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GEODETIC STUDIES BY LASER RANGING TO SATELLITES

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ABSTRACT

For three months in 1970, two Goddard Space Flight Center laser tracking systems were used in an experiment to try and detect the motion of the pole of rotation of the earth. One tracking station was located at Goddard Space Flight Center and and the other was about 408 km due north near Seneca Lake in New York State. Over two hundred passes of the Beacon Explorer C spacecraft were observed as it passed between the two stations, fifty of which were simultaneous, and these data have been used to determine the orbital inclination of the spacecraft during the experiment. The analysis of the data from the experiment required the accurate determination of the relative positions of the two tracking stations and the identification of the perturbations to the spacecraft orbit, in particular, those due to the gravitational fields of the earth, sun and moon and those caused by the solid-earth tides. The results to date indicate that the current GSFC laser systems have the capability of determining intersite distances with a repeatability of about 25 centimeters and that a new value of the Love Number, k_2, which represents the distortion of the earth's gravity field caused by the tidal deformation of the earth is 0.35 ± 0.05.

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GEODETIC STUDIES BY LASER RANGING TO SATELLITES

1. INTRODUCTION

During the summer of 1970 Goddard Space Flight Center conducted a preliminary polar motion experiment using its two precision laser tracking stations. The prime purpose of the experiment was to determine if, with the present systems, it was possible to detect the motion of the pole of rotation of the Earth on its axis from the analysis of laser observations of Earth satellites and, if so, to what accuracy this could be derived. Additional very important objectives, were the determination of the capability of the two laser systems to measure inter-station distances over hundreds of kilometers using satellites and also to make a recovery of the second degree Love Number, k_2, from the analysis of the solid Earth tidal perturbations. The analysis of the data collected during this experiment is still continuing but the results to date, those concerned with the inter-station distance measurement and the tidal perturbations, are presented here.

The principle behind the polar motion experiment was the following. If a satellite is tracked with considerable precision through the apex position of its orbit (the point of maximum latitude) then a very precise determination of the osculating orbital inclination can be made. Further, if the tracking is continued on a regular basis then changes in the latitudes of the tracking stations due to motion of the pole of rotation of the Earth in the meridians containing the stations will appear as perturbations in the observed osculating orbital inclinations. Careful analysis of the osculating inclinations obtained during the experiment should then permit a recovery of motion of the Earth's pole of rotation.

For the experiment the two GSFC laser tracking stations were located (see Fig. 1) on the same meridian, one at the Goddard Optical Site (GODLAS) and the other about 408 km due north near Seneca lake in New York State (SENLAS). The geodetic latitudes of GODLAS and SENLAS were 39° 01' N and 42° 42' N respectively and, as such, were well placed to simultaneously track three satellites through the northern apex position of the orbits. These three satellites were Beacon Explorer C (BE-C), and two French satellites D1-C and D1-D, all with orbital inclinations of about 41 degrees so that the maximum northerly latitudes reach by them was slightly north of the latitude of GODLAS but slightly south of SENLAS. All three satellites carried corner cube reflectors and could therefore be tracked by lasers.

The observation program was started on June 17th, 1970 and continued three months. The tracking stations were operated on a 10 hours-a-day, 6 days-a-week

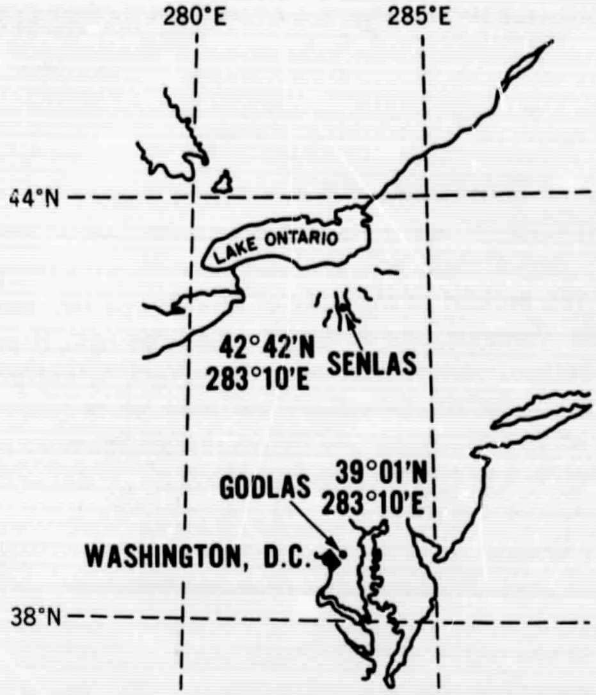

Figure 1. Laser Locations

basis with the time of the shift being determined by the time of passage of the prime spacecraft, BE-C. The two GSFC laser systems are pulsed ruby lasers with a power output of about 1 joule and a pulse rate of 1 per second (Johnson, et al, 1967).

During the observation period only the BE-C spacecraft was tracked with reliability. Both D1-C and D1-D were very difficult spacecraft to observe due entirely to our inability to provide the tracking stations with predictions of sufficient accuracy. The predictions for BE-C were considerably better and this is believed to be due partly to the availability of Minitrack observations on BE-C (the spacecraft was deliberately re-activated for this experiment) and also to the much smaller drag perturbations of BE-C than either of the other spacecraft. Both D1-C and D1-D have perigee heights below 600 km compared to nearly 1000 km for BE-C and are therefore much more sensitive to changes in air density. The GEOS 2 spacecraft was also tracked during the experiment, on a non-interference basis, for purposes unconnected with the main objectives. A summary of the data collected is shown in Table 1. So far only the data on BE-C have been analyzed.

Table 1

Summary of Observational Data

SPACECRAFT	PASSES OBSERVED	OBSERVATIONS	SIMULTANEOUS PASSES
BE-C	168	52,264	48
D1-C and D1-D	18	3,766	1
GEOS 2	18	6,157	8

On a clear day or night the two tracking stations could hope to observe four consecutive passes of the BE-C spacecraft and, indeed, on one occasion SENLAS observed five consecutive passes but, in general, weather did not permit such extensive coverage. However, one day or night's observations was a convenient length of arc for determining the spacecraft orbit and these arcs were referred to as mini-arcs. From the observations collected during the experiment 22 mini-arcs were formed containing 4 to 6 hours of observations from both sites. In addition, a further 15 arcs of the same length were formed from observations at only one of the two sites when weather did not permit simultaneous tracking.

2. LASER RANGE DATA

The basic quantity that is measured by the laser ranging system is the time interval between the outgoing and returning pulse. This time interval is adjusted according to calibration measurements taken immediately before the pass, transformed into a range measurement and corrected for atmospheric refraction. In addition, electronic and mathematical corrections are applied to ensure that the time interval measurement is taken between identical positions on the outgoing and returning pulse. These procedures, which are of fundamental importance to the interpretation of the measurements, will be described in detail in a separate publication in preparation which describes the complete laser tracking system, including the pre-processing of the data and the final production of a range measurement.

Part of the pre-processing stage of a pass of laser observations is to fit an orbit through the data to indicate if any of the individual range measurements are outside the acceptance level of 5σ, where σ is the root-mean-square (rms) of the residuals about the 5 to 10 minute short arc pass of data. Such a procedure provides a very strong indication of the precision of the laser system on a pass by pass basis and separately for each station. At the beginning of the polar motion experiment the GODLAS laser ranges had an rms noise of 30 to 40 cm but toward the end of the experiment this increased to 50 to 60 cm due to unintentional changes in the transmitted pulse length. SENLAS, however, began the experiment with an rms noise of about 1 meter but after modifications were incorporated to make the system similar to the GODLAS laser the rms fell to 20 to 30 cm. These figures only reflect the inherent precision of the system, it is almost impossible to assess the size of any bias that may exist in either of the systems. However, because of the lack of evidence of any tangible biases in these present systems it has been assumed that the biases are a few tens of centimeters, that is, comparable to the rms noise values. There is further discussion of possible biases in the system in later sections which describe some of the results.

Figure 2 shows the ground track of mini-arc 18 on September 2, 1970, when four consecutive passes were observed simultaneously at GODLAS and SENLAS.

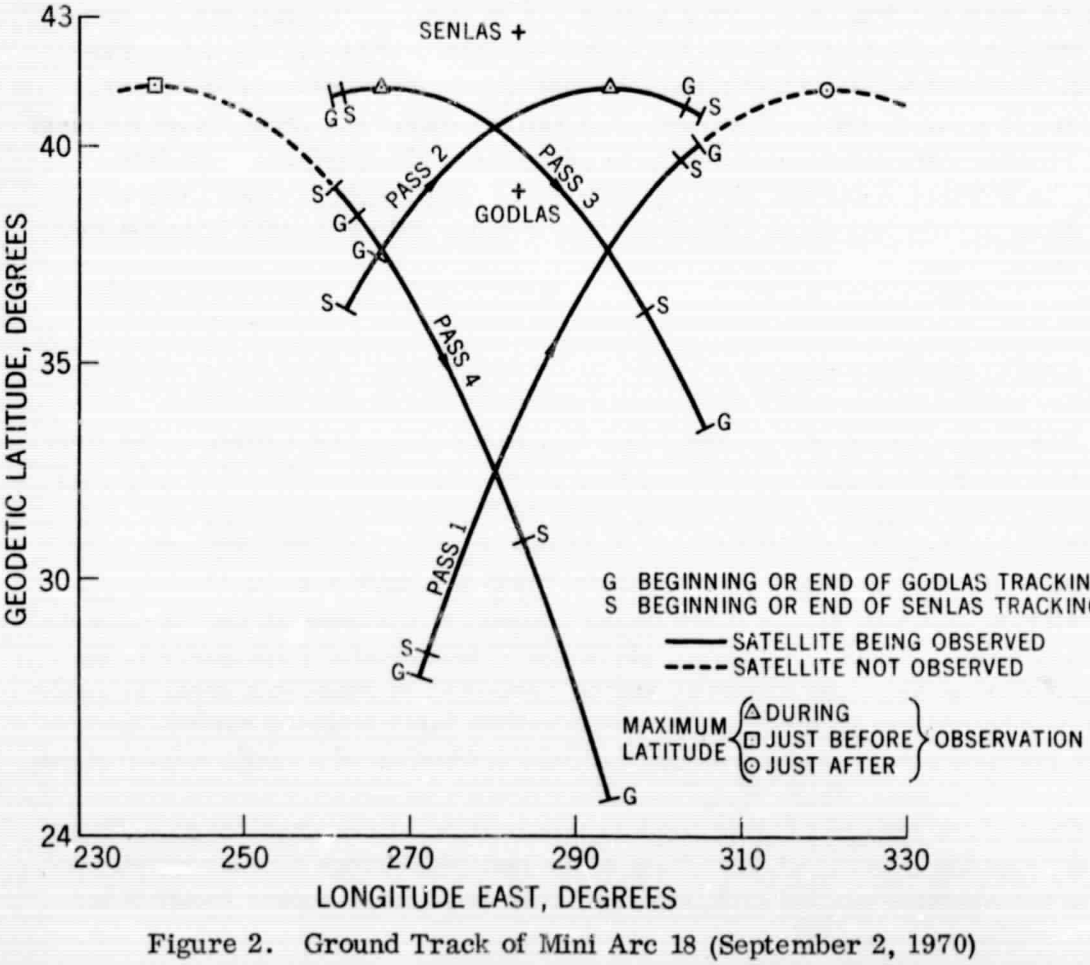

Figure 2. Ground Track of Mini Arc 18 (September 2, 1970)

Mini-arc 18 is certainly not typical because of its remarkable coverage from both stations but it does demonstrate the successive drift of the passes westward each day past the tracking stations. Figure 3 shows the range residuals from GODLAS and SENLAS to an orbital fit through mini-arc 18. The rms noise fit of 30 to 40 cm to each individual pass is clearly evident but so also is the very large quasi-periodic variation of the residuals about the fit to the mini-arc. The rms noise

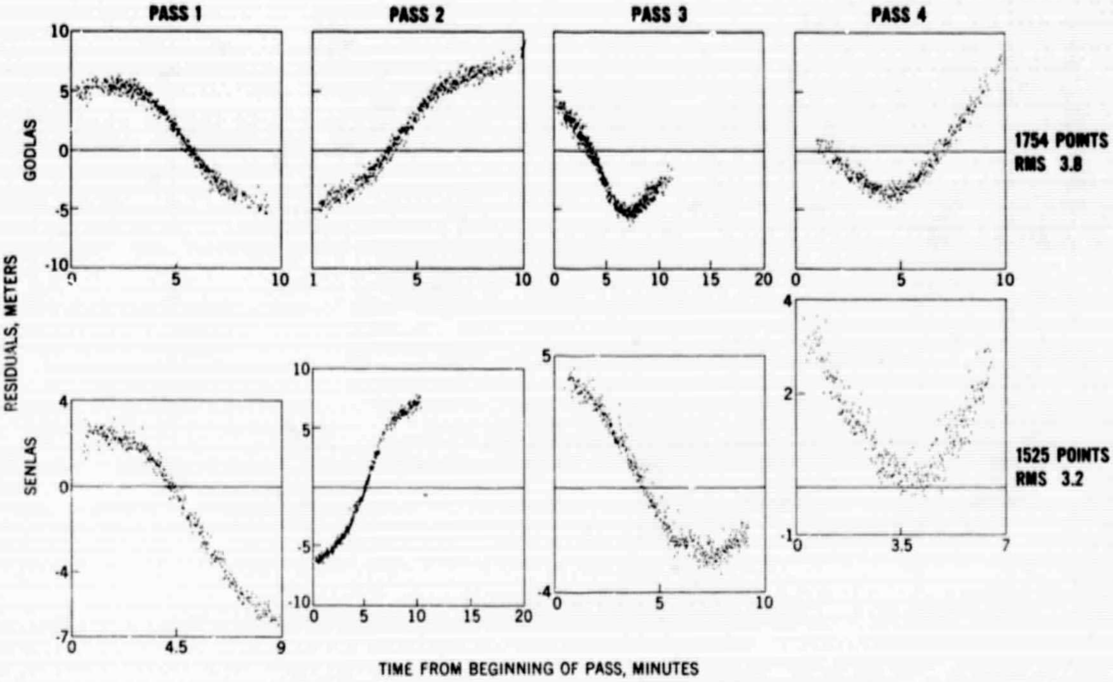

Figure 3. Range Residuals for Mini Arc 18.

from the mini-arc was about 3 meters for both GODLAS and SENLAS; but Figure 3 shows the peak-to-peak variation to be about 12 meters. Note that the scales in Figure 3 and 4 are not the same from pass to pass. These residuals are presumably due largely to deficiencies in the gravitational model which is used in the orbit determination program; but there may also be contributions from errors in station location and measurement biases. However, it is difficult to see how they could introduce the patterns and amplitudes shown in Figure 3, particularly since the coordinates of the two stations have been adjusted to deliberately minimize the contributions from station errors to the orbital fits (see Section 3).

The gravitational model used in the orbit determination program is the 1969 Standard Earth (II) model of the Smithsonian Astrophysical Observatory (Gaposchkin and Lambeck, 1970); and it naturally is of considerable interest to know what coefficients, if any, are causing the patterns shown in Figure 3. The patterns of the residuals suggest that each pass sees about one half of a cycle and

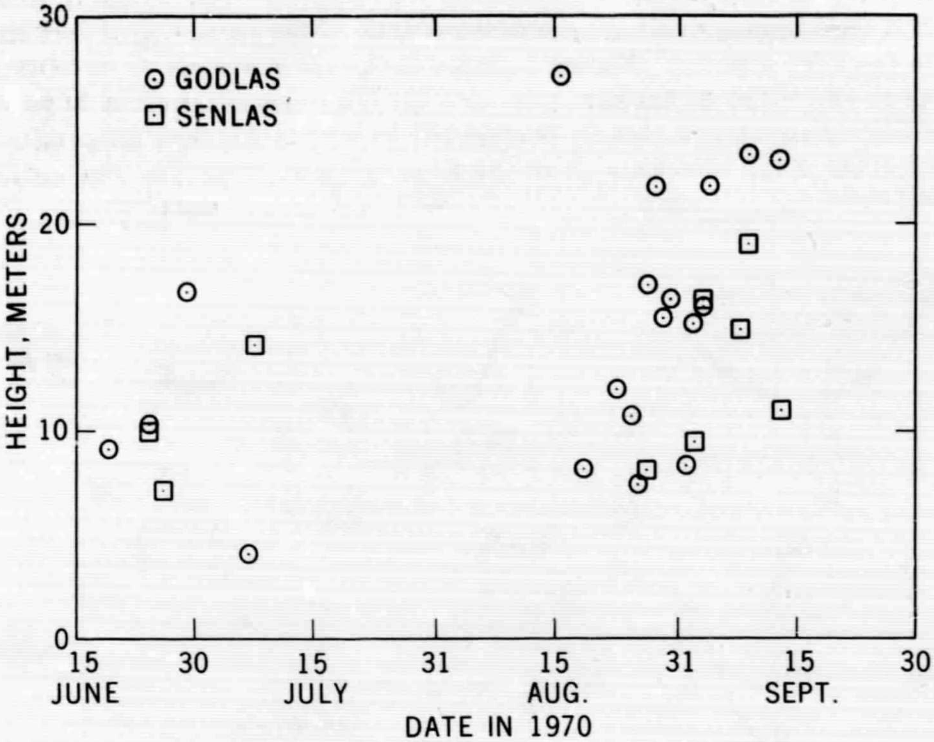

Figure 4. GODLAS and SENLAS Height Adjustments

therefore the period of the perturbation may be about 20 minutes implying that coefficients of order five or six may be the cause. However, the least squares process in the orbit determination program will minimize the residuals and in so doing may appear to artificially reduce the period of the perturbation. The coefficients responsible could therefore be of even lower than order five. An examination of the residual patterns in all of the mini-arcs is currently in progress and it is already evident that the pattern seen in mini-arc 18 is in no way unique.

The GSFC laser systems also provide a measure of the direction of the returning pulse but only to the extent of being within the beam width of the system. For GODLAS this is about 3 minutes of arc and for SENLAS about 1 minute of arc but because of the obvious possibility of a bias in these measurements they have not been used in any of the analyses. A cursory comparison of the observed angles with angles computed from a range only solutions indicate that the rms noise of the angles is about 45 arc seconds for both systems and with biases of about 30 arc seconds.

3. NORMALIZED STATION COORDINATES

In order to take maximum advantage of the accuracy of the laser observations, the best available model of the Earth's gravitational field and best locations of the stations have to be used. The present gravitational models have been generated from observational data that are not as accurate as the laser range measurements and, as Figure 3 demonstrates, the laser observations show clearly the limitations of the present SAO model (Gaposchkin and Lambeck, 1970). Also, only limited knowledge of the positions of the GODLAS and SENLAS stations was available. The GODLAS position which had been determined dynamically by Marsh et al (1971) from laser and optical observations, is probably accurate to 5 to 10 meters and is in general agreement with the position obtained by the SAO (Gaposchkin and Lambeck, 1970). For SENLAS which was a new tracking location, only the North American Datum (NAD) coordinates, obtained by the Field Facilities Branch GSFC, were available (NASA, unpublished report, 1970).

Because neither the gravitational field nor the tracking station locations were known with accuracies comparable to the laser measurements an attempt was made to recover "normalized" station coordinates for both GODLAS and SENLAS from the data obtained during the polar motion experiment. These normalized positions would only be valid when used in conjunction with the gravitational field used to derive them (Standard Earth II) and only for the BE-C spacecraft but these positions would have absorbed components of the systematic errors in the orbit caused by errors in the gravitational model.

The method of deriving these normalized coordinates was as follows. Short arc orbits were fitted through each individual pass of BE-C GODLAS data for days when three or more consecutive passes were observed. Short arc orbits were then fitted through the first two consecutive passes of each observing period and it was evident in all cases that the orbital fit through two consecutive passes was equally as good as through each pass separately. For GODLAS, this was typically an rms of 30 to 50 cm. The orbital fit was then extended to three passes and then to four passes where possible. For three passes the rms fit was nearly always larger than a meter and for four passes jumped to about four meters. In addition to the larger rms fits the three and four pass orbits also showed the average residual to be several tens of centimeters, compared to near zero for the shorter arcs, indicating that the residuals were no longer random and that there was a biasing of the orbit caused by the dynamical constraints of the gravitational model.

The recovery of GODLAS's latitude as well the orbital elements through the 3 and 4 pass orbital arcs had little or no effect on the rms or the average residual but the recovery of the GODLAS height reduced the rms of the range residuals to about 80 cm on 3-pass orbital arcs and to 2 or 3 meters on 4-pass orbital arcs. In both cases the average residual fell to a few centimeters. Twenty-one 3 and 4 pass orbital arcs from GODLAS have been analyzed in this way and the height adjustments that have been obtained have ranged from a low of +4 meters to a high of +27 meters with the exception of two values which are completely different for, as yet, unknown reasons. The significance of the adjustment being so large and all values being positive is unclear but the possibility of the radius of the Earth at GODLAS being several meters larger than expected cannot be ruled out. The contributions to the errors in height recovery are discussed later.

In order to obtain one best height adjustment to GODLAS from the GODLAS data a multi-arc solution was performed for all 21 three and four pass orbital arcs in which 6 orbital elements were recovered for each arc together with one parameter common to all arcs, the GODLAS height. The multi arc solution for the adjustment of the GODLAS height was +14.8 meters above the ellipsoid of the Standard Earth II. The values of the GODLAS height obtained by the Smithsonian Astrophysical Observatory (Gaposchkin and Lambeck 1970; Marsh et al, 1971), and the normalized value obtained here, all based on the Standard Earth II, are shown in Table 2.

Table 2

Values of GODLAS Height Based On
Standard Earth II.

AUTHOR	GODLAS HEIGHT
Gaposchkin and Lambeck (1970)	+9 meters
Marsh et al (1971)	+4 meters
This paper	+18.8 meters

A similar treatment of the data collected at SENLAS provided 9 adjustments to the SENLAS height from 3 and 4 pass orbital arcs, all of which lay between +7 meters and +19 meters with a multi-arc solution from all 9 arcs of +15.4 meters. This adjustment is almost identical to that obtained for GODLAS and probably indicates that these adjustments are due to errors in the present Earth model rather than in the measurement systems themselves. A plot of the GODLAS and SENLAS height adjustments that were obtained from the data are shown in Figure 4.

The dependence of the normalized height on the latitude of the station is shown in Figure 5 for GODLAS. The latitude that was adopted throughout the calculations was 39° 01' 13.''88 but the normalized height was recovered using several values of the latitude and Figure 5 shows the 15 meter height adjustment required by both sets of laser data cannot be explained by merely changing the latitude by GODLAS by any reasonable amount.

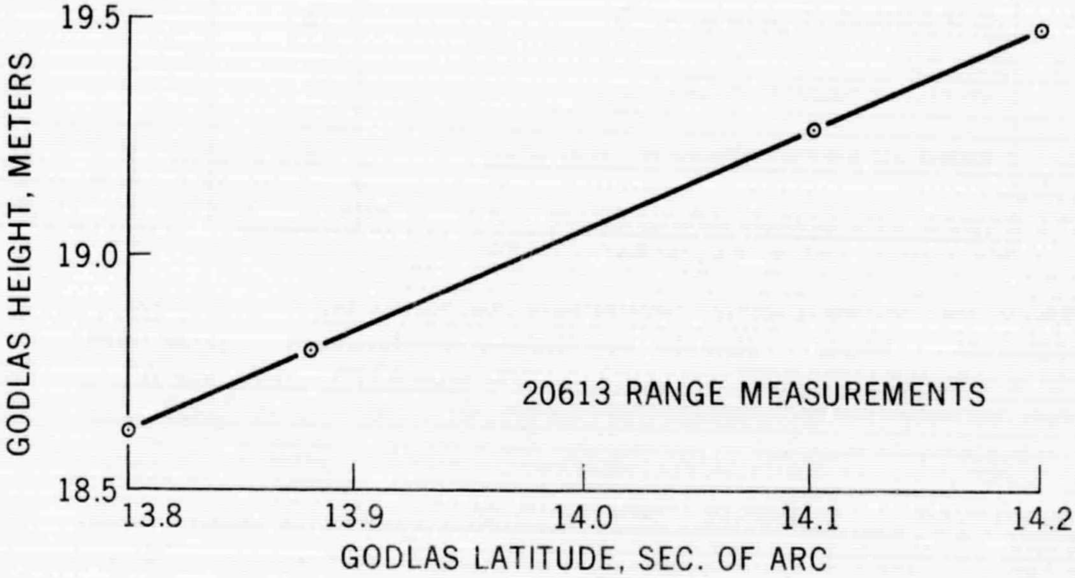

Figure 5. GODLAS Height Recovery

The effects of dynamic and observational errors on the adjustments of the GODLAS and SENLAS height have been estimated using an error analysis program and also by making additional solutions for the heights with different values of some of the parameters in the Earth model. The results of this investigation are shown in Table 3 where the largest contributor to height error at GODLAS is seen to be the gravity field. The measure of the gravity model error that was used was a quarter of the difference between the SAO 1966 Standard Earth (Lundquist and Veis, 1966) model and the APL 3.5 model (Guier and Newton, 1965). This difference has been shown by Martin and Roy (1971) to be a reasonable measure of the geopotential error of the Standard Earth II model for the Geos 2 orbit. Althought we must be cautious in using the same error model for BE-C, it probably sufficies to indicate the general size of the error.

If the errors in the models shown in Figure 3 are reasonable then it is difficult to account for the 15 meter adjustment at both sites. Furthermore, the range bias of 1 meter is probably rather generous and is more probably of the order of 25 cm. Thus there appears some evidence to suggest that the radius to

Table 3

Estimated Contributions to Errors in GODLAS Height

GRAVITY MODEL ERROR (1/4 x SAO-APL)	6.7 m.
GM ERROR (1 part in 10^{-6})	2.8 m.
GODLAS RANGE BIAS (1 m.)	3.8 m.
GODLAS REFRACTION ERROR (5%)	1.3 m.
GODLAS LATITUDE ERROR (10 m.)	0.7 m.

the geoid at the stations is larger than is indicated by the Earth model. However, it is important to remember that the SAO solution for the GODLAS height (see Table 2) is 5 meters larger than that obtained by Marsh et al (1971) and that the difference between the SAO solution and that given here is only 10 meters and could be explainable from the error contributions shown in Table 3.

At this stage the coordinates of GODLAS were fixed at the apriori values for latitude and longitude and with the recovered height of 18.8 meters; these were the "normalized" GODLAS coordinates for this experiment.

"Normalized" coordinates for SENLAS were derived from a multi-arc solution of 29 short arcs no longer than 6 hours composed of GODLAS and SENLAS observations. The normalized coordinates of GODLAS were used and kept fixed in the calculations and from the multi-arc solution values of the SENLAS latitude, longitude and height were obtained and designated the SENLAS normalized coordinates. The final normalized coordinates of GODLAS and SENLAS were

GODLAS	39° 01' 13" .880 N	283° 10' 18" .500 E	18.8 m
SENLAS	42° 42' 04" .881 N	283° 19' 17" .203 E	200.0 m

on the Earth model of the Standard Earth II (Gaposchkin and Lambeck, 1970). As a check on the precision of the recovery of the SENLAS position the data were divided into two sections; those observations that were obtained during the first six weeks of the experiment and those obtained during the second six weeks; and a new position for SENLAS was determined from each section of data. In addition, the data collected in two consecutive eight day periods, at the end of August and the beginning of September, 1970, were analysed to provide estimates of the SENLAS position. The importance of the latter two data sets was that they constituted the data collected during a period of relatively fine weather with each data set containing five mini-arcs of data and could be expected to provide an indication of the results obtainable under normal working conditions in a good climate. The five sets of cartesian coordinates for SENLAS are shown in Table 4. Solution 1 is that obtained from all the data (normalized position), solutions 2 and 3 are from the first and second halves of the data, and solutions 4 and 5 from the two 8-day periods.

Table 4

Coordinates of the SENLAS Tracking Station

Solution	X (m)	Y (m)	Z (m)
1	1069755.44	-4571171.67	4303326.88
2	1069754.69	-4571172.56	4303327.04
3	1069755.62	-4571171.27	4303326.75
4	1069755.75	-4571171.81	4303327.38
5	1069755.26	-4571171.34	4303326.18

Table 4 shows the range of values of each of the three coordinates is spread over about 1 to 1.3 meters with the "best" solution using all the data in the middle. From these figures it appears the SENLAS position has been determined with a precision of about 60 cm in each coordinate. A fact, however, that is not obvious from Table 4 is immediately evident in Table 5 which gives the chord-distance between GODLAS and SENLAS for each of the solutions. This is that the five SENLAS positions lie between two concentric spheres differing in radius by only 53 cm. Thus, although SENLAS appears to be equally well determined in each of the three cartesian coordinates it is, in effect, better determined in radial distance from GODLAS than in the tangential directions by at least a factor 2. For determining the orbital inclination of the BE-C satellite the chord distance between the stations is a fundamental parameter of the experiment and from Table 5 this appears to have been determined with a precision (or repeatability) of 25 to 30 cm.

Also shown in Table 5 is the estimate of the chord distance between the stations obtained by the Field Facilities Branch of GSFC[5] by ground survey. This value differs by only 43 cm from the best solution (Solution 1) obtained here and is probably less than the accuracy of either result. The estimated contributions to the error in the satellite solution are shown in Table 6 where the largest contributor is seen to be the gravity field and from which we estimate the total

Table 5

GODLAS-SENLAS Chord Distance

Solution	Chord Distance	Survey Difference
1	408699.20 m	+43 cm
2	408698.87 m	+10 cm
3	408699.33 m	+56 cm
4	408699.44 m	+67 cm
5	408698.91 m	+14 cm
Ground Survey	408698.77 m	

Table 6

Estimated Contributions to Baseline Errors

GRAVITY MODEL ERROR (1/4 x SAO-APL)	2.5 m.
GM ERROR (1 part in 10^{-6})	0.2 m.
GODLAS RANGE BIAS (1 m.)	0.3 m.
SENLAS RANGE BIAS (1 m.)	-0.3 m.
GODLAS REFRACTION ERROR (5%)	0.1 m.
SENLAS REFRACTION ERROR (5%)	0.1 m.
GODLAS HEIGHT (10 m.)	1.6 m.

error of the recovered chord distance to be 2 to 3 meters. Table 6 also indicates why the chord distance measurement has such a good repeatability. For a near circular orbit, such as BE-C, all but the range biases will contribute in the same way by about the same amount in every solution. Further, it is interesting to note that if the range biases are the same and constant then they have no effect on the solution.

Tables 3 and 6, giving the estimated errors in the station heights and the chord distances, tell us something about the biases of the laser systems. Independent recoveries of the GODLAS and SENLAS heights gave the same answer to within a meter and therefore the range biases (see Table 3) were either small or very similar in magnitude. Table 6, however, indicates that the biases must also be reasonably constant for the result to have a repeatability of 30 cm.

4. LOVE'S NUMBER, k_2

The locations of the laser tracking stations were chosen so as to optimize the tracking of the BE-C spacecraft through its most northerly section of its orbit thus providing the opportunity of precisely measuring the osculating inclination. The intial analysis of the laser ranges was based on fitting an orbit through 4 to 6 hours of data (a mini-arc) and then determining the maximum latitudes reached by the spacecraft from an ephemeris generated through the data based on the recovered orbit. Thus, for a four pass mini-arc four values of the maximum latitude were obtained but usually the spacecraft was only actually under observation at the time of passing through the maximum latitude position on one or two of those passes. On approximately 30 days sufficient data was available to form 4 to 6 hour orbital arcs from which the osculating values of the inclination were obtained. The values of maximum geodetic latitude (directly related to the osculating inclination) are shown in Figure 6 from which two effects are apparent. Firstly, there is a large variation in maximum latitude*, amounting to over 30 seconds of arc, during the three months of the experiment and secondly, there is a large change in maximum latitude from one pass to the next. The symbols used to show if the point of maximum latitude was actually under observation at the time of passing through that position are the same as those used in Figure 2. The spread of maximum latitudes representing a systematic increase in the orbital inclination (during a 4 to 6 hour period) from one pass to the next as the Earth rotates underneath the orbit is clearly evident in nearly all the arcs. This increase in the inclination amounts to 5 to 10 seconds of arc in about 6 hours.

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* For the purposes of the discussions "variations in maximum latitude" are synonymous with "variations in osculating inclination".

Both the long and the short term changes in the orbital inclination can be ascribed to gravitational forces; the large long period effect is caused by the sun and moon and the short term effect by the low degree and order tesseral harmonics of the Earth. Fortunately, these effects can be calculated with adequate accuracy

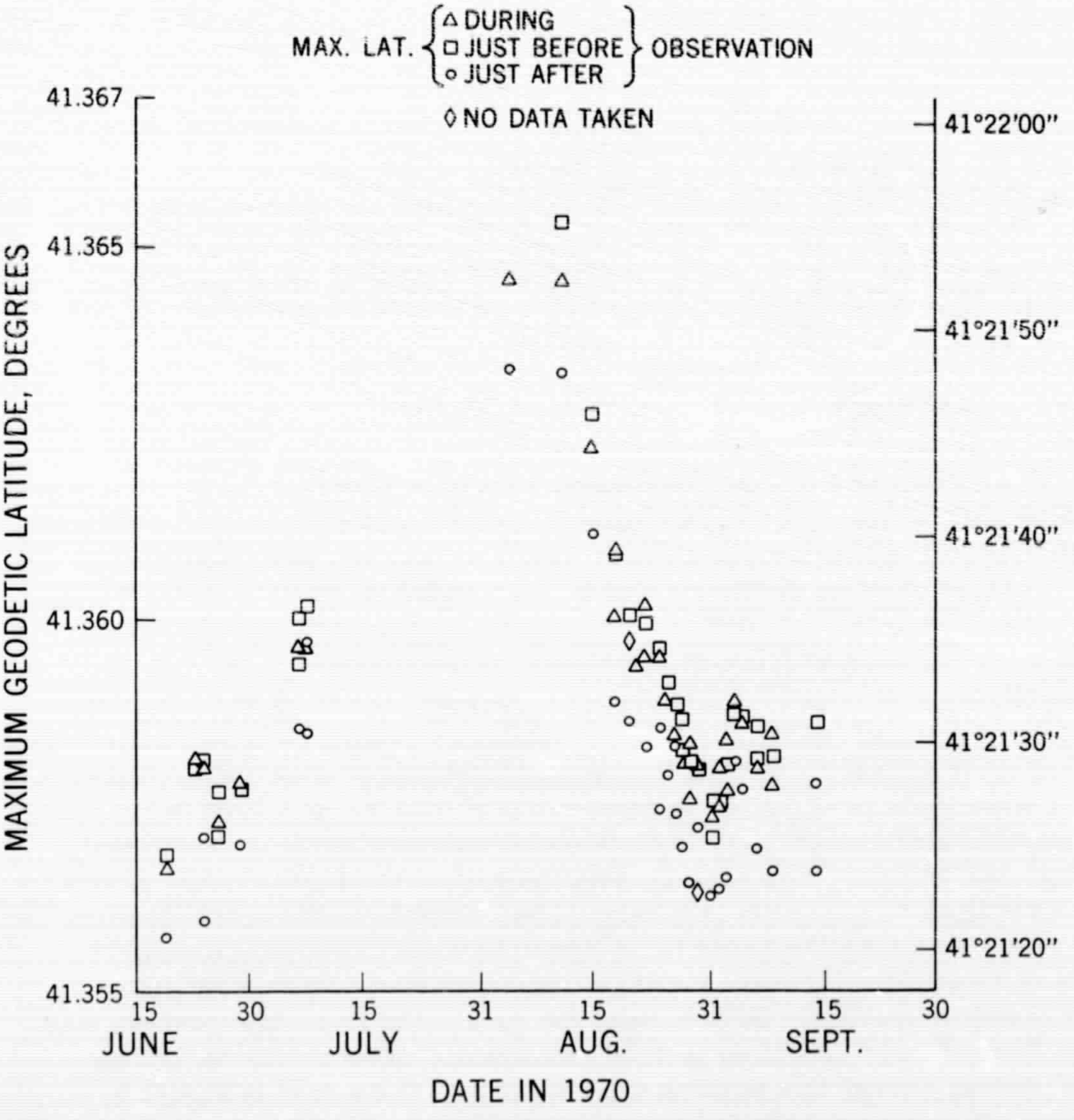

Figure 6. Mini Arc Maximum Latitudes

so that they do not substantially inhibit the study of the unmodeled perturbations, such as Earth tides. However, this is not to say that our knowledge of the Earth's gravitational field is not one of the limiting factors in studies of unmodeled forces. On the contrary, it is believed to be an important limitation but it is not thought to be a limitation on the short 4 to 6 hour arcs, only on the long 3 month arc.

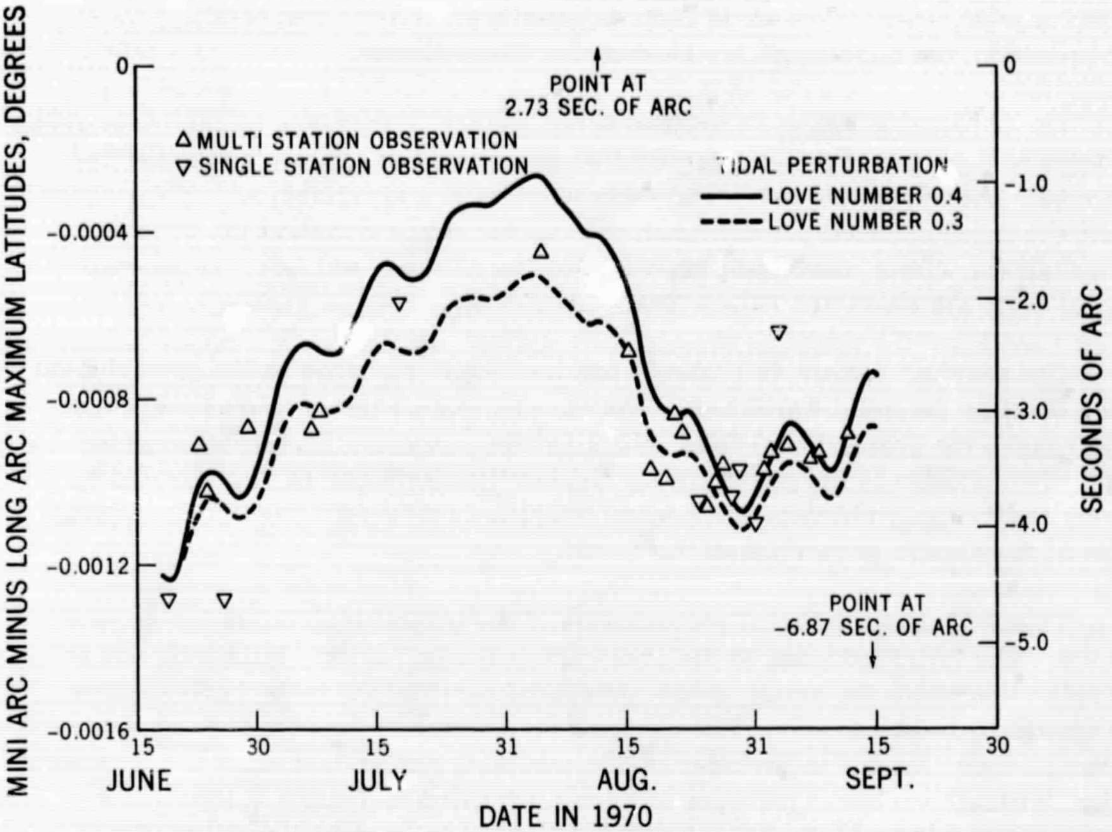

Figure 7. Difference in Maximum Latitudes Compared with the Tidal Perturbation in the Inclination

This fact is demonstrated in Figure 7 in which the differences between the maximum latitudes obtained from one single long arc of 3 months are shown. The long term variation in inclination shown in Figure 6 has now been reduced by a factor of about 10 (Figure 7) but it is important to notice that the basic shape of the two curves (Figures 6 and 7) are the same. In addition, only one value is given in Figure 7 for each short arc; this is because after having subtracted the long arc maximum latitudes from the short arc maximum latitudes the resulting differences are the same for each pass of a short arc to within about a hundredth of a second of arc (about 30 cm on the Earth's surface).

The single three month long arc fitted to all the data was obtained after considerable difficulty had been experienced with the convergence of the solution. These difficulties, which were overcome by increasing the arc length by about a week at a time, were probably caused by errors in the force models, the observations and, to a lesser extent, by the precision of the computational procedures and are reflected in the poor fit of the laser ranges to the orbit which had an rms of the residuals of nearly 2 km. With this size of rms the precision of the laser observations is completely lost and, in fact, an equally good orbit was obtained for the same period from mini-track interferometer observations.

The purpose of fitting an orbit to three months of data was to derive an orbit for the whole period of the experiment that smoothed through all the unmodeled perturbations. When this orbit was subtracted from a short arc, which was forced to absorb the unmodeled perturbation, the residuals would reflect the unmodeled perturbations. Thus, maximum latitudes for the long arc were generated, subtracted from the short arc values and the residuals shown in Figure 7.

The residual pattern in Figure 7 can be largely ascribed to the perturbation of the orbit by the solid Earth tide. The sun and moon raise a tide on the Earth that changes the gravitational field, which in turn, causes a perturbation of the orbit. This change in the gravitational field of the Earth can be represented by Love's number, k_2, which is a dimensionless parameter conveniently describing some of the elastic properties of the Earth.

Figure 7 shows the tidal perturbation of the orbital inclination for $k_2 = 0.3$ and 0.4. The large increase in the residuals at the end of July is largely due to the solar tide while the small, quasi-sinusoidal oscillations in early September are caused by the lunar tide. The cause of the scatter of the points about the two curves is uncertain but is probably due to modeling errors and to lack of geometry in the short arc orbits. Two types of result are given in Figure 7; the first contains data from both laser stations and is referred to as a multi-station observation and the second is a single station observation, composed of only GODLAS or SENLAS data. The latter is obviously the weaker solution because with range only data from one station there is much greater freedom for the adjustment of the orbit with almost no constraint except for the gravitational field. It is therefore not surprising that the majority of "bad" points in Figure 7; including those 2 actually off the Figure, are single station solutions.

An analysis of the residuals in Figure 7 leads to the preliminary result

$$k_2 = 0.35 \pm 0.05$$

as the best value for the second degree Love number. This method of measuring k_2 provides a mean global value of the parameter since it is sensing the change in the gravitational field at an altitude of about 1000 km and by a technique that integrates a perturbation over many revolutions of the spacecraft. The value obtained here compares favorably with the two other global measures of k_2, 0.29 (Kozai, 1968) and 0.34 (Newton, 1968). However, the Love number k_2 is related to the period of the Chandler wobble of the Earth on its axis by the relationship (Munk and MacDonald, 1960).

$$\frac{\sigma_0}{\sigma_r} = 1 - \frac{k_2}{k_{2s}}$$

where σ_0 is the frequency of the Chandler wobble, σ_r is the frequency of the free nutation of a rigid Earth and k_{2s} is the secular Love number. σ_r is related to the moments of inertia of the Earth about its principal axes and is determinable with considerable precision while k_{2s}, the secular Love number, is believed to be about 0.95 based on recent Earth models. Thus, a value of k_2 implies a Chandler period and the value of k_2 obtained here suggests a period about 10% larger than is actually observed if the secular Love number is taken to be about 0.95. A value of k_2 nearer 0.30 would satisfy present Earth models and is just within the acceptable range of one sigma from the preliminary result obtained here.

5. CONCLUSIONS

The initial analysis of the laser tracking data obtained during the experiment has shown that the present GSFC lasers have the capability of providing data of sufficient quality to permit the determination of site locations with a repeatability of better than a meter and to detect and measure the distortion of the Earth's gravitational field by the solid Earth tides. These results, obtained with only two ground stations, demonstrate the important contribution that laser tracking of Earth satellites can be expected to make to our knowledge of the Earth's shape, gravitational field and dynamical behavior in the next few years.

Our present ability to determine interstation distances with a precision of 25 to 30 cm, as demonstrated during this experiment, opens up the possibility of measuring the motions of tectonic plates and the movements along fault lines over a number of years. Such experiments become even more attractive when one remembers that the present GSFC laser systems could improve by a factor of 2 or 3 in the next few years and that the largest source of error, the gravity field, can be expected to undergo comparable improvements.

The analysis of the data collected during the preliminary polar motion experiment is continuing and will provide an indication of our ability to observe the motion of the pole of rotation of the Earth in the meridian containing the station. However, we feel that the results to date already indicate that the present systems and the techniques described here can profitably be used in our study of the solid Earth.

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