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SOLUTION OF THE POTENTIAL IN A SEMICONDUCTOR WITH EXPONENTIALLY DEPTH DEPENDENT CONDUCTIVITY AND APPLICATION TO FOUR-POINT-PROBE MEASUREMENTS

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16. Abstract

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SOLUTION OF THE POTENTIAL IN A SEMICONDUCTOR WITH EXPONENTIALLY DEPTH-DEPENDENT CONDUCTIVITY AND APPLICATION TO FOUR-POINT-PROBE MEASUREMENTS

By R. K. Franks* and J. B. Robertson
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SUMMARY

An exact solution has been obtained for the potential due to a point current source on a semiconductor whose conductivity varies exponentially with depth. The solution was obtained by solving the continuity equation in cylindrical coordinates and applying the appropriate boundary conditions. This solution was then applied to the interpretation of the "four-point-probe" method of measuring semiconductor conductivity.

INTRODUCTION

The four-point probe is a favorite tool for measuring conductivity because it eliminates the problem of contact resistance by use of null-measurement techniques. However, previous theory has permitted the interpretation of measurements of bulk electrical conductivity by the four-point-probe method only for media of homogeneous conductivity (ref. 1). Some approximations have been made for nonhomogeneous conductivity such as small-amplitude sinusoidal variations (ref. 2). Extension of the theory to permit four-point-probe measurements of conductivity of nonhomogeneous media is very desirable, especially for persons studying diffusion of electrically active impurities.

This paper presents a solution for the problem of a semiconductor of finite thickness whose conductivity depends exponentially upon depth. This solution may also serve as a basis for approximation in many of the actual situations involving depth-dependent conductivity.

Included in the present paper is an appendix by Carl L. Fales, Jr., of the Langley Research Center, which explains the behavior of the electric field at small values of current-probe radius.

*NASA-ASEE Summer Faculty Research Fellow from Applachian State University.

SYMBOLS

a	exponential coefficient
A_1, A_2, A_3	constants
B, B_1, B_2, B_3	constants
C, C_1, C_2, C_3, C_4	constants
D_1, D_2, D_3, D_4	constants
e	base of natural system of logarithms, 2.718
$\vec{E}(r,z)$	electric field vector
$E_r(r,z)$	r-component of electric field
$E_z(r,z)$	z-component of electric field
$F(a,S,T)$	correction factor
h	depth of current probe
I	current
I_o	modified Bessel function of the first kind, zero order
J_o	Bessel function of the first kind, zero order
$\vec{J}(r,z)$	current density
$J_r(r,z)$	r-component of current density
K_o	modified Bessel function of the second kind, zero order
r	cylindrical coordinate
r_c	radius of current probe

R	solution to radius-dependent portion of equation for the potential
S	probe spacing
t	time
T	thickness of semiconductor
V	potential difference between inner probes
Y_o	Weber's Bessel function of the second kind, zero order
z	cylindrical coordinate
z'	point on vertical axis
Z	solution to depth-dependent portion of equation for the potential
α^2	constant of separation
δ	Dirac delta function
θ	cylindrical coordinate
$\rho(r,z)$	free-charge density
$\sigma(z)$	conductivity
$\Phi(r,z)$	coulombic potential
ψ_n	set of eigenfunctions ($n = 1, 2, . . ., \infty$)

POTENTIAL OF A POINT CURRENT SOURCE

The essential part of the problem is to determine the exact solution for the potential due to the presence of a point current source on the surface of a semiconductor whose conductivity depends exponentially upon depth; that is,

$$\sigma(z) = \sigma(0)e^{az} \tag{1}$$

Cylindrical coordinates (r, θ, z) are used and the semiconductor is bound at the planes $z = 0$ and $z = T$ and at the cylinder $r = r_c$. (See fig. 1.)

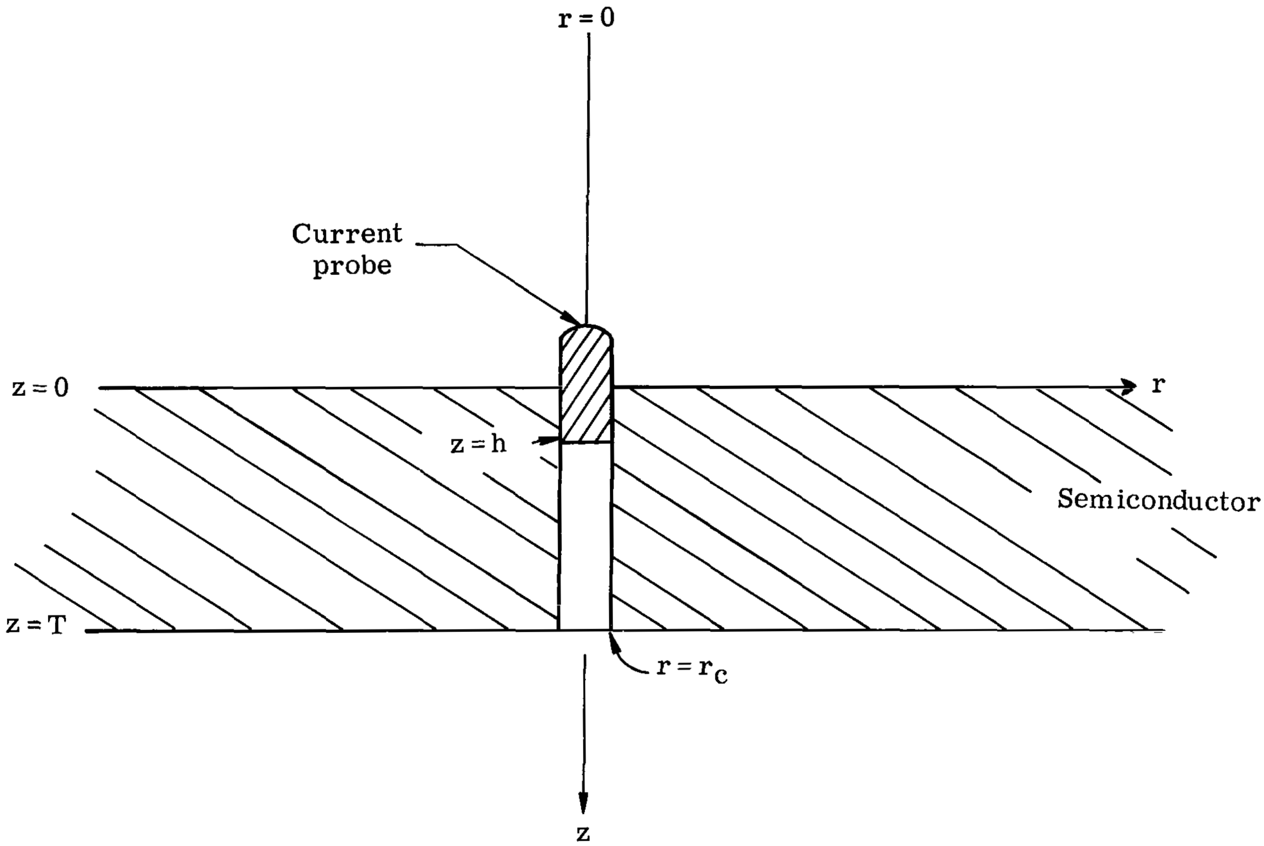

Figure 1.- Semiconductor with current probe inserted distance $z = h$ into cylindrical cavity.

The cylindrical current probe is inserted a distance h into the cavity as shown in figure 1. With the assumption of uniform current density, the boundary conditions on the cylindrical surface are

$$\left.\begin{aligned} E_r(r_c, z) &= \frac{I}{2\pi r_c h \sigma(z)} && (0 < z < h) \\ E_r(r_c, z) &= 0 && (z \geqq h) \end{aligned}\right\} \tag{2}$$

where $E_r(r_c, z)$ is the radial component of the electric field at the surface r_c, I is the current carried by the probe, and $\sigma(z)$ is the conductivity.

The radial component of the electric field must go to zero as r approaches infinity; that is,

$$E_r(\infty, z) = 0 \tag{3}$$

The z-component of the electric field must be zero on the planes $z = 0$ and $z = T$; that is,

$$E_z(r,0) = E_z(r,T) = 0 \tag{4}$$

In the limit, as both r_c and h approach zero, a situation arises which is equivalent to a point current source on the surface of the semiconductor. It has been assumed that if minority carriers are injected, they will recombine close enough to the electrode so that the conductivity will not be affected.

The continuity equation now applied is

$$\nabla \cdot \vec{J}(r,z) = \nabla \cdot \left[\sigma(z)\vec{E}(r,z)\right] = -\frac{\partial}{\partial t}\rho(r,z) \tag{5}$$

where $\vec{J}(r,z)$ is the current density and $\rho(r,z)$ is the free-charge density. By using equation (1) for the conductivity and expressing the electric field as the negative gradient of the potential ($E = -\nabla\Phi(r,z)$), equation (5) under time-independent conditions becomes

$$\nabla^2\Phi(r,z) + a\frac{\partial}{\partial z}\Phi(r,z) = 0 \tag{6}$$

Assumption of a product solution of equation (6) of the form $\Phi(r,z) = R(r)Z(z)$ and separation of variables yield

$$r^2\frac{d^2}{dr^2}R + r\frac{d}{dr}R - \alpha^2r^2R = 0 \tag{7}$$

and

$$\frac{d^2}{dz^2}Z + a\frac{d}{dz}Z + \alpha^2Z = 0 \tag{8}$$

where α^2 is the constant of separation.

There are three physically possible solutions to equation (7), each corresponding to a different value of α^2. These solutions (ref. 3, sec. 4.8) are

$$\left.\begin{aligned} R_1 &= A_1I_o(\alpha r) + B_1K_o(\alpha r) \qquad && (\alpha^2 > 0) \\ R_2 &= A_2 + B_2 \ln r && (\alpha^2 = 0) \\ R_3 &= A_3J_o(\alpha r) + B_3Y_o(\alpha r) && (\alpha^2 < 0) \end{aligned}\right\} \tag{9}$$

where I_o, K_o, J_o, and Y_o are zero-order Bessel functions. The boundary condition $E_r(\infty,z) = 0$ requires that $A_1 = 0$.

There are four possible solutions to equation (8) corresponding to different values of α^2. These solutions are

$$Z_1 = e^{-az/2}\left(C_1 \sin kz + D_1 \cos kz\right) \qquad \left(\alpha^2 > \frac{a^2}{4}\right) \tag{10a}$$

where $k \equiv \sqrt{\alpha^2 - \frac{a^2}{4}}$,

$$Z_2 = C_2 e^{-az/2} + D_2 z e^{-az/2} \qquad \left(\alpha^2 = \frac{a^2}{4}\right) \tag{10b}$$

$$Z_3 = e^{-az/2}\left(C_3 e^{gz} + D_3 e^{-gz}\right) \qquad \left(\frac{a^2}{4} > \alpha^2 > 0\right); \quad (\alpha^2 < 0) \tag{10c}$$

where $g \equiv \sqrt{\frac{a^2}{4} - \alpha^2}$, and

$$Z_4 = C_4 + D_4 e^{-az} \qquad (\alpha^2 = 0) \tag{10d}$$

The boundary condition $E_z(r,0) = E_z(r,T) = 0$ requires that C_2, C_3, D_2, D_3, and D_4 be zero, that $C_1 = \frac{a}{2k} D_1$, and that $k = \frac{n\pi}{T}$ where $n = 1, 2, \ldots, \infty$. Therefore,

$$\left.\begin{aligned} Z_{1,n} &= e^{-az/2} D_n\left(\frac{aT}{2n\pi} \sin \frac{n\pi}{T} z + \cos \frac{n\pi}{T} z\right) \\ Z_2 &= 0 \\ Z_3 &= 0 \\ Z_4 &= C_4 \end{aligned}\right\} \tag{11}$$

The solution of equation (6) is a linear combination of all product solutions

$$\Phi(r,z) = \sum_{n=1}^{\infty} A_n e^{-az/2} K_o(\alpha_n r)\left(\frac{aT}{2n\pi} \sin \frac{n\pi}{T} z + \cos \frac{n\pi}{T} z\right) + B \ln r + C \tag{12}$$

where

$$\alpha_n = \sqrt{\left(\frac{n\pi}{T}\right)^2 + \frac{a^2}{4}} = \frac{1}{2T}\sqrt{4n^2\pi^2 + a^2T^2}$$

By using boundary conditions at $r = r_c$ and the orthogonality of the terms in $E_r(r_c, z)$ with respect to the weighting function e^{az} over the interval $z = 0$ to $z = T$, it can be shown (ref. 3, sec. 5.6) that in the limit of small r_c and h

$$B = \frac{aI}{2\pi\sigma(0)\left(1 - e^{aT}\right)} \tag{13}$$

and

$$A_n = \frac{4n^2\pi^2 I}{\pi\sigma(0)T\left(a^2T^2 + 4n^2\pi^2\right)} \qquad (n = 1, 2, \ldots, \infty) \tag{14}$$

Substituting for A_n and B in equation (12) gives

$$\Phi(r,z) = \frac{Ie^{-az/2}}{\pi\sigma(0)T} \sum_{n=1}^{\infty} \frac{4n^2\pi^2}{4n^2\pi^2 + a^2T^2} K_o\left(\frac{r}{2T}\sqrt{4n^2\pi^2 + a^2T^2}\right)\left(\frac{aT}{2n\pi} \sin \frac{n\pi}{T} z + \cos \frac{n\pi}{T} z\right)$$

$$+ \frac{aI \ln r}{2\pi\sigma(0)\left(1 - e^{aT}\right)} + C \tag{15}$$

Equation (15) is the solution which has been sought. It is an equation for the potential in cylindrical coordinates, independent of θ and in terms of the probe current, semiconductor thickness, and variation in conductivity. The behavior of the electric field, as determined from this expression for the potential, in the limit of small r, is treated in the appendix.

This solution for the potential inside a semiconductor should be of interest to designers of semiconductor devices since it describes the field variations within the bulk material. The authors' prime interest in this solution is its use in interpreting four-point-probe measurements.

APPLICATION TO FOUR-POINT PROBE

The present solution (eq. (15)) is now applied to the case of a four-point probe on a flat semiconductor. The common four-point probe consists of four colinear, equispaced point probes (fig. 2). The present treatment is justified if the probe contact radius is small compared with the probe spacing and if the lateral boundary distance is large compared with the probe spacing.

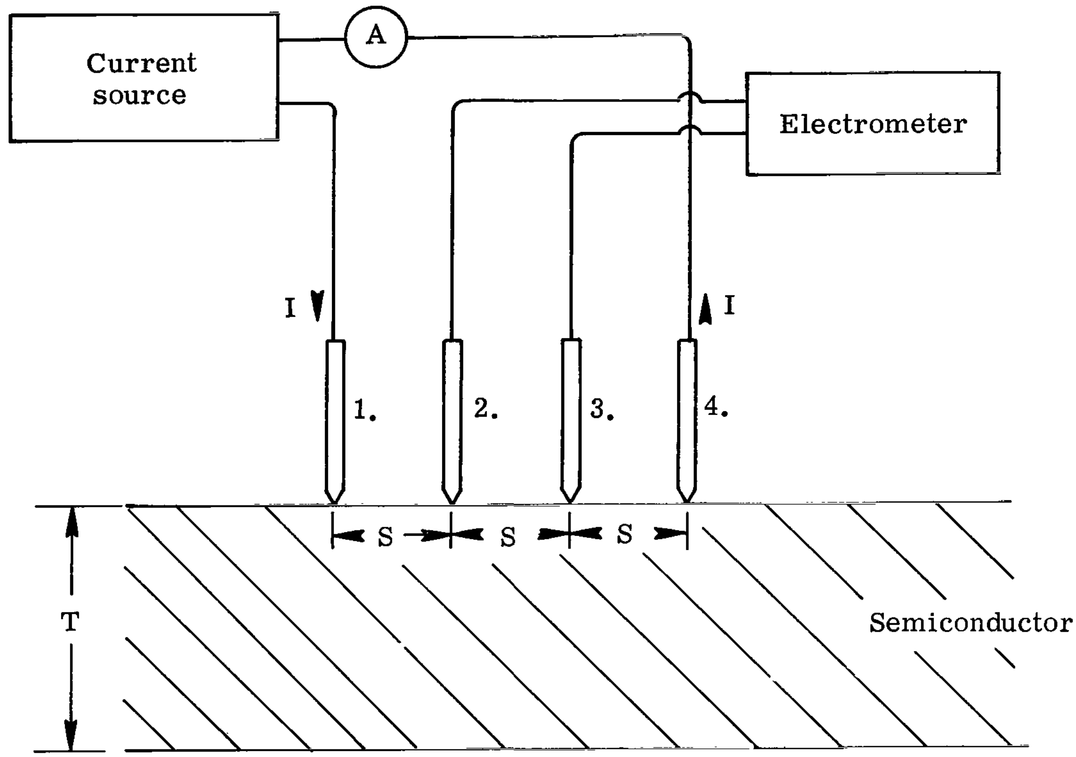

Figure 2.- Schematic diagram of four-point probe and associated circuit in contact with semiconductor surface.

If probe 1 introduces current $+I$ and if probe 4 introduces current $-I$, then the potential difference between probes 2 and 3 due to the current source at probe 1 will be

$$V^+ = \Phi(S,0) - \Phi(2S,0)$$

and the potential difference between probes 2 and 3 due to the current source at probe 4 will be

$$V^- = -\left[\Phi(2S,0) - \Phi(S,0)\right]$$

The total potential difference V between the two inner probes will then be

$$V = V^+ + V^- = 2\left[\Phi(S,0) - \Phi(2S,0)\right] \tag{16}$$

Use of the present solution for the potential (eq. (15)) gives

$$V = \frac{2I}{\pi\sigma(0)}\left\{\frac{1}{T}\sum_{n=1}^{\infty}\frac{4n^2\pi^2}{4n^2\pi^2 + a^2T^2}\left[K_o\left(\frac{S}{2T}\sqrt{4n^2\pi^2 + a^2T^2}\right)\right.\right.$$

$$\left.\left. - K_o\left(\frac{S}{T}\sqrt{4n^2\pi^2 + a^2T^2}\right)\right] + \frac{a \ln \frac{1}{2}}{2\left(1 - e^{aT}\right)}\right\} \tag{17}$$

which can be solved for the conductivity at the surface $\sigma(0)$. The resulting equation is

$$\sigma(0) = \frac{I}{2\pi SV} F(a,S,T)$$

where

$$F(a,S,T) = \frac{4S}{T}\sum_{n=1}^{\infty}\frac{4n^2\pi^2}{4n^2\pi^2 + a^2T^2}\left[K_o\left(\frac{S}{2T}\sqrt{4n^2\pi^2 + a^2T^2}\right)\right.$$

$$\left. - K_o\left(\frac{S}{T}\sqrt{4n^2\pi^2 + a^2T^2}\right)\right] + \frac{2aS \ln \frac{1}{2}}{1 - e^{aT}} \tag{18}$$

The factor $F(a,S,T)$ is a correction factor taking into account the semiconductor thickness, the probe spacing, and the variation of the conductivity.

The equation for conductivity (eq. (18)) reduces, appropriately, to the equation for conductivity in a sheet conductor (ref. 4) for very small semiconductor thickness T and reduces to the equation for homogeneous bulk conductivity (ref. 1) when the coefficient a goes to zero. The summation term in $F(a,S,T)$ converges rapidly.

Figure 3 presents $F(a,S,T)$ as a function of semiconductor thickness T in terms of probe spacing S and for various values of the exponential coefficient a.

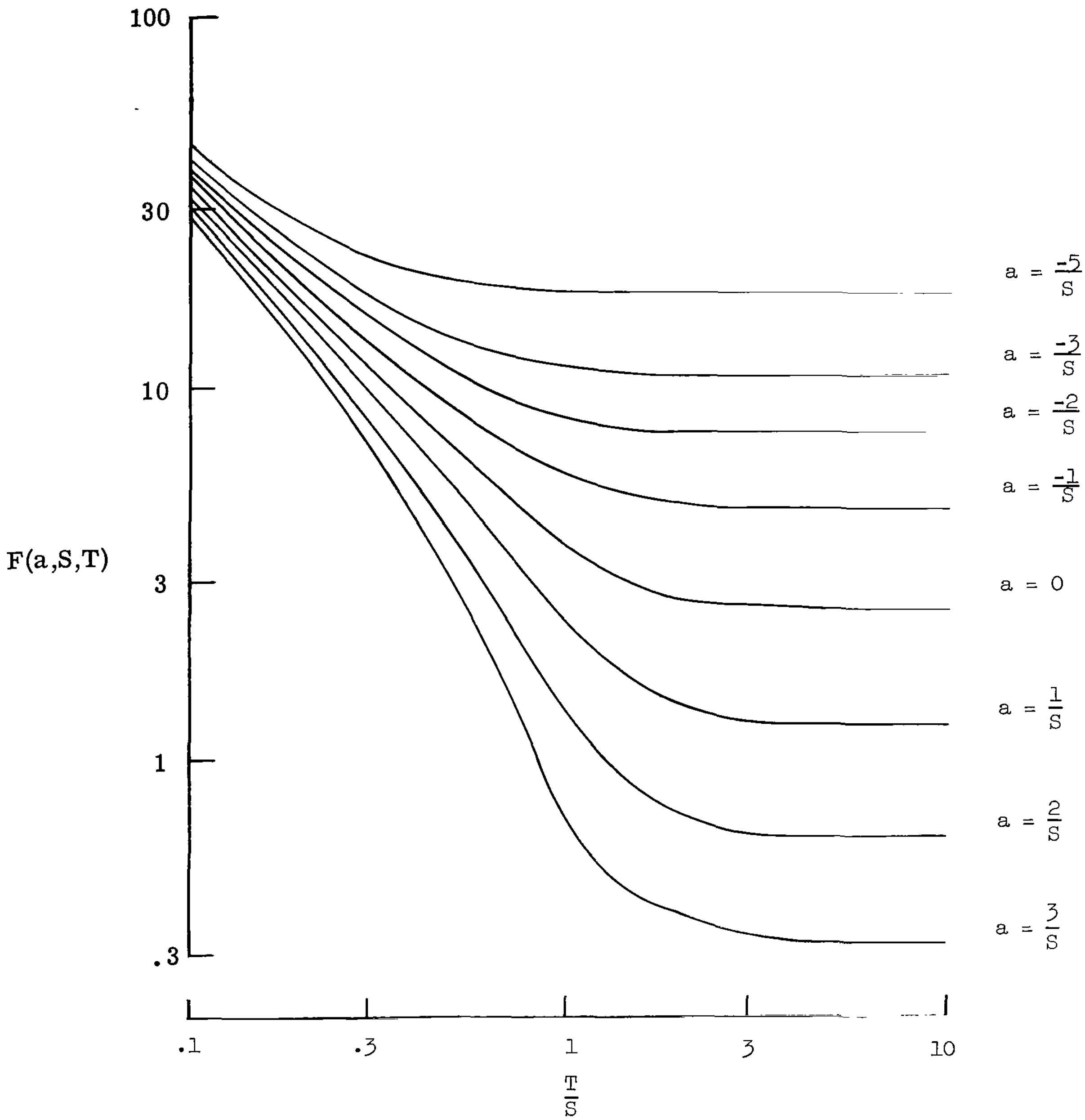

Figure 3.- Correction factor F(a,S,T) as a function of semiconductor thickness T in terms of probe spacing S and for various values of exponential coefficient a.

CONCLUDING REMARKS

The solution for the potential in a semiconductor with exponentially depth dependent conductivity allows for the first exact interpretation of four-point-probe resistivity measurements on semiconductors of nonhomogeneous conductivity. Although a semiconductor with a true exponential conductivity dependence may not be encountered, the treatment presented herein serves as a basis for approximation for other depth dependent conductivities such as those resulting from diffusion of impurities. This solution for the potential inside a semiconductor should also be of interest to designers of semiconductor devices since it describes the field variations within the bulk material.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va. September 1, 1971.

APPENDIX

ELECTRIC FIELD AT SMALL VALUES OF r_c

By Carl L. Fales, Jr.
Langley Research Center

A comment about the behavior of the r-component of the electric field in the limit of small r seems desirable.

In the limits $h \to 0$ and $r_c \to 0$,

$$E_r(r,z) = \frac{-\partial\Phi(r,z)}{\partial r}$$

$$= \frac{I}{2\pi\sigma(0)}\left\{\sum_{n=1}^{\infty} \frac{2}{T}\left[1 + \left(\frac{aT}{2n\pi}\right)^2\right]^{-1} e^{-az/2}\left[-\frac{d}{dr} K_o(\alpha r)\right]\left(\frac{aT}{2n\pi} \sin \frac{n\pi}{T} z + \cos \frac{n\pi}{T} z\right)\right.$$

$$\left. + \frac{1}{r} \frac{a}{e^{aT} - 1}\right\}$$

For r finite, there exists a number N such that $\alpha_n r >> 1$ for all $n > N$. Therefore,

$$-\frac{d}{dr} K_o(\alpha r) \approx \text{Constant} \times \frac{1}{\sqrt{\alpha_n r}} e^{-\alpha_n r}$$

and the series surely converges.

On the other hand, it is expected that $E_r(0,z) = 0$ for $z > 0$ by symmetry. However, in the limit of small r, $\frac{d}{dr} K_o(\alpha r) \to \frac{1}{r}$ and hence

$$E_r(r,z) = \frac{I}{2\pi\sigma(0)} \frac{1}{r}\left\{\sum_{n=1}^{\infty} \frac{2}{T}\left[1 + \left(\frac{aT}{2n\pi}\right)^2\right]^{-1} e^{-az/2}\left(\frac{aT}{2n\pi} \sin \frac{n\pi}{T} z + \cos \frac{n\pi}{T} z\right) + \frac{a}{e^{aT} - 1}\right\}$$

$$\equiv \frac{I}{2\pi\sigma(0)} \frac{1}{r} \sum_{n=1}^{\infty} \psi_n(z)$$

Clearly since $\psi_n(z) \not\to 0$ as $n \to \infty$, this series does not converge, at least in the sense of numerically summing a finite number of terms.

From equation (8), the operator $L = \frac{d^2}{dz^2} + a\frac{d}{dz}$ can be shown to be Hermitian (due to the self-adjointness of L and the homogeneous boundary conditions on $E_r(r,z)$). As a result, the set of eigenfunctions $\psi_n(z)$ of the eigenvalue equation

$$L\psi_n(z) = -\alpha^2\psi_n \qquad (n = 1, 2, \ldots, \infty)$$

are orthonormal with respect to the weighting function e^{az} and form a complete set (for functions obeying the same homogeneous boundary conditions). Hence, the representation for the Dirac delta function can be written as

$$\delta(z - z') = \sum_{n=1}^{\infty} e^{az'}\,\psi_n(z)\,\psi_n(z') \qquad (\psi_n \text{ real})$$

or

$$\delta(z) = \sum_{n=1}^{\infty} \psi_n(0)\,\psi_n(z)$$

The properly normalized eigenfunctions are

$$\psi_0(z) = \left(\frac{a}{e^{aT} - 1}\right)^{1/2}$$

and

$$\psi_n(z) = \left\{\frac{T}{2}\left[1 + \left(\frac{aT}{2n\pi}\right)^2\right]\right\}^{1/2} e^{-az/2}\left(\frac{aT}{2n\pi}\sin\frac{n\pi}{T}z + \cos\frac{n\pi}{T}z\right)$$

Therefore,

$$\delta(z) = \sum_{n=1}^{\infty} \frac{2}{T}\left[1 + \left(\frac{aT}{2n\pi}\right)^2\right]^{-1} e^{-az/2}\left(\frac{aT}{2n\pi}\sin\frac{n\pi}{T}z + \cos\frac{n\pi}{T}z\right) + \frac{a}{e^{aT} - 1}$$

and

$$\lim_{r\to 0} E_r(r,z) = \frac{I}{2\pi\sigma(0)}\,\delta(z)$$

Thus, $E_r(0,z) = 0$ for $z > 0$ as had been expected on physical grounds. Also, the component of current density

$$J_r(r,z) = \frac{I}{2\pi r}\,\delta(z)$$

gives the correct total current

$$\lim_{r \to 0} \int 2\pi r\,dz\,J_r(r,z) = I$$

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