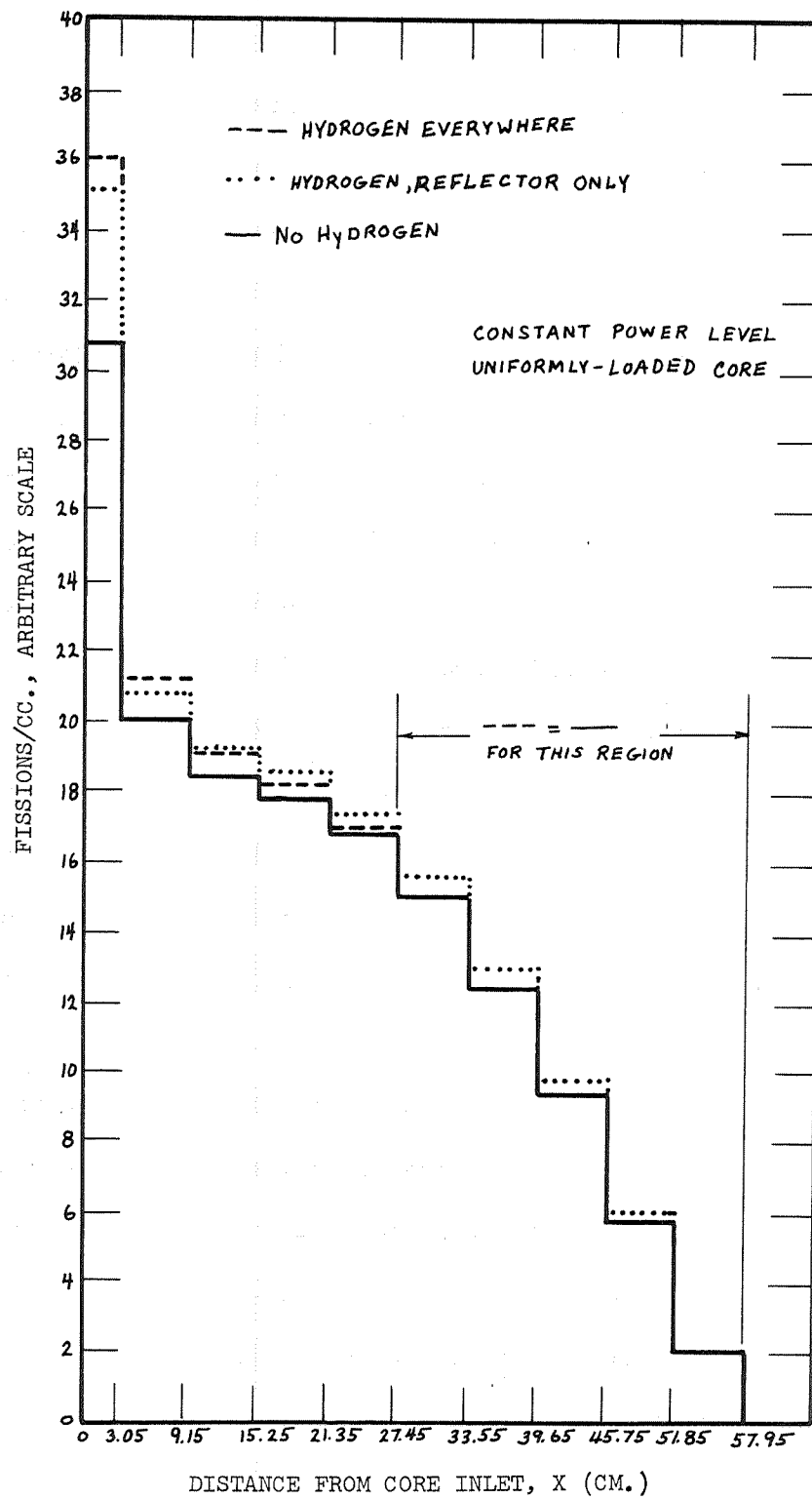


FIGURE 5-14 AXIAL POWER PROFILES AT THE RADIAL CORE-REFLECTOR
 INTERFACE, CASE A.15 ALL THREE HYDROGEN SITUATIONS

in a bootstrap operation (29).

Figure 5-14 shows that the biggest reactivity change for case A.15, which is due to the reflector hydrogen, also causes the largest spatial change. Subsequent core hydrogen introduction increases the inlet power density a little and reduces the remainder of the power profile back to the no-hydrogen condition. The reason for this behavior is that Figure 5-14 shows the axial power profile at the radial core-reflector interface. When hydrogen is introduced into the reactor core, relatively fewer thermal neutrons reach the radial edge from the interior of the core because of the presence of hydrogen. The profiles in Figure 5-14 are for a constant power level so that only the relative changes in power shape are shown. The total change for the reactor core entrance is +17%.

Figure 5-15 shows the axial power distributions along the reactor core center. The reactor core entrance has a smaller total variation, +14%, but the rest of the final power shape is generally below the no-hydrogen distribution. Thus the center tends to behave in the opposite manner as the radial core-reflector interface.

The reflector density and total amount of hydrogen in the reflector will affect the maximum variation in spatial power shape. Table 5-13 lists the relative spatial power variation at the reactor core entrance for all of the reactor cases investigated in this thesis. The core entrance region has the largest spatial power variation because the core hydrogen distribution is densest there.

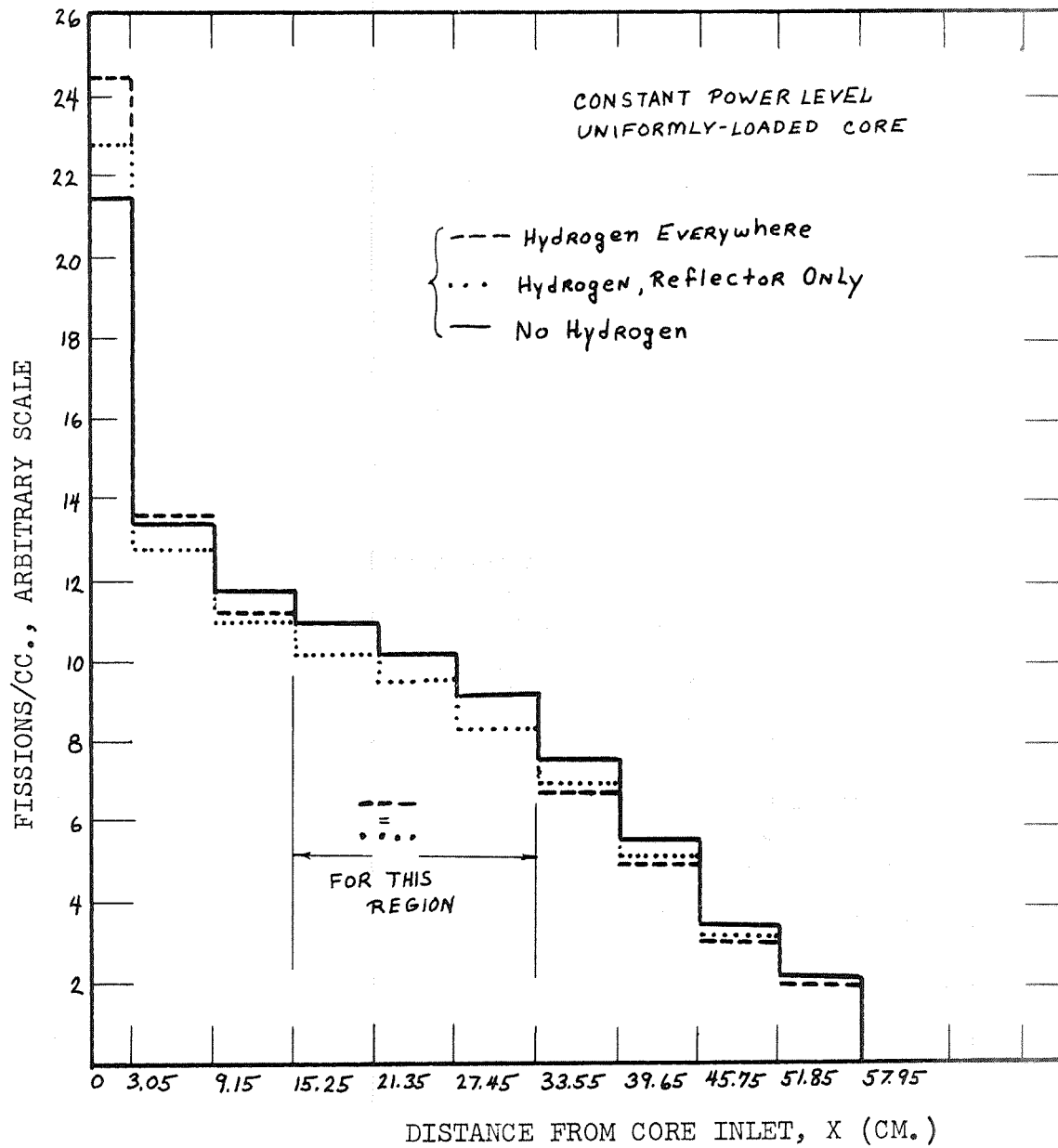


FIGURE 5-15 AXIAL POWER PROFILES AT THE REACTOR CENTER,
CASE A.15, ALL THREE HYDROGEN SITUATIONS

Table 5-13

Relative Spatial Power Tilting at the Reactor Core Entrance

$$\Delta P = [P(\text{All Hydrogen}) - P(\text{No Hydrogen})] / P(\text{No Hydrogen})$$

<u>Case</u>	<u>ΔP, % Center</u>	<u>ΔP, % Core-Reflector Radial Interface</u>
A.15	14	17
B.15	22	23
C.15	24	21
D.15	26	20
E.15	9	11
F.15	21	19
A.30	39	60
B.30	43	61
C.30	32	52

As can be seen from Table 5-13, the reactors having reflectors with reflector void fraction $\phi = 0.30$ are less desirable than those with $\phi = 0.15$ because: (1.) reflectors with $\phi = 0.30$ have the largest magnitude of power tilt, 52-60%; and (2.) they also have the largest variation of power tilt across the core entrance section. Previous authors (9, 10) have assumed that the neutron flux distributions and power distributions can be taken as spatially invariant. Time dependent models of nuclear rocket startup for the compact reactors investigated in this thesis might have to take into account the power tilt shown in Table 5-13. No calculations were performed to evaluate the differences in predicted reactivity change with and without power tilt adjustment.

5.4.2 Effect of a Non-Uniform Uranium Distribution Upon Hydrogen Reactivity Worth

Implicit in the investigation of heat-transfer optimum power profiles in Chapter 4 is the assumption that every calculated power distribution is possible to construct in an actual nuclear rocket reactor. Plebuch (10)

has demonstrated that with a suitable reflector thickness he was able to calculate suitable uranium distributions for several reactor cores in order to match his "desirable" power distributions. However, the results of Chapter 4 have shown that existing turbulent hydrogen heat transfer correlations differ substantially as to predicted optimum power shapes. Consequently, there is a large uncertainty in the exact fuel distributions which must be achieved in order to obtain maximum nuclear rocket performance.

In this chapter it has been assumed that the hydrogen reactivity worth is not sensitive to local distributions of uranium. The results of Section 5.3 show that the reactors investigated in this thesis are very fast. The high mean energy of the neutrons causing fission, $\bar{E}_p$, indicates that local details of the uranium distribution are unimportant for reactivity calculations because of the long mean free path at these energies. The results of Section 5.2.3 demonstrate that the critical uranium concentration and the specific hydrogen reactivity worth are both strongly correlated with the geometrical buckling, B^2 . This is indirect evidence that the hydrogen specific worth is probably dependent upon the total number of uranium atoms in the reactor system.

In order to determine whether the hydrogen reactivity worth is dependent upon local details of the uranium distribution, the following reactor case was considered:

- (1) The uniform uranium distribution for case A.15 was changed so that the first axial region, at the reactor core entrance, 3.05 cm. long, had exactly twice the uniform uranium concentration as the rest of the reactor core. This first axial region is the first axial hydrogen region

listed in Table 5-2 for case A.

(2) The same hydrogen distributions as used in case A.15 were fully present.

(3) Since the arbitrary addition of this extra uranium caused the total reactivity to increase, the entire uranium distribution was scaled down until $k = 1.00$ was achieved. All concentrations were scaled by 0.942.

(4) The total reactivity contribution from the reflector hydrogen and the core hydrogen were then calculated. The results were $\Delta k_{tr} = 4.19\%$ and $\Delta k_{tc} = 2.73\%$ which are only a few cents off from the values listed in Table 5-5 for A.15. Therefore it was concluded that hydrogen reactivity worth is nearly independent of the local uranium distribution.

5.5 Reactivity Losses During Startup and Steady-State Operation

5.5.1 Core Radial Expansion

In Section 5.2.3 it was shown that the total hydrogen reactivity contribution can range from 4.15% to 11.29%. During startup the reactor core can also expand because the core materials increase in temperature. This represents a small change in the geometrical buckling and consequently the neutron leakage. Because these compact reactors are very sensitive to neutron leakage, the effect of a small expansion can be significant in terms of reactivity change. Several cases were studied for which the core radius was assumed to increase by approximately 1.1% due to thermal expansion. An expansion ΔT of 4100°R was assumed for these several cases because it was found that approximately a dollar's worth of reactivity change was involved--a convenient reference value. From Table 3-3 the value of the thermal expansion coefficient for the core is taken to be $\alpha = 2.75 \times 10^{-6}/[^\circ\text{R}]$. For a $\Delta T = 4100^\circ\text{R}$, then

$\alpha\Delta T = 1.1275 \times 10^{-2}$. For case A this meant a change in the core radius of 0.303 cm; case B, 0.340 cm; and case C, 0.37 cm. All uranium and carbon concentrations were changed to reflect the volumetric change. Table 5-14 lists the results for these reactivity changes due to core thermal expansion. Extrapolating the temperature coefficients in this table to a $\Delta T = 5000^\circ\text{R}$ gives a total $\Delta k = -0.78\$$ for A.30, the lowest value, and $\Delta k = -1.45\$$ for B.30, the highest value. Values for thermal expansion reactivity loss cited in the literature for Rover nuclear rocket reactors are: $-1\$$, Kiwi (48), $-1.54\$$, NRS-A2 (1120MW) (47), and $-0.50\$$, Phoebus-2A (46). Since the values in Table 5-14 are based upon one $\$ = 0.0064 \frac{\Delta k}{k}$ while the literature values are based upon a computed delay neutron fraction of one $\$ = 0.0074 \frac{\Delta k}{k}$, then the values in Table 5-11 should be multiplied by a factor of 0.865 for comparison. When this is done, a $\Delta T = 5000^\circ\text{R}$ core expansion gives $\Delta k = 0.67\$$ to $1.25\$$ which compare quite well with the literature values.

5.5.2 Carbon Deletion

Hydrogen can react corrosively with the graphite in the reactor core channels of a nuclear rocket. This results in a loss of carbon

Table 5-14

Reactivity Change Due to Core Thermal Expansion
 $\Delta T = 4100^\circ\text{R}$

Case	ΔR_c [cm]	Δk [\\$]	$\Delta k / \Delta T$ [\$/°R]
A.15	0.303	-1.13	-2.756×10^{-4}
B.15	0.340	-1.08	-2.634×10^{-4}
C.15	0.370	-1.02	-2.488×10^{-4}
A.30	0.303	-0.64	-1.561×10^{-4}
B.30	0.340	-1.19	-2.902×10^{-4}
C.30	0.370	-1.14	-2.780×10^{-4}

which can significantly limit the operating lifetime of a nuclear rocket. Carbon moderates the neutron spectrum so that loss of carbon represents a reactivity loss for the system. As discussed in Section 5.2.3, carbon also has some inelastic scattering which may change hydrogen's effectiveness as a moderator. Coating materials such as niobium carbide (26, 27) have been developed to reduce this hydrogen corrosion to tolerable levels. However, values for carbon loss in the operating Rover nuclear rocket reactors have not been reported to date in the literature. To gain some estimate of the relative importance of carbon-related reactivity losses, several cases were assumed to undergo a 3% total carbon loss from the reactor system. Results of these calculations are reported in Table 5-15. Since 3% total carbon loss represents unequal amounts of carbon lost when small systems are compared to larger systems, the specific worth of carbon is also tabulated. As Table 5-15 shows, there is a qualitative difference in the effectiveness of carbon that can be compensated for by a quantitative increase of lost carbon atoms, just as in the case of hydrogen discussed in Section 5.2.3. Carbon seems to decrease in specific worth in the same manner as hydrogen; that is, the presence of uranium may increase carbon's effectiveness. On the other hand, because carbon

Table 5-15

Reactivity Loss Due to Carbon Deletion
3% total carbon loss

<u>Case</u>	<u>Δk [\$]</u>	<u>$\overline{\Delta k}$ [\$/KgC]</u>
A.15	-0.85	-0.3077
B.15	-1.04	-0.2161
A.30	-0.87	-0.3150
B.30	-1.09	-0.2265

has inelastic scattering in the highest energy group (37) it may be that part of the specific worth decrease for case B versus case A may be due to decreasing effectiveness of the hydrogen that is present in the two cases.

In any event, loss of carbon can significantly decrease reactivity if corrosion is not prevented. MacMillian (49) reports that unprotected carbon can be corroded by 5000°R hydrogen at the rate of 1000 mg/cm²/5 min. For case A, one reactor core channel has 46.22 cm² surface area and for case B, a reactor core channel has 51.07 cm² surface area. Using the calculated values for A.15 and B.15 in Table 5-15, then one core channel for case A loses 2.84×10^{-3} \$/min. and one core channel for case B loses 2.21×10^{-3} \$/min. Table 4-7 shows that the total number of channels, N, for these reactor cores are N(A) = 27,164 and N(B) = 27,323. Therefore, unprotected reactor core channels would mean a loss of 77.15\$/min for case A.15 and 60.38\$/min for case B.15! This is a tremendous rate of reactivity decrease. Nuclear rockets may be required to thrust continuously for 60 minutes (12, 26). For a designed carbon reactivity loss of, say 5\$ (provided by excess uranium at start of thrusting), then a 60 minute operating life would require that corrosion be restricted to less than 0.10 - 0.14% of the unprotected corrosion rate. However, this estimate cannot be compared with the actual Rover nuclear rocket reactors since no carbon deletion rates have been reported in the available literature.

5.6 Nuclear Results Summary

For the nuclear rocket reactors investigated in this thesis, these nuclear characteristics have been calculated:

- (1) Total hydrogen reactivity contribution from the reflector

hydrogen can be several times the total contribution from the core hydrogen. Reactor reflectors with the largest reflector void fractions have the largest total hydrogen contribution. Table 5-5 lists these values. The range reported is: (a) $\Delta k_{tr} = 1.79\$ - 8.18\$$, (b) $\Delta k_{tc} = 2.36\$ - 4.63\$$, (c) $\Delta k = 4.15\$ - 11.26\$$, where Δk_{tr} = total reactivity worth for reflector hydrogen, Δk_{tc} = total reactivity worth for core hydrogen, and $\Delta k = \Delta k_{tr} + \Delta k_{tc}$. The core values compare well with values cited in the literature (46, 47, 48).

(2) The critical masses calculated for these reactors fall into the range 66Kg - 254 Kg, with the exception of the "pancake" configuration which has a critical mass of 737 Kg. The reactors with larger reflector void fractions incur a 30% critical mass disadvantage. These critical masses are all lower than the 300 Kg cited for the Phoebus-2A reactor (46). However, the Phoebus-2A reactor is reported to have a large amount of reactivity tied up in niobium which was not investigated in this thesis. The critical mass values calculated in Section 5.2.2 are given in Table 5-4.

(3) The critical uranium concentration and specific hydrogen reactivity worths [$\$/KgH_2$] exhibit very definite trends when plotted against geometrical buckling. Several straight-line curve fits for $\overline{\Delta k}$ and $\overline{\Delta k}_{tr}$ were derived:

$$\overline{\Delta k}_{tr} = 1.413 + 2.845 \times 10^2 B^2, [\$/KgH_2], \phi = 0.15 \quad (5.10)$$

$$\overline{\Delta k} = 2.758 + 3.545 \times 10^2 B^2, [\$/KgH_2], \phi = 0.15 \quad (5.11)$$

$$\overline{\Delta k}_{tr} = 0.900 + 3.000 \times 10^2 B^2, [\$/KgH_2], \phi = 0.30 \quad (5.12)$$

$$\text{and } \overline{\Delta k} = 1.370 + 3.514 \times 10^2 B^2, [\$/KgH_2], \phi = 0.30 \quad (5.13)$$

where $\overline{\Delta k}_{tr}$ = specific worth of reflector hydrogen, $\overline{\Delta k}$ = specific worth of all of the reactor hydrogen, and ϕ = reflector void fraction. These curve-fits could eliminate extensive multigroup calculations in the future when fairly accurate estimates are needed of hydrogen reactivity worth.

(4) Because critical uranium concentration and hydrogen specific reactivity worth are both strongly correlated with B^2 , and thus indirectly correlated with each other, it is concluded that the trends shown in Figure 5-6 are the result of hydrogen becoming more effective as a neutron moderator when in the presence of an inelastic scatterer, such as uranium. Experimental evidence concerning neutron age in water-uranium systems (39) confirms this.

(5) The total reactivity worth of hydrogen in the reflector of a reactor with a fixed core size is directly proportional to the average hydrogen concentration in the reflector. This is shown in Figure 5-7. The specific worth of hydrogen is also affected by the density of beryllium in the reflector.

(6) The neutron energy spectra for these compact reactors are very fast with the mean energy of the neutron causing fission being in the range 62KeV - 540KeV. These values are listed in Table 5-12.

(7) Perturbations in the spatial power shape can be very large at the core entrance. Values for the power tilt are reported in Table 5-13 and are in the range 9% - 61% for the reactor cases in this thesis. Reflectors having the largest reflector void fraction, and therefore the largest amount of hydrogen, cause the largest spatial power tilt.

(8) The reactivity worth of hydrogen was found to be nearly independent of local uranium distributions when one specific case was

examined. This is reported in Section 5.4.2.

(9) Reactor core thermal expansion can cause a loss of reactivity. The values calculated are -1.561×10^{-4} to -2.902×10^{-4} [$\$/^{\circ}\text{R}$] and are reported in Table 5-14. These agree well with values cited in the literature (46, 47, 48).

(10) Carbon loss by hydrogen corrosion can cause very significant reactivity losses. Values calculated are in the range -0.2161 [$\$/\text{KgC}$] to -0.3150 [$\$/\text{KgC}$] and are listed in Table 5-15. No comparison with literature values can be made. It is estimated that reactor core channels must reduce hydrogen corrosion to approximately 0.10% of the unprotected hydrogen corrosion rate on graphite.

6. Conclusions and Recommendations for Further Study

These conclusions have been reached as a result of the calculations in Chapters 4 and 5:

(1) The predicted heat-transfer-optimum power profile depends very strongly upon the mathematical form of the heat-transfer correlation used. Therefore, the detailed uranium distribution necessary for optimum nuclear rocket performance cannot be reliably determined. However the optimum reactor core dimensions and Isp are relatively insensitive to the form of the heat-transfer correlation used.

(2) For such compact nuclear rocket reactors, the reactivity worth of hydrogen in the reflector can be significantly greater than the reactivity worth of hydrogen in the core.

(3) The total worth of reflector hydrogen is directly proportional to the average hydrogen concentration in the reflector.

(4) Very definite correlations with B2 have been demonstrated for critical uranium concentration and hydrogen reactivity worth. These trends could be very useful in future optimization studies since an accurate estimate of critical mass and total hydrogen reactivity worth can be had without extensive multigroup calculations. For the reactors in this thesis, the smallest reactor cores have the highest Isp, highest critical mass, and lowest total hydrogen worth.

(5) Pancake (low flow rate) reactor cores suffer two serious disadvantages in comparison with "square" cores: (a) greatly increased core radius and (b) greatly increased critical mass.

(6) Increasing the reflector void fraction for a given size reactor core has three disadvantages: (a) critical mass increases, (b) total hydrogen reactivity worth increases, and (c) spatial power tilting increases.

This investigation is certainly only a tentative start toward a complete optimization of a solid-core nuclear rocket system. Several areas of further research suggest themselves:

(1) Because the reflector hydrogen worth can be a major contribution to the total hydrogen reactivity, the reflector steady-state hydrogen distribution should be examined in greater detail. This would involve an accurate determination of gamma heating in the reflector and the rocket nozzle.

(2) The effect of neutron absorbers in the reflector upon hydrogen worth should be studied. Present NERVA reactors have rotating poison plates (27) in the reflector for control. Little (9) has shown that a uniformly distributed poison in the reactor core will reduce the worth of an external control system by competing for neutrons that would otherwise have leaked out of the core.

(3) Only one reflector thickness was used in this study. The effect of other thicknesses upon the hydrogen reactivity worth should be examined since it has been demonstrated that the amount of beryllium has an effect upon the effectiveness of hydrogen.

(4) Throughout this study, diffusion theory has been assumed. At the high energies found for the neutron spectra of these reactors, this is not exactly valid. Transport calculations should be made to check the validity of these results.

(5) And finally, it would appear that optimum nuclear rocket start-up could be accurately simulated without recourse to extensive multigroup calculations by use of the B^2 correlations and other results in Chapter 5. For instance, the influence of hydrogen flow perturbations upon core thermal power could use the result that

Δk_{tr} is directly proportional to hydrogen average concentration. The reactor core expansion or contraction in response to changes in power level could be represented as $\overline{\Delta K}$ changes in hydrogen worth as shown in the correlations of B^2 with $\overline{\Delta K}$, $\overline{\Delta k_{tr}}$, and $\overline{\Delta k_{tc}}$. In other words, for transient analyses, one could use these steady-state density and expansion coefficients of reactivity.

Appendix A. A Simplified Analysis of Three Power Profiles

An approximate analysis of three axial power profiles: (1) Linear-decreasing, (2) Cosine, and (3) Uniform (constant), will illustrate the advantages of shaping the axial power along a nuclear rocket reactor channel in order to shorten the required length for a given desired exit temperature or to increase the achievable exit temperature for a fixed channel length. The equation relating the maximum wall temperature, T_{WMAX} , to the parameter $y = \frac{4fL_c}{D}$, where f = average Fanning friction factor along the channel, L_c = total channel length, and D = channel hydraulic diameter, will be derived in detail for the Linear-decreasing power profile and the corresponding relations for the other two power shapes will only be stated.

The reactor core coolant channel may be taken to be a hollow rod, cooled at the inside radius as shown in Figure 3-1. A "Linear-decreasing" power profile is given by Equation (A.1):

$$P(x) = P_{01} \left(1 - \frac{x}{L_c}\right) \quad (A.1)$$

where x = axial distance from the core channel entrance. An energy balance along the coolant channel will relate the increase in bulk coolant temperature, T , to the power produced in the coolant channel wall:

$$wcpdT = P(x)A_Fdx \quad (A.2)$$

where w = mass flow rate per channel, cp = specific heat, and A_F = cross sectional area of the reactor core channel occupied by the nuclear fuel.

Substituting Equation (A.1) into Equation (A.2) and integrating gives Equation (A.3):

$$T(x) - T_0 = \frac{P_{01} A_F}{wcp} \left(x - \frac{x^2}{2L_C} \right) \quad (A.3)$$

where T_0 = bulk coolant temperature at the channel entrance. The channel coolant exit temperature, T_e , is found by setting $x = L_C$ in Equation (A.3) and normalizing Equation (A.3) will give a more useful form, Equation (A.3.1):

$$\frac{T - T_0}{T_e - T_0} = 2 \left(\frac{x}{L_C} \right) \left(1 - \frac{x}{2L_C} \right) \quad (A.3.1)$$

Fourier's law at any channel position gives Equation (A.4):

$$Dh (T_w - T) = P(x) A_F \quad (A.4)$$

where T_w = wall temperature and h = heat transfer coefficient. Using Reynold's Analogy between friction and heat transfer:

$$\frac{h}{pvc_p} = \frac{f}{2} \quad (A.5)$$

where p = coolant bulk density and v = coolant velocity, and substituting Equations (A.3) and (A.3.1) into Equation (A.4) one finds the normalized wall temperature distribution, Equation (A.6):

$$\frac{T_w - T_0}{T_e - T_0} = 2 \left(\frac{x}{L_C} \right) - \left(\frac{x}{L_C} \right)^2 + \frac{4}{y} \left(1 - \frac{x}{L_C} \right) \quad (A.6)$$

where $y = 4 \frac{fL_c}{D}$.

The maximum wall temperature, T_{WMAX} , occurs at x^* such that

$$\frac{d}{dx} \left(\frac{T_w - T_o}{T_e - T_o} \right) = \left(\frac{2}{L_c} \right) - \left(\frac{2x^*}{L_c^2} \right) - \frac{4}{yL_c} = 0 \quad (A.7)$$

which easily yields

$$\frac{x^*}{L_c} = 1 - \frac{2}{y} \quad (A.8)$$

If $y < 2$ then x^* from Equation (A.8) is outside the channel length and the maximum wall temperature occurs at $x^* = 0$, the channel entrance. Substituting back into Equation (A.6) and inverting so that the channel exit temperature, T_e , is expressed in terms of T_{WMAX} , T_o , and y , gives Equations (A.9.1) and (A.9.2):

$$\text{Linear-decreasing: } \frac{T_e - T_o}{T_{WMAX} - T_o} = \frac{1}{1 + 4/y^2}, \quad (y \geq 2) \quad (A.9.1)$$

$$\text{Linear-decreasing: } \frac{T_e - T_o}{T_{WMAX} - T_o} = \frac{y}{4}, \quad (y < 2) \quad (A.9.2.)$$

Similarly the relationships for a cosine power profile,

$$P(x) = P_{o2} \cos \left(\frac{\pi}{2} \left(1 - \frac{x}{L_c} \right) \right), \text{ and a uniform power profile,}$$

$P(x) = P_{o3}$, can be derived and are given in Equations (A.10) and (A.11);

$$\text{Cosine: } \frac{T_e - T_o}{T_{WMAX} - T_o} = \frac{2}{\left(1 + \sqrt{1 + \left(\frac{2\pi}{y}\right)^2}\right)}, \text{ (all } y) \quad (\text{A.10})$$

$$\text{Uniform: } \frac{T_e - T_o}{T_{WMAX} - T_o} = \frac{1}{1 + 2/y}, \text{ (all } y) \quad (\text{A.11})$$

Notice that the maximum power densities P_{o1} , P_{o2} , P_{o3} for each particular profile are not identical and will be determined by T_{WMAX} through Equations (A.4) and (A.3).

Table A-1 compares $T_e - T_o$ for these 3 power shapes, assuming T_{WMAX} and T_o are the same for all power shapes and

Linear/cosine = $\frac{\text{Equation (A.9.1) or (A.9.2)}}{\text{Equation (A.10)}}$ and

Linear/Uniform = $\frac{\text{Equation (A.9.1) or (A.9.2)}}{\text{Equation (A.11)}}$

Table A-1

Comparison of $T_e - T_o$ for 3 Power Profiles

<u>y</u>	<u>Linear/Cosine</u>	<u>Linear/Uniform</u>
0	0.393	0.500
1	0.795	0.750
2	1.074	1.000
3	1.150	1.154
4	1.145	1.200
5	1.123	1.207
6	1.102	1.198
7	1.083	1.189
8	1.000	1.000

It is apparent from Table A-1 that for values of $y > 2$ and only a limited wall temperature constraint, the Linear-decreasing power profile will allow higher channel exit temperatures than either the Cosine or Uniform power profiles. The Linear-decreasing profile has its maximum power density at the channel entrance where the coolant temperature is lowest. Values of $y > 2$ place the position of maximum wall temperature beyond the channel entrance as Equation (A.8) shows, and therefore Table A-1 illustrates that when the position of the maximum wall temperature is not at the channel entrance, then it is more efficient to add the maximum heat to the coolant at the coolant's lowest temperature. This result is confirmed by the detailed numerical investigation reported in Chapter 4. For practical nuclear rocket friction factors, channel lengths, and channel hydraulic diameters, the value of y will indeed be greater than 2. For example, for $f = 5 \times 10^{-3}$, $D = 0.1$ in., and $L_c = 2.0$ ft. (representative nuclear rocket parameters from Chapter 4), then $y = 4.8$.

Equations (A.9.1), (A.9.2), (A.10), and (A.11) can also be used to show that there can be a trade-off between high I_{sp} and short channel length. Assume $T_{WMAX} = 5000^\circ R$, $T_o = 300^\circ R$, $D = 0.1$ in., and $f = 5 \times 10^{-3}$ and that the exit temperature, $T_e = 4500^\circ R$, is desired. The following channel lengths are required:

$L_c (A.9.1) = 2.42 \text{ ft.}$, $L_c (A.10) = 3.59 \text{ ft.}$, and $L_c (A.11) = 7.00 \text{ ft.}$ If the channel length is fixed instead at $L_c = 2.42 \text{ ft.}$, then $y = 5.808$ and $T_e (A.9) = 4500^\circ\text{R}$, $T_e (A.10) = 4100^\circ\text{R}$, and $T_e (A.11) = 3800^\circ\text{R}$. Table A-1 shows that the Linear-decreasing power profile will always be superior to the other two power shapes. If the exit temperature $T_e = 4990^\circ\text{R}$ had been desired instead, the required channel lengths: $L_c(A.9.1) = 18.6 \text{ ft.}$, $L_c (A.10) = 292.7 \text{ ft.}$, and $L_c (A.11) = 416.7 \text{ ft.}$ indicate that even the fixed linear shape can require excessively long channel lengths. For the TWMAX constraint alone, an optimum power profile that is steeper-than-linear is needed to further reduce the channel length. Of course, this simple analysis cannot allow for stress and fuel-centerline constraints which is the subject of Chapters 3 and 4.

Appendix B. The Stress Model and Derivation of the
Stress Relationships

The model for thermal stresses used in this thesis is identical to the model commonly used in nuclear rocket heat-transfer analyses, (6, 9, 44, 45). The following assumptions apply:

- (1) The flow channel equivalent cell, Figure 3-1, is a hollow cylindrical rod which is cooled at its inner radius, a , by a gas at pressure $P(x)$;
- (2) The cell is thermally isolated from neighboring cells by requiring that $\frac{dT_F}{dr} = 0$ at the outer radius, r_o , where $T_F(r) =$ internal fuel temperature;
- (3) Steady-state conduction exists;
- (4) The power density, P , in the channel walls varies axially but not radially;
- (5) There is no axial heat conduction in the channel wall;
- (6) The channel length is much larger than the channel outer diameter so that the formulas for plain-strain apply.

Equation (B.1) defines the radial temperature distribution that exists in the channel wall at position x

$$T_F(r) - T_w = \frac{P}{2k} \left(r_o^2 \ln \left(\frac{r}{a} \right) - \left[\frac{r^2 - a^2}{2} \right] \right) \quad (B.1)$$

where T_w = wall temperature at $r=a$, inner radius, k = thermal conductivity of the fuel, and P = power density at axial position x (uniform radially). The components of stress at any radius r may be derived from Equation (B.1) and the plain-strain relations referred to in assumption (6) above. However, since the maximum stress occurs at the inner radius (9, 45) these components are evaluated at $r = a$:

$$\text{maximum axial stress} = \sigma_x = \frac{(2E)}{1-\nu} \frac{(P)}{2k} \left[\frac{r_o^4 \ln(r_o/a)}{r_o^2 - a^2} - \frac{(3r_o^2 - a^2)}{4} \right], \quad (\text{B.2})$$

$$\begin{aligned} \text{maximum azimuthal stress} = \sigma_\theta = \\ -P_* + \left[\frac{(2E)}{1-\nu} \right] \frac{(P)}{2k} \left[\frac{1}{a^2} \frac{(r_o^2 - a^2)^2}{2} \right] \end{aligned} \quad (\text{B.3})$$

$$\text{and the maximum radial stress} = \sigma_r = -P_*, \quad (\text{B.4})$$

where α , E , and ν are materials properties defined and listed in Table 3-3. Defining F_o and F_1 by the following equations:

$$F_o = \left(\frac{\alpha E D^2}{16k(1-\nu)} \right) \left[\left(\frac{-\ln \eta}{\eta(1-\eta)} \right) - \frac{3(1-\eta)}{2\eta} \right], \quad (\text{B.5})$$

$$F_1 = \left(\frac{\alpha E D^2}{32k(1-\nu)} \right) \left[\frac{1-\eta}{\eta} \right]^2, \quad (\text{B.6})$$

then Equations (B.2) and B.3) are more conveniently expressed as:

$$\sigma_x = P F_1 \quad (\text{B.2.1})$$

$$\sigma_\theta = -P_* + P F_o \quad (\text{B.2.2})$$

where the relation between inner and outer radius, Equation (B.7)

has been used:

$$r_o = a / \sqrt{\eta} \quad (B.7)$$

and η = core void fraction.

When Equations (B.2.1), (B.2.2), and (B.4) are substituted into the root-mean-squared stress relation, Equation (3.10):

$$2\sigma^2(x) = (\sigma_x - \sigma_r)^2 + (\sigma_x - \sigma_\theta)^2 + (\tau - \sigma_\theta)^2. \quad (3.10)$$

and max. $\sigma^2(x)$ set equal to S^2 , then inversion of the Equation (3.10) will result in Equation (3.10.2):

$$P_s = \frac{-P_*(2F_1 - F_o)}{2(F_o^2 - F_o F_1 + F_1^2)} + \sqrt{\left(\frac{P_*(2F_1 - F_o)}{2(F_o^2 - F_o F_1 + F_1^2)} \right)^2 - \left[\frac{P^2 - S^2}{F_o^2 - F_o F_1 + F_1^2} \right]} \quad (3.10.2)$$

Appendix C. Derivation of the Rocket Nozzle Throat Area

The nozzle throat area is derived in this thesis under the assumption that only simple area change is involved and that there is no friction or heat-transfer into the gas flow. The usual treatment of one-dimensional compressible fluid flow is covered extensively in Shapiro (41) and only those results pertaining to converging-diverging rocket nozzles are presented here. The influence coefficient for simple area change expressed in terms of the mach number (41, p. 231), M , relates relative area change, $\frac{dA}{A}$, to relative mach number change, $\frac{dM^2}{M^2}$, in Equation (C.1):

$$\frac{dM^2}{M^2} = -2 \frac{\left(1 + \frac{\gamma - 1}{2} M^2\right)}{1 - M^2} \frac{dA}{A} \quad (C.1)$$

where γ = ratio of specific heats = C_p/C_v , and M = mach number = velocity of gas divided by the speed of sound at the same temperature, T . The $1 - M^2$ in the denominator of the influence coefficient in Equation (C.1) means that $dM^2 > 0$ for two situations: (1) $M^2 < 1$ and $dA < 0$, and (2) $M^2 > 1$ and $dA > 0$. In the converging section of a rocket nozzle $dA < 0$ so subsonic flow ($M^2 < 1$) will be accelerated. In the diverging section of a rocket nozzle, $dA > 0$ and supersonic flow ($M^2 > 1$) will be accelerated. Therefore, a converging-diverging rocket nozzle will continuously accelerate initially subsonic flow to

supersonic flow, thus producing very high exit velocities, V_e , provided that a sonic condition ($M^2=1$) is maintained at the nozzle "throat" area. The minimum area in the nozzle is the throat area.

Equation (C.1) may be integrated with the $M^2=1$ throat condition and Equation (C.2) defines the nozzle throat area:

$$A_T^2 = M_o^2 A_o^2 \left[\frac{\gamma+1}{2 + (\gamma-1) M_o^2} \right]^{\frac{\gamma+1}{\gamma-1}} \quad (C.2)$$

where A_T = nozzle throat area, A_o = initial subsonic entrance area, and M_o = entrance subsonic mach number. Thus, the larger the nuclear reactor exit gas velocity, the bigger the nozzle throat area. Two correlations from Sams' paper (33), Equation (C.3):

$$\begin{aligned} \text{Wn.d.s.} = & C_1 (D_T)^2 (P_c)^{1/3} \left[\left(\frac{D_{ex}}{D_T} \right)^2 - 1 \right] + \\ & C_2 P_c (D_T)^3 \left[\left(\frac{D_{ex}}{D_T} \right)^{1/3} - 1 \right] + C_3 P_c (D_T)^3 \end{aligned} \quad (C.3)$$

where Wn.d.s = weight of the nozzle diverging section, (lb)

D_T = throat diameter (ft), P_c = nuclear reactor exit pressure
($\frac{\text{lb}}{\text{ft}^2}$),

D_{ex} = diameter of the diverging cone exit area, $C_1 = 0.147$,

$C_2 = 2.68 \times 10^{-4}$, and $C_3 = 1.17 \times 10^{-5}$;

and Equation (C.4);

$$\text{Wn.c.s.} = \frac{P_m \pi P_c D_r}{1152 \sin 30^\circ \sigma} (D_r^2 - D_T^2), \quad (\text{C.4})$$

where Wn.c.s. = weight of the nozzle converging section, (30° cone)

P_m = density of converging cone material, (lb/ft³)

σ = tensile strength of the material, (lb/ft²)

D_r = total reactor core plus reflector diameter;

allowed a weight estimate to be made for the converging-diverging nozzles for these optimum cores. An area ratio for the diverging section was assumed to be 100. which corresponds to a nozzle efficiency (see Equation (2.5) in Section 2.1, the definition of I_{sp}) of $\eta_c = 0.82$, and the properties of stainless steel were assumed so that $P_m = 0.28 \text{ lb/in}^3$ (0.484 lb/ft³) and $\sigma = 40,000 \text{ psi}$ ($5.76 \times 10^6 \text{ lb/ft}^2$). For the cases in Table 4-7, $w = 1.013 \times 10^{-2} \text{ lbm/s/channel}$, $D = 0.1 \text{ in.}$, $P_o = 1200 \text{ psia}$, $T_o = 400^\circ\text{R}$, and $\eta = 0.3$ to 0.6 which have reactor core radii varying from 1.30 ft. to 0.89 ft., respectively, and nozzle areas approximately around 1.27 ft², the weight estimates using Equations (C.3) and (C.4), were in the narrow range 570 to 460 lbs in that order. Therefore it is concluded that the weight penalty of the rocket nozzle is of small relative importance to the mission weight.

Appendix D: Material Densities and Nuclear Constants

Table D-1

Material Densities* (43)

Carbon	$\rho = 1.70 \text{ (g/cm}^3\text{)}$
Beryllium	$\rho = 1.85 \text{ (g/cm}^3\text{)}$
Uranium	$\rho = 18.95 \text{ (g/cm}^3\text{)}$

*Densities reported are for a solid fraction of unity.

Table D-2

Multigroup Nuclear Constants
(From ANL-5800 (37))

<u>Energy Group</u>	<u>Neutron Energy Range</u>	<u>Lethargy</u>	<u>Average Neutron Velocity, 10^8 cm/sec</u>	<u>Fission Spectrum*</u>
1	3- MeV		28.5	0.204
2	1.4- 3 MeV	0.762	19.9	0.344
3	0.9-1.4 MeV	0.442	14.7	0.168
4	0.4-0.9 MeV	0.811	11.0	0.180
5	0.1-0.4 MeV	1.386	6.7	0.090
6	17-100 KeV	1.772	2.70	0.014
7	3- 17 KeV	1.735	1.14	0
8	0.55- 3 KeV	1.696	0.480	0
9	100-550 eV	1.705	0.206	0
10	30-100 eV	1.204	0.101	0
11	10- 30 eV	1.099	0.0566	0
12	3- 10 eV	1.204	0.0319	0
13	1- 3 eV	1.099	0.0179	0
14	0.4- 1 eV	0.916	0.0109	0
15	0.1-0.4 eV	1.386	0.00606	0
16	0.025 eV		0.00218	0

*This denotes the fraction of fission neutrons which are born into that energy group.

(Table D-2 Continued)

Cross sections in barns, 1 barn = 10^{-24}cm^2

U-235									
Group	σ_f	$\sigma_{n,\gamma}$	σ_{tr}	$\nu\sigma_f$	$\sigma_{i \rightarrow i+1}$	$\sigma_{i \rightarrow i+2}$	$\sigma_{i \rightarrow i+3}$	$\sigma_{i \rightarrow i+4}$	$\sigma_{i \rightarrow i+5}$
1	1.21	0.05	4.25	3.557	0.27	0.37	0.65	0.44	0.06
2	1.22	0.08	4.50	3.196	0.24	0.67	0.45	0.07	-
3	1.22	0.11	4.65	3.087	0.55	0.40	0.07	-	-
4	1.20	0.15	5.2	2.988	0.35	0.08	-	-	-
5	1.43	0.23	7.9	3.518	0.08	-	-	-	-
6	2.5	0.65	12.4	6.125	0.05				
7	4.2	1.3	15.1	9.55	0.05				
8	7.9	3.2	21.1	19.36	0.05				
9	18.7	8.5	37.2	45.815	0.05				
10	38	20	68	93.1	0.05				
11	55	35	100	134.75	0.05				
12	44	40	94	107.8	0.05				
13	30	9	49	73.5	0.05				
14	70	10	90	171.5	0.05				
15	185	39	234	453.25	0.04				
16	516	95	621	1264.2	-				

H^1
 (Table D-2 Continued)

Group	σ_c	σ_{tr}	$\sigma_{i \rightarrow i+1}$	$\sigma_{i \rightarrow i+2}$	$\sigma_{i \rightarrow i+3}$	$\sigma_{i \rightarrow i+4}$	$\sigma_{i \rightarrow i+5}$
1	0	0.60	0.769	0.239	0.239	0.144	0.049
2	0	0.97	0.690	0.690	0.415	0.116	0.020
3	0	1.33	1.796	1.076	0.300	0.052	0.012
4	0	1.83	2.827	0.781	0.132	0.022	0.0055
5	0	3.00	4.041	0.684	0.117	0.0216	0.0045
6	0	5.20	6.022	1.045	0.187	0.0296	0.0125
7	0	6.33	7.372	1.349	0.209	0.0608	0.0304
8	0.001	6.666	7.880	1.220	0.360	0.1220	0.0520
9	0.004	6.669	6.719	1.920	0.680	0.1920	0.0960
10	0.008	6.672	7.757	2.719	0.780	0.2319	0.1559
11	0.014	6.676	8.494	2.418	0.719	0.360	0.119
12	0.025	6.817	7.906	2.364	1.182	0.387	-
13	0.045	6.830	7.409	3.684	1.242	-	-
14	0.070	6.847	9.982	3.334	-	-	-
15	0.130	6.920	11.020	-	-	-	-
16	0.29	7.09	-	-	-	-	-

(Table D-2 Continued)

Group	Be^9			
	σ_c	σ_{tr}	$\sigma_{i \rightarrow i+1}$	$\sigma_{i \rightarrow i+2}$
1	0.041	1.291	0.818	0.35
2	0.032	1.475	0.509	0.12
3	0	2.38	1.207	-
4	0	3.31	0.913	-
5	0	3.94	0.634	-
6	0	5.18	0.655	-
7	0	5.28	0.68	-
8	0	5.37	0.71	-
9	0	5.42	0.71	-
10	0	5.46	1.01	-
11	0	5.46	1.11	-
12	0	5.46	1.01	-
13	0.001	5.46	1.11	-
14	0.002	5.46	1.33	-
15	0.004	5.46	0.88	-
16	0.009	5.569	-	-

(Table D-2 Continued)

C12

<u>Group</u>	<u>σ_c</u>	<u>σ_{tr}</u>	<u>$\sigma_{i \rightarrow i+1}$</u>
1	0	1.23	0.515
2	0	1.42	0.314
3	0	2.26	0.856
4	0	2.93	0.604
5	0	3.59	0.433
6	0	4.25	0.401
7	0	4.44	0.428
8	0	4.34	0.428
9	0	4.34	0.428
10	0	4.34	0.603
11	0	4.34	0.662
12	0	4.44	0.616
13	0	4.44	0.677
14	0	4.44	0.813
15	0.001	4.44	-.536
16	0.003	4.44	-

Appendix E. Nomenclature

$A_{noz.}$	= converging-diverging rocket nozzle throat area [ft ²]
A_O	= converging-section inlet area for nozzle [ft ²]
A_T	= rocket nozzle throat area [ft ²]
B^2	= geometrical buckling [cm ⁻²]
C/U	= carbon-to-uranium ²³⁵ ratio
C_{st}	= propellant tankage weight-scaling coefficient [$\frac{lbm}{cu. ft}$] ^{2/3}
c_p	= specific heat of propellant [$\frac{ft - lbf}{lbm - ^\circ R}$]
$\bar{c}_p$	= average specific heat of propellant [$\frac{ft - lbf}{lbm - ^\circ R}$]
D	= channel hydraulic diameter [ft]
D_O	= outer diameter of flow channel equivalent cell [ft]
D_C	= core diameter [ft]
D_T	= nozzle throat diameter [ft]
D_{ex}	= exit diameter of diverging-section of rocket nozzle [ft]
E	= Young's modulus of elasticity [psia]
E_L	= mean neutron leakage energy [KEV]
E_p	= mean neutron production energy [KEV]
$\bar{E}_i$	= neutron group average energy [KEV]
F	= total thrust [lbf]
f	= Fanning friction factor; as a subscript means evaluation at film temperature
G	= flow rate per channel per unit flow area [$\frac{lbm/s/channel}{ft^2}$]
g_O	= conversion constant = 32.2 ft/sec
g_C	= gravitational acceleration = 32.2 $\frac{ft - lbf}{lbm - sec^2}$

h	= heat transfer coefficient $[\frac{\text{ft} - \text{lb f}}{\text{ft}^2 - \text{sec} - ^\circ\text{R}}]$
$[H]$	= concentration of hydrogen atoms $[\frac{\text{atoms H}}{\text{cc}}]$
I_{sp}	= specific impulse [sec]
k	= effective multiplication constant
K_H	= thermal conductivity of hydrogen $[\frac{\text{ft} - \text{lb f}}{\text{ft} - \text{sec} - ^\circ\text{R}}]$
K_g	= thermal conductivity of graphite $[\frac{\text{ft} - \text{lb f}}{\text{ft} - \text{sec} - ^\circ\text{R}}]$
Δk	= change in effective multiplication constant [\$]
Δk_{tc}	= reactivity worth of total core hydrogen [\$]
$\overline{\Delta k}_{tc}$	= specific worth of core hydrogen [\$/KgH ₂]
Δk_{tr}	= reactivity worth of total reflector hydrogen [\$]
$\overline{\Delta k}_{tr}$	= specific worth of reflector hydrogen [\$/KgH ₂]
$\overline{\Delta k}$	= specific worth of total hydrogen [\$/KgH ₂]
L_c	= core length [ft]
L_i	= neutron group leakage [neutrons/sec]
M	= Mach number; molecular weight of propellant
m_p	= mass of propellant [lbm]
m_b	= mass of rocket stage after propellant thrusting [lbm]
m_o	= total mass of rocket stage before propellant thrust [lbm]
m_t	= propellant tankage mass [lbm]
M_{235}	= critical mass of uranium ²³⁵
M_o	= Mach number at entrance to converging-diverging nozzle
N	= total number of reactor core channels
N_u	= Nusselt number = $\frac{hD}{K_H}$
P	= power density [MW/cu. ft.]

- P_* = channel hydrogen pressure [psia]
 ΔP_* = pressure drop across reactor core [psia]
 P_o = core entrance pressure [psia]
 P_m = mean core pressure = $\frac{1}{2} (P_o + P_{exit})$ [psia]
 P_c = rocket nozzle chamber pressure [psia]
 P_e = rocket nozzle exit pressure [psia]
 P_r = Prandtl number = $\frac{\mu c_p}{K_H}$
 P_i = neutron group productions [neutrons/sec]
 $P_{tot.}$ = total reactor power [MW]
 P_W = wall-temperature-limited maximum power density [MW/cu. ft.]
 P_C = fuel centerline-temperature-limited maximum power density [MW/cu. ft.]
 P_S = stress-limited maximum power density [MW/cu. ft.]
 P_S^0 = zero-pressure stress-limited maximum power density [MW/cu. ft.]
 p = channel hydrogen density [lbm/cu. ft.]
 p_m = channel hydrogen density [atoms H/cc]
 p_r = region hydrogen density = p_m ; [atoms H/cc. core]
 when replaced by ϕ , then p_r is also reflector region hydrogen density
 p_p = propellant density [lbm/cu. ft.]
 p_{in} = entrance propellant density [lbm/cu. ft.]
 p_{out} = exit propellant density [lbm/cu. ft.]
 R_c = core radius [ft]
 R_e = Reynolds number = $\frac{GD}{\mu}$

R_u	= Universal gas constant = $1545.33 \left[\frac{\text{ft lbf}}{\text{lbm} - \text{mole} - ^\circ\text{R}} \right]$
r_o	= outer radius of flow channel equivalent cell [ft]
S	= maximum thermal stress [psia]
T_o	= reactor core entrance hydrogen temperature [$^\circ\text{R}$]
T	= bulk hydrogen temperature [$^\circ\text{R}$]
T_f	= film temperature = $\frac{1}{2} (T + T_w)$ [$^\circ\text{R}$]
T_w	= channel wall temperature [$^\circ\text{R}$]
T_w'	= adiabatic wall temperature [$^\circ\text{R}$]
T_m, T_m'	= mean reflector hydrogen temperature [$^\circ\text{R}$]
TFMAX	= maximum fuel centerline temperature [$^\circ\text{R}$]
TWMAX	= maximum channel wall temperature [$^\circ\text{R}$]
t_B	= "burn" time for propellant thrusting [sec]
T_C	= nozzle chamber temperature [$^\circ\text{R}$]
TCL	= fuel centerline temperature [$^\circ\text{R}$]
TF	= internal fuel temperature [$^\circ\text{R}$]
Q/A	= power input to the propellant [MW/ft ²]
U	= uranium ²³⁵
v	= propellant velocity [ft/sec]
v_e	= rocket exit velocity [ft/sec]
Δv	= required velocity increment [ft/sec]
Δv_c	= "characteristic" velocity increment [ft/sec]
V_c	= reactor core volume [cu. ft.]
w	= flow rate per channel [lbm/s/channel]; when used as subscript means evaluation at wall temperature
$\langle w \rangle$	= total flow rate [lbm/sec]

WIEO = Weight In Earth Orbit

x = distance from reactor core entrance [ft]

x_0 = distance from core entrance to beginning of core hydrogen region [ft]

x_1 = distance from core entrance to end of core hydrogen region [ft]

x = incremental channel distances used in computer program MAXPOW [ft]; also used as $= x_1 - x_0$ above.

x_u = volume fraction of uranium

x_c = volume fraction of carbon

$y = \frac{4fL_c}{D}$ = Stenning simplified analysis parameter

η = reactor core void fraction

η_c = rocket nozzle efficiency

ν = Poisson's ratio; also used as the number of neutrons given off per fission.

Σ_f = macroscopic fission cross section [cm^{-1}]

$\gamma = c_p/c_v$ = ratio of propellant specific heats

α = coefficient of thermal expansion [$^{\circ}\text{R}^{-1}$]

σ = tensile strength of rocket nozzle material [psia]

$\sigma_{x,r,\theta}$ = axial, radial, and azimuthal stress components [psia]

σ_f = microscopic fission cross section [cm^2]

σ_c = microscopic capture cross section [cm^2]

σ_a = microscopic absorption cross section [cm^2]

σ_s = total microscopic scattering cross section [cm^2]

$\sigma_{i \rightarrow i+1}$ = microscopic scattering cross section from group i to group $i+1$ [cm^2]

μ = coefficient of viscosity $[\frac{\text{lbm}}{\text{sec} - \text{ft}}]$

ϕ = reflector void fraction

$\$$ = delayed neutron fraction = $0.0064 \frac{\Delta k}{k}$

Appendix F. Sensitivity Calculations

The following Table F-1 lists the results of varying $S = 4000 - 6000$ psia, $TFMAX = 5400 - 5600^\circ R$, and $TWMAX = 4900 - 5100^\circ R$. In all cases $T_o = 300^\circ R$, $P_o = 1200$ psia, $D = 0.1$ in, $w = 1.013 \times 10^{-2}$ lbm/s/channel, and Correlation (4.6) has been used.

Table F-1

Sensitivity of Optimum Reactor Core Characteristics to Changes in S, TFMAX, TWMAX

 $L_c = [ft], R_c = [ft], I_{sp} = [sec], \langle w \rangle = [lbm/sec], P_{tot.} = [MW], \delta P_* = [psia], A_{noz.} = [ft^2]$
 $T_o = 300^\circ R, P_o = 1200 \text{ psia}, D = 0.1 \text{ in.}, w = 1.013 \times 10^{-2} \text{ lbm/s/channel}$
 $S = 6000 \text{ psia}, TFMAX = 5500^\circ R, TWMAX = 5000^\circ R$

η	L_c	R_c	I_{sp}	$\langle w \rangle$	N	$P_{tot.}$	δP_*	$A_{noz.}$
0.3	2.8	1.28	878	285	28,100	4327	497	1.31
0.4	2.2	1.10	893	280	27,643	4433	444	1.22
0.5	2.0	0.97	906	276	27,251	4528	465	1.26
0.6	1.8	0.89	907	276	27,198	4534	438	1.21

 $S = 4000 \text{ psia}, TFMAX = 5500^\circ R, TWMAX = 5000^\circ R$

η	L_c	R_c	I_{sp}	$\langle w \rangle$	N	$P_{tot.}$	δP_*	$A_{noz.}$
0.3	3.6	1.31	835	299	29,565	4004	527	1.37
0.4	2.7	1.11	875	286	28,212	4299	460	1.25
0.5	2.3	0.98	899	278	27,448	4485	478	1.28
0.6	2.0	0.89	906	276	27,238	4532	463	1.25

 $S = 5000 \text{ psia}, TFMAX = 5500^\circ R, TWMAX = 5100^\circ R$

η	L_c	R_c	I_{sp}	$\langle w \rangle$	N	$P_{tot.}$	δP_*	$A_{noz.}$
0.3	3.1	1.29	862	290	28,638	4203	501	1.32
0.4	2.4	1.10	891	281	27,713	4419	457	1.24
0.5	2.1	0.97	909	275	27,158	4557	473	1.27
0.6	1.9	0.88	916	273	26,953	4605	469	1.27

Table F-1 (Cont.)

S = 5000 psia, TFMAX = 5500° R, TWMAX = 4900° R

η	L_c	R_c	I_{sp}	$\angle w$	N	$P_{tot.}$	δP^*	Anoz.
0.3	3.1	1.29	862	290	28,641	4202	501	1.32
0.4	2.4	1.10	884	283	27,915	4368	452	1.24
0.5	2.1	0.98	896	279	27,350	4458	459	1.25
0.6	1.9	0.89	901	278	27,399	4484	449	1.23

S = 5000 psia, TFMAX = 5600° R, TWMAX = 5000° R

η	L_c	R_c	I_{sp}	$\angle w$	N	$P_{tot.}$	δP^*	Anoz.
0.3	3.1	1.29	863	290	28,611	4209	502	1.32
0.4	2.4	1.10	890	281	27,731	4414	458	1.25
0.5	2.1	0.97	904	276	27,294	4520	470	1.27
0.6	1.9	0.89	909	275	27,144	4552	463	1.25

240.

S = 5000 psia, TFMAX = 5400° R, TWMAX = 5000° R

η	L_c	R_c	I_{sp}	$\angle w$	N	$P_{tot.}$	δP^*	Anoz.
0.3	3.1	1.29	860	291	28,686	4192	500	1.32
0.4	2.4	1.10	885	282	27,875	4378	452	1.24
0.5	2.1	0.97	901	277	27,377	4499	463	1.25
0.6	1.9	0.89	907	275	27,195	4539	456	1.24

References

1. Taylor, M. F.; "A Method of Predicting Heat Transfer Coefficients in the Cooling Passages of Nerva and Phoebus-2 Rocket Nozzles", AIAA-68-608.
2. Klein, M.; "Nuclear Energy in Space", Nuclear News, Dec. 1968.
3. Little, W., Hansen, K. F., Mason, E. A.; "Control Problems During Startup of Solid-Core Nuclear Rockets", AIAA-64-390.
4. Taylor, M. F., Miller; "An Improved Method of Predicting Surface Temperatures in Hydrogen-Cooled Nuclear Rocket Reactors at High Surface-to-Bulk Temperature Ratios", NASA-TN D-2594 (1965).
5. Taylor, M. F.; "A Method of Correlating Local and Average Friction Coefficients for Both Laminar and Turbulent Flow of Gases Through a Smooth Tube With Surface to Fluid Bulk Temperature Ratios from 0.35 to 7.35", Int. J. Mass & Heat Trans., Vol. 10, pp. 1123-1128 (1967).
6. Cooper, R.; "Fast Reactor Rocket Engines - Heat Transfer", LA-2746 (1962).
7. Cooper, R.; "Fast Reactor Rocket Engines - Criticality", Nuc. Sci. Eng., 13, 355-365 (1962).
8. Yasui, G.; "Reactor Heat Removal Limitations of Nuclear Rockets", Am. Roc. Soc. J., 31 (1961), 1195-1204.
9. Little, Winston; "Reactivity Changes During Startup in Large Nuclear Rockets", Sc.D. Thesis, Dept. of Nuclear Engineering, M.I.T., 1964.
10. Plebuch, Richard; "Reactor Physics of Nuclear Rocket Reactors",

- Sc.D. Thesis, Dept. of Nuclear Engineering, M.I.T., 1963
11. Bussard and DeLauer; Fundamentals of Nuclear Flight, McGraw-Hill (1965).
 12. Chovit, A. R., and Kylstra, C. D.; "Optimization of Manned Interplanetary Stopover Missions" AIAA-65-513 (1965).
 13. Chovit, A. R., Kylstra, C. D., Plebuch, R. K.; "The Performance Characteristics and Mission Capabilities of the Nuclear Rocket Engine", Post Apollo Space Exploration: Advances in the Astronautical Sciences, Vol. 20, May 1965, pp. 977-1002.
 14. Chovit, A. R., Kylstra, C. D., Plebuch, R. K.; "Defining Nuclear-Rocket Performance", Astronautics and Aeronautics, June 1965.
 15. Battin, R. H.; Astronautical Guidance, McGraw-Hill (1964).
 16. Johnson, P. G.; "Early Nuclear Rocket Applications", AIAA-68-653 (1968).
 17. Stenning, A. H.; "Rapid Approximate Method for Analyzing Nuclear Rocket Performance", ARS Journal, Vol. 30, Feb. 1960, pp. 169-72.
 18. Bogart, D., and Lantz, E.; "Nuclear Physics of Solid-Core Gas-Cooled Rocket Propulsion Reactors", Nuclear Rocket Propulsion, NASA-SP-20, December 1962.
 19. Ellerbrock, H. H., Livingood, J. N. B., Straight, D. M.; "Fluid-Flow and Heat-Transfer Problems in Nuclear Rockets", Nuclear Rocket Propulsion, NASA-SP-20, December 1962.
 20. Reshotko, E.; "An Analysis of the 'Laminar Instability' Problem in Gas-Cooled Nuclear Reactor Passages", AIAA Journal,

Sept. 1967, 5 (9), pp. 1606-1615.

21. Harry, D. P.; "A Steady-State Analysis of the 'Laminar-Instability' Problem Due to Heating Para-Hydrogen in Long, Slender Tubes", NASA-TN D-2084 (1964).
22. Turney, Smith, Juhasz; "Steady-State Investigation of Laminar-Flow Instability Problem Resulting from Relatively Large Increases in Temperature of Normal Hydrogen Gas Flowing in Small Diameter Heated Tube", NASA-TN D-3347 (1966).
23. Mohler and Perry; "Nuclear Rocket Engine Control", Nucleonics, 19 (4), April 1961.
24. Spence, R. W.; "The Rover Nuclear Rocket Program", Science, 160 (3831), 31 May 1968, p. 953ff.
25. MIT course 22.232 (1969) notes: "Nuclear Engineering for Space Applications".
26. Corrington, L. C.; "The Nuclear Rocket Program - Its Status and Plans", AIAA-68-610.
27. Durham, F. P.; "Status of the Los Alamos Nuclear Rocket Program", AIAA-68-609.
28. Gronich, S., Holl, R. J., Johnson, K. P.; "Nonintegral Burn of Nuclear Rockets - An Approach to Low Cost Space Exploration", AIAA-68-589.
29. Rice, C. M., and Arnold, W. H.; "Recent Nerva Technology Development", AIAA-68-612.
30. Schroeder, R. W.; "Status of the NERVA Program", AIAA-67-487.
31. Spence, R. W.; "Status of the Phoebus Program", AIAA-67-486.
32. Chovit, A. R., Plebuch, R. K., Ridolphi, F., Simmons, L. D.;

- "Nuclear Propulsion for Manned Mars Expedition", AIAA-67-510.
33. Sams, Eldon W.; "Performance of Nuclear Rockets for Large-Payload, Earth-Satellite Booster", IAS-59-94 (June 1959).
 34. "Pewee I is Tested", The Atom (6) No. 1 (Jan. 1969).
 35. Lombardo, J. J., and Funk, C. W.; "Nuclear Rocket Materials Program", AIAA-68-611.
 36. Campbell, W. E., and Beveridge, J. H.; "NPSP Selection for a Nuclear Rocket", AIAA-67-476.
 37. Reactor Physics Constants, Argonne National Laboratory Report ANL-5800 (1958).
 38. Fowler, T. B., Tobias, M. L., Vondy, D. R.; EXTERMINATOR-2: A Fortran IV Code for Solving Multigroup Neutron Diffusion Equations in Two Dimensions, Argonne Code Center Reference Material, Abstract 156.
 39. Lamarsh, John R.; Introduction to Nuclear Reactor Theory, Addison-Wesley Publishing Company, Inc., 1966.
 40. Meghreblian and Holmes; Reactor Analysis, McGraw-Hill, 1960.
 41. Shapiro, Ascher H.; The Dynamics and Thermodynamics of Compressible Fluid Flow, Volume I, Ronald Press Company (1953).
 42. Buden, D.; "Operational Characteristics of Nuclear Rockets", Journal of Spacecraft and Rockets, 7, 832-836 (1970).
 43. Handbook of Chemistry and Physics, The Chemical Rubber Co., 45th ed. (1964).
 44. Shen, C. N., and Liu, T. C.; "Transient Heat Transfer and Thermal Stress for a Nuclear Rocket due to Sudden Increase of Coolant Flow", 66-WA/NE-4, ASME Winter Meeting, 1966.

45. Christie, J. D.; "Performance of Large Payload Nuclear Rockets", Sc.D. Thesis (1964), Mechanical Engineering Dept., M.I.T.
46. Sapir, J. L., and Orndoff, J. D.; "Neutronics of the Phoebus 2 Reactor", Transactions of the American Nuclear Society, 12 (2), Nov. 1969.
47. Ortenberg, L. I., Maivery, A. L., and Hill, D. J.; "Comparison of Analytical and Experimental Operating Reactivity of NERVA Reactors", Trans. ANS, 8 (2), Nov. 1965.
48. Mills, C. B.; "Kiwi Temperature Coefficient", Trans. ANS, 8 (2), Nov. 1965.
49. MacMillian, D. P.; "High Temperature Materials for Rocket Reactors", Nucleonics, 19 (4), April 1961.