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AN APPROXIMATE SOLUTION FOR THE RADAR ECHO PULSE RESPONSE FOR PLANETARY RADAR ALTIMETERS

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SUMMARY

An approximate solution for the radar echo pulse response has been developed to compute the expected return pulse amplitude and shape for planetary radar altimeters. Results obtained with the solution show excellent agreement with results obtained by numerical integration; in addition, this present solution requires much less computer time. The new solution is applicable over a wider range of altitudes and pulse widths than the solution previously presented by Harrington and Stanley in NASA TN D-5220; the present solution is also more accurate at low altitudes and for long pulse widths.

INTRODUCTION

For pulse-type radar altimeters, the arrival time of the beginning of the received-pulse leading edge is a measure of the shortest distance to the surface. This time of arrival, however, is difficult to measure because most pulse-radar-altimeter range trackers measure the arrival time of some other point on the received pulse, frequently some point on the leading edge. However, the shape and rise time of the leading edge is a function of the surface backscatter characteristics, which vary with incidence angle and terrain. These variations in the shape of the return pulse produce errors in the altitude measurements known as terrain bias errors.

Knowledge of the radar echo pulse response as a function of altitude and surface backscatter characteristics can be used to determine variations in the altimeter terrain bias error. In reference 1 the radar pulse response was derived using the model of reference 2 for the surface backscatter characteristics. The solution, as shown in equation (C4) of reference 1 and in equation (A34) of appendix A of this report, is an approximation which is valid for certain ranges of normalized time and altitude. In some cases it is desirable to know the response under conditions which violate the assumptions made in reference 1. Therefore, the purpose of this report is to offer a more general solution which is less restrictive than that of reference 1, as well as to make a comparison between predicted radar return waveforms for the two solutions.

For the convenience of the reader, a slightly modified version of Harrington and Stanley's derivation of the radar echo step response (ref. 1) is presented in appendix A of this report, and a review of Muhleman's model for the radar backscatter function (ref. 2) is presented in appendix B.

SYMBOLS

For simplicity, the letters F and g have not been modified when expressed as functions of time. For example, the quantities $F(\tau)$ and $g(\tau)$ are obtained by replacing the angles θ and ϕ, respectively, by their appropriate functions of time. The same convention applies when the time scale is normalized for F, g, and S.

A,B,C,D,E coefficients in partial fraction expansion

a normalized altitude $\left(\frac{\text{Radar altitude}}{\text{Radius of planet}}\right)$

$b = \sqrt{1 + a}$

c speed of light, 2.998×10^8 meters/second (9.836×10^8 feet/second)

$F(\theta)$ normalized Muhleman backscatter function (from ref. 2) expressed in terms of incidence angle

$F(\tau)$ function obtained from $F(\theta)$ when θ is related to time

$F(v)$ function obtained from $F(\tau)$ by normalizing time scale

$f_H(h)$ probability density function for h

$f_L(l)$ probability density function for l

$f_{HL}(h,l)$ joint probability density function for h and l

G absolute antenna gain

G_o maximum antenna gain

$g(\phi)$ normalized antenna gain function

$g(\tau)$	function obtained from $g(\phi)$ when ϕ is related to time
$g(v)$	function obtained from $g(\tau)$ by normalizing time scale
H	radar altitude, meters (feet)
h	vertical length of a scattering element
$h(\tau)$	received signal resulting from transmitted impulse
K	amplitude factor for pulse echo
l	horizontal length of a scattering element
M	amplitude factor, $(1 + a)\alpha^2 K$
$P(v)$	pulse response, received signal resulting from transmitted pulse
P_r	received power, watts
P_t	transmitted power, watts
R	radius of planet, meters (feet)
r	distance between radar and arbitrary point on surface, meters (feet)
$S(t)$	step response, received signal resulting from transmitted step function
T	two-way time delay between radar and closest point on surface, seconds
t	time, seconds
t_p	transmitted pulse width, seconds
$u(v)$	unit step function
v	normalized time variable
w	transformation variable used in integration

x distance in excess of H between radar and planet along arbitrary ray, meters (feet)

α mean surface slope, σ_h/σ_l

γ planetary angle, degrees

θ incidence angle, degrees

λ wavelength, centimeters

ρ polar coordinate

σ radar cross section, meters^2 (feet^2)

σ_h standard deviation of h

σ_l standard deviation of l

σ_o radar backscatter function, or radar cross section per unit area

$\hat{\sigma}_o$ value of σ_o at normal incidence

τ dummy time variable

τ_L time of return from horizon of transmitted impulse

ϕ look angle, degrees

GENERAL DEVELOPMENT

The general solution for the radar echo step response

$$S(v) = K \int_0^v \frac{g^2(v)\ F(v)}{(1 + v)^3}\,dv$$

is obtained from equation (A21) of the appendix. Equation (A33)

$$S(v) = K \int_0^v \frac{g^2(v)\,dv}{(1 + v)\left[1 + \left(1 + \frac{a}{2}\right)\left(1 + \frac{v}{4}\right)\frac{\sqrt{2v}}{\alpha}\right]^3}$$

gives Harrington and Stanley's expression (ref. 1) for the step response in integral form (assuming $v << 1$ and $a \leqq 1$) where

$$v = \frac{\tau}{T} = \frac{x}{H}$$

and

$$a = \frac{H}{R}$$

The approximate solution to the integral for an omnidirectional antenna ($g(v) = 1$) is given in equation (A34):

$$S(v) = \frac{K\alpha^2}{(1 + v)\left(1 + \frac{a}{2}\right)^2}\left\{\frac{1}{2} - \frac{1}{\left[1 + \left(1 + \frac{a}{2}\right)\frac{\sqrt{2v}}{\alpha}\right]} + \frac{1}{2\left[1 + \left(1 + \frac{a}{2}\right)\frac{\sqrt{2v}}{\alpha}\right]^2}\right\}$$

The present development starts with equation (A24) of appendix A:

$$\sin \theta = \frac{1}{v + 1}\left\{v(v + 2)\left[1 + a - \frac{a^2 v(v + 2)}{4}\right]\right\}^{1/2}$$

from which will be derived an integral for the step response that is less restrictive than equation (A33). A solution in closed form is then found for the integral.

Completion of the multiplication in equation (A24) gives

$$\sin \theta = \frac{1}{1 + v}\sqrt{2v + v^2 + 2av + av^2 - a^2 v^2 - a^2 v^3 - \frac{a^2 v^4}{4}} \tag{1}$$

Since $\cos \theta = \sqrt{1 - \sin^2\theta}$

$$\cos \theta = \frac{1}{1 + v}\sqrt{1 - 2av - av^2 + a^2 v^2 + a^2 v^3 + \frac{a^2 v^4}{4}} \tag{2}$$

Under the conditions $v^2 + 2v \leqq 4\left(\frac{1 + a}{a^2}\right)10^{-2}$ and $v^2 + 2v \leqq \frac{10^{-2}}{a}$, $\sin \theta$ can be approximated as

$$\sin \theta = \frac{\sqrt{(2v + v^2)(1 + a)}}{1 + v} = \frac{b\sqrt{2v + v^2}}{1 + v} \tag{3}$$

and $\cos\,\theta$ can be approximated as

$$\cos\,\theta = \frac{1}{1 + v} \tag{4}$$

where $b = \sqrt{1 + a}$. The condition $v^2 + 2v \leqq \frac{10^{-2}}{a}$ is more restrictive than $v^2 + 2v \leqq 4\left(\frac{1 + a}{a^2}\right)10^{-2}$ and will be used henceforth.

Using the foregoing approximations for $\sin\,\theta$ and $\cos\,\theta$, equations (3) and (4) can be substituted into equation (A22) for Muhleman's model to obtain

$$F(v) = \frac{\alpha^3(1 + v)^2}{\left(\alpha + b\sqrt{2v + v^2}\right)^3} \tag{5}$$

where α is the mean surface slope. Substituting equation (5) into equation (A21) gives the step response

$$S(v) = K\alpha^2 \int_0^v \frac{\alpha g^2(v)\,dv}{(1 + v)\left(\alpha + b\sqrt{2v + v^2}\right)^3} \tag{6}$$

In equation (6)

$$K = \frac{G_o^2\lambda^2\hat{\sigma}_o}{32\pi^2H^2(1 + a)} \tag{7}$$

as in equation (A23) of reference 1. Now, let

$$M = (1 + a)\alpha^2K = \frac{G_o^2\lambda^2\hat{\sigma}_o\alpha^2}{32\pi^2H^2} \tag{8}$$

and the step response becomes

$$S(v) = \frac{M}{1 + a}\int_0^v \frac{\alpha g^2(v)}{(1 + v)\left(\alpha + b\sqrt{2v + v^2}\right)^3}\,dv \tag{9}$$

As a result, equation (9), the solution in integral form, corresponds to equation (C2) of Harrington and Stanley (ref. 1). At this point, their solution is slightly more accurate for $v < 1$ whereas equation (9) is slightly more accurate for $v \geqq 1$. However, Harrington and Stanley continued for $g(v) = 1$ by finding an approximate solution (eq. (C4)) to equation (C2). (See ref. 1.) On the other hand, equation (9) can be solved exactly as follows:

By letting $w = \sqrt{2v + v^2}$, equation (9) becomes

$$S(w) = \frac{M}{1 + a} \int_0^w \left[\frac{\alpha w}{(1 + w^2)(\alpha + bw)^3}\right] dw \tag{10}$$

The integrand can be expanded in partial fractions to give

$$S(w) = \frac{M}{1 + a} \int_0^w \left[\frac{Aw + B}{w^2 + 1} + \frac{C}{(bw + \alpha)^3} + \frac{D}{(bw + \alpha)^2} + \frac{E}{bw + \alpha}\right] dw \tag{11}$$

where the coefficients must satisfy the following set of equations:

$$\begin{bmatrix} b^3 & 0 & 0 & 0 & b^2 \\ 3\alpha b^2 & b^3 & 0 & b & 2\alpha b \\ 3\alpha^2 b & 3\alpha b^2 & 1 & \alpha & (\alpha^2 + b^2) \\ \alpha^3 & 3\alpha^2 b & 0 & b & 2\alpha b \\ 0 & \alpha^3 & 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \alpha \\ 0 \end{bmatrix} \tag{12}$$

Equation (11) can now be integrated to give

$$S(w) = \frac{M}{1 + a} \left[\frac{A}{2} \log (w^2 + 1) + B \tan^{-1} w - \frac{C}{2b(\alpha + bw)^2} - \frac{D}{b(\alpha + bw)} + \frac{E}{b} \log (bw + \alpha) + \frac{C}{2\alpha^2 b} + \frac{D}{\alpha b} - \frac{E}{b} \log \alpha\right] \tag{13}$$

Solving equation (12) for the coefficients A, B, C, D, and E and substituting equation (12) into equation (13) yields the following solution for the integral in equation (10):

$$S(w) = \frac{M}{1 + a} \left[\frac{\alpha^2(\alpha^2 - 3b^2)}{2(\alpha^2 + b^2)^3} \log (w^2 + 1) + \frac{\alpha b(3\alpha^2 - b^2)}{(\alpha^2 + b^2)^3} \tan^{-1} w + \frac{\alpha^2}{2(\alpha^2 + b^2)(\alpha + bw)^2} + \frac{\alpha(\alpha^2 - b^2)}{(\alpha^2 + b^2)^2 (\alpha + bw)} - \frac{\alpha^2(\alpha^2 - 3b^2)}{(\alpha^2 + b^2)^3} \log (\alpha + bw) - \frac{1}{2(\alpha^2 + b^2)} - \frac{\alpha^2 - b^2}{(\alpha^2 + b^2)^2} + \frac{\alpha^2(\alpha^2 - 3b^2)}{(\alpha^2 + b^2)^3} \log \alpha\right] \tag{14}$$

The solution for the step response in terms of normalized time and altitude can now be found by rearranging equation (14) and transforming back to the original variables to give the final result:

$$S(v) = \frac{M}{1 + a}\left(\frac{\alpha^2(\alpha^2 - 3a - 3)}{2(\alpha^2 + a + 1)^3}\log\left\{\frac{\alpha^2(v^2 + 2v + 1)}{\left[\alpha + \sqrt{(2v + v^2)(1 + a)}\right]^2}\right\}\right.$$

$$+ \frac{\alpha\sqrt{1 + a}(3\alpha^2 - a - 1)}{(\alpha^2 + a + 1)^3}\tan^{-1}\left(\sqrt{2v + v^2}\right)$$

$$+ \frac{\alpha^2}{2(\alpha^2 + a + 1)\left[\alpha + \sqrt{(2v + v^2)(1 + a)}\right]^2}$$

$$\left. + \frac{\alpha(\alpha^2 - a - 1)}{(\alpha^2 + a + 1)^2\left[\alpha + \sqrt{(2v + v^2)(1 + a)}\right]} - \frac{3\alpha^2 - a - 1}{2(\alpha^2 + a + 1)^2}\right) \tag{15}$$

As in reference 1, the pulse response can now be found by taking the difference of two step responses, that is,

$$P(v) = S(v)\,u(v) - S\left(v - \frac{t_p}{T}\right)u\left(v - \frac{t_p}{T}\right) \tag{16}$$

where t_p is the transmitted pulse width.

Restrictions

As previously stated, Harrington and Stanley (ref. 1) assumed $a \leqq 1$ and $v << 1$ (e.g., $v \leqq 10^{-2}$). The solution in equation (15) assumes $v^2 + 2v \leqq \frac{10^{-2}}{a}$. These restrictions are plotted in figure 1 in terms of the normalized variables a and v to show the domains in which the two solutions are valid. In figure 2 the restrictions are plotted in terms of the variables t (time) and H (altitude) for a specific case, namely the planet Mars. As shown by figure 2, the solution of equation (15) is valid for long pulse widths whereas the previous solution of Harrington and Stanley (ref. 1) is not.

RESULTS

In order to demonstrate the utility of the present solution and to provide examples for comparison between this solution and the previous one of Harrington and Stanley (ref. 1), the expected values of the received pulse amplitude as a function of time were calculated for several values of altitude H and the mean surface slope α. The results are plotted in figures 3 to 5. (A rectangular transmitted pulse of 10-μsec duration and a planet radius of 3370 km ($\approx 1.1 \times 10^7$ ft) were used in all cases, and square-law detection was assumed.) The figures show the results obtained using the present solution (eq. (15)), using the previous solution of Harrington and Stanley (eq. (A34)), and using numerical integration of the exact integral solution (eq. (A29)). (Integration intervals of 2.5 nsec for $\alpha = 1.0$ and 1.25 nsec for $\alpha = 0.01$ were used in the numerical integration.) Note that the planet radius used is that of Mars; thus, the restrictions on time and altitude shown in figure 2 are applicable to the examples in figures 3 to 5.

The results for an altitude of 1.52 km (5000 ft) and for α of 1.0 and 0.01 are shown in figures 3(a) and 3(b). Good agreement exists between the results of the present solution and the results of numerical integration, the only discernible difference occurring near the pulse peak for $\alpha = 0.01$ (fig. 3(b)). However, the previous results of Harrington and Stanley (ref. 1) are quite different in both cases. It can be seen from figure 2 that the previous restriction on time for $H = 1.52$ km (5000 ft) is exceeded at $t = 0.1$ μsec, or long before the pulse peak occurs. It can be seen from figure 3(b) that the previous solution begins to decrease for time less than one pulse width, but such a decrease does not occur physically. This anomaly accounts for the trailing-edge amplitude becoming negative in that case.

The results in figure 3(c) are for an altitude of 152 km (500 000 ft) and for $\alpha = 0.01$. In this case, Harrington and Stanley's previous restriction on time is not exceeded until $t = 10$ μsec; and their result closely agrees with the results of the other two solutions.

The amplitudes in figure 3 have all been normalized by dividing by the factor M. Therefore, comparisons of peak amplitude, as well as pulse shape, are valid among the three solutions for a given α and H.

In figure 4 the results for $\alpha = 1.0$ and $H = 1.52$ km (5000 ft) are plotted after the amplitude at the end of one pulse duration has been normalized to unity. Such a presentation emphasizes the differences in pulse shape among the three solutions.

The solutions for the same return pulse are plotted in figure 5, in this case after passage through a third-order low-pass Butterworth filter with a 3-dB cutoff frequency equal to $0.5/t_p$. The filtering was accomplished using the technique described by Harrington and Stanley in reference 1. A slight difference between the previous solution of Harrington and Stanley and the other two solutions can again be seen.

The advantage of the present solution over the previous solution of Harrington and Stanley is improved accuracy for cases where their restrictions on time and amplitude are violated. The advantage of the present solution over the solution by numerical integration is a reduction in computer time. By way of illustration, the computer central processing unit time required at $H = 1.52$ km (5000 ft) and $\alpha = 0.01$ was 4.9 seconds for the solution by numerical integration compared to 0.15 second for the present solution.

CONCLUSIONS

The solution presented can be used to compute the expected return pulse amplitude and shape for planetary radar altimeters, where the surface backscatter characteristics are described by Muhleman's model. The solution is valid over a wide range of altitude and pulse widths, and results obtained with this solution show excellent agreement with results obtained by numerically integrating the exact integral solution. Computations using this solution require significantly less computer time than computations using numerical integration. In addition, the solution is less restrictive on the normalized altitude and time than the solution previously presented in NASA TN D-5220 by Harrington and Stanley. Furthermore, the new solution is more accurate at low altitudes and for long pulse widths.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., July 28, 1971.

APPENDIX A

DERIVATION OF RADAR ECHO STEP RESPONSE

The derivation of the radar echo step response in this appendix is a slight modification of the one presented by Harrington and Stanley in reference 1. Since the electromagnetic wave transmission properties of a vacuum and of the atmosphere and the reflection characteristics of a planetary surface are independent of power (for reasonable power levels), the atmosphere and surface can be treated as a linear system, and linear system theory can be applied to the problem of finding the power received at the altimeter (system output) in response to a transmitted pulse (system input). The first step is to find the system impulse response, that is, the received power as a function of time due to an impulse transmitted at time $t = 0$. Note that in this case the input and output functions are in units of power rather than amplitude, as is the more usual case.

From the fundamental radar range equation the received power is given by

$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 r^4} \tag{A1}$$

For the radar altimeter the geometry of the model is shown in figure 6. Consider a unit impulse transmitted at time $t = 0$. The energy reflected by the annular ring dA, which is at a constant range $r = H + x$ from the altimeter, will be received by the altimeter at time $t = T + \tau$, where

$$T = \frac{2H}{c} \tag{A2}$$

and

$$\tau = \frac{2x}{c} \tag{A3}$$

The magnitude of the energy received from dA is

$$h(\tau)\,d\tau = \frac{G^2 \lambda^2 \sigma_o\,dA}{(4\pi)^3 (H + x)^4} = \frac{G^2 \lambda^2 \sigma_o\,dA}{(4\pi)^3 H^4 \left(1 + \frac{\tau}{T}\right)^4} \tag{A4}$$

Now the antenna gain is a function of the look angle ϕ and can be expressed as

$$G = G_o g(\phi) \tag{A5}$$

where

$$0 \leqq g(\phi) \leqq 1$$

For an isotropic surface the backscatter function σ_o is a function of the incidence angle θ, but not of the azimuth angle. Thus the function σ_o can be expressed as

$$\sigma_o = \hat{\sigma}_o F(\theta) \tag{A6}$$

where $\hat{\sigma}_o$ is the backscatter per unit area at normal incidence. The differential area dA is

$$dA = 2\pi R^2 \sin\gamma \, d\gamma \tag{A7}$$

Combining equations (A4) to (A7) results in

$$h(\tau)\, d\tau = \frac{G_o{}^2 g^2(\phi)\, \lambda^2 \hat{\sigma}_o F(\theta)\, R^2 \sin\gamma \, d\gamma}{32\pi^2 H^4 \left(1 + \frac{\tau}{T}\right)^4} \tag{A8}$$

Applying the law of cosines to triangle POQ in figure 6 gives

$$\cos\gamma = 1 - \frac{x^2 + 2xH}{2R(H + R)} \tag{A9}$$

By differentiating equation (A9),

$$\sin\gamma \, d\gamma = \frac{(x + H)\, dx}{R(H + R)} = \frac{H^2\left(1 + \frac{\tau}{T}\right) d\tau}{TR(H + R)} \tag{A10}$$

is obtained. Substituting equation (A10) into equation (A8), and letting

$$a = \frac{H}{R} \tag{A11}$$

and

$$K = \frac{G_o{}^2 \lambda^2 \hat{\sigma}_o}{32\pi^2 H^2 (1 + a)} \tag{A12}$$

gives finally

$$h(\tau)\, d\tau = \frac{K g^2(\phi)\, F(\theta)\, d\tau}{T\left(1 + \frac{\tau}{T}\right)^3} \tag{A13}$$

Dividing by $d\tau$ and expressing $g(\phi)$ and $F(\theta)$ as functions of τ gives the following system impulse response which corresponds to equation (A24) of reference 1:

$$h(\tau) = \begin{cases} \dfrac{Kg^2(\tau)\,F(\tau)}{T\left(1 + \dfrac{\tau}{T}\right)^3} & \left(0 \leqq \tau \leqq \tau_L\right) \\ 0 & \left(\tau < 0,\ \tau > \tau_L\right) \end{cases} \tag{A14}$$

where

$$\tau_L = T\left(-1 + \sqrt{1 + \frac{2}{a}}\right) \tag{A15}$$

The limit $\tau = 0$ is the time when the transmitted impulse first returns from the surface. The limit τ_L corresponds to reflection at the horizon ($\theta = 90^\circ$).

From linear-system theory, the power received from an arbitrary transmitted signal $P_t(t)$ can be found by convolution as

$$P_r(t) = \int_{-\infty}^{\infty} h(\tau)\,P_t(t - \tau)\,d\tau \tag{A16}$$

Upon substitution of equation (A14) this becomes

$$P_r(t) = \frac{K}{T}\int_0^{\tau_L} \frac{g^2(\tau)\,F(\tau)}{\left(1 + \dfrac{\tau}{T}\right)^3} P_t(t - \tau)\,d\tau \tag{A17}$$

To find the step response $S(t)$, let $P_t(t) = u(t)$. Then

$$S(t) = \frac{K}{T}\int_0^{\tau_L} \frac{g^2(\tau)\,F(\tau)}{\left(1 + \dfrac{\tau}{T}\right)^3} u(t - \tau)\,d\tau \tag{A18}$$

Equation (A18) can be written as follows:

$$S(t) = \begin{cases} \dfrac{K}{T}\displaystyle\int_0^{t} \frac{g^2(\tau)\,F(\tau)}{\left(1 + \dfrac{\tau}{T}\right)^3}\,d\tau & \left(t \leqq \tau_L\right) \\ \dfrac{K}{T}\displaystyle\int_0^{\tau_L} \frac{g^2(\tau)\,F(\tau)}{\left(1 + \dfrac{\tau}{T}\right)^3}\,d\tau & \left(t > \tau_L\right) \end{cases} \tag{A19}$$

Now, introduce the normalized time variable v, where

$$v = \frac{\tau}{T} = \frac{x}{H} \tag{A20}$$

Then

$$S(v) = \begin{cases} K \displaystyle\int_0^v \frac{g^2(v)\,F(v)}{(1+v)^3}\,dv & \left(v \leqq \dfrac{\tau_L}{T}\right) \\ K \displaystyle\int_0^{\tau_L/T} \frac{g^2(v)\,F(v)}{(1+v)^3}\,dv & \left(v > \dfrac{\tau_L}{T}\right) \end{cases} \tag{A21}$$

Thus far only backscatter functions which are an arbitrary function of the incidence angle θ have been considered. Henceforth, attention will be restricted to the Muhleman model (ref. 2) for $F(\theta)$, namely

$$F(\theta) = \frac{\alpha^3 \cos\theta}{(\sin\theta + \alpha\cos\theta)^3} \tag{A22}$$

Applying the law of sines to triangle QOP in figure 6 gives the relationship

$$\frac{\sin\gamma}{H + x} = \frac{\sin\theta}{H + R}$$

or

$$\sin\theta = \frac{1 + a}{a(1 + v)} \sin\gamma \tag{A23}$$

Using the identity $\sin\gamma = \sqrt{1 - \cos^2\gamma}$ and equation (A9) results in

$$\sin\theta = \frac{1}{1+v}\left\{v(v+2)\left[1 + a - \frac{a^2 v(v+2)}{4}\right]\right\}^{1/2} \tag{A24}$$

which corresponds to equation (B5) of reference 1.

Now let

$$A(v) = \left\{v(v+2)\left[1 + a - \frac{a^2 v(v+2)}{4}\right]\right\}^{1/2} \tag{A25}$$

so that

$$\sin\ \theta = \frac{A(v)}{1 + v} \tag{A26}$$

and

$$\cos\ \theta = \frac{1}{1 + v}\sqrt{(1 + v)^2 - A^2(v)} = \frac{B(v)}{1 + v} \tag{A27}$$

Then Muhleman's model becomes

$$F(v) = \frac{\alpha^3(1 + v)^2 B(v)}{\left[A(v) + \alpha B(v)\right]^3} \tag{A28}$$

By substituting equation (A28) for $F(v)$ in the integral of equation (A21), the step response becomes

$$S(v) = K\alpha^3 \int_0^v \frac{g^2(v)\ B(v)}{(1 + v)\left[A(v) + \alpha B(v)\right]^3}\, dv \tag{A29}$$

Thus far no approximations have been made. However, the integral in equation (A29) is quite unwieldy and has not been integrated in closed form. It could, of course, be integrated using numerical-integration techniques, and this was done to check the validity of the approximate solutions (as discussed in the main body of this report).

At this point, Harrington and Stanley (ref. 1) made some assumptions in order to obtain an analytic approximation to the step response. With the assumptions that $a \leqq 1$ and $v << 1$, Harrington and Stanley approximated $\sin\ \theta$ by equation (B7) of reference 1 as

$$\sin\ \theta \approx \frac{\left(1 + \frac{a}{2}\right)\left(1 + \frac{v}{4}\right)\sqrt{2v}}{1 + v} \tag{A30}$$

and $\cos\ \theta$ by equation (B6) of reference 1 as

$$\cos\ \theta \approx \frac{1}{1 + v} \tag{A31}$$

With these approximations the Muhleman backscatter function becomes (eq. (B8) of ref. 1)

$$F(v) \approx \frac{(1 + v)^2}{\left[1 + \left(1 + \frac{a}{2}\right)\left(1 + \frac{v}{4}\right)\frac{\sqrt{2v}}{\alpha}\right]^3} \tag{A32}$$

Substitution of equation (A32) into equation (A21) gives equation (C2) of reference 1 for the step response as follows:

$$S(v) \approx K \int_0^v \frac{g^2(v)\, dv}{(1 + v)\left[1 + \left(1 + \frac{a}{2}\right)\left(1 + \frac{v}{4}\right)\frac{\sqrt{2v}}{\alpha}\right]^3} \tag{A33}$$

For an omnidirectional antenna ($g(v) = 1$), the step response integral (eq. (A33)) was approximated in equation (C4) of reference 1 by the following analytic expression:

$$S(v) \approx \frac{K\alpha^2}{(1 + v)\left(1 + \frac{a}{2}\right)^2}\left\{\frac{1}{2} - \frac{1}{\left[1 + \left(1 + \frac{a}{2}\right)\frac{\sqrt{2v}}{\alpha}\right]} + \frac{1}{2\left[1 + \left(1 + \frac{a}{2}\right)\frac{\sqrt{2v}}{\alpha}\right]^2}\right\} \tag{A34}$$

APPENDIX B

REVIEW OF MUHLEMAN'S MODEL FOR THE RADAR BACKSCATTER FUNCTION

In reference 2, Muhleman derived an expression for the radar backscatter function from a random surface using the principles of geometric optics. Muhleman assumed that the surface consisted of individual plane scattering elements, all of which are long compared with the wavelength of the incident radiation. By geometric optics a scatterer will reflect in a given direction only if the normal to the scattering surface is coplanar with the incident and reflected rays and if the normal bisects the angle between these rays. For radar backscatter this amounts to the scattering element normal being parallel to the incident ray. Muhleman utilized the assumed statistical properties of the surface scattering elements to determine the probability of the normal meeting this requirement. In his derivation Muhleman assumed an isotropic surface, and he did not consider the effects of polarization.

The surface is characterized by length l parallel to the mean surface and a height h perpendicular to the mean surface. If h and l are assumed to be independent random variables, then joint probability density function $f_{HL}(h,l)$ can be expressed as the product of the individual density functions $f_H(h)$ and $f_L(l)$:

$$f_{HL}(h,l) = f_H(h)\, f_L(l) \tag{B1}$$

By transforming to polar coordinates ρ and θ and integrating over ρ, the probability density function for the normal can be obtained in terms of θ, the angle between the scattering element normal and the mean surface normal. The angle θ is also the angle of incidence relative to the mean surface.

Muhleman (ref. 2) derived the backscatter function for two cases. In one case he assumed a Gaussian distribution for the height variable h and a Rayleigh distribution for the length variable l. In the other case, he used exponential and Poisson distributions for the height and length variables, respectively. The resulting scattering functions for both cases were fitted by Muhleman to existing radar backscatter data from the Moon and Venus. Although both models exhibited good agreement with the data, the exponential model produced the best fit; hence, it was utilized by Harrington and Stanley (ref. 1) and in the work described in this report.

Muhleman's model for the backscatter function derived for the exponential case is

$$F(\theta) = \frac{\alpha^3 \cos\theta}{(\sin\theta + \alpha\cos\theta)^3} \tag{B2}$$

where

α mean surface slope, σ_h/σ_l

σ_h standard deviation of h

σ_l standard deviation of l

The function has been normalized to unity at vertical incidence $(\theta = 0)$; thus, the radar cross section per unit area can now be expressed as

$$\sigma_o(\theta) = \hat{\sigma}_o F(\theta) \tag{B3}$$

where $\hat{\sigma}_o$ is the cross section per unit area at vertical incidence.

In comparing his theoretical model to the radar reflection data for the Moon, Muhleman (ref. 2) adjusted the value of the parameter α to obtain the best fit between his model and the measured data. At wavelengths of 68 cm, 10 cm, and 3.6 cm, he obtained values for α of 0.145, 0.28, and 0.35, respectively. Using data from radar observations of Venus, Muhleman estimated that the value of α lies between 0.04 and 0.12 for that planet at a wavelength of 68 cm. In order to span the values of α obtained by Muhleman (ref. 2) and to facilitate comparison between Harrington and Stanley's results (ref. 1) and the results contained in this report, values for α of 0.01 and 1.0 were selected.

REFERENCES

1. Harrington, Richard F.; and Stanley, William D.: An Analysis of Terrain Bias Error in Planetary Radar Altimeters. NASA TN D-5220, 1969.

2. Muhleman, D. O.: Radar Scattering From Venus and the Moon. Astron. J., vol. 69, no. 1, Feb. 1964, pp. 34-41.

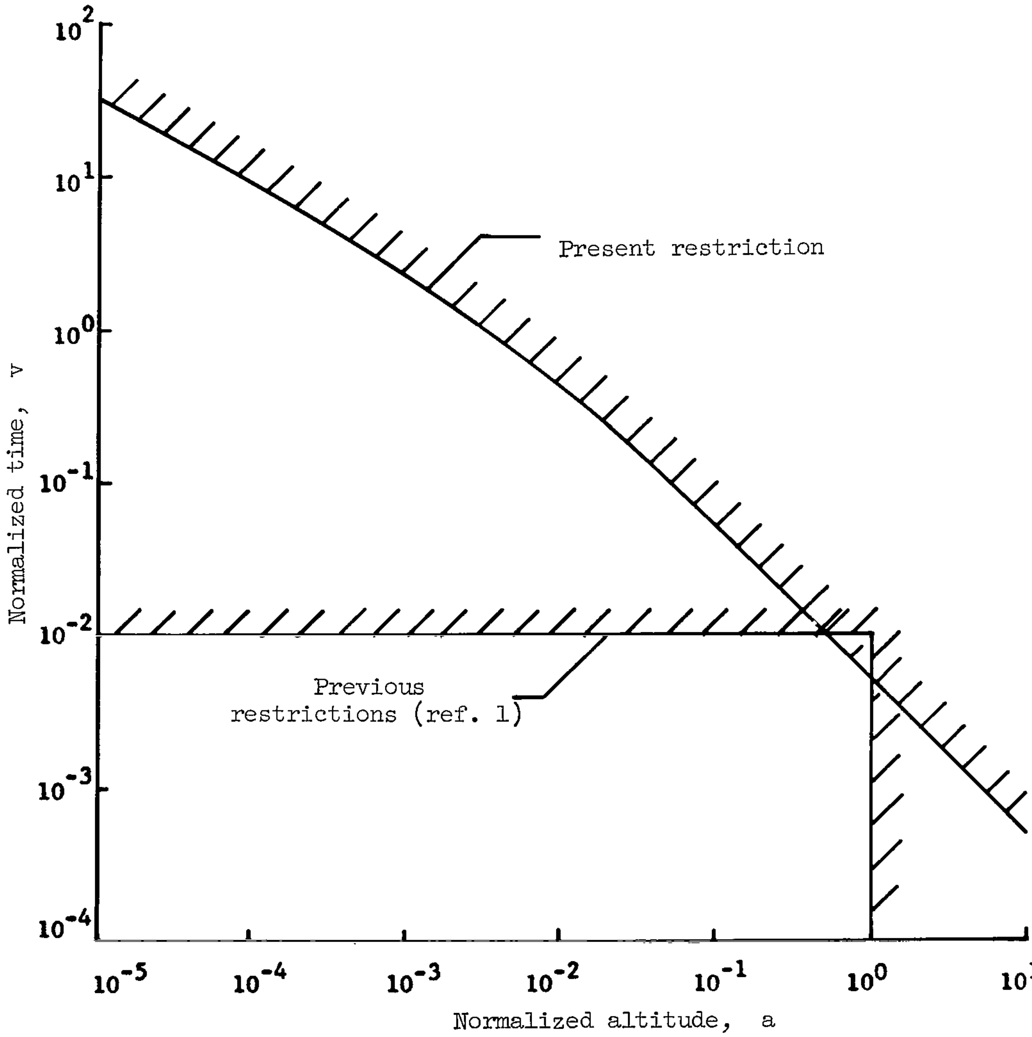


Figure 1.- Restrictions on normalized time and altitude.

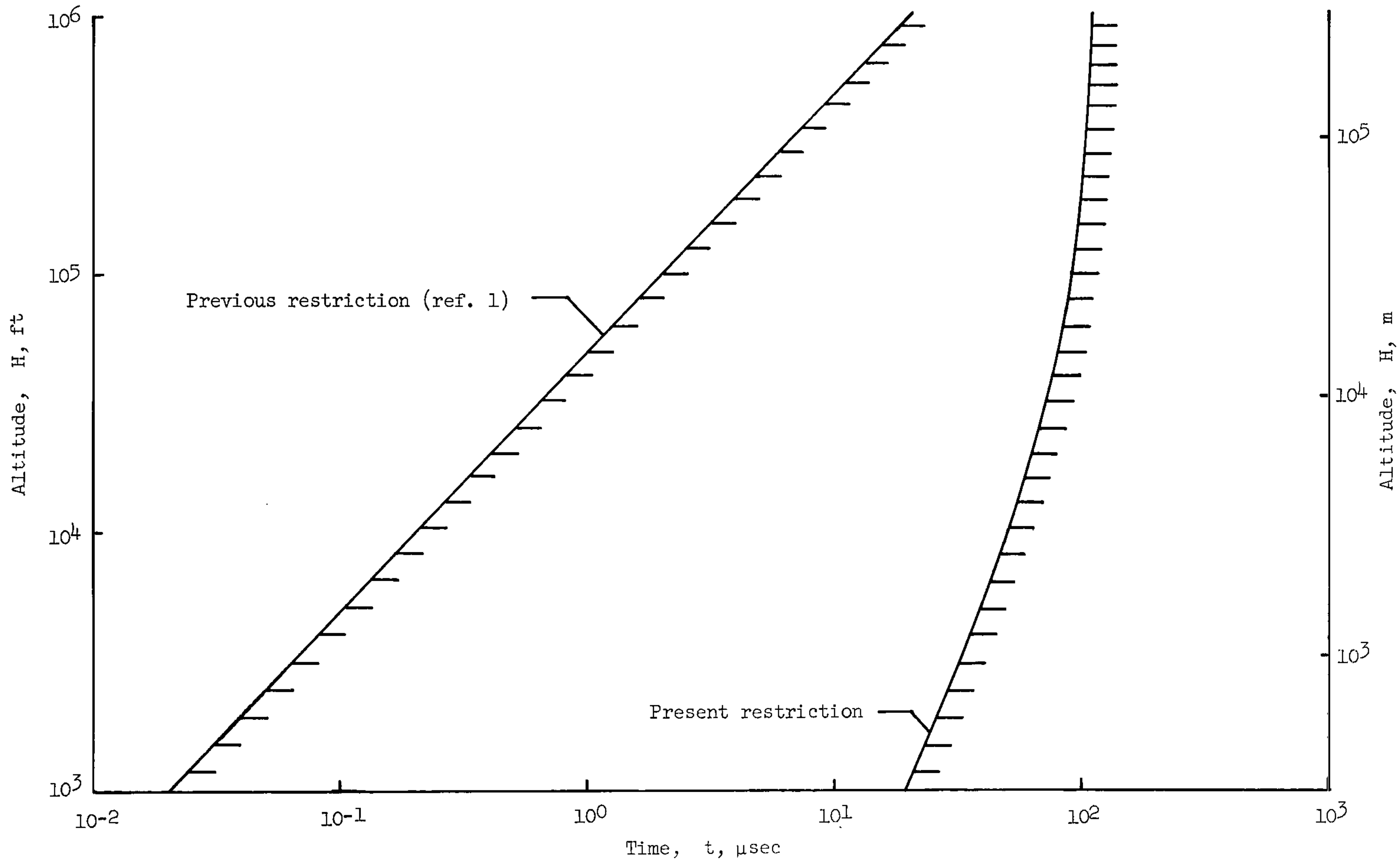
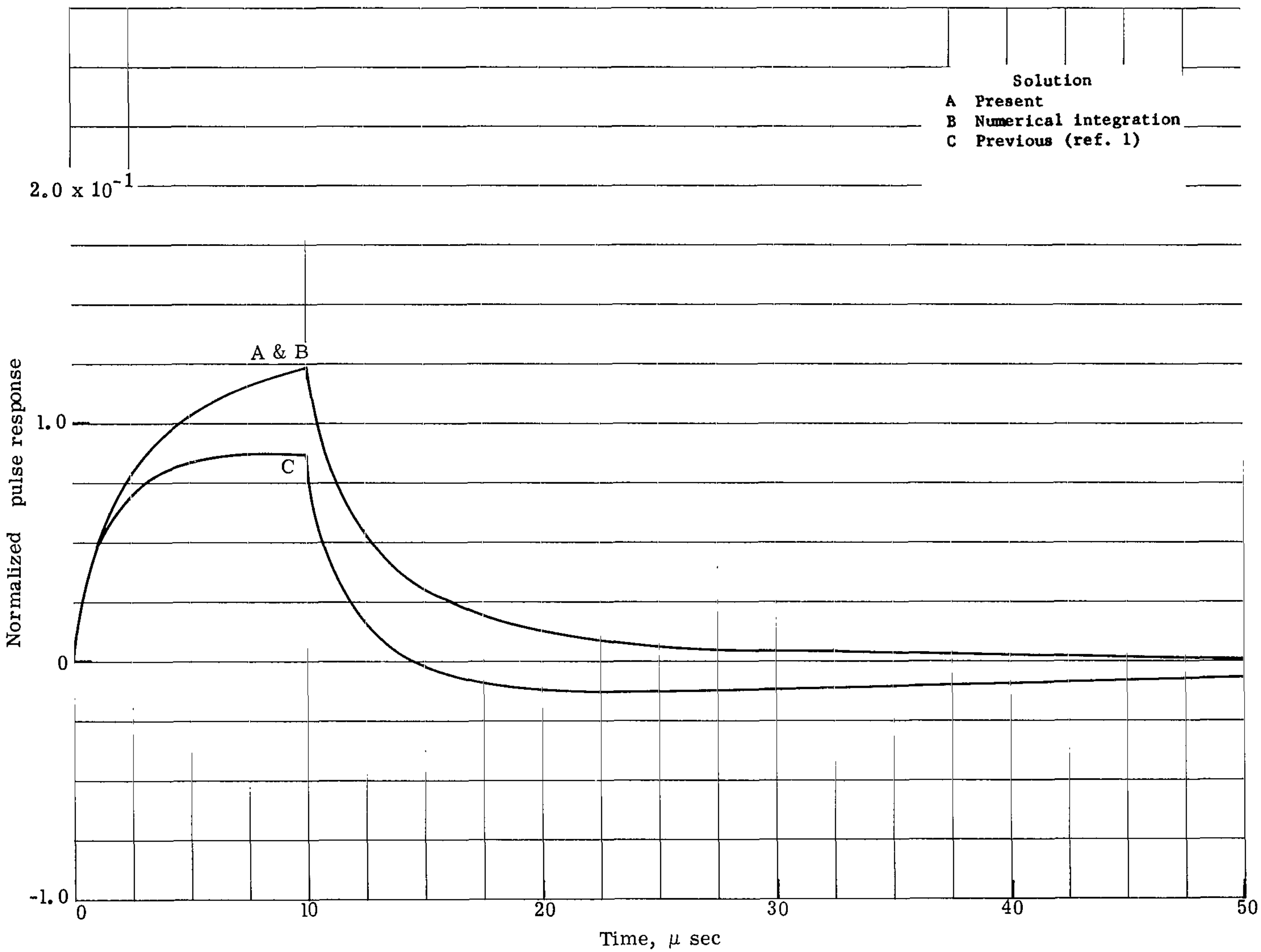
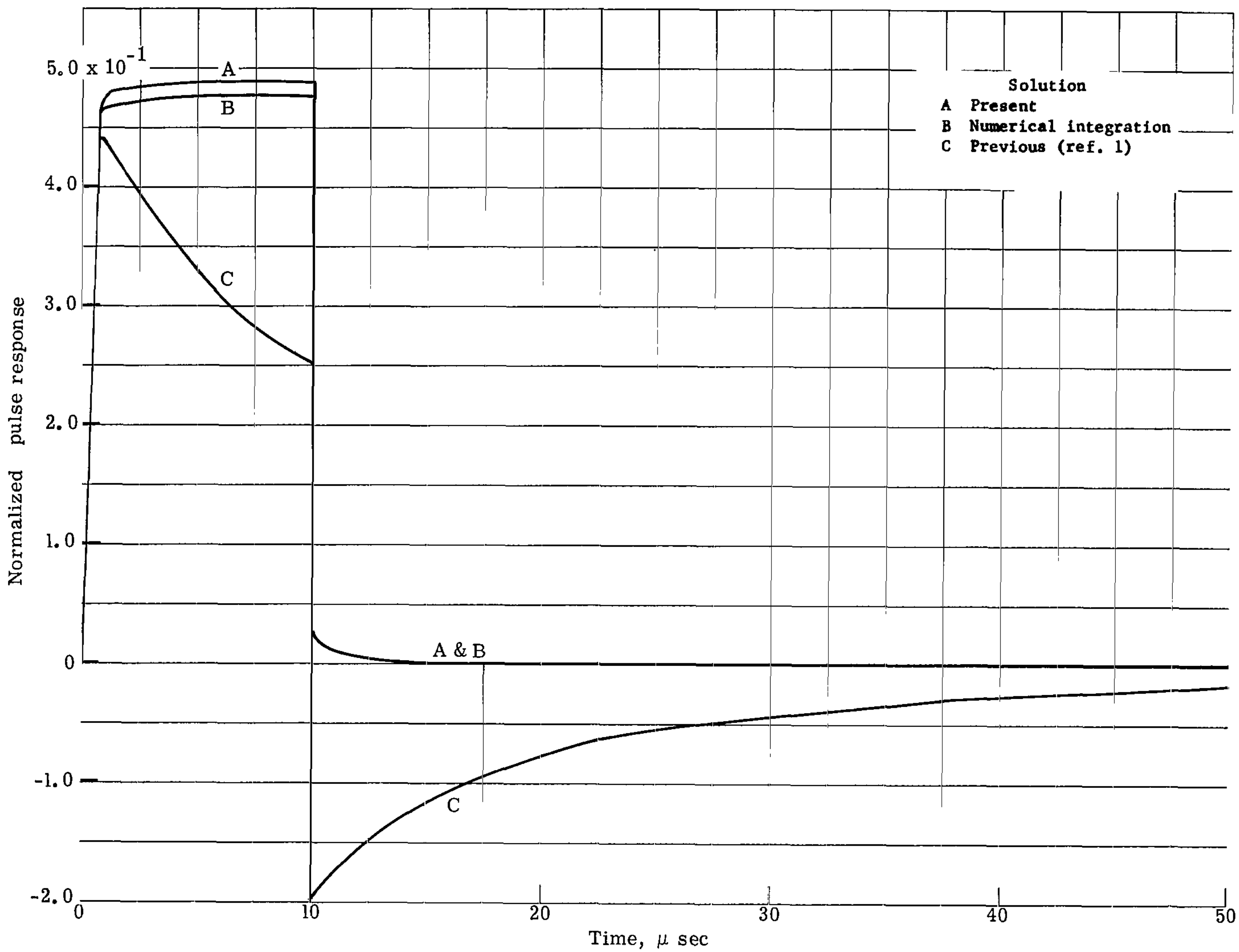


Figure 2.- Restrictions on time and altitude for Mars.



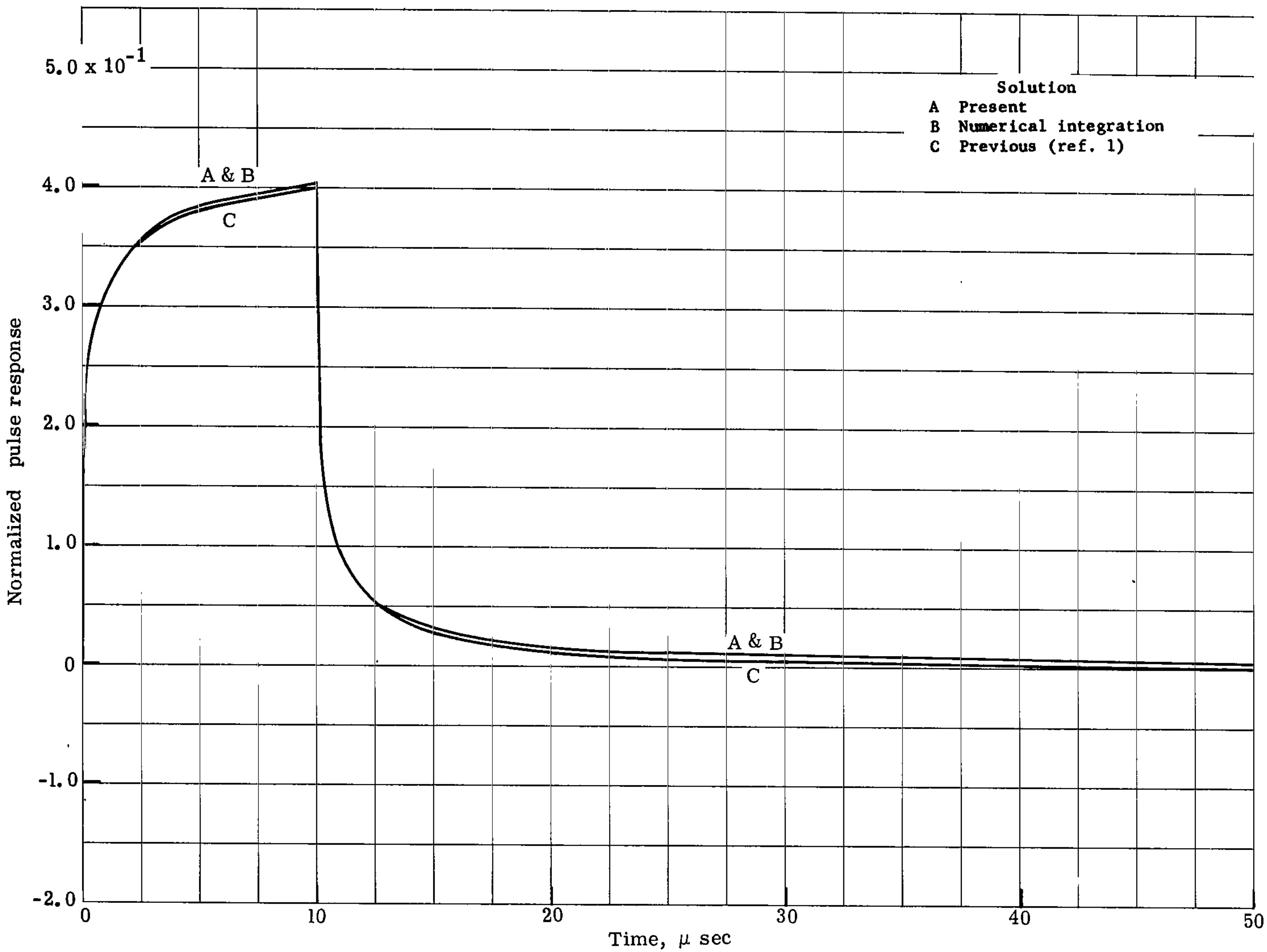
(a) $\alpha = 1.0$; H = 1.52 km (5000 ft).

Figure 3.- Comparison of return pulse shapes and amplitudes.



(b) $\alpha = 0.01$; H = 1.52 km (5000 ft).

Figure 3.- Continued.



(c) $\alpha = 0.01$; H = 152 km (500 000 ft).

Figure 3.- Concluded.

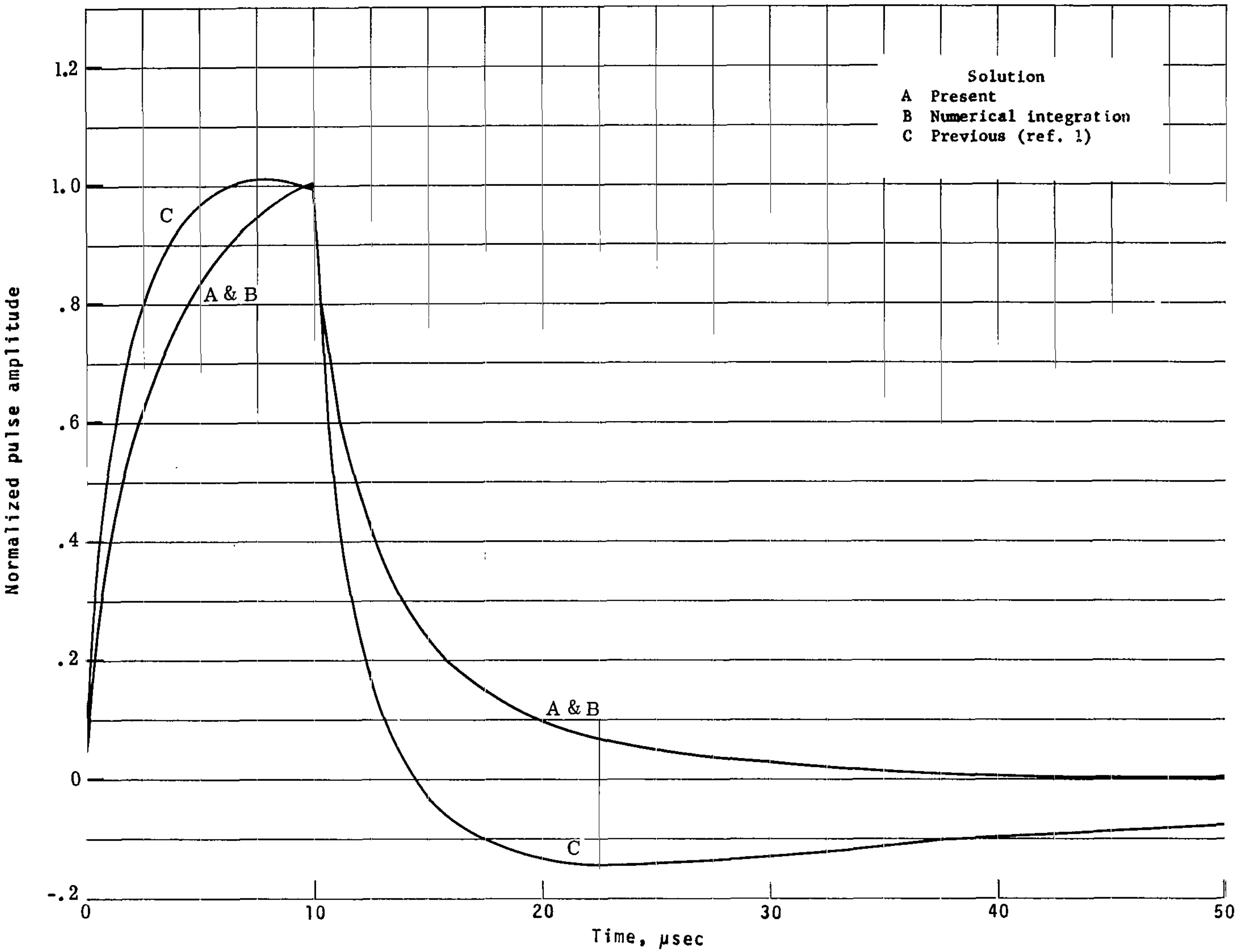


Figure 4.- Comparison of return pulse shapes for $\alpha = 1.0$ and H = 1.52 km (5000 ft).

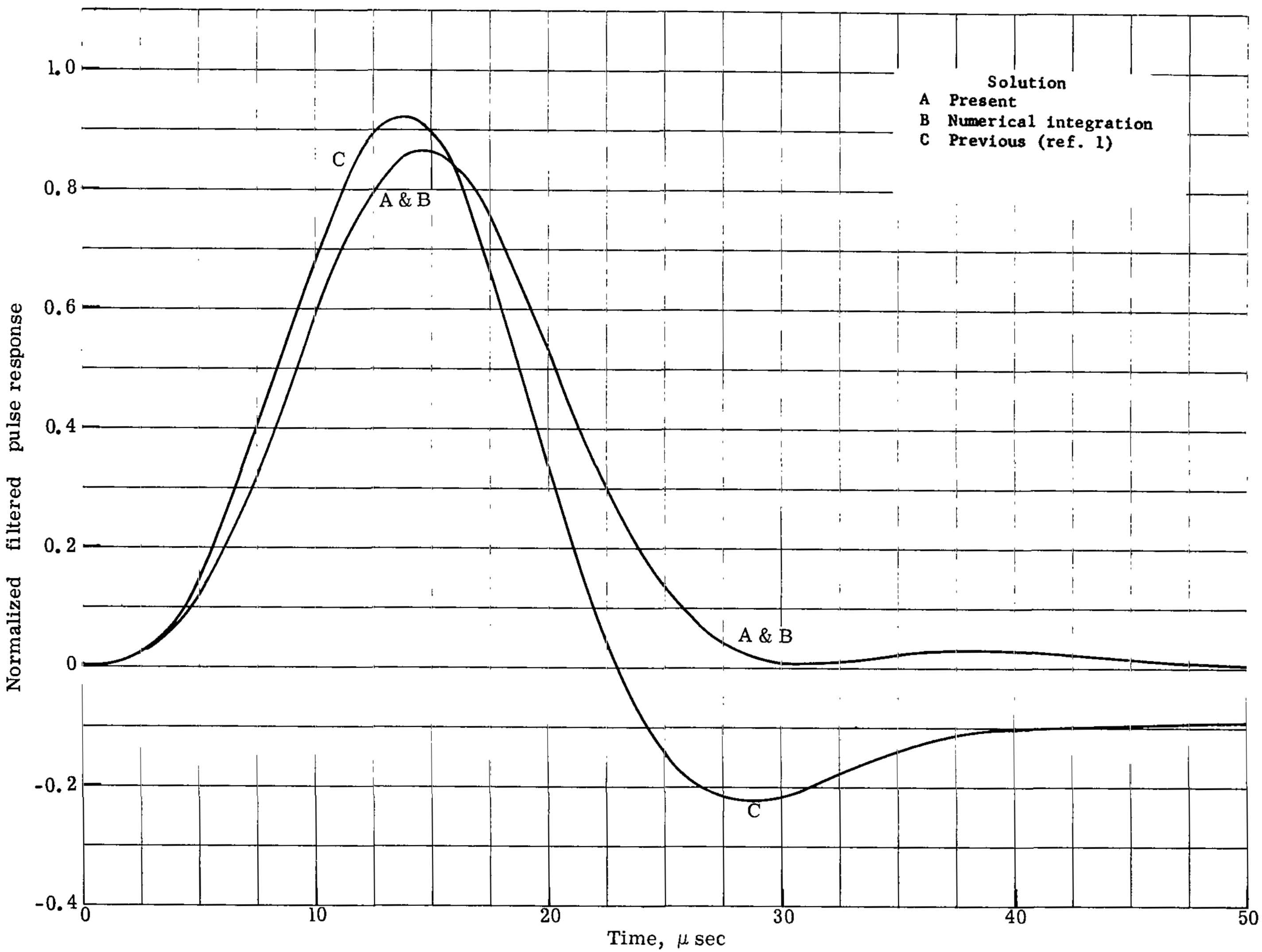


Figure 5.- Comparison of filtered pulses for $\alpha = 1.0$ and $H = 1.52$ km (5000 ft).

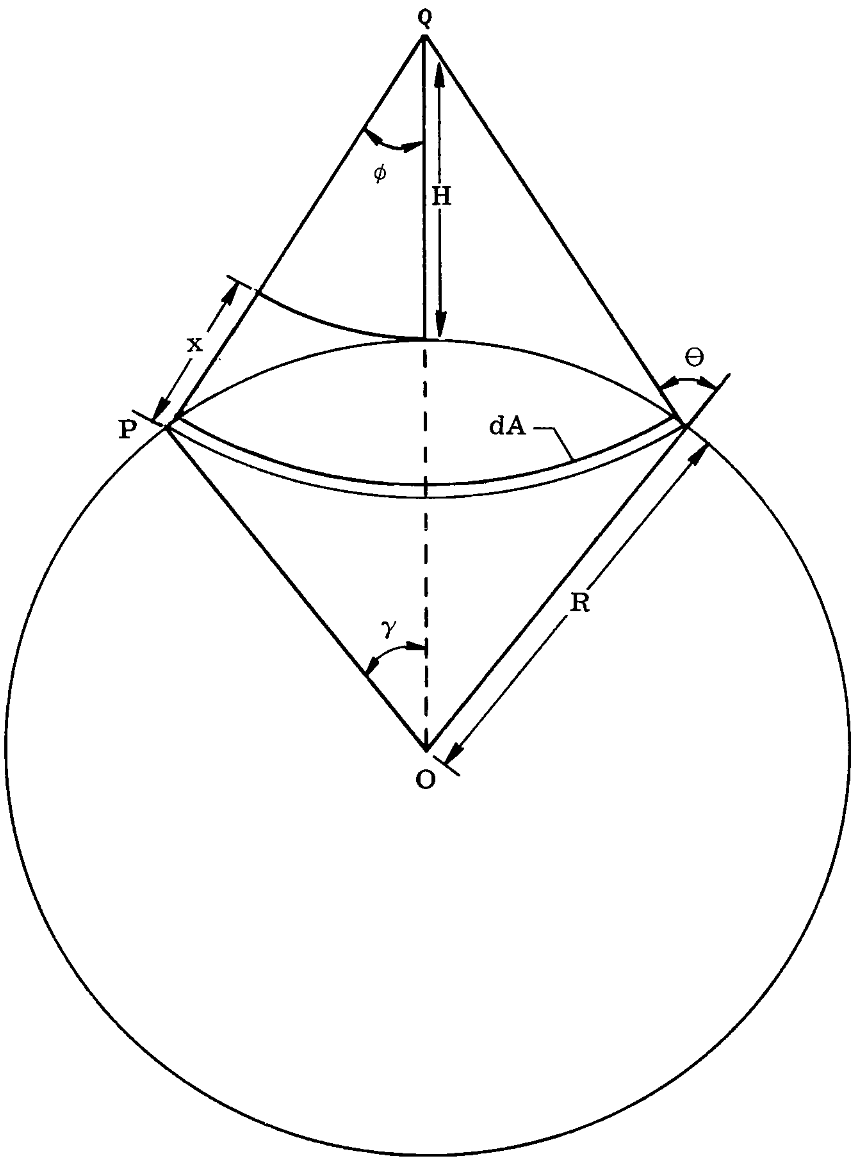


Figure 6.- Geometry of model of planetary radar altimeter.

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