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THE RELATIVE MOTION OF UNRESTRAINED BODIES WITHIN ORBITING SPACE STATIONS

by

William N. Lee

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Abstract

Recent analyses of several experiments proposed for an earth orbiting space station have shown it advantageous to allow the experiments to float freely inside the spacecraft. Perturbating forces and/or experimental locations at other than the center of mass, however, will cause relative motion between the experimental apparatus and the spacecraft.

This work sets forth a linearized perturbation method for calculating the relative motion, including the prediction of possible collision with the spacecraft. The theory is then applied in detail to two probable attitude orientation modes of a NASA Skylab vehicle in earth orbit.

Acknowledgment

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The publication of this report does not constitute approval by the National Aeronautics and Space Administration of the findings or conclusions contained therein. It is published only for the exchange and stimulation of ideas.

TABLE OF CONTENTS

I. INTRODUCTION

With the advent of earth orbiting workshops, the possibility of permitting objects within the spacecraft to "drift" for long periods of time becomes of special interest. In this manner, an experiment package, for example, could be kept free of the effects of aerodynamic forces, radiation pressure, and the zero order gravitational field of the spacecraft environment.

An obvious restriction on the trajectory of the object is collision with the walls of the spacecraft. This event is dependent upon the force field experienced by the bodies, the motion of the spacecraft about its center of mass, and the initial position and velocity of the particle. In order to avoid a collision for the longest period of time, or to reduce the maneuvering of the spacecraft to a minimum, it is necessary to determine the initial conditions and orientation of the vehicle which produce the optimum trajectory.

In this preliminary analysis, the steps necessary for treating the general problem are indicated, but, due to the complexity of the problem, only the special cases possessing the following characteristics are considered in detail:

a) The oblateness of the earth is neglected.

b) Initially the spacecraft is assumed to be in a circular earth orbit of about 435 km.

c) The spacecraft gravity field is neglected. Only the first order (gravity gradient) force due to the earth is assumed to act on the particle.

d) Only the zero and first order gravitational forces due to the earth and aerodynamic forces are assumed to act on the spacecraft.

e) Two orientations of the spacecraft are considered: a "vertical hold," and an "inertial hold," as to be explained in Section II.1.

f) The variational theory utilized to describe the motion of the object relative to a circular reference orbit is a linearized perturbation theory. It is applicable due to the small differences anticipated between the two orbits.

In conjunction with items (d) and (f), a subtlety in the application of the theory should be explained. The net aerodynamic force affecting the spacecraft is considered mathematically as acting on the particle and in the opposite direction as the actual force acting on the spacecraft. Because of the linear theory used, this has no effect on the description of the motion of the particle relative to the center of mass of the spacecraft. With this approach, the center of mass remains in a perfectly circular orbit. Hence, the only error introduced is in the description of the orbit of the spacecraft, a matter of no importance here, as long as the deviations remain small.

II. LINEARIZED VARIATIONAL EQUATIONS OF MOTION

II.1 Coordinate Systems

As mentioned in I, the description of the relative motion between the particle and the spacecraft depends, in part, on the orientation of the spacecraft throughout the flight. For the two particular orientations considered here, it is advantageous in each case to adopt a coordinate system which remains fixed relative to the body of the spacecraft.

In the so-called "vertical hold" mode, the spacecraft rotates at orbital frequency in the plane of the orbit such that the same side of the spacecraft continually faces the earth. A conventional rotating coordinate system (r,s,z) is defined by

$$\hat{\underline{r}} = \frac{\underline{r}}{r}$$

$$\hat{\underline{s}} = \hat{\underline{h}} \times \hat{\underline{r}} \qquad \text{where } \hat{\underline{h}} = \frac{\underline{r} \times \underline{v}}{|\underline{r} \times \underline{v}|}$$

$$\hat{\underline{z}} = \hat{\underline{h}}$$

with origin at the attractive center (Figure I). For a circular reference orbit, $\hat{\underline{s}} = \underline{v} \, / \, |\underline{v}|$.

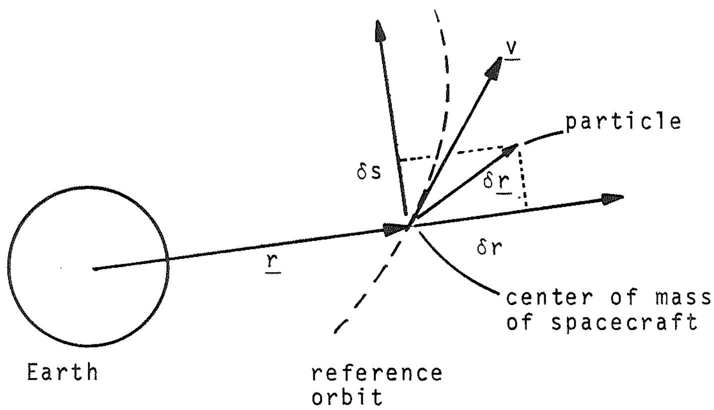


Figure I. Coordinate System for the Vertical Hold Mode.

The relative particle position and velocity (according to an observer rotating with the system) will be denoted by

$$\delta\underline{r} = \begin{bmatrix} \delta r \\ \delta s \\ \delta z \end{bmatrix} \quad \text{and} \quad \delta\dot{\underline{r}} = \begin{bmatrix} \delta\dot{r} \\ \delta\dot{s} \\ \delta\dot{z} \end{bmatrix}.$$

Because these quantities represent deviations from the reference orbit of the spacecraft, they may be thought of as position and velocity in a rotating coordinate system with origin located at the center of mass of the spacecraft. The associated state vector at time t_j is defined as

$$\underline{x}_j = \begin{bmatrix} \delta\underline{r} \\ \delta\dot{\underline{r}} \end{bmatrix}_{t = t_j}. \tag{1}$$

In the inertial hold mode of operation, the spacecraft is kept from rotating relative to inertial space. In this case, nonrotating coordinates are selected, and deviations in position and velocity (a nonrotating observer in this case) are

$$\delta\underline{r} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} \quad \text{and} \quad \delta\underline{v} = \begin{bmatrix} \delta v_x \\ \delta v_y \\ \delta v_z \end{bmatrix}$$

and the state vector is defined as

$$\underline{y}_j = \begin{bmatrix} \delta\underline{r} \\ \delta\underline{v} \end{bmatrix}_{t = t_j}. \tag{2}$$

These quantities represent the motion of the particle relative to a nonrotating coordinate system with origin at the spacecraft center of mass. Note that the z axes are the same in both coordinate systems and the other two pairs of axes are defined so as to coincide at time t_i (Figure II).

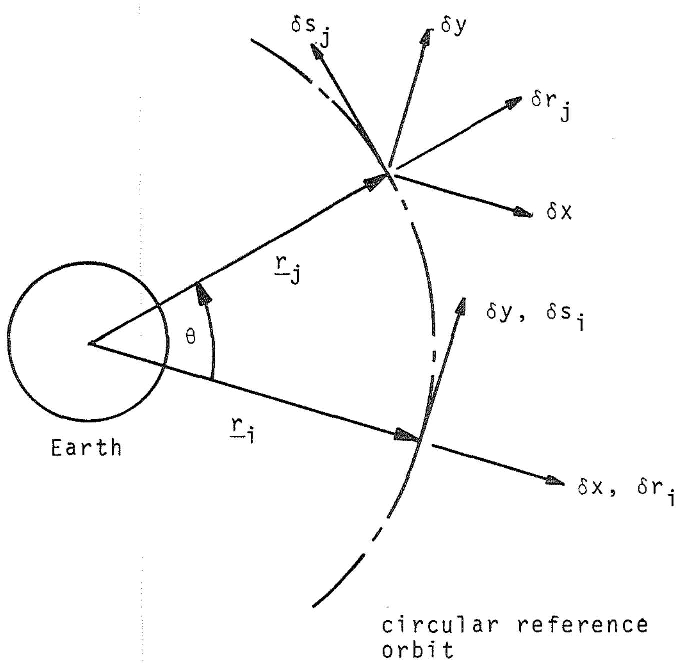


Figure II. The Local Coordinate Systems at Two Different Times, t_i and t_j.

II.2 The Differential Variational Equations of Motion in Rotating Coordinates

The linearized variational matrix equation for the motion of the particle relative to the center of mass is

$$\delta \underline{a} = P\delta \underline{r} + \underline{d}, \tag{3}$$

where $\delta \underline{a}$ - acceleration of the particle

P - gravity gradient matrix

$\underline{d}$ - disturbing accelerations (aerodynamic, electrostatic radiation, oblateness of the earth, tension from wires connected to the particle, spacecraft gravity field, etc.)

If the spacecraft gravity gradients are neglected, and the earth is considered as a point mass,

$$P = \frac{\mu}{r^3} \left(\frac{3\underline{r}\ \underline{r}^T}{r^2} - I_3 \right).$$

For all practical purposes, μ equals the product of the universal gravitational constant (G) and the mass of the earth; I_3 represents a three-by-three identity matrix. In the (r,s,z) system,

$$P = \frac{\mu}{r^3} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{4}$$

If the operator $\frac{d}{dt}$ is denoted by D, and f is the true anomaly of the reference orbit, $\delta \underline{a}$ can be shown (Reference 1) to be

$$\delta \underline{a} = \begin{bmatrix} D^2 - \dot{f}^2 & -2\dot{f}D - \ddot{f} & 0 \\ 2\dot{f}D + \ddot{f} & D^2 - \dot{f}^2 & 0 \\ 0 & 0 & D^2 \end{bmatrix} \delta \underline{r} \tag{5}$$

in a rotating (r,s,z) system, as observed from a nonrotating system.

Equation (3) can then be analytically or numerically integrated to yield the motion of the particle relative to a two-body reference trajectory. This has been carried out analytically for the case when $\underline{d} = \underline{0}$ by Stern (Refernce 1).

When the reference orbit is circular and of frequency ω, it is convenient to define the operator $F = \frac{d}{df}$ (Reference 2). Equation (5) can then be written as

$$\delta\underline{a} = \omega^2 \begin{bmatrix} F^2 - 1 & -2F & 0 \\ 2F & F^2 - 1 & 0 \\ 0 & 0 & F^2 \end{bmatrix} \delta\underline{r}. \qquad (6)$$

Using (4) and (6), equation (3) can be rewritten as

$$\omega^2 \begin{bmatrix} F^2 - 3 & -2F & 0 \\ 2F & F^2 & 0 \\ 0 & 0 & F^2 + 1 \end{bmatrix} \delta\underline{r} = \underline{d}\ (f). \qquad (7)$$

II.3 Integration of the Variational Equations of Motion in Rotating Coordinates

In order to integrate (7), the disturbing acceleration $\underline{d}$ must be specified. The aerodynamic forces depend on the orientation of a nonspherical spacecraft, but might be nearly constant in vertical hold except for the dependence of atmospheric density on orbital parameters, including a periodic variation at orbital frequency. A nonrotating spacecraft would acquire additional variations at twice orbital frequency.

As a simple example, however, $\underline{d}$ is assumed to consist of only a constant aerodynamic drag acceleration, so that in rotating coordinates

$$\underline{d} = \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix}.$$

Even with this assumption, however, the analysis should reveal something about the motion of a particle in a vertical hold mode.

Integration of (7) yields the positions and their derivatives with respect to the true anomaly f, in terms of six arbitrary constants:

$$\begin{bmatrix} \delta r \\ \delta s \\ \delta z \\ (\delta r)' \\ (\delta s)' \\ (\delta z)' \end{bmatrix} = \begin{bmatrix} 0 & 0 & \sin f & \cos f & 2 & 0 \\ 0 & 0 & 2\cos f & -2\sin f & -3f & 1 \\ \sin f & \cos f & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos f & -\sin f & 0 & 0 \\ 0 & 0 & -2\sin f & -2\cos f & -3 & 0 \\ \cos f & -\sin f & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} + \frac{d}{\omega^2} \begin{bmatrix} 2f \\ -3/2\ f^2 \\ 0 \\ 2 \\ -3f \\ 0 \end{bmatrix}. \qquad (8)$$

The constants may be replaced by the initial values of the state vector, and (8) may be expressed as

$$\begin{bmatrix} \delta r \\ \delta s \\ \delta z \\ (\delta r)' \\ (\delta s)' \\ (\delta z)' \end{bmatrix}_{t = t_j} = Q_{ji}(\theta) \begin{bmatrix} \delta r \\ \delta s \\ \delta z \\ (\delta r)' \\ (\delta s)' \\ (\delta z)' \end{bmatrix}_{t = t_i} + \underline{T}_{ji}(d,\theta) \tag{9}$$

where $\theta \equiv f_j - f_i$, and $Q_{ji}(\theta)$ is a 6×6 matrix. Due to the circular reference orbit,

$$\frac{d}{df} = \frac{1}{\omega}\frac{d}{dt},$$

and equation (9) can be transformed into the time domain again. Consequently, the state vector $\underline{X}_j$, which was previously defined by (1), is

$$\underline{X}_j = \Phi_{ji}\,\underline{X}_i + \underline{R}_{ji} \tag{10}$$

where Φ_{ji} is the state transition matrix:

$$\Phi_{ji} = \begin{bmatrix} 4 - 3\cos\theta & 0 & 0 & \frac{1}{\omega}\sin\theta & \frac{2}{\omega}(1-\cos\theta) & 0 \\ 6(\sin\theta-\theta) & 1 & 0 & -\frac{2}{\omega}(1-\cos\theta) & \frac{-3\theta+4\sin\theta}{\omega} & 0 \\ 0 & 0 & \cos\theta & 0 & 0 & \frac{1}{\omega}\sin\theta \\ 3\omega\sin\theta & 0 & 0 & \cos\theta & 2\sin\theta & 0 \\ 6\omega(\cos\theta-1) & 0 & 0 & -2\sin\theta & -3+4\cos\theta & 0 \\ 0 & 0 & -\omega\sin\theta & 0 & 0 & \cos\theta \end{bmatrix}, \tag{11}$$

and $\underline{R}_{ji}$ contains the effects of the disturbance force:

$$\underline{R}_{ji} = \frac{d}{w^2} \begin{bmatrix} 2(\theta-\sin\theta) \\ -3/2\ \theta^2 + 4(1-\cos\theta) \\ 0 \\ 2\omega(1-\cos\theta) \\ \omega(-3\theta + 4\sin\theta) \\ 0 \end{bmatrix}. \tag{12}$$

To convert to the nonrotating system, the position and velocities must be transformed in the following manner, keeping in mind that the coordinate axes coincide at $t_i (\theta = 0)$:

$$\underline{Y}_j = E_j\ \underline{X}_j \quad \text{and} \quad \underline{X}_i = E_i^{-1}\ \underline{Y}_i$$

where

$$E_j = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\omega\sin\theta & -\omega\cos\theta & 0 & \cos\theta & -\sin\theta & 0 \\ \omega\cos\theta & -\omega\sin\theta & 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

It follows that $\underline{Y}_j$, as defined by (2), can be written as

$$\underline{Y}_j = \Psi_{ji}\,\underline{Y}_i + \underline{S}_{ji} \tag{13}$$

where

$$\underline{S}_{ji} = \frac{d}{\omega^2}\begin{bmatrix} \frac{3}{2}\theta^2\sin\theta + 2\theta\cos\theta + \sin\theta(2\cos\theta-4) \\ -\frac{3}{2}\theta^2\cos\theta + 2\theta\sin\theta + 4\cos\theta - 2(1+\cos^2\theta) \\ 0 \\ \omega\left[\frac{3}{2}\theta^2\cos\theta + \theta\sin\theta + \cos\theta(4\cos\theta-2) - 2\right] \\ \omega\left[\frac{3}{2}\theta^2 - \theta\cos\theta + \sin\theta(4\cos\theta-2)\right] \\ 0 \end{bmatrix} \tag{14}$$

and

$$\Psi_{ji} = \begin{bmatrix} 2\cos\theta-1-\sin^2\theta+3\theta\sin\theta & \sin\theta(1-\cos\theta) & 0 & \frac{1}{\omega}\sin\theta(2-\cos\theta) & \frac{2}{\omega}(\cos\theta-1-\sin^2\theta)+\frac{3\theta}{\omega}\sin\theta & 0 \\ \sin\theta(2+\cos\theta)-3\theta\cos\theta & 1-\cos\theta+\cos^2\theta & 0 & \frac{1}{\omega}(1-\cos\theta)^2 & \frac{2}{\omega}\sin\theta(1+\cos\theta)-\frac{3\theta}{\omega}\cos\theta & 0 \\ 0 & 0 & \cos\theta & 0 & 0 & \frac{\sin\theta}{\omega} \\ \omega\sin\theta(1-2\cos\theta)+3\omega\theta\cos\theta & \omega(\cos\theta-1)+2\omega\sin^2\theta & 0 & 2(\cos\theta+\sin^2\theta)-1 & \sin\theta(1-4\cos\theta)+3\theta\cos\theta & 0 \\ \omega\cos\theta(2\cos\theta-1)-\omega+3\omega\theta\sin\theta & \omega\sin\theta(1-2\cos\theta) & 0 & 2\sin\theta(1-\cos\theta) & -2-\cos\theta+4\cos^2\theta+3\theta\sin\theta & 0 \\ 0 & 0 & -\omega\sin\theta & 0 & 0 & \cos\theta \end{bmatrix} \quad (15)$$

III. VARIANT MOTION RELATIVE TO AN INERTIALLY NONROTATING SPACECRAFT

With an inertially nonrotating spacecraft, the drag force complicates the equations of motion (14) to such an extent that the drag is ignored for now. As it turns out, it is very difficult to restrain the motion of the particle even in the absence of disturbing forces.

Inspection of (15) reveals that initial displacements out of the orbital plane are not coupled to the x-y motion, and vice versa. Hence, the two cases can be considered independently. In the out-of-plane case,

$$\delta z = \cos\theta \, \delta z_i + \sin\theta \, \frac{\delta V_{z_i}}{\omega} .$$

Unfortunately, the particle will always oscillate between $\pm \delta z_i$, or farther. As will be shown, these are much larger excursions than are necessary with the appropriate choice of initial conditions in the x-y plane.

Further inspection of (15) for the in-plane case reveals that the excursions from the center of mass will grow with time unless the secular terms are eliminated by setting

$$\delta V_{y_i} = - \omega \delta x_i . \tag{16}$$

This leaves three independent initial conditions to be determined so that in some sense the motion of the particle is minimized.

It may be desirable to keep the particle as close to its initial position as possible. To achieve this, it is necessary to minimize the magnitude of the vector $\underline{\varepsilon}$ for a given $|\delta \underline{r}_i|$ where $\underline{\varepsilon} = \delta \underline{r} - \delta \underline{r}_i$ (Figure III).

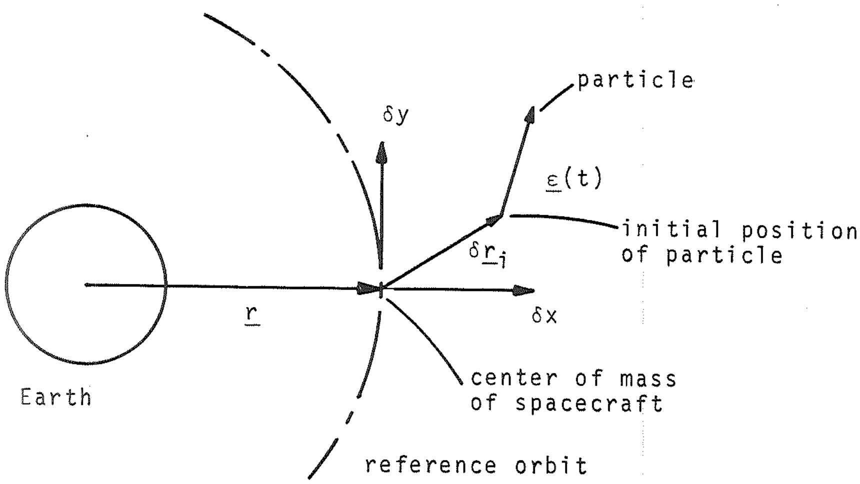


Figure III.

From (15),

$$\varepsilon_x = \delta x - \delta x_i$$
$$= \delta x_i \left[\sin^2\theta\right] + \delta y_i \left[\sin\theta(1 - \cos\theta)\right] + \frac{\delta V_{x_i}}{\omega}\left[\sin\theta(2 - \cos\theta)\right]$$

and

$$\varepsilon_y = \delta y - \delta y_i$$
$$= \delta x_i \left[-\sin\theta\cos\theta\right] + \delta y_i \left[\cos\theta(\cos\theta - 1)\right] + \frac{\delta V_{x_i}}{\omega} \left[(1 - \cos\theta)^2\right] .$$

The results of the minimization of $\dfrac{|\underline{\varepsilon}|}{|\delta \underline{r}_i|}$ are shown in Figure IV. For $\delta V_{x_i} = -0.43\ \omega\delta y_i$ and $\delta x_i = 0$, the particle can be kept within 0.45 of its initial displacement from the center of mass. The corresponding trajectory, relative to the δx, δy coordinates, is indicated in Figure V.

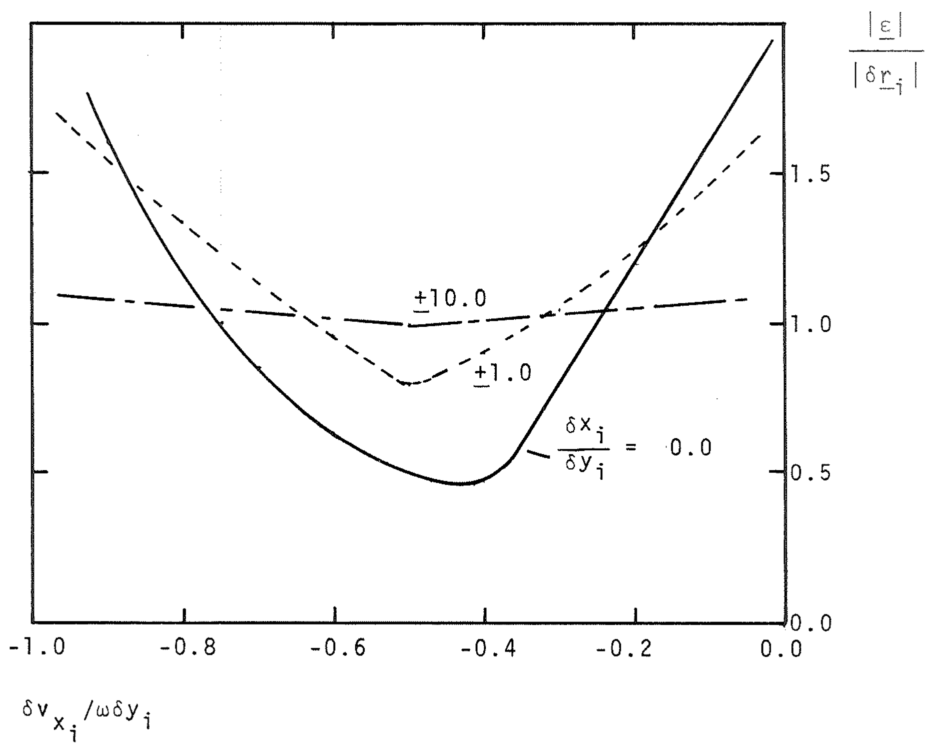


Figure IV. Maximum Excursions from Initial Position in Inertial Hold with $\delta v_{y_i} = -\omega \delta x_i$.

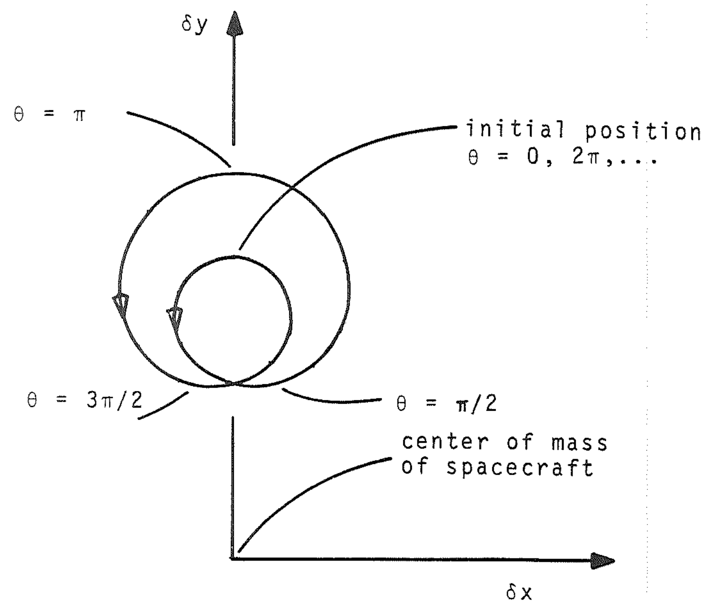


Figure V. Trajectory in Inertial Hold with
$\delta \ddot{x}_i = \delta v_{y_i} = 0,\ \delta v_{x_i} = -.43\omega\delta y_i.$

Since it is advantageous to locate the package as close to the center of mass as possible, it is necessary to consider the initial position as well as the excursions from it. The spacecraft is assumed to be an elongated body, as is Skylab, with the work area located a large distance from the center of mass (Figure VI). The particle is also assumed to be initially located at some point along the longitudinal axis which passes through the center of mass. For large "a," the angle ϕ will not change much even if the assumption is not quite true.

Two displacements are then examined: ε_{perp}, which is the excursion perpendicular to the longitudinal axis; and ε_{axis}, the displacement from the initial position measured along the axis. From Figure VI,

$$\varepsilon_{perp} = \varepsilon_x \sin\phi - \varepsilon_y \cos\phi$$

and

$$\varepsilon_{axis} = \varepsilon_x \cos\phi + \varepsilon_y \sin\phi.$$

Let

$$\eta = \frac{\varepsilon_{perp}}{|\delta \underline{r}_i|} \quad \text{and} \quad \xi = \frac{\varepsilon_{axis}}{|\delta \underline{r}_i|},$$

and the quantities of interest are then

$$\varepsilon_{perp} = \frac{a|\eta|_{max}}{1 + \xi_{min}}$$

and

$$\Delta\varepsilon_{axis} = \frac{a(\xi_{max} - \xi_{min})}{1 + \xi_{min}} .$$

These represent the range of displacements, for a given "a," perpendicular to the axis and along it, respectively. The dimensionless ratios ε_{perp}/a and $\Delta\varepsilon_{axis}/a$ can be compared for various initial conditions, as shown in Figure VII.

ε_{perp}/a reaches a minimum value of 0.5 at $\delta V_{x_i} = -0.5\ \omega\delta y_i$ and either $\delta x_i = 0$ or $\delta y_i = 0$. The ratio $\frac{\Delta\varepsilon_{axis}}{a}$ is also minimized ($\Delta\varepsilon_{axis}/a = 1.0$) for these same conditions. Interestingly, the trajectory for each case is a circle at twice orbital frequency (Figure VIII).

For $\delta x_i = 0$ and $\delta V_{x_i} = -0.43\omega\delta y_i$ (the conditions previously found for mimimizing $|\underline{\varepsilon}|/|\delta\underline{r}_i|$), $\varepsilon_{perp}/a = 0.74$ and $\Delta\varepsilon_{axis}/a = 1.33$. The new optimizations are clearly an improvement over this case in terms of restricting the range of the particle.

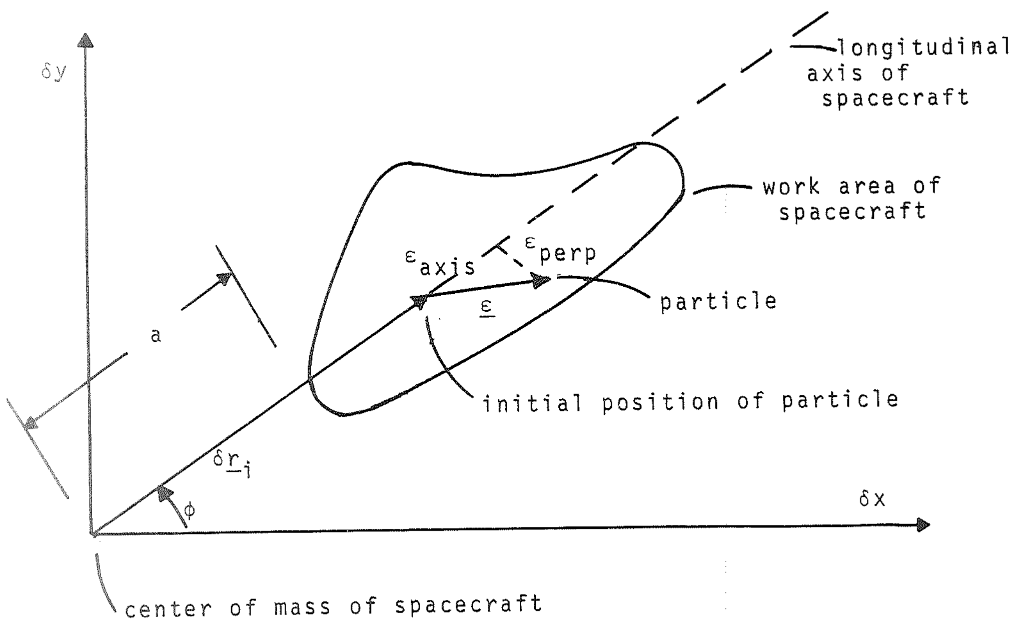
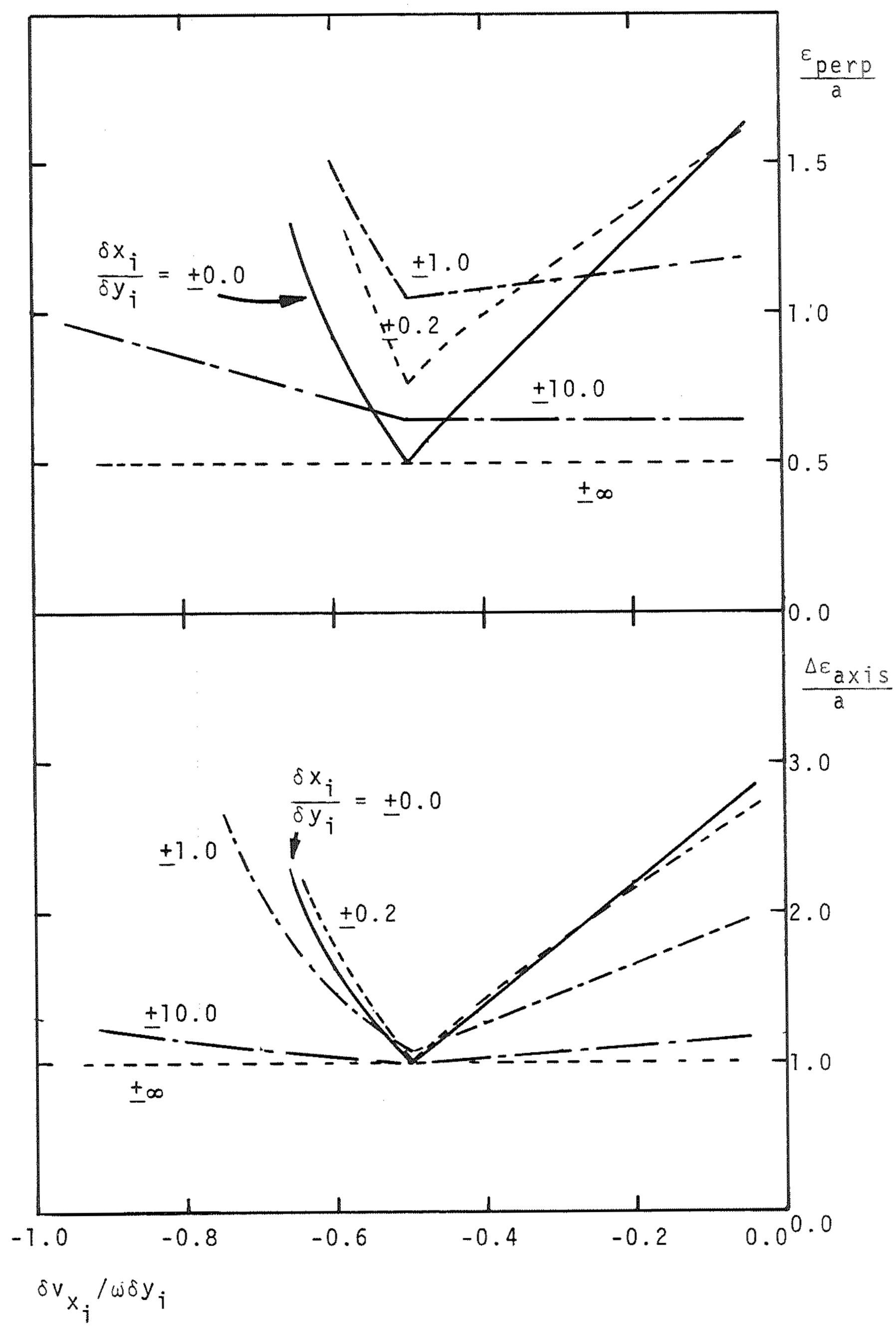


Figure VI. Geometry When the Work Area Is a Large Distance from the Center of Mass.

Figure VII. Maximum Excursions Perpendicular and Parallel to Longitudinal Axis of Spacecraft with $\delta v_{y_i} = -\omega\delta x_i$.



An an example of how large these excursions are, assume "a" to be 30 feet, approximately that for Skylab. This means that, at best, the object will float 15 feet away from the longitudinal axis and 30 feet along the axis. With the dimensions of the work area of Skylab on the order of 7 to 10 feet, it appears as if it is impossible to allow an experiment package to float inside a Skylab in inertial hold unless 1) an enormous amount of space is made available, 2) the center of mass is moved much nearer the work area, or 3) the spacecraft is maneuvered occasionally to "follow" the package.

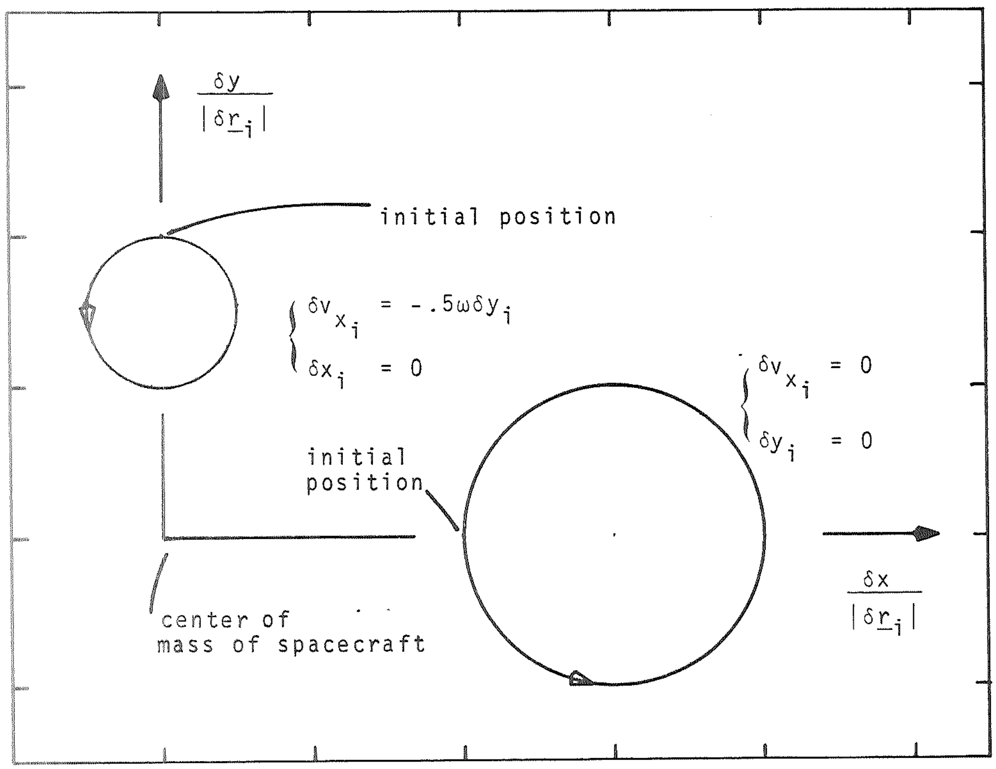


Figure VIII. Trajectories in Inertial Hold with $\delta v_{y_i} = -\omega\delta x_i$.

IV. VARIANT MOTION RELATIVE TO A SPACECRAFT IN VERTICAL HOLD

In a vertical hold mode, it is much easier to control the relative motion of the particle. In the absence of drag, for example, the particle remains motionless at $\delta s = \delta s_i$ for initial conditions $\delta r_i = \delta V_{r_i} = \delta V_{s_i} = \delta z_i = \delta V_{z_i} = 0$ (11).

The effect of constant drag, however, is to introduce secular terms which grow as time squared (12); the secular terms due to gravity gradients only increase linearly with time. Hence, the best that can be done is to trade off one against the other for as long as possible, i.e., until the drag dominates.

As before, the out-of-plane case can be disregarded due to the large excursions which result from initial out-of-plane displacement and velocity (11). Inspection of (11) reveals that many of the oscillatory terms cancel with the choice of $\delta V_{s_i} = -3/2\omega\delta r_i$. The components of the vector $\underline{\varepsilon} = \delta\underline{r} - \delta\underline{r}_i$ can then be written as

$$\left.\begin{aligned} \varepsilon_r &= \left[\frac{1}{\omega}\sin\theta\right]\delta V_{r_i} + \frac{d}{\omega^2}\left[2(\theta - \sin\theta)\right] \\ &\text{and} \\ \varepsilon_s &= \left[-3/2\;\theta\right]\delta r_i - \left[\frac{2}{\omega}(1-\cos\theta)\right]\delta V_{r_i} \\ &\quad + \frac{d}{\omega^2}\left[-3/2\;\theta^2 + 4(1-\cos\theta)\right]. \end{aligned}\right\} \qquad (17)$$

Note that both ε_r and ε_s are independent of δs_i, and with the preceeding choice of δV_{s_i}, ε_r is independent of δr_i. The first observation is important because the spacecraft may well be oriented with zero angle of attack in order to minimize the effects of drag (Reference 3). The δs axis then aligns with the longitudinal axis; hence, the location of the center of mass becomes of little practical importance.

A further observation of (17) is that δV_{r_i} does not affect any of the secular terms. It therefore plays a minor role in the attempt to confine the particle for as long as possible. For this reason, and in order to simplify the analysis, δV_{r_i} is assumed to be zero. In

Figure IX. Length of Time for which $0 \leq \varepsilon_s \leq \varepsilon_{s_{max}}$.

$\delta v_{s_i} = -3/2\ \omega \delta r_i$

$\delta v_{r_i} = 0$

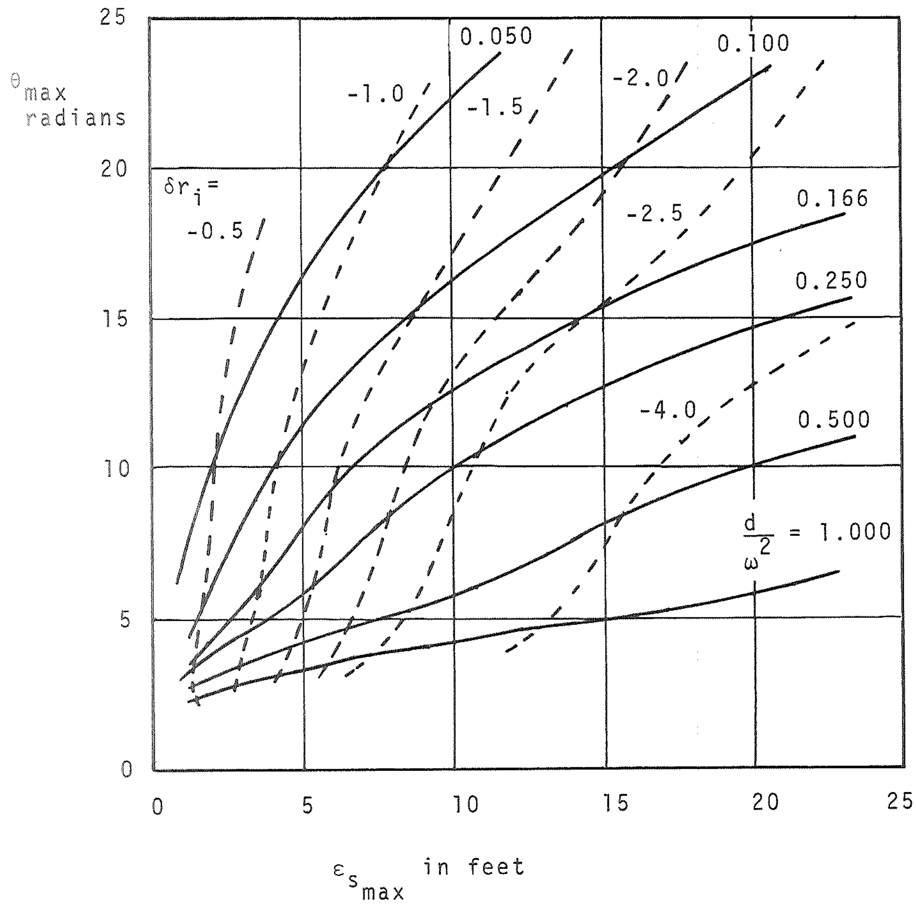
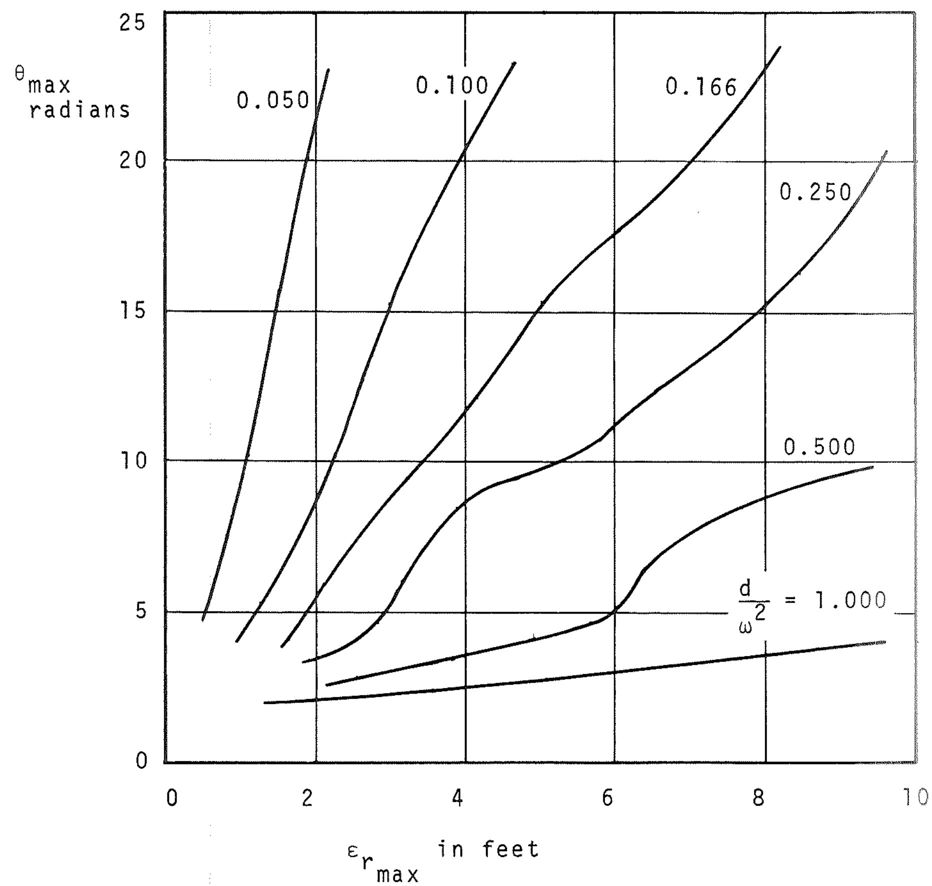


Figure X. Length of Time for which
$0 \leq \varepsilon_r \leq \varepsilon_{r_{max}}$.

$\delta v_{s_i} = -3/2\ \omega \delta r_i$

$\delta v_{r_i} = 0$



some cases, however, a non-zero value of δV_{r_i} can improve the performance slightly.

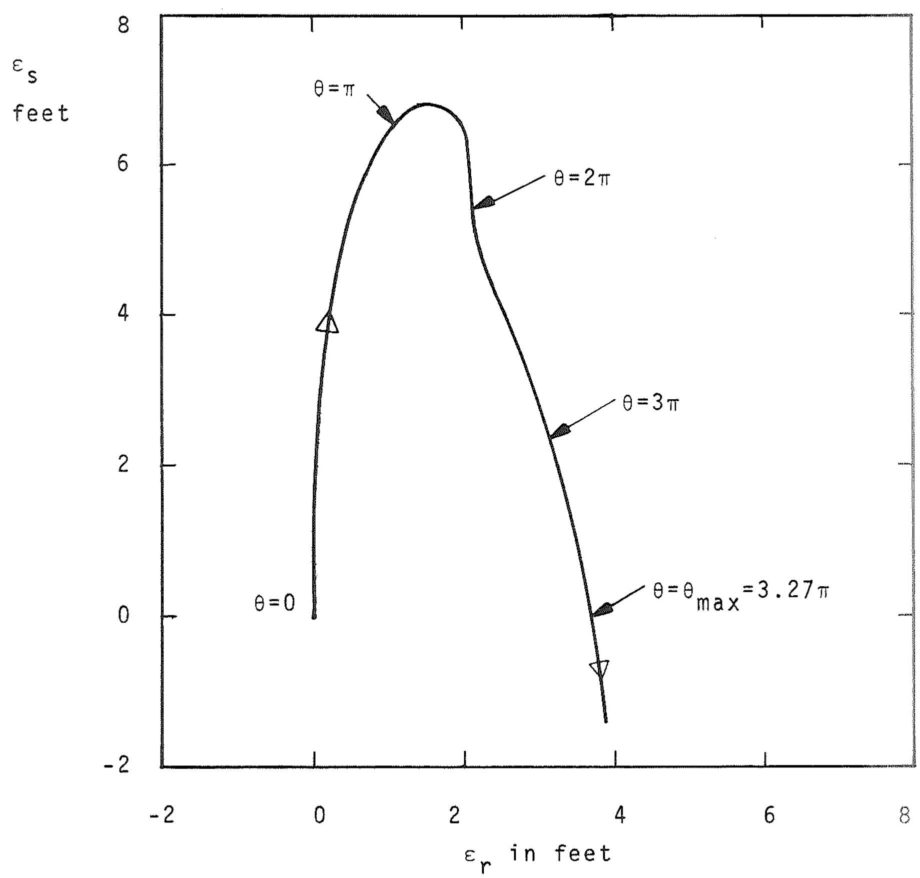
Since the drag will ultimately cause the particle to accelerate in the negative δs direction, a negative δr_i must be chosen in order to initially get the particle moving in the positive δs direction (17). $\varepsilon_{r_{max}}$ and $\varepsilon_{s_{max}}$ will denote the maximum excursions in the positive δr and δs directions, respectively, and θ_{max} is the angle the orbit sweeps through before ε_s returns to zero. In order to determine θ_{max}, values for $\varepsilon_{s_{max}}$ (or $\varepsilon_{r_{max}}$) and d/ω^2 must be specified. $\varepsilon_{s_{max}}$ is assumed to be about 7 feet, with $\varepsilon_{r_{max}}$ at least that large. For 1974, a mean value of $d/\omega^2 = 0.166$ feet is selected for Skylab at zero angle of attack at 435 km. (Reference 4).

Figures IX and X indicate the results of the investigation of the relationships among θ_{max}, $\varepsilon_{s_{max}}$, $\varepsilon_{r_{max}}$, d/ω^2, and δr_i for values near the nominal values selected. For the specific case where $\varepsilon_{s_{max}} = 7$ feet and $d/\omega^2 = 0.166$ feet, $\theta_{max} = 10.29$ radians and δr_i must be -1.64 feet (Figure IX). From Figure X, $\varepsilon_{r_{max}}$ is found to be 3.7 feet. Thus, the excursions in the δr direction are less than in the longitudinal direction, and the particle can be confined for slightly more than $1\frac{1}{2}$ orbits. The trajectory is shown in Figure XI.

Figure IX reveals the great advantage of operating at low levels of drag (high altitude, low solar activity, small angle of attack), as the curves of constant d/ω^2 not only shift to the left, but the slopes increase significantly, allowing one to operate for much longer periods of time with excursions of only a few feet.

It should be kept in mind, however, that the drag, which has been assumed to be constant, may actually vary a great deal during the course of an orbit, although the major variation will be periodic. It should also be emphasized that the results shown in Figures IX and X depend on the particular values of δV_{r_i} and δV_{s_i} selected earlier.

Figure XI. Trajectory in Vertical Hold for $\varepsilon_{s_{max}} = 7$ feet with $\frac{d}{\omega^2} = 0.166$ feet, $\delta v_{r_i} = 0$, $\delta v_{s_i} = -3/2\ \omega\delta r_i$, $\delta r_i = -1.64$ feet.



As mentioned, positive δV_{r_i} may improve the results slightly; larger δV_{s_i} will, in general, trade off increased excursions in the δr direction for decreases in the δs direction, while θ_{max} is lengthened slightly in some cases.

Considering then, the many assumptions involved and the focusing of this analysis on one particular set of initial velocities, no attempt has been made to select the optimum conditions for all circumstances. But the results do represent an approximation to the kind of performance to be expected during a Skylab mission in vertical hold.

V. SUMMARY AND CONCLUSIONS

The foregoing analysis has outlined the steps necessary to determine the perturbed motion of a particle relative to the center of mass of a spacecraft in earth orbit. Two simple cases, which could be treated analytically, were examined in detail, the purpose being to keep a freely floating experiment package from colliding with the environs.

The problem was to determine the initial conditions necessary for the desired trajectory. In general, this would require inversion of six-by-six matrices like (11) and (15), a task which was avoided here. Instead, a number of trials were run to obtain a feeling for the appropriate initial conditions, for a given circumstance. For an inertially nonrotating Skylab in circular earth orbit, where the work area is located an appreciable distance from the center of mass (compared to the work lab dimensions), it was learned that it is highly unlikely that a collision during the first orbit could be avoided, unless the spacecraft was to be manuevered from time to time.

In the vertical hold mode, however, the experiment package would not move at all relative to the spacecraft, when given the proper initial conditions, if it were not for perturbing forces such as aerodynamic drag. Assuming constant drag for a projected Skylab flight in 1974, it was shown that the package could be contained within a 4 foot $\times$ 7 foot area for about $1\frac{1}{2}$ orbits.

The primary conclusion is, then, that within the present designs of Skylab and its principal modes of operation (vertical and inertial hold), it would be very difficult to operate a free fall experiment inside the spacecraft for an extended length of time unless the spacecraft was maneuvered occasionally, or additional perturbating forces were applied. The major difficulties appear to be 1) the remote location of the work area with respect to the center of mass, and 2) the relatively high level of drag encountered at 435 km.

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