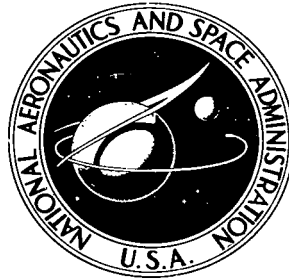


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**OPTIMAL DESIGN PROCEDURE FOR
TWO-LEVEL FRACTIONAL FACTORIAL
EXPERIMENTS GIVEN PRIOR
INFORMATION ABOUT PARAMETERS**

by Steven M. Sidik and Arthur G. Holms

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Cleveland, Ohio 44135

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • NOVEMBER 1971



0133462

1. Report No. NASA TN D-6527	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle OPTIMAL DESIGN PROCEDURE FOR TWO-LEVEL FRACTIONAL FACTORIAL EXPERIMENTS GIVEN PRIOR INFORMATION ABOUT PARAMETERS		5. Report Date November 1971	
7. Author(s) Steven M. Sidik and Arthur G. Holms		6. Performing Organization Code	
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		8. Performing Organization Report No. E-6223	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		10. Work Unit No. 129-04	
15. Supplementary Notes		11. Contract or Grant No.	
16. Abstract <p>Any prior information an experimenter has should be incorporated into the design of further experiments. Assume such information is expressed as probabilities that each parameter of the full factorial model is nonzero. This report develops optimal design procedures by posing the experimental design problem as a finite decision problem. Bays and mini-max design strategies are then derived and their application illustrated. The major computational step is the evaluation of all possible matchings of physical variables to the abstract variables of all potential designs. The technique of telescoping sequences of blocks permits the consideration of experiments to be performed in stages.</p>		13. Type of Report and Period Covered Technical Note	
17. Key Words (Suggested by Author(s)) Experimental design; Factorial design; Sequential experiments; Factorial arrangements; Bayesian design of experiments; Decision theory; Block designs; Confounding; Aliases; Orthogonality; Fractional replicates; Telescoping designs		14. Sponsoring Agency Code	
18. Distribution Statement Unclassified - unlimited		15. Supplementary Notes	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 39	22. Price* \$3.00

OPTIMAL DESIGN PROCEDURE FOR TWO-LEVEL FRACTIONAL FACTORIAL EXPERIMENTS GIVEN PRIOR INFORMATION ABOUT PARAMETERS

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SUMMARY

In many cases in practice an experimenter has some prior knowledge of indefinite validity concerning the main effects and interactions which would be estimable from a two-level full factorial experiment. Such information should be incorporated into the design of the experiment.

This report presents a method for incorporating such knowledge when the prior information can be expressed as prior probabilities that each main effect or interaction will be nonzero.

A review of fractional factorials, blocking of factorial designs, and telescoping sequences of blocks is presented. It is shown how, by using these techniques, the experimental design problem may be posed as a finite decision problem. Then for the finite decision problem, Bayes and mini-max strategies are developed and their application to the design problem illustrated. The major computational step is the evaluation of all possible matchings of physical variables to the abstract variables of all potential designs.

The technique of telescoping sequences of blocks permits the consideration of experiments to be performed and analyzed in stages.

INTRODUCTION

Confounded full and fractional factorial experimental designs have become a widely used and valuable experimental tool since their development by Fisher (ref. 1) and Finney (ref. 2). They are often used where there are several factors (independent variables) which may jointly influence the level of a response (dependent variable). The assumed relation between the dependent variable Y and the independent variables X_A , X_B , . . . is of the form

$$Y = \beta_I + \beta_A X_A + \beta_B X_B + \beta_{AB} X_A X_B + \beta_C X_C + \beta_{AC} X_A X_C + \beta_{BC} X_B X_C + \beta_{ABC} X_A X_B X_C + \dots + \beta_{ABC \dots} X_A X_B X_C \dots + \delta \quad (1)$$

where for repeated observations the δ are independent random variables with mean zero and finite variance, and the β 's are unknown parameters some of which the experimenter wishes to estimate. (All symbols are defined in appendix A.)

If a fractional replicate of a full factorial experiment is run, not all the parameters whose existence is implied by equation (1) can be estimated. Instead, only certain linear combinations of these parameters can be estimated.

For example, a one-half replicate experiment on four independent variables would provide eight estimates which might (depending on the fractional plan chosen) be estimates of paired combinations of parameters as follows:

$$\begin{array}{ll} \beta_I + \beta_{ABCD} & \beta_D + \beta_{ABC} \\ \beta_A + \beta_{BCD} & \beta_{AB} + \beta_{CD} \\ \beta_B + \beta_{ACD} & \beta_{AC} + \beta_{BD} \\ \beta_C + \beta_{ABD} & \beta_{AD} + \beta_{BC} \end{array}$$

From such estimates, nothing can be inferred about any single parameter without making assumptions about, or having prior information about, the other parameter of a paired combination.

In the general situation the experimenter often has some prior beliefs about which parameters are nonzero, and he can state such beliefs with varying shades of conviction, according to the particular parameter being considered. If pressed, he should state those odds that he would be willing to take on either side of a bet that a particular parameter is nonzero. If his odds are $w:z$ that an interaction between X_2 and X_3 is nonzero, his subjective prior probability that β_{23} is nonzero is $p = w/(w + z)$.

The experimenter could match his physical variable names X_1, X_2, X_3, X_4 to the design variable names X_A, X_B, X_C, X_D so that the lower prior probability parameters of the paired combinations could be dropped, under the following conditions. The experimenter's prior probabilities of nonzero three- and four-factor interactions were all less than 0.01. He believed that prior probabilities of nonzero two-factor interactions between X_1 and any other independent variable and X_2 and any other independent variable were less than 0.05. This matching is as follows:

$$\left. \begin{aligned} X_1 &= X_C \\ X_2 &= X_D \\ X_3 &= X_A \\ X_4 &= X_B \end{aligned} \right\} \quad (2)$$

If the experiment is performed and numerical values are computed for the maximum number of parameter estimates, the equation for Y could be written

$$Y = b_0 + b_3X_3 + b_4X_4 + b_1X_1 + b_2X_2 + b_{34}X_3X_4 + b_{31}X_3X_1 + b_{32}X_3X_2 \quad (3)$$

Equation (3) thus represents a combination of the experimenter's prior knowledge with the information that could be extracted from the one-half replicate experiment.

As illustrated by the preceding example, the experimenter can maximize the information obtained about parameters, while minimizing the number of experimental units performed, if he uses a judicious matching of the physical variables X_1, X_2, X_3, \dots to the design variables X_A, X_B, X_C, \dots . Thus, one problem of constructing an optimal design is to find a "best" matching for a given design.

After presenting a survey of some pertinent topics such as fractional factorials, blocking, and telescoping blocks, this report establishes definitions for "best" designs and methods for finding the best or nearly best designs. Appendix B contains an illustrative example.

The reader is assumed to have a knowledge of two-level factorial designs as might be found in Davies (ref. 3), Peng (ref. 4), or Kempthorne (ref. 5). Some results from elementary decision theory and game theory are used. Some references on this subject are Raiffa (ref. 6), Raiffa and Schlaifer (ref. 7), and McKinsey (ref. 8).

A method for systematically comparing the possible combinations and choices of interest and a computer program for performing some of the calculations have been developed and are presented in Sidik (ref. 9).

STEPS IN THE DESIGN OF FACTORIAL EXPERIMENTS

The decisionmaking in the design of an experiment can be subdivided into steps as follows:

(1) What quantities to vary: The quantities that will enter into the investigation as controlled independent variables must be decided upon. In taking this step, there should be no intention that other variables will be added later. The full list of potentially important variables should be incorporated into the initial experiments so that interactions among the variables can be observed.

(2) In what way to vary the variables: The variables are sometimes introduced through linearizing transformations. For example, the velocity of a fluid is the dependent variable and differential pressure is an independent variable. Then the levels of the differential pressure chosen for the investigation might be a uniform sequence of values of the square root of the differential pressure. Thus approximately linear response would be achieved on that particular independent variable.

(3) Starting region of the experimentation: This is specified so that the midpoint of the values of the independent variables is located at the point in the experimental space that is thought to be of the greatest interest or importance.

(4) Amounts by which the independent variables are varied: The increments between successive levels of the independent variables are ideally large enough so that the dependent variable will respond by an increment significantly larger than the random error. On the other hand, the increments of the independent variables should be small enough so that the resulting observations will not be too irregular to be fitted to the intended model (e.g., eq. (1)). Furthermore the increments should be small enough so that the independent variables do not leave the range of the experimental space that is of practical interest.

(5) Number of levels for the independent variables: In this connection the composite designs that were closely studied by Box and Hunter (ref. 10) are known to be highly efficient in providing the data for fitting model equations of considerable usefulness. The beginning point of the composite design is the full or regular fraction of the two-level factorial experiment.

(6) Block size: One approach to designing experiments so that they will have good statistical properties and still be no larger than necessary is to design them as telescoping sequences as in Addelman (ref. 11) and Holms and Sidik (ref. 12). In such sequences the first block is conveniently one that is just large enough to estimate all the coefficients of a first-degree model equation. (Physical limitations might occasionally dictate that a smaller block size must be used.)

(7) Choice of defining contrasts and matching of physical variables to design variables: This is the main subject of this report.

BASIS OF DESIGN OF TWO-LEVEL FACTORIAL EXPERIMENTS

Notations for Treatments, Contrasts, Parameters, and Estimates

A two-level factorial experiment with n independent variables X_A, X_B, X_C, \dots is an experiment in which the response or dependent variable is assumed to depend upon the independent variables as indicated by equation (1) of the INTRODUCTION.

A combination of the levels of the independent variables is called a treatment combination or just a treatment. A treatment represents an experimental condition under which an observation might be made. If there are n independent variables and the variables are restricted to assuming only two possible values, there are 2^n possible treatment combinations. If observations are made at each of the 2^n treatment combinations, all the parameters of equation (1) can be estimated.

The values of the independent variables can be coded so that they take on the values +1 or -1 only. The "upper" level or "high" level is represented by $X_A = +1$, $X_B = +1$, and so forth. The "lower" or "low" level is represented by $X_A = -1$, $X_B = -1$, and so forth. Since a treatment combination is a specified combination of the levels of the independent variables, a treatment combination may be identified as a vector of +1's and -1's; for example,

$$(X_D, X_C, X_B, X_A) = (-1, -1, +1, -1)$$

Another frequently used notation involves the use of combinations of the lower case letters of the alphabet. The presence of a letter denotes that the independent variable having the associated upper case of that letter as a subscript is at its high level. The symbol (1) is used to denote the low level. Thus, the preceding treatment combination may be written

$$(X_D, X_C, X_B, X_A) = (-1, -1, +1, -1) = (1)(1)b(1) = b$$

A second example is

$$(X_D, X_C, X_B, X_A) = (+1, +1, -1, -1) = dc(1)(1) = dc$$

In what follows it is convenient to be able to order the treatment combinations and parameters in a standard order. For convenience, the independent variables are ordered alphabetically from right to left as X_E, X_D, X_C, X_B, X_A . This order is the opposite of that used in the INTRODUCTION and SUMMARY and is used in the remainder of this report.

Let the v^{th} treatment combination be denoted

$$t_v = t_{m_n m_{n-1} \dots m_1}$$

The subscript on the right is v written as a binary number. Then t_v is identified as the treatment combination which has the i^{th} independent variable (from the right) at its low level if $m_i = 0$ and has the i^{th} independent variable at its high level if $m_i = 1$. Thus the preceding examples can be written

$$(X_D, X_C, X_B, X_A) = (-1, -1, +1, -1) = (1)(1)b(1) = b = t_{0010} = t_2$$

$$(X_D, X_C, X_B, X_A) = (+1, +1, -1, -1) = dc(1)(1) = dc = t_{1100} = t_{12}$$

Table I presents the design matrix for a 2^3 factorial. The column labeled "Treatment" gives both the t_v and alphabetic notation for the eight treatment combinations. The column labeled "Observed response" gives the eight y values observed. The remaining columns give the values of X_A , X_B , X_C , and their products.

The same kind of ordering as has just been developed for the treatment combinations can be developed for the parameters of a model equation for a factorial experiment. In fact, it can easily be seen that the terms of equation (1) are already arranged in standard order with the parameters having alphabetic subscripts. Suppose the independent variables are ordered from right to left. Then let the v^{th} parameter be denoted

$$\beta_v = \beta_{m_n m_{n-1} \dots m_1}$$

where the subscript on the right is v written as a binary number. Then if m_{i_1} , m_{i_2} , \dots , m_{i_k} designate the digits that equal 1, β_v is the coefficient of the k -factor interaction between the i_1, i_2, \dots, i_k independent variables, recalling that they are ordered from the right. An interaction represents the fact that the effect of the change of level and some independent variable on the response might depend upon the levels of other independent variables. A two-factor interaction between two independent variables (e.g., X_A and X_B) indicates that the amount by which a change in X_A changes the dependent variable depends upon the particular value of X_B . Likewise, the amount by which a change in the level of X_B changes the dependent variable depends upon the particular value of X_A . Interaction among three or more independent variables is defined similarly. The only difference being that there are dependencies on more variables.

To estimate the parameters of equation (1) the method of least squares may be used. In this special case of two-level factorial designs the method of least squares leads to a

simple rule: the parameter associated with the heading of a column (such as in table I) is estimated by the linear combination of the y 's indicated by the +1's and -1's in that column divided by the number of observations. For example, from the data of table I, the estimate of β_A is

$$b_A = (-y_1 + y_2 - y_3 + y_4 - y_5 + y_6 - y_7 + y_8) \frac{1}{8}$$

All the columns are linear combinations and are called contrasts. In a full two-level factorial design there is a one-to-one correspondence between the contrasts and the parameters.

Use of Elementary Group Theory

Some elementary notions from group theory are useful in simplifying the notation and later discussions.

A group is a set G of elements and an operation \otimes defined upon any two elements from G which satisfies the following properties:

- (1) If $a \in G$ and $b \in G$, then $a \otimes b \in G$.
- (2) If $a \in G$, $b \in G$, and $c \in G$, then $(a \otimes b) \otimes c = a \otimes (b \otimes c)$.
- (3) There is some element of G denoted as (1) , such that $a \otimes (1) = (1) \otimes a = a$ for all $a \in G$.
- (4) For every $a \in G$, there is some element we denote as a^{-1} such that $a \otimes a^{-1} = a^{-1} \otimes a = (1)$.

Let $T = \{t_v\}$ represent the full set of treatment combinations and $B = \{\beta_v\}$ represent the full set of parameters. A group structure may be imposed upon both sets by introducing the product \otimes defined by

$$\beta_{m_n \dots m_1} \otimes \beta_{k_n \dots k_1} = \beta_{j_n \dots j_1}$$

and

$$t_{m_n \dots m_1} \otimes t_{k_n \dots k_1} = t_{j_n \dots j_1}$$

where

$$j_i = (m_i + k_i) \pmod{2}$$

(4)

This notation is completely analogous to the more common notations using the letters of the alphabet where, for the treatments,

$$a^{m_1} b^{m_2} \dots \otimes a^{k_1} b^{k_2} \dots = a^{j_1} b^{j_2} \dots$$

Note that this notation does not imply that parameter values are to be multiplied but is used to indicate the relations between parameters, treatments, and contrasts.

Suppose $n = 4$. Then we may use the alphabetic notation for T to illustrate how T and \otimes form a group by showing that the properties 1 to 4 hold. T contains all possible combinations of the four letters a, b, c, d including the "combination" (1) where none are present. Thus we may say $T = \left\{ d^{i_4} c^{i_3} b^{i_2} a^{i_1} : i_j = 0 \text{ or } 1 \text{ in all possible combinations} \right\}$ and define $a^0 = b^0 = c^0 = d^0 = (1)$. Then any two elements w_1 and w_2 of T will be of the form

$$w_1 = d^{i_4} c^{i_3} b^{i_2} a^{i_1}$$

$$w_2 = d^{j_4} c^{j_3} b^{j_2} a^{j_1}$$

and

$$w_1 \otimes w_2 = d^{(i_4+j_4) \pmod 2} c^{(i_3+j_3) \pmod 2} b^{(i_2+j_2) \pmod 2} a^{(i_1+j_1) \pmod 2} \quad (5)$$

It is clear that since the exponents are reduced modulo 2, no new combinations of letters other than those in T can be formed. This satisfies the first requirement of a group. The second requirement is clearly true by the way \otimes was defined. Now note that since $(1) = d^0 c^0 b^0 a^0$, multiplying any element by (1) will not change the values of the exponents so that $(1) \otimes w = w$. Thus the third requirement is satisfied. Suppose $w = d^{i_4} c^{i_3} b^{i_2} a^{i_1}$ is any arbitrary element of T . Then $w \otimes w = d^{2i_4 \pmod 2} c^{2i_3 \pmod 2} b^{2i_2 \pmod 2} a^{2i_1 \pmod 2} = d^0 c^0 b^0 a^0 = (1)$. Thus every element serves as its own inverse and the fourth requirement is satisfied.

A subgroup of a group is a subset of the full set which is also a group under the same operation. For example, consider the set $\{(1), a\}$ which is a subset of $\{(1), a, b, ab\}$. It can be verified that under the operation \otimes as previously defined, $\{(1), a\}$ is a group and $\{(1), a, b, ab\}$ is a group and hence $\{(1), a\}$ is a subgroup of $\{(1), a, b, ab\}$.

Let G^* be a subgroup of G under the operation \otimes . Let w be some arbitrary element of G and let $w \otimes G^*$ denote the set of elements $\{w \otimes t_1, w \otimes t_2, \dots, w \otimes t_n\}$, where the t_i are the elements of G^* . Such a set is called a coset of G^* .

Two elements of a group will be called orthogonal if in their alphabetic representations they have an even number of letters in common. They will be orthogonal also if in the binary subscript notations they have an even number of 1's in common locations.

Thus, β_{CBA} is orthogonal to β_{DCB} , dcb is orthogonal to dca , and t_{1110} is orthogonal to t_{0111} . Orthogonality may also be defined between different groups so that for example, $\beta_{CBA} = \beta_{0111}$ is orthogonal to $dca = t_{1101}$.

Fractional Factorials

It is often impractical to perform all 2^n treatment combinations of a full factorial experiment. In many cases in practice there are seven or eight or more independent variables so that the full factorial with 2^n treatments might be too expensive to perform. Also, it is often assumed that only some of the possible parameters of equation (1) are nonzero. In particular, it is generally assumed that higher order interactions either do not exist or else are associated with effects of small magnitude. Thus, it is of interest to determine how the number of observations and the number of estimated coefficients can be reduced from a full factorial in a manner consistent with the experimenter's objectives.

A common way of reducing the number of treatment combinations is to perform a regular fractional replicate. A regular fractional replicate is any coset of a subgroup T_1 of T . The subgroup T_1 is sometimes referred to as the treatment subgroup or principal fraction.

Finney (ref. 2) has shown that for every subgroup T_1 of T there exists a subgroup B_1 of B called the complete orthogonal subgroup which contains every element of B which is orthogonal to every element of T_1 . The complete orthogonal subgroup will be referred to as the defining parameter group (d.p.g.) and its elements called the defining parameters. It is also true that for every subgroup B_1 of B there is a subgroup T_1 of T such that T_1 contains every element of T which is orthogonal to every element of B_1 .

It has been previously stated that if a regular fractional replicate is performed, not all the parameters of equation (1) can be independently estimated. Only certain linear combinations of the parameters can be estimated. If there are 2^{n-r} observations in a regular fraction, there can be only 2^{n-r} contrasts that are linearly independent. Each of these contrasts is an estimator of a linear combination of the parameters appearing in a single coset of the complete orthogonal subgroup.

All subgroups of T will have 2^q treatment combinations in them for some $q = 0, 1, \dots, n$. If a 2^{n-r} fractional factorial is performed, it will consist of 2^{n-r} treatment combinations and there will be 2^{n-r} cosets each containing 2^r parameters.

As an example of these points consider a one-half replicate of a 2^3 experiment with defining parameter group B_1 where

$$B_1 = \{\beta_I, \beta_{CBA}\} = \{\beta_{000}, \beta_{111}\} = \{\beta_0, \beta_7\}$$

Then by the "rule of even numbers" (ref. 3), it can readily be shown that

$$T_1 = \{(1), ba, ca, cb\} = \{t_{000}, t_{011}, t_{101}, t_{110}\} = \{t_0, t_3, t_5, t_6\}$$

The three remaining distinct cosets of B_1 are

$$\beta_A \otimes B_1 = \{\beta_A, \beta_{CB}\}$$

$$\beta_B \otimes B_1 = \{\beta_B, \beta_{CA}\}$$

$$\beta_{BA} \otimes B_1 = \{\beta_{BA}, \beta_C\}$$

A summary of this experiment is given by the first four lines of table II. Let us apply the simple estimation rule previously stated for the full factorial case but simply ignore the observations not available. That is, the parameter associated with the heading of a column (contrast) is estimated by the linear combination of the available y 's indicated by the +1's and -1's in that column divided by the number of observations; for example,

$$b_A = \frac{-y_1 + y_2 + y_3 - y_4}{4}$$

However, note that

$$b_{CB} = \frac{y_1 - y_2 - y_3 + y_4}{4} = -b_A$$

Actually, $(-y_1 + y_2 + y_3 - y_4) \frac{1}{4}$ estimates $\beta_A - \beta_{CB}$. If β_A is assumed to be zero, it

may be said that the preceding linear combination of y 's estimates $-\beta_{CB}$. If β_{CB} is assumed to be zero, it may be said that the linear combination estimates β_A . The name assigned to the estimator depends upon the assumptions made. Thus, in a simplified sense it may be said that each independent estimator may be assigned more than one name. This situation arises in the following way. For a full factorial design, the contrasts which estimate each parameter are linearly independent of each other. When a regular fractional replicate is performed, there will be a reduced number of linearly independent contrasts. Each of these contrasts will provide an estimate for a linear combination of certain of the parameters. This was illustrated previously where the contrast $\frac{1}{4}(-y_1 + y_2 + y_3 - y_4)$ estimates $\beta_A - \beta_{CB}$. It was shown by Finney (ref. 2) that the linear combination of parameters that a given contrast estimates will contain all the parameters in the complete orthogonal subgroup or one of its cosets, and no other parameters. This may be illustrated as follows:

$$(1) \frac{1}{4}(y_1 + y_2 + y_3 + y_4) \text{ estimates } \beta_I - \beta_{CBA}$$

$$(2) \frac{1}{4}(-y_1 + y_2 - y_3 + y_4) \text{ estimates } \beta_B - \beta_{CA}$$

$$(3) \frac{1}{4}(-y_1 - y_2 + y_3 + y_4) \text{ estimates } \beta_C - \beta_{BA}$$

as can be readily checked from table II.

For each parameter in a 2^n factorial there can be uniquely associated one contrast which estimates it from the data of a full factorial. From a fractional factorial there is a smaller number of observations. If the components of the contrasts corresponding to the observations not made are dropped, each parameter in a coset will have the same contrast of the available observations (except for possibly a change of sign). The d.p.g. "defines" the experiment in the sense that the treatment subgroup (or some coset of the treatment subgroup) which contains every element of T which is orthogonal to every element of the d.p.g. will constitute a regular fractional replicate of the full factorial. The alias sets resulting from the estimation rule given previously will be cosets of the d.p.g.

Blocked Factorial Designs

A problem often encountered in practice is that of maintaining uniform experimental conditions throughout the period of experimenting. A method called block confounding was developed (ref. 1) to help minimize biasing the parameter estimates of a factorial experiment when it is known or suspected that certain conditions which are not of particular interest or not readily measurable change.

Suppose an experimenter wishes to perform a 2^{-I} fractional factorial experiment on n independent variables, but cannot assure that conditions will be uniform for all

2^{n-r} treatment combinations. He can assure, however, that 2^w ($w < (n - r)$) treatment combinations can be run under the same conditions.

The procedure for constructing a blocked fractional factorial is as follows: Choose a subgroup B_1 of B that defines a fractional replicate with 2^{n-r} treatment combinations. Let these treatment combinations be a subgroup T_1 of T . Choose a subgroup B_2 of B containing B_1 so that B_2 defines a fractional replicate with 2^w treatments. Let these treatments be a subgroup T_2 of T such that T_2 is a subgroup of T_1 . Then the subgroup T_2 and its cosets are called blocks. The expectation of the population responses over a block is called the block mean. It was shown in Holms and Sidik (ref. 12) that the estimators for those alias sets which contain parameters which are elements of B_2 but not elements of B_1 are also estimators of linear combinations of the block means. All other estimators are linearly independent of the block means. An aliased set of parameters whose estimator also estimates a block effect (linear combination of block means) is said to be confounded with blocks.

As an example of this confounding, consider the experiment summarized in table II, where $n = 4$, $r = 0$, and $w = 2$. The two blocks are the sets of treatment combinations $\{(1), ba, ca, cb\}$ and $\{a, b, c, cba\}$. It can be shown that the d.p.g. for the fractional replicate corresponding to those treatments in block 1 is $B_2 = \{\beta_I, \beta_{CBA}\}$. In this example, $B_1 = \{\beta_I\}$. Thus the estimator for β_{CBA} is confounded with any block effect. This may be easily seen by recalling the previously stated estimation rule that

$$b_{CBA} = \frac{(y_5 + y_6 + y_7 + y_8) - (y_1 + y_2 + y_3 + y_4)}{8}$$

$$= \frac{1}{2} \left(\frac{y_5 + y_6 + y_7 + y_8}{4} - \frac{y_1 + y_2 + y_3 + y_4}{4} \right) = \frac{1}{2} (\hat{\mu}_2 - \hat{\mu}_1)$$

where $\hat{\mu}_i$ is the estimated block mean of block i . More detailed developments of this aspect can be found in references 3, 4, 5, and 12. If a 2^n factorial is divided into a 2^{n-r} fractional factorial and then further divided into blocks with 2^w ($w < (n - r)$) treatments per block, there are 2^{n-r-w} blocks in the fractional factorial.

Telescoping Blocks

The previous two concepts of fractional replication and blocking of factorial experiments can be combined to provide a means of sequentially adding blocks to the factorial while retaining the property of orthogonal blocking. This technique is called telescoping

and a description of double and multiple telescoping is given by Holms and Sidik in reference 12. Addelman (ref. 11) has tabulated many single telescoping designs.

An experimenter may begin experimenting with a very small fractional replicate as one block. The treatment combinations used and the alias sets available are defined by a choice of defining parameter group B_1 . This is a subgroup of B , the full parameter group. When the experimenter completes the first block, he has the option to continue experimenting or to stop. If he stops, he has a small number of estimators each of which is associated with a large alias set. If he elects to continue with regular fractional replicates, he must at least double the current size of the experiment. He may do this by choosing a subgroup B_2 of B_1 and performing a second block of treatment combinations. The added treatment combinations are given by those which are orthogonal to B_2 but have not already been performed. In addition to a reduction in the variance of the estimates, the gain from continuing to the new stage will be that there will be twice as many estimators available and each alias set will have only one-half as many elements as the alias sets of the one-block experiment. An important fact is that one estimator will be confounded with the block difference between blocks 1 and 2.

In general, suppose experimenting is started with a 2^{-r} replicate and that it is also a single block. Then an experiment will be said to be at the l^{th} stage if it contains 2^{l-1} blocks. Let the treatment group at the l^{th} stage be written as $T(l)$ and the d.p.g. at the l^{th} stage as $B(l)$. Let $T(r+1)$ be the full treatment group and $B(0)$ be the full parameter group. The number of elements of $B(l)$ is one-half that of $B(l-1)$ and the number of elements of $T(l)$ is twice that of $T(l-1)$. The parameters confounded with blocks at any stage are given by the elements of $B(1)$ that are not elements of $B(l)$. This rule is valid only for single telescoping. More complex rules are indicated in reference 12 for the case of multiple telescoping.

USE OF GAME AND DECISION THEORY

Estimability of Parameters

An estimator for an alias set is generally useless unless all except one parameter in the alias set are known. The conventional rules for estimability use the basic assumption that the response function is "smooth" to justify assuming all the high-order interactions are zero. Thus, common practice is to assign the estimator to the parameter in the alias set which represents the lowest order interaction. If there are ties, the estimator is either discarded or assigned to a particular parameter based upon other considerations. Thus, for example, the estimator for $\{\beta_A, \beta_{EDCB}, \beta_{CB}, \beta_{EDA}\}$ would normally be assigned to β_A , whereas specific prior knowledge would be required for

the estimator for $\{\beta_D, \beta_{ECBA}, \beta_{DCBA}, \beta_E\}$ to be assigned to β_D or β_E .

Suppose now that the experimenter has some knowledge which enables him to assign a prior probability to each parameter. The prior probability measures his prior degree of belief that the parameter is nonzero. This knowledge may have been obtained from experiments in more or less similar situations, or it may be in the form of some theoretical relations he knows of. Such knowledge can indicate which parameters are more likely to be nonzero than others.

Consider the problem of matching estimators and parameters at the h^{th} stopping point. The d.p.g. is $B(h)$, and the alias sets are all those distinct cosets of the form $\beta_i \otimes B(h) = \{\beta_{i_1}, \beta_{i_2}, \dots, \beta_{i_m}\}$. Let $P(\beta_{i_j} \neq 0) = p_{i_j}$. If the estimator for the alias set $\beta_i \otimes B(h)$ is assigned to the parameter β_{i_k} , then assuming independence, the prior probability that the estimator will be unbiased is

$$\prod_{\substack{j \in S(i, h) \\ j \neq i_k}} (1 - p_j) \quad (6)$$

This probability is used to help find a "best" matching of estimators to parameters.

Formulation of the Problem

All the preceding discussion concerning fractional factorials, blocking, telescoping designs, and estimability of parameters from these designs can now be put together to represent the steps of the design and performance of an experiment as a finite game between nature and the experimenter.

The experimenter is free to choose any of the parameters in an alias set to be assigned the estimator. The experimenter is also free to choose the design-variable-to-physical-variable matching. It is very likely that different matchings will obtain different values to the experimenter since changes in the physical-design variable matching will cause changes in the elements of the alias sets.

The experimenter is free to choose a telescoping design. This involves the choice of initial d.p.g. and the sequence of subgroups (or he might choose a multiple telescoping design with branch point decisions restricted by nature). Thus the decision space for the experimenter is composed of the choice of initial d.p.g., choice of sequence or sequences of subgroups, choice of physical-design variable matching, and choice of parameter-estimator matching.

Nature has the choice of which of the β 's will be nonzero and the choice of the path and the stopping point of the telescoping.

A decision "tree" is a graphical representation of the possible sequences of choices the experimenter and nature can make. A decision tree representing all the preceding choices would, in general, be much too large to be conveniently represented. A flow diagram is presented in figure 1 which indicates the sequence of choices to be made by the experimenter and nature.

It should be noted that the order of presentation shown in figure 1 of the choices of nature and the experimenter are not necessarily in temporal order. The choice of d.p.g. and physical-design variable matching are interchangeable but must be made before the actual execution of the experiment. Nature's choice of the parameter values would normally be made prior to anything the experimenter does. However, it does no harm to conceive of nature being permitted to choose the parameter values after the experiment is performed. In fact, this concept is more convenient in describing the Bayes decision procedure for the experimenter.

In order to evaluate all these possible outcomes, the relative value of each possible outcome to the experimenter should be specified. These relative values are defined as utilities.

Once given the structure of a game or a decision problem and the utilities of the outcomes, the statistician must devise a method for choosing the possible alternatives so that a desirable outcome is finally attained. The possible methods for making decisions are called strategies. More precisely, a strategy for the experimenter is a set of rules which specify what choice is to be made from the alternatives available as a function of all previous choices at every possible point where a choice is to be made.

There are two general principles by which strategies are ordered which are in current use. One is called the Bayes principle and the other is called the mini-max principle.

The Bayes Solution

The Bayes procedure assumes that nature is an indifferent participant in the game. Thus, the choices that nature makes are independent of the choices available to the experimenter, but instead are made according to a specified probability distribution. Any information the experimenter may have concerning the strategy (specified probability distribution) nature will follow is useful information that should be incorporated into the decision procedure. The strategy which maximizes the utility given the assumed strategy for nature is called the Bayes strategy. Thus, let us assume the experimenter has prior information which he can express as probabilities of each parameter being

nonzero and probabilities of stopping exactly at a given stage of the experiment. Let β_v be the v^{th} parameter in the standard ordering and let $P(\beta_v \neq 0) = p_v$. Let

$$P \left(\begin{array}{l} \text{Experiment is terminated} \\ \text{at exactly the } h^{\text{th}} \\ \text{stopping point} \end{array} \right) = P_{hs}$$

Let us restrict the utilities so that $u_i(h)$ is the utility of obtaining an unbiased estimator for β_i at the h^{th} stopping point and assume $u_i(h) = 0$ if the estimator is biased. We allow it to be some finite value greater than or equal to zero if the estimator is not biased.

Suppose at the box in figure 1 marked with a star (\star), l stages have been completed. Then there are $2^{n-r+l-1}$ alias sets of the form $\beta_i \otimes B(l) = \{\beta_{i_1}, \beta_{i_2}, \dots, \beta_{i_m}\}$, where $m = 2^{r-l+1}$. From each alias set, one parameter may be chosen to be assigned the estimator. Independently of these assignments, nature now chooses which of the parameters are zero. By the previously stated assumptions, the experimenter cannot be sure what nature will do for this particular instance, but he does know with what probabilities the parameters will be nonzero. Thus, the proper thing to do is to compute the expected utility for each possible assignment from the alias sets.

Let the utility placed upon an unbiased estimate of β_i at the h^{th} stopping point be denoted by $u_i(h)$. Let the expected utility to the experimenter gained by assigning the estimator for the alias set $\beta_i \otimes B(h)$ to the parameter β_k be denoted by $U(i, k)$. Then

$$U(i, k) = u_k(h) \prod_{\substack{j \in S(i, h) \\ j \neq k}} (1 - p_j) \quad (7)$$

The experimenter should thus assign the estimator to the parameter β_k which maximizes this quantity. A special case requires separate discussion. Suppose that an alias set is confounded with some block effect. In this case there are $2^{r-l+1} + 1$ parameters in the alias set. Assume that prior probabilities can be assigned to the event that each block effect will be nonzero and assume the utility of an unbiased estimate can be specified. Then this information can be incorporated into the decision procedure by computing the $m + 1$ expected utilities:

$$U(i, k) = u_k(h) \left[\prod_{\substack{j \in S(i, h) \\ j \neq k}} (1 - p_j) \right] (1 - p_b) \quad (8)$$

$$U(i, b) = u_b \left[\prod_{j \in S(i, h)} (1 - p_j) \right] \quad (9)$$

where

$$p_b = P \{ \text{the block effect} \neq 0 \}$$

and u_b is the utility of an unbiased estimate of the block effect and $U(i, b)$ is the expected utility gained by assigning the estimator for $\beta_i \otimes B(h)$ to the block effect.

Since the alias sets are disjoint and the estimators for each alias set are independent of each other, it follows that the choice of parameter from one alias set need not influence the choice from another alias set. Thus, the restriction on the utility function that sets it equal to zero for biased estimators and the use of the expected utility imply that the number of choices at the boxes marked with a \star and a $*$ is $(2^{r-l+1})(2^{n-r+l-1}) = 2^n$, where l is the number of stages comprising the h^{th} stopping point. This is the number of alias sets times the number of choices that could be made from each alias set, and is equal to the number of factorial parameters, ignoring block parameters.

Thus, let the total utility to the experimenter at the h^{th} stopping point be denoted $U(h) = \sum U(i, k)$, where the summation is over all the distinct cosets at the h^{th} stopping point. Let

$$U = \sum_{h=1}^s p_{hs} U(h) \quad (10)$$

Then U represents the expected utility over all stopping points for a fixed choice of initial d.p.g., telescoping sequence or sequences, and physical-design variable matching. The formal Bayes solution to the decision problem would then be to compute the value U for each of the possible choices of initial d.p.g., naming, and telescoping sequences, and then choose the combination which gave the maximum U . This involves a large number of alternatives.

Modified Bayes Solution

As a practical procedure, it seems highly unlikely that all the possible d.p.g.'s will be evaluated. There would be far too many alternatives to compute the formal Bayes solution in many problems. Common sense may, however, indicate some reasonable

ways to reject certain types of d.p.g.'s depending upon the given utilities and probabilities.

Many experimental situations satisfy the condition that the higher the order of interactions, the greater the probabilities of their being small or absent, and therefore the smaller their expected utilities. This is the typical situation and it seems likely that the "best" design in this case will be one of the designs derived on the basis of resolution level. (For a given number of independent variables, these kinds of designs occur at the beginnings of the lists in Addelman (ref. 11).)

A different situation might conceivably arise when there are essentially two sets of independent variables under consideration. One of these sets is known to contain elements interactive among themselves. The second set, however, is known to have first-order effects upon the dependent variable; but these elements do not have much tendency to interact. (For such situations, the appropriate designs might occur at the ends of the lists in Addelman (ref. 11).)

Mini-Max Solution

To derive a mini-max strategy, the statistician assumes nature to be an aggressive player who will choose alternatives available to her which will minimize the maximum utility the statistician may gain. Thus the experimenter is the maximizing player, and nature is the minimizing player.

A strategy for nature involves two component choices: the choice of which parameters will be nonzero and the choice of the path and stopping point of the telescoping. One choice available to nature which the experimenter has no influence upon is the choice of nonzero parameters. In fact, it is possible that nature may choose to let every parameter be nonzero. In this instance it will not be possible to obtain unbiased estimates for any parameter until the full replicate is run. If the utility function is restricted to be zero when the estimator is biased, there is zero utility except for the full replicate. Thus, the only possible way for the experimenter to obtain a gain is to design for the full replicate. However, nature also may choose the stopping stage; and so to minimize the experimenter's utility, any stage other than the full replicate may be chosen. Thus, any strategy at all that the experimenter uses will be a mini-max strategy, for nature can always assure that the experimenter will receive zero utility.

Two less aggressive strategies for nature are conceivable. The first assumes that nature will still choose to have every parameter nonzero but will stop the experiment at one of the stages according to prior probabilities known by the experimenter. Then the only reasonable approach for the experimenter is to choose a design which maximizes the utility of the full factorial. Clearly, this only involves minimizing losses due to block confounding.

The second of these less aggressive strategies allows that nature will choose the parameters to be nonzero according to probabilities known to the experimenter but will choose the stopping stage so as to minimize the experimenters maximum utility. To find the mini-max strategy, the tree form of the game can be reduced to the form of a rectangular game, and the technique of linear programming can be applied to determine the optimal strategies. Considering the size of the tree in this problem, it does not seem likely that this approach would be computationally feasible.

It is true that mini-max strategies are in general randomized strategies. Rather than adopting such a randomized strategy it may be more desirable to adopt what might be called a security strategy. This would be the strategy of the experimenter which maximizes the minimum possible utility. This could be done by examining the expected utilities at each of the stopping points and would involve much the same computations as the Bayes procedure.

Assignment of Utility Functions

The utility function is a function defined upon the space of all possible outcomes of a decision problem. This function describes the absolute or relative value to the decisionmaker of each of these outcomes. Methods of specifying the utility function based upon the axioms of utility theory are given in Raiffa (ref. 6) and DeGroot (ref. 13). These methods involve choices of lotteries and would be extremely impractical and cumbersome for the current problem. An alternative procedure is often used which attempts to place some common evaluation upon each outcome, such as the monetary value, man-years of expended energy, and so forth. This approach also seems unlikely of being applicable in the present problem. In fact, in only the most trivial of experiment design problems does it seem likely that a utility function can be specified which will completely describe the relative values of each outcome. This does not at all imply that the decision theory approach to statistical decision problems is worthless. As in almost any branch of applied mathematics, concessions are made to practicality, and a utility function is chosen which is both mathematically tractable and at least roughly representative of the decisionmaker's preferences. For example, two such concessions used in the theory of estimation are the absolute-error and squared-error loss (negative of utility) functions.

In keeping with the preceding comments, the following five utility functions are offered as being both practical and somewhat representative of the decisionmaker's preferences in the appropriate instances:

$$(1) u_i(h) = \begin{cases} 1 & \text{if estimator unbiased} \\ 0 & \text{if not} \end{cases}$$

This is a utility function which simplifies the expected utility to be the probability of getting an unbiased estimate of the parameter. This might be a useful utility function in the more basic scientific exploration where relative evaluations of the importance of specific parameters are not possible.

$$(2) u_i(h) = \begin{cases} p_i & \text{if unbiased} \\ 0 & \text{if biased} \end{cases}$$

This simple utility function can be thought of as representing the prior probability that an estimator will be unbiasedly estimating a nonzero quantity. Such a utility function might be used in response surface or optimum seeking experiments. This is true because a zero parameter contributes nothing toward being able to change the response by changing the levels of the independent variables.

$$(3) u_i(h) = \begin{cases} \sqrt{n_h} & \text{if unbiased} \\ 0 & \text{if biased} \end{cases}$$

where n_h is the number of treatment combinations at the h^{th} stopping point. This utility function could be useful in the situation where the experimental error is large and the cost of many observations is not much more than the cost of a few observations. The rationale for this is that the variance of an estimator is proportional to the inverse of the number of observations. Thus, one way to weight the value of an unbiased estimate is to weight it proportionately to the inverse of the standard deviation, that is, to $\sqrt{n_h}$.

$$(4) u_i(h) = \begin{cases} \frac{1}{n_h} & \text{if unbiased} \\ 0 & \text{if biased} \end{cases}$$

In the opposite situation to function (3), suppose the experimental error is negligible but the cost of each observation is large so that the cost of a stage is a first-degree function of the number of treatments. Then this utility function would weight the estimators more heavily at the early stages of the experiment and penalize the later stages.

$$(5) u_i(h) = \begin{cases} \frac{1}{\sqrt{n_h}} & \text{if unbiased} \\ 0 & \text{if biased} \end{cases}$$

This is an elementary combination of functions (3) and (4) and is intended for situations where both the costs of the observations are large and the experimental error is large.

Further simple utility functions might consist of multiplicative combinations of (2) with one of (3), (4), or (5).

CONCLUDING REMARKS

The two-level fractional factorial designs represent a class of designs of experiments yielding a large number of estimates of first-degree effects and interactions for a small amount of experimentation. The main disadvantage of this class of designs is that the estimates are always estimates of aliased combinations of parameters. To make conclusions about single parameters it is necessary to have some information about the parameters from a source other than the experiment. If such information is available before the experiment is performed, it may be incorporated into the design of the experiment.

There are many situations in practice in which an experimenter may have varying amounts of information concerning the variables he wishes to investigate. This report has developed some optimal design procedures to be used when the prior information is

- (1) For each parameter the experimenter states his prior probability that it is not zero
- (2) For each parameter the experimenter states what it is worth to him to obtain an unbiased estimate of it

The information and decisions were formulated as a finite decision problem, and Bayes and mini-max procedures were developed.

The main components of the problem were (1) the choice of estimator-parameter matching, (2) the choice of physical-design variable matching, and (3) the evaluations of all possible defining parameter groups and sequences of subgroups.

Modified Bayes and mini-max procedures which are computationally feasible were developed, and some potential utility functions were presented.

The procedures presented herein should prove to be of considerable value in application since the information required is of a nature that is easily specified and the computations required are amenable to being programmed for a digital computer.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, July 21, 1971,
129-04.

APPENDIX A

SYMBOLS

B	complete set of parameters or coefficients
$B(h)$	subgroup of B used at the h^{th} stopping point of experiment
B_1, B_2	subgroups of B
b_A, \dots	estimates of parameters β_A, \dots
b_1, \dots	estimates of parameters β_1, \dots
G	group
G^*	subgroup of G
h	stopping point of experiment
l	stage of experiment
n	number of factors (independent variables)
n_h	number of treatment combinations at h^{th} stopping point
p_b	prior probability of a block effect not being zero
p_{ht}	conditional probability of proceeding to the $h + 1$ stopping point given that the h^{th} stopping point is reached
p_{hs}	probability experiment terminates at exactly h^{th} stopping point
p_i	prior probability that $\beta_i \neq 0$
r	first stage contains 2^{n-r} treatment combinations; it is a $(1/2)^r$ fractional factorial
$S(i, h)$	set of standard-order subscripts of elements of $\beta_i \otimes B(h)$
T	complete set of treatment combinations
$T(h)$	subgroup of T used at h^{th} stopping point of experiment
T_1, T_2	subgroups of T
t_v	v^{th} treatment combination when treatment combinations are arranged in their standard order
U	total expected utility for a given strategy
$U(h)$	total expected utility of h^{th} stopping point for a given strategy
$U(i, b)$	expected utility gained by assigning estimator for $\beta_i \otimes B(h)$ to the block effect that alias set is confounded with

$U(i, k)$	expected utility gained by assigning the estimator for $\beta_i \otimes B(h)$ to β_k
$u_i(h)$	utility assigned to an unbiased estimate of β_i at h^{th} stopping point of experiment
v	standard-order subscript of a parameter or treatment combination for a full factorial experiment
w	there are 2^w treatments in a block
w_i	used to denote abstractly some combination of the letters a, b, c, \dots
X_A, X_B, \dots	independent variables (design)
X_0, X_I	dummy independent variables which are identically equal to 1 for all treatment combinations
X_i	independent variables (physical)
Y	random response (dependent variable)
y	observed value of Y
β_A, β_B, \dots	parameters of a model equation in design variables
β_0, β_I	constant terms of model equation
β_1, β_2, \dots	parameters of a model equation in physical variables
$\beta_i \otimes B(h)$	coset obtained by multiplying all elements of $B(h)$ by β_i
ϵ	element of
δ	random error
\otimes	group operation
(1)	treatment combination with all independent variables at their low levels

APPENDIX B

A HYPOTHETICAL EXPERIMENT WITH FIVE FACTORS

Consider a five-factor experiment involving

X_1 = temperature

X_2 = pressure

X_3 = time

X_4 = velocity

X_5 = angle

Suppose that the experimenter's facilities are such that he can only perform four treatment combinations at one time and be reasonably sure that experimental conditions are homogeneous. Thus his experiment should be designed as a blocked factorial design with blocks of size 4. Assume also that he has enough materials at one time to perform eight treatment combinations but no more and that batches of uniform material are not available in quantities that will supply more than eight treatment combinations. Then the blocks of the experiment might be as shown in the following illustration, where the two columns represent two different test facilities and the four rows represent four different batches of raw material.

		Column blocks (test facilities)	
		1	2
Row blocks (batches)	1	1, 1	1, 2
	2	2, 1	2, 2
	3	3, 1	3, 2
	4	4, 1	4, 2

The differences between the first block and the second block in a row are due to performing the experiment in two different test facilities. The differences between rows are due to possible effects of new batches of materials. Suppose the experimenter feels that there is a probability of 0.50 of there actually being a test facility block effect. Let the probability of there being an effect due to differing batches of raw materials be 1.0. Assume further that probability of an interaction between these block effects is specified as zero. The stopping points of the experiment are after completion of

- (1) Stage one: block (1, 1)
- (2) Stage two: blocks (1, 1), (1, 2)
- (3) Stage three: blocks (1, 1), (1, 2), (2, 1), (2, 2)
- (4) Stage four: the full factorial

Let p_{ht} be the conditional probability of going on to the next stage, given that the h^{th} stage has been completed. Based upon his available resources and upon past histories of some similar projects the experimenter has worked on, the following probabilities are thought to be appropriate:

$$p_{1t} = 0.90$$

$$p_{2t} = 0.80$$

$$p_{3t} = 0.70$$

$$p_{4t} = 0.0$$

Then the probability of stopping at the h^{th} stage is given by $p_{hs} = (1 - p_{ht}) \prod_{j=0}^{h-1} p_{jt}$, where $p_{0t} = 1.0$ and the symbol $\prod_{j=0}^0$ is taken to be 1.0. Then

$$p_{1s} = (1 - 0.90) = 0.10$$

$$p_{2s} = (1 - 0.80)(0.90) = 0.18$$

$$p_{3s} = (1 - 0.70)(0.80)(0.90) = 0.216$$

$$p_{4s} = (1 - 0.0)(0.70)(0.80)(0.90) = 0.504$$

Based on past experience and physical considerations, the experimenter claims the following:

Coefficient of-	Standard-order subscript	Prior probability of being nonzero
X_0	0	1.00
X_1	1	0.80
X_2	2	↓
X_2X_1	3	
X_3	4	
X_3X_1	5	
X_3X_2	6	
$X_3X_2X_1$	7	
X_4	8	
X_4X_1	9	.50
X_4X_3	12	.50
$X_4X_3X_1$	13	.40
X_5	16	1.0
X_5X_1	17	.40
X_5X_3	20	.30

All the other coefficients have zero prior probability. All that remains to be specified is the choice of utility function. Let us assume the investigation is basic research of an exploratory nature so that there are no clearly defined consequences of making inappropriate estimates. In this case, it seems reasonable that the experimenter should simply try to maximize his probability of making correct decisions. The utility function

$$u_i(h) = \begin{cases} 1 & \text{if unbiased estimate} \\ 0 & \text{if biased} \end{cases}$$

will serve the purpose. It is assumed that the purpose of the experiment is to determine the effects of changes in the levels of the independent variable and not necessarily the mean response. Thus β_0 is assumed to have zero utility.

Rather than investigating all the possible nonequivalent d.p.g.'s and their telescoping options, only the following three will be investigated for the best matching of physical to design variables.

(1) Design I:

$$B(1) = \{\beta_I, \beta_{CBA}, \beta_{DCB}, \beta_{DA}, \beta_{EDC}, \beta_{EDBA}, \beta_{EB}, \beta_{ECA}\}$$

$$B(2) = \{\beta_I, \beta_{CBA}, \beta_{EDC}, \beta_{EDBA}\}$$

$$B(3) = \{\beta_I, \beta_{EDBA}\}$$

$$B(4) = \{\beta_I\}$$

Using the rules presented in reference 12, it may be seen that the following assignment of treatment combinations to the blocks of the experiment will have the block confounding as follows:

Block	Treatments
(1, 1)	(1), dca, ecb, edba
(1, 2)	ba, dcb, eca, ed
(2, 1)	db, cba, edc, ea
(2, 2)	da, c, edcba, eb
(3, 1)	a, dc, ecba, edb
(3, 2)	b, dcba, ec, eda
(4, 1)	dba, cb, edca, e
(4, 2)	d, ca, edcb, eba

(a) At stage 2 - The estimator for the alias set $\{\beta_{DA}, \beta_{DCB}, \beta_{ECA}, \beta_{EB}\}$ is confounded with the test facility block effect which has prior probability 0.50.

(b) At stage 3 - $\{\beta_{DA}, \beta_{EB}\}$ is confounded with the test facility block effect. $\{\beta_{CBA}, \beta_{EDC}\}$ is confounded with the raw material block effect which has prior probability 1.0.

(c) At stage 4 - $\{\beta_{DA}\}$ is confounded with the test facility block effect. $\{\beta_{CBA}\}$, $\{\beta_{EDBA}\}$, and $\{\beta_{EDC}\}$ are confounded with the raw material block effect.

(2) Design II:

$$B(1) = \{ \beta_I, \beta_{DCBA}, \beta_{EDCA}, \beta_{EB}, \beta_{EDBA}, \beta_{DA}, \beta_{CB}, \beta_{EC} \}$$

$$B(2) = \{ \beta_I, \beta_{DCBA}, \beta_{EDCA}, \beta_{EB} \}$$

$$B(3) = \{ \beta_I, \beta_{DCBA} \}$$

$$B(4) = \{ \beta_I \}$$

(3) Design III:

$$B(1) = \{ \beta_I, \beta_{BA}, \beta_{CB}, \beta_{CA}, \beta_{EDA}, \beta_{EDB}, \beta_{EDCBA}, \beta_{EDC} \}$$

$$B(2) = \{ \beta_I, \beta_{BA}, \beta_{EDC}, \beta_{EDCBA} \}$$

$$B(3) = \{ \beta_I, \beta_{EDCBA} \}$$

$$B(4) = \{ \beta_I \}$$

Table III shows the matching between the physical parameters and the design parameters and the appropriate probabilities when the matching of independent variables is

$$X_1 = X_D$$

$$X_2 = X_B$$

$$X_3 = X_C$$

$$X_4 = X_E$$

$$X_5 = X_A$$

For the utility function chosen, computation of the expected utility for each parameter-estimator matching need not be performed. The choice that yields the maximum utility will be to assign the estimator to the parameter which has the largest prior

probability of being nonzero. If there is more than one parameter in an alias set with the same largest prior probability, it is immaterial which is chosen since each will result in the same expected utility. The specification that β_0 has zero utility and $P(\beta_0 \neq 0) = 1.0$ also assures that the estimator for the d.p.g. will always have zero expected utility. Thus the computations need not be done for the d.p.g.

Tables IV to VI present the results of the computations for each of the first three stages of the experiment. The d.p.g. for each stage is listed: the cosets, the elements of each of the cosets, and the appropriate probabilities. The element $\beta_{K_{\max}}$ to which the estimator for each coset should be assigned is footnoted. The expected utility for this assignment is given for each coset as computed from equations (8) and (9), and the total expected utility for each stage is then given.

For stage 4, note that there will be one unit of utility for each estimator except for the estimator of β_0 and those estimators which are confounded with blocks. The estimator for β_{DA} is confounded with a block effect with probability 0.50. Hence, the utility of that estimator is 0.50. The estimators for β_{CBA} , β_{EDBA} , and β_{EDC} are confounded with a block effect with probability 1.0; and, hence, they have utility zero. The estimator for β_0 has zero utility. The remaining 27 estimators all have utility of 1.0. Thus

$$U(4) = 27.00 + 0.50 = 27.5$$

The total expected utility according to equation (10) is then

$$\begin{aligned} \sum_{h=1}^4 U(h)p_{hs} &= U(1)(0.10) + U(2)(0.18) + U(3)(0.216) + U(4)(0.504) \\ &= (0.148)(0.10) + (3.40)(0.18) + (13.5)(0.216) + (27.5)(0.504) \\ &= 17.4028 \end{aligned}$$

The calculations just illustrated are quite straightforward but tedious. Furthermore, the three designs proposed are not equivalent under any permutation of the letters since they have different numbers of parameters with the same length subscripts. Thus, just to evaluate these three designs would involve $3 \times 5! = 360$ such computations. The program NAMER in reference 9 was developed to perform the permutations and to evaluate them for a given choice of d.p.g. and telescoping sequence. The three designs given previously were run on NAMER, with the results summarized in table VII. For each of the three designs under consideration is presented a table which shows the

physical-design variable matchings which optimize the total expected utility over all stages and the matchings which optimize the expected utilities of each of the stages.

The modified Bayes solution with respect to these three designs would then be to use the matching shown in the column headed "Total" for either design 2 or design 3. The reason is that the total expected utility for either of these is 17.4298, which is greater than the maximized total expected utility for design 1.

A modified mini-max solution may also be found. In table VII the minimum in each column corresponding to each stopping point is footnoted. The maximum among these is seen to be 0.510 and occurs under the matchings for stages 1, 3, and 4 of design 3 (all of which are the same for this particular example). Thus the matching for stage 1 of design 3 may be used as a security strategy.

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TABLE I. - DESIGN MATRIX FOR A 2^3 FULL FACTORIAL EXPERIMENT

Treatment		Observed response	Values of independent variables							
			X_I	X_A	X_B	$X_B X_A$	X_C	$X_C X_A$	$X_C X_B$	$X_C X_B X_A$
t_0	(1)	y_1	+1	-1	-1	+1	-1	+1	+1	-1
t_1	a	y_2	↓	+1	-1	-1	↓	-1	+1	+1
t_2	b	y_3	↓	-1	+1	-1	↓	+1	-1	+1
t_3	ba	y_4	↓	+1	+1	+1	↓	-1	↓	-1
t_4	c	y_5	↓	-1	-1	+1	+1	-1	↓	+1
t_5	ca	y_6	↓	+1	-1	-1	↓	+1	↓	-1
t_6	cb	y_7	↓	-1	+1	-1	↓	-1	+1	-1
t_7	cba	y_8	↓	+1	+1	+1	↓	+1	+1	+1

TABLE II. - DESIGN MATRIX FOR A 2^3 FACTORIAL RUN IN TWO BLOCKS

[Each block is a one-half replicate of the full factorial.]

Treatment		Observed response	Values of independent variables							
			X_I	X_A	X_B	$X_B X_A$	X_C	$X_C X_A$	$X_C X_B$	$X_C X_B X_A$
Block 1										
t_0	(1)	y_1	+1	-1	-1	+1	-1	+1	+1	-1
t_3	ba	y_2	↓	+1	+1	+1	-1	-1	-1	↓
t_5	ca	y_3	↓	+1	-1	-1	+1	+1	-1	↓
t_6	cb	y_4	↓	-1	+1	-1	+1	-1	+1	↓
Block 2										
t_1	a	y_5	+1	+1	-1	-1	-1	-1	+1	+1
t_2	b	y_6	↓	-1	+1	-1	-1	+1	-1	↓
t_4	c	y_7	↓	-1	-1	+1	+1	-1	-1	↓
t_7	cba	y_8	↓	+1	+1	+1	+1	+1	+1	↓

TABLE III. - MATCHING BETWEEN PHYSICAL PARAMETERS AND DESIGN

PARAMETERS AND APPROPRIATE PRIOR PROBABILITIES WHEN

MATCHING OF INDEPENDENT VARIABLES IS $X_1 = X_D$,

$X_2 = X_B$, $X_3 = X_C$, $X_4 = X_E$, $X_5 = X_A$

Physical-variable parameters	Design-variable parameters	Parameter prior probability	Physical-variable parameters	Design-variable parameters	Parameter prior probability	
β_0	β_I	1.00	β_{16}	β_A	1.00	
β_1	β_D	.80	β_{17}	β_{DA}	.40	
β_2	β_B	↓	β_{18}	β_{BA}	0	
β_3	β_{DB}		β_{19}	β_{DBA}	0	
β_4	β_C		β_{20}	β_{CA}	.30	
β_5	β_{DC}		β_{21}	β_{DCA}	0	
β_6	β_{CB}		β_{22}	β_{CBA}	↓	
β_7	β_{DCB}		β_{23}	β_{DCBA}		
β_8	β_E		1.00	β_{24}		β_{EA}
β_9	β_{ED}		.50	β_{25}		β_{EDA}
β_{10}	β_{EB}		0	β_{26}		β_{EBA}
β_{11}	β_{EDB}		0	β_{27}		β_{EDBA}
β_{12}	β_{EC}	.50	β_{28}	β_{ECA}		
β_{13}	β_{EDC}	.40	β_{29}	β_{EDCA}		
β_{14}	β_{ECB}	0	β_{30}	β_{ECBA}		
β_{15}	β_{EDCB}	0	β_{31}	β_{EDCBA}		

TABLE IV. - EVALUATION AT STAGE 1

$$[B(1) = \{\beta_1, \beta_{CBA}, \beta_{DCB}, \beta_{DA}, \beta_{EDC}, \beta_{EDBA}, \beta_{EB}, \beta_{ECA}\} = \{\beta_0, \beta_{22}, \beta_7, \beta_{17}, \beta_{13}, \beta_{27}, \beta_{10}, \beta_{28}\}.]$$

Coset		Elements and probabilities								$\prod(1 - p_j)$	$U(i, k_{\max})$
$\beta_A \otimes B(1) = \beta_{16} \otimes B(1)$	Element	$\beta_A = \beta_{16}$	$\beta_{CB} = \beta_6$	$\beta_{DCBA} = \beta_{23}$	$\beta_D = \beta_1$	$\beta_{EDCA} = \beta_{29}$	$\beta_{EDB} = \beta_{11}$	$\beta_{EBA} = \beta_{26}$	$\beta_{EC} = \beta_{12}$		
	p_j	1.0	0.80	0.00	0.80	0.00	0.00	0.00	0.50		
	$1 - p_j$	0.0	0.20	1.00	0.20	1.00	1.00	1.00	0.50	0.020	0.020
	(a)										
$\beta_B \otimes B(1) = \beta_2 \otimes B(1)$	Element	$\beta_B = \beta_2$	$\beta_{CA} = \beta_{20}$	$\beta_{DC} = \beta_5$	$\beta_{DBA} = \beta_{19}$	$\beta_{EDCB} = \beta_{15}$	$\beta_{EDA} = \beta_{25}$	$\beta_E = \beta_8$	$\beta_{ECBA} = \beta_{30}$		
	p_j	0.80	0.30	0.80	0.00	0.00	0.00	1.00	0.00		
	$1 - p_j$	0.20	0.70	0.20	1.00	1.00	1.00	0.00	1.00	0.028	0.028
	(a)										
$\beta_{BA} \otimes B(1) = \beta_{18} \otimes B(1)$	Element	$\beta_{BA} = \beta_{18}$	$\beta_C = \beta_4$	$\beta_{DCA} = \beta_{21}$	$\beta_{DB} = \beta_3$	$\beta_{EDCBA} = \beta_{31}$	$\beta_{ED} = \beta_9$	$\beta_{EA} = \beta_{24}$	$\beta_{ECB} = \beta_{14}$		
	p_j	0.00	0.80	0.00	0.80	0.00	0.50	0.00	0.00		
	$1 - p_j$	1.00	0.20	1.00	0.20	1.00	0.50	1.00	1.00	0.100	0.100
	(a)										
$U(1) = \sum U(i, k_{\max}) = 0.148$											

^aDenotes the element $\beta_{K_{\max}}$ to which estimator for each coset should be assigned.

TABLE V. - EVALUATION AT STAGE 2

$$[B(2) = \{\beta_I, \beta_{CBA}, \beta_{EDC}, \beta_{EDBA}\} = \{\beta_0, \beta_{22}, \beta_{13}, \beta_{27}\}.$$

Coset		Elements and probabilities				$\prod(1 - p_j)$	$(1 - p_b)\prod(1 - p_j)$	$U(i, k_{\max})$
$\beta_A \otimes B(2) =$ $\beta_{16} \otimes B(2)$	Element	$\beta_A = \beta_{16}$	$\beta_{CB} = \beta_6$	$\beta_{EDCA} = \beta_{29}$	$\beta_{EDB} = \beta_{11}$			
	p_j	1.00	0.80	0.00	0.00			
	$1 - p_j$	0.00	0.20	1.00	1.00	0.20	-----	0.20
	(a)							
$\beta_B \otimes B(2) =$ $\beta_2 \otimes B(2)$	Element	$\beta_B = \beta_2$	$\beta_{CA} = \beta_{20}$	$\beta_{EDCB} = \beta_{15}$	$\beta_{EDA} = \beta_{25}$			
	p_j	0.80	0.30	0.00	0.00			
	$1 - p_j$	0.20	0.70	1.00	1.00	0.70	-----	0.70
	(a)							
$\beta_{BA} \otimes B(2) =$ $\beta_{18} \otimes B(2)$	Element	$\beta_{BA} = \beta_{18}$	$\beta_C = \beta_4$	$\beta_{EDCBA} = \beta_{31}$	$\beta_{ED} = \beta_9$			
	p_j	0.00	0.80	0.00	0.50			
	$1 - p_j$	1.00	0.20	1.00	0.50	0.50	-----	0.50
	(a)							
$\beta_D \otimes B(2) =$ $\beta_1 \otimes B(2)$	Element	$\beta_D = \beta_1$	$\beta_{DCBA} = \beta_{23}$	$\beta_{EC} = \beta_{12}$	$\beta_{EBA} = \beta_{26}$			
	p_j	0.80	0.00	0.50	0.00			
	$1 - p_j$	0.20	1.00	0.50	1.00	0.50	-----	0.50
	(a)							
$\beta_{DA} \otimes B(2) =$ $\beta_{17} \otimes B(2)$	Element	$\beta_{DA} = \beta_{17}$	$\beta_{DCB} = \beta_7$	$\beta_{ECA} = \beta_{28}$	$\beta_{EB} = \beta_{10}$			
	p_j	0.40	0.80	0.00	0.00			
	$1 - p_j$	0.60	0.20	1.00	1.00	0.60	$(0.50)(0.60)=0.30$	0.30
	(a)							
$\beta_{DB} \otimes B(2) =$ $\beta_3 \otimes B(2)$	Element	$\beta_{DB} = \beta_3$	$\beta_{DCA} = \beta_{21}$	$\beta_{ECB} = \beta_{14}$	$\beta_{EA} = \beta_{24}$			
	p_j	0.80	0.00	0.00	0.00			
	$1 - p_j$	0.20	1.00	1.00	1.00	1.00	-----	1.00
	(a)							
$\beta_{DEA} \otimes B(2) =$ $\beta_{19} \otimes B(2)$	Element	$\beta_{DEA} = \beta_{19}$	$\beta_{DC} = \beta_5$	$\beta_{ECBA} = \beta_{30}$	$\beta_E = \beta_8$			
	p_j	0.00	0.80	0.00	1.00			
	$1 - p_j$	1.00	0.20	1.00	0.00	0.20	-----	0.20
	(a)							

$U(2) = 3.40$

^aDenotes the element $\beta_{K_{\max}}$ to which the estimator for each coset should be assigned.

TABLE VI. - EVALUATION AT STAGE 3

$$[B(3) = \{\beta_I, \beta_{EDBA}\} = \{\beta_0, \beta_{27}\}]$$

Coset	Elements and probabilities						$\pi(1 - p_j)$	$(1 - p_b)\pi(1 - p_j)$	$U(i, k_{\max})$
	Element	p_j	$1 - p_j$	Element	p_j	$1 - p_j$			
$\beta_A \otimes B(3) =$	$\beta_A = \beta_{16}$ (a)	1.00	0.00	$\beta_{EDB} = \beta_{11}$	0.00	1.00	1.00	-----	1.00
$\beta_B \otimes B(3) =$	$\beta_B = \beta_2$ (a)	0.80	0.20	$\beta_{EDA} = \beta_{25}$	0.00	1.00	1.00	-----	1.00
$\beta_{BA} \otimes B(3) =$	$\beta_{BA} = \beta_{18}$	0.00	1.00	$\beta_{ED} = \beta_9$ (a)	0.50	0.50	1.00	-----	1.00
$\beta_C \otimes B(3) =$	$\beta_C = \beta_4$ (a)	0.80	0.20	$\beta_{EDCBA} = \beta_{31}$	0.00	1.00	1.00	-----	1.00
$\beta_{CA} \otimes B(3) =$	$\beta_{CA} = \beta_{20}$ (a)	0.30	0.70	$\beta_{EDCB} = \beta_{15}$	0.00	1.00	1.00	-----	1.00
$\beta_{CB} \otimes B(3) =$	$\beta_{CB} = \beta_6$ (a)	0.80	0.20	$\beta_{EDCA} = \beta_{29}$	0.00	1.00	1.00	-----	1.00
$\beta_{CBA} \otimes B(3) =$	$\beta_{CBA} = \beta_{22}$	0.00	1.00	$\beta_{EDC} = \beta_{13}$ (a)	0.40	0.60	1.00	(0.0)(1.00)	0.00
$\beta_D \otimes B(3) =$	$\beta_D = \beta_1$ (a)	0.80	0.20	$\beta_{EBA} = \beta_{26}$	0.00	1.00	1.00	-----	1.00
$\beta_{DA} \otimes B(3) =$	$\beta_{DA} = \beta_{17}$ (a)	0.40	0.60	$\beta_{EB} = \beta_{10}$	0.00	1.00	1.00	(0.50)(1.00)	0.50
$\beta_{DB} \otimes B(3) =$	$\beta_{DB} = \beta_3$ (a)	0.80	0.20	$\beta_{EA} = \beta_{24}$	0.00	1.00	1.00	-----	1.00
$\beta_{DBA} \otimes B(3) =$	$\beta_{DBA} = \beta_{19}$	0.00	1.00	$\beta_E = \beta_8$ (a)	1.00	0.00	1.00	-----	1.00
$\beta_{DC} \otimes B(3) =$	$\beta_{DC} = \beta_5$ (a)	0.80	0.20	$\beta_{ECBA} = \beta_{30}$	0.00	1.00	1.00	-----	1.00
$\beta_{DCA} \otimes B(3) =$	$\beta_{DCA} = \beta_{21}$ (a)	0.00	1.00	$\beta_{ECB} = \beta_{14}$	0.00	1.00	1.00	-----	1.00
$\beta_{DCB} \otimes B(3) =$	$\beta_{DCB} = \beta_7$ (a)	0.80	0.20	$\beta_{ECA} = \beta_{28}$	0.00	1.00	1.00	-----	1.00
$\beta_{DCBA} \otimes B(3) =$	$\beta_{DCBA} = \beta_{23}$	0.00	1.00	$\beta_{EC} = \beta_{12}$ (a)	0.50	0.50	1.00	-----	1.00

$U(3) = 13.5$

^aDenotes the element $\beta_{K_{\max}}$ to which estimator for each coset should be assigned.

TABLE VII. - OPTIMAL MATCHINGS

(a) Design 1

Physical variable	Total	Stage 1	Stage 2	Stage 3	Stage 4
Physical-design variable matchings which optimize the expected utilities					
Temperature	D	A	D	A	A
Pressure	B	C	A	B	B
Time	C	D	B	C	C
Velocity	E	B	C	D	D
Angle	A	E	E	E	E
Stage	Expected utilities of matchings				
Total	17.4028	17.1851	17.2062	17.310	17.310
1	.148	^a .221	^a .144	^a .12	^a .12
2	3.400	.347	3.93	2.90	2.90
3	13.50	12.40	12.15	13.50	13.50
4	27.50	27.50	27.50	27.50	27.50

(b) Design 2

Physical variable	Total	Stage 1	Stage 2	Stage 3	Stage 4
Physical-design variable matchings which optimize the expected utilities					
Temperature	C	A	C	C	A
Pressure	D	C	A	D	B
Time	E	D	D	E	C
Velocity	B	B	B	B	D
Angle	A	E	E	A	E
Stage	Expected utilities of matchings				
Total	17.4298	16.9979	17.0578	17.4298	16.9963
1	.148	^a .221	^a .19	^a .148	^a .124
2	3.55	3.75	4.10	3.55	3.795
3	13.50	11.30	11.30	13.50	11.30
4	27.50	27.50	27.50	27.50	27.50

^aMinimum in each column corresponding to a single stage.

TABLE VII. - Concluded. OPTIMAL MATCHINGS

(c) Design 3

Physical variable	Total	Stage 1	Stage 2	Stage 3	Stage 4
Physical-design variable matchings which optimize the expected utilities					
Temperature	C	A	C	A	A
Pressure	D	B	D	B	B
Time	A	C	A	C	C
Velocity	B	D	B	D	D
Angle	E	E	E	E	E
Stage	Expected utilities of matchings				
Total	17.4298	17.4210	17.4298	17.4210	17.4210
1	.148	^a .510	^a .148	^a .510	^a .510
2	3.55	3.30	3.55	3.30	3.30
3	13.50	13.50	13.50	13.50	13.50
4	27.50	27.50	27.50	27.50	27.50

^aMinimum in each column corresponding to a single stage.

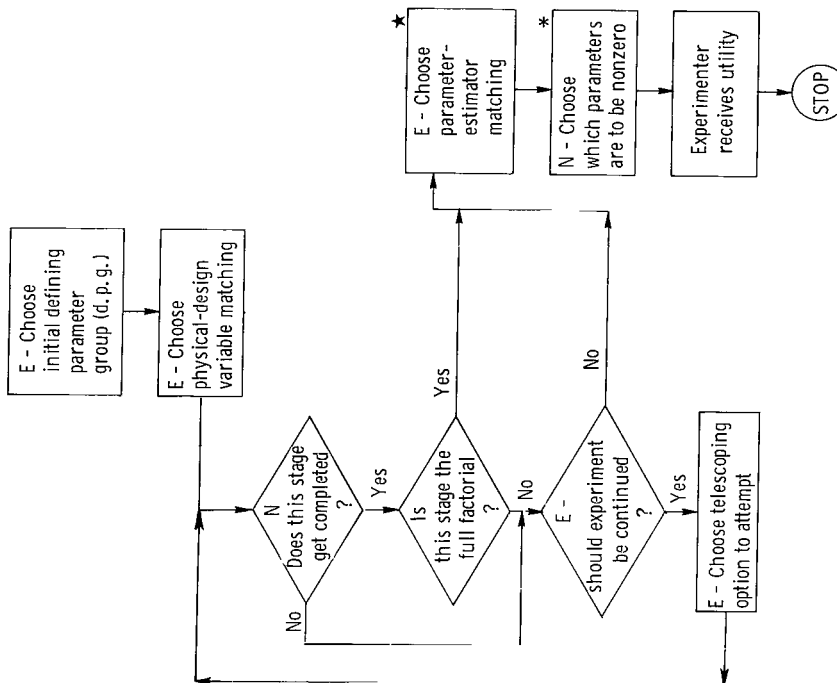


Figure 1. - Flow chart representing sequence of choices made by experimenter and nature. (E denotes a choice made by experimenter; N denotes a choice made by nature.)



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