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NOZZLE AND CAVITY WALL COOLING LIMITATIONS ON SPECIFIC IMPULSE OF A GAS-CORE NUCLEAR ROCKET

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Abstract

The uranium plasma nuclear rocket is a proposed device which features a high specific impulse. The objective of this study was to find the upper limit of the ability of the seeded hydrogen propellant to intercept the thermal radiation from the high temperature fuel and thus protect the reactor cavity wall from excessive heat flux. This limit fixes the maximum reactor power that can be used for a fixed propellant flow rate; and determines the maximum achievable cavity specific impulse. This maximum cavity propellant temperature can then be used to find the amount of nozzle wall seeded transpiration flow that is necessary. This required nozzle coolant flow reduces the maximum cavity specific impulse to some lower value that is the maximum specific impulse such as an engine system can provide. The model used for the heat-transfer calculation is as follows: flow enters the cavity wall at a given temperature, and picks up the amount of radiant heat deposited on the inner surface of the wall as it flows through it. The propellant then leaves the wall at the solid-wall-limited temperature and begins to flow radially inward inside the cavity. Between the wall and the central uranium plasma there is an energy balance between the absorbing hydrogen flow toward the plasma and the radiant heat flux toward the wall. The same basic heat-transfer analysis is used in the cavity and in the nozzle. However, in the nozzle, one must additionally specify the boundary layer thickness caused by the injection of the absorbing, transpiration coolant flow. The thickness of this coolant layer in the nozzle is calculated from conservation of mass and momentum.

This study showed that, for an eight foot cavity diameter uranium plasma nuclear rocket operating at a pressure of 1000 atm and a propellant mass flow rate of 10 lbm/s, the cavity wall could be cooled up to a power level of 7400 MW. This corresponded to a maximum cavity specific impulse of 5800 s. The wall heat flux was very low for any reactor power below this limit of 7400 MW. The reason for this was that there was a relatively cool, opaque insulating layer of seeded propellant between the hot plasma and the solid wall. The degradation effect of the transpirationally cooled nozzle on the cavity specific impulse resulted in a maximum specific impulse of 5200 seconds at a reactor power of 7400 MW. Approximately 12 percent of the total propellant was used to cool the nozzle wall.

Introduction

The uranium plasma nuclear rocket is a proposed device which features a high specific impulse (2000-7000 sec) and a moderately high thrust (10,000-100,000 lb).(1) A conceptual design of current interest is shown in Fig. 1 and is called a "spherical" or "porous-wall" reactor. Energy is generated by nuclear fission in a uranium plasma fuel. This energy is radiated to a seeded hydrogen propellant. The propellant flows through the cavity wall, around the fuel and is then exhausted through the nozzle. The objective of this study was to determine the upper limits on specific impulse placed by the wall cooling requirements of both the reactor cavity and the exhaust nozzle.

As the propellant flows through the cavity wall, it is heated from cryogenic temperatures to solid-wall-limited temperatures. Thus, the propellant transpirationally cools the cavity wall. The amount of energy deposited in the propellant as it flows through the wall equals the amount of energy deposited on the cavity side of the wall by the impinging radiant and conductive heat flux. (This assumes the gamma heating of the wall is small.) Thus, specifying inside and outside wall temperatures (and the mass flow through the cavity wall) fixes the allowable cavity wall heat flux. Using the radial component of the energy equation, a modified diffusion approximation for the radiant heat flux and an assumed radial mass flow profile inside the cavity, the radial temperature profile and heat flux profile from the edge of the fuel to the cavity wall can be calculated. From these profiles the reactor power, the cavity I_{sp}, and the thrust can be calculated.

As the propellant leaves the reactor cavity and flows through the nozzle, it begins to heat the nozzle wall by forced convection and by thermal radiation. A layer of seeded coolant gas must be injected through the nozzle wall. This coolant transpirationally cools the nozzle wall in much the same way as the propellant cools the cavity wall. In the nozzle the heat-transfer dimension is much smaller than in the cavity; therefore, the coolant flow rate per unit area must be much larger in the nozzle than in the cavity.

The approach used in this study was first to find the cavity wall cooling limitation; this will determine the maximum reactor power and the maximum cavity specific impulse. Then, using the corresponding propellant outlet temperature, the amount of required nozzle wall coolant is determined, and the cavity specific impulse is reduced due to this additional flow of cooler gas. The resulting specific impulse is the maximum that can be produced by the engine without burning out either the reactor cavity wall or the nozzle wall.

Symbols

A area, m^2

a absorption coefficient, m^{-1}

E_n expodential integral of order n, $E_n(z) = \int_1^\infty \frac{e^{-zt}}{t^n}\, dt$

h enthalpy, J/kg

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I intensity, $w/(m^2\text{-sr-Hz})$

k thermal conductivity, w/m

L cooling surface length, m

M mass flow rate, kg/s

$\hat{n}$ normal unit vector

n index used in Taylor Series

P pressure, N/m^2 or atm

q heat flux, w/m^2

r radius, m

T temperature, K

v velocity, m/s

z axial length, m

δ distance from solid wall towards plasma, m

θ angles in nozzle, rad

ν frequency, Hz

ρ density, kg/m^3

σ Stefan-Boltzmann constant, 5.6697×10^{-8} w/ $(m^2 {}^oK^4)$

τ optical depth from solid wall towards plasma

ω solid angle, sr

Subscripts:

c conduction

g gas

N nozzle

o cool side of solid wall

p plasma

R radiation

t nozzle throat

w plasma side of wall

O up stream of nozzle throat

1 down stream of nozzle throat

Analysis

The model used for the heat-transfer calculation is shown in Fig. 2. Flow enters a wall at a temperature, T_o, then picks up the amount of heat deposited on the wall, and then leaves the wall at a temperature, T_w. In terms of enthalpy, h, this heat balance is:

$$q_w = -\rho v(h_w - h_o) \tag{1}$$

Inside the cavity, there is an energy balance between the radially inward convection toward the plasma and the radially outward heat flux toward the wall. In vector notation this can be written as:

$$0 = (\rho\vec{v}) \cdot \nabla h + \nabla \cdot \vec{q} \text{ (using only the radial terms)} \tag{2}$$

The heat flux can be written as the sum of two terms: conduction and radiation.

$$\vec{q} = \vec{q}_c + \vec{q}_R \tag{3}$$

The conductive heat flux is proportional to the local temperature gradient.

$$\vec{q}_c = -k\nabla T \tag{4}$$

The radiative heat flux is given in Ref. 2 as

$$\vec{q}_R = \int_0^\infty \int (\vec{I}_\nu \cdot \hat{n}) d\omega \, d\nu \, \hat{n}$$

For a grey gas in local thermodynamic equilibrium, in cartesian coordinates, the radiative heat flux is

$$q_R = 2\sigma T_w^4 E_3(\tau) - 2\sigma T_p^4 E_3(\tau_p - \tau)$$

$$+ \int_0^\tau 2\sigma T^4 E_2(\tau - \tau')d\tau' - \int_\tau^{\tau_p} 2\sigma T^4 E_2(\tau' - \tau)d\tau' \tag{5}$$

An approximation to this heat flux can be made if the radiation from the propellant can be neglected. Thus if

$$2\sigma T_p^4 E_3(\tau_p) - \sigma T_w^4 \gg \sigma T_g^4(1 - 2E_3(\tau_p))$$

then

$$q_R = 2\sigma T_w^4 E_3(\tau) - 2\sigma T_p^4 E_3(\tau_p - \tau) \tag{5a}$$

(This approximation is used when τ_p is small.) Another approximation to this heat flux relation can be made if the source function for the gas can be represented by a Taylor Series. Thus

$$T^4 = T^4(\tau) + \frac{dT^4(\tau)}{d\tau}(\tau' - \tau) + \ldots$$

$$= \sum_{n=0} \frac{(\tau' - \tau)^n}{n!} \frac{d^nT^4(\tau)}{d\tau^n}$$

Substituting this into Eq. (5) and integrating gives:

$$q_R = 2\sigma T_W^4 E_3(\tau) - 2\sigma T_p^4 E_3(\tau_p - \tau)$$

$$+ 2\sigma \sum_{n=0} \frac{d^n T^4(\tau)}{dt^n} \left\{ \sum_{L=0}^{n} \frac{1}{(n-L)!} \left[(\tau_p - \tau)^{n-L} E_{L+3}(\tau_p - \tau) \right.\right.$$

$$\left.\left. -(-1)^n \tau^{n-L} E_{L+3}(\tau) \right] - \frac{(1-(-1)^n)}{(n+2)} \right\}$$

Neglecting the terms of the series for which n is greater than one yields

$$q_R = 2\sigma T_W^4 E_3(\tau) - 2\sigma T_p^4 E_3(\tau_p - \tau) + 2\sigma T^4 (E_3(\tau_p - \tau)$$

$$- E_3(\tau)) + 2\sigma \frac{dT^4}{d\tau} \left[(\tau_p - \tau) E_3(\tau_p - \tau) \right.$$

$$\left. + \tau E_3(\tau) + E_4(\tau_p - \tau) + E_4(\tau) - 2/3 \right]$$

If the optical thickness (optical depth from the wall to the radiating plasma) is large,

$$\tau_p \gg 1$$

then

$$q_R = 2\sigma T_W^4 E_3(\tau) - 2\sigma T^4 E_3(\tau)$$

$$+ 2\sigma \frac{dT^4}{d\tau} \left[\tau E_3(\tau) + E_4(\tau) - \frac{2}{3} \right] \quad (5b)$$

If the local propellant optical depth is large,

$$\tau \gg 1$$

then

$$q_R = - \frac{4\sigma}{3} \frac{dT^4}{d\tau} \quad (5c)$$

which is just the standard diffusion approximation.

The optical depth is related to the distance from the cavity wall by

$$d\tau = a \, d\delta \quad (6)$$

Using the given wall temperature, the calculated wall heat flux from Eq. (1), and the given wall distance as boundary conditions, the differential Eqs. (2), (3), and (6) coupled with the auxiliary Eqs. (4), (5) and an assumed variation of mass flow rate, ($\rho v(\delta)$ equals a constant), can be integrated numerically from the wall into the cavity.

The heat-transfer analysis used in the nozzle is the same as that used in the cavity. A flow model used in the calculation of the coolant boundary layer thickness is shown in Fig. 3. Conservation of mass requires

$$(\rho v)_w L = (\rho v)_1 \delta \quad (7)$$

Conservation of momentum, (neglecting compressible and viscous effects) requires

$$\Delta P \, \delta \geq (\rho v^2)_1 \delta \quad (8)$$

Combining Eqs. (7) and (8) yields

$$\delta \geq (\rho v)_w L / \sqrt{\rho \, \Delta P'} \quad (9)$$

An analogous equation for a convergent-divergent nozzle, Fig. 4, is

$$\delta \geq \frac{1}{2} r_t (\rho v_w) \left[\frac{(A_0/A_t - 1)}{\sin \theta_0} + \frac{(A_1/A_t - 1)}{\sin \theta_1} \right] \Bigg/ \left[\sqrt{\rho P_c \left(\frac{P_0}{P_c} - \frac{P_1}{P_c} \right)} \sqrt{\left(\frac{A_0}{A_t} \right)\left(\frac{A_1}{A_t} \right)} \right] \quad (10)$$

where Eqs. (8) and (9) have been rewritten in cylindrical geometry, and the control volume is a convergent-divergent shell next to the nozzle wall. The total mass flow through the nozzle wall is

$$\overset{\circ}{M}_N = \left[\frac{(A_0/A_t - 1)}{\sin \theta_0} + \frac{(A_1/A_t - 1)}{\sin \theta_0} \right] (\rho v)_w A_t \quad (11)$$

Properties for uranium vapor were taken from Refs. 3 and 4. Properties for hydrogen were taken from Refs. 5, 6, 7, and 8. The absorption coefficient for the solid seed (uranium 238) was assumed to be 50,000 cm^2/g.

Discussion

A cross section of the uranium plasma nuclear rocket is shown on Fig. 1. The propellant flows through the cavity wall, transpirationally cooling the wall. Then the propellant flows from the wall toward the fissioning uranium plasma convecting the radiant heat that it absorbs back toward the fuel. Near the fuel the propellant turns and flows tangentially out of the cavity by way of the nozzle. In the nozzle the propellant acts as a source of radiant heat. Additional coolant must be used to transpirationally cool the nozzle wall and absorb the radiant heat.

Figure 5 illustrates the complexity of the problem. It shows the "Rosseland Mean Absorption Coefficient" versus temperature. At low temperatures the solid seed is the dominant absorber. As the temperature increases, the solid-seed absorption coefficient decreases because the propellant density decreases and, therefore, the number of solid particles per unit volume also decreases. At some temperature (~5000° K) the solid seed begins to vaporize and the absorption coefficient decreases very sharply. In the temperature region between 7000-10,000° K, the absorption is due mainly to seed vapor which in this case is uranium vapor, since in these calculations we have used depleted uranium as the seed material. At temperatures above 10,000° K, the absorption is due mainly to the hydrogen becoming absorptive.

Figure 6 shows results of some typical reactor cavity calculations. Figure 6(a) shows the I_{sp} and inside cavity wall temperature versus reactor power for a fixed propellant flow rate of 10 lbm/s, cavity diameter of 8 ft, and seed concentration of 10 percent by weight. The I_{sp} is proportional to the square root of the reactor power. For a reactor power below about 7400 MW, the cavity inside wall temperature remains nearly constant at 944° K. At a reactor power equal to about 7400 MW, the wall temperature has increased rapidly to a value that approaches and would soon exceed solid wall temperature limits. This value of reactor power would be the maximum allowable reactor power, and for this particular set of input parameter values corresponds to a maximum attainable cavity specific impulse of 5800 seconds. Because reactor power could fluctuate, a "nominal operating power" level would probably be fixed at 90 percent or so of the maximum operating power.

Figure 6(b) shows a radial temperature profile for both the maximum and the nominal reactor power situations. Both temperature profiles are similar except near the cavity wall. For the nominal case there is a cold opaque layer of propellant between the hot propellant and the solid wall. For the maximum case this layer has been reduced to almost zero thickness.

The model used to estimate the wall cooling requirements of the nozzle is shown in Fig. 4. Propellant flows from the cavity (which is at a pressure of 1000 atm) through the nozzle, and is discharged into space. Transpiration cooling with an absorbing gas is necessary in the high acceleration (throat) region of the nozzle, a region taken to correspond to local static pressure varying from 900 atm to 100 atm. The thickness of the injected boundary layer gas was calculated from a momentum balance flow model assuming negligible viscous shear.

Figure 7 shows the result of nozzle cooling limitations on I_{sp}. It shows the nozzle I_{sp} (calculated using the propellant flow from cavity plus the nozzle coolant flow) versus cavity I_{sp} for a seed concentration of 40 percent and a wall temperature of 1100° K. The spread in the results is due to uncertainties related to what extent the nozzle coolant would mix down stream of the nozzle throat with the main propellant. The cavity-wall-limited I_{sp} for this case is about 5800 seconds, while the actual I_{sp} including nozzle cooling degradation is between 5100 and 5500 seconds.

These results are not intended to be the final word on the complicated subject of I_{sp} limitations of a gas-core rocket engine due to thermal protection requirements. These results were obtained by a "first-cut" look at the problem. The answers appear encouraging, but more information on seeded hydrogen opacity and on transpiration flow in the presence of thermal radiation absorption is required.

Conclusions

This study has shown that for an eight foot cavity diameter uranium plasma nuclear rocket operating at a pressure of 1000 atm, and a propellant mass flow rate of 10 lbm/sec, the cavity wall could be cooled up to a reactor power level of 7400 MW. This corresponded to a maximum cavity specific impulse of 5800 s. The wall heat flux was very low for any reactor power below this limit of 7400 MW. The reason for this was that there was a relatively cool, opaque insulating layer of seeded propellant between the hot plasma and the solid wall.

The additional coolant flow required to protect the nozzle reduced the maximum cavity specific impulse value of 5800 seconds to a value of 5200 seconds. Approximately 12 percent of the total propellant was used to cool the nozzle wall.

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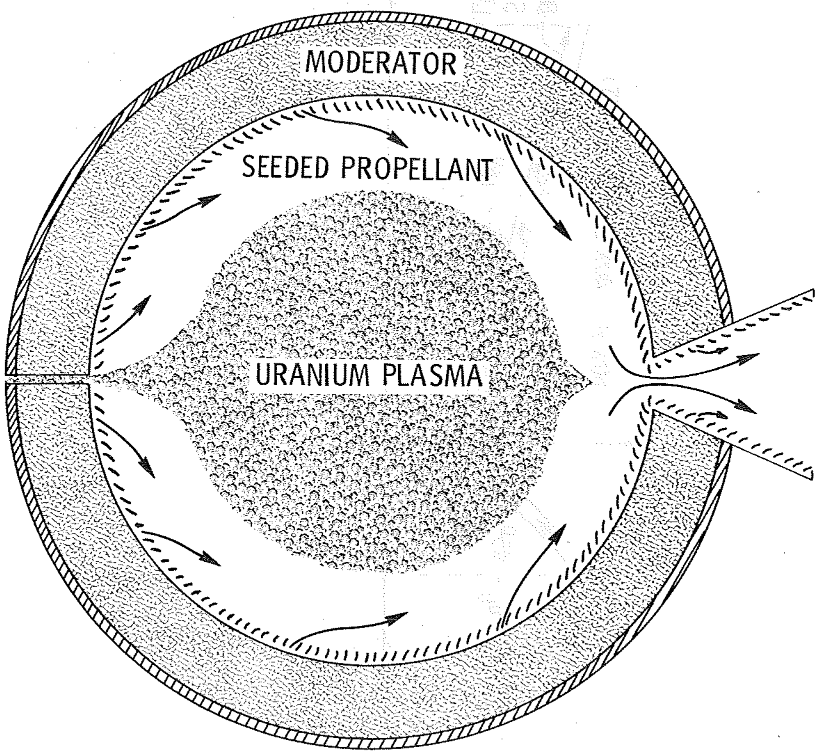

Figure 1. - Heat transfer model of spherical reactor.

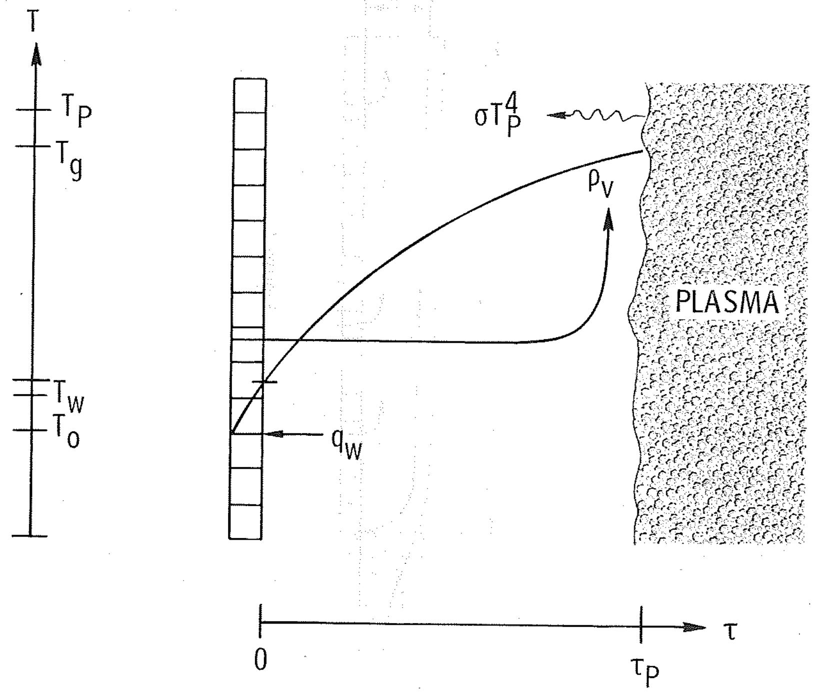

Figure 2

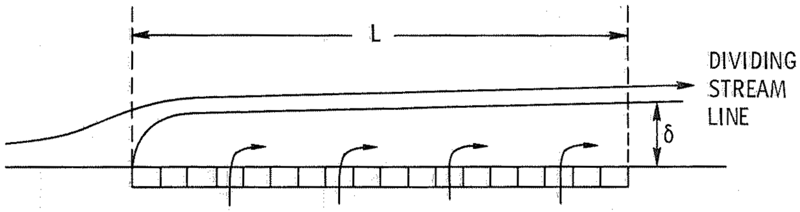

Figure 3

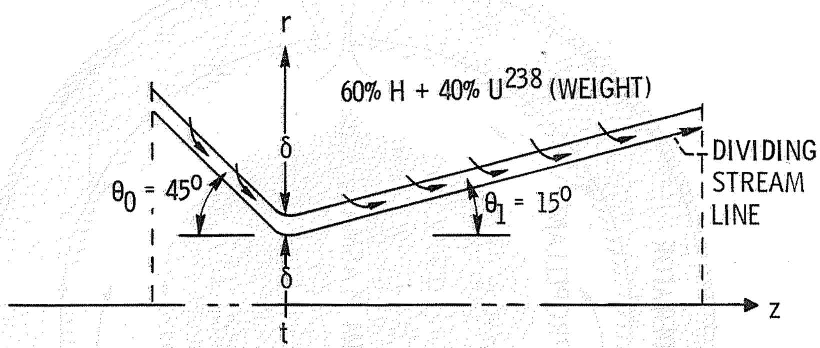

Figure 4

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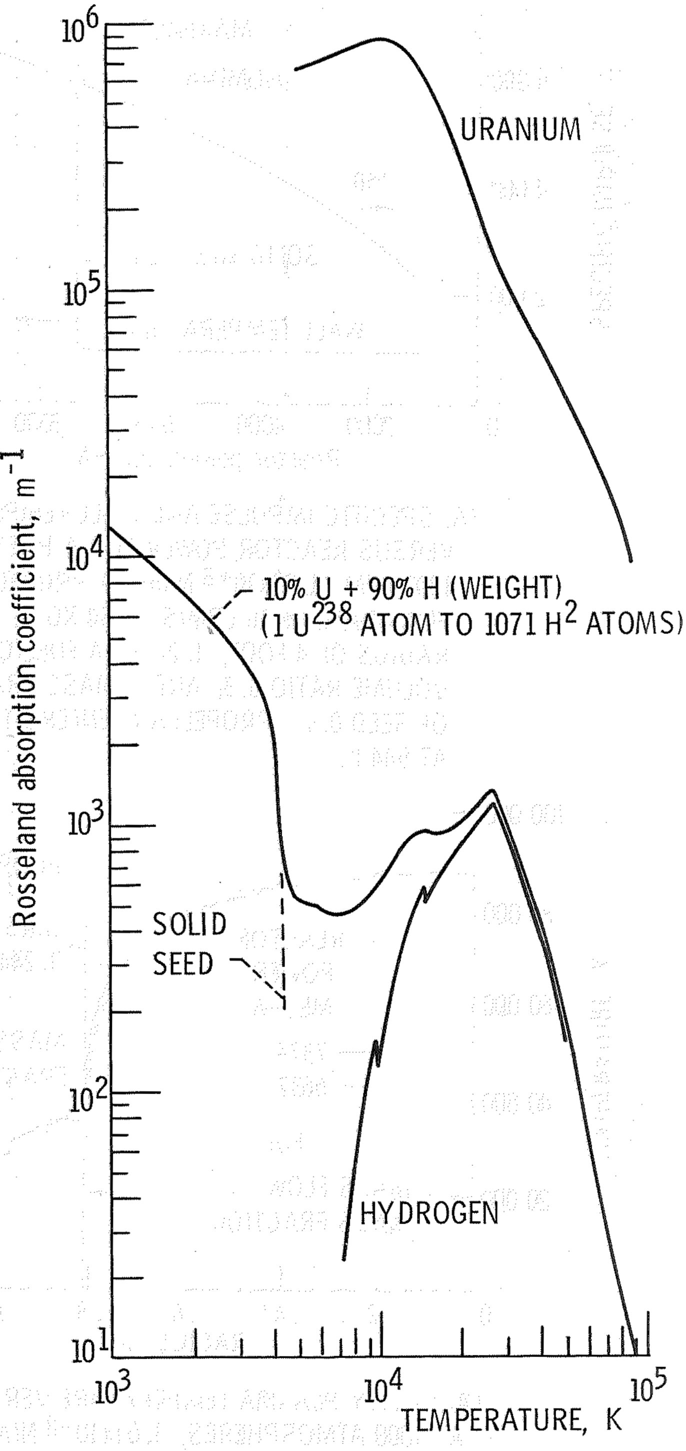
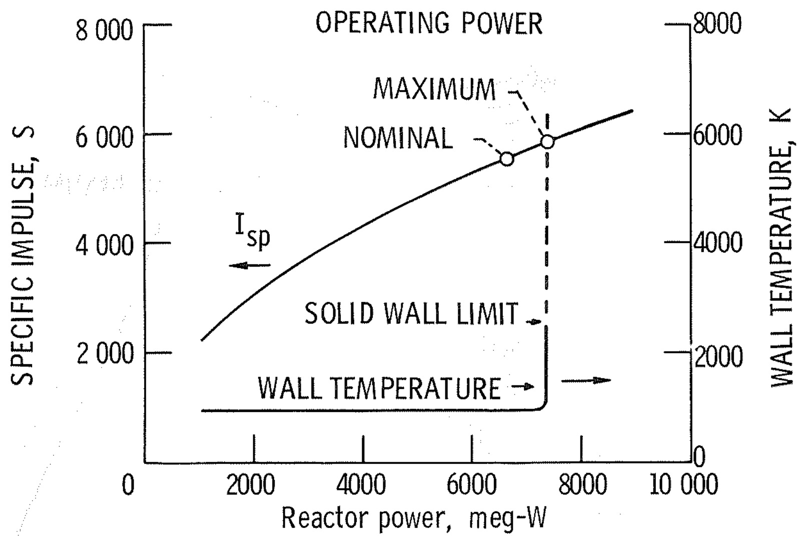
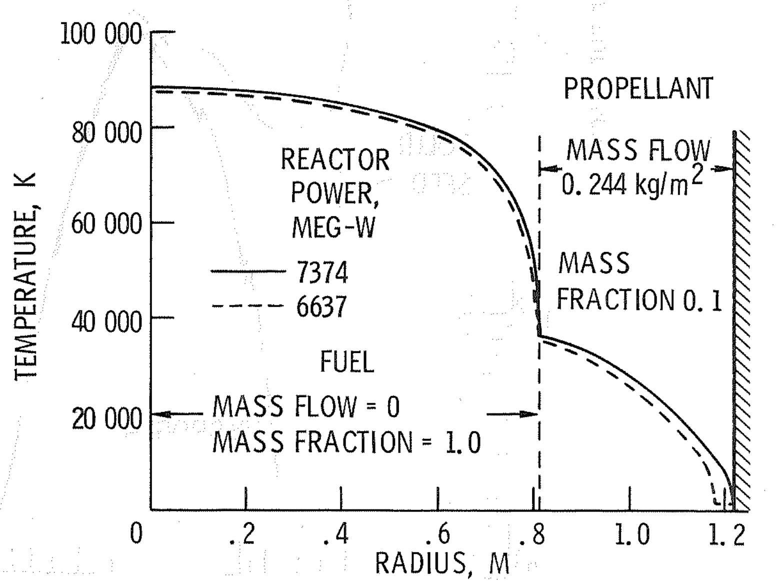


Figure 5. - Rosseland absorption coefficient versus temperature at 1000 atmosphere, 1.01×10^{-8} N/m^2.



(A) SPECIFIC IMPULSE AND WALL TEMPERATURE VERSUS REACTOR POWER FOR A PRESSURE OF 1000 ATM, $1.01 \times 10^{+8}$ N/m^2; A PROPELLANT FLOW RATE OF 10 LBM/S, 4.54 KG/S; A CAVITY RADIUS OF 4 FOOT, 1.22 m; A FUEL/CAVITY VOLUME RATIO 0.3; AND A MASS FRACTION OF SEED 0.1. PROPELLANT ENTERS THE WALL AT 944 K.



(B) CAVITY PLASMA TEMPERATURE VERSUS RADIUS, AT 1000 ATMOSPHERES, $1.01 \times 10^{+8}$ N/m^2, FOR MAXIMUM AND NOMINAL REACTOR POWER; PROPELLANT-FLOW RATE = 10 LBM/S, 4.54 kg/S. PROPELLANT ENTERS THE WALL AT 944 K.

Figure 6

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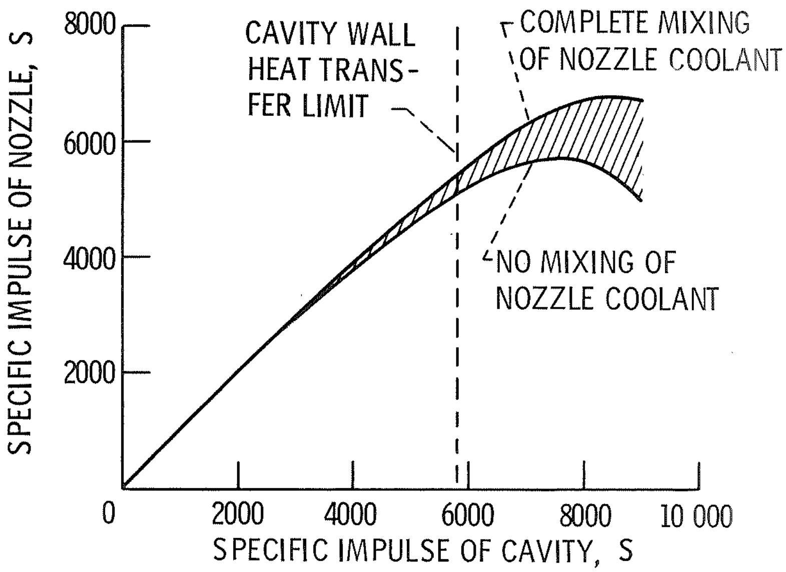

Figure 7. - Specific impulse degradation due to nozzle coolant flow, (40% U^{238} + 60% H_2). Coolant enters the wall at 300 K and leaves the wall at 1100°K.