STELLAR CALIBRATION OF L-/S-BAND

AND VHF RECEIVING SYSTEMS

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RALPH E. TAYLOR

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STELLAR CALIBRATION OF L-/S-BAND AND VHF RECEIVING SYSTEMS

This paper describes a stellar calibration method for the L-/S-Band and VHF frequencies, discusses associated calibration errors, and presents stellar calibration measurements made in the National Aeronautics and Space Administration's space tracking and data acquisition network (STADAN) stations at 137 MHz, 402 MHz and 1702 MHz.

Telemetry data acquisition receiving stations located in the northern hemisphere can be calibrated accurately using the absolute flux density from either Cassiopeia A (Cas A) or Cygnus A (Cyg A). Effective antenna gain, system noise temperature, and the gain-to-noise-temperature ratio (G/T) can be determined to better than one decibel, using these stellar sources. This calibration accuracy is possible since the Cas A and Cyg A noise flux densities are known within several tenths a decibel from radio astronomy observations.

Cas A and Cyg A provide sufficient noise flux density, at the L-Band and S-Band frequencies, to calibrate a paraboloidal dish antenna as small as 20-feet (6-m) in diameter; dish antennas ranging up to 85-ft. (26-m) in diameter can be calibrated without the necessity of a beam correction factor.

Furthermore, a VHF (137 MHz) phase array antenna, with an effective area as small as fifty square meters (50 m^2), has been calibrated accurately from both Cas A and Cyg A.

From an operational standpoint, it is desirable to calibrate field station antennas using an on-site radio-frequency (RF) telemetering receiver.

To accomplish this, the NASA references[1-3] describe a calibration method that utilizes a conventional total-power[4] receiver for stellar calibration; typical investigators[5,6] utilized a switched Dicke[4] radiometer receiver. Being much simpler, a total-power receiver consists of a front-end receiver (disabled AGC), and a square-law detector followed by a low-pass-amplifier RC integrator, and d-c recorder.

The primary advantage of a Dicke receiver is freedom from predetection gain instability; however, by carefully stabilizing all supply voltages to a total-power receiver, excellent gain stability can be achieved, especially over short-time intervals such as 5-minutes, required for a stellar calibration. For the case of a receiver operating where the system noise temperature, T_{sys} = 400K, a predetection gain change by 0.1 percent gives a change in noise temperature of only 0.4K. A 0.4K noise temperature instability is sufficient for calibrating a 20-ft. (6-m) dish antenna at L-/S-Band frequencies.

By way of comparison, the stellar flux density from Cas A or Cyg A is relatively low compared to solar flux density at L-/S-Band frequencies (Slide 1). For instance, the solar flux from the quiet sun is three orders of magnitude greater than the Cas A flux density at S-Band. Even though the stellar flux is low, it is nevertheless constant within several tenths of a decibel. On the other hand, the larger solar flux can vary several decibels, subsequently requiring a prediction for accurate solar calibration.[8]

In Slide 2, the antenna noise temperature increase, T_{star}, due to a point-source radio star crossing the antenna's boresight axis, is defined for a single polarization as

$$T_{star} = \frac{G\lambda^2 F}{8\pi k} \text{ degrees K}$$

where G = effective antenna power gain, above isotropic

λ = free-space wavelength, meters (m), at same frequency as G is measured

F = radio-star flux density, watt/meter^2/Hz ($\text{w m}^{-2}\ \text{Hz}^{-1}$)

k = Boltzmann's constant = 1.38×10^{-23} J/K.

The value of T_{star} was computed for a parabolic dish antenna diameter ranging from 20-ft. (6-m) to 85-ft. (26-m), to be calibrated at L-Band with either Cas A, Cyg A or the quiet sun. T_{star} ranges from 10K to 270K for the stellar sources, and 6200K to 125,000K for the quiet sun (Slide 2).

Also shown is the corresponding predetection RF power increase, ΔP_{if}, due to T_{star}, for a typical receiving system noise temperature, T_{sys} = 400K. Values of ΔP_{if} range from only 0.1dB to 2.2dB for the stellar sources; whereas correspondingly higher values result for the quiet sun which range from 12.2dB to 25.0dB.

For calibrating a 20-ft. (6-m) diameter dish antenna at L-Band, the total-power receiver is capable of resolving a 10K noise temperature increase with an accuracy of $\Delta T_{min} \leq 0.5$K. For example, in a receiving system where T_{sys} = 400K, with a predetection noise bandwidth, Δf = 1.0 MHz, and a postdetection time constant, RC = 1.0 sec., the noise temperature accuracy for a total-power receiver with constant predetection gain is defined by Kraus[4] as

$$\Delta T_{min} = \frac{T_{sys}}{\sqrt{2\ RC\ \Delta f}} \text{ degrees K}$$

or

$$\Delta T_{min} = \frac{400K}{\sqrt{2(1.0s)(10^6 Hz)}} \cong 0.3K .$$

Slide 3 shows the essential elements of a total-power receiver, used for stellar calibration, that includes: low-noise preamplifier, station RF receiver (manual gain control), square-law detector, RC postdetection filter, and d-c amplifier.

Two output voltage measurements from a square-law detector are required: an "off-star" voltage, V_{DC}, and the change in detector voltage, ΔV_{DC}, that results upon shifting the antenna mainlobe from the "off-star" to the "on-star" position. Two precautions are necessary for the "off-star" position; first, V_{DC} must be measured upon shifting the mainlobe along constant brightness-temperature contour lines; and second, the radio star, essentially a point source, must be positioned within the first null of the antenna's radiation pattern.

Another necessary condition is that the receiving system parameters, consisting of RF gain, dc gain, predetection bandwidth, and the detector constant, must remain fixed for the two positions of the antenna mainlobe. A final condition is that the Ko[9] beamwidth correction factor must be applied to stellar calibration for antennas larger than 85-ft (26-m) in diameter, operating at S-band and above.

The isotropic antenna gain, G, referenced to the preamplifier input, can then be determined from the relationship[1-3]

$$G = \frac{8\pi k}{\lambda^2 F}\left[\frac{\Delta V_{DC}}{V_{DC}}\right]\{\varepsilon\ [T_A - T_0] + T_0 + T_R\}$$

for a single polarization, where

k = Boltzmann's constant = 1.38×10^{-23} J/K

λ = free-space wavelength, m

F = radio star's noise flux density, watts/meter^2/hertz, w m^{-2} Hz^{-1}

$\Delta V_{DC}/V_{DC}$ = amplified output voltage ratio from square-law detector

V_{DC} = dc amplifier output voltage for "off-star" position of mainlobe, vdc

ΔV_{DC} = change in dc amplifier output voltage upon shifting mainlobe from "off-star" to "on-star" position, vdc

ε = antenna-to-preamplifier transmission line power loss, $0 < \varepsilon \leq 1$

and

T_0 = ambient physical temperature of antenna-to-preamplifier transmission line, K.

In general, the terms within the braces in the above expression for G constitute the receiving system's kelvin noise temperature, T_{sys}, for the "off-star" position, where

$$T_{sys} = \varepsilon \, [T_A - T_0] + T_0 + T_R \, .$$

But more generally,

$$T_A = \sum_{1}^{n} T_{ant}$$ = total antenna noise temperature for n noise sources, K

and

$$T_R = \sum_{1}^{N} \frac{T_i}{\pi_1^i G_j}$$ = receiver noise temperature, referenced to preamplifier input terminals, for N cascaded networks, K.

The square-law detector is ideal since it makes possible an output calibration that is independent of the detector input power level, for fixed predetection gain. When the system noise temperature, T_{sys}, drifts slowly over an incremental range, ΔT, because of small variations in background noise, the detector input power will change correspondingly. However, for $\Delta T << T_{sys}$, and constant postdetection gain, the "on-star" voltage, ΔV_{DC}, for a given radio star, is constant and proportional to T_{star}, being independent of T_{sys} (Slide 4).

Thus, in this respect, the square-law detector is self-calibrating; furthermore, the receiver dynamic range need be linear only from 0dB up to about 3dB, linearity being defined as a linear relationship existing between the input RF signal power and the output d-c voltage from the square-law detector. The postdetection gain ranges from 25dB to 50dB (constant at a fixed value), this amount of d-c gain being required to amplify the low signal output from the square-law detector to a level suitable for recording.

By substituting $T_{star} = FG\,\lambda^2/8\pi k$ in the equation

$$T_{sys} = T_{star}\left(\frac{V_{DC}}{\Delta V_{DC}}\right) ,$$

the gain-to-noise-temperature ratio, G/T_{sys} (Slide 5) can be obtained from the expression

$$\frac{G}{T_{sys}} = \frac{8\pi k}{\lambda^2 F}\left[\frac{\Delta V_{DC}}{V_{DC}}\right]$$

This expression does not require a knowledge of antenna gain G, however, a knowledge of the true design value of G/T_{sys} should be available with which to compare the station measurements.

When the effective antenna gain, G, is known accurately, the system noise temperature, referenced to the "off-star" position, is obtained from

$$T_{sys} = \frac{G\lambda^2 F}{8\pi k}\left[\frac{V_{DC}}{\Delta V_{DC}}\right], \text{ K} .$$

A stellar calibration error analysis has been performed on the following gain expression (Slide 6), for a 20-ft. (6-m) diameter dish antenna operating at L-Band, where

$$G = \frac{8\pi k}{\lambda^2 F}\left[\frac{\Delta V_{DC}}{V_{DC}}\right] T_{sys}$$

$T_{sys} = \varepsilon[T_A - T_0] + T_0 + T_R$

= system noise temperature in "off-star" position, K

ε = RF line loss $0 < \varepsilon \leq 1$

$T_A \cong 40 \pm \sigma_{T_A} = 40 \pm 20$ K

T_0 = RF line ambient temperature, K

T_R = **receiver noise temperature, K.**

The primary assumptions (Slide 7) for the error analysis exclude the degrading effects of local radio-frequency interference (RFI), especially ignition noise. A stellar calibration can best be performed at night, or at other times when on-site RFI is low. Receiver gain variations, and multipath effects, have not been a problem. Ionospheric disturbance, resulting in star signal scintillation, in generally negligible at VHF (137 MHz), and above, except for brief time intervals in the equatorial and polar regions.

A statistical error analysis, for a typical L-Band receiving system using a 20-ft. (6-m) dish antenna, shows that absolute antenna gain, G, can be determined from a stellar calibration with an accuracy of $\pm$ 0.65 dB (Slides 8-11, and Appendix). Similarly, uncertainties in system noise temperature, $\sigma_{T_{sys}}$, and $\sigma C/T_{sys}$, can be determined within $\pm$ 0.4 dB at 1440 MHz (L-Band).

At VHF (137 MHz), the Appendix shows that $\sigma_G = \pm\ 0.8$ dB, $\sigma_{T_{sys}} = \pm\ 0.6$ dB, and $\sigma_{G/T_{sys}} = \pm 0.40$ dB, for a 50 m^2 phase array antenna. Somewhat reduced accuracy results at VHF compared to L-Band, except for the G/T_{sys} ratio.

Although a limited number of stellar antenna-gain calibrations have been made at 402 MHz and 1702 MHz in the NASA space tracking and data acquisition network (STADAN), the first objective has been to obtain antenna-gain calibrations at 136 MHz...the primary NASA spacecraft data acquisition frequency.

The summary results from 53 stellar antenna-gain calibrations, and 42 comparable aircraft-measured antenna gain calibrations-making a total of 95 independent gain determinations-are compared in Slide 12. STADAN antennas calibrated at 136 MHz include: 40-ft. (12-m) diameter parabolic dish antennas, phase-array antennas with an effective area of approximately 50 m^2 (21 dBi), and an 85-ft. (26-m) diameter parabolic dish antenna. These antennas are deployed at a total of nine world-wide STADAN station locations.

Except for several calibration points, the mean values of the stellar and aircraft antenna gain measurements agree closely, as shown in Slide 12. The solid-line curve represents the mean value of a comparable aircraft antenna-gain calibration for 42 out of the 53 stellar antenna-gain measurements; in 11 instances, antenna design handbook data was substituted for missing aircraft measurements. The aircraft measurements reference antenna gain to a standard-gain 136 MHz antenna.

A histogram plot of the 53 stellar-gain calibrations, from Slide 12, shows that the point scatter approximates a Normal (Gaussian) distribution; furthermore 60 percent (32 out of 53) of the stellar calibration points fall within a range of ± 1.0 dB. Since 68 percent of the points would fall within a $\pm 1\sigma$ range for a Normal distribution, this infers that $\sigma_G \leq \pm 1.0$ dB, at 136 MHz. This result agrees closely with the value of $\sigma_G \leq \pm 0.8$ dB, for VHF, obtained earlier.

Finally, two of the stellar antenna-gain measurements shown in Slide 12, for the $50m^2$ 136 MHz phase array antenna, were made using Taurus A (Tau A)...a weak radio star, or the Crab Nebula. The Tau A noise flux density[12] is 1.47×10^{-23} $w\ m^{-2}\ Hz^{-1}$, at 137 MHz, which compares to 13.8×10^{-23} $w\ m^{-2}\ Hz^{-1}$ for Cas A (1970.5 epoch), and 11.2×10^{-23} $w\ m^{-2}$ Hz^{-1} for Cyg A. Both Cas.A and Cyg A were used for the remaining 51 stellar antenna-gain calibrations in Slide 12.

A limited number of stellar measurements, using Cas A at 402 MHz, were made at the NASA Alaska STADAN station with a 40-ft. (12-m) diameter parabolic dish antenna (Slide 13). The test results show excellent repeatability; (G/T_{sys}), T_{sys} and G all repeat with 0.5 dB, five days later. Receivers 1 and 2 refer to two independent radio receiving systems, including low-noise preamplifiers, operating from the same antenna for polarization diversity.

Finally, a more extensive set of stellar measurements, using Cas A at 1702 MHz, were made at the NASA Alaska STADAN station (Slide 14), both with a 40-ft. (12-m) diameter dish, and an 85-ft. (26-m) dish antenna.

For the 40-ft. dish, G repeats on two different days within 0.8 dB; the G/T_{sys} ratio repeats within 0.2 dB. However, a deviation of 2.5 dB was observed for Receiver 2, between the first set of measurements (9 April 1971), and the second set on 14 April 1971. It was concluded that a 2.0 dB reduction in effective antenna gain, G, and a 0.5 dB increase in T_{sys} resulted in the 2.5 dB G/T_{sys} ratio reduction.

The 85-ft. (26-m) dish antenna measurements, at 1702 MHz, also show excellent repeatability; the G/T_{sys} ratio is within 0.4 dB, T_{sys} within 0.2 dB, and G within 0.8 dB. The gain value G = 44.7 dBi, for 10 April 1971, was verified as 44.0 dBi from an independent aircraft measurement employing a standard-gain reference antenna.

APPENDIX

L-BAND AND VHF STELLAR CALIBRATION ERROR ANALYSIS

A statistical error analysis, considering the propagation of precision indexes, is performed as follows on the expression for effective antenna gain,

$$G = \frac{a}{FV} \cdot T_{sys} .$$

where $a = 8\pi k\lambda^{-2}$ = constant,

$$V = \frac{V_{DC}}{\Delta V_{DC}} ,$$

and

$$T_{sys} = \varepsilon(T_A - T_0) + T_0 + T_R .$$

When the component quantities are independent, the mean-squared one-sigma error in antenna gain, σ_G^2, can be obtained from a general expression by Worthing and Geffner[10] as

$$\sigma_G^2 = \left(\frac{\partial G}{\partial F}\right)^2 \sigma_F^2 + \left(\frac{\partial G}{\partial V}\right)^2 \sigma_V^2 + \left(\frac{\partial G}{\partial T_{sys}}\right)^2 \sigma_{T_{sys}}^2$$

where σ represents the one-sigma uncertainty in each variable, and F, V and T_{sys} are mean values.

Taking partial derivatives,

$$\frac{\partial G}{\partial F} = -\frac{G}{F}, \quad \frac{\partial G}{\partial V} = -\frac{G}{V}, \quad \text{and} \quad \frac{\partial G}{\partial T_{sys}} = \frac{G}{T_{sys}} .$$

The minus signs can be ignored since the above terms will be squared. Similarly,

$$\sigma_{T_{sys}}^2 = \left(\frac{\partial T_{sys}}{\partial \varepsilon}\right)^2 \sigma_\varepsilon^2 + \left(\frac{\partial T_{sys}}{T_A}\right)^2 \sigma_{T_A}^2 + \left(\frac{\partial T_{sys}}{\partial T_0}\right)^2 \sigma_{T_0}^2$$

$$+ \left(\frac{\partial T_{sys}}{\partial T_R}\right)^2 \sigma_{T_R}^2 .$$

where

$$\frac{\partial T_{sys}}{\partial \varepsilon} = T_A - T_0, \quad \frac{\partial T_{sys}}{\partial T_A} = \varepsilon, \quad \frac{\partial T_{sys}}{\partial T_0} = 1 - \varepsilon, \text{ and } \frac{\partial T_{sys}}{\partial T_R} = 1.$$

From the above,

$$\sigma^2_{T_{sys}} = (T_A - T_0)^2 \, \sigma^2_\varepsilon + (\varepsilon\sigma_{T_A})^2 + (1 - \varepsilon)^2 \, \sigma^2_{T_0} + \sigma^2_{T_R} \, .$$

Substitution of the appropriate partial derivatives into the expression for σ^2_G results in a general expression for the rms, one sigma (1σ) gain error, normalized to antenna gain G, as

$$\frac{\sigma_G}{G} = \pm \left[\left(\frac{\sigma_F}{F}\right)^2 + \left(\frac{\sigma_V}{V}\right)^2 + \left(\frac{\sigma_{T_{sys}}}{T_{sys}}\right)^2 \right]^{1/2}$$

Finally,

$$\frac{\sigma_G}{G} = \pm \left\{ \left(\frac{\sigma_F}{F}\right)^2 + \left(\frac{\sigma_V}{V}\right)^2 + \frac{1}{T_{sys}^2} \left[(T_A - T_0)^2 \, \sigma_\varepsilon^2 + (\varepsilon \sigma_{T_A})^2 + (1 - \varepsilon)^2 \, \sigma_{T_0}^2 + \sigma_{T_R}^2 \right] \right\}^{1/2}$$

The significance of the above expression is described as follows. We shall assume that V, ε, T_0 and T_R can be measured independently in a typical receiving system with a worst-case accuracy of ten percent (i.e., σ_V, σ_ε, σ_{T_0}, and $\sigma_{T_R} \leq \pm$ 10%).

It should be noted that σ_G/G is insensitive to the uncertainty, σ_{T_A}, since in general, $\sigma_{T_A} << T_{sys}$. In fact σ_{T_A} can assume at 1440 MHz a value of $\sigma_{T_A} \cong \pm 0.5\ T_A$ without affecting significantly the value of σ_G/G. Furthermore, for 136 MHz, it is shown that σ_{T_A} may be set to equal the large value of 870 $\pm$ 200K, and yet maintain $\sigma_G < \pm$ 1 dB. Therefore, only a rough estimate of the antenna noise temperature, T_A, is needed to obtain an accurate determination of antenna gain.

The following approximation, given by Starker[11], can be used to obtain such an estimate of antenna-noise temperature, T_A, in kelvin, for a practical antenna, where

$$T_A \triangleq T_{sky}\, G_M \frac{\Omega_M}{4\pi} + \overline{T}_{sky}\, \overline{G}_s \left(0.50 - \frac{\Omega_M}{4\pi}\right) + 0.50\, T_E\, \overline{G}_s$$

mainlobe + side lobes + back lobe

and the condition

$$G_M \frac{\Omega_M}{4\pi} + \overline{G}_s \left[1 - \frac{\Omega_M}{4\pi}\right] = 1 \text{ is valid, and}$$

Ω_M = mainlobe solid angle, steradians

G_M = mainlobe power gain, above isotropic

$\overline{G}_s$ = mean value of antenna gain outside the mainlobe

T_{sky} = mean value of sky-brightness temperature, within half-power beamwidth of mainlobe, for "off-star" position, K

$\overline{T}_{sky}$ = mean value of sky-brightness temperature within secondary antenna lobes, K

T_E = effective noise temperature of the earth, K.

From Kraus[4] (p. 221),

$$\Omega_M = 1.13 \frac{\theta^\circ_{HP}\ \phi^\circ_{HP}}{(57.3)^2}$$

for a Gaussian power pattern, where θ°_{HP} and ϕ°_{HP} are the half-power beam widths, in degrees, for the θ and ϕ planes, respectively.

Again from Kraus[4] (p. 158),

$$G_M \simeq \frac{30{,}000}{\theta^\circ_{HP}\ \phi^\circ_{HP}}$$

Also, it can be shown that

$$\overline{G}_s \simeq 0.25 \text{ (-6.0 dBi).}$$

The above expressions give an estimate of the antenna noise temperatures as

$$T_A \simeq 0.82\ T_{sky} + 0.13\ (\overline{T}_{sky} + T_E),\ K$$

that is valid for a solid beam's half-power beamwidth, $\theta^{\circ}_{HP} = \phi^{\circ}_{HP} \leq 25°$.

Appropriate values for T_{sky} and $\overline{T}_{sky}$ can be obtained readily from a radio-sky map of noise-brightness temperature at the calibration frequency.

<u>L-Band Case (Stellar Calibration)</u>:

Type antenna	20-ft. (6-m) diameter parabolic dish
Frequency	1440 MHz (λ = 0.208 m)
Radio star	Cas A
Cas A flux density (1970.5 .. epoch, assuming 1.1 percent per year secular decrease)	$F \pm \sigma_F = (2.17 \pm 0.06) \times 10^{-23}$ w m^{-2} Hz^{-1}, Findlay[5], et al.
Antenna noise temperature ... for T_{sky} = 5K, $\overline{T}_{sky}$ = 4K and T_E = 290K	$T_A \pm \sigma_{T_A} \simeq (4 + 36) \pm 20K \simeq 40 \pm 20K$
Assumed system noise .... temperature ("off-star" position)	$T_{sys} \simeq 400K$ (mean value)

<u>Measured parameters (worst-case $1\sigma = \pm 10\%$)</u>:

$V \pm \sigma_V = 25.0 \pm 2.5$, $T_0 \pm \sigma_{T_0} = 290 \pm 29K$,

$\varepsilon \pm \sigma_\varepsilon = 0.8 \pm 0.08 = 1.0$ dB $\pm$ 0.4 dB loss,

and

$T_R \pm \sigma_{T_R} = 300 \pm 30K = 3.1$ dB $\pm$ 0.2 dB noise figure @ 290K.

Also,

$$\frac{\sigma_F}{F} = \frac{0.06 \times 10^{-23}}{2.17 \times 10^{-23}} = 0.0276 = 2.76\%$$

and

$$\frac{\sigma_V}{V} = \frac{2.5}{25} = 0.100 = 10.0\% \ .$$

From the expression for $\sigma^2_{T_{sys}}$,

$$\left[\frac{\sigma_{T_{sys}}}{T_{sys}}\right]^2 = \left[\frac{(T_A - T_0)\sigma_\varepsilon}{T_{sys}}\right]^2 + \left[\frac{\varepsilon\sigma_{T_A}}{T_{sys}}\right]^2 + \left[\frac{(1-\varepsilon)\sigma_{T_0}}{T_{sys}}\right]^2 + \left[\frac{\sigma_{T_R}}{T_{sys}}\right]^2 =$$

$$\left[\frac{(40-290)(0.08)}{400}\right]^2 + \left[\frac{(0.8 \times 20)}{400}\right]^2 + \left[\frac{(1-0.8)(29)}{400}\right]^2 + \left[\frac{30}{400}\right]^2$$

$$\left[\frac{\sigma_{T_{sys}}}{T_{sys}}\right]^2 = \underbrace{[0.050]^2}_{5.0\%} + \underbrace{[0.040]^2}_{4.0\%} + \underbrace{[0.0145]^2}_{1.45\%} + \underbrace{[0.075]^2}_{7.5\%}$$

or

$$(\sigma_{T_{sys}}/T_{sys} = \pm\sqrt{0.010} = \pm\ 0.10 = \pm\ 10 \text{ percent.}$$

Substituting the appropriate values in

$$\frac{\sigma_G}{G} = \pm \left[\left(\frac{\sigma_F}{F} \right)^2 + \left(\frac{\sigma_V}{V} \right)^2 + \left(\frac{\sigma_{T_{sys}}}{T_{sys}} \right)^2 \right]^{1/2}$$

gives

$$\frac{\sigma_G}{G} = \pm \left[(0.0276)^2 + (0.100)^2 + (0.100)^2 \right]^{1/2} = \pm\ 0.15$$

$$= \pm\ 15 \text{ percent}$$

or $\sigma_G = \pm\ 0.15\ G$ (worst-case at L-band).

For example, the above 20-ft. (6-m) diameter parabolic dish would have a mean value of antenna gain equal to about 5000, power gain above isotropic, at 1440 MHz. Expressed in dB above isotropic (dBi),

$$G \pm \sigma_G = 5000 \pm 0.15\ (5000) = 5000 \pm 750$$

$$= 37 \text{ dBi} \pm 0.65 \text{ dB } (1\sigma).$$

Finally, the above value of $\sigma_G = \pm 0.15G$ is different significantly from that obtained by a simple root-mean-square (rms) addition of the 1σ values for the individual parameters. Strictly speaking, an rms addition applies only to a sum or difference function; whereas the propagation of precision indexes applies generally, including a product or quotient function.

<u>VHF Case (Stellar Calibration)</u>:

Type antenna	Phase array with 50 m^2 effective area
Frequency	137 MHz ($\lambda = 2.2$ m)
Radio star	Cas A
Cas A flux density (1970.5 epoch, assuming 1.1 percent per year secular decrease) ..	$F \pm \sigma_F = (13.8 \pm 0.50) \times 10^{-23}$ w m^{-2} Hz^{-1}, (Parker[12]).
Antenna noise temperature for $T_{sky} = 950K$, $T_{sky} = 400K$ and $T_F = 290K$ ...	$T_A \pm \sigma_{T_A} = (780 + 90) \pm 200K = 870 \pm 200K$
Assumed system noise temperature ("off-star" position) ...	$T_{sys} \simeq 1100K$ (mean value).

Measured parameters (worst-case $1\sigma = \pm 10\%$):

$V \pm \sigma_V = 10.0 \pm 1.0$, $T_0 \pm \sigma_{T_0} = 290 \pm 29K$

$\varepsilon \pm \sigma_\varepsilon = 0.80 \pm 0.08 = 1.0$ dB $\pm$ 0.4 dB loss,

and

$T_R \pm \sigma_{T_R} = 300 \pm 30K = 3.1$ dB $\pm$ 0.2 dB noise figure @ 290K.

Also,

$$\frac{\sigma_F}{F} = \frac{0.50 \times 10^{-23}}{13.8 \times 10^{-23}} = 0.036 = 3.6\%$$

and

$$\frac{\sigma_V}{V} = \frac{1.0}{10.0} = 0.10 = 10\% \; .$$

From the expression for $\sigma^2_{T_{sys}}$,

$$\left[\frac{\sigma_{T_{sys}}}{T_{sys}}\right]^2 = \left[\frac{(T_A - T_0)\sigma_\varepsilon}{T_{sys}}\right]^2 + \left[\frac{\varepsilon\sigma_{T_A}}{T_{sys}}\right]^2 + \left[\frac{(1-\varepsilon)\sigma_{T_0}}{T_{sys}}\right]^2 + \left[\frac{\sigma_{T_R}}{T_{sys}}\right]^2 =$$

$$\left[\frac{(870-290)(0.08)}{1100}\right]^2 + \left[\frac{0.80 \times 200}{1100}\right]^2 + \left[\frac{(1-0.80)(29)}{1100}\right]^2 + \left[\frac{30}{1100}\right]^2$$

$$\left[\frac{\sigma_{T_{sys}}}{T_{sys}}\right]^2 = \underbrace{[0.042]^2}_{4.2\%} + \underbrace{[0.145]^2}_{14.5\%} + \underbrace{[0.005]^2}_{0.5\%} + \underbrace{[0.027]^2}_{2.7\%} = 0.0235$$

or

$$(\sigma_{T_{sys}}/T_{sys}) = \pm\sqrt{0.0235} = \pm\ 0.153 = \pm\ 15.3 \text{ percent.}$$

Substituting the above values,

$$\frac{\sigma_G}{G} = \pm\left[\left(\frac{\sigma_F}{F}\right)^2 + \left(\frac{\sigma_v}{V}\right)^2 + \left(\frac{\sigma_{T_{sys}}}{T_{sys}}\right)^2\right]^{1/2}$$

$$\frac{\sigma_G}{G} = \pm\ [(0.036)^2 + (0.10)^2 + 0.0235]^{1/2}$$

$$\frac{\sigma_G}{G} = \pm\ 0.187 = \pm\ 18.7 \text{ percent}$$

or

$$\sigma_G = \pm\ 0.187\ G \text{ (worst case).}$$

For example, the above 50 m^2 phase array would have a mean value of antenna gain equal approximately to 125, power gain above isotropic, at 137 MHz. Expressed in dB above isotropic (dBi),

$$G \pm \sigma_G = 125 \pm 0.187\ (125) = 125 \pm 23$$

or

$$G = 21 \text{ dBi} \pm 0.8 \text{ dB } (1\sigma \text{ worst case}).$$

Statistical Error Analysis of Stellar G/T Ratio Determination:

It is of interest that a G/T_{sys} determination, for the stellar calibration method, is more accurate than a determination of antenna gain. From the expression,

$$\frac{G}{T_{sys}} = \frac{a}{FV}$$

where

$$a = 8\pi k\lambda^{-2} = \text{constant}$$

$$V = \frac{V_{DC}}{\Delta V_{DC}}$$

V_{DC} = dc amplifier output voltage for "off-star" position of mainlobe, vdc.

ΔV_{DC} = change in dc amplifier output voltage upon shifting mainlobe from "off-star" to "on-star" position, vdc.

Applying the propagation of precision indexes,

$$\left[\frac{\sigma_{G/T_{sys}}}{G/T_{sys}}\right]^2 = \left(\frac{\sigma_F}{F}\right)^2 + \left(\frac{\sigma_v}{V}\right)^2$$

for stellar calibration.

The quantities on the right-hand side of the above equation were determined earlier for the stellar L-band case as:

$$\frac{\sigma_F}{F} = 0.0276 = 2.76\%$$

$$\frac{\sigma_v}{V} = 0.100 = 10.0\%$$

Substituting the above quantities,

$$\left[\frac{\sigma_{G/T_{sys}}}{G/T_{sys}}\right] = \pm\ [(0.0276)^2 + (0.100)^2]^{1/2}$$

$$= \pm\ \sqrt{0.0108} = \pm 0.104 = \pm\ 10.4 \text{ percent } (1\sigma \text{ worst case})$$

or

$$(\sigma_{G/T_{sys}}) = \pm\ 0.104\ (G/T_{sys}) = \left(\frac{G}{T_{sys}}\right) \pm 0.4 \text{ dB } (1\sigma \text{ worst case})$$

for stellar calibration at L-band.

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SLIDE 1. Cas A, Cyg A AND SOLAR FLUX DENSITIES AT L-BAND & S-BAND.

	1) Cas A		2) Cyg A		3) QUIET SUN	
	$(F \pm \sigma_F) \cdot 10^{-23}$ $wm^{-2} Hz^{-1}$	σ_F (dB)	$(F \pm \sigma_F) \cdot 10^{-23}$ $wm^{-2} Hz^{-1}$	σ_F (dB)	$F \cdot 10^{-23}$ $wm^{-2} Hz^{-1}$	ΔF (dB)
1440MHz	2.17 ± 0.06	0.12	1.54 ± 0.06	0.17	1000	< 3
1705MHz	1.90 ± 0.05	0.10	1.35 ± 0.05	0.15	–	–
2250MHz	1.54 ± 0.04	0.10	1.00 ± 0.04	0.17	1500	< 2

1) 1970.5 EPOCH; 1.1% PER YR. SECULAR DECREASE, FINDLAY, et al [5].

2) HEESCHEN [7].

3) TYPICAL SAGAMORE HILL RADIO OBSERVATORY DATA AT 1415 MHz (L–BAND) AND 2695 MHz (S-BAND), WHERE ΔF IS PEAK–TO–PEAK VARIATION OF SOLAR FLUX DATA.

SLIDE 2. PARABOLIC DISH NOISE-TEMPERATURE RISE FOR Cas A, Cyg A & QUIET SUN AT L-BAND.

	PARABOLIC DISH DIAMETER					
	85-FT.		40-FT.		20-FT.	
	T_{star}	ΔP_{if} (dB)	T_{star}	ΔP_{if} (dB)	T_{star}	ΔP_{if} (dB)
Cas A	270K	2.2	50K	0.50	14K	0.15
Cyg A	190K	1.7	36K	0.40	10K	0.10
QUIET SUN	125,000K	25.0	23,000K	17.7	6,200K	12.2

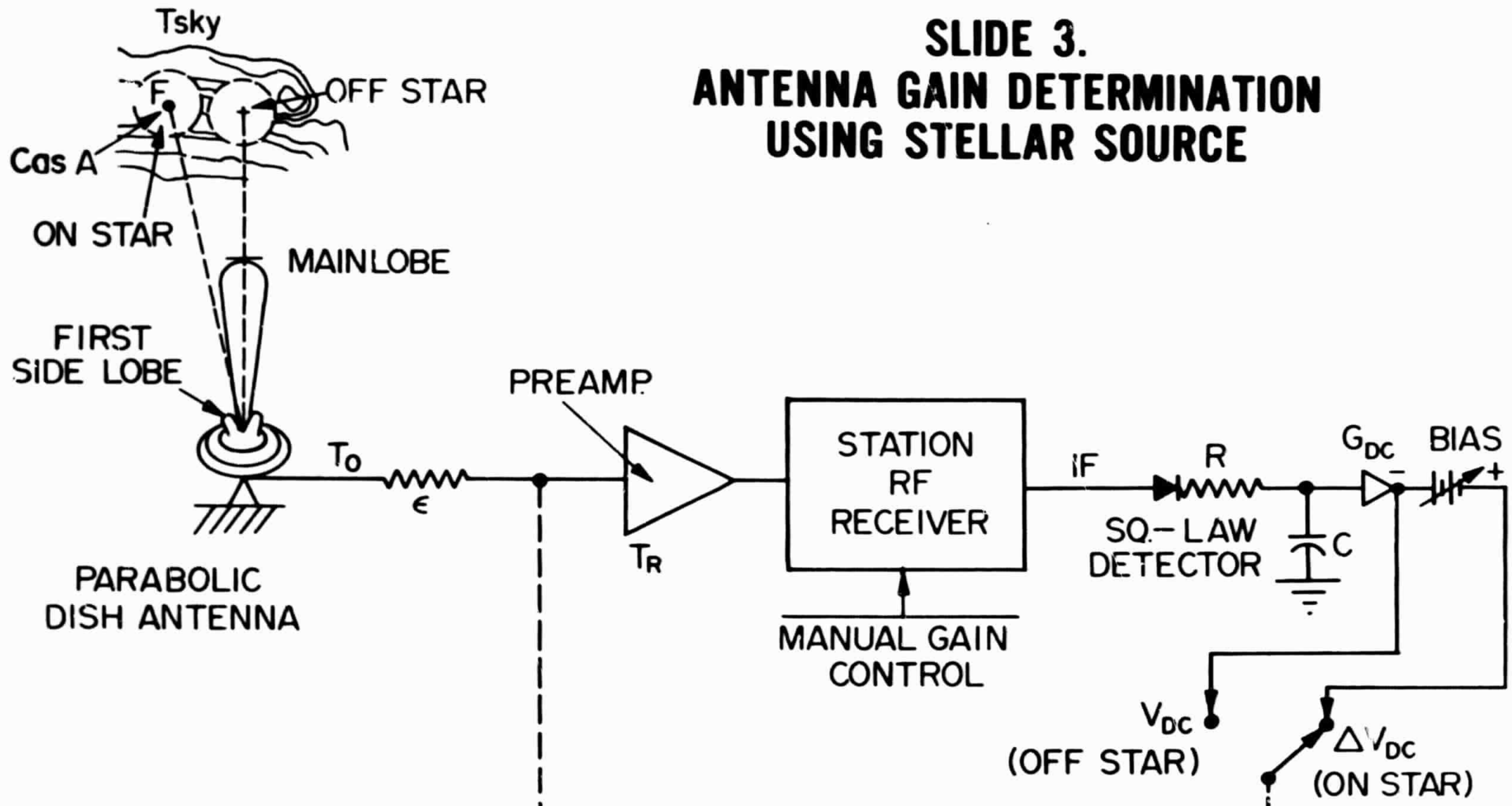
$T_{star} = \frac{G\lambda^2 F}{8\pi k}$ = ANTENNA NOISE TEMPERATURE RISE, K (FOR SINGLE POLARIZATION).

G = ANT. POWER GAIN, 55% EFFICIENCY, AT 1440 MHz.

F = STAR NOISE FLUX DENSITY, $wm^{-2}Hz^{-1}$.

$\Delta P_{if} = 10 \log \left(\frac{T_{star}}{T_{sys}} + 1\right)$ dB = PREDETECTION IF POWER RISE, FOR T_{sys} = 400K (TYPICAL).

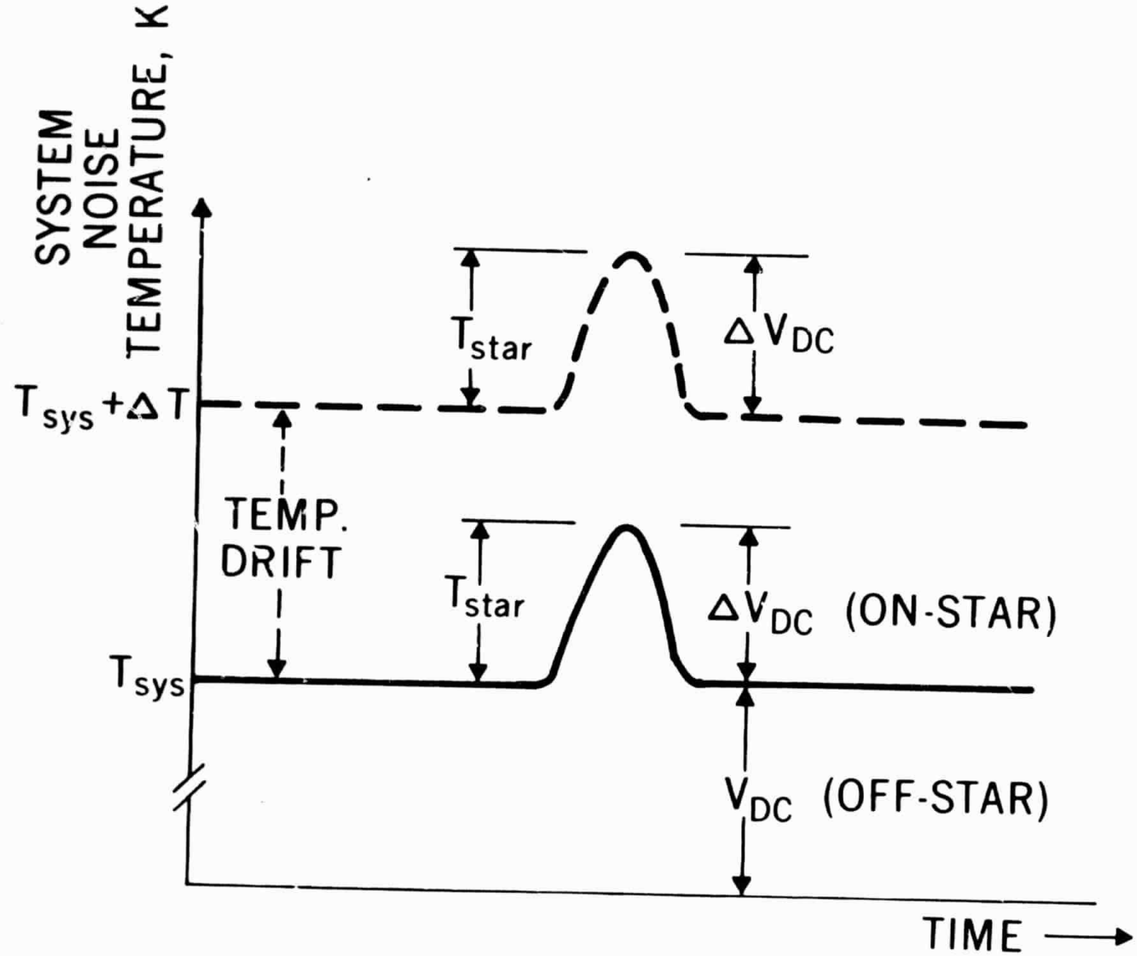
SLIDE 3.
ANTENNA GAIN DETERMINATION USING STELLAR SOURCE



$$G = \frac{8\pi k}{\lambda^2 F}\left[\frac{\Delta V_{DC}}{V_{DC}}\right]\underbrace{\left\{\epsilon\left[T_A - T_O\right] + T_O + T_R\right\}}_{T_{sys}}$$

$$\sigma_G = \pm\, 0.65 \text{ dB AT } 1440 \text{ MHz}$$

SLIDE 4. SELF-CALIBRATION PROPERTIES OF SQUARE-LAW DETECTOR.



$$T_{star} \triangleq \frac{G \lambda^2 F}{8 \pi k}$$

$$T_{sys} = T_{star} \left(\frac{V_{DC}}{\Delta V_{DC}} \right)$$

WHEN $\Delta T << T_{sys}$

& $G_{RF} = \text{CONST.}$,

$\therefore \Delta V_{DC} = \text{CONST.}$

SLIDE 5. STELLAR RELATIONSHIP FOR G/T_{sys}

- $$\frac{G}{T_{sys}} = \frac{8\pi k}{\lambda^2 F}\left[\frac{\Delta V_{DC}}{V_{DC}}\right]$$

$\frac{\Delta V_{DC}}{V_{DC}}$ = AMPLIFIED VOLTAGE RATIO FROM SQ.-LAW DETECTOR

G = EFFECTIVE ANTENNA POWER GAIN, ABOVE ISOTROPIC

λ = FREE-SPACE WAVELENGTH, m

F = RADIO-STAR FLUX DENSITY, $wm^{-2}Hz^{-1}$

k = BOLTZMANN'S CONSTANT, J/K

SLIDE 6. STELLAR RELATIONSHIPS FOR G AND T_{sys}

- $$T_{sys} = \frac{G\lambda^2 F}{8\pi k}\left[\frac{V_{DC}}{\Delta V_{DC}}\right]$$

- $$G = \frac{8\pi k}{\lambda^2 F}\left[\frac{\Delta V_{DC}}{V_{DC}}\right] T_{sys}$$

$T_{sys} = \epsilon\,(T_A - T_O) + T_O + T_R$ = SYSTEM NOISE TEMP.

ϵ = RF LINE LOSS, $0 < \epsilon \leq 1$

$T_A \cong 40 \pm \sigma_{T_A} = 40 \pm 20\,K$, AT L-BAND AND S-BAND.

T_O = RF LINE AMBIENT TEMP., K.

T_R = RECEIVING SYSTEM NOISE TEMPERATURE, K.

SLIDE 7. PRIMARY ASSUMPTIONS FOR STELLAR ERROR ANALYSIS.

- LOCAL RFI IS NEGLIGIBLE (e.g., NO ON-SITE IGNITION NOISE)
- RECEIVER SYSTEM GAIN REMAINS CONSTANT FOR "OFF-STAR" AND "ON-STAR" OBSERVATIONS
- MULTIPATH EFFECTS ARE NEGLIGIBLE (i.e., RADIO STAR'S ELEVATION ANGLE ≥30° ABOVE HORIZON)
- IONOSPHERIC DISTURBANCE, RESULTING IN STAR SIGNAL SCINTILLATION, IS NEGLIGIBLE.

SLIDE 8. ANTENNA-GAIN ERROR ANALYSIS (TYPICAL L-BAND 20-FT. DISH)

$$G = \frac{a}{FV} \bullet T_{sys}$$

$$a = 8 \pi k \lambda^{-2}$$

$$V = \frac{V_{DC}}{\Delta V_{DC}} = \text{VOLTAGE RATIO}$$

$$F = \text{STAR FLUX DENSITY}$$

$$T_{sys} = \epsilon (T_A - T_O) + T_O + T_R = \text{SYSTEM NOISE TEMPERATURE}$$

SLIDE 9. ERROR ANALYSIS DEFINITIONS

$$T_{sys} \triangleq \epsilon \left[T_A - T_O \right] + T_O + T_R$$

$\sigma_{F/F} = \pm\ 2.76\%$ @ L-BAND (1440 MHz) &

$\pm\ 3.6\%$ @ VHF (136 MHz)

FOR Cas A (1970.5 EPOCH).

$$\frac{\sigma_V}{V} = \frac{\sigma_{T_R}}{T_R} = \frac{\sigma_{T_O}}{T_O} = \frac{\sigma_\epsilon}{\epsilon} = \pm\ 10\% \text{(WORST-CASE)}$$

$T_A \pm \sigma_{T_A} \cong 40 \pm 20K$ @ L-BAND

$\cong 870 \pm 200K$ @ VHF

SLIDE 10. PROPAGATION OF PRECISION ERROR INDEXES.

$$\frac{\sigma_G}{G} = \pm \left\{ \left(\frac{\sigma_F}{F} \right)^2 + \left(\frac{\sigma_V}{V} \right)^2 + \frac{1}{T_{sys}^2} \left[(T_A - T_O)^2 \sigma_\epsilon^2 + (\epsilon \sigma_{T_A})^2 + (1-\epsilon)^2 \sigma_{T_O}^2 + \sigma_{T_R}^2 \right] \right\}^{1/2}$$

$$= \pm \left\{ \underbrace{(0.0276)^2}_{2.76\%} + \underbrace{(0.10)^2}_{10\%} + \underbrace{\left[\underbrace{(0.050)^2}_{5\%} + \underbrace{(0.040)^2}_{4\%} + \underbrace{(0.0145)^2}_{1.45\%} + \underbrace{(0.075)^2}_{7.5\%} \right]}_{10\%} \right\}^{1/2}$$

FOR TYPICAL $T_{sys} = 400K$,

$$\frac{\sigma_G}{G} = \pm\, 0.15 = \pm\, 15\%$$

$\therefore G \pm \sigma_G = 37\text{dBi} \pm 0.65\text{dB}$,

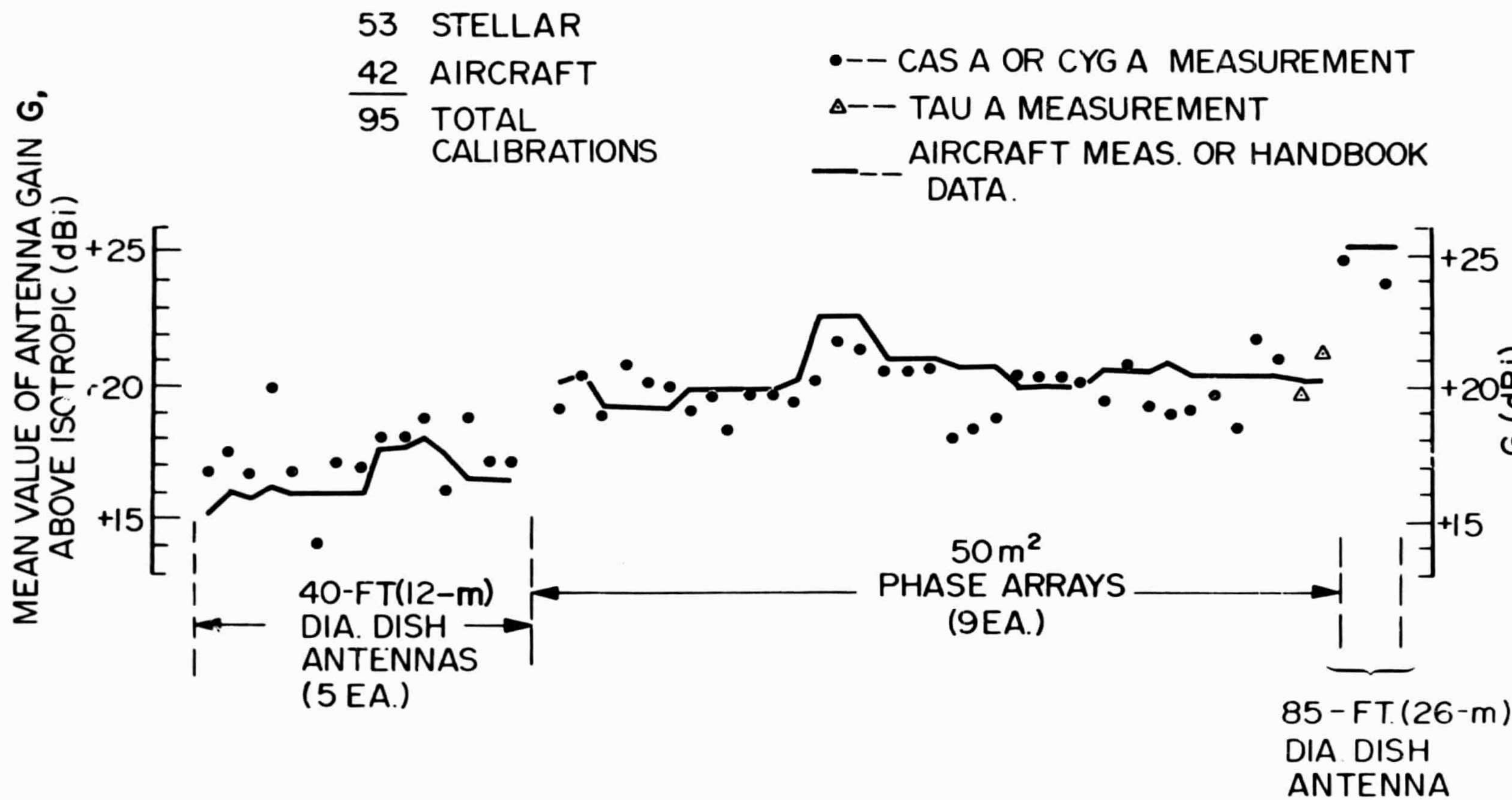
FOR 20 – FT(6-m) DISH, 55% EFFICIENCY,

AT 1440 MHz (L-BAND).

SLIDE 11. COMPUTED STANDARD ERROR IN G, T_{sys} & G/T_{sys} FOR Cas A AT L-BAND & VHF

	COMPUTED STANDARD ERROR		
	σ_G (dB)	$\sigma_{T_{sys}}$ (dB)	$\sigma_{G/T_{sys}}$ (dB)
L-BAND 20-FT.(6-m) DISH	± 0.65	± 0.40	± 0.40
VHF $50 m^2$ PHASE ARRAY	± 0.80	± 0.60	± 0.40

SLIDE 12. STADAN 136 MHz ANTENNA GAIN CALIBRATIONS



SLIDE 13. STELLAR MEASUREMENTS, ALASKA STATION 402 MHz, Cas A

	ANTENNA	RECEIVER	G/T_{sys} (dB)	T_{sys}	G (dBi)
9 APRIL '71	40' DISH	1	3.6	327K	27.1
9 APRIL '71	40' DISH	2	4.8	381K	28.3
14 APRIL '71	40' DISH	1	3.4	356K	27.2
14 APRIL '71	40' DISH	2	4.9	419K	28.7

SLIDE 14. STELLAR MEASUREMENTS, ALASKA STATION 1702 MHz, Cas A

	ANTENNA	RECEIVER	G/T_{sys} (dB)	T_{sys}	G (dBi)
9 APRIL '71	40' DISH	1	17.9	309K	40.5
9 APRIL '71	40' DISH	2	19.7	309K	42.5
14 APRIL '71	40' DISH	1	18.1	338K	41.3
14 APRIL '71	40' DISH	2	17.4	338 K	40.5
10 APRIL '71	85' DISH	1	20.0	395K	*44.7
10 APRIL '71	85' DISH	2	20.0	407K	44.9
11 APRIL '71	85' DISH	1	19.6	385K	43.9
11 APRIL '71	85' DISH	2	19.6	396K	44.3

*44.0 dBi FROM AIRCRAFT MEASUREMENT.