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Optimization of Structures to
Satisfy Aeroelastic Requirements

by

Carl S. Rudisill
May 31, 1971

Work Completed

Equations for finding the derivatives of the flutter velocity of an aircraft with respect to structural parameters were derived. A numerical procedure was developed for determining the values of the structural parameters such that a specified flutter velocity constraint is satisfied and the structural mass is a relative minimum. A search procedure was developed which utilized two gradient search methods and a projected gradient method. The procedure was applied to the design of a cantilevered box beam. This work will be published in the "AIAA Journal" in July of 1971.

A method for the optimization of a complex structure to satisfy a divergence velocity constraint has been developed and tested on a cantilevered box beam. This method is very similar to the method of optimization for a flutter velocity. An unpublished paper based on this work may be found in the Appendix. The abstract of this paper is as follows:

Analytical expression for evaluating the partial derivatives of the torsional divergence velocity of an aircraft structure with respect to design variables is derived. An optimization procedure to satisfy a specified divergence velocity is illustrated, using gradient methods and finite-element representation, for a box beam with the lower and upper values specified for the design variables. It is shown that there is a possibility of serious designer judgement error due to inefficient performance of optimization methods against multiple constraints. A "dimension reduction technique" is proposed to help in such situations.

Work in Progress

The programs for the optimization of a structure subject to flutter velocity and divergence velocity constraints are being revised along the following lines:

1. The programs are being modified to use the stiffness method instead of the flexibility method of structural analysis. This will be a minor modification.
2. The first n eigenfunctions associated with the lowest natural frequencies of the structure will be computed and used as assumed modes in the flutter and divergence velocity analysis; thus, reducing the order of the problem. After several re-design cycles new assumed modes will be computed.
3. Design parameters P_i will be selected such that the stiffness matrix $[K]$ and mass matrices are linear functions of the design parameters then new values of the stiffness matrix and mass matrix will be determined by the relations

$$[K]^* = [K] + \sum_{i=1}^m \frac{\partial [K]}{\partial P_i} \Delta P_i \quad (1)$$

$$[M]^* = [M] + \sum_{i=1}^m \frac{\partial [M]}{\partial P_i} \Delta P_i \quad (2)$$

where $[K]^*$ and $[M]^*$ are the new values.

For the box beam the design parameters can be found such that the stiffness, mass matrix and total mass are linear functions of the design parameters. Since the values of $\frac{\partial [K]}{\partial P_i}$ and $\frac{\partial [M]}{\partial P_i}$ are constants then they will be computed only once and it will

be necessary to perform the stiffness analysis but one time. Initially values of $[K]$, $[M]$, $\frac{\partial [K]}{\partial P_i}$ and $\frac{\partial [M]}{\partial P_i}$ may be supplied by a separate program. The aerodynamic matrix and its derivatives with respect to the reduced frequency will be recomputed for each re-design cycle by a subroutine called by the optimization program.

In the original optimization procedures for flutter and divergence velocity constraints the flexibility method was used and a flexibility analysis was performed for each re-design cycle; hence the computer program was very time consuming.

The real eigenanalysis (for determining the assumed modes) may be performed by using the IBM scientific subroutine NR00T which will find the eigenvalues and vectors from the equation

$$([M] - \lambda[K])\{u\} = 0 \quad (3)$$

or

$$([K]^{-1}[M] - \lambda[I])\{u\} = 0 \quad (3a)$$

where the inverse of $[K]$ is not actually calculated.

Since, the real eigenanalysis will be repeated after several re-design cycles to improve accuracy, it may be more economical to invert $[K]$ initially, compute $[K]^{-1}[M]$ and then use equation (3a) for the eigenanalysis. New values of $[K]^{-1}[M]$ could

then be found from the relation

$$([K]^{-1}[M])^* = [K]^{-1} \left\{ [M] - \left(\sum_{i=1}^m \frac{\partial [K]}{\partial P_i} \right) [K]^{-1}[M] + \sum_{i=1}^m \frac{\partial [M]}{\partial P_i} \Delta P_i \right\} \quad (4)$$

where $([K]^{-1}[M])^*$ is a new value of $([K]^{-1}[M])$.

For the optimization of a structure subject to a deflection constraint the inverse of $[K]$ may be

determined from the relationship

$$([K]^{-1})^* = [K]^{-1} \left([I] - \sum_{i=1}^m \frac{\partial [K]}{\partial P_i} \Delta P_i \right) [K]^{-1} \quad (5)$$

where $[I]$ is the identity matrix. Equations (4)

and (5) may be derived by differentiating

$$[K]^{-1}[K] = [I] \quad (6)$$

to find

$$\frac{\partial [K]^{-1}}{\partial P_i} = -[K]^{-1} \frac{\partial [K]}{\partial P_i} [K]^{-1} \quad (7)$$

and then substituting into the equations

$$([K]^{-1}[M])^* = [K]^{-1}[M] + \sum_{i=1}^m \frac{\partial ([K]^{-1}[M])}{\partial P_i} \Delta P_i \quad (8)$$

$$([K]^{-1})^* = [K]^{-1} + \sum_{i=1}^m \frac{\partial [K]^{-1}}{\partial P_i} \Delta P_i \quad (9)$$

It should be observed that $[K]^{-1}[M]$ and $[K]^{-1}$ are nonlinear functions of P_i when $[K]$ and $[M]$ are

linear functions of P_i . When this is true exact values of $[K]$ and $[M]$ may be computed from equations (1) and (2) and approximate values of $[K]^{-1}$, $[M]$ and $[K]^{-1}$ may be computed from equations (4) and (5) for a given set of ΔP_i without repeating the stiffness analysis and inverting $[K]$.

4. The P-K method for solving the flutter equation will be incorporated into the optimization procedure and a comparison will be made with the K method.

A subroutine for computing the stiffness matrix and its derivatives for a cantilevered uniform box beam with n number of bays is nearing completion.

A procedure for finding the minimum mass structure subject to natural frequency, stress and deflection constraints has been worked out. The loading is static and the arrangement of structural members is held constant. A computer program has not been written and a start on this work can not be made until the flutter and divergence optimization programs are revised.

Work (unfunded) is being continued on the project until July 1 under the assumption that the grant will be renewed; it is understood that NASA is in no way obligated to renew the grant.

APPENDIX

OPTIMIZATION OF COMPLEX STRUCTURES
TO SATISFY DIVERGENCE VELOCITY CONSTRAINT

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OPTIMIZATION OF COMPLEX STRUCTURES
TO SATISFY DIVERGENCE VELOCITY CONSTRAINT^{*}

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ABSTRACT

Analytical expression for evaluating the partial derivatives of the torsional divergence velocity of an aircraft structure with respect to design variables is derived. An optimization procedure to satisfy a specified divergence velocity is illustrated, using gradient methods and finite-element representation, for a box beam with the lower and upper values specified for the design variables. It is shown that there is a possibility of serious designer judgement error due to inefficient performance of optimization methods against multiple constraints. A "dimension reduction technique" is proposed to help in such situations.

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1. INTRODUCTION

In the recent past, there have been several publications dealing with optimization with respect to a static aeroelastic constraint on the torsional divergence velocity [1,2,3,4]*. McIntosh and Eastep [2] presented a calculus of variations formulation for a tapered cantilevered wing with torsional stiffness dominated by contributions from a thin outer skin. McIntosh and Weissshaar [3], and Armand and Vitte [4] used transition-matrix approach of optimal control theory. McIntosh and Weissshaar [3] concluded that early optimism concerning the use of transition-matrix approach must be tempered somewhat, and it may in the long run prove most useful to adopt more sophisticated steepest ascent or gradient methods. Further, it appears that even for the gradient methods to be computationally attractive for practical problems, without excessive penalty in the computer time used to arrive at an optimum solution, closed form analytical expressions should be derived for the partial derivatives involved so that their computation does not involve individual perturbation of each design variable and corresponding evaluation of the objective function. This would be a very significant consideration for problems with a large number of design variables, and/or where evaluation of the objective function is expensive in terms of computer time. In the present paper, the authors' aim is, therefore, to develop a closed form expression for the partial derivatives of torsional divergence velocity with respect to design variables, and to present a practical method for optimum weight design of an aircraft wing structure subject to torsional divergence velocity constraint.

* Numbers in square brackets indicate references listed at the end of paper.

II. PARTIAL DERIVATIVES OF DIVERGENCE VELOCITY

The governing characteristic equation for torsional divergence of an aircraft wing structure in a static neutral state can be expressed as, (see Ref. (5))

$$([K] - \lambda[A]) \{U\} = 0 \quad (1)$$

where

$[K]$ = torsional stiffness matrix, function of design variables P_i , symmetric matrix.

$[A]$ = torsional air-force matrix, function of air density and wing geometry, constant and real.

λ = eigenvalue, equal to the divergence velocity squared.

$\{U\}$ = angular displacement of the wing

To define an associated row vector $\{V\}'$ of the eigenvector $\{U\}$, consider

$$([K] - \lambda[A])' \{V\} = 0 \quad (2a)$$

Taking the transpose of equation (2a) and using the symmetry property of $[K]$ yields

$$\{V\}' ([K] - \lambda[A]) = 0 \quad (2)$$

Differentiation of the characteristic equation (1) with respect to a design variable P_i yields

$$\left(\frac{\partial [K]}{\partial P_i} - \frac{\partial \lambda}{\partial P_i} [A] \right) \{U\} + ([K] - \lambda[A]) \frac{\partial \{U\}}{\partial P_i} = 0 \quad (3)$$

Premultiplying equation (3) by $\{V\}'$ and simplifying by substituting equation (2), results in the equation

$$\frac{\partial \lambda}{\partial P_i} = \frac{\{V\}' \frac{\partial [K]}{\partial P_i} \{U\}}{\{V\}' [A] \{U\}} \quad (4)$$

A similar expression can be derived using the flexibility matrix instead of the stiffness matrix, in which case the characteristic equation is given by

$$([C][A] - 1/\lambda[I]) \{U\} = 0 \quad (5)$$

where $[C]$ is the inverse of $[K]$.

From equation (5) it can be shown that

$$\frac{\partial \lambda}{\partial p_i} = - \frac{\lambda^2 \{V\}^T \left(\frac{\partial [C]}{\partial p_i} \right) [A] \{U\}}{\{V\}^T \{U\}} \quad (6)$$

The partial derivatives of the divergence velocity can be quickly computed from either equation (4) or (6). The use of these derivatives will be discussed in section III.

III. OPTIMIZATION PROCEDURE

Two general approaches to optimization have evolved thus far: maximizing a certain eigenvalue for fixed mass, or minimizing the mass for a fixed eigenvalue [2]. The authors use the first approach, which is equivalent to projected gradient method with eigenvalue as the objective function, in conjunction with gradient mass and gradient velocity methods. This procedure is described in detail in the authors' earlier paper [6]. The optimum design with divergence velocity constraint will be illustrated for a three bay box beam (See Figure 1) with twelve design variables having upper and lower constraints.

The optimization program is independent of the formulation used for the air-force matrix. However, significant simplification is obtained when the air-force matrix $[A]$ is symmetric, in which case equation (4) is simplified to

$$\frac{\partial \lambda}{\partial P_i} = \frac{\{U\}' \left[\frac{\partial K}{\partial P_i} \right] \{U\}}{\{U\}' [A] \{U\}} \quad (7)$$

This simplification would not be achieved using the flexibility matrix. Thus there is substantial computational advantage for the case of symmetric air-force matrix, if the stiffness approach is used. For the numerical example presented in this paper, a diagonal air-force matrix was used [5].

Table 1 lists the results obtained from the optimization program for the box-beam of Figure 1. Two sets of values for the design

variables were used as initial input to the optimization program.

The mass-optimization was subject to a torsional divergence velocity constraint of 600 ft./sec. with upper and lower constraints specified for all the 12 design variables. These constraints were same for both the cases of initial design values.

For the case 1, an optimum design was very quickly arrived at in two design cycles. A design cycle here defines a step taken in the multi-dimensional space of design variables during an execution of projected gradient, mass gradient or velocity gradient search; each design cycle involves computation of stiffness matrix and evaluation of the eigenvalue problem in addition to computation of derivatives. The final values for P_2 , P_3 and P_{10} seem to be numerically same as the corresponding initial values. Actually the final values are slightly higher than the initial values, but due to a very small numerical difference this is not apparent from the number of digits listed in the table. Therefore, the optimum arrived at for case 1, appears to be a free optimum, i.e., the design parameters are not against any constraint.

For the case 2, the lower constraints for P_7 , P_8 and P_9 slowed down the optimization procedure considerably, and it took 47 design cycles to reach a mass approximately 40% higher and a divergence velocity approximately 0.45% higher than for the previous optimum attained. This suggests the possibility of serious designer judgement error due to inefficient performance of optimization methods against multiple constraints. To circumvent this, the authors suggest the use of "dimension reduction technique". The proposed technique would monitor the partial derivatives of the relevant eigenvalue with respect to the design parameters which are against the constraints, and from

these determine the design parameter which would tend to violate the constraints if a step in the desired direction were taken. Such parameters would be then held constant for the next step, thus effectively reducing the dimension of the design parameter space for this step. At the new point thus reached, a new design cycle would begin and partial derivatives of the relevant eigenvalue with respect to all the design variables would be computed, and the above process repeated. It is expected that such a technique would reduce the number of design cycles required for problems where constraints are encountered.

CONCLUSIONS

The closed form analytical expression derived for the partial derivatives of divergence velocity with respect to a design parameter is useful in the gradient type search procedures. The example solution illustrates its use. It seems that where lower constraints are specified for the design variables, a logical initial point should preferably include applicable lower constraints. This view is substantiated by authors experience that gradient velocity procedure operating in the neighborhood of lower constraints is very fast and effective in increasing the velocity to the desired value, since in increasing the velocity it would usually tend to move away from the constraints. For the case where the constraints slow down the gradient methods, the dimension reduction technique may improve their performance. The authors hope to explore the potential of this method in their future work.

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FOLDOUT FRAME

TABLE 1 Initial and Final Parameters with Co

		Area of Longitudinals (Sq. in.)			Front & Back Web Thickness ($\times 10^{-1}$ in.)			Top & Bottom Web ($\times 10^{-1}$ in.)	
		Bay 1 P_1	Bay 2 P_2	Bay 3 P_3	Bay 1 P_4	Bay 2 P_5	Bay 3 P_6	Bay 1 P_7	Bay 2 P_8
Initial Values	Case 1	0.33264	0.33264	0.33264	0.13332	0.13332	0.13332	0.0666	0.066
	Case 2	2.000	2.000	2.000	0.800	0.800	0.800	0.400	0.400
Constraints	Lower	0.33264	0.33264	0.33264	0.13332	0.13332	0.13332	0.0666	0.066
	Upper	8.0064	8.0064	8.0064	3.204	3.204	3.204	1.596	1.596
Final Values	Case 1	0.33336	0.33264	0.33264	0.1362	0.13464	0.133368	0.081228	0.082
	Case 2	0.3865	0.3865	0.3865	0.35184	0.35184	0.3184	0.0666	0.066

* Beyond the range of aerodynamic theory

a) Number of design cycles: Case 1 - 2

Case 2 - 47

1 Initial and Final Parameters with Constraints

FOLDOUT FRAME 2

Back Web Thickness 10^{-1} in.)			Top & Bottom Web Thickness ($\times 10^{-1}$ in.)			Rib Thickness ($\times 10^{-1}$ in.)			Total Mass (Slugs)	Divergence Velocity (ft./sec.)
Bay 2 P_5	Bay 3 P_6		Bay 1 P_7	Bay 2 P_8	Bay 3 P_9	Bay 1 P_{10}	Bay 2 P_{11}	Bay 3 P_{12}		
0.13332	0.13332		0.0666	0.0666	0.0666	0.0666	0.0666	0.0666	0.92	559.23
0.800	0.800		0.400	0.400	0.400	0.400	0.400	0.400	5.54	1370.41*
0.13332	0.13332		0.0666	0.0666	0.0666	0.0666	0.0666	0.0666	0.92	559.23
3.204	3.204		1.596	1.596	1.596	1.596	1.596	1.596	22.15	2738.71*
0.13464	0.133368		0.081228	0.082284	0.072672	0.0666	0.066756	0.067008	0.95	559.99
0.35184	0.3184		0.0666	0.0666	0.0666	0.3066	0.3066	0.3066	1.30	602.65

Beyond the range of aerodynamic theory used.

Number of design cycles: Case 1 - 2

Case 2 - 47

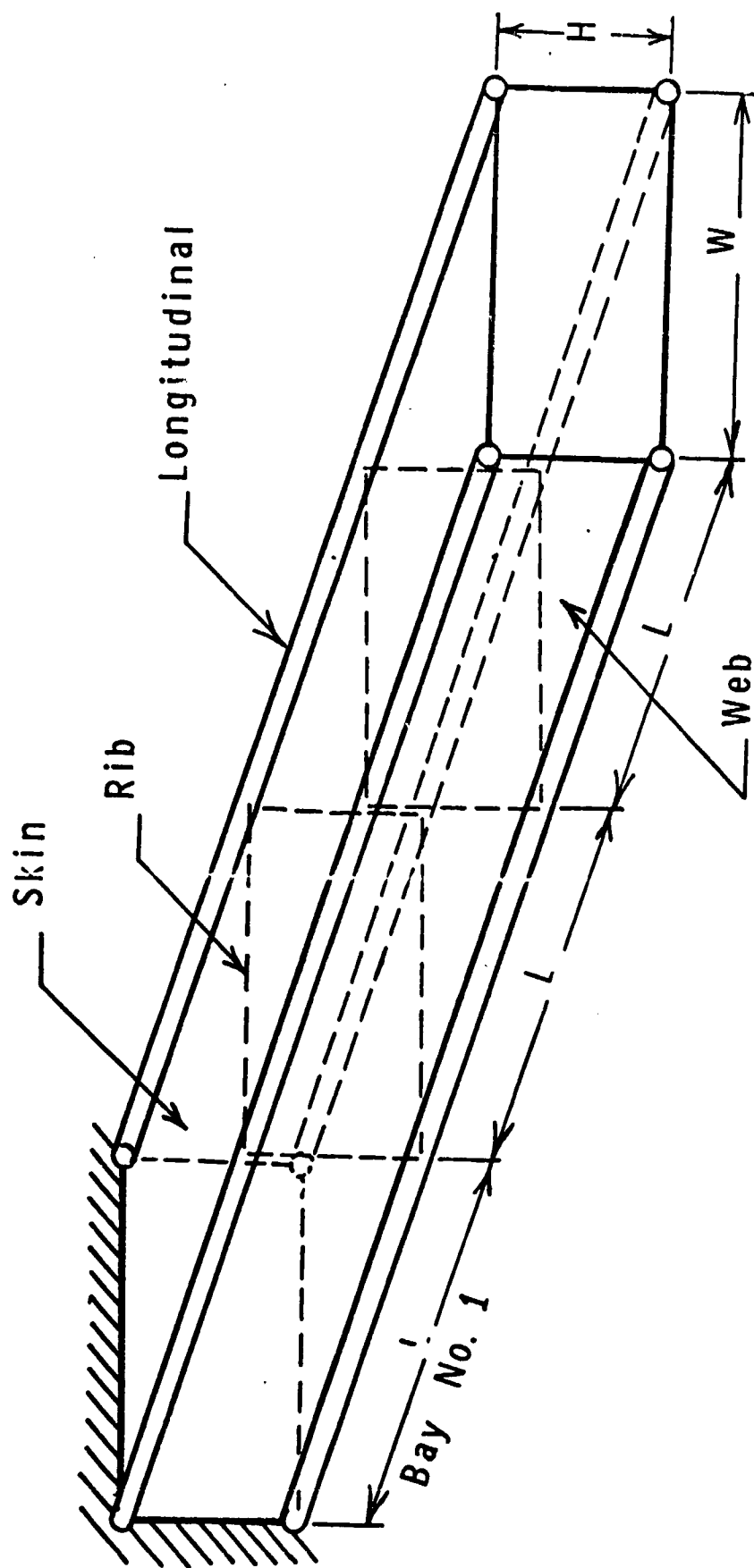


Fig. 1