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ABSTRACT

A technique is presented for the design of switching thresholds for use in the lateral guidance of an entry vehicle. Lateral maneuvers are accomplished by roll modulation about the relative-velocity vector. The problem is formulated as an optimal stochastic control problem, using the projected lateral miss and the direction of roll as state variables. Two different cost functions are used to implement the tradeoff between miss-distance and roll maneuvers: (1) the expected value of the weighted sum of the number of rolls and the square of the miss-distance, and (2) the probability that the miss exceeds a preselected threshold with the maximum number of roll maneuvers specified a priori. The effects of navigation errors, environmental uncertainties, and variations in vehicle aerodynamic characteristics are included in the formulation. Dynamic programming is used to generate the optimal switching thresholds as functions of projected lateral miss-distance. The first cost function leads to a single threshold function, whose values depend on the relative weighting between miss-distance and number of roll maneuvers. The second cost function provides a better description of the problem, but requires as many threshold functions as the maximum number of roll maneuvers permitted. Simulation results are presented for a low-cross-range vehicle, showing the important system-design tradeoffs. In the cases studied, satisfactory terminal accuracies were obtained with as few as three roll reversals over a typical entry trajectory.

1. INTRODUCTION

This paper is concerned with the lateral guidance of a shuttle vehicle during the period of hypersonic entry. A low-cross-range orbiter is considered (L/D = 0.5), entering the atmosphere at a relatively shallow flight-path angle (-1.5 degrees). To minimize heating effects, the vehicle maintains a fixed, high angle-of-attack during entry (about 60 degrees).

Entry guidance under the above conditions is accomplished by roll modulation, i.e. by rolling the lift vector around the vehicle's velocity vector from one side to the other. This technique has been used successfully for entry guidance during all Apollo flights[1,2,3]. The longitudinal and lateral guidance problems can be effectively decoupled. The vertical-plane aerodynamic forces necessary for solving the down-range guidance problem determine the magnitude of the roll angle. Lateral-guidance requirements are then satisfied by periodic reversals of the direction of roll.

The problem is essentially to develop the best procedure for deciding when to switch the direction of roll from one side to the other, recognizing that uncertainties will be present in the operational environment, the navigation system, and the aerodynamic characteristics of the vehicle. A basic tradeoff is involved between the number of roll maneuvers (switchings) and the lateral miss distance at the terminal time. From the viewpoint of terminal accuracy, frequent roll reversals are desirable. Passenger comfort and roll-control-system fuel requirements, on the other hand, are best satisfied by a minimum number of roll reversals. These factors must be balanced off against each other.

The approach adopted here is to formulate the problem as an optimal stochastic control problem. This provides a framework for systematically trading-off the number of roll maneuvers vs. the miss distance. Two methods have been considered for performing this important tradeoff:

1.) Minimize the expected value of the weighted sum of the number of roll maneuvers and the mean-square-miss distance.
2.) Minimize the probability that the miss is greater than a specified value, given a fixed maximum permissible number of roll maneuvers.

Optimal switching thresholds have been found for both of these cases, using dynamic programming techniques. Important factors such as vehicle lateral-maneuver capability, atmospheric-density variations, navigation-system errors, variations in the vehicle's L/D, and attitude-control system oscillations are all included in the system-design process.

2. FORMULATION OF THE PROBLEM

As stated above, the lateral entry control problem may be effectively decoupled from the in-plane guidance system. The in-plane nominal guidance policy is determined first, yielding a trajectory which attains the required range to the target, within the maximum lift capability of the

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vehicle. At each point on the trajectory a certain vertical component of lift is required. The spacecraft supplies the required lift by roll modulation (i.e. the vehicle is rolled to attain the appropriate vertical component of lift). Rolling the vehicle right or left produces lateral trajectory control. In effect, the lateral control capability is determined by the in-plane nominal trajectory. Since the rominal trajectory is specified at the onset of entry, the lateral control capability is also known at that time.

Let $\phi(t)$ be the roll angle* magnitude obtained from the nominal in-plane trajectory and let $u(t_n)$ denote the lateral control input at time t_n. This control is a scalar quantity constrained to take only the values +1, 0, -1. Application of control +1 or -1 commands the vehicle attitude-control system to roll the vehicle to angles $+\phi$ or $-\phi$ respectively. Application of zero control produces no change in vehicle attitude. Vehicle roll rate is constrained to ($\leq 10°$/sec), so the commanded roll angle is not attained immediately. For the purpose of simplifying the optimization problem, it is assumed that the interval between control decisions exceeds the time required to roll the vehicle through an angle 2ϕ. In effect, the vehicle is committed to complete a roll maneuver once the command is given. As a result, at all control decision times t_n, the vehicle roll angle is either $+\phi(t_n)$ or $-\phi(t_n)$.

A convenient state variable for the lateral control problem is the direction of roll. Hence, the state variable $x_1(n)$ is chosen to indicate the sense of vehicle roll angle at the control decision time t_n.

$$x_1(n) = \begin{cases} +1 \text{ indicates roll angle } +\phi(t_n) \\ -1 \text{ indicates roll angle } -\phi(t_n) \end{cases}$$

State $x_1(n)$ satisfies the following difference equation

$$x_1(n+1) = x_1(n) + 2u(n); \quad x_1(0) = \pm 1 \tag{2-1}$$

with control u(n) constrained as follows:

$$u(n) = \begin{cases} 0 \text{ or } -1 & \text{if } x_1(n) = +1 \\ 0 \text{ or } +1 & \text{if } x_1(n) = -1. \end{cases} \tag{2-2}$$

A second state variable for the lateral control problem is the projected lateral miss-distance (x_2) that would result if a zero roll attitude (no lateral aerodynamic forces) was maintained from the present time to the end of the problem. The state x_2 satisfies the following difference equation:

$$x_2(n+1) = x_2(n) + a(n)x_1(n) + b(n)u(n) + v(n). \tag{2-3}$$

The initial state $x_2(0)$ is a normally distributed random variable whose statistics are known. Parameter a(n) specifies the magnitude of the change in x_2 in the interval t_n to t_{n+1} resulting from a roll angle of $\phi(t_n)$, and $x_1(n)$ denotes the sense of the roll angle. Thus, the second term on the right of (2-3) is the change in terminal miss distance produced by the roll angle of the vehicle. Also, b(n) is the magnitude of the effect of a roll maneuver commanded at t_n, so the third term on the right accounts for the effect of a roll maneuver commanded at time t_n. Finally, v(n) is an independent gaussian sequence with mean and autocorrelation given as:

$$E[v(n)] = 0 \quad E[v(n)v(m)] = \begin{cases} V(n) & n=m \\ 0 & n \neq m \end{cases}$$

The quantity v(n) serves to model random disturbances to the vehicle. Included are the effects of atmospheric density variations, high frequency vehicle oscillations, uncertainties in vehicle lift and drag characteristics, and random winds.

There is an on-board inertial measurement unit (IMU) which provides measurements of the non-gravitational accelerations of the vehicle. These measurements may contain additive random errors which are assumed to be gaussian. The controller processes this data and utilizes it as feedback information to determine the control.

With these definitions and assumptions two optimal control problems can be posed.

Problem #1

Minimize the expected total number of roll maneuvers plus the weighted mean-square lateral miss distance at the terminal time. The corresponding cost function is

$$J_1 = E\left[\sum_{n=0}^{q} |u(n)| + \frac{\lambda}{2} x_2^2(q+1)\right],$$

where λ is a weighting on mean-square lateral miss distance, t_q is the time at which the last control decision is made, and t_{q+1} is the fixed terminal time.

Problem #2

Minimize the probability that terminal lateral miss-distance exceeds a given value (ℓ), under the condition that the total number of roll maneuvers be constrained to no more than a given positive integer (M). The cost is

$$J_2 = E\left[g(x_2(q+1))\right],$$

where g is the indicator function,

$$g(x) = \begin{cases} 0 \text{ if } |x| \leq \ell \\ 1 \text{ if } |x| > \ell, \end{cases}$$

and the constraint on roll maneuvers is

$$\sum_{n=0}^{q} |u(n)| \leq M.$$

Cost J_2 represents the probability that the lateral miss-distance magnitude is greater than the miss-tolerance ℓ.

Both of these problems will be solved in the sequel. In Problem #1 the expected cost in fuel for rolling the vehicle is balanced, by appropriate weighting, against the mean-square lateral miss

*Roll angles are defined about the relative wind vector.

distance. Problem #2 yields minimum probability of a miss distance greater than the given value ℓ, under the constraint that no more than M roll maneuvers are permitted. In Problem #1 the designer must determine, by choice of λ, the importance of roll fuel relative to miss distance. In Problem #2 the designer chooses the permissible miss tolerance and the maximum permissible number of roll maneuvers.

The two cost functions have certain advantages and disadvantages. Cost J_1 requires a subjective choice of weighting between fuel and miss distance, whereas J_2 requires choice of miss tolerance and the total roll control fuel allotment. Since the designer usually knows the tolerance on miss distance and the fuel allotment, it is felt that cost function J_2 allows a more objective choice of parameter values than does J_1. It will be shown however, that the optimal control for J_2 is much more complex than the optimal control for J_1. The relative merits of the two optimal controls will be discussed in some detail.

3. METHOD OF SOLUTION

It is the task of the controller to utilize the available measurement information to determine the optimal control to be applied at each decision point. As formulated above, the lateral entry dynamics are described by linear difference equations driven by an additive white gaussian noise sequence. Further, the measurements available to the controller are linear with additive gaussian errors. It has been shown[4,5] that for such a linear gaussian system, the mean of the state conditioned on the measurement history is a sufficient statistic for determining the optimal control. That is to say, the conditional mean summarizes all the available information necessary to determine the optimal control.

An onboard navigation system processes the measurements to obtain estimates of vehicle position and velocity via the familiar discrete time estimation formulas of Kalman.[6] These are also the mean position and velocity, conditioned on the measurements. Upon projecting these estimations forward to the terminal time, a recursion formula for the conditional mean of $x_2(n)$ is obtained.

$$\hat{x}_2(n) = \hat{x}_2(n-1) + a(n-1)\,x_1(n-1) + b(n-1)\,u(n-1) + K(n)\left[m(n) - \hat{m}(n)\right], \quad (3\text{-}1)$$

where $m(n)$ represents the IMU measurement and $\hat{m}(n)$ is the estimate of $m(n)$ based on previous measurements. Hence, the difference $m(n) - \hat{m}(n)$ is the so-called measurement residual, and $K(n)$ is the optimal weighting of the residual. Defining

$$\hat{x}'_2(n) = \hat{x}_2(n-1) + a(n-1)\,x_1(n-1) + b(n-1)\,u(n), \quad (3\text{-}2)$$

$$s(n) = K(n)\left[m(n) - \hat{m}(n)\right]. \quad (3\text{-}3)$$

Equation (3-1) becomes:

$$\hat{x}_2(n) = \hat{x}'_2(n) + s(n), \quad (3\text{-}4)$$

where $\hat{x}'_2(n)$ is the extrapolated estimate of $x_2(n)$ and $s(n)$ represents new data obtained from $m(n)$. It is known that $s(n)$ is a white gaussian sequence[4,5] with zero mean and covariance $S(n)$ given as:

$$S(n) = P_2(n-1) - P_2(n) + V(n), \quad (3\text{-}5)$$

where P_2 is the covariance of the error in the estimate of x_2. The corresponding probability density for $s(n)$ in terms of the dummy variable (ζ) is:

$$f_{s(n)}(\zeta) = \frac{1}{\sqrt{2\pi S(n)}} \exp\left\{-\frac{\zeta^2}{2S(n)}\right\}. \quad (3\text{-}6)$$

With these definitions in hand it is possible to proceed to the determination of optimal controls. In the previous section two optimization problems were posed. The method of solution for each will be presented in turn.

A. Minimization of J_1*

In References [4,5] a general recursion formula is developed for the minimum expected cost to complete the problem from time t_n, conditioned on the measurements up to time t_n. Since the conditional mean of the state is a sufficient statistic, this cost may equivalently be conditioned on $x_1(n)$ and $\hat{x}_2(n)$. Hence define

$C^*(x_1(n), \hat{x}_2(n), n)$ = minimum expected cost to complete the problem from time t_n, conditioned on the state $x_1(n)$ and estimated state $\hat{x}_2(n)$.

For the problem at hand (i.e. minimize J_1) the appropriate recursion formula is:

$$C^*(x_1(n), \hat{x}_2(n), n) = \min_{u(n)} \left\{ |u(n)| + \int_{-\infty}^{\infty} f_{s(n+1)}(\zeta)\, C^*(x_1(n+1), \hat{x}'_2(n+1) + \zeta, n+1)\, d\zeta \right\}, \quad (3\text{-}7)$$

subject to the control constraint (2-2) and with the terminal condition

$$C^*(x_1(q+1), \hat{x}_2(q+1), q+1) = E\left[\frac{\lambda}{2} x_2^2(q+1) \,\middle|\, x_1(q+1), \hat{x}_2(q+1)\right] = \frac{\lambda}{2}\left[\hat{x}_2^2(q+1) + P_2(q+1)\right]. \quad (3\text{-}8)$$

Assuming the statistics of the IMU information are known a priori, the error variances $P_2(n)$ can be determined a priori and densities $f_{s(n+1)}(\zeta)$ can be determined a priori from (3-5) and (3-6). With these functions Eqs. (3-7), (3-8), (3-2) and (2-1) are solved via a backward step-by-step dynamic programming process, starting at the terminal time. By performing the minimization in (3-7) for all possible values of $x_1(n)$ and $\hat{x}_2(n)$, the optimal feedback control is obtained as a function of $x_1(n)$, $\hat{x}_2(n)$ and n. Details of the solution method may be found in [6]. Numerical results for the shuttle entry problem are presented in Section IV below.

Results of the dynamic programming solution reveal that the control is determined by thresholds. It is found that if $\hat{x}_2(n)$ exceeds the threshold, then the optimal control commands the vehicle to change the direction of roll. If $\hat{x}_2(n)$ does not exceed the threshold, then the roll direction is not altered.

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* This solution was presented in [6]. It is summarized here for completeness.

B. Minimization of J_2

Although two state variables are sufficient to solve problem J_1, an additional state is necessary in the solution of J_2. Since the total number of roll maneuvers is constrained in this problem, the minimum expected cost to complete the process and the optimal control depend upon the number of roll maneuvers remaining. Therefore, a third state is defined as

$x_3(n)$ = number of roll maneuvers expended before time t_n.

The state $x_3(n)$ satisfies the recursion formula:

$$x_3(n+1) = x_3(n) + |u(n)| ; \quad x_3(0) = 0. \tag{3-9}$$

The minimum expected cost to complete the problem is then redefined as:

$C^*(x_1(n), \hat{x}_2(n), x_3(n), n)$ = minimum expected cost to complete the problem from time t_n, conditioned on states $x_1(n)$, $x_3(n)$ and estimated state $\hat{x}_2(n)$.

Since no more than M roll maneuvers are permitted, the control constraint (2-2) must be modified as follows:

$$u(n) = \begin{cases} 0 & \text{if } x_3(n) = M \\ 0 \text{ or } +1 & \text{if } x_3(n) < M,\ x_1(n) = -1 \\ 0 \text{ or } -1 & \text{if } x_3(n) < M,\ x_1(n) = +1. \end{cases} \tag{3-10}$$

The appropriate recursion formula for the minimum expected cost to complete the problem is then

$$C^*(x_1(n), \hat{x}_2(n), x_3(n), n) = \min_{u(n)} \left\{ \int_{-\infty}^{\infty} f_{s(n+1)}(\zeta) C^*(x_1(n+1), \hat{x}_2(n+1) + \zeta, x_3(n+1), n+1)\, d\zeta \right\}, \tag{3-11}$$

with the terminal condition

$$C^*(x_1(q+1), \hat{x}_2(q+1), x_3(q+1), q+1) = E\left[g(x_2(q+1)) \,|\, x_1(q+1), \hat{x}_2(q+1), x_3(q+1)\right]$$

$$= \int_{-\infty}^{\infty} f_{x_2}(\zeta|\hat{x}_2) g(\zeta) d\zeta = 1 - \int_{-\ell}^{+\ell} f_{x_2}(\zeta|\hat{x}_2) d\zeta, \tag{3-12}$$

where

$f_{x_2}(\zeta|\hat{x}_2)$

is the probability density of $x_2(q+1)$ conditioned on the estimate $\hat{x}_2(q+1)$. The relation for $f_{x_2}(\zeta|\hat{x}_2)$ is:

$$f_{x_2}(\zeta|\hat{x}_2) = \frac{1}{\sqrt{2\pi P_2(q+1)}} \exp\left\{-\frac{1}{2} \frac{(\zeta - \hat{x}_2(q+1))^2}{P_2(q+1)}\right\}. \tag{3-13}$$

As was the case for J_1, the solution of (3-11), (3-9), (3-2) and (2-1) with the control constraint (3-10) and terminal condition (3-12) is accomplished via dynamic programming. The minimization in (3-11) yields the optimal control as a function of $x_1(n)$, $\hat{x}_2(n)$, $x_3(n)$ and n.

As before, the control is determined by thresholds. In this case, however, the thresholds are different for different values of $x_3(n)$, the number of roll maneuvers expended. It is found that as $x_3(n)$ increases (i.e. fewer maneuvers remaining), the thresholds become larger so $\hat{x}_2(n)$ must be larger before a maneuver is made. In this way the controller tends to conserve the remaining roll maneuver capability.

4. APPLICATION OF DESIGN TECHNIQUE TO SHUTTLE ENTRY PROBLEM

The particular case chosen for study was the entry of a low-cross-range orbiter on a nominal descent trajectory from a 270 nmi circular orbit. To minimize heating effects, it was assumed that the vehicle maintained an angle-of-attack of 60 degrees during the entry trajectory, resulting in an L/D of about 0.5 and a W/C_DA of about 22 lbs/ft^2. The vehicle was assumed to have a flight-path angle of -1.5 degrees at entry interface (h = 400,000 feet). With a maximum deceleration limit of 2.5 g's, the useable footprint under these conditions was about 1500 nmi from front to back, and about 200 nmi maximum to either side[3].

The assumed down-range guidance concept was a modified perturbation guidance system[2] similar to the one used for the final phase of Apollo entry[1,3]. The basic equations are given in Appendix A for the required roll-angle magnitude.

In order to apply the techniques of the preceding section, it is first necessary to determine the coefficients a(n) and b(n) used in Eq. (2-3). To accomplish this, simulated entry trajectories were run to various points along the footprint centerline using the relations of Appendix A, but with all lateral aerodynamic forces set equal to zero. The roll angle (ϕ) required to obtain the in-plane aerodynamic forces was then used to compute a(n) from the following relation:

$$a(n) = \int_{t_n}^{t_{n+1}} \frac{L \sin\phi\, r_{GO}}{v_h} dt, \tag{4-1}$$

where L represents the aerodynamic lift force per unit mass acting on the vehicle, r_{GO} is the range-to-go to the end of the problem, and v_h is the vehicle's horizontal velocity. Consecutive control decision times in the simulations were spaced 10 seconds apart, so that each a(n) represents the change in predicted miss $x_2(n)$ during a 10-second period.

For the purpose of determining b(n), an average roll angle of 50 degrees was assumed, which is reasonable for a center-of-the-footprint trajectory. In addition, a 10 deg/sec roll rate was assumed during all roll-reversal maneuvers. Under these assumed conditions with a control-decision interval of 10 seconds, the value of the projected miss-distance is unchanged over an interval where the roll angle is reversed. Thus b(n) = a(n) in Eq. (2-3).

The rms value of the random variable v(n) in Eq. (2-3), which is used to account for the effect of random variations in the environment and the vehicle's aerodynamic characteristics, was computed from the relation:

$$\sigma_v(n) = c_v |a(n)/\sin\phi|,$$

where the angle ϕ used here is the average value of the roll angle over the interval from t_n to t_{n+1}. The same functional form is used throughout the trajectory to model v(n) as for a(n). The coefficient c_v is selected to give a specified total integrated rss error for the overall trajectory, i.e. to specify

$$\sqrt{\sum_{n=0}^{q} [\sigma_v(n)]^2}.$$

A typical value used for the integrated rss error was 3.7 nmi. A representative set of switching thresholds computed by the methods of the preceding section, for the first type of cost function (J_1), is shown in Fig. 1. A center-of-the-footprint trajectory is assumed here. In this particular case a λ of 0.08 was used, which weights one roll maneuver equally with a projected miss of 5 nmi. To use this type of threshold function for lateral guidance, a computation is first made of projected miss distance, using the best available estimates of the vehicle's state. The estimated miss ($\hat{x}_2$) is then compared with the stored threshold (x_{2s}) and the following control law applied.

$$u(n) = \begin{cases} +1 & \text{if } x_1(n) = -1,\ \hat{x}_2(n) < -x_{2s}(n) \\ -1 & \text{if } x_1(n) = +1,\ \hat{x}_2(n) > x_{2s}(n) \\ 0 & \text{otherwise} \end{cases}$$

The shape of the curve shown in Fig. 1 reflects the manner in which lateral maneuver capability decreases during entry as the vehicle speed is decreased. At very low speeds where the maneuver capability is small compared to the rms value of projected miss, the thresholds again increase.

The effect of variations in the relative weighting function between miss distance and roll maneuvers (λ) for J_1 is shown in Fig. 2. As can be seen, increasing λ will decrease the threshold levels and, hence, increase the expected number of roll reversals. The effect of the rss error in x_2 due to random variations in the characteristics of the vehicle and the environment are shown in Fig. 3. The thresholds are seen to decrease as the magnitude of the random disturbances increase.

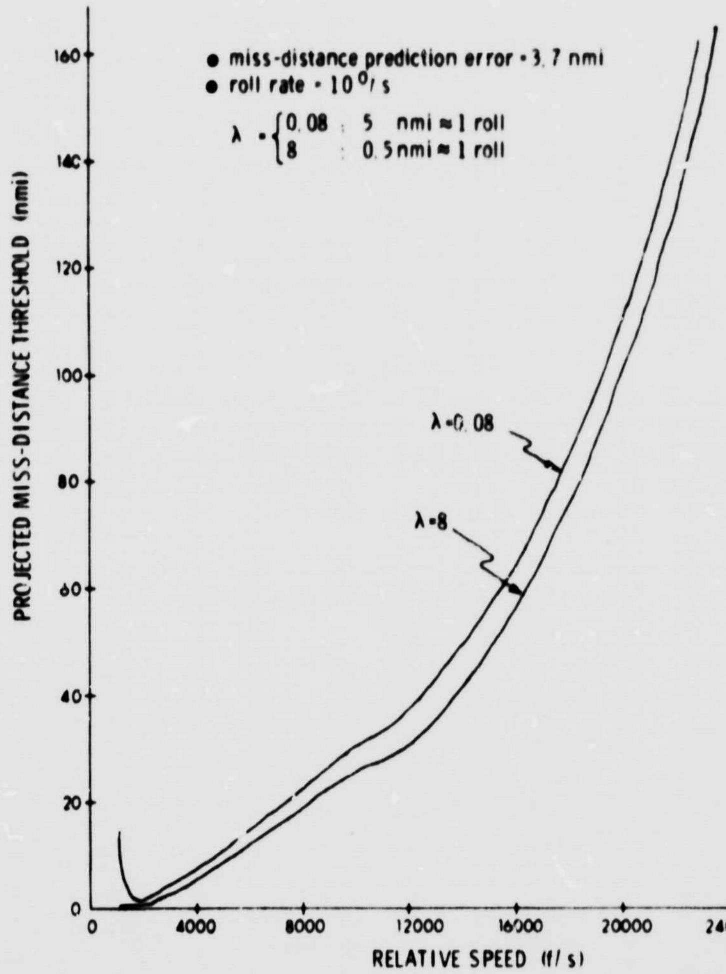

Fig. 2 Threshold Variation with Miss-Distance Penalty in Terminal Cost

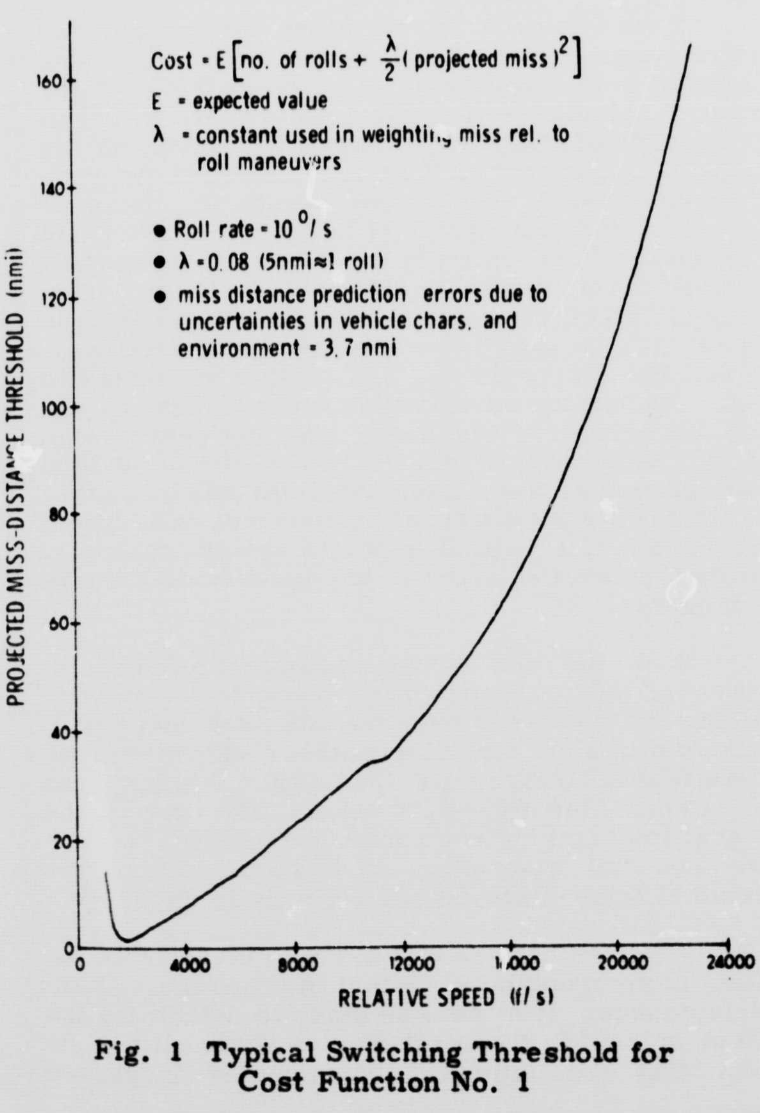

Fig. 1 Typical Switching Threshold for Cost Function No. 1

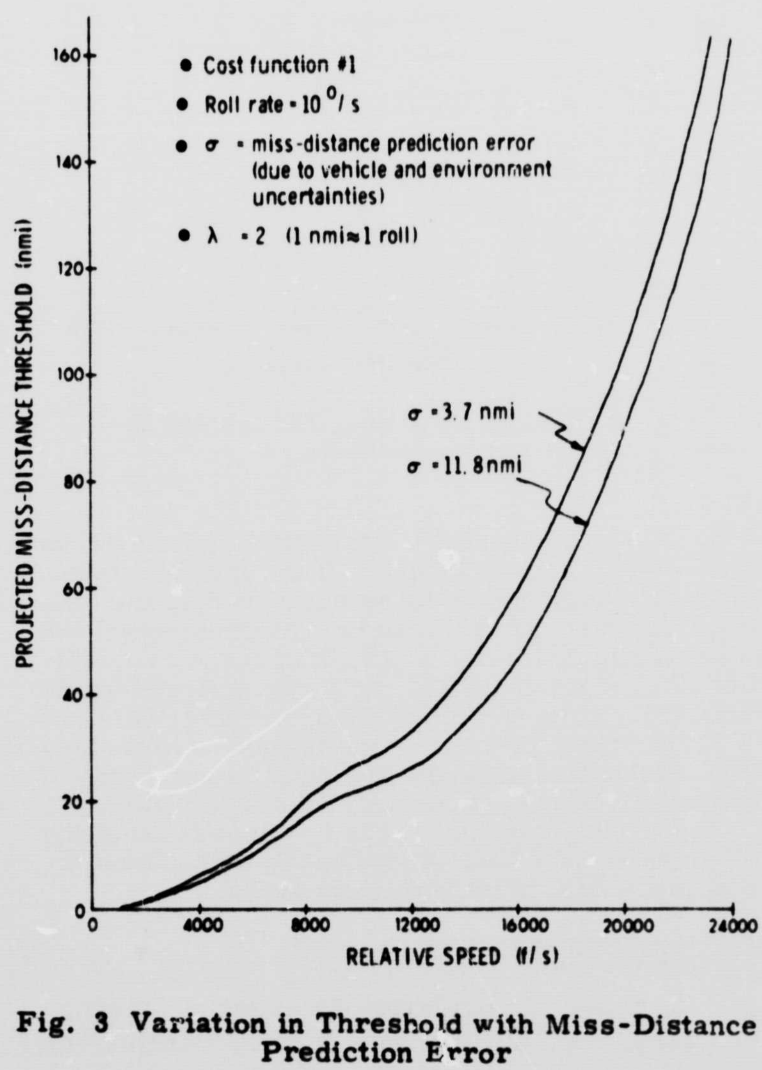

Fig. 3 Variation in Threshold with Miss-Distance Prediction Error

A set of switching thresholds for the second type of cost function (J_2) is shown in Fig. 4, assuming that up to four roll reversals are permitted. With this particular cost function a different threshold curve is used, depending on the number of roll maneuvers remaining. As can be seen, the threshold values increase as the number of rolls remaining is decreased. For the particular conditions assumed in Fig. 4, essentially no performance improvement is obtained by permitting more than three roll maneuvers.

A summary of simulation results for the second cost function (J_2) is given in Table 1. The form of the data is the probability that x_2 will be smaller than a specified level for up to four roll maneuvers permitted. A center-of-the-footprint trajectory is used in all cases.

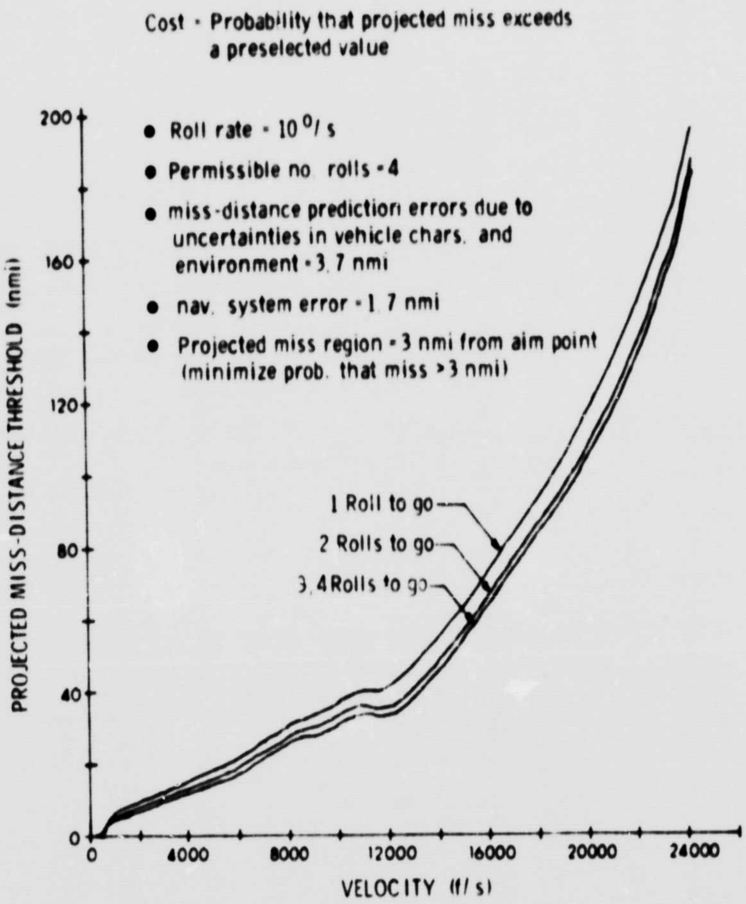

Fig. 4 Typical Switching Thresholds for Cost Function No. 2

There are several interesting points to be seen from the data of Table 1. If the level (ℓ) below which x_2 should lie is large compared to the rms navigation errors and random environmental or aerodynamic characteristics, then large probabilities can be obtained with only two roll reversals (Run 137). If, on the other hand, the level (ℓ) is of the same order as the rms navigation errors, then the probabilities attained are small regardless of the number of roll maneuvers (Runs 138 and 139). For the cases shown in Table 1 it appears that the probabilities are not significantly increased by using more than three roll maneuvers.

5. SIMULATION RESULTS

In order to evaluate the lateral guidance techniques, a series of reentry trajectory simulations was made using the optimal thresholds. All runs were made over a rotating earth with the vehicle traveling East in the equatorial plane.

Environmental disturbances have a significant effect on the ability of the lateral guidance system to meet the objective of satisfactory terminal accuracy with a minimum of roll maneuvers. In order to test the statistical design in a realistic manner, real-world models of these disturbances were included in the simulation of the actual trajectories.

The wind model used was the 95%-design steady-state wind profile for the Eastern Test Range[8]. In summary, the wind speed (assumed cross-track here) was assumed to vary linearly from a maximum of 400 ft/sec above 200,000 ft. altitude to 80 ft/sec at 80,000 ft. altitude. The density variation model, also taken from the same reference, was based on data taken at Cape Kennedy. The deviations, given in percent, are of the same sign at all altitudes and are taken with respect to the 1963 Patrick Reference Atmosphere. The one-sigma deviation magnitude varies linearly from 2.6% at 90,000 ft. to 9% at 210,000 ft., and then to 3.5% at 270,000 ft. Above 270,000 ft. the deviation is assumed to be constant at 3.5%.

The combined effects of vehicle attitude oscillations due to aerodynamic effects and attitude control deadbands were simulated by varying the vehicle roll angle by a fixed amount over a 4-second interval, the value of the variation being determined independently for each interval by a pseudo-random number generator. A nominal value of one deg. rms, normally distributed, was assumed.

A summary of the terminal down-range and cross-range miss-distances, along with the required roll maneuvers, is given in Table 2 for several selected runs. All data are presented at an altitude of 100,000 ft. and are based on the quadratic terminal cost function. The first two runs compare results for a conservatively designed threshold and one designed using the methods of this paper. The case using the optimized thresholds reduced the number of required roll switchings from seven to three without a significant degradation in terminal accuracy. The value of the projected miss x_2 and the threshold x_{2s} are plotted vs. speed in Fig. 5 for both the conservative and optimized cases. The conservative threshold was computed as a quadratic in speed. From the figure, it can be seen that the use of the conservative threshold causes the first roll maneuver to occur too soon. In addition, subsequent roll switching points are not optimally located, since the shape of the conservative curve is incorrect.

With the use of the optimized threshold, however, the projected miss-distance essentially follows the threshold after the first roll switching, thus minimizing the total number of switchings. The effect of flying to an extreme out-of-plane point is shown in Run #3, which went to the edge of the lateral footprint (180 nmi out-of-plane). In this case only two roll maneuvers were required and the terminal miss-distance was essentially unchanged.

A series of runs was made to investigate the effect of environmental disturbances on the system performance. Run #4 was made to determine the effect of a 95% steady-state side wind. It can be seen that the lateral miss-distance increased

TABLE 1

STATISTICAL DATA FROM THRESHOLD-DESIGN SIMULATION RUNS

Cost function J_2 (minimum probability)

Trajectory range from interface (h = 400,000 ft) ≈ 2550 nmi

Description	Run No.	rms miss-distance navigation error nmi	rms miss-distance prediction error nmi	width of terminal region (2) nmi	Initial Probability of Being Inside Region Roll Maneuvers-to-Go			
					1	2	3	4
Nominal	135	3.0	3.7	6	0.49	0.85	0.89	0.91
Vary rms terminal miss-distances	136	3.0	11.8	6	0.25	0.73	0.85	0.88
	139	10.0	3.7	6	0.43	0.63	0.65	0.66
Vary region width	137	3.0	3.7	12	0.81	0.9956	0.9987	0.9992
	138	1.0	3.7	2	0.18	0.51	0.60	0.66

slightly to one nmi, but only two roll maneuvers were required. With the same side wind, 3-sigma density variations and L/D variations (Run #4), the lateral miss-distance increased to two nmi, while requiring two roll maneuvers. For this case, the vehicle L/D was reduced by 4% from the value used to design the thresholds, and the signs of all errors were taken in the worst sense. The effect of random vehicle oscillations is shown in Run #5; the lateral miss is less than one nmi and two roll maneuvers were required.

6. SUMMARY OF RESULTS AND CONCLUSIONS

The main result of the paper is the development of a technique for the design of switching thresholds for use in the lateral guidance of a roll-controlled reentry vehicle. The form of the basic tradeoff between number of roll maneuvers and miss distance can readily be specified by the choice of the cost function. Two different cost functions have been minimized: (1) the expected value of the weighted sum of the number of roll maneuvers and the square of the miss-distance; (2) the probability that the miss-distance exceeds a preselected value, with the maximum number of roll maneuvers specified apriori. The method developed permits factors such as vehicle maneuver capability, navigation-system errors, and environmental uncertainties to be included in an orderly manner.

The design technique is applied to the entry problem of a low-cross-range shuttle vehicle, using dynamic programming to find the optimal thresholds. Of the two cost functions considered, it is felt that the second one (J_2), which minimizes the probability of the miss exceeding a fixed value for a given number of roll maneuvers, provides a much better formulation of the key design tradeoff. The resultant switching logic, however, is more complex, since the required number of thresholds is equal to the maximum permissible number of roll maneuvers. With the first cost function (J_1),

TABLE 2

ACTUAL TRAJECTORY DATA WITH WIND, DENSITY, L/D VARIATIONS, VEHICLE ATTITUDE OSCILLATIONS

- No IMU or initial-condition errors
- Downrange from interface (h = 400,000 ft) ≈ 2550 nmi

Run No.	Description		Terminal Miss-Distance Downrange (nmi)	Terminal Miss-Distance Crossrange (nmi)	Roll Switching to h = 100,000 ft
1	Conservative Threshold (Fig. 5)		0.04	-0.72	7
2	Optimal Threshold		0.04	-0.60	3
3	Optimal Thresholds	max. lateral range (180 nmi)	-0.07	-0.69	2
4		95% wind	-0.25	-1.04	2
5		95% wind, 3 density, 4% L/D error	-1.89	2.02	2
6		random att. osc. (1 deg rms)	-0.42	-0.95	2

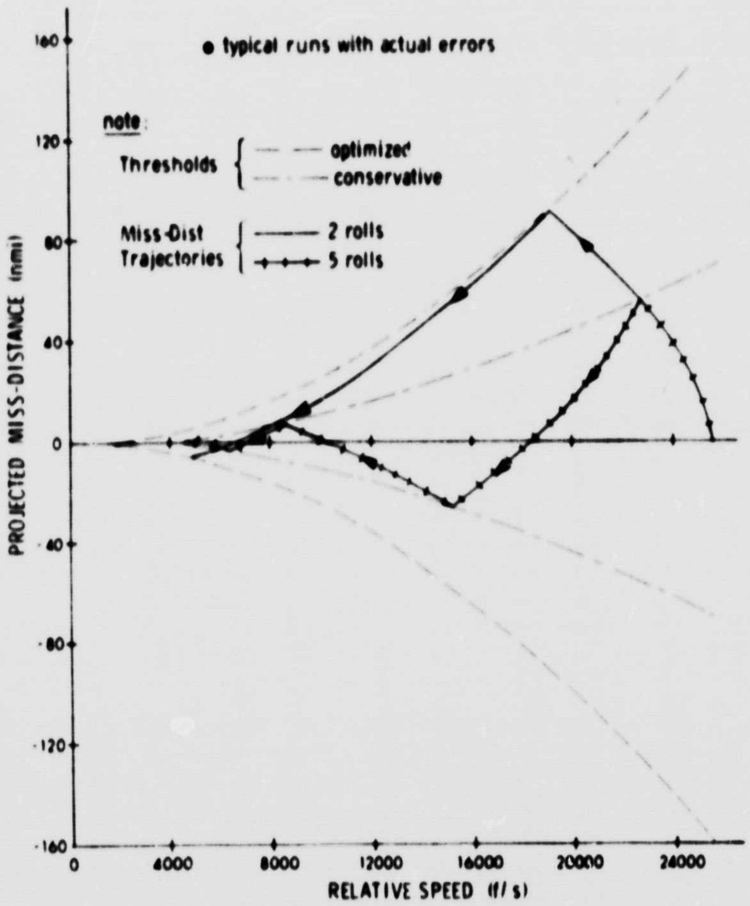

Fig. 5 Comparison of Miss-Distance Trajectories for Conservative and Optimized Thresholds

on the other hand, only a single threshold is required. The tradeoff between miss-distance and roll maneuvers in this case, however, is accomplished less directly through the weighting coefficient (λ) which must be selected by the system designer.

Simulation results are presented for various entry trajectories, including variations in the aerodynamic characteristics of the vehicle and the operational environment. For the particular cases studied, it was found that satisfactory terminal accuracies could be obtained with three roll maneuvers or less. Permitting a larger number of roll maneuvers did not significantly improve the system accuracy. In effect the paper demonstrates the application of stochastic control theory to a current problem of spacecraft control-system design and illustrates the usefulness of optimization theory as an aid in making design-tradeoff decisions.

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APPENDIX A

DOWN-RANGE ENTRY GUIDANCE EQUATIONS

The required in plane L/D, i.e. $(L/D)_V$ is based on the difference between the distance to the terminal point (θ_T) and the estimated entry range of the vehicle (θ), using the perturbation-guidance technique[2,3]. The basic relation is:

$$(L/D)_V = (L/D)_0 + 4 \left[\frac{\partial \theta}{\partial (L/D)_V}\right] (\theta_T - \theta), \qquad \text{(A-1)}$$

where $(L/D)_0$ is the preselected nominal L/D corresponding to a center-of-the-footprint entry reference trajectory. The sensitivity $\partial\theta/\partial(L/D)_V$ is obtained from a stored reference-trajectory table at the current vehicle speed. The entry range (θ) is computed from the vehicle's estimated speed (v), vertical velocity ($\dot{r}$), and drag (d), using the relation:

$$\theta = \theta_{TR} + \left(\frac{\partial \theta}{\partial \dot{r}}\right) (\dot{r} - \dot{r}_R) + \left(\frac{\partial \theta}{\partial d}\right) (d - d_R), \qquad \text{(A-2)}$$

where θ_{TR}, $\dot{r}_R$, and d_R are the reference trajectory values of range-to-go, vertical velocity, and drag at the current speed. The sensitivities ($\partial\theta/\partial r$) and ($\partial\theta/\partial d$) are also stored in a table along with the other reference-trajectory quantities required in Eqs. (A-1) and (A-2).

The overall guidance concept requires that the vehicle roll about its velocity vector to achieve the in-plane L/D determined by Eqa. (A-1) and (A-2). The required roll angle (ϕ) is given simply by

$$\phi = \cos^{-1} \left[(L/D)_V/(L/D)\right]. \qquad \text{(A-3)}$$

where the subscript V is used to differentiate between the total and in-plane (L/D).