A Two-Fluid Solar Wind Model with Anisotropic Proton Temperature

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This work was supported by the Norwegian Research Council for Science and the Humanities (E. Leer) and the National Aeronautics and Space Administration under Contract #NGR-05-009-081.

September 1971

ABSTRACT

A two-fluid model of the solar wind with anisotropic proton temperature and allowing for extended coronal proton-heating is considered for the case of a purely radial and of a spiral magnetic field. Proton-proton Coulomb-collisions together with a spiral interplanetary magnetic field are found to be sufficient to reduce the thermal anisotropy in the proton gas to a value in agreement with observations. Reasonable values are obtained for the flow-velocity, number density and the proton-temperature near the orbit of the Earth.

1. Introduction

The flow of plasma from the Sun was first analyzed by Parker (1958, 1963), using a hydrodynamic model in which the fluid has rather simple properties. This model accounts for many of the main features of observations of the solar wind made near the Earth (see reviews by Parker, 1965, 1967; Axford, 1968; and Hundhausen, 1968, 1970). However, since it is assumed in this model that the electron and proton temperatures are equal, and that the pressure is isotropic, it cannot account for the observed difference between the electron and proton temperatures nor for the anisotropy of the proton temperature. Sturrock and Hartle (1966), (see also Hartle and Sturrock, 1968), have considered a two-fluid model, in which the electron and proton temperatures are allowed to be different, but both are assumed to be isotropic. They have shown that transfer of energy between the electrons and protons by Coulomb collisions is less important than is implicitly assumed in the one-fluid model, and since heat conduction in the proton gas is negligible they obtained a relatively low proton temperature at the Earth's orbit. However the model is not completely satisfactory in that the effects of the interplanetary magnetic field are not taken into account, and the proton temperature is assumed to be isotropic. Furthermore, the model yields bulk velocities and proton temperatures that are too low, and number densities that are too high, when compared to observations made in the vicinity of the Earth. The differences between observations and the predictions of this simple two-fluid model suggest that the protons and other ions in the solar wind must be heated by some other means than Coulomb collisions with electrons.

In this paper we extend the Hartle /Sturrock analysis for a steady state, two-fluid solar wind by allowing for the presence of a heat source for the proton gas, taking into account some of the effects of the interplanetary magnetic field, and permitting the proton temperature to be anisotropic. It is assumed that the flow is radial everywhere, and we have considered both the radial and the spiral interplanetary magnetic field. We have neglected the effects of viscosity, since it is pointed out by Parker (1965), these are not important as far as the radial expansion of the solar wind is concerned, although there may be some viscous heating of the proton gas (e.g. Wolff et al., 1971). Viscosity and the magnetic field play an important role close to the Sun as far as the azimuthal component of velocity is concerned (e.g. Weber and Davis, 1970; Weber, 1970), however these effects are neglected in this analysis which is concerned more with the properties of the solar wind in the vicinity of the orbit of Earth and at greater distances from the Sun.

We have assumed a convenient analytic form for the heating function, and justify this on the grounds that the results are not sensitive to the precise form of the heating function, and there is no clear observational evidence in favor of any particular form. Heating out to some 20 solar radii was postulated by Parker (1963) in order to obtain reasonable velocities and number densities on the basis of a one-fluid model. Analyses of the one-fluid model with heat addition have been carried out by Konyukov (1967) and by Holzer and Axford (1970). A two-fluid model with proton heating, assuming the proton temperature to be isotropic and neglecting the effects of the interplanetary magnetic field has been considered by Hartle and Barnes (1970) and Barnes et al., (1971). In all of these analyses one finds, as should be expected (see Saunders, 1953), that heating in the subsonic region tends to increase the Mach number by increasing the bulk velocity, whereas heating in the supersonic region tends to decrease the Mach number by increasing the temperature. Furthermore, a heat source can make a transition from subsonic to supersonic flow possible in cases where such a transition would otherwise not occur.

It is usually assumed that the solar corona is heated as a result of the dissipation of waves which propagate upwards from the photosphere (Biermann, 1946, 1948; Alfvén, 1947; Osterbrock, 1961; Whitaker, 1963; Lighthill, 1967; Kuperus, 1965, 1969). If the wave energy is transferred entirely to the electron gas (e.g. D'Angelo, 1968, 1969) it is necessary to invoke some instability mechanism in order to transfer the energy from electrons to protons more effectively than Coulomb collisions will permit (e.g. Kennel and Scarf, 1968; Forslund, 1970; Hollweg and Volk, 1970; Scarf, 1970; Toichi, 1971). However, if the waves transmit their energy directly to the ions (e.g. Barnes, 1968, 1969) no such instability mechanism is required. We have assumed that the latter process occurs in our calculations, although we note that there is no clear evidence at present that the wave energy is not coupled to the ions via the electrons (cf. Hundhausen, 1970). Heating of the solar wind by shocks and instabilities when high velocity gas overtakes low velocity gas may also occur (Coleman, 1968; Jokipii and Davis, 1969), but this "positive velocity gradient heating" does not seem to be a dominant heat source within the orbit of Earth (Burlaga and Ogilvie, 1970). We find that with isotropic heat source near the Sun it is possible to obtain reasonable values for the number density, flow velocity and proton temperatures near the Earth. Our values for the electron temperature are too high by a factor of the order 2 in comparison with observations near the Earth. This could be corrected if, for example, we were to take into account the effects of the heat conduction instability discussed by Forslund (1970). Otherwise there appears to be no need to appeal to special mechanisms to explain any of the average properties of the solar wind observed in the vicinity of the Earth.

2. Governing equations

The equations of motion representing a steady, spherically-symmetric, radial electron proton solar wind with anisotropic proton temperatures and allowing for heat conduction are as follows (see Holzer and Axford, 1970):

$$\frac{d}{dr}(n v r^2) = 0 \quad , \tag{1}$$

$$\rho v \frac{dv}{dr} + \frac{d}{dr}\left[nk\left(T_p^{\parallel} + T_e\right)\right] + \frac{2}{r} nk\left(T_p^{\parallel} - T_p^{\perp}\right) + \rho \frac{GM_o}{r^2} = 0 \quad , \tag{2}$$

$$\frac{dT_e}{dr} = \frac{2}{3}\frac{T_e}{n}\frac{dn}{dr} + \frac{2}{3}\frac{1}{Fk}\frac{d}{dr}\left(\kappa_e' r^2 \frac{dT_e}{dr}\right) - \frac{\nu_{pe}}{v}\left(T_e - T_p\right) + \frac{2}{3}\frac{Q_e}{nvk} \quad , \tag{3}$$

$$\frac{dT_p^{\parallel}}{dr} = -\frac{2T_p^{\parallel}}{v}\frac{dv}{dr} + \frac{2}{Fk}\frac{d}{dr}\left(\kappa_p r^2 \frac{dT_p^{\parallel}}{dr}\right)$$

$$+ \frac{\nu_{pe}}{v}\left(T_e - T_p^{\parallel}\right) - \frac{2\nu_{pp}}{v}\left(T_p^{\parallel} - T_p^{\perp}\right) + \frac{2Q_p^{\parallel}}{nvk} \quad , \tag{4}$$

and

$$\frac{dT_p^{\perp}}{dr} = -2\frac{T_p^{\perp}}{r} + \frac{\nu_{pe}}{v}\left(T_e - T_p^{\perp}\right) + \frac{\nu_{pp}}{v}\left(T_p^{\parallel} - T_p^{\perp}\right) + \frac{2Q_p^{\perp}}{nvk} \quad , \tag{5}$$

It is assumed that the plasma is quasineutral [i.e. $n_e \approx n_p = n$], and thus, since the divergence of electric current is zero in a steady state, $v_e \approx v_p = v$. $F = nvr^2$ is the flux per steradian, k is the Boltzmann constant, and G is the gravitational constant. M_o is the mass of the Sun, and ρ is the mass density: $\rho = (n_e m_e + n_p m_p) \approx n m_p$.

T_e is the electron temperature, and $T_p^{\perp}$ and $T_p^{\parallel}$ are the proton temperatures perpendicular and parallel to the magnetic field respectively; the mean proton temperature is defined as $T_p = \frac{1}{3}(T_p^{\parallel} + 2T_p^{\perp})$. Q_e and $Q_p = Q_p^{\parallel} + 2Q_p^{\perp}$ are heat sources for the electrons and the protons. (There may also be a corresponding transfer of momentum to the particles associated with the radiation pressure of the waves concerned, but this has been neglected in the present work.)

Expressions for the collision frequencies (ν_{ee}, ν_{ep}, ν_{pp}) and thermal conductivities parallel to the magnetic field (κ_e, κ_p) have been given by Braginskii (1965):

$$\nu_{ee} = \tau_e^{-1} = \frac{4}{3}\left(\frac{2\pi}{m_e}\right)^{\frac{1}{2}} \frac{\lambda e^4 n}{(kT_e)^{3/2}} ,$$

$$\nu_{ee} : \nu_{pp} : \nu_{ep} = 1 : \left(\frac{m_e}{2m_p}\right)^{\frac{1}{2}} \left(\frac{T_e}{T_p}\right)^{3/2} : \frac{2m_e}{m_p} ,$$

$$\kappa_e = 3.16\, k^2\, T_e\, \tau_e\, n/m_e ,$$

and

$$\kappa_p = \left(\frac{m_e}{m_p}\right)^{\frac{1}{2}} \left(\frac{T_p}{T_e}\right)^{5/2} \kappa_e .$$

τ_e is the relaxation time for the electron gas. e is the charge and m is the mass of an electron. In conditions typical of a quiet solar wind, the Coulomb logarithm λ can be taken as $\lambda = 25 \pm 1$. As a result of the electrostatic field in the solar wind the thermal conductivity of the electron gas is reduced to $\kappa_e' = C\kappa_e$, where $C \approx 0.42$ (Spitzer and Härm, 1953). The skewed, anisotropic electron distribution can produce instabilities which result in an increase in the effect of collision frequency of electrons which in turn suppresses heat conduction and changes the form of κ_e (cf. Forslund, 1970).

The thermal conductivity of the proton gas is much smaller than that of the electron gas. In the case with no extended heat source, the heat conduction term in Equation (4) produces a minor correction to the temperature profile near the Sun (Hartle and Sturrock, 1968). With an extended heat source both the proton temperature and hence the proton thermal conductivity increase. However the effects of heat conduction are small compared with those of the heat source itself close to the Sun, and at great distances from the Sun the effect of heat conduction on $T_p^{\parallel}$ is small compared with those of proton-proton collisions. Skewed, and anisotropic proton distributions have been observed (e.g. Hundhausen et al., 1967; Hundhausen, 1968, 1970); from the third moment of such distributions one can calculate the effects of heat conduction and viscosity in the proton gas, but they clearly cannot have any significant effects on the flow.

3. The case of a radial interplanetary magnetic field.

For an electron gas dominated by heat conduction Equation (3) reduces to

$$r^2 \, T_e^{5/2} \, \frac{dT_e}{dr} = \text{constant.}$$

By assuming that the heat conduction dominated region extends to infinity, where $T_e = 0$, we obtain (cf. Chapman, 1957)

$$T_e = T_e^o \left(\frac{r_o}{r}\right)^{2/7} \quad . \tag{6}$$

Combining Equations (1), (2) and (4) (neglecting heat conduction for the protons) we obtain

$$\frac{dv}{dr}\left[m_p v - \frac{k}{v}\left(3T_p^{\parallel} + T_e\right)\right] = \frac{2k}{r}\left(T_e + T_p^{\perp}\right)$$

$$- k\,\frac{dT_e}{dr} - \frac{\nu_{pe}}{v}\,k\left(T_e - T_p^{\parallel}\right) + \frac{2\nu_{pp}}{v}\,k\left(T_p^{\parallel} - T_p^{\perp}\right)$$

$$- \frac{2Q_p^{\parallel}}{n v} - \frac{m_p G M_o}{r^2} \quad , \tag{7}$$

This equation has a singular point in the (v, r) plane at which the flow is exactly sonic.

We have solved Equations (1), (4), (5), (6) and (7) simultaneously for a given flux F, and given values of the temperatures at $R = r/R_o = 2$; (R_o is the radius of the Sun). The solution has been chosen so that there is a smooth transition at the critical point. The Adams-Moulton method was used to integrate the equations (Hildebrand, 1956). Results for a case with isotropic heating of the protons (i.e. $Q_p^{\parallel} = Q_p^{\perp}$) are shown in Figures (1a) and (1b). For R < 2 we have assumed a collision-dominated proton gas. This is well justified, as is evident from Figure (1c) which shows the collision frequencies (ν_{ee}, ν_{pp}) together with the expansion rate, $\nu_{exp} = -\frac{v}{n}\frac{dn}{dr}$.

4. Spiral magnetic field.

In the case of a spiral interplanetary magnetic field the equations of motion become rather complicated. Since we are mainly concerned with the behavior of the electron and proton temperatures, we make the following approximations: v = constant and radial, $B_r = B_o (r_o/r)^2$, $B_\varphi = \left(\frac{\Omega r}{v} \sin\theta\right) B_r$ and $B_\theta = 0$. Here $\Omega = 2 \times 10^{-6} \sec^{-1}$ is the angular velocity of the Sun, and θ is the helio-latitude. The magnitude of the magnetic field is

$$B = \left(B_r^2 + B_\varphi^2\right)^{\frac{1}{2}} = B_o\left(\frac{r_o}{r}\right)^2 \left(1 + \left(\frac{r\Omega}{v}\sin\theta\right)^2\right)^{\frac{1}{2}} \quad . \tag{8}$$

The $\underline{j} \times \underline{B}$ force associated with the spiral magnetic field is very small, and consequently our assumption that the flow is radial remains valid. Close to the Sun the magnetic field plays a very important role in determining the behavior of the solar wind (e.g. Weber and Davis, 1967, 1970; Weber, 1970; and Grzedzielskii, 1970). However we will ignore the $\underline{j} \times \underline{B}$ force entirely, and hence it should be noted that our results are valid only for R >> 20.

For an arbitrary magnetic field direction, Equations (3), (4) and (5) must be amended as follows (Nishida, 1969):

$$\frac{dT_e}{dr} = \frac{2}{3}\frac{T_e}{n}\frac{dn}{dr} - \frac{\nu_{pe}}{v}\left(T_e - T_p\right) + \frac{2}{3}\frac{1}{kF}\frac{d}{dr}\left[\kappa_e' r^2 \cos^2\psi \frac{dT_e}{dr}\right] \quad , \tag{9}$$

$$\frac{dT_p^{\parallel}}{dr} = \frac{T_p^{\parallel}}{n^2} B^2 \frac{d}{dr}\left(\frac{n^2}{B^2}\right) + \frac{\nu_{pe}}{v}\left(T_e - T_p^{\parallel}\right) - \frac{2\nu_{pp}}{v}\left(T_p^{\parallel} - T_p^{\perp}\right) + \frac{2Q_p^{\parallel}}{nvk} \quad , \tag{10}$$

$$\frac{dT_p^{\perp}}{dr} = \frac{T_p^{\perp}}{B}\frac{dB}{dr} + \frac{\nu_{pe}}{v}\left(T_e - T_p^{\perp}\right) + \frac{\nu_{pp}}{v}\left(T_p^{\parallel} - T_p^{\perp}\right) + \frac{2Q_p^{\perp}}{nvk} \quad . \tag{11}$$

As before we neglect the heat source for the electrons and heat conduction in the proton gas. Since there is a temperature gradient perpendicular to the magnetic field there should be an additional term in Equation (9); however, the thermal conductivity perpendicular to the magnetic field is very small (Braginskii, 1965) and the term is accordingly neglected.

The electron-proton collision term is negligible in this situation, hence the equation for the electron temperature can be integrated independently of the others. In the heat conduction dominated region we have

$$T_e^{7/2} = T_e^{o\,7/2} + \frac{7}{2}C_1\left[\frac{1}{r_o} - \frac{1}{r} + \left(\frac{\Omega}{v}\sin\theta\right)^2 (r - r_o)\right] \quad . \tag{12}$$

Equation (12) shows that the electron temperature can be dominated by heat conduction everywhere only for $\theta = 0$ (i.e. in the polar region). $T_e(r)$ is determined by assigning the temperatures at two different points, or alternatively the temperature at one point and the gradient at the same or another point. The temperature gradient near the Sun is determined by processes occurring in the lower corona where heat conduction into the chromosphere is very important (Kuperus and Athay, 1967; Kopp and Kuperus, 1968; Kuperus, 1969). In order to proceed without becoming involved in the complexities of the chromosphere-corona interface, we will assume that the coronal temperature distribution is spherically symmetric near the Sun, and that the temperature gradient is given by Equation (6) for all heliolatitudes (cf. Hartle and Sturrock, 1968). Thus, $\left(\frac{dT_e}{dr}\right)_{r = r_o} = -\frac{2}{7} T_e^o / r_o$.

We will ignore the effect of the spiral magnetic field in a region $r < r_1$, and hence

$$T_e^{7/2} = T_e^{o\,7/2} \left(\frac{r_o}{r}\right), \text{ for } r < r_1 \quad , \tag{13}$$

by requiring that T_e and $\frac{dT_e}{dr}$ be continuous at $r = r_1$, we obtain from Equation (12)

$$T_e^{7/2} = T_e^{o\,7/2} \frac{1}{1 + \left(\frac{r_1 \Omega}{v} \sin\theta\right)^2} \left\{ \frac{r_o}{r} - \left(\frac{r_o \Omega}{v} \sin\theta\right)^2 \left(\frac{r}{r_o} - 2\frac{r_1}{r_o}\right) \right\} \quad ,$$

$$\text{for } r \geq r_1 \quad . \tag{14}$$

Beyond the heat conduction dominated region the gas expands adiabatically, and to find the electron temperature distribution we must integrate Equation (8) inwards from $r = r_2$, where $T_e = T_2$ and $\frac{dT_e}{dr} = -\frac{4}{3} \frac{T_2}{r_2}$. The solutions are then matched to solutions of Equation (14) at a convenient intermediate point. By assuming that the magnetic field is radial in $R < 100$, and allowing for a spiral magnetic field in $R > 100$, we obtain the electron and proton temperature distributions in the ecliptic plane shown in Figure (2a). The proton temperatures are obtained by integrating Equations (10) and (11) outwards from $R = 100$, with initial values taken from Figures (1a) and (1b). The electron temperature is obtained by matching solutions in the adiabatic region ($R \gtrsim 300$) to solutions in the heat conduction dominated region ($R \lesssim 300$). The corresponding expansion rate and collision frequencies are shown in Figure (2b).

Similar calculations can be carried out for different heliolatitudes. As we neglect the coupling between the electron and the proton temperatures, the integration of the equations for $T_p^{\parallel}$ and $T_p^{\perp}$ is quite straightforward. Figure (3a) shows the proton temperature distribution for a solar wind with a constant, radial velocity. The anisotropy in the proton temperature is shown in Figure (3b). The electron temperature is found by integrating Equation (9) inwards starting in the adiabatic region. The results shown in Figure (3c) were obtained by matching these solutions to the solution given by Equation (14), at points where $(r - r_1) \frac{\Omega}{v} \sin\theta \sim 0.3$.

5. Conclusions.

Assuming a particle flux $F = 4 \times 10^{34}$ sec^{-1} ster^{-1}, and isotropic proton heating near the Sun, we have obtained the following results near the Earth for a steady radial solar wind with a radial magnetic field: $n = 5.5$ cm^{-3}, $v = 340$ km/sec^{-1}, $T_p = 4.2 \times 10^4$ K, $T_p^{\parallel}/T_p^{\perp} = 3.9$ and $T_e = 3.9 \times 10^5$ K. These results are in fairly good agreement with observations made under quiet conditions (e.g. Hundhausen, 1968, 1970). It should be noted that the electron-electron collision frequency is larger than, or comparable to the expansion rate; thus the electron gas is collision dominated. The proton-proton collision frequency is smaller than the expansion rate beyond a few solar radii, and hence the thermal anisotropy must be included in models that give reasonable values for the proton temperature near the Earth. It is interesting to note that it is sufficient to include Coulomb collisions only to obtain thermal anisotropies comparable to those observed.

If the same amount of heat was fed into the proton gas in an anisotropic manner (e.g. $Q_p^{\parallel} = 0$ and $Q_p^{\perp} \neq 0$, corresponding to "collisionless cyclotron resonance heating") we would obtain approximately the same values for the number density, flow velocity and temperatures. However, the thermal anisotropy, $T_p^{\parallel} / T_p^{\perp}$, would be reduced to 2.5 near the Earth. For the case $Q_p^{\parallel} \neq 0$ and $Q_p^{\perp} = 0$ (corresponding to "collisionless hydrodynamic heating", e.g., Barnes, 1969), the anisotropy would be larger. The introduction of a spiral magnetic field (for $R > 100$) causes the electron temperature near the Earth to be reduced to

3.4×10^5 K, the proton temperature to 3.7×10^4 K, and $T_p^{\parallel}/T_p^{\perp}$ from 3.9 to 2.5. The reduction of the proton temperature and of the temperature anisotropy is the result of "inertial cooling" (Khoklov, 1967); this effect can be easily explained by considering a collision free proton gas in a spiral magnetic field: for a constant flow velocity and a small "garden hose" angle, ψ, Equations (10) and (11) yield: $T_p^{\perp} \propto r^{-2}$ and $T_p^{\parallel}$ = constant. Thus, $T_p \rightarrow$ constant and $(T_p^{\parallel} - T_p^{\perp})/T_p \rightarrow 3$ as $r \rightarrow \infty$. For large "garden hose" angles we have $T_p^{\perp} \propto r^{-1}$ and $T_p^{\parallel} \propto r^{-2}$, and hence $T_p \propto r^{-1}$ and $(T_p^{\parallel} - T_p^{\perp})/T_p \rightarrow -2/3$ as $r \rightarrow \infty$. The rapidly decreasing proton temperature parallel to a spiral magnetic field is clearly evident in Figure (2a). As shown in Figure (2b) the proton gas becomes collision dominated for $R \gtrsim 800$, where the proton temperature anisotropy is negative and quite small. The condition for the "firehose" instability cannot be satisfied beyond this heliocentric distance (Eviatar and Schulz, 1970).

The reduction of the electron temperature shown in Figure (2a) is caused by the reduced thermal conductivity in the radial direction in regions where ψ is not small. In the polar regions of the Sun the magnetic field is almost radial, and for a constant flow velocity the proton temperature decreases so slowly that the gas becomes collisionless. In this case $T_p^{\parallel}$ is almost constant, and the temperature anisotropy increases from 1.55 at r = 1 a.u. to 2.1 at r = 50 a.u. (see Figure (3b)). For $\theta > 0$ the "garden hose" angle, ψ, increases with heliocentric distance, and the proton temperature decreases rapidly enough for collisions to become important eventually. Because of the dominance of the collision term we find that $(T_p^{\parallel} - T_p^{\perp})/T_p \gtrsim - 0.15$ (see Figure (3b)). With lower particle densities and higher proton temperatures it will be possible to achieve more negative values of the anisotropy. However, the condition for the "mirror" instability cannot be satisfied in a typical quiet solar wind (e.g. Brandt, 1969).

The electron temperature shown in Figure (3c) is too high in comparison with observations made near the Earth (e.g. Hundhausen, 1968, 1970). There are three possible reasons for this discrepancy: (1) the gradient in the electron temperature assumed here may be incorrect; (2) the expression for the thermal conductivity of the electron gas may be invalid (e.g. Forslund, 1970); (3) the coupling between the electrons and the protons could be stronger than implied by Coulomb collisions only (e.g. Nishida, 1969; Toichi, 1971; Cuperman and Harten, 1971).

By combining the results shown in Figures (3a) and (3c) with the magnetic field strength given by Equation (8), we can calculate the ratio of the particle pressure and the magnetic field pressure: $\beta = 8\pi nk(T_e + T_p)/B^2$. Since the electron temperatures obtained here are too high, the values of β shown in Figure (3d) are also too high in comparison with observations made near the Earth (Burlaga and Ogilvie, 1971). It is interesting to note that at low heliolatitudes β decreases outwards beyond the orbit of Earth. If the stability of interplanetary magnetic field sector boundaries against reconnection is dependent on β, the sector structure may break up somewhere beyond the orbit of Earth where β is small. Since the magnetic field is almost azimuthal in this region and the electron gas expands adiabatically, such an instability would have no effect on the electron temperature profile.

To improve the present calculation other heat sources should be included. As pointed out by Wolff et al. (1971) the effects of viscosity on the proton temperature may be important in 0.1 a.u. $\lesssim r \lesssim$ 1 a.u., and heating of ions beyond 1 a.u. should also be taken into account (cf. Coleman, 1968; Jokipii and Davis, 1969; Siscoe et al., 1971). However the least satisfactory part of this work is the calculation of the electron temperature distribution. The heat conduction coefficient used here may be valid near the Sun where the mean free path is short. However in the vicinity of the orbit of Earth, the mean free path for electrons is in the order of 1 a.u., and it seems likely that the particles would be scattered by some means other than Coulomb collisions before travelling this distance (cf. Forslund, 1970). A reduced value for the heat conduction coefficient would result in a smaller electron temperature some distance away from the Sun. As the flow velocity and particle density are determined mainly by the temperature near the Sun, the effect of a reduced electron temperature near the Earth on these quantities would be relatively small. Because of the weak coupling between the electrons and protons there would be no significant change in the proton temperatures.

A significant improvement could be obtained by combining these calculations with an analysis of the structure of the lower corona (cf. Kuperus, 1969). The cold chromospheric gas is a very effective heat sink for the electrons in particular, and heat conduction towards the Sun is likely to control the electron temperature distribution out to several solar radii. Finally, we wish to point out that if the heat source acts directly on the proton gas as we have assumed, then regardless of the behavior of the electron temperature, the proton temperature should reach a maximum at a distance of a few solar radii and this maximum temperature may exceed the electron temperature by a substantial factor. This could provide a means of determining whether the proton heating takes place directly or via the electron gas (cf. Hundhausen, 1970), since in the latter case the proton temperature could never exceed the electron temperature.

ACKNOWLEDGMENTS

The authors wish to thank J. A. Fejer and T. E. Holzer for their assistance in preparing this paper.

Figure Captions

Figure 1. Solar wind properties for the case of a radial interplanetary magnetic field: (a) Electron-temperature, T_e, and parallel and perpendicular proton-temperature; $T_p^{\parallel}$ and $T_p^{\perp}$; (b) number density, n, and flow velocity, v; (c) collision frequencies, ν_{ee} and ν_{pp}, and the expansion rate, ν_{exp}, versus distance, r. The protons are assumed to be heated isotropically such that $Q_p^{\parallel} = Q_p^{\perp} = \left[0.03 \frac{ev}{sec}\right] \frac{n}{R} \exp(-0.2R)$. The initial conditions are $T_e = T_p^{\perp} = T_p^{\parallel} = 1.5 \times 10^6 K$ at $R = 2$, and $F = 4 \times 10^{34} \sec^{-1} \text{ster}^{-1}$. The corresponding parameters for the same initial conditions, but without heating (cf. Hartle and Sturrock, 1968) are given by broken curves.

Figure 2. Solar wind properties for the case of a spiral interplanetary magnetic field: (a) Electron temperature, T_e, and parallel and perpendicular proton temperature, $T_p^{\parallel}$ and $T_p^{\perp}$; (b) collision frequencies, ν_{ee} and ν_{pp}, and the expansion rate, ν_{exp}, versus distance, r, in the ecliptic plane. The protons are assumed to be heated isotropically, such that $Q_p^{\parallel} = Q_p^{\perp} = \left[0.03 \frac{ev}{sec}\right] \frac{n}{R} \exp(-0.2R)$. In $R > 100$ there is a spiral magnetic field and a constant, radial flow velocity, $v = 320$ km/sec.

Figure 3. Heliocentric distance, r, versus latitude, θ, for some values of: (a) The proton temperature, $T_p = \frac{1}{3}(T_p^{\parallel} + 2 T_p^{\perp})$; (b) the anisotropy in the proton temperature, $(T_p^{\parallel} - T_p^{\perp})/T_p$; (c) the electron temperature, T_e, and (d) the ratio of thermal and magnetic pressure,

$\beta = 8\pi nk\,(T_e + T_p)/B^2$. The protons are heated near the Sun, such that $Q_p^{\parallel} = Q_p^{\perp} = \left[0.03\,\frac{ev}{sec}\right]\,\frac{n}{R}\,\exp(-0.2R)$. In $R < 100$ the magnetic field is taken to be radial, and in $R \geq 100$ there is a constant velocity ($v = 320$ km sec^{-1}), and a spiral magnetic field. The initial conditions are $T_e = T_p^{\parallel} = T_p^{\perp} = 1.5 \times 10^6$K at $R = 2$, and $F = 4 \times 10^{34}\ sec^{-1}\,ster^{-1}$.

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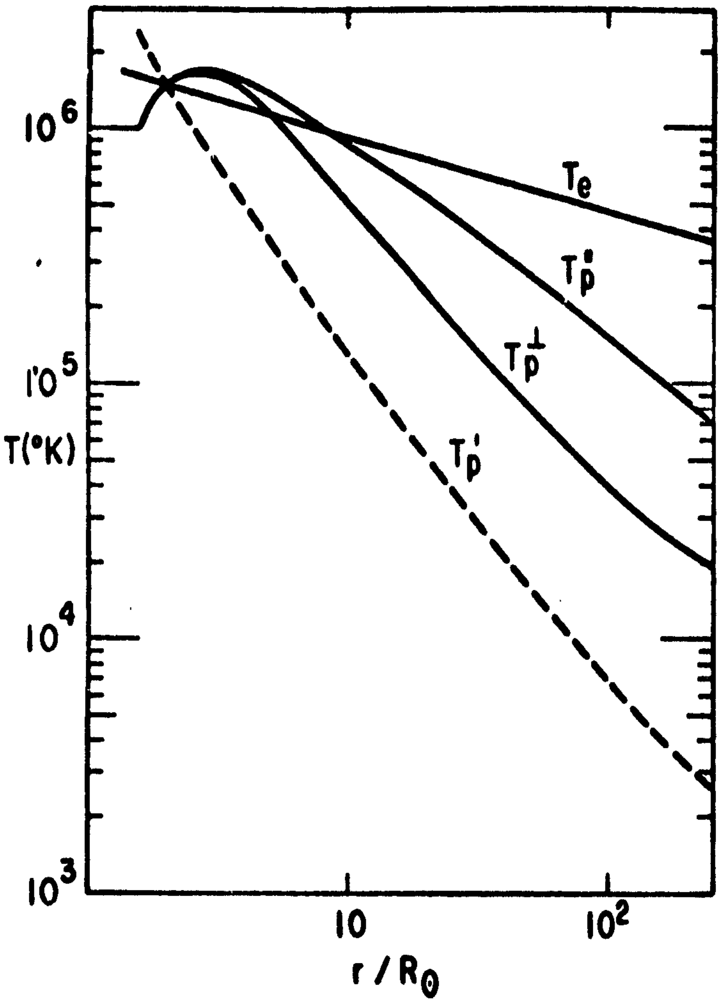

10^6
10^5
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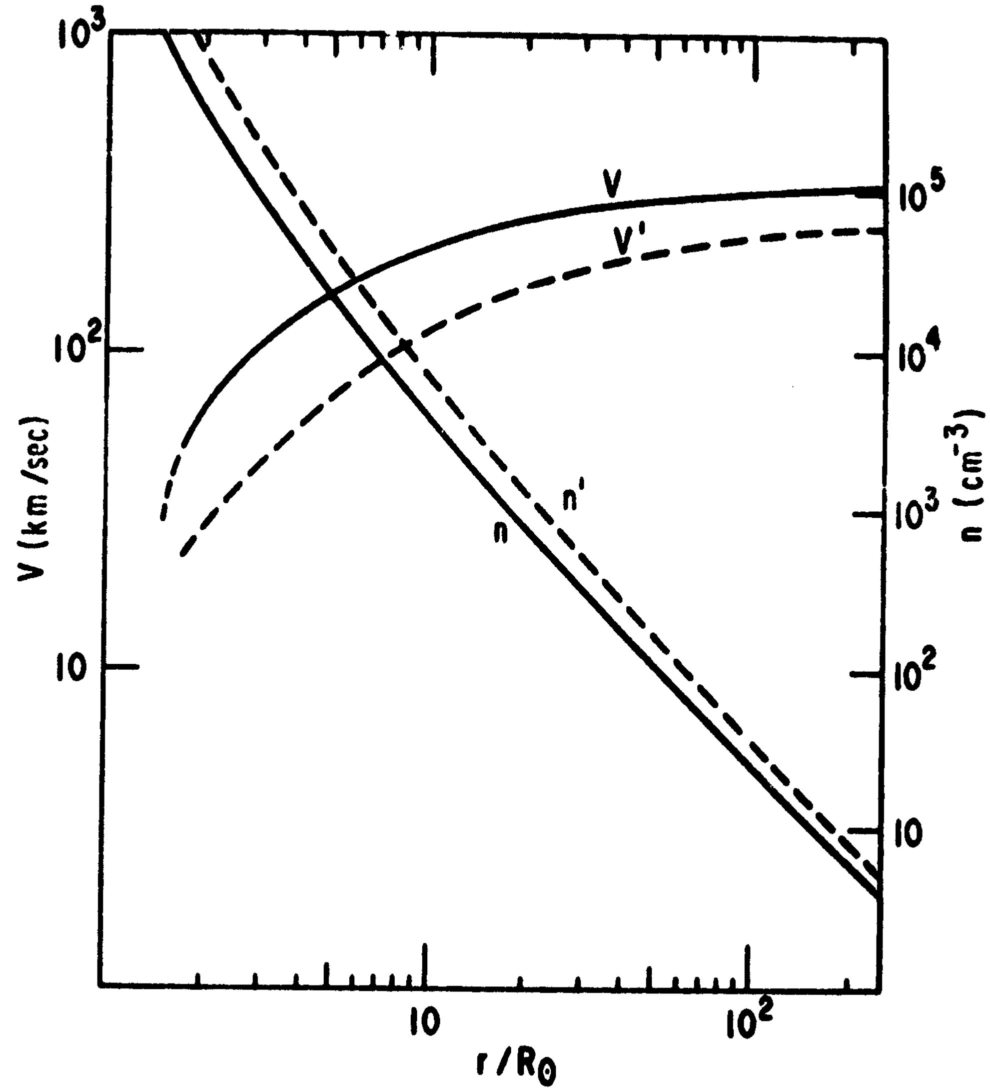
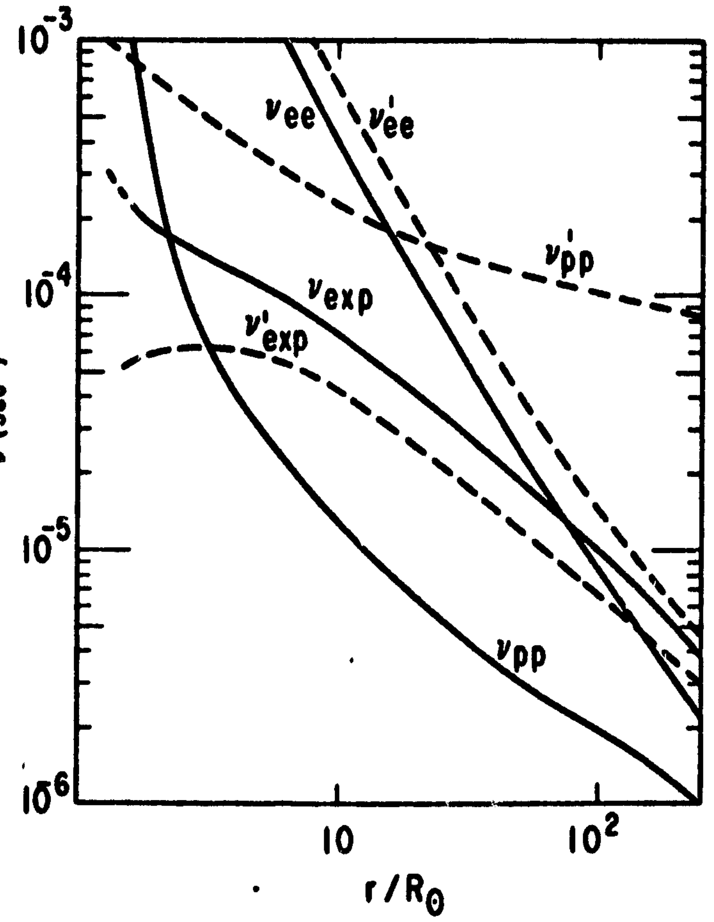
10^3
T(°K)
Te
Tp∥
Tp⊥
Tp'
10
10^2
r / R⊙

a



10^3
10^2
10
V (km/sec)
n (cm^-3)
10^5
10^4
10^3
10^2
10
V
V'
n
n'
10
10^2
r / R⊙

b



10^-3
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10^-5
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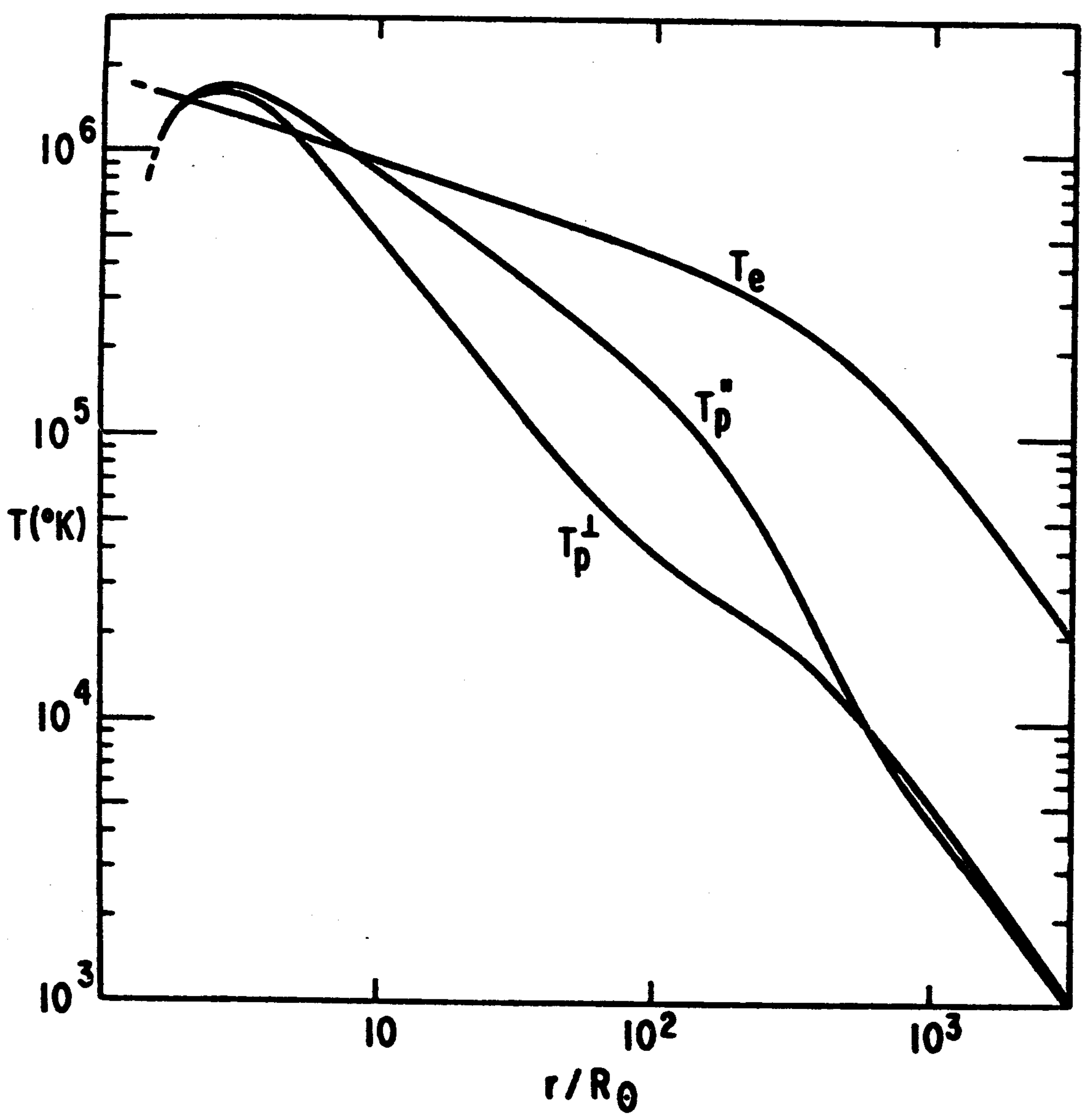
ν (sec^-1)
νee
ν'ee
ν'pp
νexp
ν'exp
νpp
10
10^2
r / R⊙

c



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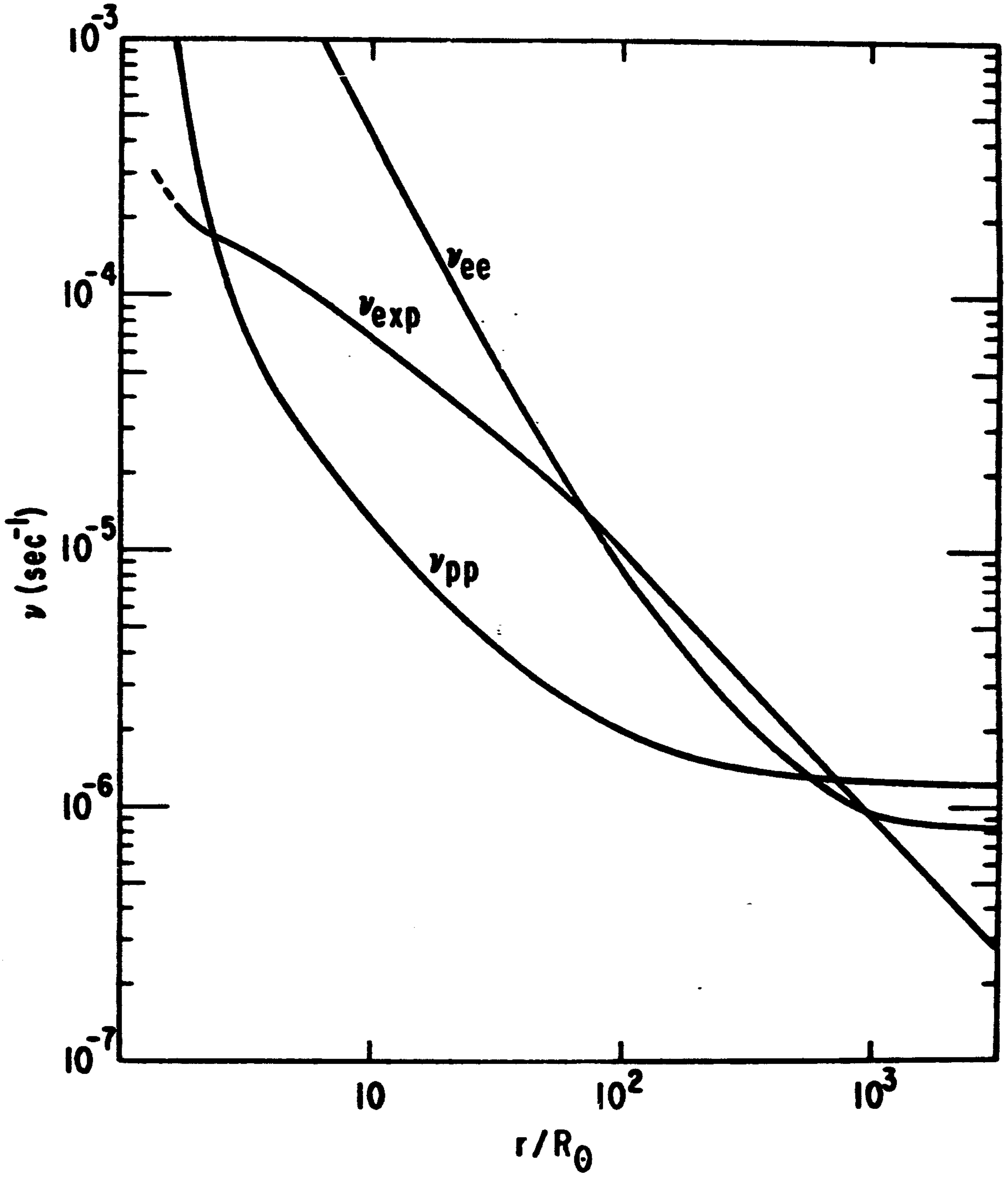
10^3
$r/R_\odot$

a



10^{-3}
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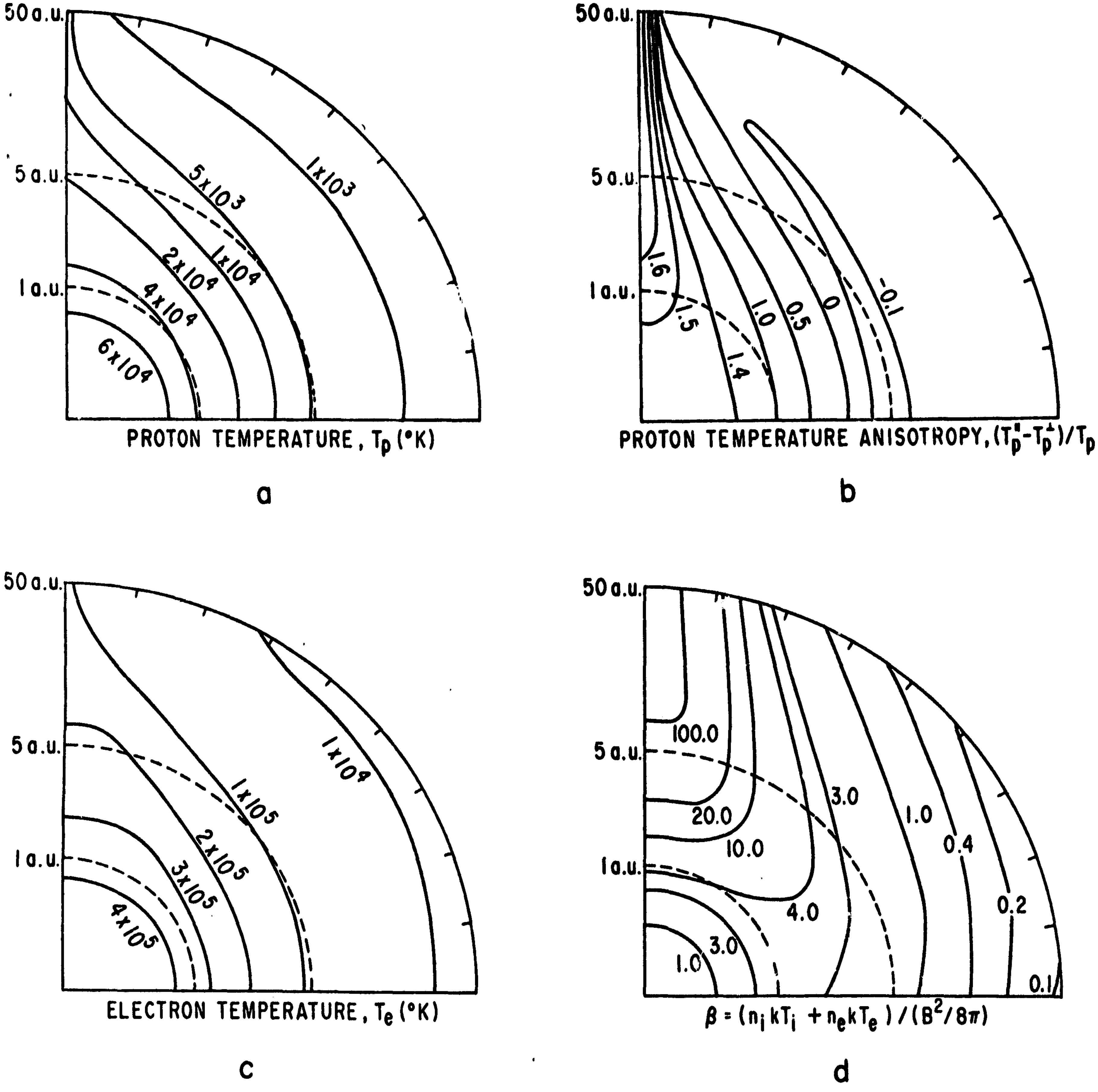
10^3
$r/R_\odot$

b


50 a.u.
5 a.u.
1 a.u.
5x10³
1x10³
1x10⁴
2x10⁴
4x10⁴
6x10⁴
PROTON TEMPERATURE, T_p (°K)
a
50 a.u.
5 a.u.
1 a.u.
1.6
1.5
1.4
1.0
0.5
0
-0.1
PROTON TEMPERATURE ANISOTROPY, $(T_p^{\parallel} - T_p^{\perp})/T_p$
b
50 a.u.
5 a.u.
1 a.u.
1x10⁴
1x10⁵
2x10⁵
3x10⁵
4x10⁵
ELECTRON TEMPERATURE, T_e (°K)
c
50 a.u.
5 a.u.
1 a.u.
100.0
20.0
10.0
3.0
4.0
1.0
0.4
0.2
0.1
3.0
1.0
$\beta = (n_i k T_i + n_e k T_e)/(B^2/8\pi)$
d