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THE EFFECT OF GALACTIC MAGNETIC FIELD LINE WANDERING ON COSMIC-RAY PARAMETERS

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Abstract

The local configuration of the galactic magnetic field is an important factor in determining local cosmic-ray parameters such as density, mean age and streaming velocity. According to the theory of Jokipii and Parker the flux tube that confines the cosmic rays seen at earth undergoes a random wandering about the average field direction. It continues this wandering until it reaches a distance sufficiently far from the central plane of the galaxy such that the bubble blowing instability occurs and it thus comes to an end as a container of cosmic rays. In a previous paper we have calculated a probability density for the end points of the flux tube passing through the earth employing the theory of Gaussian processes. We have now used this result to calculate the probability densities of the above mentioned cosmic-ray parameters based on a smoothed out source, one dimensional diffusion model. It is seen that these parameters have a very wide range of fluctuation but that even when the mean age is confined to a narrow range the mean and most probable value of the streaming velocity is zero. From this it is argued that a very small observed value for the cosmic-ray anisotropy requires no special explanation any more than would any other value compatible with the probability distribution.

Paper OG-111, 12th International Conference on Cosmic Rays, Hobart, 1971, Conference Papers (University of Tasmania), 1, 396.

The Effect of Galactic Magnetic Field Line

Wandering On Cosmic Ray Parameters

Frank C. Jones

It has long been realized that the strength and configuration of the magnetic field of the galaxy is of great importance in determining many of the parameters of galactic cosmic rays. Since the cyclotron radius of a typical (100 GeV) cosmic ray particle in the galactic field ($\sim 3\mu$ gauss) is approximately 3×10^{-5} pc the particles are very strongly constrained to follow the field lines from their source to the point where they escape from the galaxy. Thus it is not hard to see that such cosmic ray parameters as density, mean lifetime and streaming velocity are profoundly affected by the galactic magnetic field.

Until recently, however, attention has been paid only to the average effects of the field. Average trapping times for the galaxy as a whole have been deduced from assumed diffusive properties of the field combined with an overall regular field of one form or another. However, in recent years it has been realized that if the local galactic field departs in a significant way from the average galactic field the cosmic ray parameters that we observe at the earth may have little or no relation to average values computed for the galaxy as a whole. In other words the usual treatment of the galactic field in theoretical cosmic-ray research has concentrated its attention on averages and ignored (for the most part) fluctuations, insofar as our very local observations are concerned the fluctuations may be of utmost importance.

The major treatment of the deviation of the galactic magnetic field from its average configuration is that of Jokipii and Parker (1969a,b) in their development of the ergodic field line picture. In this picture a given field line, or flux tube, does not follow the smooth average configuration of the field but rather, due to the turbulent motion of the ionized gas in the galaxy, it wonders in a random manner about this average configuration such that it resembles a position versus time plot of a Brownian particle.

The important aspect of this picture, insofar as cosmic rays are concerned, is that when a field line wanders sufficiently far from the central plane of the galaxy an instability, the bubble blowing instability (Parker 1965), can occur and at this point the cosmic rays that are trapped on this field line or flux tube can escape the galaxy. Therefore, according to this picture the two points, on either side of the earth, where the flux tube that passes through the earth wanders to the "edge" of the galactic disk and allows the cosmic rays to escape define the trapping volume of the cosmic rays that we see here on earth. Such parameters of the cosmic rays as the density, mean age and streaming velocity will be determined by, among other things, the total length of this tube and the position of the earth with respect to the ends. Figure 1 illustrates a possible configuration.

Unfortunately we do not know the configuration of our local flux tube. We are therefore forced to approach the problem probabilisticly. In a previous paper (Jones 1971) the author has derived a probability density function for the effective "end points" of the flux tube passing through the earth. This derivation was based on the theory of Gaussian processes rather than on a diffusion or Fokker-Planck approach because the micro- and macroscopic scale lengths (which must be clearly separated if a diffusion type equation is to be valid) are here both characterized by the correlation length L of the underlying gas motions.

The results of this derivation are shown in Figure 2 for various values of $Z_1$the height above (and below) the central plane of the instability level or "edge" of the disk. In this figure δ is proportional to Z_1in units of $L\ (\tan\theta\ )_{RMS}$ where θ is the angle a typical field line makes with respect to the direction of the average field. $L(\tan\theta)_{RMS}$ is thus related to the RMS distance a typical field line wanders away from its average position. The distance X is just proportional to the distance down the mean field direction in units of the correlation length L.

We now consider essentially the same situation as that treated by Kulsrud and Pearce(1969) namely a flux tube running from $-y_1$ to $+y_2$ with cosmic rays undergoing one dimensional diffusion with coefficient D along this flux tube. At the end points $-y_1$ and $+y_2$ the particles freely escape and hence the density must be zero here. Furthermore, as a first approach to this problem we shall consider the cosmic ray particles to be injected uniformly in space and time with strength S. We hereby ignore all of the complications that I discussed at the last meeting in this series in Budapest (Jones 1970 a) namely that, in fact, the sources of cosmic rays are very likely random discrete events in space and time (supernovae)andthis fact itself implies a stochastic aspect of cosmic ray parameters (Ramaty et al 1970; Jones 1970 b). This complication will have to wait for later.

In this model the above cosmic ray parameters have essentially simple expressions (Kulsrud and Pearce 1969). Introducing $X \equiv y/L$ we have

$$\rho = (SL^2/2D) \quad (X_1 X_2) = \tilde{\rho}\ (X_1 X_2)$$

$$v = (D/L) \left(\frac{1}{X_2} - \frac{1}{X_1} \right) = \tilde{v} \left(\frac{1}{X_2} - \frac{1}{X_1} \right)$$

$$\tau = (L^2/12D)\ (X_1{}^2 + X_2{}^2 + 3X_1 S_2) = \tilde{\tau}\ (X_1{}^2 + X_2{}^2 + 3X_1 X_2)$$

where in each case we have explicitly separated the dimentionless variables X_1 and X_2 from the characteristic dimensioned quantities $\tilde{\rho}$, $\tilde{v}$ and $\tilde{\tau}$.

We have chosen values of X_1 and X_2 at random from the distribution shown in Figure 2 with $\delta = 0.8$ the corresponding values of $\rho/\tilde{\rho}$, $v/\tilde{v}$ and $\tau/\tilde{\tau}$ were computed and a relative frequency distribution was obtained for a sample size of 10^5. The relative frequency distributions for $\rho/\tilde{\rho}$, $v/\tilde{v}$ and $\tau/\tilde{\tau}$ are shown in Figure 3.

The important point to be noted here is the fact that the streaming velocity distribution is strongly peaked about zero. Thus in this model a small observed value of the streaming velocity is no more in need of a special explanation than is any other. The scale velocity is $\tilde{v} = D/L = c\lambda/3L$ where λ is the mean free path for cosmic ray diffusion and since for diffusion to be valid at all we must have $\lambda \ll L$ we see that $\tilde{v} \ll c$. Also since the anisotropy $\delta \approx 4.5\ v/c$ (Forman 1970) an observed value of $\delta = 10^{-4}$ (Elliot et al. 1970) implies $-2.2 \times 10^{-5} < v/c < 2.2 \times 10^{-5}$. If we choose $\lambda \approx 10$pc we have $\tilde{v} = c/30$ and hence require $|v/\tilde{v}| \lesssim 6.7 \times 10^{-4}$.

This is an extremely narrow slice of the disbribution and this fact has led many to consider it extremely unlikely that this value could have arisen by chance. It is however <u>no more unlikely</u> than any other value defined with equal precision. In fact it is more likely than any other equally precise value. One may therefore not rule out a chance origin of the small observed anisotropy.

One might well ask, however, whether adding information might not change this picture. For example we know the mean age of cosmic rays near the earth to within say $\pm$ 50%, might not adding this fact to the problem affect the streaming velocity distribution?

A glance at Figure 4 should help to allay this suspicion. What we see here are the relative frequency plots for another 10^5 cases where we have specified that $235 \leq \tau/\tilde{\tau} \leq 315$. We see that although the distribution of $\rho/\tilde{\rho}$ is profoundly affected the distribution of $v/\tilde{v}$ is changed only slightly and in fact is more peaked than before.

The reason for this is straightforward. The streaming velocity depends strongly on the point of observation relative to the end point whereas the density and especially the particle age is far less sensitive to the observation point being more dependant on the overall length of the flux tube. This statement is not in contradiction to previous remarks that have been made concerning the leakage lifetime of particles and its dependance on position (Jokipii and Meyer 1968; Jones 1970c). The leakage lifetime of previous discussions is actually a "life expectancy" for the particles whereas the present concept is an actual age related to the past of a particle rather than its future.

We see, therefore, that specifying the age in no way changes the picture that we have discussed. In any statistical system that is not _a priori_ asymetric zero will be the _most probable_ value of the streaming velocity and it should not b e considered impossible that its observation is due to chance.

Future work along these lines should take into account the random, discreet nature of the cosmic ray sources as well as the possibility that the propagation of the particles is by some mode other than diffusion. Furthermore some account should be taken of the fact that the true length of a flux tube is not the distance y along the mean field that we employed here but rather the integral $\int(dy^2 + dz^2)^{\frac{1}{2}}$. Also the variables x_1 and x_2 are not completely independant but are slightly correlated by the ballistic propagation of initial conditions that was discussed in the previous paper (Jones 1971).

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Figure Captions

Figure 1. Schematic representation of a possible configuration of the magnetic flux tube that passes through the earth. The "edge" of the galactic disk is to be taken to mean only that distance sufficiently far from the central plane such that the bubble blowing instability takes place.

Figure 2. Outward first crossing rate of field line at Z_1. Probability density per unit $x = (\pi)^{\frac{1}{2}} y/2L$ for three values of $\delta = (Z_1/L)(\pi/8)^{\frac{1}{2}}/(\tan\theta)_{RMS}$.

Figure 3. Relative frequency **plots** for cosmic-ray parameters for 10^5 cases. **Relative** frequency is in arbitrary units and the parameters are $\rho/\widetilde{\rho}$, $v/\widetilde{v}$ and $\tau/\widetilde{\tau}$.

Figure 4. Relative frequency plots for cosmic-ray parameters for 10^5 cases with the constraint $235 \le \tau/\tau \le 315$. Relative frequency is in arbitrary units and the parameters are $\rho/\widetilde{\rho}$, $v/\widetilde{v}$ and $\tau/\widetilde{\tau}$.

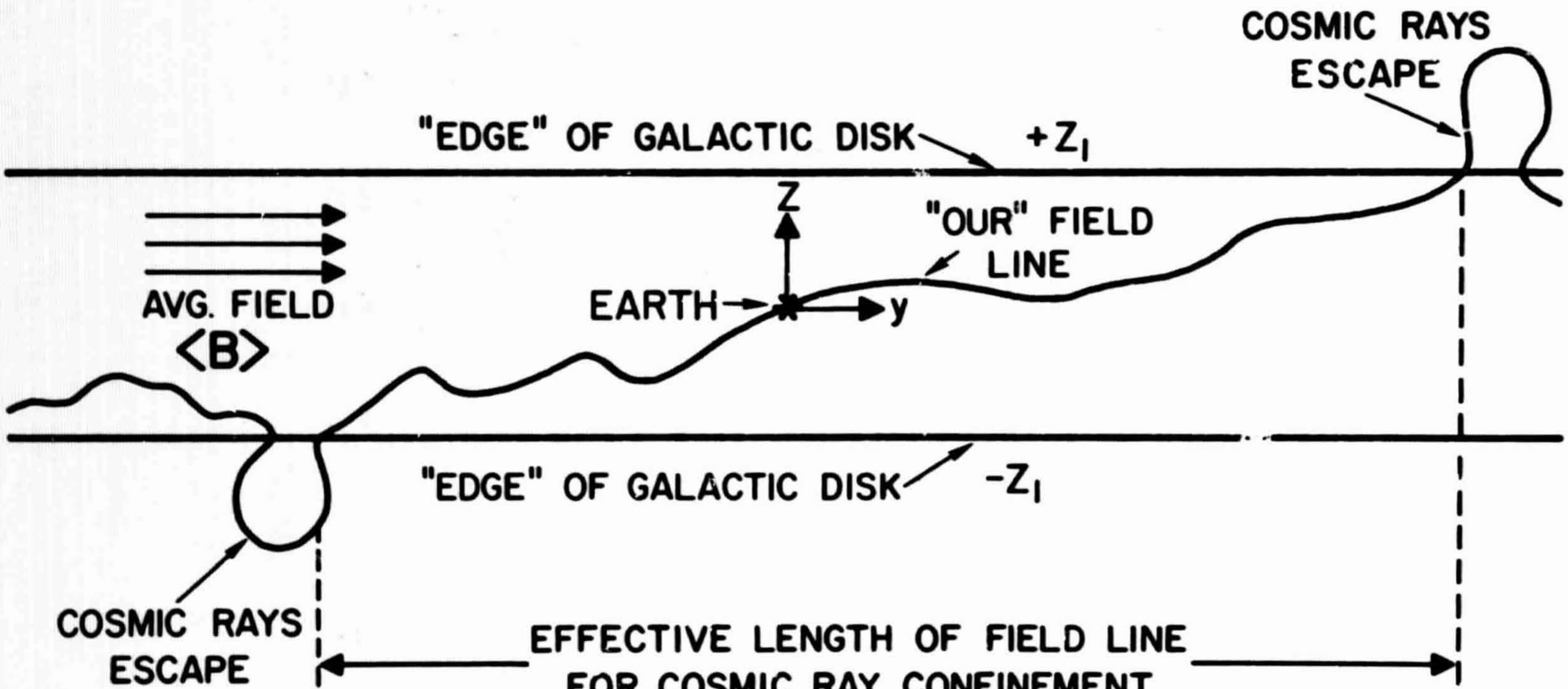
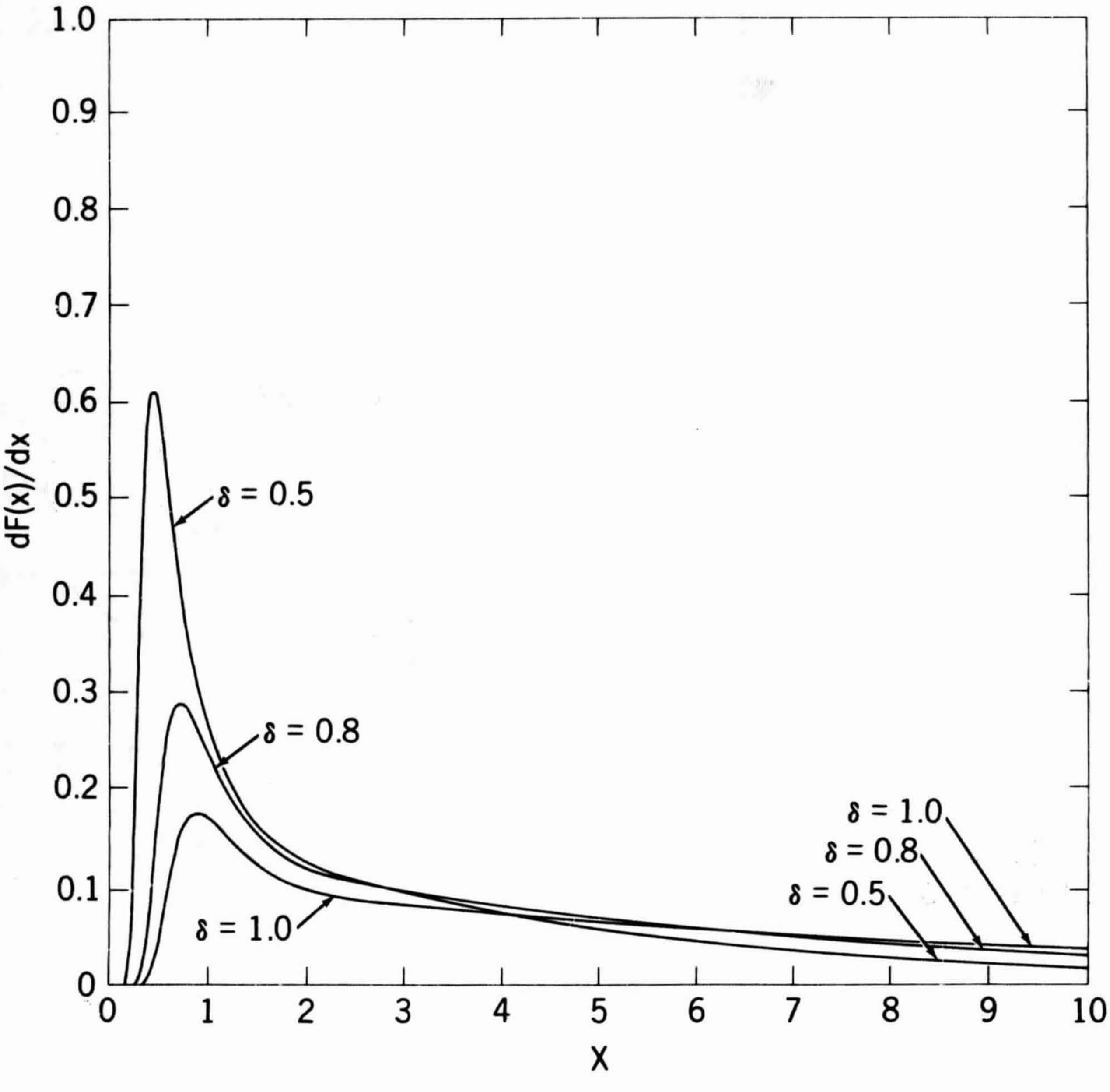

COSMIC RAYS
ESCAPE
"EDGE" OF GALACTIC DISK
$+Z_1$
Z
"OUR" FIELD
LINE
AVG. FIELD
⟨B⟩
EARTH
y
"EDGE" OF GALACTIC DISK
$-Z_1$
COSMIC RAYS
ESCAPE
EFFECTIVE LENGTH OF FIELD LINE
FOR COSMIC RAY CONFINEMENT

Figure 1



dF(x)/dx
X
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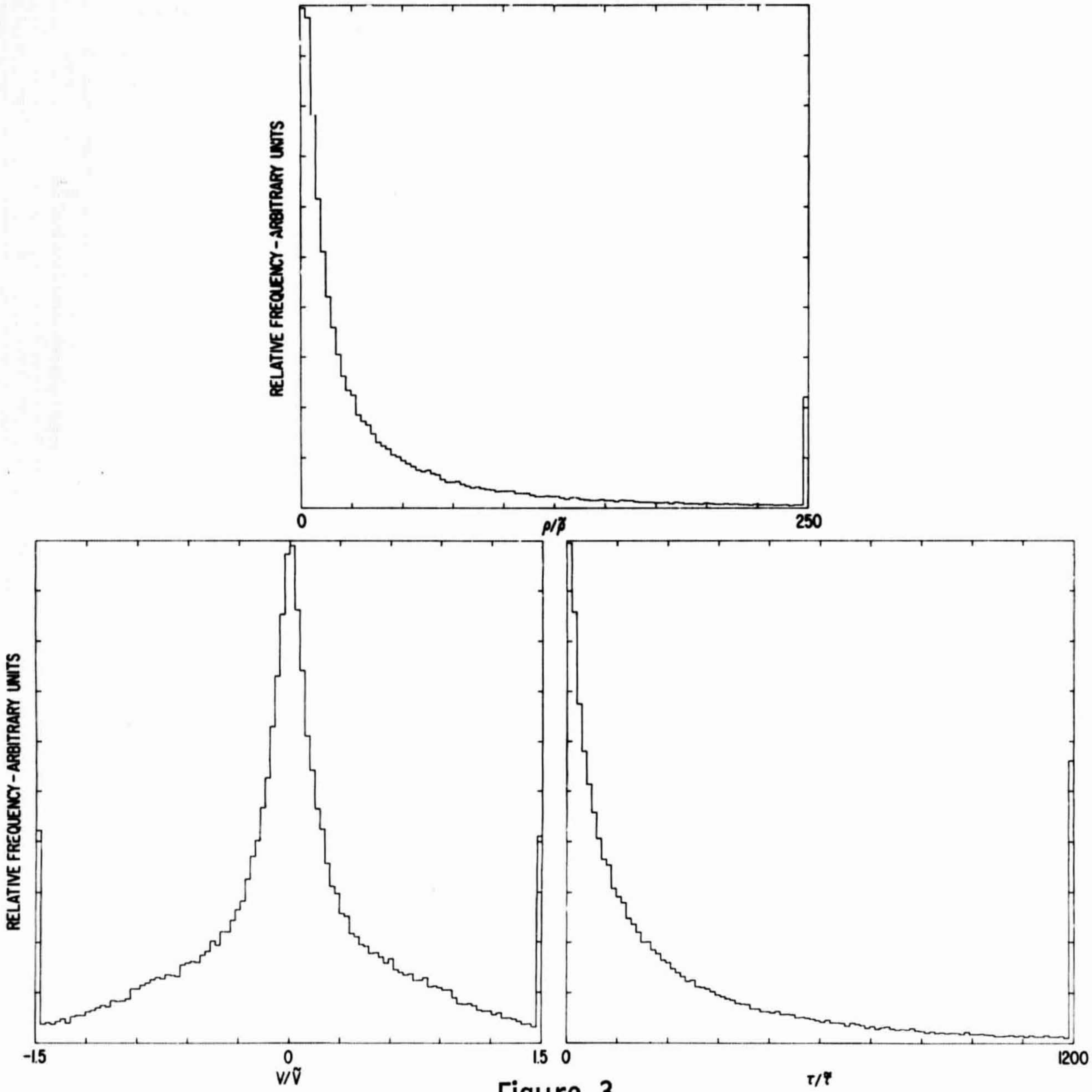
δ = 0.8
δ = 1.0
δ = 1.0
δ = 0.8
δ = 0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0
1
2
3
4
5
6
7
8
9
10

Figure 2


RELATIVE FREQUENCY - ARBITRARY UNITS
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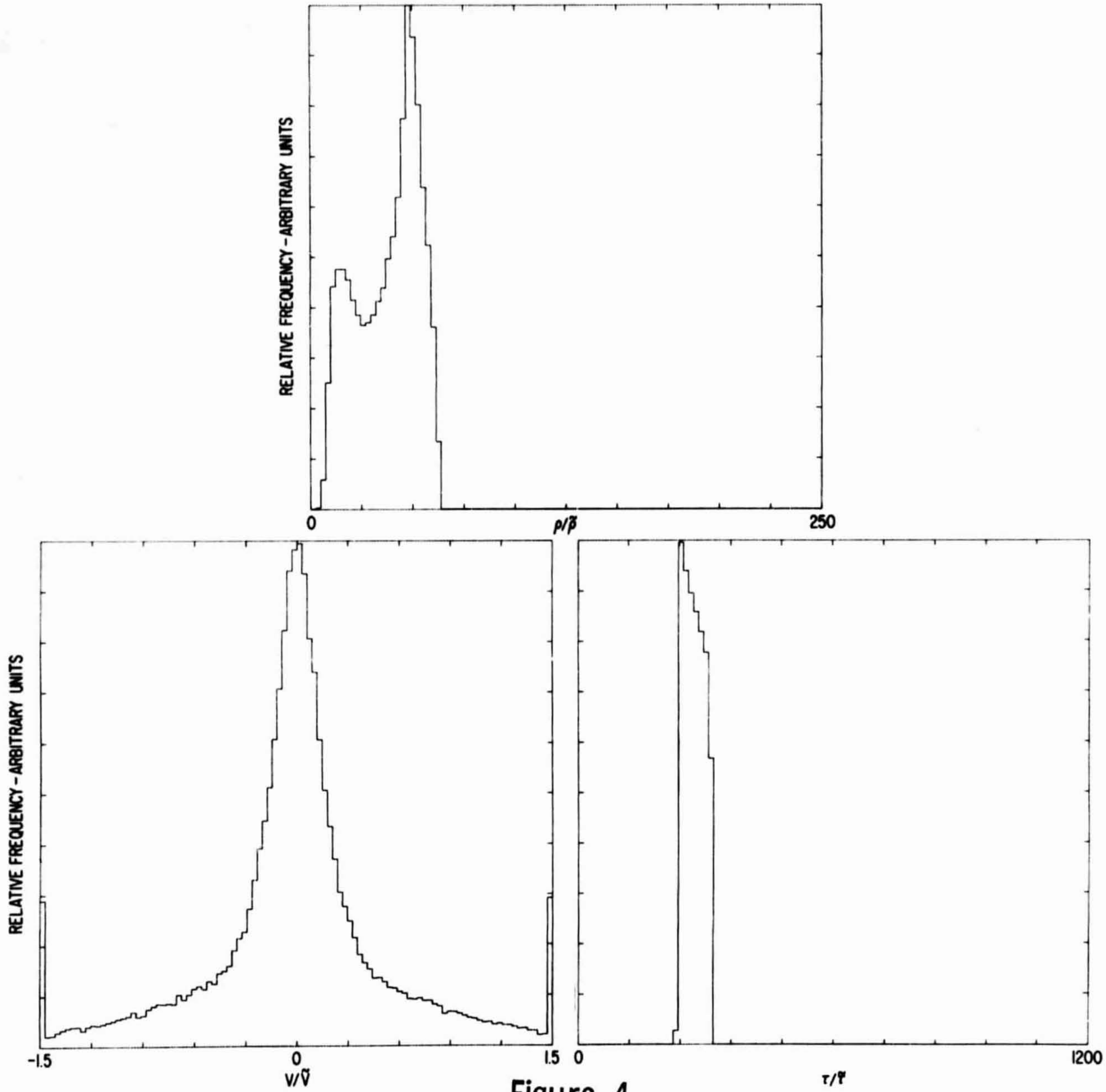
250
RELATIVE FREQUENCY - ARBITRARY UNITS
-1.5
0
1.5
0
1200

Figure 3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0
ρ/ρ̄
250
RELATIVE FREQUENCY - ARBITRARY UNITS
-1.5
0
1.5
V/V̄
0
1200
τ/τ̄

Figure 4