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ABSTRACT

Absorption and group delay data from an equatorial station have been used to model the electron density profile in the D and E regions of the ionosphere. The model profiles were computed by numerical inversion of the propagation integrals using the generalised magnetoionic theory. The profiles so obtained are found to be in good agreement with electron density profiles obtained at the same time at a nearby location with a rocket experiment.

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THE ELECTRON DENSITY PROFILE IN THE D AND E REGIONS OF THE EQUATORIAL IONOSPHERE

1. Introduction

Groundbased radio propagation experiments such as the wave interaction technique and v.l.f. and l.f. measurements yield electron density profiles of the D-region up to about 80 km. Above about 100 km ionograms can be used to deduce the electron density profile using assumptions about the ionization below this height. Piggott and Thrane (1966) made the first attempt to deduce the shape of the ionization profile between 80 and 100 km using ground based measurements of vertical incidence absorption and virtual height. Using the electron density profiles from the cross modulation experiments of Barrington et al., (1963) and assuming models of electron density above 100 km they adjusted the shape of the profile in the intervening region until the absorption and virtual height computed with the profile agreed with the observed values on several frequencies. This technique has since been used by Thomas (1968) to deduce electron density profiles during anomalous days in winter.

Beynon and Rangaswamy (1969) have outlined a technique to obtain a two-parameter model of the electron density profile in the region 60 to 100 km using as input data absorption and virtual height at frequencies near 2 MHz. A model profile of the form

$$N(h) = N(h_o) \cdot \exp(a \cdot (h-h_o)^2) \qquad (1)$$

was used where $N(h_0)$ and 'a' were parameters obtained by iteration so that the computed and observed values of absorption and virtual height agreed to the required tolerance.

It is possible to add successive segments of such profiles by using data for higher frequencies. However such a two-parameter model will have the lower most segment necessarily monotonic due to the form of height variation used. This constraint can be removed by using a larger number of parameters and simultaneously solving for them using multi-frequency observations. The use of a larger number of parameters will also provide a test of convergence in the power series expression for the electron density profile and thus check the usefulness of the two-parameter model.

In the present paper a numerical technique to obtain a four-parameter model of the electron density profile in the D and lower-E regions using absorption and virtual height data on two frequencies is presented. The resultant profile is compared with the two-parameter model as well as profiles obtained by rocket experiments concurrent with the ground based measurements.

2. Experimental data

Kane (1969) and Aikin et al, (1971) have deduced the electron densities at Tumba (lat. 8°N) by a rocket experiment measuring the height variation of absorption for the two magnetoionic components. Kane's measurement was made around noon on 8 March 1968 and covered the height range 58 to 84 km. Aikin et al. made their measurements on 19 March 1970 at different times of the day. Their profile corresponding to 1509 hours local time (hereafter called the AGS profile) spanned the 61 to 97 km region. The ground based measurements of absorption were made by Gnanalingam (1971) at Colombo (lat. 7°N). For 8 March 1968 absorption data was available for the four frequencies 2.0, 2.2, 2.6 and 3.2 MHz. For 19 March 1970 absorption data corresponding to the AGS-profile was available for the three frequencies 2.0, 2.2 and 2.6 MHz.

3. Collision frequencies in the equatorial lower ionosphere

The collision frequency of mono-energetic electrons was computed from pressure and temperature data from atmospheric models using appropriate collision cross sections. For the height range 60 to 80 km the supplementary atmosphere of Cole et al. (1965) for 15°N was used. For the height range 80 to 110 km the supplementary atmosphere of Champion (1967) for 15°N was used. Only electron-neutral collision was considered for the entire height region. Above 80 km collisions with atomic oxygen were included. Thermal equilibrium was assumed for the region below 110 km. The computed collision frequency profile is shown in Figure 1. For comparison the collision frequency profile used by Kane and Aikin et al. is also shown in the Figure (circles).

4. Method of analysis

Since the contribution to h.f. absorption from the region below 70 km is quite small it would be better not to model this region using h.f. data. Hence the electron density for the region 60 to 72 km was taken from the profile obtained by Kane (hereafter called the K profile) for 8 March 1968. Similarly for

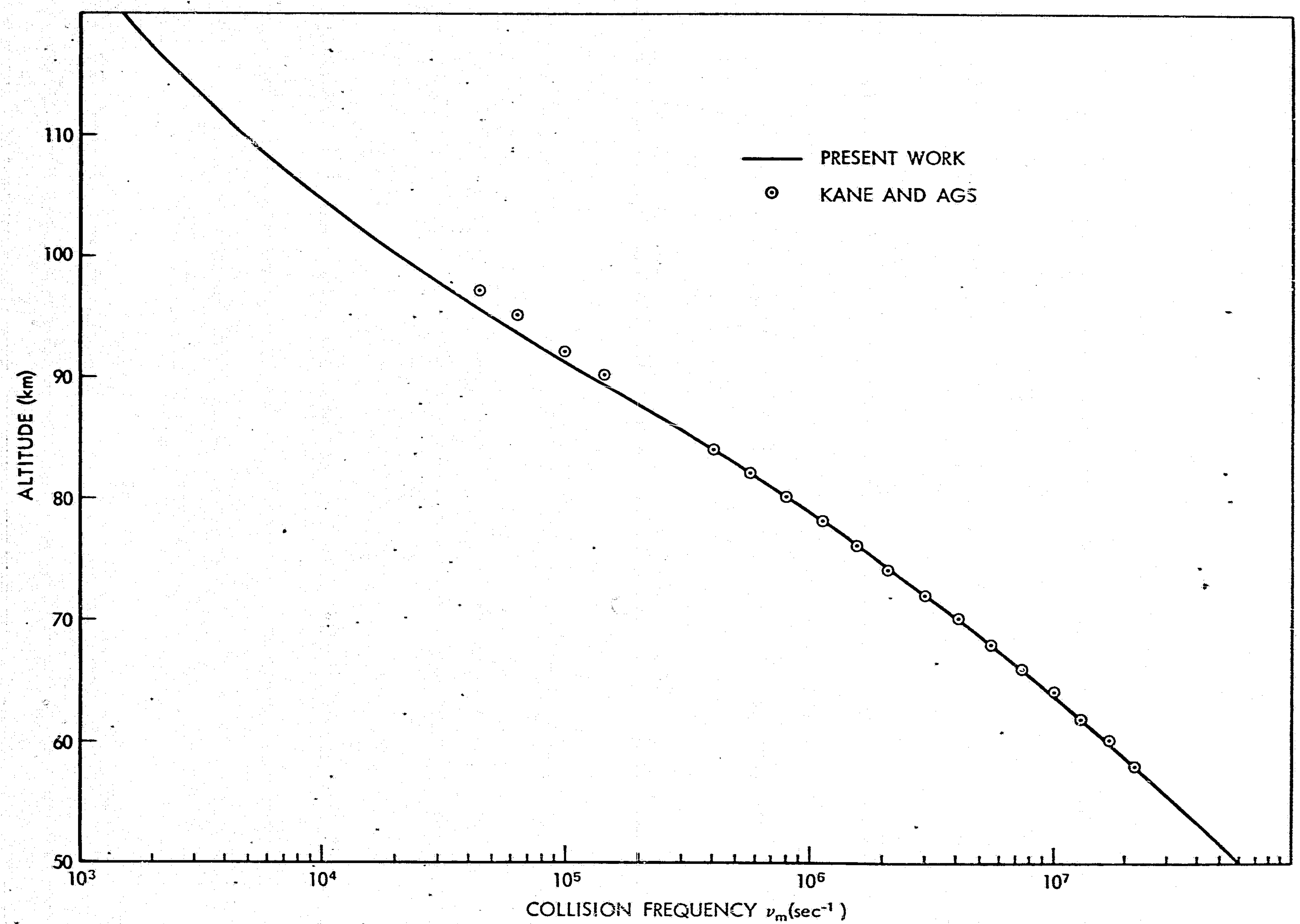

Figure 1. Collision Frequency Profile

19 March 1970 the electron density for the region 60 to 70 km was taken from the AGS profile. In each case the electron density above a base height h_o (72 km for 8 March 1968 and 70 km for 19 March 1970) was assumed to be an exponential quartic of the form

$$N(h) = N(h_o) \cdot \exp\left(\sum_{i=1}^{4} a_i \cdot (h-h_o)^i\right) \tag{2}$$

where $N(h_o)$ is the electron density at the reference base height h_o taken from the K- and AGS-profiles respectively and a_i the parameter set determining the profile above h_o. Assuming some initial values of the parameters a_i the height of reflection for an ordinary wave of frequency "f" at vertical incidence was determined by using the geometrical optics criterion X = 1 where X is the magnetoionic parameter. For an ionization profile described by equation 2 the height of reflection is given by h_r where

$$h_r = h_o + h_p \tag{3}$$

h_p is the smallest real positive root of the quartic equation

$$\sum_{i=0}^{4} a_i \cdot (h-h_o)^i = 0 \tag{4}$$

where

$$a_o = \ln\left(N(h_o)/(12400.f^2)\right) \tag{5}$$

and f is in MHz.

With the assumed electron density profile it is possible to compute the absorption and virtual height of a radiowave of frequency f. This was done by using the generalized magneto-ionic theory of Sen and Wyller (1960) for the highest and lowest frequencies for which observations were available for each of the two days respectively. The computed values of absorption and virtual height were then compared with the observed values and the parameter set a_i was adjusted iteratively until the observed and computed values agreed to the required tolerance. A modified secant technique was used for the iteration with numerical constraints to prevent divergence and to obtain real positive roots for the quartic.

5. Results

The continuous line in Figure 2 shows the exponential quartic profile (EQ profile) computed for 8 March 1968 by the technique described above. The K profile is shown by circles in the same Figure. It may be seen that the two profiles are in good agreement over the entire region of overlap. Theoretically the model profile is valid only up to the height of reflection of the higher of the two frequencies used. However extrapolation over a small range above this height showed a maximum at 108 km corresponding to a plasma frequency of 4.4 MHz. Examination of published data showed that the median value of f_oE (critical frequency of the E-layer) at noon for March 1968 at the magnetic equator was about 4.0 MHz. The deviation of only about 10% between the observed and calculated critical frequencies gives a further measure of confidence in the main trend of the computed model profile.

To check the internal consistency of the profile the absorption and virtual height at two intermediate frequencies (2.2 and 2.6 MHz) were computed using the EQ profile. Table I gives the observed and computed values on all four

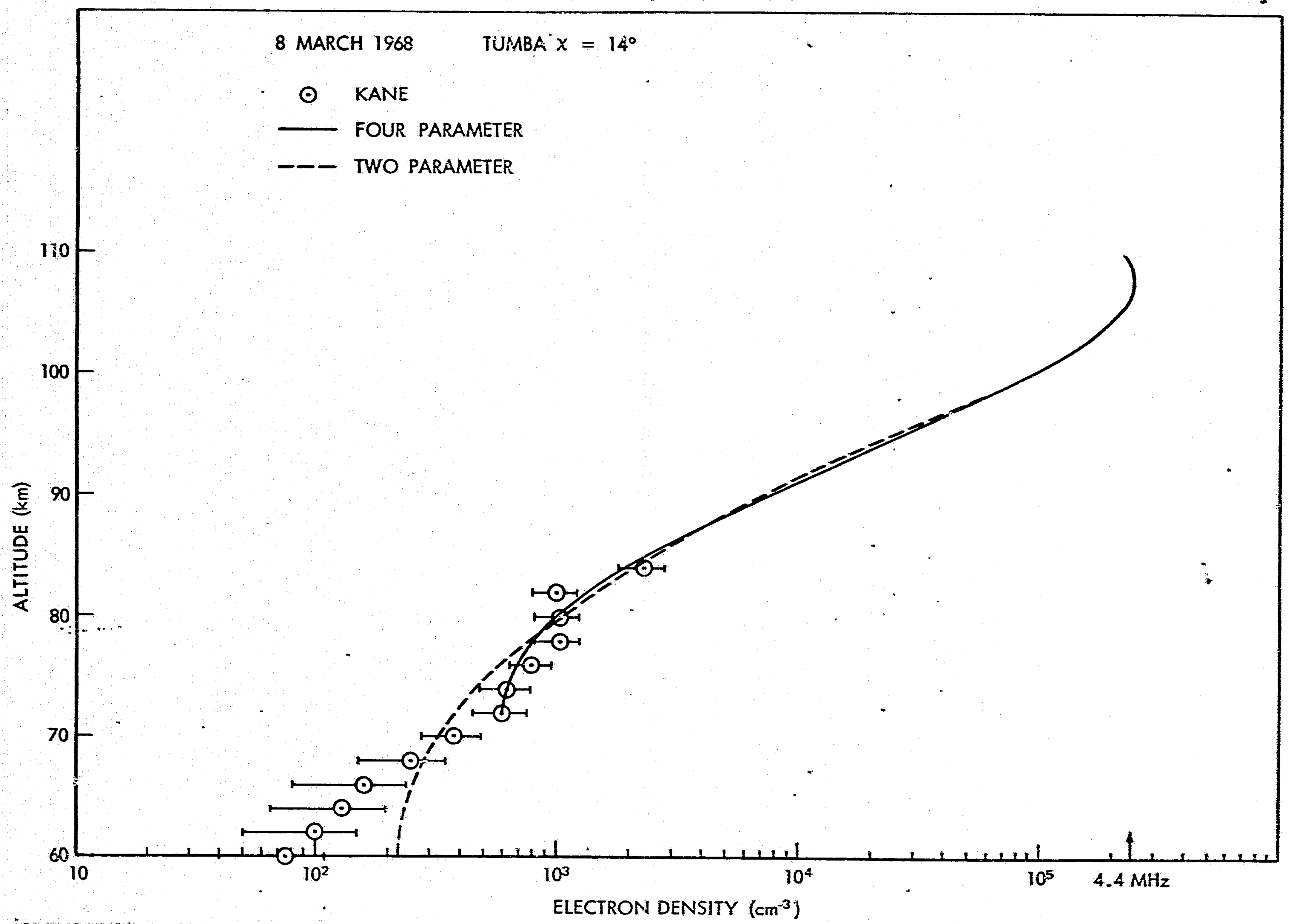

Figure 2. Electron Density Profiles for 8 March 1968

frequencies. It may be observed from the Table that the model leads to good agreement between the observed and computed values at the intermediate frequencies also.

A two-parameter model of the form given by equation 1 was also computed using absorption and virtual height data for 2.0 MHz. This is shown by the dashed line in Figure 2. It may be seen from Figure 1 that the two parameter model also shows good agreement with the K profile above 65 km. The two- and four-parameter models are in agreement up to about 100 km.

The continuous line in Figure 3 shows the exponential quartic profile computed from the data for 19 March 1970. The corresponding rocket measurements are also shown in the Figure. It may be seen from the Figure that in this case also the computed EQ profile shows quite good agreement with the rocket measurements. In fact it is possible to see the agreement between the two profiles up to about 100 km due to the availability of the rocket measurements up to this height. As in the case of 8 March 1968 the extrapolation of the EQ profile for 19 March 1970 can be seen (from Figure 3) to lead to a maximum. In this case the maximum of the model profile occurred at the height of 106 km and corresponded to a plasma frequency of about 3.4 MHz. This value agrees well with the published median value of f_0E at the equator for March 1970 as 3.6 MHz at 1500 hours L.T.

The dashed line in Figure 3 is the corresponding two-parameter profile computed from data for 2.0 MHz. In this case also it may be seen that the

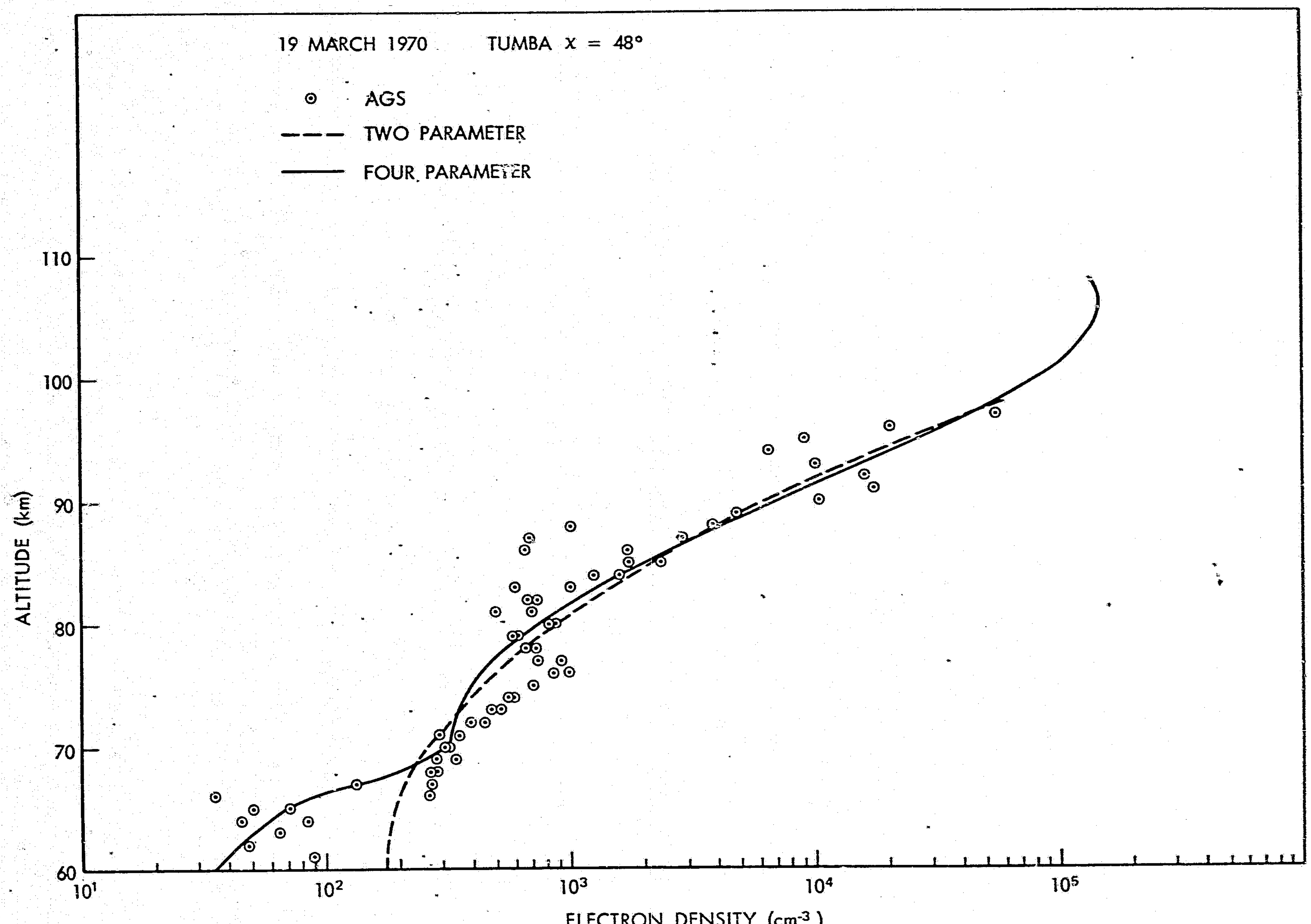

Figure 3. Electron Density Profiles for 19 March 1970

two-parameter profile is in good agreement with the AGS profile and the EQ profile from 70 km upwards.

6. Discussion

Ideally propagation data on a number of closely spaced frequencies ranging from v.l.f. to h.f. reflected in a range of heights from the D-region to the F-region is needed to define a self-consistent electron density profile of the D and E-regions. However, computation of absorption for waves reflected in the F-region requires a knowledge of the collision frequency above the E-region. But at present the collision frequency above about 100 km is not too well known (Thrane and Piggott, 1966) and thus the use of F-region data may lead to errors.

In this paper a numerical technique to deduce model electron density profiles in the D and lower E-regions using absorption data for frequencies reflected in the E-region has been demonstrated. Such a model is not unique in the sense that it reproduces the actual profile including minor flexures. However, it has been shown that it is capable of reproducing the main trend of the profile as measured by rocket experiments. The inclusion of the virtual height data as input ensures that the gradient of the profile near the reflection levels is determined correctly. The inclusion of the absorption data, on the other hand, leads to the determination of the smoothed variation of the product $N\nu$ where N is the electron density and ν is the collision frequency. Since minor flexures of the electron density profile do not contribute significantly to the absorption integral it will not be possible to invert the integral equations numerically to obtain these flexures. The electron density profile obtained by this technique will therefore be a smoothed profile weighted by the height variation of the absorption integral.

The agreement between the two-parameter and the four-parameter models shows that the former can be used to estimate the electron density variation over limited height ranges. The model profiles described in this paper cannot be expected to be accurate in the lowest regions of the ionosphere (60 to 70 km) due to the small contribution of these regions to the absorption integral. However, above this region the model profile can yield the smoothed height variation of the ionization up to E-region heights. Such a model can be used to correct true-height analyses of ionograms for underlying ionization even at locations where the extra-ordinary ray is heavily attenuated. It can also be used with multi-frequency absorption data and concurrent ionogram to estimate the height variation of collision frequency above 100 km. Work is in progress to determine the collision frequency above the E-region by this technique.

Acknowledgement

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Table I

Frequency (MHz)	Observed		Computed	
	Absorption (db)	Virtual Height (km)	Absorption (db)	Virtual Height (km)
2.0	53.0	102	54.5	101
2.2	47.0	103	48.4	104
2.6	41.0	105	39.3	105
3.2	32.0	108	30.5	108