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PSEUDO-DIAMAGNETIC SUSPENSION

by

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ABSTRACT

This report describes a device that develops forces in a magnetic field as though it were composed of diamagnetic material. Since a diamagnetic material can be stably suspended (i.e. levitated) in a properly shaped magnetic field, this device or its principles may be useful for suspension applications. As such, it is called pseudo-diamagnetic suspension. The forces developed can be substantially larger than those developed in conventional diamagnetic suspension systems.

This device requires electrical components and an energy source, but it can be completely self contained in that it need sense only its internal conditions. In some designs, this energy requirement can be made significantly small. A laboratory experiment is described where a stable suspension was provided at the rate of 232 kg/watt.

The analogous electric field case is also analyzed, where a device develops forces in an electric field as though it were composed of material with a permittivity less than that of free space. This is called "pseudo-diadielectric suspension". It simulated no known natural phenomenon.

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I. INTRODUCTION

Force field suspension* of devices can have the extremely attractive feature of vanishingly small friction. For instance, this friction is negligible compared with gas friction on freely suspended spinning rotors, such that the deceleration of the rotor can be used as a measure of the gas pressure down to pressures of the order of 5×10^{-8} torr[1]. More recently, the drag torques on a decelerating magnetically suspended spinning steel sphere has been measured to a level approaching that required to detect relativistic effects[2]. In addition, these suspensions can be arranged to have low force and torque levels in some degrees of freedom and, simultaneously, high force and torque levels in others. All force field suspensions but one are unstable under static conditions[3] and hence require energy input and state sensing-control. The only exception is diamagnetic suspension, where a material whose permeability is less than that of free space will develop a force (or reaction) in a magnetic field towards the minimum-energy-density direction. There would be another exception (an electric field analogy) if materials with a permittivity that is less than that of free space existed. (These are called "diadielectrics" in this report.)

Diamagnetic suspension has the advantages of simplicity and low (or no) power consumption. Unfortunately, the diamagnetic susceptibility of known materials at normal temperatures and the magnetic fields available from permanent (or electro) magnets restricts the application of this principle to very small or low-force devices.[4] Diamagnetic suspension utilizing superconductivity significantly raises the force capability but adds the expense of cryostatic operations.

This report describes a device that behaves, under certain conditions, as though it were diamagnetic. As such, it or its principles may be useful for suspension applications. It requires active components and an energy source, but it can be completely self-contained (it need only sense its internal conditions). Under some conditions, the energy requirement can be made significantly small. The analogous electric field case can also be made, providing for "diadielectric" suspension which, unlike the "diamagnetic" device, simulates no known natural phenomenon.

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* In this report, we use the term <u>suspension</u> in the sense of <u>levitation,</u> i.e. suspension without contact. Hence we mean suspension of all 3 degrees of linear motion with force fields only.

II. HISTORICAL OUTLINE OF MAGNETIC AND ELECTRIC SUSPENSIONS

For an excellent survey of magnetic and electric suspensions up to 1964, the reader's attention is directed to Reference (5). Pertinent to diamagnetic suspension, we note that S. Earnshaw (1839) showed that a monopole is unstable in a static inverse-square-law field[(6)]. Extending this analysis for static electric and magnetic fields, Von W. Braunbek (1939) concluded that suspension is possible only for diamagnetic ($\mu/\mu_o < 1$) or "diadielectric" ($\varepsilon/\varepsilon_o < 1$) materials[(3)]. He demonstrated diamagnetic suspension of small pieces of bismuth (8×10^{-6} kg) and graphite (7.5×10^{-5}kg)[(7)]. In 1956, A. Boerdijk also suspended a small piece of graphite in a static permanent-magnetic field[(8)]. A.D. Waldron (1965) stably suspended a graphite bearing (10^{-3}kg) with permanet magnets[(9)], and in 1968, I. Simons constructed an extremely sensitive tiltmeter utilizing a graphite seismic mass suspended in a permanent magnet field[(10)].

Suspension utilizing superconductors was first achieved in Russia by V. Arkadiev (1945)[(11)], followed by others in this country, including P.K. Chapman and S. Ezekiel who constructed a low-level accelerometer by suspending a bar magnet over a superconducting surface[(12)].

The other suspension technique pertinent to this report is time variation of the magnetic field strength, where position sensing of the suspended object controls the suspension current in an electromagnet. The position sensing can be achieved by detecting changes in position optically, inductively, or capacitively and appropriately modifying the d.c. or a.c. suspension current. Historically, this is well described in Ref. (5). Of further interest is the work done by Beams[(1)(13)], Gilinson, et al.[(14)] and the Cambridge Thermionic Corporation[(15)].

III. CONCEPT OF PSEUDO-DIAMAGNETIC SUSPENSION

As noted previously, diamagnetic suspension is very simple; if one shapes a permanent magnet field properly, it can suspend a piece of diamagnetic material indefinitely. In concept this occurs for two reasons: 1) magnetic fields can be shaped to have a flux density (hence energy density) minimum in free-space, (but not a flux density maximum); 2) the magnetic field within a diamagnet is less than that which would otherwise exist in free space (the magnetic induction is negative). Under these two conditions, the minimum energy configuration of the magnetic field and diamagnet requires that the diamagnet be positioned at the energy density minimum, hence it can be suspended in free space.

To imitate this characteristic of a diamagnetic material (at the expense of energy input) would seem to be straight-forward; namely, measure the field that exists (e.g. with a Hall sensor) and drive current through an electromagnet coil oriented to oppose the measured flux. Of course the field that exists is the sum of the free space field plus the generated field, but nevertheless it is possible to reduce the resultant field to any extent desired. Since the above discussion applies to only one component of the free space field, we are required, in general, to do this for each orthogonal direction. The next section shows that this procedure does indeed imitate this characteristic of a diamagnetic material. We have demonstrated a suspension using this approach experimentally in the laboratory (see Section VI).

Whether this technique is useful or not depends upon the application and design. True diamagnetic suspension can operate indefinitely (although there are few applications which demand that capability), but with very limited force. Pseudo-diamagnetic suspension can provide a relatively large force capability but only for a finite time (the product of force and time being proportional to the energy available). Some configurations are examined in Section V.

To increase the force beyond the limit inherent in the straight-forward design, would require that the flux density within the device be not merely reduced to zero, but in fact, reversed in polarity. That this can be done is developed in Section VII. Also in Section VII and Appendix B is the analysis to support the electric field analogy leading to the diadielectric suspension.

IV. THEORY OF OPERATION

Consider an orthogonal set of circular current-carrying coils with coincident centers in an external magnetic field. (See Fig. 1). Let us neglect time dependent effects of a rotation of the coils relative to the field. An elemental section of a coil (ds) will experience a force* given by

$$d\underline{F} = NI d\underline{s} \times \underline{\beta} \tag{1}$$

Selecting a coordinate system (x,y,z with unit vectors, respectively, $\hat{i},\hat{j},\hat{k}$) centered and aligned with the coils, and neglecting gradients of the magnetic field flux density higher than the first, the moment ($\underline{M}_n$) on each coil will be

$$\underline{M}_x = \pi R_x^2 N_x I_x [\quad - \beta_{zo}\hat{j} + \beta_{yo}\hat{k}] \tag{2}$$

$$\underline{M}_y = \pi R_y^2 N_y I_y [\beta_{zo}\hat{i} \quad - \beta_{xo}\hat{k}] \tag{3}$$

$$\underline{M}_z = \pi R_z^2 N_z I_z [-\beta_{yo}\hat{i} + \beta_{xo}\hat{j} \quad] \tag{4}$$

and the force ($\underline{F}_n$) on each coil will be

$$\underline{F}_x = \pi R_x^2 N_x I_x [-(\frac{\partial \beta_z}{\partial z} + \frac{\partial \beta_y}{\partial y})\hat{i} + \frac{\partial \beta_x}{\partial y}\hat{j} + \frac{\partial \beta_x}{\partial z}\hat{k}] \tag{5}$$

$$\underline{F}_y = \pi R_y^2 N_y I_y [\frac{\partial \beta_y}{\partial x}\hat{i} - (\frac{\partial \beta_x}{\partial x} + \frac{\partial \beta_z}{\partial z})\hat{j} + \frac{\partial \beta_y}{\partial z}\hat{k}] \tag{6}$$

$$\underline{F}_z = \pi R_z^2 N_z I_z [\frac{\partial \beta_z}{\partial x}\hat{i} + \frac{\partial \beta_z}{\partial y}\hat{j} - (\frac{\partial \beta_y}{\partial y} + \frac{\partial \beta_x}{\partial x})\hat{k}] \tag{7}$$

where, for the n^{th} coil

R_n = radius of coil

N_n = no. of turns

I_n = current

$\beta_{xo}, \beta_{yo}, \beta_{zo}$ are the values of the external magnetic field at the origin

*SI Units are used throughout this report

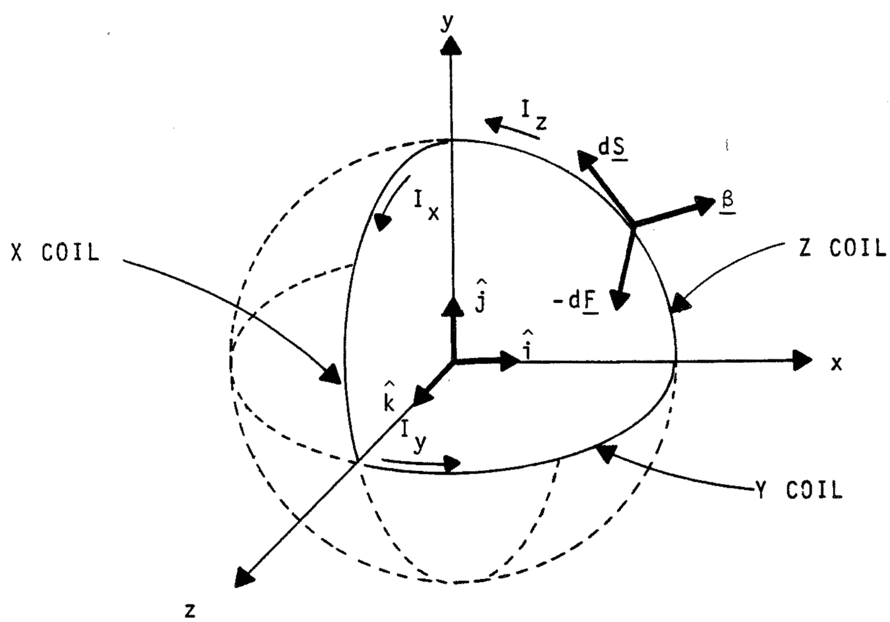

Figure 1 Coil Orientation

Note that ($\underline{M}_n'$,$\underline{F}_n'$) are the total moment and force on coil n and not components.

Let us consider a device where we sense the magnitude of the total magnetic field flux density component (β_{tn}) which is normal to the plane of each coil (at the origin) and control the current through that coil such that the current is proportional to that measurement. (See Figure 2)

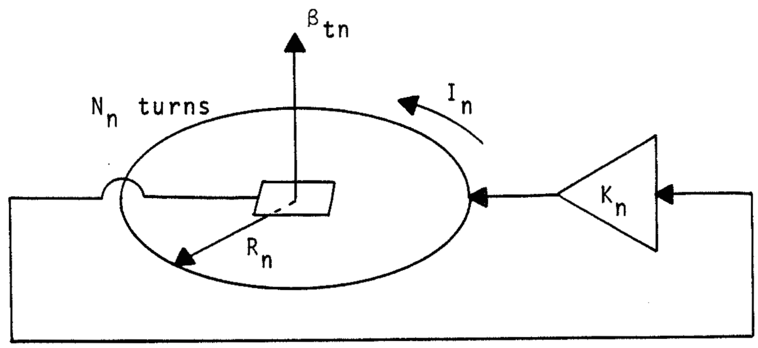

Figure 2 Coil n Feedback Configuration

Thus

$$I_n = -K_n\beta_{tn} \tag{8}$$

where K_n is the amplifier-sensor gain.

Noting that the total field is the sum of the external field (β_n) plus the field due to the coil current (β_c), namely

$$\beta_{tn} = \beta_n + \beta_c = \beta_n + \frac{\mu_o N_n I_n}{2R_n} \tag{9}$$

we see that

$$I_n = - \frac{K_n}{1 + \frac{\mu_o N_n K_n}{2R_n}} \beta_n \tag{10}$$

Defining the feedback gain as

$$C_n \equiv \frac{1}{1 + \frac{2R_n}{\mu_o N_n K_n}} \tag{11}$$

we have

$$N_n I_n = - \frac{2C_n R_n \beta_n}{\mu_o} \tag{12}$$

which can be substituted into equations (2-7) to yield the moments and forces on the coils under this feedback arrangement. Note that the field due to any coil does not have any net cross-coupling effects on the other sensors or coils. Also noting that

$$\nabla \cdot \underline{\beta} = 0$$

we have

$$\underline{M}_x = \frac{2\pi C_x R_x^3}{\mu_o} \beta_{xo} \left[\qquad + \beta_{zo}\hat{j} - \beta_{yo}\hat{k} \right] \tag{13}$$

$$\underline{M}_y = \frac{2\pi C_y R_y^3}{\mu_o} \beta_{yo} \left[-\beta_{zo}\hat{i} \qquad + \beta_{xo}\hat{k} \right] \tag{14}$$

$$\underline{M}_z = \frac{2\pi C_z R_z^3}{\mu_o} \beta_{zo} \left[\beta_{yo}\hat{i} - \beta_{xo}\hat{j} \qquad \right] \tag{15}$$

$$\underline{F}_x = -\frac{2\pi C_x R_x^3}{\mu_o} \beta_{xo} \left[\frac{\partial \beta_x}{\partial x} \hat{i} + \frac{\partial \beta_x}{\partial y} \hat{j} + \frac{\partial \beta_x}{\partial z} \hat{k}\right] \quad (16)$$

$$\underline{F}_y = -\frac{2\pi C_y R_y^3}{\mu_o} \beta_{yo} \left[\frac{\partial \beta_y}{\partial x} \hat{i} + \frac{\partial \beta_y}{\partial y} \hat{j} + \frac{\partial \beta_y}{\partial z} \hat{k}\right] \quad (17)$$

$$\underline{F}_z = -\frac{2\pi C_z R_z^3}{\mu_o} \beta_{zo} \left[\frac{\partial \beta_z}{\partial x} \hat{i} + \frac{\partial \beta_z}{\partial y} \hat{j} + \frac{\partial \beta_z}{\partial z} \hat{k}\right] \quad (18)$$

if the coils are attached to each other, then the total moment and force on the device is

$$\begin{aligned} \underline{M} = \frac{2\pi}{\mu_o} [& (C_z R_z^3 - C_y R_y^3) \beta_{yo} \beta_{zo} \hat{i} \\ & + (C_x R_x^3 - C_z R_z^3) \beta_{xo} \beta_{zo} \hat{j} \\ & + (C_y R_y^3 - C_x R_x^3) \beta_{xo} \beta_{yo} \hat{k}] \end{aligned} \quad (19)$$

$$\begin{aligned} \underline{F} = -\frac{2\pi}{\mu_o} [& (C_x R_x^3 \beta_{xo} \frac{\partial \beta_x}{\partial x} + C_y R_y^3 \beta_{yo} \frac{\partial \beta_y}{\partial x} + C_z R_z^3 \beta_{zo} \frac{\partial \beta_z}{\partial x}) \hat{i} \\ & + (C_x R_x^3 \beta_{xo} \frac{\partial \beta_x}{\partial y} + C_y R_y^3 \beta_{yo} \frac{\partial \beta_y}{\partial y} + C_z R_z^3 \beta_{zo} \frac{\partial \beta_z}{\partial y}) \hat{j} \\ & + (C_x R_x^3 \beta_{xo} \frac{\partial \beta_x}{\partial z} + C_y R_y^3 \beta_{yo} \frac{\partial \beta_y}{\partial z} + C_z R_z^3 \beta_{zo} \frac{\partial \beta_z}{\partial z}) \hat{k}] \end{aligned} \quad (20)$$

Specific Cases

Eqs. (19) and (20) are of particular interest in three specific cases of coil parameter values (coil radius cubed times feedback gain) and coil orientation. These cases are: Case 1) - One of the coils is oriented for maximum flux density; Case 2) - Two of the coil parameters are identical and the other coil is oriented for zero flux density; and Case 3) - All three coil parameters are equal. In these three cases, the moment on the device will be zero* and the force can be described

* The rotational stability of these 3 cases is examined in Appendix A. In summary; Case 1 is stable provided that the smallest coil parameter is aligned to the field (the device will have 1 degree of rotational freedom), Case 2 is stable provided that the zero flux coil parameter is larger than the other two (the device will have 2 degrees of rotational freedom), and Case 3 is stable regardless (the device will have 3 degrees of rotational freedom).

in simple vector form.

Case 1 conditions are met by having (for example) the X coil aligned with the field. Therefore

$$\beta_{yo} = \beta_{zo} = 0 \qquad (21)$$

hence from Eq. (20)

$$\underline{F}_1 = -\frac{2\pi}{\mu_o} C_x R_x^3 \beta_{xo} \left(\frac{\partial \beta_x}{\partial x}\hat{i} + \frac{\partial \beta_x}{\partial y}\hat{j} + \frac{\partial \beta_x}{\partial z}\hat{k}\right) \qquad (22)$$

Utilizing Eq. (C8) and recognizing that specific coil selection is immaterial, we can express the force on the device* (Case 1) as

$$\underline{F}_1 = -\frac{\pi}{\mu_o} C_1 R_1^3 \nabla \beta_1^2 \qquad (23)$$

where $C_1 R_1^3$ is the parameter of the coil aligned to the field.

Case 2 conditions are met by having (for example) the Z coil normal to the field and the other two coil parameters equal.

$$\beta_{zo} = 0 \qquad (24)$$

$$C_x R_x^3 = C_y R_y^3 \equiv C_2 R_2^3 \qquad (25)$$

hence

$$\underline{F}_2 = -\frac{2\pi}{\mu_o} C_2 R_2^3 \left[\left(\beta_{xo}\frac{\partial \beta_x}{\partial x} + \beta_{yo}\frac{\partial \beta_x}{\partial x}\right)\hat{i} + \left(\beta_{xo}\frac{\partial \beta_x}{\partial y} + \beta_{yo}\frac{\partial \beta_y}{\partial y}\right)\hat{j}\right] \qquad (26)$$

Utilizing Eq. (C10) and generalizing, we have for Case 2

$$\underline{F}_2 = -\frac{\pi}{\mu_o} C_2 R_2^3 \nabla \beta_2^2 \qquad (27)$$

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*See Appendix C for the pertinent vector relations.

where $C_2R_2^3$ is the value of the parameter of one of the "equal" coils.

Case 3 conditions provide that

$$C_xR_x^3 = C_yR_y^3 = C_zR_z^3 \equiv C_3R_3^3 \tag{28}$$

and we have from Eq. (20) and (C6)

$$\underline{F}_3 = - \frac{\pi}{\mu_o} C_3R_3^3\nabla\beta_3^2 \tag{29}$$

where $C_3R_3^3$ is the value of any of the equal coil parameters.

Thus we see that in every case where zero moment rotational stabiliby exists, the force on the device will be

$$\underline{F} = - \frac{\pi}{\mu_o} CR^3\nabla\beta^2 \tag{30}$$

where CR^3 is the value associated with the coil most closely aligned to the magnetic field.

Since

$$\underline{\beta} = \mu_o\underline{H} \tag{31}$$

Eq. (30) can be expressed as

$$\underline{F} = - \pi CR^3\mu_o\nabla H^2 \tag{32}$$

Braunbek[7] has shown that the force on a diamagnetic material in a static magnetic field will be (in SI units)

$$\underline{F} = - \frac{\mu_o}{2} (1 - \frac{\mu}{\mu_o})V \ \nabla H^2 \tag{33}$$

where μ is the magnetic permeability and V the volume of the material.

In comparing Eq's. (32) and (33) we note that the term C in Eq. (32) is dimensionless and its value is in the range

$$0 < C < 1 \tag{34}$$

as the amplifier-sensor gain K_n (coil current/mag. field strength) takes any positive value (See Eq. (11)). Note that in Eq. (33) the term $(1 - \mu/\mu_o)$ is dimensionless and its value is in the range

$$0 < (1 - \frac{\mu}{\mu_o}) < 1 \tag{35}$$

for any diamagnetic material. Thus we see that these terms are similar in range and dimension.

Hence

$$(1 - \frac{\mu}{\mu_o}) \to C \tag{36}$$

To complete the comparison between Eqs. (32) and (33), we merely need to set

$$\frac{1}{2} V \to \pi R^3 \tag{37}$$

which is, of course, dimensionally compatible. Note that in this analogy, the relative permeability is related to

$$\frac{\mu}{\mu_o} = (1-C) = \frac{1}{1 + \frac{\mu_o NK}{2R}} \tag{38}$$

In summary, we have shown that a device consisting of an orthogonal set of circular coils, with negative feedback currents proportional to the respective component of magnetic field, will develop forces in that magnetic field identically to a diamagnetic material of relative permeability $1/(1+\mu_o NK/2R)$ and volume $(2\pi R^3)$, where N is the number of turns, R the radius and K the amplifier-sensor gain of the coil most closely aligned to the magnetic field; provided that the device is zero moment-rotationally stable. The conditions for zero moment rotational stability and the related degrees of rotational freedom are shown in Table 1. The additional dynamic behavior of this device, due to currents being induced in the coils by rapid angular motions, is beyond the scope of this report. Possibly, forces due to these induced currents, which may dissipate energy in the coil circuit resistance, may resemble forces due to eddy current losses.

Coil Parameters ($CR^3 = \frac{\mu_o NKR^2}{2+\mu_o NKR}$)	Conditions for Rotational Stability	Degrees of Rotational Freedom
All different or two similar, one smaller	Coils orthogonal to field and the coil with the smallest parameter aligned with the field	1
Two similar, one larger	Coil with the larger parameter aligned normal to field	2
All similar	No alignment conditions required	3

Table 1. Stability Conditions

V. SUSPENSION CONFIGURATIONS

Having developed a device capable of providing a stabilized suspension force*, we can see that it may be utilized in a number of modes. For example, 1) the magnetic field may be fixed and the device suspended, 2) the device fixed and the magnetic field source (magnet) suspended - with or without an additional fixed magnetic field, 3) in either 1) or 2) above some degrees of rotational or linear motion may be restrained by gravitational or electric fields or by other restraints. Generally speaking, each configuration which differs from those in Section IV must be fully analyzed for stability, and these analyses are outside the scope of this report. However some potentially useful configurations will be discussed.

1) Device Suspended

Although a true diamagnet can remain suspended passively in a magnetic field, a pseudo-diamagnetic device requires active components and an energy supply. This energy can be supplied, for example, from a fixed storage (e.g. battery) or from photovoltaic cells (provided a suitable illumination is available). In comparing these two sources we note that if a typical specific mass density for battery (K_b) is 1.25×10^{-5} Kg/watt-sec and for photovoltaic cells (K_e) is 2×10^{-2} Kg/watt, that for suspension times greater than 1,600 seconds, a device with a photovoltaic supply will be lighter.

If we postulate that the weight of the device is due to the energy supply (W_e), the coils (W_c), and the supporting structure (W_s), then we can draw some interesting design conclusions. To simplify this analysis we set

$$
\begin{aligned}
N_x &= N_y = N_z = N && \text{(no. of turns)} \\
R_x &= R_y = R_x = R && \text{(rad. of coils)} \\
C_x &= C_y = C_z = C && \text{(feedback constant)} \qquad (39) \\
K_e &= K_b t = K && \text{(mass density of supply)}
\end{aligned}
$$

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* in a quasi-static sense

It can be shown that the weight of the energy supply will be

$$W_e = \frac{32gC^2R^3\beta^2K\rho}{\mu_o^2D^2N} \tag{40}$$

that the weight of the coils (neglecting insulation) will be

$$W_c = 1.5\ \pi^2 gD^2NR\sigma_c, \tag{41}$$

and that the weight of the structure (assuming that the structure consists of 3 solid disks of radius R) will be approximately

$$W_s = 3\pi gR^2T_s\sigma_s \tag{42}$$

where

g = accel. of gravity

ρ = resistivity of coil material

σ_c= mass density of coil material

σ_s= mass density of coil support material

T_s= thickness of support material

D = diameter of coil wire

t = time of suspension (for battery operation)

The optimum wire diameter for minimum total weight is

$$D' = \sqrt{\frac{8CR\beta}{\pi\mu_o N}}\quad \sqrt[4]{\frac{\rho K}{3\sigma_c}} \tag{43}$$

Thus the total weight with optimum wire diameter is

$$W' = \frac{8\sqrt{3}\ \pi g C R^2 \beta\ \sqrt{K\sigma_c \rho}}{\mu_o} + 3\pi g\ R^2 T_s \sigma_s \qquad (44)$$

For the case where the magnetic field is symmetrical about and aligned to the vertical-down (Z) direction, and a force balance achieved, we have

$$\beta_x = \beta_y = \frac{\partial \beta_z}{\partial x} = \frac{\partial \beta_z}{\partial y} = 0$$

$$\beta = \beta_z$$

$$\underline{F} = -\frac{\pi}{\mu_o} C R^2 \nabla \beta^2 = -\frac{2\pi C R^3 \beta}{\mu_o} \frac{\partial \beta}{\partial z} \hat{k} \qquad (45)$$

$$\underline{F} = -\underline{W}' = -W'\hat{k}$$

Hence

$$\frac{2\pi C R^3 \beta}{\mu_o} \frac{\partial \beta}{\partial z} = W' = \frac{8\sqrt{3}\ \pi g C R^2 \beta\ \sqrt{K\sigma_c \rho}}{\mu_o} + 3\pi g R^2 T_s \sigma_s \qquad (46)$$

which sets the requirement for the coil radius namely

$$R' = (g/\frac{\partial \beta}{\partial z})(4\sqrt{3}\ \sqrt{K\sigma_c \rho} + \frac{3 T_s \sigma_s \mu_o}{2C\beta}) \qquad (47)$$

Under the minimum weight condition, the power requirement is

$$P' = \frac{W'_e}{Kg} = \frac{32 \rho C^2 \beta^2 R'^3}{\mu_o^2\ ND'^2}$$

$$= \frac{12\pi g^2 C\beta\sqrt{\sigma_c \rho}}{\mu_o \sqrt{K} (\frac{\partial \beta}{\partial z})^2} (4\ \sqrt{K\sigma_c \rho} + \frac{\sqrt{3}\ T_s \sigma_s \mu_o}{2C\beta})^2 \qquad (48)$$

to minimize this power requirement, the optimum feedback gain is

$$C'' = \frac{\sqrt{3}\ T_s \sigma_s \mu_o}{8\beta\sqrt{K\sigma_c \rho}} \tag{49}$$

and thus the minimum power level will be

$$P'' = \frac{288\pi g^2 T_s \sigma_s \sigma_c \rho}{(\frac{\partial \beta}{\partial x})^2} \tag{50}$$

Note however that in Eq. (49) the limitation, $C'' \leq 1$, must apply, hence under these conditions the minimum flux density is

$$\beta_{min} = \frac{\sqrt{3}\ T_s \sigma_s \mu_o}{8\sqrt{K\sigma_c \rho}} \tag{51}$$

In summary, under the condition of minimum weight and minimum power consumption, the coil radius requirement is

$$R'' = \frac{8\sqrt{3}\ g\ \sqrt{K\sigma_c \rho}}{\frac{\partial \beta}{\partial z}} \tag{52}$$

and in terms of this radius, the weight of the device is

$$W'' = 6\pi g T_s \sigma_s R''^2 \tag{53}$$

and the wire diameter is

$$D'' = \sqrt{\frac{T_s \sigma_s R''}{\pi N \sigma_c}} \tag{54}$$

(Note that this defines the active cross-sectional area of the coil, namely

$$A''_c = \frac{\pi D''^2}{4}\ N = \frac{T_s \sigma_s R''}{4\sigma_c}\ ) \tag{55}$$

The power consumption is

$$P'' = \frac{3\pi T_s \sigma_s R''^2}{2K} \tag{56}$$

and the feedback parameter (and minimum flux density) are given by

$$C'' = \frac{\beta_{min}}{\beta} = \frac{\sqrt{3}\ T_s \sigma_s \mu_o}{8\beta\sqrt{K\sigma_c \rho}} \leq 1 \tag{57}$$

From Eq. (52) we note that the difference in flux density from the top of the device to the bottom is

$$\Delta\beta_c'' = 2R'' \frac{\partial\beta}{\partial z} = 16\sqrt{3}\ g\sqrt{K\sigma_c \rho} \tag{58}$$

for the minimum weight - minimum power case, and that this flux density difference is independent of the coil size. If we wish to reduce the flux density difference at the expense of power conservation, we note from Eq. (47) and (48) that at best (unlimited power) we can cut the flux density difference (i.e. flux density gradient) in half.*

Selecting aluminum for the coil wire material to reduce the flux density gradient requirement), styrofoam for the structural material, and taking, as typical values:

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* Under the condition of <u>unequal</u> coil parameters (viz Case 2 or 3) it can be shown that the <u>flux density</u> difference can be reduced, without increasing the power consumption; but at the expense of increasing the minimum required flux density. At best (unlimited flux density) the flux density difference can be reduced to $1/\sqrt{3} = 0.577$ of the value required for the equal parameter case (i.e. Eq. (58)). For the condition of unlimited flux density (with unequal coil parameters) <u>and</u> unlimited power, the flux density difference can, at most, be <u>reduced</u> to $1/2\sqrt{3} = 0.289$ of the value given by Eq. (58).

$R'' = 10^{-2}$ m

$g = 10$ m/s

$\rho = 2.8 \times 10^{-8} \Omega$-m

$\sigma_c = 2.7 \times 10^3$ kg/m^3

$\sigma_s = 10^2$kg/m^2

$T_s = 10^{-3}$m

$K = 2 \times 10^{-2}$ kg/watt

We have

$A_c'' = 9.3 \times 10^{-8}$m (approx. #28 AWG, N = 1)

$W'' = 1.9 \times 10^{-3}$new (0.19 grams)

$P'' = 2.4 \times 10^{-3}$ watt

$\partial\beta/\partial z = 17.0$ tesla/m

$\beta_{min} = 2.2 \times 10^{-5}$ tesla

$\Delta\beta_c'' = 0.34$ tesla

Under the stated conditions, we note that these values indicate that the most stringent condition appears to be the flux density gradient, (i.e. the flux density at the bottom of the device must be 0.34 tesla (3.4 kilogauss) more than at the top).

2) Device Fixed-Magnet Suspended

Since a force balance must exist between the stabilized device and the magnetic field sources, we can consider either one fixed and the

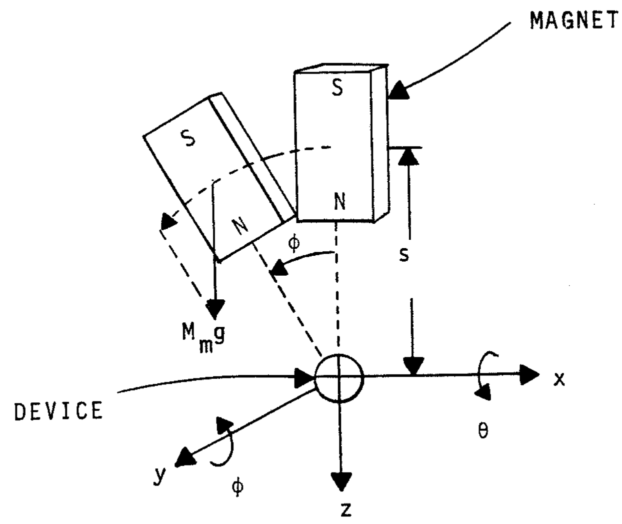

Figure 3a Suspended Magnet

other supported. If we have a fixed magnet stably supporting a device in a gravity field, and we invert the orientations, then the magnet will be stably supported; provided that (refer to the configuration in Fig. 3a)

$$\frac{2\pi C_z R_z^2 \beta_z}{\mu_o} \frac{\partial \beta_z}{\partial z} = M_m g \tag{59}$$

and

$$\phi \frac{\partial M_y}{\partial \phi} s > M_m g \phi$$

$$\theta \frac{\partial M_x}{\partial \theta} s > M_m g \theta \tag{60}$$

where

M_m = magnet mass

s = distance from magnet center of mass to device center.

Utilizing the analysis in Appendix A (viz. Eq. (A12)), Eq. (60) may be restated as

$$C_z R_z^3 - C_y R_y^3 > \frac{M_m g \mu_o}{2\pi s} \tag{61}$$

$$C_z R_z^3 - C_x R_x^3 > \frac{M_m g \mu_o}{2\pi s}$$

Note that the configuration in Fig. 3a has one degree of rotational freedom.

To avoid the restriction of a vertical orientation, we may use two devices, as shown in Fig. 3b. This arrangement also provides the magnet with one degree of rotational freedom

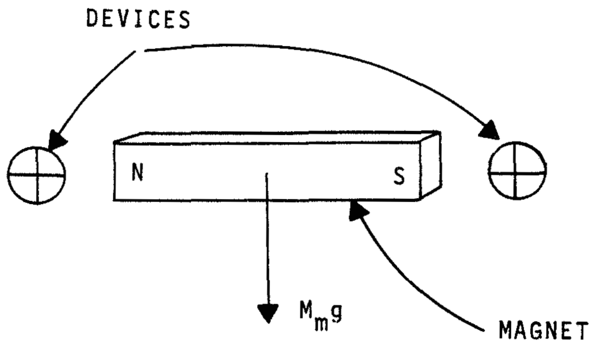

Figure 3b Suspended Magnet

To suspend a magnet and also provide some linear motion freedom, one may provide a planar array of devices as shown in Fig. 3c. In this arrangement

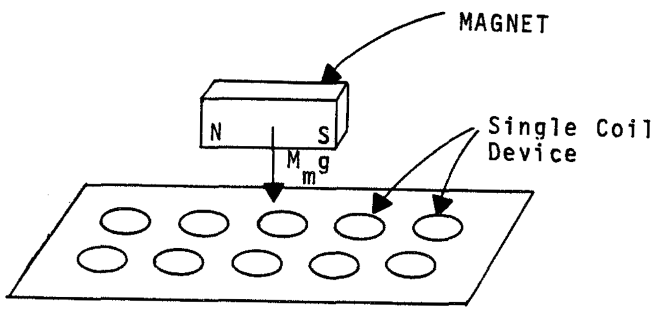

Figure 3c Suspended Magnet

each device requires only the coil which is parallel to the plane. This multidevice configuration is analogous to a superconducting sheet.

In addition to suspension configurations utilizing magnet and device alone, arrangements consisting of a fixed device, suspended magnet and fixed magnet are of considerable interest. For example, the arrangement shown in Fig. 3a can be improved with the addition of an upper fixed magnet to provide some of the suspension force. (This, of course, has an unstabilizing effect in the vertical direction that the device must compensate.) In fact, we note that the upper magnet and the gravity field can provide all the required suspension force and also provide stabilizing forces in all degrees of freedom except for rotation about the vertical (which is neutrally stable) and except for stabilizing forces for vertical motion. Thus in order to stably support the lower magnet, the only function required of the device is to provide a stabilizing force in the vertical direction. To do this, we require only one coil component of the device, as shown in Fig. 3d. Since no supporting force is required of the device, the average current through the coil (in the absence of disturbing forces) can be reduced to zero by summing in a bias voltage (V_B) to the amplifier. Thus the power consumption in the coil is related to only the disturbance forces, and in their absence, the power consumption of the device is only due to the quiescent operation of the electronics.

This configuration is directly analogous to that used by Boerdijk[8] in 1956 when he levitated a small cylindrical magnet with a diameter of 10^{-3}m and 0.3×10^{-5}m thick.

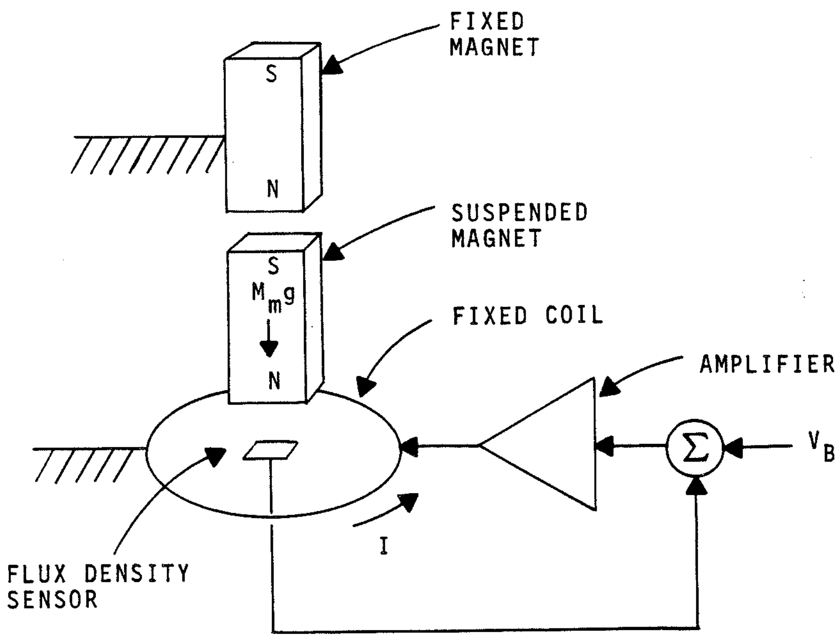

Figure 3d Suspended Magnet

VI. EXPERIMENTAL RESULTS

During August 1970, the suspension configuration shown in Fig. 3d was built and tried with components selected from "equipment of opportunity" in our laboratory. Of particular interest, the flux sensor was a Bell, Inc. Hall Sensor Model BH700; the fixed magnet was a cylinder 2.54×10^{-2}m. diameter, 10^{-1}m. length, the suspended magnet was a cylinder 1.3×10^{-2} m. diameter, 3.9×10^{-3} m. length, 8.36×10^{-2} kg weight, both Alnico V magnets; the coil had an inside diameter of 1.9×10^{-2}m., 10^4 turns, with a resistance of 40 Ω. The amplifier-sensor gain was 10 amp/tesla. The initial operation provided suspension - and also a dynamic oscillation of the suspended magnet. This dynamic oscillation was easily corrected by the addition of a velocity sensing coil (of 200 turns) placed around the fixed magnet and appropriately summed into the amplifier. The resulting operation was entirely satisfactory and was achieved with a minimum of difficulty.

In particular, we were able to reduce the coil current to less than 3 ma. in our laboratory bench environment. In other words, we were providing stable suspension at the rate of 232 Kg/watt, (exclusive of sensor-amplifier quiescent power). Additionally, we determined the vertical stiffness to be 3.68 new/amp.

VII. ALTERNATIVE DESIGNS

a. Super Pseudo-Diamagnets

It is interesting to note that the actual flux density at the center of the coil is given by (See Eqs. (8), (9), and (10))

$$\beta_{tn} = \frac{\beta_n}{1 + \frac{\mu_o N K_n}{2R_n}} \tag{62}$$

For very large values of amplifier-sensor gain (K_n), this flux goes to zero, and the analogous relative permeability (Eq. (38)) also goes to zero (and C increases to one). In a sense, this simulates a perfect diamagnet.

If this flux density could be driven further and made negative, then our analogous relative permeability would also be negative and C would increase beyond one. From Eq. (34) we see that the force on the device could then be increased beyond that of the analogous perfect diamagnet.

Since feedback devices can do no more than drive the sensor output to null, it is clear that the sensor location must be shifted to a location such that the flux density due to the coil current is less than that at the center (but of the same polarity). If the new location is along a principle axis of the device, and normal to the coil then cross-coupling with the flux from the other coils will be avoided.

Let us define the flux density ratio as the flux density at the new location compared to the flux density at the center, (both due to the coil only)

$$k_\ell \equiv \beta_\ell / \beta_c \tag{63}$$

If we modify the analysis of Section IV by including Eq. (63), it can be shown that the feedback gain will be redefined as

$$C_{n\ell} = \frac{1}{k_\ell + \frac{2R_n}{\mu_o N_n K_n}} \tag{64}$$

and the force on the device will be increased by

$$\frac{F_\ell}{F} = \frac{1 + \dfrac{2R_n}{\mu_o N_n K_n}}{k_\ell + \dfrac{2R_n}{\mu_o N_n K}} = \frac{1 + \dfrac{\mu_o N_n K_n}{2R_n}}{1 + \dfrac{k_\ell \mu_o N_n K_n}{2R_n}} \tag{65}$$

Along the axis normal to the coil, the flux density ratio is given by (See for example Ref. (16))

$$k_{\ell n} = \frac{1}{[1 + (\frac{\ell}{R})^2]^{3/2}} \tag{66}$$

which is plotted in Figure 4, together with the flux density ratio along the radius of a one-turn coil ($k_{\ell r}$). At a distance from the center, $\ell = R$, we note than $k_{\ell n} = 0.35$ while $k_{\ell r}$ is nowhere (outside of the wire) positive and less than one.

Hence, with one-turn coils and with the sensor location restricted to the interior of the device, the best location for the sensors would be not at the center, but displaced out a distance R. (Using the sum of the outputs of two sensors for each coil, one sensor on each side, would eliminate the first order effects of external field flux-density gradients.) For very large amplifier-sensor gains, the force would be increased by

$$F_\ell = \sqrt{8}\ F = 2.83F \tag{67a}$$

and the total flux density in the center of the coil would be opposite in direction to the applied external field. The magnitude of this flux density would be

$$|\beta_{tn}| = [\sqrt{8} - 1]\beta_n = 1.83\beta_n \tag{67b}$$

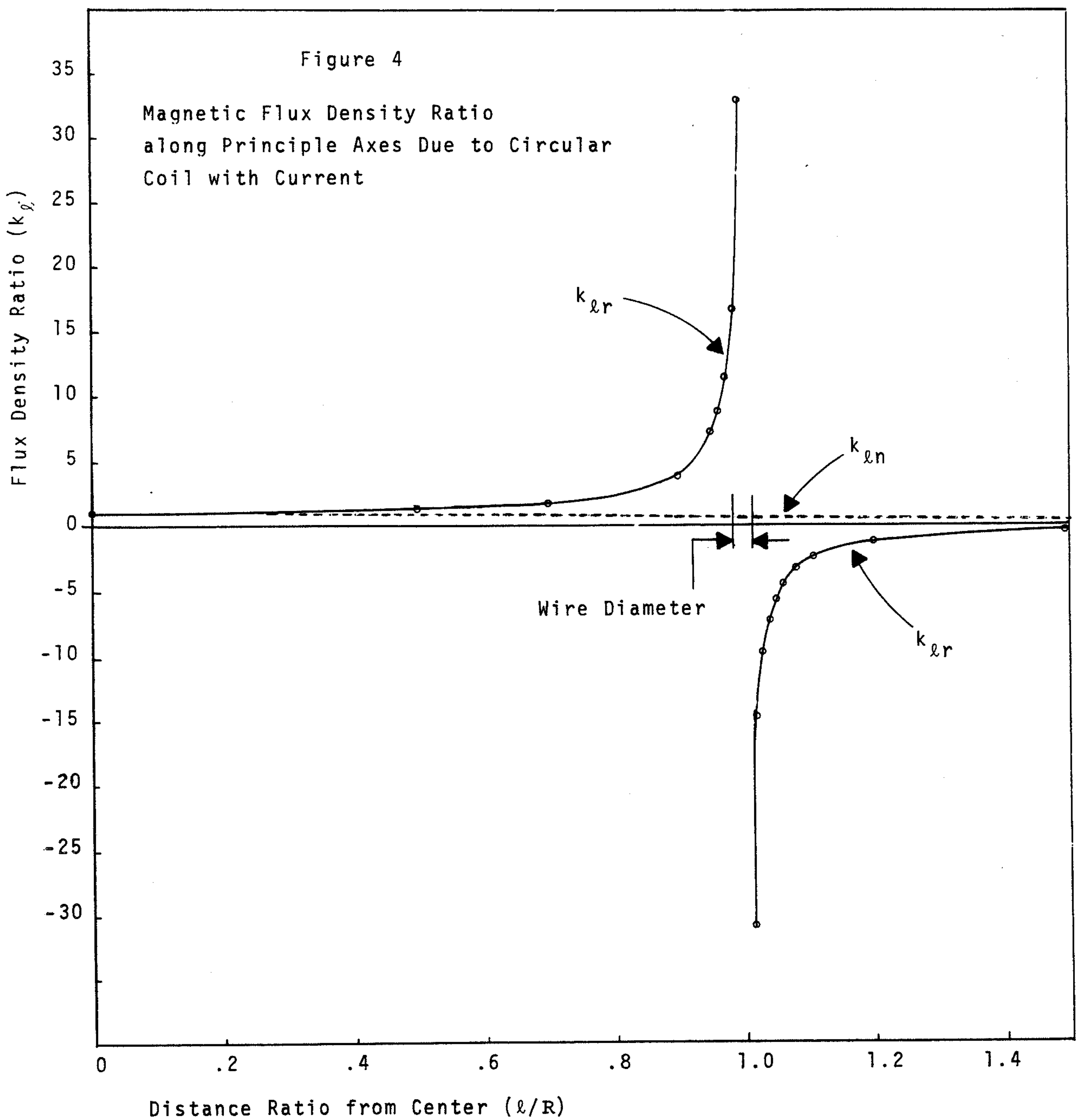
Figure 4
Magnetic Flux Density Ratio
along Principle Axes Due to Circular
Coil with Current
Flux Density Ratio (k_ℓ)
35
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
$k_{\ell r}$
$k_{\ell n}$
Wire Diameter
$k_{\ell r}$
0
.2
.4
.6
.8
1.0
1.2
1.4
Distance Ratio from Center (ℓ/R)

Even smaller values of k_ℓ may be achieved with multiturn coils. For example, Fig. 5 shows the relative flux density along the radius of a coil ($k_{\ell r}$) consisting of two concentric turns (with radii 1.0 R and 1.1 R and wire of diameter .02 R). The place of interest is between the coils. This section is enlarged in Fig. 6. ($k_{\ell n}$ is also plotted for comparison). We note that in a small region approximating midway between the turns, the relative flux density is positive <u>and</u> less than one and also less than the corresponding value along the normal axis ($k_{\ell n}$). Thus, Fig. 6 shows that we can locate a region (in a two-turn coil) where we can select k_ℓ to be as small as we wish. Hence the force on the device can be arbitrarily large. However, it is apparent that the motional stability of a sensor placed here will be very critical since the value of k_ℓ is very sensitive to radial position. It must be noted, also, that for the design developed in Section V, (see Eqs. (52) through (58)), the only apparent advantage of this "super pseudo-diamagnet" is to reduce the minimum flux density required to achieve the optimum conditions.

b. Pseudo Diadielectrics

As noted in Section II, Braunbek showed that stable suspensions can be achieved in properly shaped electric fields with materials that have a relative permittivity less than one. (here called "diadielectric" materials). Since no known materials have this property, this conclusion had no practical merit. In light of the approach used with magnetic fields in this report, the question arises as to whether diadielectric behavior can be simulated by electric fields. This question can be answered by considering a device whose design is suggested by the magnetic field approach.

As we contemplate an electric analog to the magnetic case, we are immediately confronted with a difficult problem. Whereas we are able to sense and generate magnetic fields with materials (i.e. conductors) that themselves affect the magnetic fields very little ($\mu \simeq \mu_o$) the materials that we must use to control large amounts of charge (again conductors) affect the electric fields severely ($\varepsilon \approx \infty$). Since we are attempting to design a device that behaves as though $\varepsilon = 0$ and we must use materials with $\varepsilon = \infty$, we must proceed carefully.

It may be instructive to review some pertinent relations in static electromagnetic theory (see for example Ref. 16)

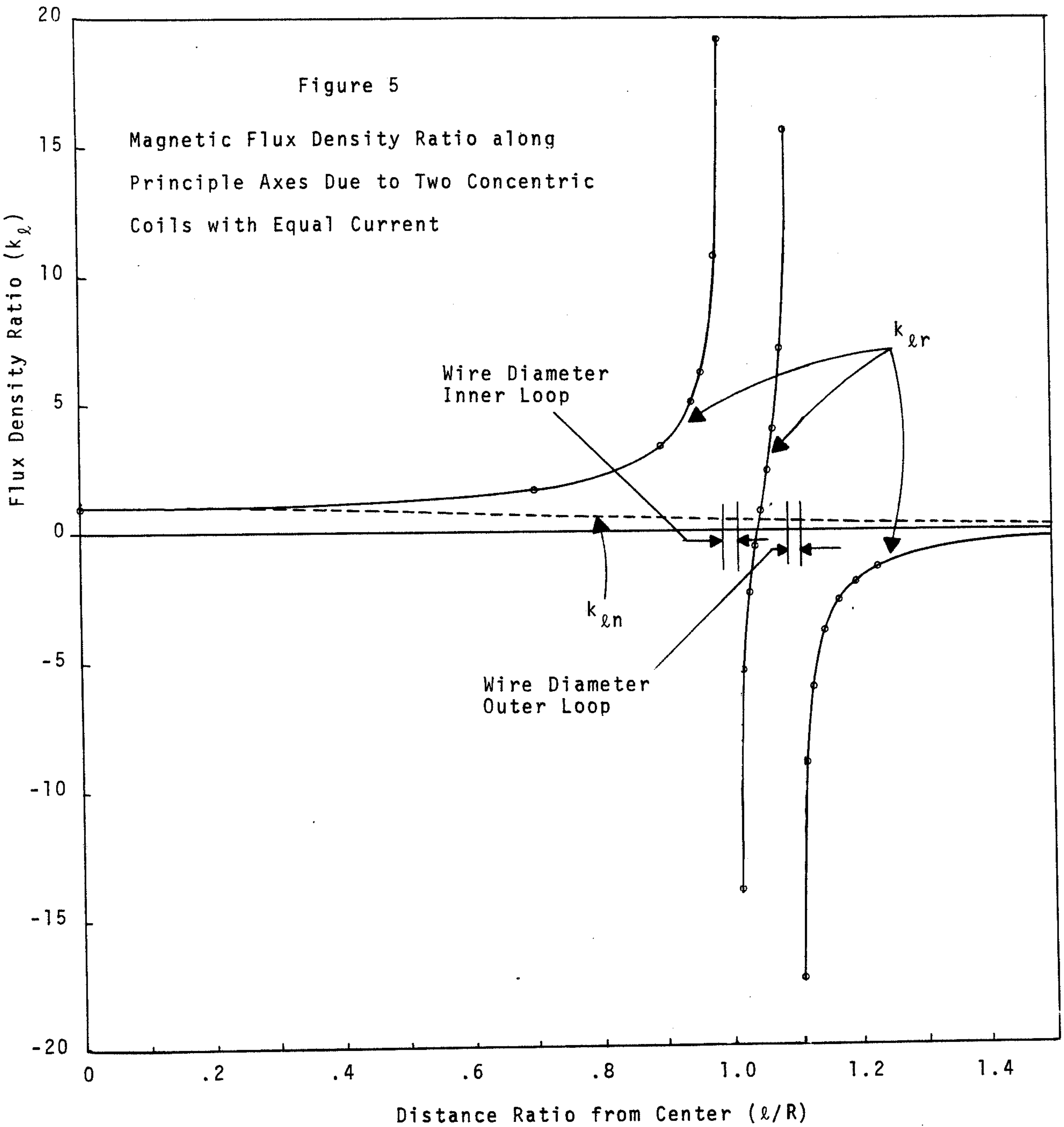
Figure 5
Magnetic Flux Density Ratio along
Principle Axes Due to Two Concentric
Coils with Equal Current
Flux Density Ratio (k_ℓ)
$k_{\ell r}$
Wire Diameter
Inner Loop
$k_{\ell n}$
Wire Diameter
Outer Loop
Distance Ratio from Center (ℓ/R)
20
15
10
5
0
-5
-10
-15
-20
0
.2
.4
.6
.8
1.0
1.2
1.4

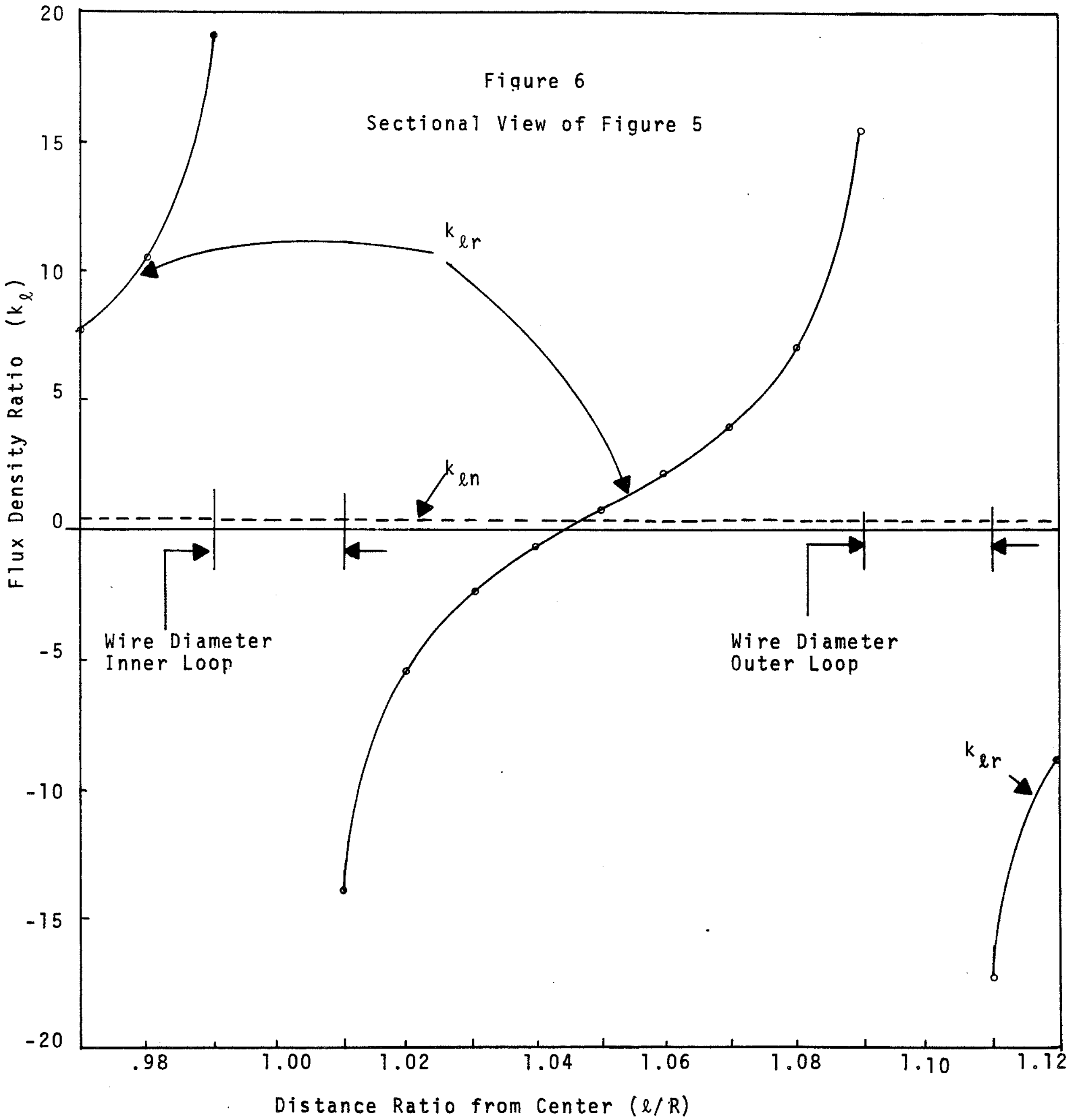
Figure 6
Sectional View of Figure 5
20
15
10
5
0
-5
-10
-15
-20
Flux Density Ratio (k_ℓ)
$k_{\ell r}$
$k_{\ell n}$
Wire Diameter Inner Loop
Wire Diameter Outer Loop
$k_{\ell r}$
.98
1.00
1.02
1.04
1.06
1.08
1.10
1.12
Distance Ratio from Center (ℓ/R)

$$\underline{B} = \mu_o\underline{H} + \underline{M} = \mu_o\underline{H} + \chi_m\mu_o\underline{H} = (1+\chi_m)\mu_o\underline{H} = \mu\underline{H} \qquad (68)$$

$$\underline{D} = \varepsilon_o\underline{E} + \underline{P} = \varepsilon_o\underline{E} + \chi_e\varepsilon_o\underline{E} = (1+\chi_e)\varepsilon_o\underline{E} = \varepsilon\underline{E} \qquad (69)$$

$$U_m = \frac{1}{2}\int \mu H^2 dv \qquad (70)$$

$$U_e = \frac{1}{2}\int \varepsilon E^2 dv \qquad (71)$$

$$\underline{F} = q(\underline{E} + \underline{V} \times \underline{\beta}) \qquad (72)$$

Although the Maxwell equations for the general case are

$$\nabla\cdot\underline{\beta} = 0 \qquad (73)$$

$$\nabla\cdot\underline{D} = \rho \qquad (74)$$

$$\nabla\times\underline{E} = -\frac{\partial\underline{\beta}}{\partial t} \qquad (75)$$

$$\nabla\times\underline{H} = \underline{J} + \frac{\partial\underline{\beta}}{\partial t} \qquad (76)$$

if we are dealing with fields that are time independent and we are not in regions with real charge and conduction current, these equations become

$$\nabla\cdot\underline{\beta} = 0 \qquad (77)$$

$$\nabla\cdot\underline{D} = 0 \qquad (78)$$

$$\nabla\times\underline{H} = 0 \qquad (79)$$

$$\nabla\times\underline{E} = 0 \qquad (80)$$

A conclusion of Eq. (77) is that the normal component of β is continuous and a conclusion of Eq. (78) is that the normal component of D is continuous. A conclusion of Eq. (79) is that the tangential component of H is continuous and a conclusion of Eq. (80) is that the tangential component of E is continuous .

Thus we see that β (magnetic flux density) is analogous to D (electric flux density) and H (magnetic field strength) is analogous to E (electric field strength). If the pseudo-diadielectric design were to duplicate the pseudo-diamagnetic design, we would measure D in a region and control real charges such as to null the measurement. However from Eq. (72) we note that whereas we can measure β (and not H) we cannot measure D (but rather E). Although D and E are usually linearly related, this is not true within a conductor, where $\varepsilon = \infty$.* Thus we are led to the conclusion that the pseudo diadielectric sensor (measuring E)must not be shielded by the conducting surfaces which will support the control charges.

Noting from Eq. (78) that the normal component of D is continuous in regions with no real charge, we conclude that the sensor (which measures E) for the diadielectric design should be located normal to and in the <u>free</u> space just <u>outside</u> of a conducting surface. Now, if the real charge on that surface is adjusted to null the output of that sensor, then D in that region will be nulled in the same fashion as it would be nulled with a material of zero permittivity. The design implied by this reasoning is shown in Fig. 7. Each axis consists of a pair of parallel plates driven by a voltage that is proportional to the sum of the sensor output. The sensors provide a voltage proportional to the normal component of E at the outside surface of the plates.

If the sensors are displaced away from the plates, they are less sensitive to the applied field. Hence the total electric field near the outside surface of the plates can be driven to the opposite polarity of the applied field. This would provide us with a "super pseudo-diadielectric" condition analogous to the "super pseudo-diamagnet" condition of Part a of this Section.

The detailed analysis of the design of the pseudo-diadielectric device outlined here is given in Appendix B. The analysis confirms the analogy to the magnetic field design.

It is interesting to refer the conclusions of this electric field case back to the magnetic field device. To have a truly analogous situation we would require, of the magnetic design, the inclusion of a (weightless) sphere of high permeability in the center of the coils. (The β sensors would be orthogonally paired on the outer surface of the sphere). Following the line of reasoning presented for the electric

* This statement is made in the sense that, in a conductor, although $E = 0$, D may have a non-zero value.

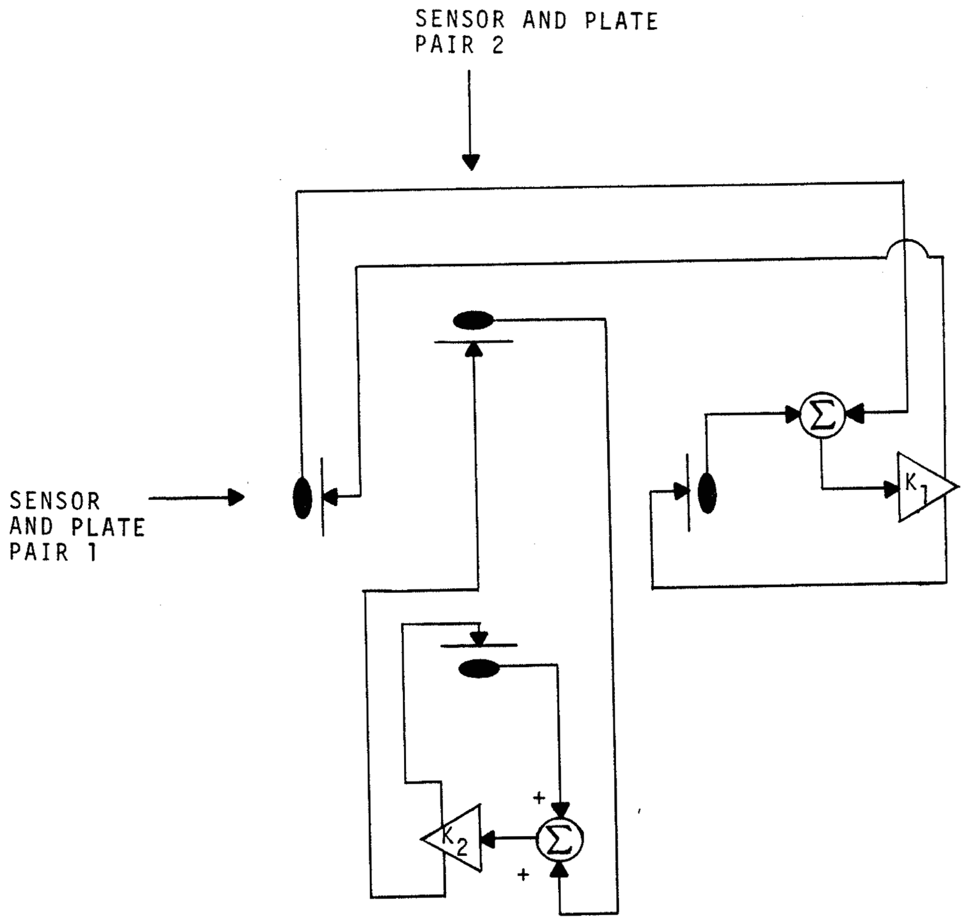

Figure 7 Pseudo-Diadielectric Configuration

field design, we conclude that the inclusion of the high permeability sphere would have no first order effect on the performance of the pseudo-diamagnetic device, although a second order effect is due to the fact that the device-generated magnetic field internal to the coil is not uniform.

APPENDIX A

Rotational Stability

A device at orientation $\underline{\alpha}$ will be in rotational stability if its moment $\underline{M}(\underline{\alpha})$ is zero and, if when it is disturbed by arbitrary infinitesimal rotation, $\underline{\delta}$,

$$\underline{\delta}\cdot\underline{M}(\underline{\alpha}+\underline{\delta}) < 0 \qquad \text{(A1)}$$

This states that the torque at the displaced orientation tends to return the system to the orientation $\underline{\alpha}$. Since

$$\underline{\delta}\cdot\underline{M}(\underline{\alpha}+\underline{\delta}) = \underline{\delta}\cdot\underline{M}(\underline{\alpha})+\underline{\delta}\cdot\left|\frac{\partial\underline{M}}{\partial\underline{\alpha}}\right|\underline{\delta} \qquad \text{(A2)}$$

$$= \underline{\delta}\cdot\underline{M}(\underline{\alpha})+\underline{\delta}^T\left|\frac{\partial\underline{M}}{\partial\underline{\alpha}}\right|\underline{\delta}$$

we require for stability that

$$\underline{\delta}^T\left|\frac{\partial\underline{M}}{\partial\underline{\alpha}}\right|\underline{\delta} < 0 \qquad \text{(A3)}$$

which will be the case if $\left|\frac{\partial\underline{M}}{\partial\underline{\alpha}}\right|$ is negative definite. Since (See Fig. A1)

$$\left|\frac{\partial\underline{M}}{\partial\underline{\alpha}}\right| = \begin{vmatrix} \frac{\partial M_x}{\partial\theta} & \frac{\partial M_x}{\partial\phi} & \frac{\partial M_x}{\partial\xi} \\ \frac{\partial M_y}{\partial\theta} & \frac{\partial M_y}{\partial\phi} & \frac{\partial M_y}{\partial\xi} \\ \frac{\partial M_z}{\partial\theta} & \frac{\partial M_z}{\partial\phi} & \frac{\partial M_z}{\partial\xi} \end{vmatrix}$$

$$\equiv \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} \qquad \text{(A4)}$$

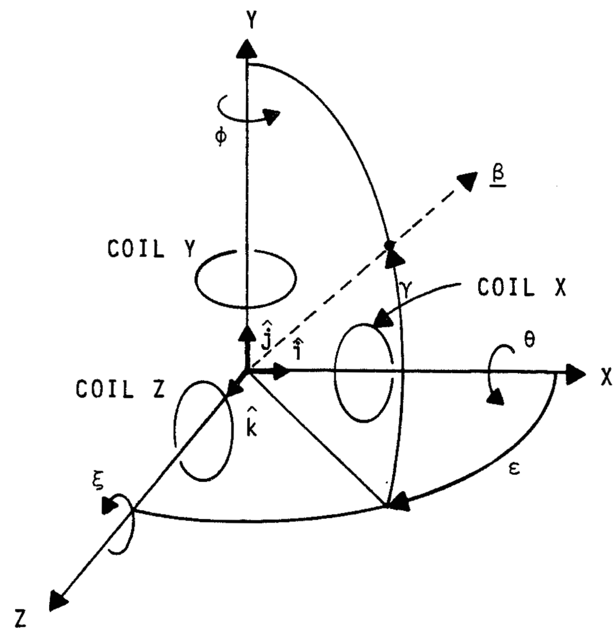

Figure A1 Coordinate System

and the criteria for a negative definite matrix is given in terms of a symmetric matrix where

$$a_{11} = b_{11}, \qquad a_{22} = b_{22}, \qquad a_{33} = b_{33}$$

$$a_{12} = \frac{b_{12}+b_{21}}{2}; \; a_{13} = \frac{b_{13}+b_{31}}{2}; \; a_{23} = \frac{b_{23}+b_{32}}{2} \qquad \text{(A5)}$$

then $\left|\frac{\partial \underline{M}}{\partial \underline{\alpha}}\right|$ is negative definite if and only if[17]

$$a_{11} + a_{22} + a_{33} < 0$$

$$(a_{11}a_{22}-a_{12}^2)+(a_{22}a_{33}-a_{23}^2)+(a_{11}a_{33}-a_{13}^2) > 0 \qquad \text{(A6)}$$

$$|a_{ij}| < 0$$

where $|a_{ij}|$ is the determinant of Eq. (A4).

If any of the conditions are identically zero, then the matrix is semidefinite and the device will have neutrally stable modes.

Defining coil parameters as

$$\frac{2\pi}{\mu_o} C_x R_x^3 = X$$

$$\frac{2\pi}{\mu_o} C_y R_y^3 = Y$$

$$\frac{2\pi}{\mu_o} C_z R_z^3 = Z \qquad \text{(A7)}$$

$$Z - Y = a$$

$$X - Z = b$$

$$Y - X = c$$

Eq. (19) can be rewritten as

$$M_x = a\beta_{yo}\beta_{zo}$$

$$M_y = b\beta_{xo}\beta_{zo} \qquad \text{(A8)}$$

$$M_z = c\beta_{xo}\beta_{yo}$$

If the flux density is oriented as shown in Fig. A1, we see that

$$\beta_{xo} = \beta \cos \varepsilon \cos \gamma$$

$$\beta_{yo} = \beta \sin \gamma$$

$$\beta_{zo} = \beta \sin \varepsilon \cos \gamma \qquad \text{(A9)}$$

Under small angle rotation (θ,ϕ,ξ), the field components become

$$\beta_x = \beta_{xo} + \xi\beta_{yo} - \phi\beta_{zo}$$

$$\beta_y = -\xi\beta_{xo} + \beta_{yo} + \theta\beta_{zo} \qquad \text{(A10)}$$

$$\beta_z = \xi\beta_{xo} - \theta\beta_{yo} + \beta_{zo}$$

and hence

$$\frac{\partial\beta_x}{\partial\theta} = 0 \qquad \frac{\partial\beta_y}{\partial\theta} = \beta_{zo} \qquad \frac{\partial\beta_z}{\partial\theta} = -\beta_{yo}$$

$$\frac{\partial\beta_x}{\partial\phi} = -\beta_{zo} \qquad \frac{\partial\beta_y}{\partial\phi} = 0 \qquad \frac{\partial\beta_z}{\partial\phi} = \beta_{xo} \qquad \text{(A11)}$$

$$\frac{\partial\beta_x}{\partial\xi} = \beta_{yo} \qquad \frac{\partial\beta_y}{\partial\xi} = -\beta_{xo} \qquad \frac{\partial\beta_z}{\partial\xi} = 0$$

Therefore we have

$$a_{11} = \frac{\partial M_x}{\partial \theta} = a\beta^2(\sin^2\varepsilon \cos^2\gamma - \sin^2\gamma)$$

$$a_{12} = \frac{1}{4}(a-b)\beta^2 \cos\varepsilon \sin 2\gamma$$

$$a_{13} = \frac{1}{4}(c-a)\beta^2 \sin 2\varepsilon \cos^2\gamma$$

$$a_{23} = \frac{1}{4}(b-c)\beta^2 \sin\varepsilon \sin 2\gamma \qquad \text{(A12)}$$

$$a_{22} = b\beta^2 \cos 2\varepsilon \cos^2\gamma$$

$$a_{33} = c\beta^2(\sin^2\gamma - \cos^2\varepsilon \cos^2\gamma)$$

<u>Case 1 (One of the coils is oriented for maximum flux density)</u>

Case 1 conditions can be met by having (for example)

$$\varepsilon = \gamma = 0 \qquad \text{(A13)}$$

Hence

$$\underline{M}_1 = \beta_{yo} = \beta_{zo} = a_{11} = a_{12} = a_{13} = a_{23} = 0 \qquad \text{(A14)}$$

and

$$a_{11} + a_{22} + a_{33} = \beta_1^2(b-c) \qquad \text{(A15)}$$

$$a_{11}a_{22} - a_{12}^2 + a_{22}a_{33} - a_{23}^2 + a_{11}a_{33} - a_{12}^2 = -bc\beta_1^2 \qquad \text{(A16)}$$

$$|a_{ij}| = 0 \qquad \text{(A17)}$$

Taking Eq. (A16) first and expressed in terms of coil parameters, this stability requirement is that

$$-bc = (\underline{X}-\underline{Z})(\underline{X}-\underline{Y}) > 0 \qquad \text{(A18)}$$

In other words the coil parameter X must be either the largest or the smallest. Eq. (A15) requires that

$$b - c = (X-Z) - (Y-X) = 2X - (X+Y) < 0 \tag{A19}$$

and we see that the X coil parameter must be less than the average of the other two. These conditions can be met if and only if the X coil parameter (which is aligned to the field) is the smallest of the three, i.e.

$$Y > X < Z \tag{A20}$$

As expected, these results also apply if the field is aligned with either of the other coils (with an appropriate change in coil label). Note that it is immaterial whether the zero flux coils are equal or not.

Case 2 (Two of the coil parameters are equal and the other is oriented for zero flux density)

Case 2 conditions can be met by having (for example)

$$C_x R_x^3 = C_y R_y^3 \equiv V \tag{A21}$$

$$\varepsilon = 0 \tag{A22}$$

Hence

$$\underline{M}_2 = c = \beta_{zo} = a_{13} = a_{23} = a_{33} = 0 \tag{A23}$$

$$a = -b = Z - V \tag{A24}$$

and

$$a_{11} + a_{22} + a_{33} = -a\beta_2^2 \sin^2\gamma + b\beta_2^2 \cos^2\gamma \tag{A25}$$

$$= -a\beta_2^2$$

$$a_{11}a_{22} - a_{12}^2 + a_{22}a_{33} - a_{23}^2 + a_{11}a_{33} - a_{13}^2 = 0 \tag{A26}$$

$$|a_{ij}| = 0 \tag{A27}$$

thus the only requirement for stability is

$$a > 0 \tag{A28}$$

Since

$$a = Z - V = Z - X = Z - Y \tag{A29}$$

the condition for stability is that

$$Y < Z > X \tag{A30}$$

or that the Z coil parameter be larger than the other two. Note that this result does not conflict with Case 1.

Case 3 (Equal coil parameters)

If all coil parameters are equal namely

$$C_x R_x^3 = C_y R_y^3 = C_z R_z^3 \tag{A31}$$

we see that

$$a = b = c = \underline{M}_3 = \left| \frac{\partial \underline{M}_3}{\partial \underline{\alpha}} \right| = 0 \tag{A32}$$

and the device is neutrally stable.

Stability Summary

The device will be rotationally stable if and only if one of the following conditions are met:

1) the smallest coil parameter is aligned with the magnetic field (1 degree of rotational freedom)

or 2) the largest coil parameter is normal to the magnetic field and the other two coil parameters are equal (2 degrees of rotational freedom).

or 3) all three coil parameters are equal (3 degrees of rotational freedom).

APPENDIX B

Pseudo-Diadielectric Analysis

Let us calculate the moments and forces on an orthogonal set of charged dipoles as shown in Fig. B1.

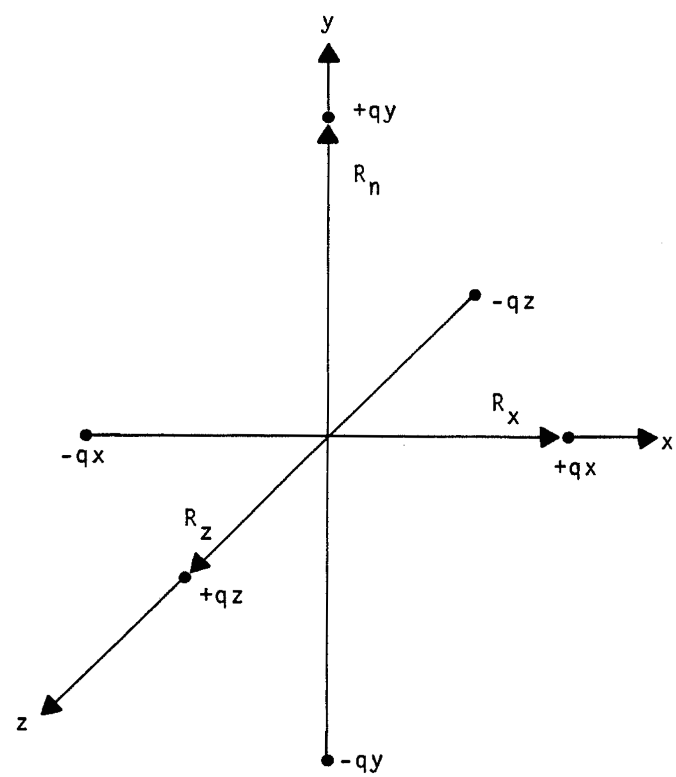

Figure B1 Orthogonal Dipole Configurations

The force ($\underline{E}$) on a charge (q) in terms of the field at the center ($\underline{E}_o$) and the first order gradients $|\partial\underline{E}/\partial\underline{\alpha}|$ will be

$$\underline{F}(\underline{\alpha}+\underline{\delta}) = q\underline{E}(\underline{\alpha}+\underline{\delta}) = q[\underline{E}_o + \left|\frac{\partial \underline{E}}{\partial \underline{\alpha}}\right| \underline{\delta}] \tag{B1}$$

where

$$\frac{\partial \underline{E}}{\partial \underline{\alpha}} = \begin{vmatrix} \frac{\partial E_x}{\partial x} & \frac{\partial E_x}{\partial y} & \frac{\partial E_x}{\partial z} \\ \frac{\partial E_y}{\partial x} & \frac{\partial E_y}{\partial y} & \frac{\partial E_y}{\partial z} \\ \frac{\partial E_z}{\partial x} & \frac{\partial E_z}{\partial y} & \frac{\partial E_z}{\partial z} \end{vmatrix} \equiv [\underline{E}] \qquad \text{(B2)}$$

Since the dipoles are aligned to the axes, we have

$$q = \pm q_n$$

$$\underline{\delta}_n = \pm R_n \hat{n} \qquad \text{(B3)}$$

The vector moment and force on any dipole will be

$$\underline{M}_n = R_n(\hat{n} \times \underline{F}^+ + \hat{n} \times \underline{F}^-)$$

$$= q_n R_n \hat{n} \times [(\underline{E}_o + R_n[\underline{E}]\hat{n}) + (\underline{E}_o - R_n[\underline{E}]\hat{n})] \qquad \text{(B4)}$$

$$= 2q_n R_n(\hat{n} \times \underline{E}_o)$$

$$\underline{F}_n = \underline{F}^+ + \underline{F}^-$$

$$= q_n(\underline{E}_o + R_n[\underline{E}]\hat{n}) - q_n(\underline{E}_o - R_n[\underline{E}]\hat{n}) \qquad \text{(B5)}$$

$$= 2q_n R_n[\underline{E}]\hat{n}$$

If we place pairs of electric field sensors on either side of the dipole charge at distance X (X > R) from the center of the dipoles, and aligned to dipole axis, the summed output of a pair of sensors will be

$$V_{sn} = K_{sn}[\underline{E}_s(+X)\cdot\hat{n} + \underline{E}_s(-X)\cdot\hat{n}] \equiv 2K_{sn}E_{tn} \qquad \text{(B6)}$$

$$\underline{E}_s(+X)\cdot\hat{n} = (\underline{E}_0+X[\underline{E}]\hat{n})\cdot\hat{n} + E_d \qquad \text{(B7)}$$

where $\underline{E}_s$ is the total electric field at the sensor, K_{sn} is the sensitivity of the sensors, E_{tn} is the <u>average</u> electric field component aligned with the dipole axis, and E_d is the electric field component at the sensor due to the dipole itself.

Since*

$$E_d = \frac{q_n}{4\pi\varepsilon_o}\left[\frac{1}{(X-R_n)^2} - \frac{1}{(X+R_n)^2}\right]$$

$$= \frac{q_n}{4\pi\varepsilon_o R_n^2}\left[\frac{1}{(X/R_n-1)^2} - \frac{1}{(X/R_n+1)^2}\right] \qquad \text{(B8)}$$

and defining

$$K_p \equiv \left[\frac{1}{(X/R_n-1)^2} - \frac{1}{(X/R_n+1)^2}\right] \qquad \text{(B9)}$$

we have

$$2E_{tn} = (\underline{E}_o + X[\underline{E}]\hat{n} + \underline{E}_o - X[\underline{E}]\hat{n})\cdot\hat{n} + \frac{2q_nK_p}{4\pi\varepsilon_o R_n^2} \qquad \text{(B10)}$$

$$E_{tn} = E_n + \frac{K_p}{4\pi\varepsilon_o R_n^2}\, q_n \qquad \text{(B11)}$$

If we servo the charge on the dipoles to be proportional to the output of the appropriate sensor pair we have

$$q_n = -K_{an}V_{sn} = -2K_{an}K_{sn}E_{tn}$$

$$= -\frac{2K_{an}K_{sn}}{1 + \frac{K_nK_p}{4\pi\varepsilon_o R_n^2}}\, E_n \qquad \text{(B12)}$$

* Note that there is no cross-coupling of the electric field of one dipole to the sensor of another

Defining*

$$C_n \equiv \frac{1}{K_p + \frac{4\pi\varepsilon_o R_n^2}{K_n}} \tag{B13}$$

$$K_n \equiv 2K_{an}K_{sn}$$

we have

$$q_n = -4\pi\varepsilon_o C_n R_n^2 E_n \tag{B14}$$

Also note that in free space

$$\underline{D} = \varepsilon_o \underline{E} \tag{B15}$$

Substituting Eqs. (B14) and (B15) into Eqs. (B4) and (B5) we have

$$\underline{M}_n = -\frac{8\pi}{\varepsilon_o} C_n R_n^3 D_n (\hat{n} \times \underline{D}_o) \tag{B16}$$

$$\underline{F}_n = -\frac{8\pi}{\varepsilon_o} C_n R_n^3 D_n [\underline{D}]\hat{n} \tag{B17}$$

Expanded, Eqs. (B16) and (B17) are

$$\begin{aligned}
\underline{M}_x &= \frac{8\pi C_x R_x^3}{\varepsilon_o} D_{xo} [\qquad\qquad D_{zo}\hat{j} - D_{yo}\hat{k}\,] \\
\underline{M}_y &= \frac{8\pi C_y R_y^3}{\varepsilon_o} D_{yo} [-D_{zo}\hat{i} \qquad\qquad + D_{xo}\hat{k}\,] \\
\underline{M}_z &= \frac{8\pi C_z R_z^3}{\varepsilon_o} D_{zo} [\,D_{yo}\hat{i} - D_{xo}\hat{j} \qquad\qquad]
\end{aligned} \tag{B18}$$

*Note that $K_p = 1$ at $X = 1.95$

$$\underline{F}_x = \frac{8\pi C_x R_x^3}{\varepsilon_o} D_{xo} [\frac{\partial D_x}{\partial x} \hat{i} + \frac{\partial D_x}{\partial y} \hat{j} + \frac{\partial D_x}{\partial z} \hat{k}]$$

$$\underline{F}_y = \frac{8\pi C_y R_y^3}{\varepsilon_o} D_{yo} [\frac{\partial D_y}{\partial x} \hat{i} + \frac{\partial D_y}{\partial y} \hat{j} + \frac{\partial D_y}{\partial z} \hat{k}] \qquad \text{(B19)}$$

$$\underline{F}_z = \frac{8\pi C_z R_z^3}{\varepsilon_o} D_{zo} [\frac{\partial D_z}{\partial x} \hat{i} + \frac{\partial D_z}{\partial y} \hat{j} + \frac{\partial D_z}{\partial z} \hat{k}]$$

The force on dielectric material in a static electric field is[5]

$$\underline{F} = - \frac{\varepsilon_o}{2} (1 - \frac{\varepsilon}{\varepsilon_o}) V \nabla E^2$$

$$= \frac{1}{2\varepsilon_o} (1 - \frac{\varepsilon}{\varepsilon_o}) V \nabla D^2 \qquad \text{(B20)}$$

A comparison of Eqs. (B18), (B19) and (B20) with Eqs. (14) through (18) and (33) shows that this pseudo-diadielectric device is entirely analogous to the pseudo-diamagnetic device.

APPENDIX C

Vector Relations

From the definition of gradient we have

$$\nabla S = \frac{\partial S}{\partial x}\hat{i} + \frac{\partial S}{\partial y}\hat{j} + \frac{\partial S}{\partial z}\hat{k} \qquad (C1)$$

If

$$S = u + v + w \qquad (C2)$$

$$\begin{aligned} \nabla S = & \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\right)\hat{i} \\ & + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial y}\right)\hat{j} \\ & + \left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial z} + \frac{\partial w}{\partial z}\right)\hat{k} \end{aligned} \qquad (C3)$$

If

$$S = \beta^2 = \beta_{xo}^2 + \beta_{yo}^2 + \beta_{zo}^2 \qquad (C4)$$

and

$$\begin{aligned} u &= \beta_{xo}^2 \\ v &= \beta_{yo}^2 \\ w &= \beta_{zo}^2 \end{aligned} \qquad (C5)$$

then we see that

$$\begin{aligned} \frac{1}{2}\nabla\beta^2 = & \left(\beta_{xo}\frac{\partial\beta_x}{\partial x} + \beta_{yo}\frac{\partial\beta_y}{\partial x} + \beta_{zo}\frac{\partial\beta_z}{\partial x}\right)\hat{i} \\ & + \left(\beta_{xo}\frac{\partial\beta_x}{\partial y} + \beta_{yo}\frac{\partial\beta_y}{\partial y} + \beta_{zo}\frac{\partial\beta_z}{\partial y}\right)\hat{j} \\ & + \left(\beta_{xo}\frac{\partial\beta_x}{\partial z} + \beta_{yo}\frac{\partial\beta_y}{\partial z} + \beta_{zo}\frac{\partial\beta_z}{\partial z}\right)\hat{k} \end{aligned} \qquad (C6)$$

For Case 1, where two of the components of the field are zero, (e.g.)

$$\beta_{yo} = \beta_{zo} = 0 \tag{C7}$$

we see that (e.g.)

$$\frac{1}{2}\nabla\beta_1^2 = \beta_{xo}\left(\frac{\partial\beta_x}{\partial x}\hat{i} + \frac{\partial\beta_x}{\partial y}\hat{j} + \frac{\partial\beta_x}{\partial z}\hat{k}\right) \tag{C8}$$

For Case 2, where one of the components of the field is zero, (e.g.)

$$\beta_{zo} = 0 \tag{C9}$$

we see that (e.g.)

$$\frac{1}{2}\nabla\beta_2^2 = \left(\beta_{xo}\frac{\partial\beta_x}{\partial x} + \beta_{yo}\frac{\partial\beta_y}{\partial x}\right)\hat{i} + \left(\beta_{xo}\frac{\partial\beta_x}{\partial y} + \beta_{yo}\frac{\partial\beta_y}{\partial y}\right)\hat{j} \tag{C10}$$

and for Case 3

$$\frac{1}{2}\nabla\beta_3^2 = \text{Eq. (C6)} \tag{C11}$$

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NEW TECHNOLOGY APPENDIX

PSEUDO-DIAMAGNETIC SUSPENSION

This report is published in the belief that it constitutes an improvement in the state of the art of force field suspension (levitation). In particular, the concept of using force fields sensors and electronic components in such a fashion as to imitate the magnetic (or electric) properties of materials, which have the characteristic that they can be suspended (levitated) in properly shaped fields, is novel. The analysis in this report supports the validity of this hypothesis and describes a laboratory experiment which further confirms the concept.