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ON ESTIMATING THE VENUS SPIN VECTOR FROM DATA OBTAINED DURING THE PLANETARY EXPLORER MISSION

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SUMMARY

This paper demonstrates the feasibility of obtaining improved estimates of the Venus Spin Vector through a least squares processing of range rate data from P.E. probes on the surface of Venus. It is shown that the probes need transmit data only for a few moments after landing in order to obtain accurate estimates. This procedure estimates the right ascension of the spin vector better than the declination. The estimation procedure remains viable even if a few of the probes do not survive impact. Some improvement in the estimates of probe locations is also obtainable.

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INTRODUCTION

The Planetary Explorer (P.E.) mission is designed to land a total of eight probes on the surface of Venus during the 1977 Venus opportunity. All these probes will be equipped with devices for returning range rate data to the Earth. These devices are expected to survive impact and to continue transmitting data from the planet's surface. The Venus spin vector as well as the positions of the probes on the surface are observable in this data. Thus the correct processing of this data should provide at least some improvement in our knowledge of these parameters. This prospect is an attractive one because of the considerable interest in the spin vector of Venus and because the data necessary for the estimation is obtained as a byproduct of the P.E. mission and thus its acquisition imposes no further constraints or compromises on mission planning.

The purpose of this paper is to indicate the sort of improvement in spin vector and probe position estimates one may reasonably expect from the processing of such data. This was done by duplicating the ensemble calculations associated with a weighted least squares with a-priori estimation technique applied to range rate data which was assumed to be unbiased and uncorrelated. The weighting matrix was assumed to be the inverse of the covariance matrix of the noise on the data. Attention is focused primarily on the spin vector estimation. After a brief discussion of previous efforts to estimate the Venus spin vector, a presentation and analysis of the Authors' results will be given.

PREVIOUS ESTIMATES OF VENUS SPIN VECTOR

Several radar determinations of the Venus spin vector have been reported in the literature. This technique involves the transmission of a cw signal to the planet and the analysis of the power spectrum of the return signal. If the transmitted frequency corrected for relative motion between Venus and the tracking station is used as the zero point of the spectrum, then every other frequency can be interpreted as a doppler shift, and the intensity associated with that frequency can be related to a line of constant radial velocity on the planet. These lines are easy to describe. A plane going through the tracking station and parallel to the spin axis of Venus and intersecting the planet's surface describes a circle. That segment of the circle visible to the tracking station is a member of the family of lines of constant radial velocity. A frequency at which a peak in the spectra occurs corresponds to a region on Venus which is rougher than adjacent regions. Thus a cw spectrum may be viewed as a type of map of the radar brightness of the surface of the planet. By obtaining such spectra at different times it should be possible to track specific surface features and from such information to infer the spin vector of the planet under investigation. Carpenter [1] reproduces several of these radar spectra and identifies several surface features as peaks in the spectra. The mathematics of how one might utilize such information in order to estimate a spin vector is given in some detail by Shapiro [2]. Basically a weighted least squares estimation procedure is used with the surface features associated with peaks in the spectra treated as point sources of range rate data. This method has been applied by Carpenter [3], and [4], Smith [5]. Goldstein [6] and Shapiro [2]. Their various estimates of the rotation period of Venus are quite close to the so-called synodic resonance period of 243.16 days. If Venus were to have this period, it would rotate backward relative to the Earth four times between each inferior conjunction and thus present the same side of the Earth at each inferior conjunction. The uncertainties associated with present estimates are such that the hypothesis that Venus is in synodic resonance is still unsettled.

Another interesting aspect of the radar determinations of the Venus spin vector is that statistically speaking they are mutually incompatible. Table 1 exhibits spin vector estimates and associated statistics obtained from four

Table 1

Recent Radar Determinations and Associated Standard Deviations of Venus Spin Vector

Determination	Rotation Period (In Days)	Right Ascension in Degrees	Declination in Degrees
1. Carpenter (1970	242.98 ± .04	94 ± 3	-71.5 ± 1
2. Shapiro (1967)	243.09 ± .18	84.7 ± 1.8	-65.8 ± 1.2
3. Dyce, Pettengill, and Shapiro (1967)	244.3 ± 2	90.9 ± 1	-66.4 ± 1
4. Goldstein (1967)	242.6 ± .6	98 ± 5	-69 ± 2

different radar determinations; Carpenter [1], Shapiro [7], Dyce, Pettengill, and Shapiro [8], and Goldstein [9].

Clearly the standard deviations of Table 1 give a too optimistic representation of the accuracy of the radar determination technique. The reason is that ensemble calculations giving standard deviations of the least squares estimation process do not reflect the impact of modeling errors. The most obvious modeling error associated with the technique is that of treating the source of a peak in a frequency spectrum as a point. Any surface feature on Venus capable of causing a significant perturbation in the return spectrum of a radar scan must be quite extensive. A more subtle though possibly less important modeling error is the assumption that the return rate data is uncorrelated. Corrections must be made to the doppler data before it can be used for least squares estimation. These corrections are responsible for a portion of the noise on the data. If this portion is significant, then it is improper to model the noise on the data as an uncorrelated or "white" random process. The correct procedure would then be to solve for a bias and perhaps a scale factor error in the data. If this were done the ensemble calculation would yield a more accurate though quantitatively less impressive figure for the standard deviation of the Venus spin vector estimate. Another possibility is that the cause of the peaks on the return spectra may be a complex combination of phenomena rather than a fixed surface feature.

It is not easy to obtain reasonable estimates of the quality of the radar determination technique. Carpenter [1] suggests that the formal standard deviation numbers associated with radar determinations should be increased several times. This is quite vague, but it would be imprudent to be more precise.

SPIN VECTOR ESTIMATION WITH P.E. PROBES

The P.E. mission will consist of two separate launchings of multiprobe spacecraft. Each launch will land a main probe equipped with a two way doppler devices and three mini probes each equipped with a one way doppler device. The geometric distribution of the probes is given by Figure 2. The succeeding analysis rests on these modeling assumptions.

(1) For the duration of their transmission of doppler data, the probes remain stationary relative to the Venus surface.

(2) For this same duration the Venus spin vector is constant.

(3) The noise on the range rate data transmitted by the probes is stationary and uncorrelated.

(4) Unbiased estimates of the Venus spin vector and the locations and effective radii of the probes are available. It is also assumed that these estimates are statistically uncorrelated and that their standard deviations are known.

Assumptions one and two should disturb no one. Assumption three is somewhat more troublesome. We defer discussion of this assumption to a later stage of the analysis. Concerning assumption four, the estimate of the Venus spin vector and its associated uncertainty would presumably be borrowed from a radar determination. The probe positions and associated uncertainties would be obtained from extensive post-flight analysis of all relevant data gathered during the P.E. mission. This would include accelerometer, temperature and pressure measurements and range rate data.

The conventional way to process the doppler data from the P.E. probes is to form the usual weighted least squares with a-priori loss function and choose the spin vector and probe positions which minimize this loss function. To be specific, let X be a vector of dimension twenty seven and whose elements are estimates of the spin vector and positions of each of the eight probes. Let the vector $\tilde{Y}$ be the range rate measurements, arranged in some sequence, which would be obtained in the absence of noise if X contained the true values of the parameters in question. The vector $\tilde{Y}$ is a known function of X symbolized by

$$\tilde{Y} = f(X) \tag{1}$$

Let Q be the covariance matrix of the noise on the observations and let P be the covariance matrix of an a-prori estimate X' of the parameters. Then the loss function is

$$L(X) = (Y - f(X))^T Q^{-1} (Y - f(X)) + (X' - X)^5 P^{-1} (X' - X) \tag{2}$$

where Y is the vector of measured values of range rate data obtained from the probes. Notice that by assumptions three and four the matrices Q and P are diagonal – a fact which greatly simplifies following computations. The least squares estimate is defined as that value of X which minimizes the loss function of Equation 2. Since the data necessary to implement this procedure is not yet available, interest is focused on just the statistical properties of this estimation procedure. The covariance matrix of the least squares estimator may be obtained if one more assumption is imposed. It must be supposed that the function of equation 1 can be accurately represented as a first order Taylor series about $\hat{X}$ where $\hat{X}$ is the least squares estimate. Thus we assume that equation 1 can be written as

$$Y - f(\hat{X}) = A(X - \hat{X}) \tag{3}$$

The symbol A represents the variational matrix. If N be the total number of range rate measurements, then A is an N by 27 dimensional matrix. The element in the i-th row and j-th column of A is the partial derivative of the i-th component of Y with respect to the j-th component of X. It is relatively easy to obtain an analytical expression for A. The details are found in appendix one. If equation 3 is valid, then the covariance matrix of the least squares estimator is given by

$$\mathrm{COV}\,(\hat{X}) = (A^T Q^{-1} A + P^{-1})^{-1} \tag{4}$$

Equation 4 provides a mode for the performance of a parametric study of the accuracy attainable in the estimation of the Venus spin vector. Interest was focused primarily on the variation of this accuracy with respect to variations in the following parameters

(1) Length of time the probes survive on the surface

(2) Quality of a-prori information

(3) Size of the noise on the data

This paper reports on the results of such a parametric study. As usual, nominal values were established for all relevant parameters. Certain parameters were then systematically varied with the other parameters fixed at their nominal values. Equation 4 is then used to obtain the corresponding standard deviations of the spin vector and probe locations. There is a certain amount of arbitrariness involved in the selection of nominal values. It is not easy, for instance, to decide what are reasonable values for the a-priori uncertainties of the spin vector at some time several years in the future when the P.E. mission is to be executed. Also the time at which the probes land is a factor since this affects tracking sight geometry. This too is somewhat arbitrary although one would of course choose a time during which dual coverage from the D.S.N. network is possible. The data chosen for the landing is May 19, 1977. Dual coverage from Goldstone and Madrid is assumed. The nominal value of the spin vector as obtained from [1] is 242.982 days for the period and 94.1 and -71.4 degrees respectively for the right ascension and declination of the spin vector (equator of 1950). The longitude and latitude of each probe is given in target coordinates in [10]. The data acquisition rate is assumed to be one per minute. It is also assume that all biases are estimated in the least squares procedure.

A summary of a-priori standard deviations is provided in Table II.

Table 2

A Priori Standard Deviations

Parameter	Standard Deviation
Right ascension of main probes	.115°
Declination of main probes	.115°
Right ascension of mini probes	.165°
Declination of mini probes	.165°
Range rate from main probes	5 mm/sec.
Range rate from mini probes	5 cm/sec.
Period of Venus rotation	1 day
Right ascension of Venus spin vector	10°
Declination of Venus spin vector	5°
Effective radius of Venus at each probe	5 km

The usual standard deviation figure for two way range rate and for a one per minute sampling rate is 1 mm/sec. See for instance Blackshear and Williams [11] who use this figure for two way range rate data in an error study similar to this one but for the Viking project. The larger figure of 5 mm/sec. for the two way range rate data on the main probes was used to compensate for the fact that the data was assumed to have no time correlation. Since the various corrections which must be made to the data in fact do tend to introduce time correlations this assumption is a questionable one. Its legitimacy can only be defended if a bias on the data is also solved for in the estimation procedure. In this error study no provisions have been made for bias estimation. Hence it was felt that to use the usual standard deviation figure for the noise on the two way range rate data would lead to an excessively optimistic result. The use of a larger standard deviation number was an effort to compensate for this optimism. Since the presence of biases on one way doppler data is a much more serious problem, the standard deviation of the noise on the mini probe data was set at ten times the corresponding figure for the noise on the main probe data.

Figures 2, 3 and 4 display the possibilities of estimating the period, right ascension, and declination of the Venus spin vector as a function of the length of time the probe's range rate devices survive on the surface. Since it is not certain that all range rate devices will survive parachute openings or impact on the surface, the figures also demonstrate the deterioration in the estimation procedure if a mini probe per launch fails to function and also if a mini probe and a main probe on each launch fail to function. A salient feature of the figures is that most of the improvement in the estimates occurs within the first few minutes after impact. This fact suggests that the primary reason for the feasibility of this estimation procedure is the coupling provided by Equation 4 between the spin vector estimate and the relatively small standard deviations of the a priori estimates of probe locations. If this were true one would expect the quality of the estimation procedure to be far more sensitive to changes in the standard deviations of the a priori estimates of probe locations than to changes in the standard deviations of the noise on the data. This appears to be the case. For example, with regard to the nominal case with all probes lasting thirty minutes the standard deviation of the rotation period is .28 days. When the standard deviations of the noise on the data are doubled the number becomes .31

days. With the standard deviations of the noise returned to nominal values but with the standard deviations of the a priori probe location estimates doubled the standard deviation of the estimate of the period rises to .4 days.

It is apparent from a comparison of figures 3 and 4 that the right ascension of the Venus spin vector is far more observable in this experiment than the declination. A glance at Table 1 reveals that the situation is precisely the opposite with regard to the radar determination procedure. It is not obvious why this should be so. But in this sense, at least, the two estimation procedures should neatly compliment each other.

The ability of this estimation procedure to improve knowledge of probe positions is somewhat less impressive. After thirty minutes of tracking, knowledge of the positions of the main probes is improved by approximately 15%. Knowledge of the positions of the mini probes after the same period of tracking is improved by approximately 10%. After an hour of tracking these percentage improvements are respectively 18% and 15%.

CONCLUSION

This paper has demonstrated the feasibility of utilizing range rate data generated by P.E. probes in order to estimate the Venus spin vector. The estimation procedure is viable even if each probe transmits doppler data for just a few minutes after impact. The standard deviation figures associated with this estimation procedure are not dependent on questionable modeling assumptions for their validity. Consequently they are a true measure of the estimate's accuracy.

The right ascension of the spin vector is far more observable in the data than the declination. Fortuitously, the radar determination technique estimates the declination better than the right ascension. Thus the two estimation techniques compliment each other.

The improvements in probe position uncertainty are somewhat marginal.

Finally, one of the happy aspects of this proposed experiment is that it utilizes data which is generated essentially as a byproduct of the P.E. mission and hence its implementation poses no additional constraints on P.E. mission planning.

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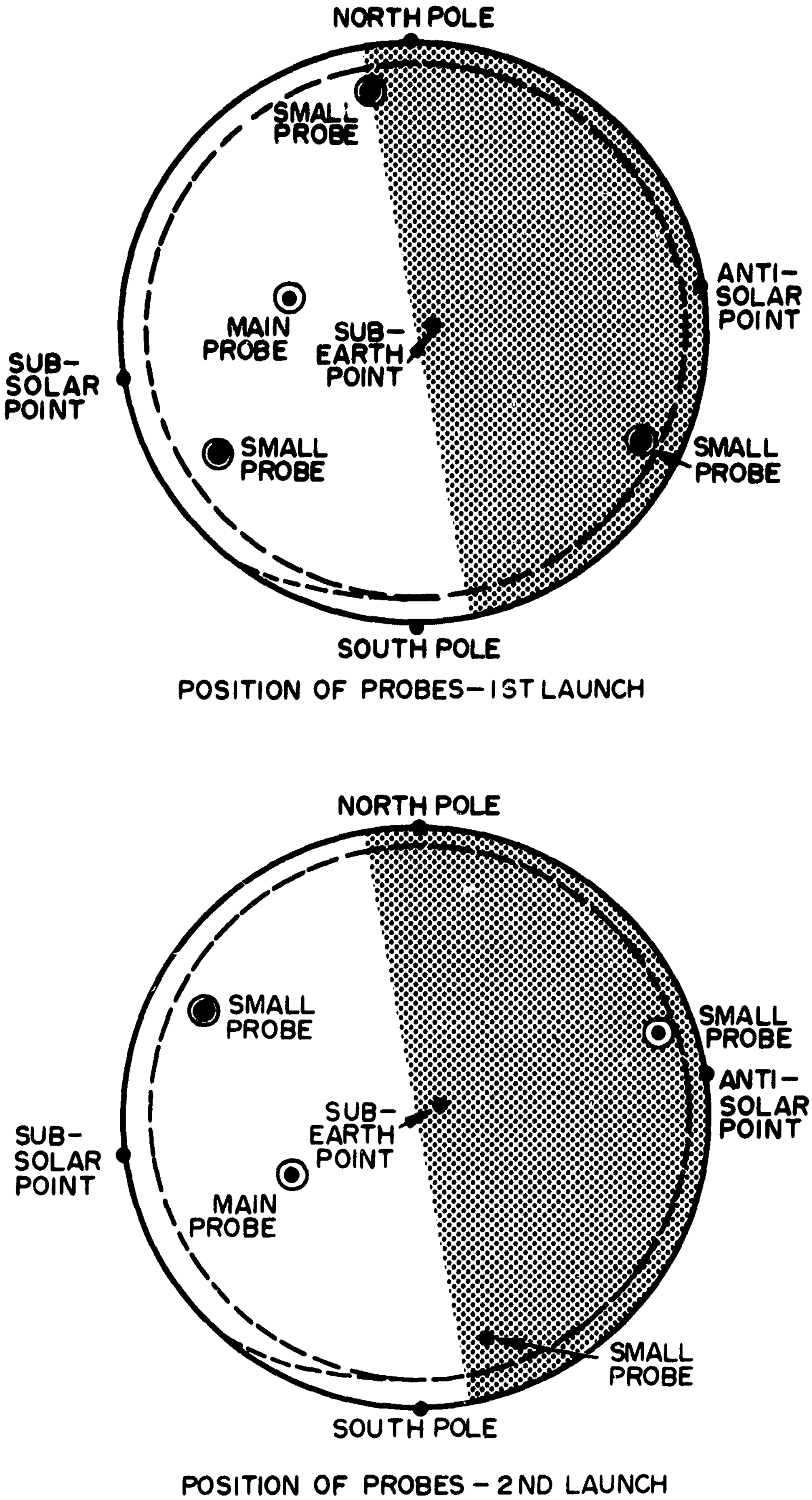

Figure 1. Position of Probes on Venus Surface

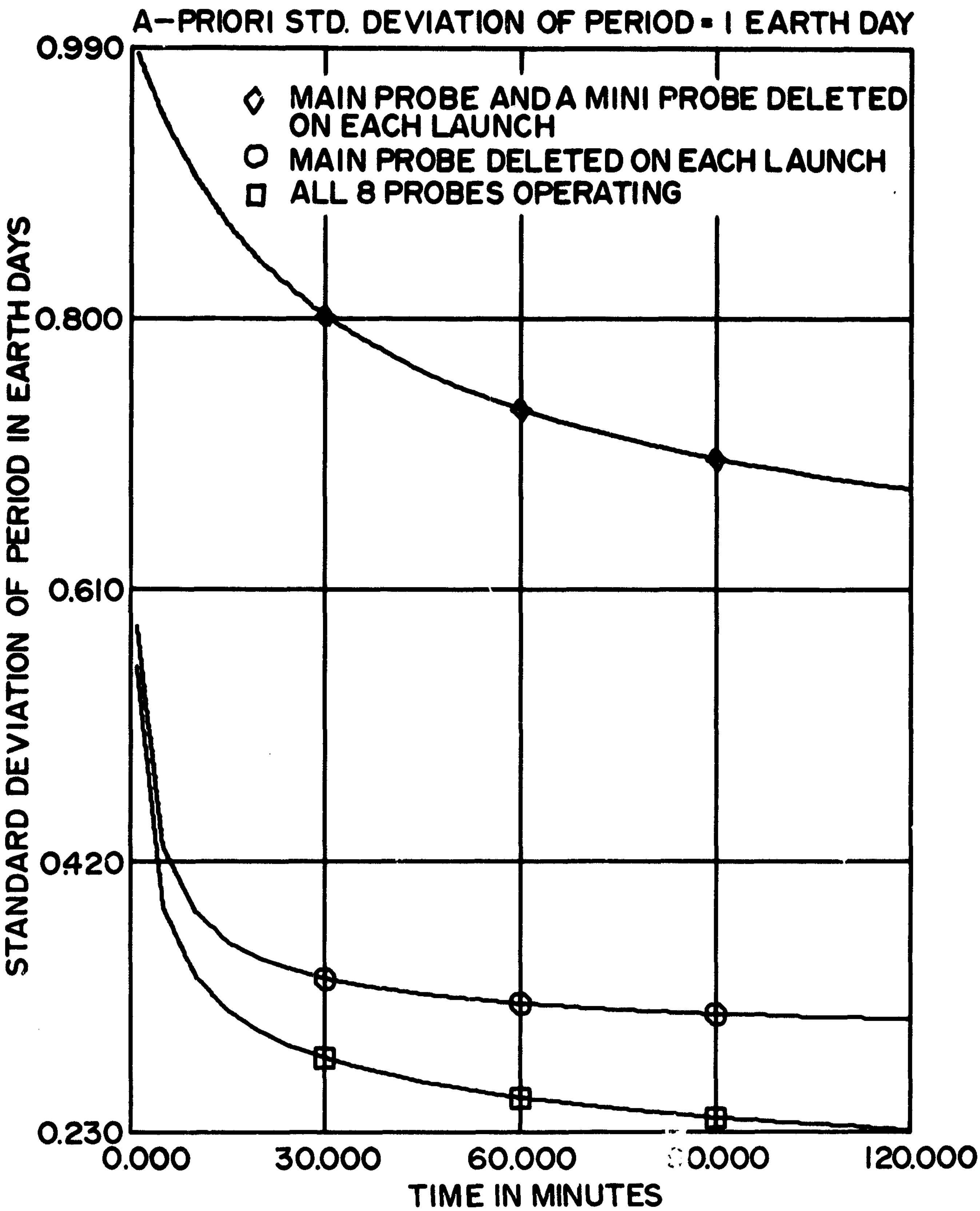

Figure 2. Standard Deviation of Period as Function of Time

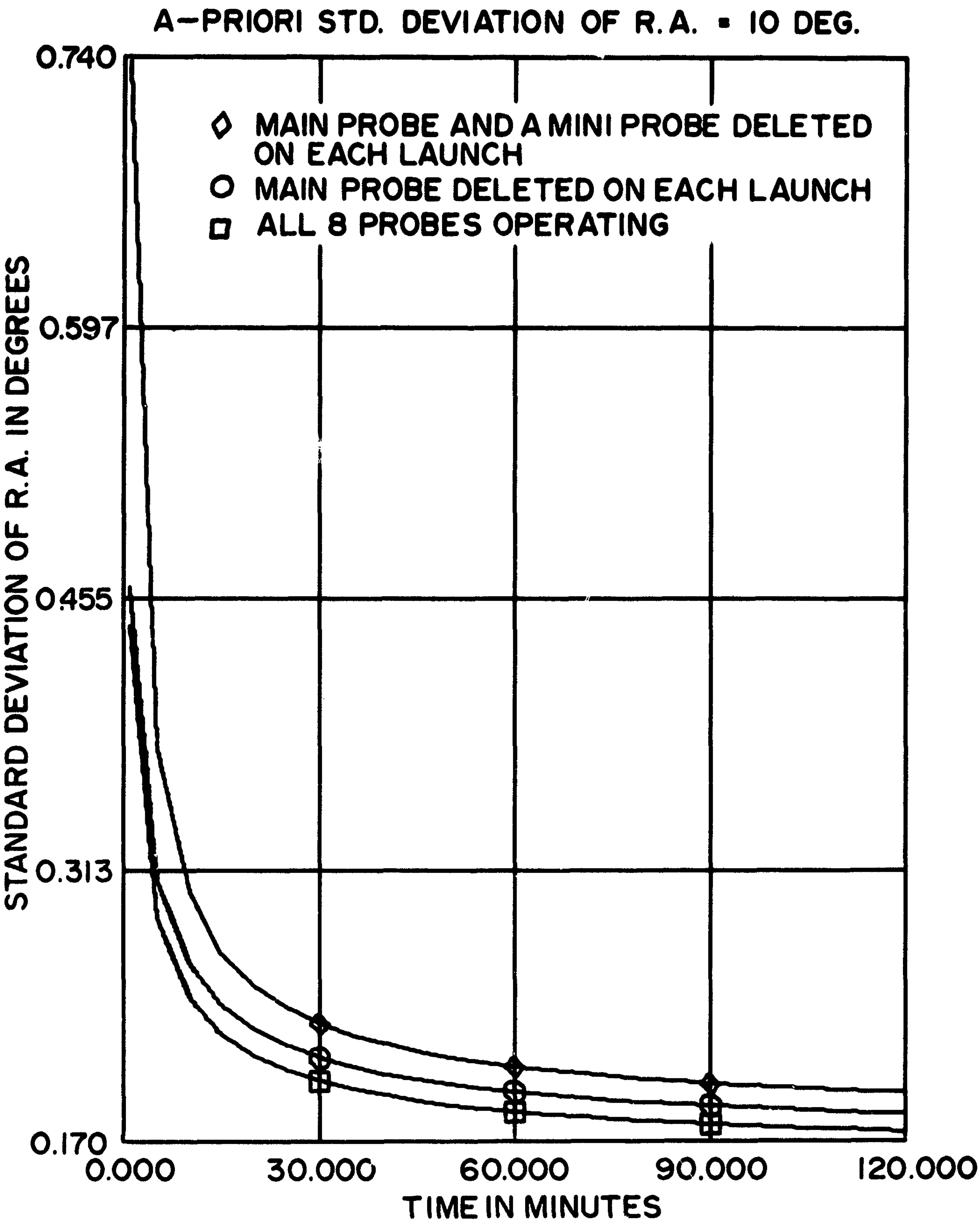

Figure 3. Standard Deviation of Right Ascension as a Function of Time

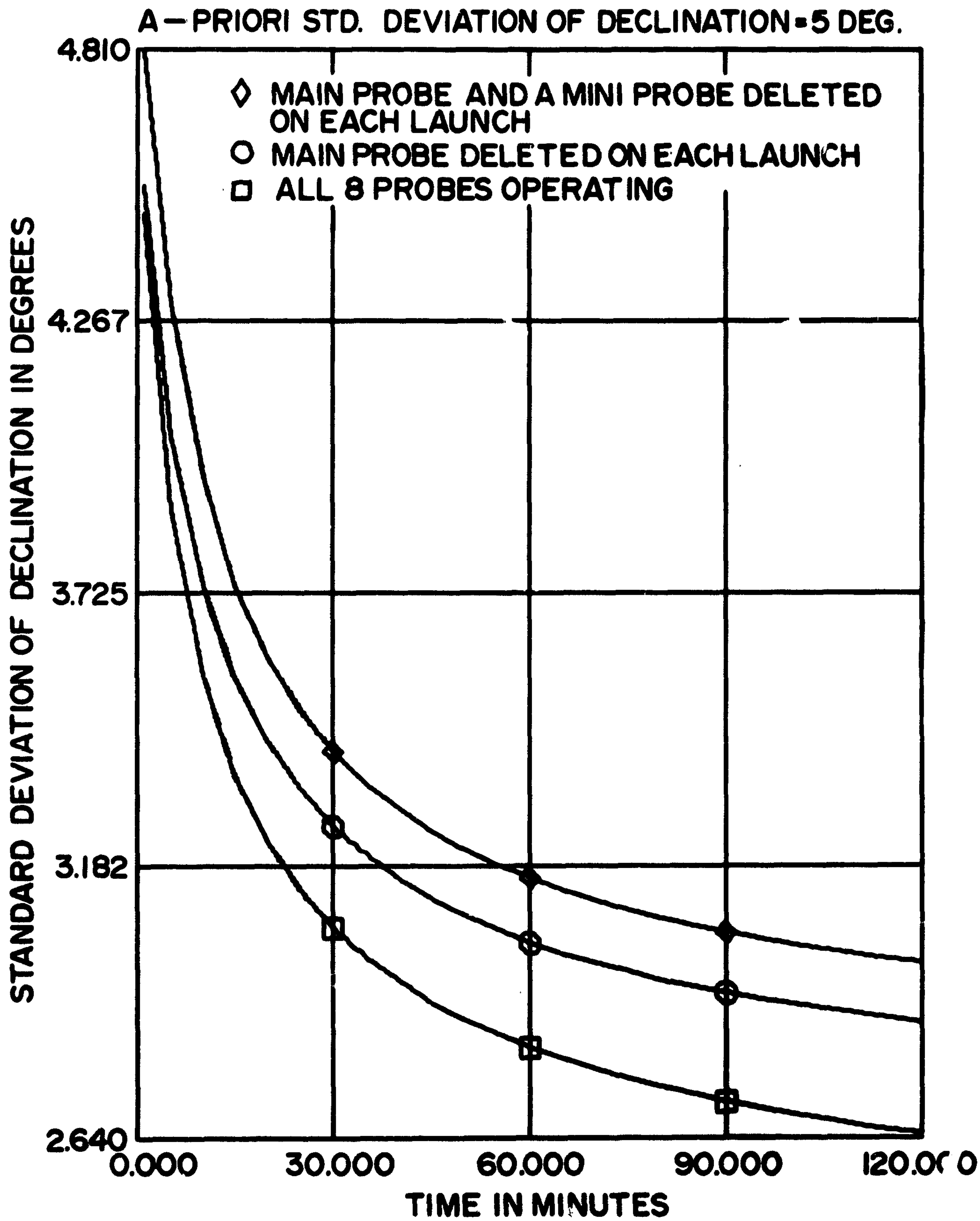

Figure 4. Standard Deviation of Declination as a Function of Time

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APPENDIX

THE VARIATIONAL MATRIX FOR SPIN VECTOR ESTIMATION

Let Y be a vector of range rate measurements obtained from P.E. probes on the surface of Venus. Let X be the set of parameters that are to be estimated. In this case X consists of the three components of the spin vector and three components for each of the probes on the surface. The variational matrix A is defined by specifying that the element in the i-th row and j-th column of A is the partial derivative, evaluated at nominal values of X, of the i-th component of Y with regard to the j-th component of X. Notice that a given range rate measurement has a non-zero partial derivative only with regard to the components of the spin vector and the position components of the probe which generated the measurement. What follows is a derivation of these six partial derivatives of a range rate measurement from any given probe and at any given time.

Let R_0 be the position vector of a probe at time of impact t_0. If R(t) represents the position of the probe at a later time t and if w represents the spin vector of Venus, then the vector equation of motion of the probe can be written as

$$\dot{R}(t) = \Omega_\omega R(t) \tag{1}$$

where

$$\Omega_\omega = \begin{pmatrix} 0 & w_z & -w_y \\ -w_z & 0 & w_x \\ w_y & -w_x & 0 \end{pmatrix}$$

The unique solution to this matrix differential equation is

$$R(t) = e^{\Omega_w (t-t_0)} R_0 \tag{2}$$

where by definition

$$e^{\Omega_w (t-t_0)} = I + \lim_{N \to \infty} \sum_{K=1}^{N} \frac{\Omega_w^k (t - t_0)^k}{k!} \tag{3}$$

The right side of Equation 3 can be written in the following closed form

$$\begin{aligned} e^{\Omega_w (t-t_0)} = {} & e^{\lambda_1 (t-t_0)} \frac{(\Omega_w - \lambda_2 I)\ (\Omega_w - \lambda_3 I)}{(\lambda_1 - \lambda_2)\ \ (\lambda_1 - \lambda_3)} \\ & + e^{\lambda_2 (t-t_0)} \frac{(\Omega_w - \lambda_1 I)\ (\Omega_w - \lambda_3 I)}{(\lambda_2 - \lambda_1)\ \ (\lambda_2 - \lambda_3)} + e^{\lambda_3 (t-t_0)} \frac{(\Omega_w - \lambda_1 I)\ (\Omega_w - \lambda_2 I)}{(\lambda_3 - \lambda_1)\ \ (\lambda_3 - \lambda_2)} \end{aligned} \tag{4}$$

where $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of Ω_w. These eigen values are known to be 0, $i|w|$, $-i|w|$. Hence after some manipulation, Equation 4 can be written as

$$e^{\Omega_w (t-t_0)} = I + \frac{\Omega_w}{|w|} \left[\frac{\Omega_w}{|w|} - \frac{\Omega_w}{|w|} \cos |w|\ (t - t_0) + \sin |w|\ (t - t_0) \right] \tag{5}$$

By differentiating Equation 2 one obtains

$$\dot{R}(t) = \Omega_w\ e^{\Omega_w (t-t_0)} R_0 . \tag{6}$$

Letting the symbol B_t represent the unit vector from the position of the probe at time t to the tracking station, the scaler range rate D(t) measured at the tracking station at time t is

$$D(t) = B_t^T \dot{R}(t) = B_t^T \Omega_w \, e^{\Omega_w (t-t_0)} R_0 . \tag{7}$$

Thus the gradient of D(t) with regard to R_0 is

$$\nabla_{R0} D(t) = B_t^T \Omega_w \, e^{\Omega_w (t-t_0)} \tag{8}$$

To obtain the gradient of D(t) with respect to W, one begins by writing Equation 1 in the following form

$$\dot{R}(t) = \Omega_{R(t)} \, W \tag{9}$$

where

$$\Omega_{R(t)} = \begin{pmatrix} 0 & -R_z(t) & R_y(t) \\ R_z(t) & 0 & -R_x(t) \\ -R_y(t) & R_x(t) & 0 \end{pmatrix}$$

Hence

$$D(t) = B_t^T \Omega_{R(t)} \, W \tag{10}$$

As long as $t - t_0$ does not represent a duration of more than a few hours, the dependence of the components of Ω_R on W can be neglected and the gradient of D(t) with regard to W can be stated as

$$\nabla_w D(t) = B_t^T \Omega_{R(t)} . \tag{11}$$

To see that this is true consider the correct equation for the partial derivative of D(t) with respect to W_x

$$\frac{\partial D(t)}{\partial w_x} = B_y R_z - B_z R_y + \left[B_y \frac{\partial R_z}{\partial w_x} - B_z \frac{\partial R_y}{\partial w_x}\right] w_x \tag{12}$$

$$+ \left[B_z \frac{\partial R_x}{\partial w_x} - B_x \frac{\partial R_z}{\partial w_x}\right] w_y + \left[B_x \frac{\partial R_y}{\partial w_x} - B_y \frac{\partial R_x}{\partial w_x}\right] w_z .$$

It is easy to show that the first two terms on the right side of Equation 12 are the only ones that need be considered. Consider for instance the third term on the right side of Equation 12. The nominal value of W_x is approximately $3\,(10)^{-4}$ radians/hr. The terms

$$\left|\frac{\partial R_z}{\partial w_x}\right| \text{ and } \left|\frac{\partial R_y}{\partial w_x}\right|$$

are bounded respectively by $|R_z|\,(t - t_0)$ and $|R_y|\,(t - t_0)$ where $t - t_0$ is measured in hours. Thus the first two terms are seen to be four orders of magnitude greater than succeeding terms. By deleting these smaller terms and performing similar deletions in the equations for the partial derivatives of D(t) with respect to W_y and W_z, one obtains the representation of Equation 11. It is interesting to observe that if the period of the Venus rotation were substantially shorter or if data from the P.E. probes were to be processed over a longer period of time, say several days, then that portion of the variational matrix obtained by Equation 11 would be substantially more complicated.