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ABSTRACT

A detailed gravimetric geoid for the United States, which has been computed from a combination of satellite derived gravity data and local gravimetric measurements, is presented herein. The detailed geoid is shown to possess a precision of ±2 meters rms. Comparisons of the detailed geoid with results of Rapp, Fischer, and Rice are presented.

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A DETAILED GRAVIMETRIC GEOID FOR THE UNITED STATES

1. INTRODUCTION

The objective of the work described within this report was to compute a gravimetric geoid for the United States having an rms accuracy of ±2 meters. This objective was achieved through the use of a combination of surface and satellite gravimetric information. The detailed geoid presented here is believed to represent the first detailed gravimetric geoid of the entire United States ever published in the open literature. Rapp (1967) has previously published a combination geoid for a portion of the United States.

Comparisons of the computed gravimetric geoid with the results of Rapp, and with the astrogeodetic geoids of Fischer (1967) and Rice (1962, 1970), indicate that the anticipated ±2 meter rms accuracy has been achieved.

2. METHOD OF COMPUTATION

The geoidal undulation at any point P on the earth can be computed using the well known Stokes' formula:

$$N(\varphi, \lambda) = \frac{R}{4\pi G} \int_{\lambda'=0}^{2\pi} \int_{\varphi'=-\frac{\pi}{2}}^{\pi/2} \Delta g_T(\varphi', \lambda')\, S(\Psi) \cos \varphi' d\varphi' d\lambda' \qquad (1)$$

where:

φ, λ = The latitude and longitude, respectively, of the computation point.

φ', λ' = The latitude and longitude, respectively, of the variable integration point.

$N(\varphi, \lambda)$ = Geoid undulation at φ, λ.

R = Mean radius of the earth.

G = Product of the universal gravitational constant and the mass of the earth.

$\Delta g_T(\varphi', \lambda')$ = Free air gravity anomaly at the variable point φ', λ'.

$$S(\Psi) = \frac{1}{\sin(\Psi/2)} - 6 \sin(\Psi/2) + 1 - 5 \cos \Psi$$

$$- 3 \cos \Psi \ln [\sin(\Psi/2) + \sin^2(\Psi/2)$$

where

$$\Psi = \cos^{-1} [\sin \varphi \sin \varphi' + \cos \varphi \cos \varphi' \cos (\lambda - \lambda')]$$

In order to combine surface and satellite gravity data for geoid computation the earth is divided into two areas, a local area (A_1) surrounding the point P, and the remainder of the earth (A_2). Also the anomalous gravity in each area is partitioned into two parts represented by the symbols Δg_s and Δg_2. The Δg_s values are defined as that part of the anomalous gravity field which can be represented by the coefficients in a satellite derived spherical harmonic expansion of the gravitation potential. The 1969 SAO Standard Earth given in Table 1 was used in all computations described in this paper. The Δg_2 values are defined as the remainder of the anomalous gravity field. Using this division of the earths surface in two areas and of the anomalous gravity each into two components one can write equation (1) in the form:

$$N(\varphi, \lambda) = N_1 + N_2 + N_3 \qquad (2)$$

where

$$N_1 = \frac{R}{4\pi G} \left[\int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Delta g_s (\varphi', \lambda') \, S(\Psi) \cos \varphi' \, d\varphi' \, d\lambda' \right]$$

$$N_2 = \frac{R}{4\pi G} \left[\iint_{A_1} \Delta g_2 (\varphi', \lambda') \, S(\Psi) \cos \varphi' \, d\varphi' \, d\lambda' \right] \qquad (3)$$

$$N_3 = \frac{R}{4\pi G} \left[\iint_{A_2} \Delta g_2 (\varphi', \lambda') \, S(\Psi) \cos \varphi' \, d\varphi' \, d\lambda' \right]$$

The following paragraphs discuss how each of the three components presented in equation (3) is handled in the computations.

Given a set of satellite derived coefficients in the spherical harmonic expansion of the gravitational potential a number of methods exist for computation of the N_1 component of the geoid undulation.

Table 1

Normalized Coefficients for the SAO-69 Standard Earth Model

(Gaposchkin and Lambeck, 1970)

n	m	$\bar{C}_{nm}$	$\bar{S}_{nm}$	n	m	$\bar{C}_{nm}$	$\bar{S}_{nm}$
2	0	-0.4842×10^{-3}		13	5	9.6637×10^{-9}	-4.7760×10^{-8}
3	0	0.8025×10^{-6}		14	5	-2.5958×10^{-8}	-2.2344×10^{-8}
4	0	0.5310×10^{-6}		15	5	3.1624×10^{-8}	1.8250×10^{-9}
5	0	0.6935×10^{-7}		16	5	-4.4201×10^{-8}	3.2222×10^{-8}
6	0	-0.1392×10^{-6}		6	6	8.8693×10^{-8}	-7.4756×10^{-7}
7	0	0.9347×10^{-7}		7	6	-2.5847×10^{-7}	1.0209×10^{-7}
8	0	0.2862×10^{-7}		8	6	-5.3403×10^{-8}	2.5930×10^{-7}
9	0	0.2294×10^{-7}		9	6	2.4186×10^{-8}	2.2028×10^{-7}
10	0	0.7725×10^{-7}		10	6	-3.2486×10^{-8}	-2.0153×10^{-7}
11	0	-0.4212×10^{-7}		11	6	3.7517×10^{-8}	6.9005×10^{-9}
12	0	0.8400×10^{-8}		12	6	-2.3868×10^{-8}	-6.6770×10^{-10}
13	0	0.2367×10^{-7}		13	6	-8.3417×10^{-8}	5.9782×10^{-9}
14	0	0.1356×10^{-7}		14	6	1.9140×10^{-8}	-5.8721×10^{-8}
15	0	0.3125×10^{-7}		15	6	7.0020×10^{-9}	-1.1872×10^{-7}
16	0	-0.3255×10^{-7}		16	6	-5.8439×10^{-8}	-4.2009×10^{-8}
17	0	-0.1437×10^{-7}		7	7	1.5916×10^{-7}	-6.7710×10^{-9}
18	0	0.3798×10^{-7}		8	7	3.4390×10^{-8}	8.9168×10^{-8}
19	0	0.3459×10^{-7}		9	7	-5.0450×10^{-8}	-1.2700×10^{-7}
20	0	0.7809×10^{-7}		10	7	5.4961×10^{-9}	3.2003×10^{-8}
21	0	-0.2196×10^{-7}		11	7	4.5726×10^{-8}	-1.3862×10^{-7}
3	1	1.9698×10^{-6}	2.6015×10^{-7}	12	7	1.4507×10^{-8}	5.9784×10^{-8}
4	1	-5.2989×10^{-7}	-4.8765×10^{-7}	13	7	-5.2217×10^{-9}	-3.2562×10^{-9}
5	1	-5.3816×10^{-8}	-9.7955×10^{-9}	14	7	1.1061×10^{-8}	8.4132×10^{-9}
6	1	-9.8984×10^{-9}	3.7652×10^{-8}	15	7	1.1856×10^{-7}	4.2690×10^{-8}
7	1	2.4142×10^{-7}	1.1567×10^{-7}	16	7	1.0591×10^{-7}	8.1008×10^{-9}
8	1	3.1254×10^{-8}	2.5696×10^{-8}	8	8	-7.7364×10^{-8}	6.7607×10^{-8}
9	1	1.3823×10^{-7}	-1.6100×10^{-8}	9	8	2.3359×10^{-7}	5.7239×10^{-8}
10	1	1.1251×10^{-7}	-1.0167×10^{-7}	10	8	7.3957×10^{-9}	-7.9706×10^{-9}
11	1	4.3900×10^{-9}	2.9751×10^{-8}	11	8	6.4546×10^{-9}	-1.6993×10^{-9}
12	1	-4.5955×10^{-9}	-3.1000×10^{-8}	12	8	-5.7854×10^{-9}	3.3752×10^{-8}
13	1	-5.6042×10^{-8}	2.6288×10^{-9}	13	8	-4.1759×10^{-9}	-2.0231×10^{-9}
14	1	-2.3090×10^{-9}	4.9064×10^{-9}	14	8	-3.0273×10^{-9}	-6.0838×10^{-8}
15	1	-3.5971×10^{-9}	4.0142×10^{-9}	15	8	-9.7657×10^{-9}	-3.5710×10^{-8}
16	1	-2.3789×10^{-9}	7.6413×10^{-8}	16	8	-8.4738×10^{-9}	-2.4677×10^{-9}
2	2	2.4129×10^{-6}	-1.3641×10^{-6}	9	9	-8.2490×10^{-8}	9.2326×10^{-8}
3	2	8.9204×10^{-7}	-6.3468×10^{-7}	10	9	-6.8563×10^{-9}	6.2499×10^{-9}
4	2	3.3024×10^{-7}	7.0633×10^{-7}	11	9	1.1750×10^{-7}	-5.9451×10^{-9}
5	2	6.1286×10^{-7}	-3.5087×10^{-7}	12	9	-3.2232×10^{-8}	4.2659×10^{-9}
6	2	5.4825×10^{-8}	-3.5175×10^{-7}	13	9	-2.5623×10^{-9}	1.0767×10^{-7}
7	2	2.8306×10^{-7}	1.5645×10^{-7}	14	9	4.9539×10^{-9}	9.2345×10^{-9}
8	2	4.8161×10^{-8}	8.4140×10^{-8}	15	9	2.2064×10^{-9}	2.6632×10^{-8}
9	2	6.6741×10^{-9}	-8.1733×10^{-9}	16	9	9.0001×10^{-9}	-1.0628×10^{-7}
10	2	-3.1225×10^{-8}	-1.0450×10^{-7}	10	10	1.2377×10^{-7}	-2.5585×10^{-8}
11	2	4.8900×10^{-8}	-3.1994×10^{-8}	11	10	-1.1736×10^{-7}	-1.8900×10^{-9}
12	2	2.7481×10^{-9}	7.5986×10^{-9}	12	10	-1.6590×10^{-8}	4.8382×10^{-9}
13	2	-4.7456×10^{-8}	1.7367×10^{-9}	13	10	8.6589×10^{-8}	-1.0528×10^{-8}
14	2	3.2120×10^{-8}	-4.5239×10^{-9}	14	10	5.3732×10^{-8}	-4.3168×10^{-8}
15	2	-4.4333×10^{-9}	-1.6056×10^{-8}	15	10	-2.0648×10^{-8}	5.3724×10^{-10}
16	2	2.1327×10^{-9}	3.0669×10^{-9}	16	10	-2.9849×10^{-9}	-5.2467×10^{-10}
3	3	6.8630×10^{-7}	1.4304×10^{-6}	11	11	1.1795×10^{-7}	-4.0688×10^{-9}
4	3	9.8943×10^{-7}	-1.5467×10^{-7}	12	11	-4.4921×10^{-9}	-4.8206×10^{-9}
5	3	-4.3083×10^{-7}	-8.6663×10^{-8}	13	11	3.3749×10^{-8}	5.8541×10^{-9}
6	3	2.7873×10^{-9}	4.4626×10^{-8}	14	11	2.7833×10^{-8}	-8.1637×10^{-9}
7	3	2.0285×10^{-7}	-2.3448×10^{-7}	15	11	-3.2585×10^{-8}	9.4052×10^{-8}
8	3	-5.7444×10^{-9}	1.8086×10^{-9}	16	11	6.8502×10^{-9}	-7.0765×10^{-9}
9	3	-9.6463×10^{-8}	-1.1817×10^{-7}	12	12	-1.9407×10^{-9}	-5.7771×10^{-9}
10	3	-2.3346×10^{-8}	-1.4137×10^{-7}	13	12	-1.3229×10^{-9}	8.2192×10^{-9}
11	3	-6.3247×10^{-8}	-1.3109×10^{-7}	14	12	1.2481×10^{-8}	-5.7314×10^{-9}
12	3	5.8386×10^{-9}	5.4784×10^{-8}	15	12	1.0524×10^{-8}	6.8726×10^{-9}
13	3	2.3833×10^{-8}	-2.8930×10^{-8}	16	12	2.2834×10^{-8}	-3.4087×10^{-9}
14	3	1.9042×10^{-8}	1.1919×10^{-9}	17	12	8.3097×10^{-8}	-3.5424×10^{-9}
15	3	8.3016×10^{-9}	-5.7218×10^{-9}	18	12	1.1662×10^{-8}	8.4724×10^{-9}
16	3	-4.7358×10^{-9}	3.2610×10^{-8}	19	12	6.7115×10^{-8}	-8.2623×10^{-9}
4	4	-7.9692×10^{-8}	3.3928×10^{-7}	13	13	-7.0288×10^{-9}	7.4643×10^{-9}
5	4	-2.6693×10^{-7}	8.3010×10^{-9}	14	13	5.1554×10^{-8}	4.5453×10^{-8}
6	4	-4.0342×10^{-10}	-4.0388×10^{-7}	15	13	-3.7348×10^{-9}	4.0249×10^{-9}
7	4	-1.9727×10^{-7}	-1.1390×10^{-7}	16	13	3.5475×10^{-9}	2.0683×10^{-8}
8	4	-1.5378×10^{-7}	7.5264×10^{-8}	17	13	3.2749×10^{-8}	4.2920×10^{-10}
9	4	5.7125×10^{-8}	1.1183×10^{-7}	18	13	4.6903×10^{-9}	-3.5547×10^{-8}
10	4	-4.8185×10^{-8}	-4.3248×10^{-9}	19	13	3.3201×10^{-8}	-6.3128×10^{-8}
11	4	-3.0193×10^{-8}	5.4317×10^{-8}	20	13	5.8374×10^{-8}	3.3320×10^{-8}
12	4	-4.3649×10^{-9}	-2.2262×10^{-8}	21	13	3.6928×10^{-9}	-1.6288×10^{-9}
13	4	-1.9980×10^{-9}	5.7030×10^{-9}	14	14	-5.2032×10^{-8}	-1.2840×10^{-8}
14	4	7.8017×10^{-9}	-3.7527×10^{-8}	15	14	1.2193×10^{-8}	-2.6786×10^{-9}
15	4	1.3916×10^{-8}	6.6644×10^{-8}	16	14	-7.3590×10^{-9}	-2.2626×10^{-9}
16	4	-1.1591×10^{-9}	4.3001×10^{-8}	17	14	-1.6058×10^{-8}	2.7286×10^{-8}
5	5	1.2593×10^{-7}	-5.9910×10^{-7}	18	14	-2.7445×10^{-9}	-4.6376×10^{-8}
6	5	-2.1143×10^{-7}	-5.2264×10^{-7}	19	14	-3.9779×10^{-9}	-2.3617×10^{-9}
7	5	-8.7024×10^{-10}	9.8461×10^{-8}	20	14	1.1130×10^{-8}	-1.6183×10^{-8}
8	5	-5.6733×10^{-8}	6.1636×10^{-8}	21	14	5.2067×10^{-9}	3.0801×10^{-10}
9	5	-6.1435×10^{-9}	3.3551×10^{-9}	22	14	-8.0549×10^{-9}	2.6440×10^{-8}
10	5	-8.0004×10^{-8}	-1.4279×10^{-7}	15	15	1.4515×10^{-9}	-1.4602×10^{-8}
11	5	3.2523×10^{-9}	1.3215×10^{-7}	16	15	-3.5485×10^{-8}	8.4126×10^{-10}
12	5	2.3375×10^{-8}	4.2637×10^{-8}	16	16	-2.9522×10^{-9}	8.6217×10^{-9}

The computation of N_1 was not carried out in the present case by using the integration indicated in equation (3). Rather the procedure described by Bacon, et al., (1970), was used. Briefly this procedure consists of fixing a value of the potential, W_o, and computing the component N_1 as

$$N_1 = R_G - R_E \tag{4}$$

where:

R_G is the radial distance of the equipotential surface defined by W_o and the potential coefficients of the SAO 1969 Standard Earth.

R_E is the radial distance to a selected reference ellipsoid defined by a semimajor axis (a_e) and flattening (f).

The radial distance, R_G, to the equipotential surface W_o at a particular latitude and longitude ϕ_1, λ_1 is determined by using the equation

$$W_o = \sum_{n=2}^{k} \sum_{m=1}^{m} \left(\frac{a_e}{r}\right)^n \{C_{nm} \sin m\lambda_1 + S_{nm} \sin m\lambda_1\} P_{nm}(\phi_1) \tag{5}$$

The only unknown in this equation is r. Using an iterative three point inverse interpolation scheme the value of r (i.e., $r = R_G$) which will make equation 5 an identity is determined. Using this value of R_G, and R_E computed using the input values of a and f of the reference ellipsoid, a geoid undulation component N_1 is computed.

For the computations described in this paper the area A_1 for a point at which the geoid was being computed was defined to consist of a twenty degree by twenty degree area centered on the computation point. The computational formula used was:

$$N_2 = \frac{R}{4 \pi G} \sum_{j=1}^{400} \overline{\Delta g}_2 (\varphi_j', \lambda_j') S(\Psi_j) \cos \varphi_j' \Delta \varphi' \partial \lambda' \qquad (6)$$

where

$\overline{\Delta g}_2 (\varphi_j' , \lambda_j')$ is the mean value of Δg_2 within the j^{th} $1° \times 1°$ square

$S(\Psi_j)$ is the value of Stokes' function at the center of the j^{th} $1° \times 1°$ square.

$\Delta \varphi' = \Delta \lambda' = 1°$.

The value of $\overline{\Delta g}_2$ used for each $1° \times 1°$ square was computed using the formula

$$\overline{\Delta g}_2 = \overline{\Delta g}_e - \overline{\Delta g}_s$$

The $\overline{\Delta g}_e$ values are mean $1° \times 1°$ free-air anomalies provided by surface gravity data. The primary source material for this gravity data is indicated in Appendix A.

The $\overline{\Delta g}_s$ values are that part of the mean $1° \times 1°$ free-air anomalies represented by the satellite harmonic coefficients used in computing N_1. The $\overline{\Delta g}_s$ values are computed using the formula

$$\overline{\Delta g_e} = \gamma_e \sum_{n=2}^{k} \sum_{m=0}^{n} (n-1) \left[\overline{C}_{nm} \cos m\lambda' + \overline{S}_{nm} \sin m\lambda'\right] P_{nm}(\varphi')$$

where

γ_e = Equatorial gravity in milligals ($0.98 \times 10^{+6}$).

$\overline{C}_{nm}$, $\overline{S}_{nm}$ = Normalized geopotential spherical harmonic coefficients C_{nm}, S_{nm} except for $\overline{C}_{20}$ and $\overline{C}_{40}$.

k = Upper limit on degree and order of the geopotential model.

n = Degree index of harmonic coefficients.

m = Order index of harmonic coefficients.

In equation (5), the $\overline{C}_{20}$ and $\overline{C}_{40}$ terms do not represent the complete coefficients but rather the difference between the complete coefficients and the coefficients compatible with the ellipsoid used in computing N_1. In order for the above described procedure to produce correct results, the quantities $\overline{\Delta g_e}$, $\overline{\Delta g_s}$, and the a and f which define the ellipsoid used to compute N_1 must all be compatible. Compatibility implies that the values of $\overline{C}_{20}$ and $\overline{C}_{40}$ used to compute the values of theoretical gravity needed to obtain $\overline{\Delta g_e}$ and $\overline{\Delta g_s}$ are the same as the values of $\overline{C}_{20}$ and $\overline{C}_{40}$ implied by the reference ellipsoid. Correct results in absolute sense are also dependent upon the value of W_0 being chosen to represent the true value of the potential of the geoid. The effects of not making $\overline{\Delta g_e}$, $\overline{\Delta g_s}$, a, and f compatible are twofold. First, all the computed geoid heights may by in error by a constant; in addition, there will be a systematic error as a function of latitude. The effect of selecting an incorrect value of W_0 would be to introduce a constant error in all geoid heights.

In the calculations described here the term N_3 in equation (2) is set equal to zero. This is equivalent to assuming that the satellite derived approximation to the gravity field is adequate for the area A_2 at a distance of greater than ten degreees from the computation point.

3. FINAL RESULTS

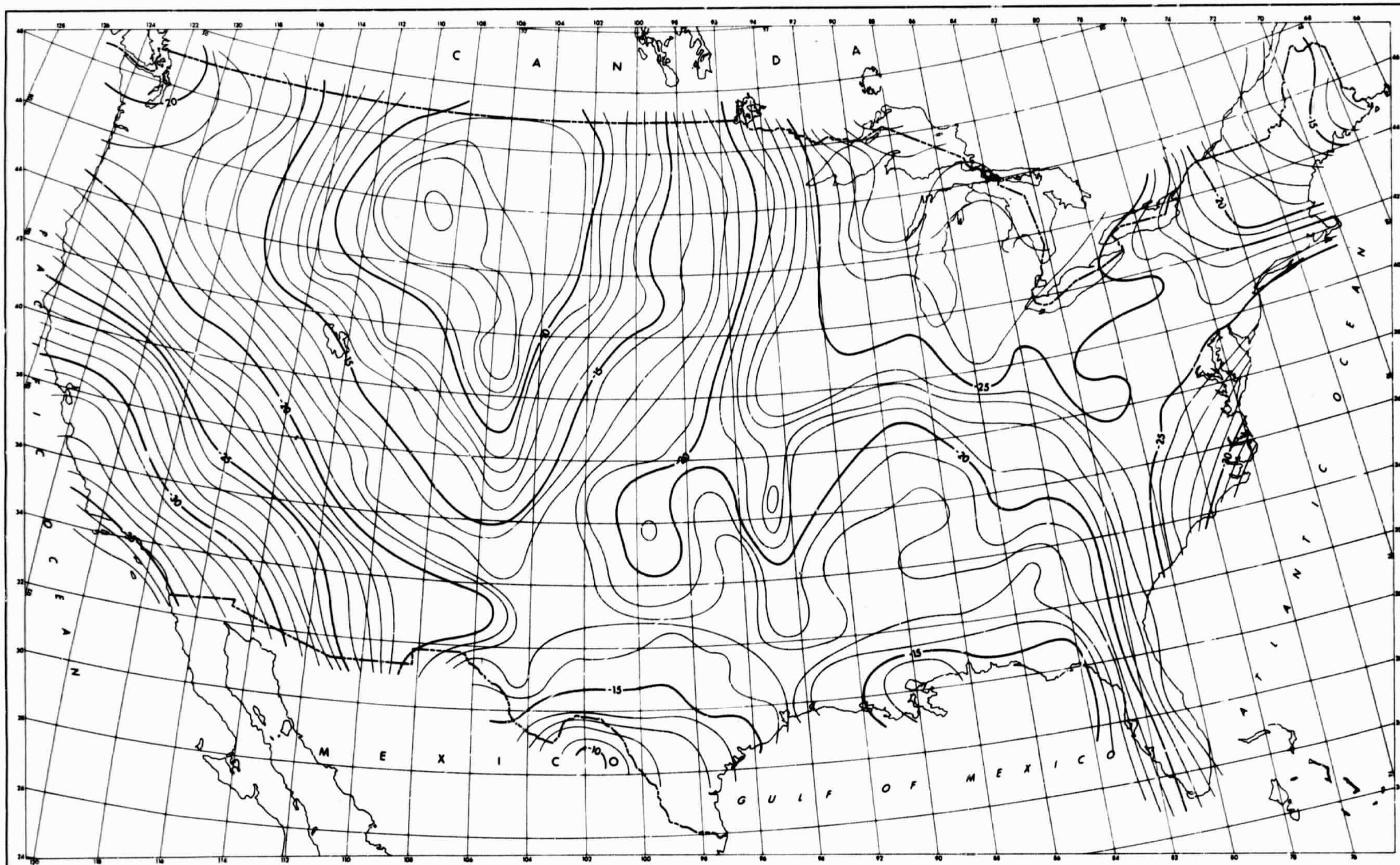
The final detailed gravimetric geoid is presented in Figure 1 using a one meter contour interval. The geoid is referenced to an ellipsoid with a flattening given by f = 1/298.255.

The primary sources of the surface gravity data used in carrying out the computations are described in Appendix A.

3.1 Parameter Values Used for Computation

In carrying out the initial geoid computations a value of W_0 = 6263675.7 kgal m taken from Rapp (1967) was used. The ellipscid parameters were given as 1/f = 298.255 and a = 6378.155. The values of Δg_s were made compatible with the chosen flattening by using $\Delta C_{20} = .08 \times 10^{-6}$ and $\Delta C_{40} = -.756 \times 10^{-6}$ when computing Δg_s. The values of Δg_e were made compatible by using a reference gravity formula of the form

$$g = 978.0421(1 + 0.00530241 \sin^2 \varphi - .0000059 \sin^2 2\varphi)$$



Contour Interval - One Meter

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MISSION & TRAJECTORY ANALYSIS DIVISION
BRANCH 552 DATE 4/6/71
BY J. G. MARSH PLOT NO. 4010

Figure 1. Detailed Gravimetric Geoid of the United States

Since it was known that the values of W_0 used was not completely correct it was anticipated that all the computed geoid undulation would be in error by a constant.

In order to establish accurate absolute values of geoid undulation the geocentric station positions obtained using dynamic analyses were employed.

3.2 Absolute Adjustment

Table 2 presents a comparison of the geoid heights, computed in the present analysis, and geoid heights computed using the geocentric x, y, z positions obtained for 12 stations in North America in the SAO 1969 Standard Earth computations (Gaposhkin and Lambeck, 1969). The SAO geoid heights are each given with reference to two ellipsoids, with semi-major axes of 6378.155, and 6378.137 km, respectively; both with flattenings of 1/298.255.

When the detailed gravimentric geoid heights at SAO station sites were compared to SAO geoid heights referenced to the 6378.155 km ellipsoid, a systematic difference of 18 meters was noted. Rather than recomputing the gravimetric geoid using an adjusted value for W_0, an equivalent adjustment was made to the value of a_e characterising the reference ellipsoid to which the gravimetric geoid is assumed to be referenced. This consisted of subtracting 18 meters from the value originally used, which resulted in the value 6378.137 km reported above.

As may be seen, the gravity geoid heights differ in a random manner by 10 meters or less from the SAO station geoid heights referenced to an ellipsoid with a semi-major axis of 6378.137 km.

Table 2

SAO - Gravity Geoid Comparisons

(1) Station No.	(2) SAO Geoid* Height (155) (meters)	(3) SAO Geoid Height (137) (meters)	(4) Gravimetric Geoid (meters)	(3) - (4) (meters)
1021	-42	-24	-25	+ 1
1034	-32	-14	-18	+ 4
1042	-49	-31	-21	-10
7037	-42	-24	-23	- 1
7045	-21	- 3	-11	+ 8
7050	-42	-24	-24	0
7075	-52	-34	-29	- 5
9001	-35	-17	-21	+ 4
9010	-39	-21	-26	+ 5
9021	-42	-24	-24	0
9050	-46	-28	-19	- 9
9113	-37	-19	-29	+10

*Computed using data taken from Gaposhkin and Lambeck (1970)

Table 3 presents a similar comparison between the results of Marsh et al. (1970) and the gravimetric geoid. If one assumes Marsh's values are correct the geoid in Figure 1 should be considered as referenced to a geoid of semi-major axis 6378.124 kms.

3.3 Comparative Evaluation

In order to evaluate the accuracy of the final geoid, a number of comparisons were made. The first comparison was made with the computations of Rapp (1967) of a combination geoid in a portion of the Western United States. Figure 2 presents a plot of these comparisons. In this comparison, nine meters has been subtracted from the results of Rapp so that a comparison of relative shape is effected. No attempt at an absolute comparison was made. The agreement as to relative geoid shape is exceptional (differences exceeding 1.5 meters occur at only one point), considering the entirely different sources of gravimetric data, and the differences in computational procedure.

Another important source of comparative data is the detailed astrogeodetic geoid data. These data should have an rms accuracy on the order of 2 meters and should therefore provide an excellent basis for comparison throughout the United States. The most accurate astrogeodetic geoid information should be that of Rice (1970) which provide point astrogeodetic geoid values based largely on closed traverse loops with deflection measurements every 10 to 15 km.

Table 3

Marsh - Gravity Geoid Comparisons

(1) Station No.	(2) Marsh Geoid* Height (155) (meters)	(3) Marsh Geoid Height (124) (meters)	(4) Gravimetric Geoid (meters)	(5) (3) -(4) (meters)
1021	-55	-24	-25	+1
1022	-50	-19	-18	-1
1030	-53	-22	-28	+6
1034	-53	-22	-18	-4
1042	-57	-26	-21	-5
7037	-60	-29	-23	-6
7045	-44	-13	-11	-2
7050	-52	-21	-24	+3
7072	-53	-22	-26	+4
7075	-59	-28	-29	+1

*Computed using data taken from Marsh et al. (1970)

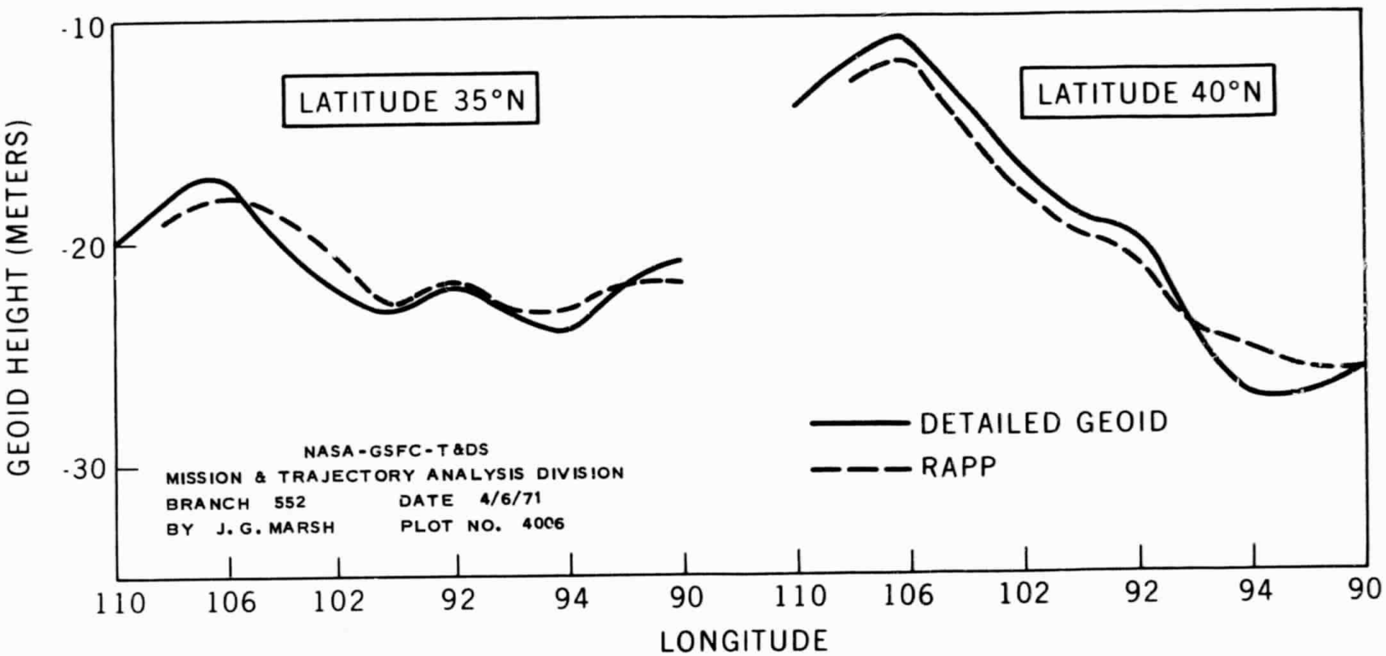

Figure 2. Detailed Geoid Vs Rapp's Western U.S. Geoid (1967)

Before any comparison can be made, the Rice Astrogeodetic geoid values must be transformed from the NAD to geoid values relative to a geocentric ellipsoid. This raises the questions of the transformation to use in converting NAD coordinates to geocentric coordinates. A number of transformations have been proposed, some involve simple transformations and others involve both transformations and rotations. Table 4 presents the differences between Rice's astrogeodetic geoid and the gravimetric geoid after using each of four different sets of translation elements, and removing the mean differences. The important point to be noted is the overall degree of agreement between the astrogeodetic and gravimetric geoid using the different translation coordinate sets. In all cases the rms differences are on the order of 2 meters.

In addition to the point geoid heights, Rice (1967) has provided a detailed astrogeodetic geoid along the 35th parallel. Figure 3 graphically presents a comparison of this astrogeodetic geoid profile with the gravity geoid using the same procedure as employed with the point values and transformation set 3 of Table 4. Again the rms agreement is of the order of 2 meters, although at one point (95° longitude) the two differ by 4 meters. It is of interest to note, in Figure 2, that Rapp's result and the present gravimetric geoid agree well with one another at this point of maximum disagreement with the astrogeodetic value.

TABLE 4

COMPARISON OF DETAILED GEOID vs RICE'S ASTROGEODETIC GEOID UNDER VARYING ASSUMPTIONS FOR TRANSFORMING ASTROGEODETIC DATA

Latitude	Longitude	1	2	3	4
34° 58' 03"0	120° 38' 05"5	4	0	5	5
35 00 38.0	119 00 48.0	3	1	5	5
38 47 23.1	121 52 15.6	2	-1	4	4
35 02 36.1	106 30 24.1	3	3	5	4
32 13 14.7	106 29 41.6	4	3	5	4
32 00 00.6	103 16 07.2	1	0	1	1
30 59 40.0	098 05 50.5	-2	-2	-1	-2
30 36 26.5	091 23 18.1	-5	-3	-4	-4
29 38 10.8	091 06 49.3	-5	-4	-4	-5
30 59 25.5	089 34 29.5	-4	-4	-4	-5
25 30 25.2	080 23 17.5	2	5	2	3
28 29 28.6	080 33 35.6	3	5	3	4
30 36 53.3	081 42 14.8	0	2	0	-1
39 28 18.9	076 05 15.2	-3	0	-3	-2
34 59 44.0	076 59 11.7	-2	1	-2	-1
33 28 42.4	091 00 08.5	-1	0	0	-1
33 34 48.5	092 50 07.2	-1	0	0	-1
34 56 47.0	093 24 18.3	-2	-2	-2	-2
37 38 08.4	094 35 46.8	-2	-1	-1	-1
35 03 04.0	097 56 52.6	0	0	1	1
39 13 26.7	098 32 30.5	0	0	1	0
43 37 10.7	096 17 52.3	0	1	1	0
35 06 16.2	103 19 55.0	3	2	3	3
34 56 32.8	096 24 55.3	-1	0	0	0
44 43 46.0	105 25 50.7	2	1	2	2
36 47 44.2	103 11 48.5	3	3	4	4
38 50 40.6	102 48 46.8	1	0	2	1
48 06 18.6	102 21 09.7	-2	-2	-2	-1
46 44 47.4	102 15 13.4	-1	-1	0	0
45 12 45.7	102 09 14.1	0	-2	-1	-1
46 21 53.1	108 59 07.3	2	0	2	2
31 03 07.3	102 56 05.8	0	0	2	1

Latitude	Longitude	1	2	3	4
41° 30' 41.9	097° 37' 23.4	1	1	1	1
30 48 49.8	093 12 26.9	-6	-4	-6	-6
47 50 28.9	110 00 46.4	1	-1	1	1

1* Corrected difference between Rice's Astrogeodetic geoid and detailed gravimetric geoid.

Using Marsh's translation values $\Delta X = -25.1$, $\Delta Y = +162.9$, $\Delta Z = +172.5$ (Marsh, et.al., 1970)

2* Corrected difference between Rice's Astrogeodetic geoid and detailed gravimetric geoid.

Using Fischer's translation values $\Delta X = -18$ $\Delta Y = +145$ $\Delta Z = +183$ (Fischer, 1968)

3* Corrected difference between Rice's Astrogeodetic geoid and detailed gravimetric geoid.

Using SAO Standard Earth '66 translation values $\Delta X = -30$, $\Delta Y = +152$, $\Delta Z = +176$.

4* Corrected difference between Rice's Astrogeodetic geoid and detailed gravimetric geoid.

Using SAO's translation values $\Delta X = -25.8$, $\Delta Y = +168.1$ $\Delta Z = +167.0$ (Lambeck, 1971)

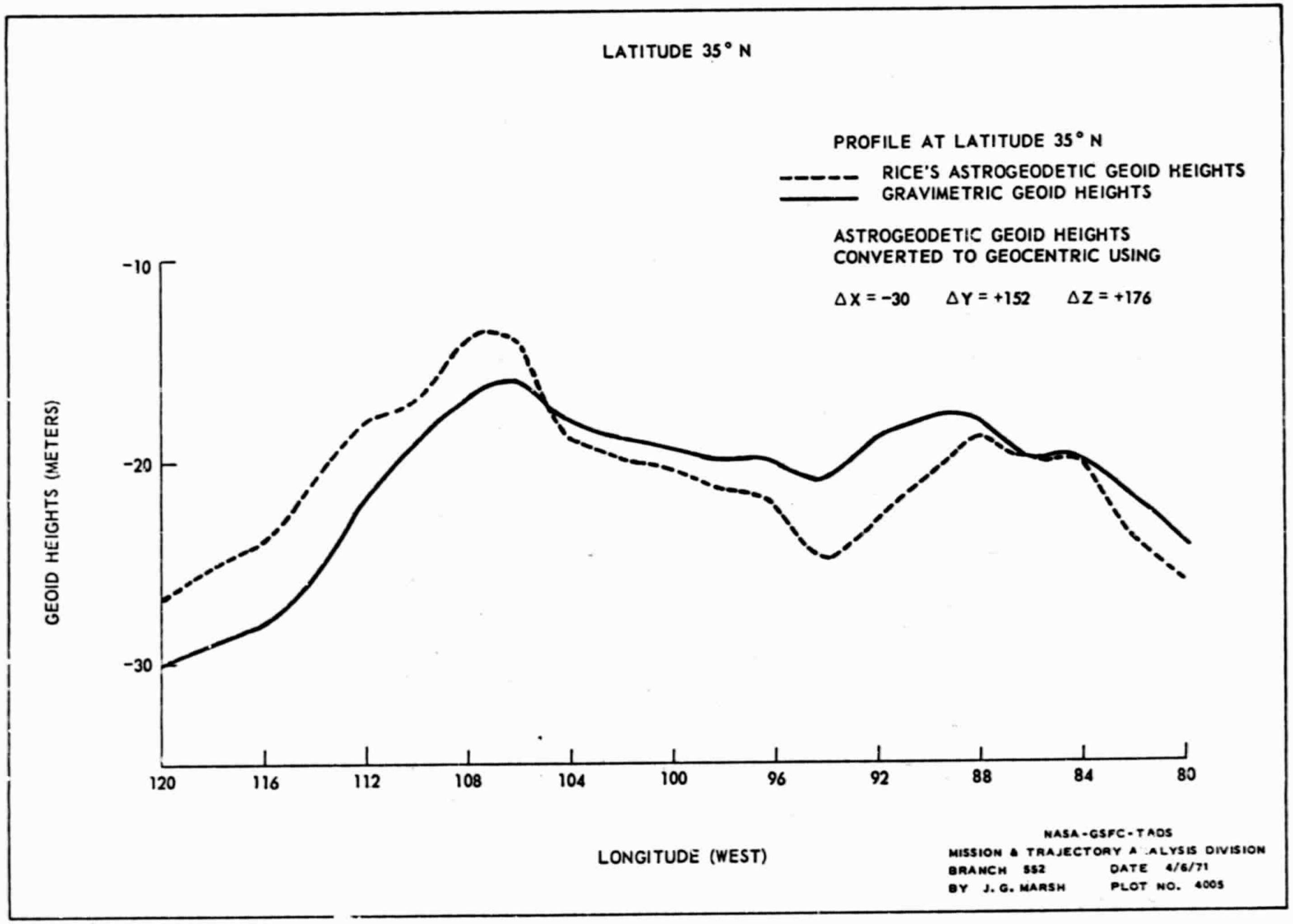

Figure 3. Detailed Gravimetric Geoid Vs Rice's Astrogeodetic Geoid Profile

A final comparison was made using transformation sets 1 and 2 from Table 4 to transform geoid profile data taken from the map of Fischer (1967) to the geocentric system. Comparisons of the transformed astrogeodetic data and the gravimetric geoid along two East-West profiles located at latitude 40°N and 45°N are presented in Figures 4 and 5. Again the relative agreement is within the ±2 meter range.

4. CONCLUSIONS

The conclusion which can be drawn, based upon comparisons made, is that the detailed gravimetric geoid presented here has precision of ±2 meters. Further study will be required to fix the absolute values of the geoid by choosing between the possibilities indicated in Tables 2 and 3 to provide an accuracy which is equivalent to the precision obtained.

An important question which can be studied using the results presented here is the question of possible rotations of the North American Dataum. Figure 6 presents plots of the difference between the Astrogeodetic geoid heights of Rice which were transformed to geocentric-by a simple translation and the detailed gravimetric geoid heights. The translation used was that of Marsh, i.e., $\Delta X = -25.1$, $\Delta y = +162.9$, $\Delta z = +172.5$. Examination of these plots indicate that there is no conclusive evidence of a rotation of the North American datum. If any datum rotation exists it is extremely small. Given the much smaller number of samples and the lesser accuracy available when

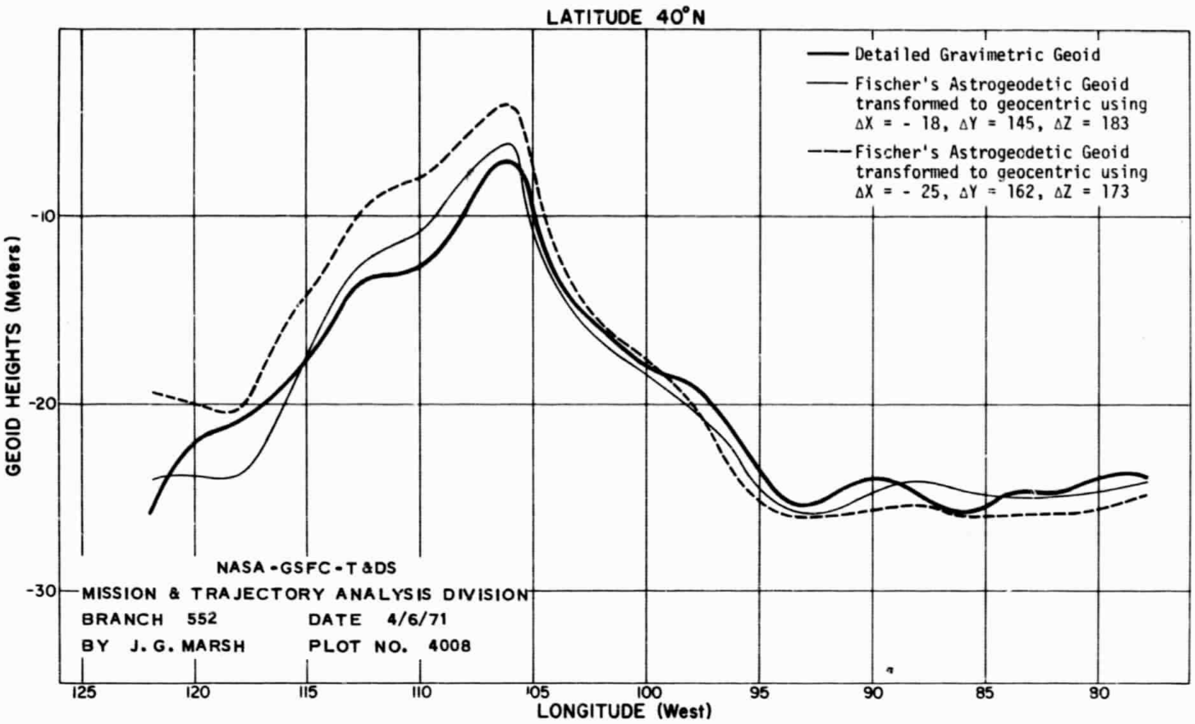

Figure 4.

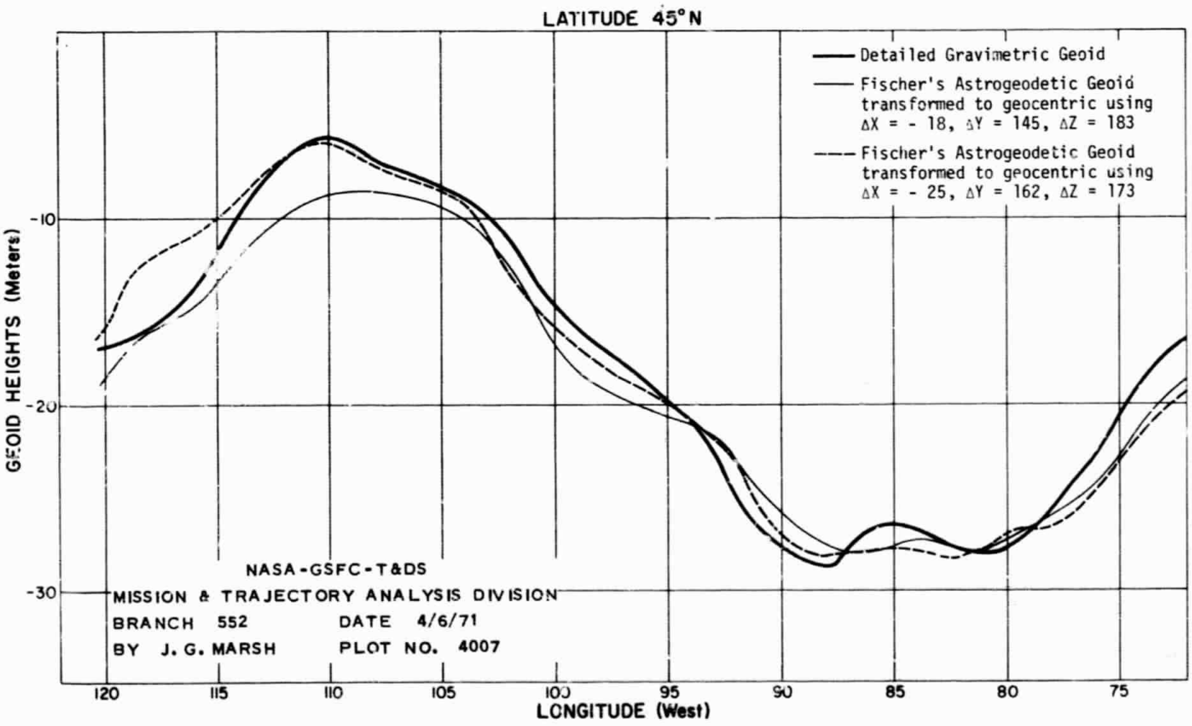

Figure 5.

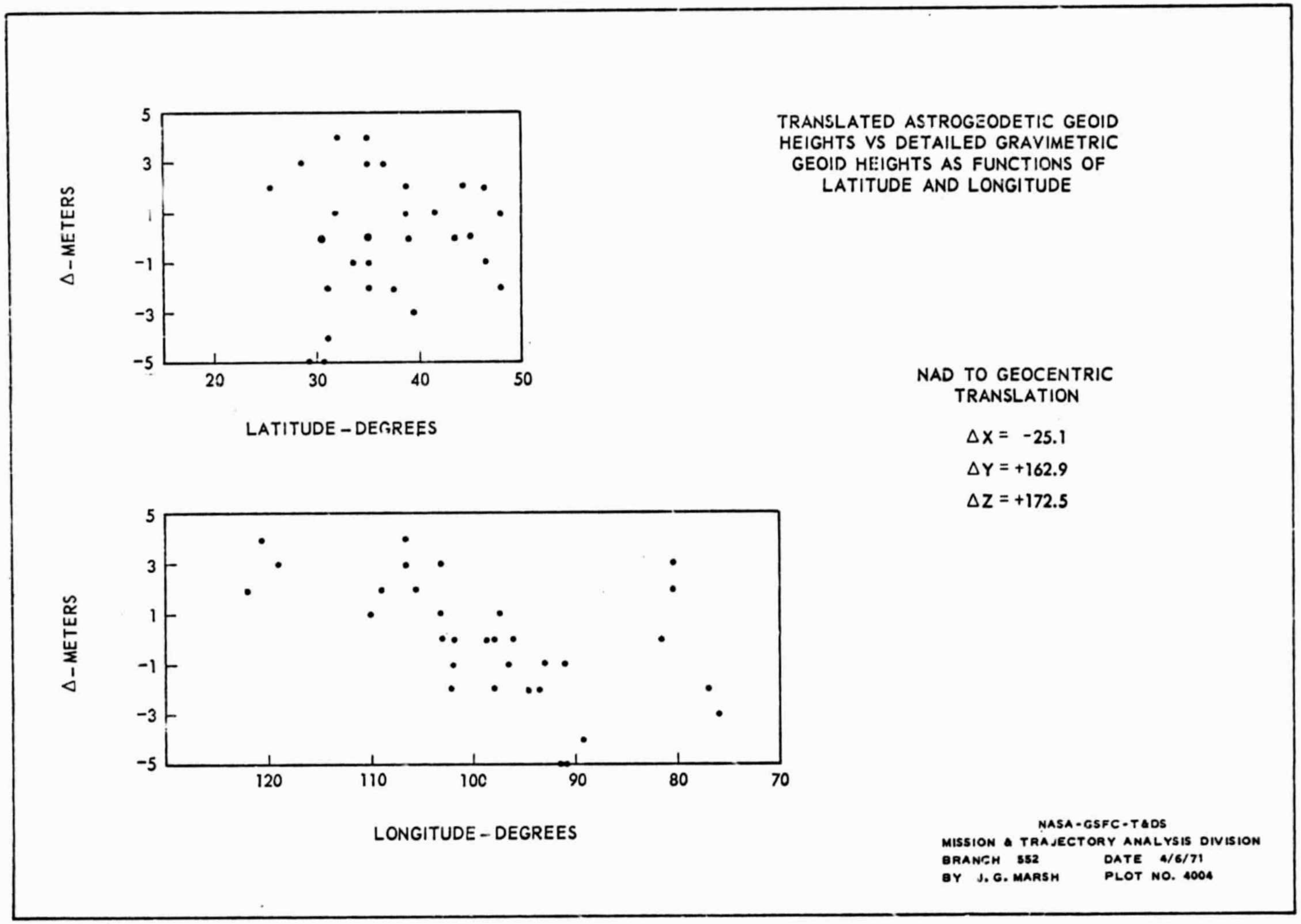

Figure 6. Detailed Geoid Vs Rice's Astrogeodetic Geoid

deriving rotations using dynamically derived geocentric station positions, any rotations proposed on the basis of such data must be seriously questioned.

5. ACKNOWLEDGMENTS

The programming for this computation was carried out by Mrs. Brenda Hill while the actual computer runs were performed by Mr. Michael Nichols.

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APPENDIX A

DERIVATION OF 1° × 1° TERRESTRIAL FREE-

AIR ANOMALIES USED IN THE DETAILED GEOID COMPUTATION

Except for portions of Canada, all of the terrestrial gravity data included herein were compiled from published sources. For many areas, particularly those outside continental margins, the data used were of a generalized nature, even though more detailed data are known to exist. These latter data were either unavailable or would require lengthy evaluation and analysis to obtain 1° × 1° mean anomaly values. Nontheless, all results are adjudged to be within the limits of accuracy required by the study.

The following paragraphs include identification of all data sources, and description of computations used for conversion and averaging of various input data types. The areas of coverage of the different sources are shown in Figure A-1.

A.1 Areas 1, 2, and 3 - Continental Portions of U.S., Canada and Mexico

Most of the coverage of these three areas is provided directly from previously published mean 1° × 1° free-air anomalies. Those of the continental U.S. (area 1) are from Strange and Woollard (1964); those of Mexico and Canada (area 2 and 3, respectively) are from Woollard (1968). A few 1° × 1° means were modified based on more recent information.

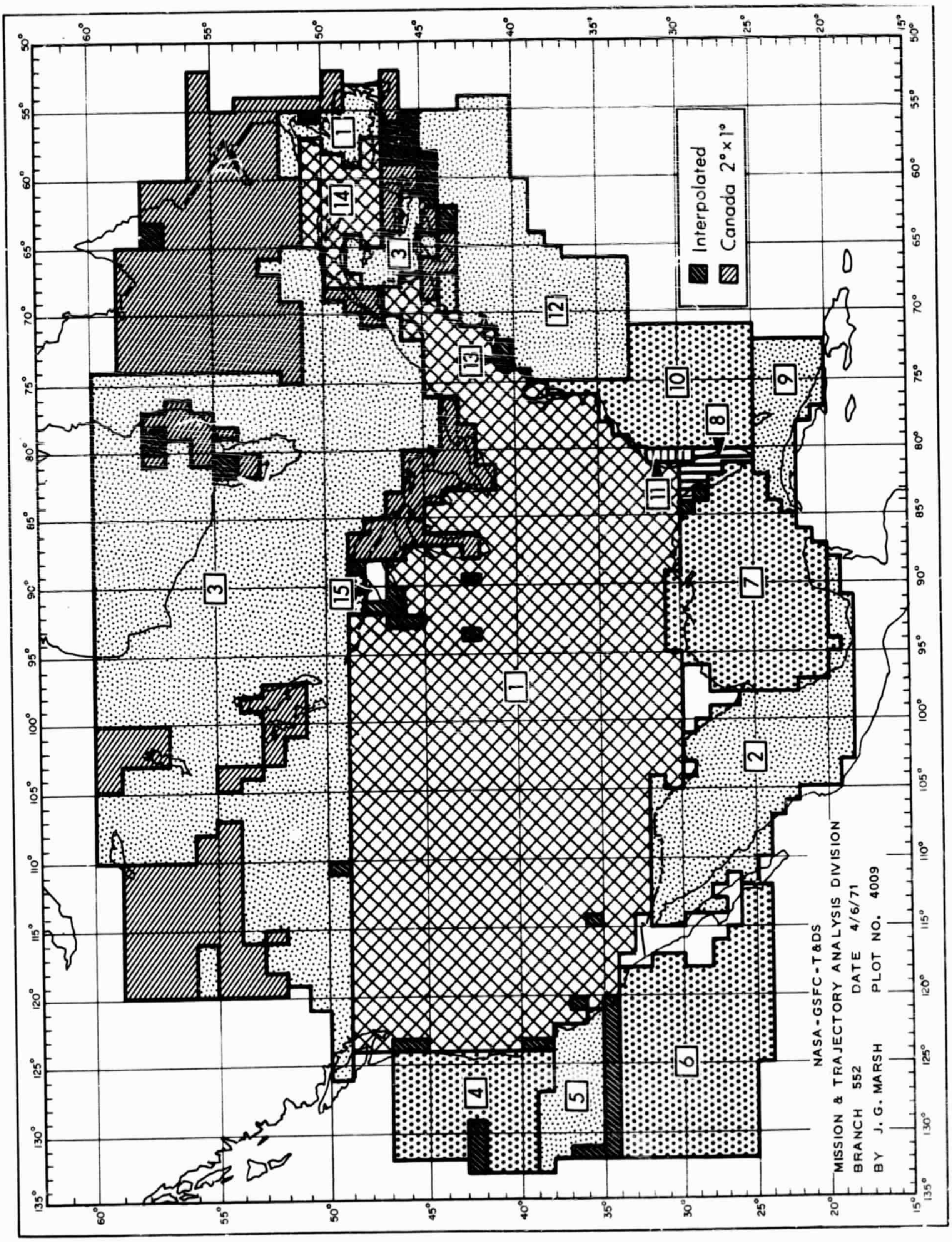

Figure 7. Area of Surface Gravity Coverage

A.2 Other Areas of Canada

Values for the lighter hachured areas in Canada were derived from unpublished $2° \times 1°$ mean free-air gravity anomalies, provided by Dr. D. Nagy, Gravity Division, Department of Energy, Mines and Resources, Ottawa. Because of their generalized nature, these values were applied only where more detailed data were lacking, or were suspect.

In most places, the $2° \times 1°$ values were applied directly without modification. However, where one of the $1° \times 1°$ squares within the $2° \times 1°$ areas was previously defined by a more accurate data source, the value of the blank $1° \times 1°$ was defined such that the average for it and the previous value equalled that of the $2° \times 1°$.

A.3 Area 4 - Washington - Oregon Coast.

The data sources for this area were free-air gravity contour maps published by Dehlinger (1969). Unless the setting of a $1° \times 1°$ square was particularly straight forward, it was subdivided into segments of approximately equal gravity. The mean value for the $1° \times 1°$ square then, was computed as the average of the mean values of the individual segments, weighted proportionately relative to the segment area.

A.4 Area 5 - Central California Coast

Free-air anomalies for $1° \times 1°$ squares of area 5 were derived from the Bouguer and bathymetric map of Lattimore, Bush and Bush (1968). Each

1° × 1° square of their map was divided into nine equal areas, from which the Bouguer gravity and water depth of the midpoint of each of the nine segments were approximated from the source maps. A free-air value at each midpoint was then computed by the following equation from Nettleton (1940, p. 54)

$$F.A. = B.A. - (.04185)\ (\sigma)(h)\ \text{milligal/meter}$$

where

F.A. = Free-air anomaly

B.A. = Bouguer anomaly

h = water depth below sea level, meters

σ = specific gravity

$= 2.84 - 1.03^{3}$

$= 1.81\ gm/cm^3$

The latter density values are those applied by Lattimore et al. to derive the Bouguer anomalies.

The mean free-air anomaly of each 1° × 1° square is the average of its nine component free-air values.

A.5 Area 6 - Baja California Coast

The free-air values of 1° × 1° squares within area 6 were derived by plotting and contouring all free-air measurements within the area as reported by Worzel (1965). Mean values for 1° × 1° squares were approximated from the 10 milligal contours by methods described for area 4.

A.6 Area 7 - Gulf of Mexico

Values for $1° \times 1°$ squares of this area were derived from the free-air gravity map of Dehlinger and Jones (1965), in the same manner as that applied for the area 4 data.

A.7 Area 8 - Florida

The free-air gravities of the ten $1° \times 1°$ squares of Florida were estimated directly from Woollard's Bouguer Gravity Map of the United States (1964), by methods described for area 4.

A.8 Area 9

The free-air values of area 9 were derived from three difference sets of data:

(a) Free-air gravity measurements of Cuba, mostly located in the northern and eastern portions, as published by Dickerson (1940), and Shubert (1957). $1° \times 1°$ squares for these portions of Cuba were visually estimated from 10 milligal contours of these data, as supplemented by off shore data, mentioned below.

(b) A Bouguer map of Southern Andros Island and shallow adjacent marine areas, computed by Richards and Malone (1949). Because of the low elevations on Andros Island and the shallowness of the sea over a broad area south of the island, test computations showed that differences between Bouguer and free-air gravity is less than a milligal. Therefore, formal conversion was unwarranted, and $1^\circ \times 1^\circ$ values for this area were visually estimated directly from the Bouguer contour map.

(c) Free-air values at sea, provided by Worzel (1965). These data were relatively sparce considering the topographic variations of the sea floor and island chains, and hence, could not be dependably contoured. Rather, where a number of readings were listed within the same general area, and apparently shared similar conditions including water dpeth, their arithmetic average was assumed to represent the general area as well as adjacent areas of similar characteristics where data was lacking. Values derived from areas 7 and 10 were similarly extrapolated into data-poor segments of the Andros-Cuba area.

A.9 Area 10 - Continental Shelf of Southeastern U.S.

$1^\circ \times 1^\circ$ mean values were interpolated directly from the free-air gravity map of the eastern U.S. continental shelf, contoured at 20 milligal intervals by Emory, et al (1970). The mean value for each $1^\circ \times 1^\circ$ square was visually estimated on the basis of proportional areas as for area 4. In areas with minimal gravitational relief, the method was aided by adding supplemental 10 milligal contours.

A.10 Area 11 - Georgia Coast

The mean values of the three $1^\circ \times 1^\circ$ squares of this area were estimated directly from the free-air 5 milligal contour map published by U.S. Coast and Geodetic Survey (1968).

A.11 Area 12 - Continental Shelf of Northeastern U.S.

Data for this area were free-air values provided by Worzel (1965), for various measurements prior to 1959, and those derived from Chain Cruises 70 and 73, published by Bowen and Aldrich (1969a, 1969b). Data from all three sources were plotted together and contoured at 10 milligal intervals. Values for each 1° × 1° within the area were then estimated by methods applied for area 4.

A.12 Area 13 - Gulf of Maine

Five of the squares within this area were estimated directly from the 5 milligal contours of the free-air gravity map prepared by Yellin (1968). The value for the sixth square in the northeast corner of the area, where contours were generally lacking, was the arithmetic average of the 1° × 1° free-air values within the square, as provided by Yellin.

A.13 Area 14 - The Gulf of St. Lawrence

The data sources for area 14 were bathymetric depths and simple Bouguer anomalies provided by Goodacre, Brule, and Cooper (1969). The Bouguer values were converted into free-air values by applying the same equation as for area 5, except that rock density was 2.67. These values were supplemented by Bouguer gravity along the major coast lines, shown by the 50 milligal controur lines of the gravity map of Goodacre, et al.

Results were plotted and contoured at 10 milligal intervals, allowing visual estimates of mean free-air values for each of the 1° × 1° squares within the area.

A.14 Area 15 - Northern Lake Superior

Free-air anomalies for area 15 were computed from the Bouguer gravity map of Lake Superior by Weber and Goodacre (1966), in conjunction with the bottom topography furnished by Wold and Ostenso (1966). Each $1° \times 1°$ square of the area on each map was divided into 16 equal portions, and the water depth and Bouguer gravity at each line intersection were tabulated. Water depths were converted into elevations assuming a lake elevation of 166 feet above sea level. Then free-air values were computed for each intersection by the equation,

$$\text{F.A.} = \text{B.A.} + \text{B.C.}$$

where

F.A. = Free-air anomaly

B.A. = Bouguer anomaly

B.C. = Bouguer correction = a + b - c

a = correction for water above sea level

= feet of water above sea level x water density x m

b = correction for rock above sea level

= feet of rock above sea level x rock density x m

c = correction for water below sea level

= (feet of water below sea level) (rock density - water density) x (m)

m = constant, .01276, after Nettleton (1940, p. 54)

Rock and water densities are 2.67 and 1.00 respectively, as assumbed by Weber and Goodacre.

The resulting free-air values were plotted and contoured at 10 milligal intervals, from which an estimate of the mean free air gravity of each $1° \times 1°$ square was made.

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