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STABLE LONGITUDES FOR 12 HOUR ECCENTRIC ORBIT SATELLITES

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Trajectory Analysis and Geodynamics Division

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ABSTRACT

The accelerated longitude drift regimes of eccentric 12 hour orbits are considered, due to the resonant geopotential. These quasi-stationary orbits are under investigation at the N.A.S.A. for large payload earth surveillance spacecraft in the small applications technology satellite (SATS) program. Inclinations near 'critical' are especially attractive because of their possible long term stability, and these are examined in detail. 'Stable' equator crossing longitudes for these satellites are found as functions of the argument of perigee. As long as the argument of perigee is between ±100°, these 'stable' longitudes lie within a narrow range. Maximum 'east-west' station keeping requirements for these satellites at non equilibrium positions are in the range of .3 to 3 m/sec/yr.

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INTRODUCTION

Among the most attractive orbits for space applications is the eccentric 12 hour orbit. This is a quasi-synchronous orbit which, with a low perigee, spends most of it's time within two small lattitude-longitude boxes at true synchronous altitudes (at about 36,000 km; see Figure 1 for example). Thus, if a mission requirement of an absolutely continuous link with a given ground station can be slightly relaxed, this orbit may be preferred over the more conventional 24 hour 'geostationary' one. In fact the U.S.S.R. has already utilized this orbit extensively for communications and other applications in their Molniya and Cosmos series of satellites. Recently, NASA has considered utilizing such orbits in its small application technology satellite program.[1] The first such 12 hour orbit, of Cosmos 41, was achieved in 1964.

The payloads in 12 hour eccentric orbits are significantly heavier than in 24 hour geostationary ones because (among other factors):

1. The orbit is closer to the earth
2. The orbit is highly eccentric (~ 0.70) compared to the circular geostationary orbit, and
3. No inclination change maneuver has to be performed for launches from high latitude sites (such as those in the Soviet Union).

On the other hand, the circular 'geostationary' orbit is inherently more stable than the 12 hour-eccentric one whose perigee is subjected to strong long term luni-solar perturbations.[2], [3] In fact many of the Molniya's have already been lost (from rapid perigee collapse) due to improper initial placement of the orbit node with respect to the moon's node.[2] However, with proper node placement, this early demise can be avoided. Cosmos 41 is still in orbit in 1971 and, according to Quinn et. al.,[4] will stay in orbit for another 39 years.

Radiation pressure also affects the eccentric orbit to a greater extent than the circular. However for the usual satellite payloads, where the projected area to mass ratio is considerably less than 1 cm^2/gm, these perturbations do not affect the orbit substantially.

The dominant perturbation affecting the longitude of the 12 hour orbit is due to resonance with longitude terms in the earth's gravity potential since the geographic trace (latitude, longitude and altitude) is repetitive every day. Because of it's eccentricity, the resonant regime is more complex than for the 24 hour circular orbit. However it's stability is only affected by the significant rotation of the line of apsides, due to the earth's oblateness.[5], [6] As a consequence of this rotation only two general 12-hour eccentric orbits exist which have long term longitude stability. These are the equatorial orbits, discussed in detail by Allan[6] in 1967, and the critically inclined orbit (at 63.4° and 116.6°) briefly described by Wagner[7] in 1968. In both of these cases a stationary disturbing potential on the orbit can be constructed. This insures that, while the resonant libration regime is more complex than in the circular orbit case, the motion under this potential is always periodic. In particular, for the equatorial and critically inclined orbit, cases of stable equilibrium can be found so that satellites placed in these orbits will not need stationkeeping to maintain a given geographic trace. For the general inclination case, rotation of the line of apsides gives rise to a non stationary potential, aperiodic motion and no stable longitude placement. This case has been discussed by Gedeon, Douglas and Palmiter[5] in 1967. It is well illustrated by the mean longitude history of the satellite 1966 96A (Intelsat 2-F1) shown in Figure 2. (The mean longitude (λ) in this figure is the combination $(M + \omega)/2 + N - \theta_e$, where M, ω and N are the satellites mean anomaly, argument of perigee and right ascension of the ascending node, and θ_e is the earth's Greenwich hour angle.) For a circular 12 hour orbit this longitude would also be the actual geographic ascending equator crossing longitude. For an eccentric 12 hour orbit there can be a considerable difference between this mean longitude and the actual equator crossing. (This difference will be described later as a function of the argument of perigee and the eccentricity.) Clearly the longitude excursions of this satellite are not periodic but are increasing with time as predicted by Gedeon et al.[5] but contrary to the expectations of Allan.[6] (Allan felt this orbit of 18 degrees inclination was sufficiently close to equatorial to have a stationary geopotential disturbing potential.) By contrast, the longitude history of 1964 49E (Cosmos 41 rocket) with an inclination near 'critical,' is nearly periodic (see Figure 3). It's excursions are characteristic of a widely swinging pendulum.

Since 1969, starting with Molniya 11, the U.S.S.R. has placed their closely commensurate 12 hour satellites in orbits with ascending equator crossings close to 112 degrees west. This longitude placement has proved to be stable for a wide range of perigee arguments. The shallowest excursions about this apparent equilibrium position was shown by Molniya 12 in 1969-1970 before it's perigee collapsed in the fall of 1970 (Figure 4).

This paper will derive all the resonant equilibrium longitudes for these satellites as functions of inclination and argument of perigee. Only the low perigee orbit (e = .725, perigee height = 930 km) is examined in detail because it is the most useful for the applications satellites.

ANALYSIS – THE DRIFT REGIME OF AN ECCENTRIC 12 HOUR SATELLITE

The equation of motion of the mean longitude (λ) of a closely commensurate resonant satellite due to the geopotential is given by Wagner[7], [8] as:

$$\ddot{\lambda} = 12\pi^2 \sum_{\ell,m \text{ relevant}} C_{\ell m} \left[m a^{-\ell} (FG)_{\ell m} \left| \begin{array}{c} \sin m\left(\lambda + \frac{\gamma_{\ell m}}{m}\right) \\ \\ -\cos\left(\lambda + \frac{\gamma_{\ell m}}{m}\right) \end{array} \right| \begin{array}{l} \ell - m \text{ even} \\ \\ \ell - m \text{ odd} \end{array} \right]$$

$$+ S_{\ell m} \left[m a^{-\ell} (FG)_{\ell m} \left| \begin{array}{c} -\cos m\left(\lambda + \frac{\gamma_{\ell m}}{m}\right) \\ \\ -\sin m\left(\lambda + \frac{\gamma_{\ell m}}{m}\right) \end{array} \right| \begin{array}{l} \ell - m \text{ even} \\ \\ \ell - m \text{ odd} \end{array} \right] \tag{1}$$

radians/sidereal day^2.

In equation (1); the $C_{\ell m}$ and $S_{\ell m}$ are the unnormalized harmonic coefficients of degree ℓ and order m ($\ell \geq m$) in the geopotential expansion,[9] a is the orbits semimajor axis in earth radii, and the $(FG)_{\ell m}$ and $\gamma_{\ell m}$ are amplitudes and phase angles of composite harmonic vectors which depend on the inclination eccentricity and argument of perigee of the orbit.

The composite vectors $(\overline{FG})_{\ell m}$ are determined from Kaula's F and G functions and his decomposition of the disturbing geopotential potential (ℓ, m) into (ℓ, m, p, q) components.[9] The amplitudes $(FG)_{\ell m}$ are given as:

$$(FG)_{\ell m} = \left\{ \left[\sum_{p,q \text{ resonant}} FG_{\ell,m,p,q} \cos(-q\omega) \right]^2 + \left[\sum_{p,q \text{ resonant}} FG_{\ell,m,p,q} \sin(-q\omega) \right]^2 \right\}^{1/2} \tag{2}$$

The phase angles are given as:

$$\gamma_{\ell_m} = \mathrm{Tan}^{-1}\left[\frac{\sum\limits_{p,q,\ \mathrm{resonant}} FG_{\ell,m,p,q}\ \sin(-q\omega)}{\sum\limits_{p,q\ \mathrm{resonant}} FG_{\ell,m,p,q}\cos(-q\omega)}\right] \tag{3}$$

The relevant resonant terms (ℓ, m, p, q) are determined from the indicial equation:

$$\ell - 2p + q = m/s, \tag{4}$$

where s is the rational fraction representing the commensurability of the orbit in revolutions per day. For the 12 hour orbit, s = 2 and the dominant components on the highly eccentric satellite (e ~ .7) are:[10] (2, 2, 1, 1), (2, 2, 0, -1), (3, 2, 1, 0), (4, 2, 1, -1) and (4, 2, 2, 1). Together these terms account for all but at most 10% of the resonant acceleration on the 12 hour satellite. All but at most 20% of the acceleration on this orbit is accounted for by just the two (2,2) components. Furthermore, for orbits in the vicinity of critical inclination all but about 25% of the acceleration is accounted for by the single term (2, 2, 1, 1). Similarly, for equatorial orbits, the single term (2, 2, 0, -1) accounts for all but about 25% of the acceleration. Therefore it is both instructive and useful to work out these two simplest cases first, for low and high inclinations, as fairly good approximations to the drift regime.

The analysis that follows will only be concerned with determining the conditions for longitude equilibrium on these satellites. It must be born in mind that only for the cases near 'critical' inclination (63.4°) and for equatorial orbits, can such equilibrium conditions be maintained for long periods of time. As illustrated in figures 2 and 3, an 18° inclined orbit does not maintain it's stability in this sense while a 68 degree orbit does.

Low Inclination Approximation

In this case, the equation of motion, (1), of the longitude can be reduced to the simple pendulum equation (for constant ω, a, I and e), involving the term (2, 2, 0, -1) only:

$$\ddot{\lambda} = [C_1 \sin (2\lambda + \omega) + C_2 \cos (2\lambda + \omega)] . \tag{5}$$

The zero's of this equation give the equilibrium conditions for the orbit. Equilibrium exists when:

$$\lambda = \frac{1}{2} \mathrm{Tan}^{-1} \frac{-[C_1 \sin \omega + C_2 \cos \omega]}{C_1 \cos \omega - C_2 \sin \omega}$$

But $C_1 = KC_{2,2}$ and $C_2 = -KS_{2,2}$, where; $K = 24\pi^2 F_{2,2,0}$ (I) $G_{2,0,-1}$ (e)/a, for the 12 hour orbit. Also, we have from the definition of the harmonic coefficients:[10] $C_{2,2} = J_{2,2} \cos 2\lambda_{2,2}$, $S_{2,2} = J_{2,2} \sin 2\lambda_{2,2}$, where, from current values,[12] $\lambda_{2,2} = -15°$. With these substitutions for C_1 and C_2, the equilibrium longitudes are given as:

$$\lambda_E = \frac{1}{2}\,\mathrm{Tan}^{-1}\left[\frac{\cos 2\lambda_{22}\sin\omega - \sin 2\lambda_{22}\cos\omega}{\cos 2\lambda_{22}\cos\omega + \sin 2\lambda_{22}\sin\omega}\right]$$

$$= \frac{1}{2}\,\mathrm{Tan}^{-1}\,\frac{\sin(2\lambda_{22}-\omega)}{\cos(2\lambda_{22}-\omega)} \qquad (6)$$

Equation (6) has four solutions:

$$\lambda_E = \lambda_{22} - \omega/2,\ \lambda_{22} - \omega/2 + 90°,\ \lambda_{22} - \omega/2 + 180° \text{ and } \lambda_{22} - \omega/2 + 270°.$$

However since the 12 hour orbit always has two mean longitudes 180° apart (corresponding to the two ascending equator crossings), only two of these solutions are distinct, for example:

$$\lambda_E = \lambda_{2,2} - \omega/2 \text{ and } \lambda_{2,2} - \omega/2 + 90°. \qquad (7)$$

One of these equilibrium orbits is stable and the other unstable. To find which is which, we note (from Figure 5) that the zeros of $\ddot{\lambda}$ where $d\ddot{\lambda}/d\lambda < 0$ are stable longitudes, while those for which $d\ddot{\lambda}/d\lambda > 0$ are unstable. Differentiating (5) with respect to λ and substituting the first solution of (7), we have:

$$\frac{d\ddot{\lambda}}{d\lambda} = 2\,[C_1 \cos 2\lambda_{2,2} - C_2 \sin 2\lambda_{2,2}] = 2KJ_{2,2}\,[\cos^2 2\lambda_{2,2} + \sin^2 2\lambda_{2,2}]$$

$$= 2KJ_{2,2}.$$

Since $J_{2,2} > 0$, $d\ddot{\lambda}/d\lambda$ for this first equilibrium solution has the sign of K, or the sign of $F_{2,2,0}$ (I) $G_{2,0,-1}$(e). But $F_{2,2,0}$ (I) = 3 (1 + cos I)^2/4 $\gtrless$ 0, and $G_{2,0,-1}$ (e) = $-e/2 + e^3/16 + \ldots \lessdot 0$ (see Reference 9). Therefore, $\lambda_E = \lambda_{2,2} - \omega/2$ is a stable equilibrium longitude and similarly it can be shown that $\lambda_E = \lambda_{2,2} - \omega/2 + 90°$ is an unstable equilibrium longitude.

R. R. Allan in 1967 placed the stable perigee longitudes for the equatorial 12 hour eccentric orbit at $\lambda_{2,2}$ and $\lambda_{2,2}$ + 180° (the redundant position).[6]

We can check this placement by the calculation above with reference to Figure 6. For an equatorial orbit, the geographic longitude of perigee is $\pi_L' = \omega + N - \theta_e$, when the satellite (s) is at perigee (or M = 0). But when the satellite is at perigee, the mean longitude is $\lambda = \omega/2 + N - \theta_e$. This longitude is stable when $\lambda = \lambda_{2,2} - \omega/2$. Therefore, the stable condition implies:

$$\lambda_{2,2} - \omega/2 = \omega/2 + N - \theta_e; \text{ or}$$

$$\lambda_{2,2} = \omega + N - \theta_e = \pi_L',$$

as first postulated by Allan.[6]

High Inclination Approximation

In this case, the equation of motion of the mean longitude, (1), involves only the term (2, 2, 1, 1) and is formally the same as (5) with $-\omega$ substituted for ω in the sine and cosine arguments. Equilibrium values are therefore at:

$$\lambda_E = \lambda_{2,2} + \omega/2, \text{ and } \lambda_{2,2} + \omega/2 + 90^\circ. \tag{8}$$

By a calculus entirely similar to the previous case, recognizing that: $F_{2,2,1}$ (I) $= 3 \sin^2 I/2 > 0$, and $G_{2,1,1}$ (e) $= 3e/2 + 27 e^3/16 + \ldots > 0$, it can be shown that $\lambda_E = \lambda_{2,2} + \omega/2$ is the unstable equilibrium longitude and $\lambda_E = \lambda_{2,2} + \omega/2 + 90°$ is the stable equilibrium longitude. Figure 7 shows these approximate locations of the stable mean longitudes for low and high inclinations.

For intermediate inclinations the situation is more complicated. As mentioned before no geostationary disturbing potential exists for the general inclination case. Therefore the equilibrium longitudes derivable for them are not valid over the long term. In fact the time of validity depends on amount of rotation of the line of apsides. This can be appreciated from Figure 7 since for each of these harmonic terms, a full range of perigee arguments implies a full range of 'stable' longitudes. For intermediate inclination 12 hour orbits, the period of apsidal rotation is of the order of a few years (Figure 8). In addition,for intermediate inclinations, more than one harmonic term has appreciable influence. But for inclinations fairly near critical, which are most useful in the applications, only the two terms (2, 2, 0, -1) and (2, 2, 1, 1) carry all but about 10% of the effect. Therefore it will be instructive and practical to calculate the approximate quasi-equilibrium longitudes from this particular combination.

Intermediate Inclination Approximation

The equation of motion (1) including the two terms (2, 2, 0, -1) and (2, 2, 1, 1) is:

$$\ddot{\lambda} = \frac{24\pi^2}{a} (FG)_{2,2} \, [C_{2,2} \sin(2\lambda + \gamma_{2,2}) - S_{2,2} \cos(2\lambda + \gamma_{2,2})], \tag{9}$$

radians/sidereal day^2,

where $(FG)_{2,2}$ and $\gamma_{2,2}$ are functions of I, e, and ω. The vector interpretation of these functions [defined in equations (2) and (3)] permits them to be easily resolved by trigonometry (Figure 9). The individual component amplitudes (FG's) in this figure all are absolute values. Thus, using the law of cosines we have:

$$(FG)_{2,2} = + \, [FG^2_{2,2,1,1} + FG^2_{2,2,0,-1} - 2\, FG_{2,2,1,1} \, FG_{2,2,0,-1} \cos\omega]^{1/2}. \tag{10}$$

Using the law of sines:

$$\frac{FG_{2,2,0,-1}}{\sin(-\gamma_{22} - \omega)} = \frac{(FG)_{2,2}}{\sin 2\omega}, \text{ determines } \gamma_{2,2} \text{ as:}$$

$$\gamma_{2,2} = -\left\{\omega + \sin^{-1}\left[\frac{FG_{2,2,0,-1} \sin 2\omega}{(FG)_{2,2}}\right]\right\} \tag{11}$$

$$= -\left\{\omega + \sin^{-1}\left[\frac{\sin 2\omega}{(X^2 + 1 - 2X\cos 2\omega)^{1/2}}\right]\right\},$$

where $X = |FG_{2,2,1,1}| / |FG_{2,2,0,-1}|$. Table 1 gives values of x as a function of inclination taken from the graphs in reference 10.

The zero's of (9), giving the 'equilibrium' longitudes, are found from the equation:

$$\cos 2\lambda_{2,2} \sin(2\lambda + \gamma_{2,2}) = \sin 2\lambda_{2,2} \cos(2\lambda + \gamma_{2,2}),$$

or;

$$\mathrm{Tan}\,(2\lambda + \gamma_{2,2}) = \mathrm{Tan}\, 2\lambda_{2,2}.$$

The 2 unique solutions to this equation are:

$$\lambda_E = \lambda_{2,2} - \gamma_{2,2}/2, \text{ and } \lambda_{2,2} - \gamma_{2,2}/2 + 90°. \tag{12}$$

It can be shown that the second solution is the one for stable equilibrium. Thus, for the intermediate inclination case (second approximation):

$$\lambda_E\,(\mathrm{Stable}) = \lambda_{2,2} + \omega/2 - \delta(\omega, I), \tag{13}$$

where δ is one half the arc sine function in equation (11). δ is a small sinusoidal function for X >> 1. For example, for the most useful case of the critical inclination satellite of high eccentricity (e = .725, perigee height = 930 km); X = 6.20 and δ (ω, 63.4°) is plotted in Figure 10.

Figure 11 shows the variation of λ_E (stable) with argument of perigee, for intermediate inclinations, under various approximations. The simplest, straight line solution is most accurate for high inclinations and considers the (2, 2, 1, 1) term only. The second approximation (including the (2, 2, 0, -1) term) is shown for the critical inclination case only. Third approximations (including the next 3 most influential terms (3, 2, 1, 0), (4, 2, 1, -1) and (4, 2, 2, 1)) for I = 50°, 60° and 70° are also shown in Figure 11. These are calculated directly from the zero's of (1) for these terms. The gravity field used for these calculations is the Smithsonian Standard Earth I, M1 Field.[11] Even at 50° inclination the maximum deviation from the straight line solution is only 17°. Consideration of higher degree and order terms will add or subtract less than 10% of 17° or less than 2° to those equilibrium longitudes. Inclinations less than 50° are not considered in Figure 11 because, as mentioned previously, the apsidal rotation is too rapid for the 'equilibrium' longitudes to be considered anything but momentary values. However, the near-equatorial satellite is a special case, as Allan found.[6] For it, the apsidal rotation is in the same sense as the earth's rotation and a stationary geographic trace is assured with proper adjustment of the mean motion. As inferred previously, near equatorial 12 hour eccentric orbits are stable if one of their periapses is within a few degrees of $\lambda_{2,2}$ or 15° west.

For the applications it remains to calculate the actually geographic position of these 'stable' orbits when the 'stable' mean longitudes are given (as in Figure 11). This will be done in the following section by finding the relation between the actual ascending equator crossing (AEC) longitude and the mean longitude (λ) for a commensurate (or resonant) orbit.

THE ASCENDING EQUATOR CROSSING LONGITUDE

Consider the arguments in the orbit and equatorial planes from Figure 12. γ is the vernal equinox, G is the position of Greenwich at epoch, when the satellite is at SAT. G_0 is the Greenwich position at a time $T_p + T_\omega$ earlier, when the satellite is at the ascending node (AN). θ_e is the right ascension of Greenwich, ω is the argument of perigee and M is mean anomaly, all at epoch. When the satellite is at AN, λ_{AEC} is the longitude of the ascending equator crossing. We make two simplifying assumptions; one, that over the time $T_p + T_\omega$ the node and perigee are stationary and two, the commensurate mean motion of the satellite, n, is $s\dot{\theta}_e$.

From Figure 12:

$$\lambda_{AEC} = N - \theta_e + \dot{\theta}_e (T_p + T\omega) .$$

But since $T_p = M/n$, and $n = s\dot{\theta}_e$,

$$\lambda_{AEC} = N - \theta_e + 1/s \ (M + T_\omega n) . \tag{13}$$

The quantity $T_\omega n$ may be called the mean anomaly equivalent of ω,(ω_M). (i.e. it is the angle swept out at the average rotation rate in the actual time for the satellite to go from AN to π.) We recall[5], [7] that the definition of the mean longitude (λ) for a general commensurate orbit is:

$$\lambda = N - \theta_e + 1/s \ (M + \omega) \tag{14}$$

Comparing (13) and (14):

$$\lambda_{AEC} = \lambda + 1/s\ (\omega_M - \omega). \tag{15}$$

Let us call the quantity $\omega_M - \omega$, the perigee excess ($\Delta\omega$). Evidently $\Delta\omega$ is zero for a circular orbit and also zero for eccentric orbits when $\omega = 0°$ and $180°$. $\Delta\omega$ is clearly some sinusoidal function of ω for a given e. The function $\Delta\omega$ can be calculated from Keplers equation:

$$M = E - e \sin E,$$

and the relationship between the true anomaly (f) and the eccentric anomaly (E),

$$E = 2\ \mathrm{Tan}^{-1}\left[\left(\frac{1-e}{1+e}\right)^{1/2}\ \mathrm{Tan}\ f/2\right].$$

In terms of the mean and eccentric anomaly equivalents of perigee (ω_M and ω_E), these relations become:

$$\omega_E = 2\ \mathrm{Tan}^{-1}\left[\left(\frac{1-e}{1+e}\right)^{1/2}\ \mathrm{Tan}\ \omega/2\right] \tag{16}$$

$$\omega_M = \omega_E - e \sin \omega_E, \text{ and} \tag{17}$$

$$\Delta\omega = \omega_M - \omega. \tag{18}$$

It is noted that $\Delta\omega$ is an 'odd' function about $\omega = 180°$. The asymptote of $\Delta\omega$, when e = 1, are the parallel lines of -1 slope through $\omega = 0°$ and 360°. For use in applications for $e << 1$, $\Delta\omega \doteq - e \sin\omega$. For any eccentric commensurate orbit, the graph of $\Delta\omega$ as a function of ω and e is given (from evaluation of (16), (17) and (18)) in Figure 13.

Finally, figures 11 and 13 are combined (with equation (15)) to give (in figure 14) the 'stable' equilibrium equator crossing longitudes on 12 hour eccentric orbits. These are given for a range of inclinations useful for applications satellites (with an eccentricity = .725, giving a perigee altitude essentially out of the atmosphere at 930 km.) In Figure 14 is also shown the observed and computed 'stable' equator crossing longitudes for Molniya 12 which confirms the validity of this chart to within a few degrees.

STATION KEEPING AT NON EQUILIBRIUM LONGITUDES

Figure 15 shows the longitude acceleration on the maximum altitude 12 hour orbit of 50° inclination for ω = 270°, a typical orbit that has been considered advantageous in the SATS program.[1] Only the dominant terms ((2, 2, 0, -1), (2, 2, 1, 1), (3, 2, 1, 0), (4, 2, 1, -1) and (4, 2, 2, 1)) have been included in the evaluation of the acceleration from equation (1). The Smithsonian Standard Earth I, M1 gravity field[11] has been used in this computation.

It is interesting that the maximum resonant acceleration on this 12 hour satellite is of the same order of magnitude (1 to 10 $\times 10^{-5}$ rad/day) as that on the 24 hour satellite.[12] This rough 'equipartition' of the disturbing force among all the 'one day' resonances was first noted by Douglas et al. in 1968.[10] 'One day resonances have ground traces which repeat in one day. In reference [12], the maximum station keeping requirements on 24 hour satellites were estimated to be about 2 m/sec/yr. Because of equipartition we can expect the requirements to be the same order of magnitude for all one day resonances. However, some fuel savings can be realized on eccentric orbits by performing the maneuvers at perigee where a given change in velocity (ΔV) has the greatest effect on the energy of the orbit. The following estimate of the requirements assumes that all station-keeping thrusting occurs at perigee.

If we ask 'what is the longitude acceleration ($\ddot{\lambda}$) in a commensurate orbit due to a continual change in orbit period (T) of Δ T each commensurate period of n' days ?,' the answer is exactly the same as for the circular case:[13]

$$\ddot{\lambda} = -2\pi n' \frac{\Delta T}{T} \text{ radians}/(n' \text{ days})^2$$

$$= -2\pi (1/n') \frac{\Delta T}{T} \text{ radians/day}^2$$

This equation could be written without specified dimensions with $\dot{\theta}_e = 2\pi$. But from Keplers 3rd law:

$$\frac{\Delta T}{T} = \frac{3}{2} \frac{\Delta a}{a},$$

so that

$$\ddot{\lambda} = \frac{-3\pi (1/n')}{a} \Delta a \text{ rad/day}^2.$$

In the above equation the negative sign means that an increase in the semimajor axis causes the orbit to fall behind the rotating earth. We now need to determine only the ΔV required to change the semimajor axis Δa to complete the station keeping specifications. For this purpose we use the two-body energy integral for elliptic motion in the vis-viva form:

$$V^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right), \tag{19}$$

where μ is the central gravitational constant (3.986×10^5 km^3/sec^2), r is the distance to the satellite from the center of the earth and V is the satellite's velocity. Taking differentials of (19) with respect to V and a (holding r constant, at perigee (p));

$$2V_p \Delta V_p = \frac{\mu \Delta a}{a^2}.$$

But from (19) again (with $r_p = a(1 - e)$);

$$V_p = \left(\frac{\mu}{a}\right)^{1/2} \left(\frac{1 + e}{1 - e}\right)^{1/2}.$$

Therefore, a perigee velocity change (ΔV_p) results in a semimajor axis change, Δa, according to the formula;

$$\Delta a = 2(a^3/\mu)^{1/2} \left(\frac{1 + e}{1 - e}\right)^{1/2} \Delta V_p.$$

Substituting this equation into the acceleration equation (and reversing the sign) gives the station keeping requirements (for perigee maneuvers) as:

$$\Delta V_p = \frac{(n')\,(\mu/a)^{1/2}}{6\pi} \left(\frac{1 - e}{1 + e}\right)^{1/2} \ddot{\lambda}. \tag{20}$$

Equation (20) gives the minimum ΔV requirements for a commensurate period of n' sidereal days in length/time units, if $\ddot{\lambda}$ is given in units of radians/sidereal day^2 and n' is given in sidereal days. The sign is reversed because the station keeping maneuver must be in an opposite sense to the ΔV rate equivalent to the acceleration, $\ddot{\lambda}$, caused by the geopotential. Therefore, the yearly minimum ΔV requirement for station keeping is $366/n'$ times ΔV_p or:

$$\Delta V(\text{minimum}) = \frac{366\,(\mu/a)^{1/2}\left(\frac{1-e}{1+e}\right)^{1/2}}{6\pi}\,\ddot{\lambda},$$

$$= 1.225 \times 10^{7}\,(a^{-1/2})\left(\frac{1-e}{1+e}\right)^{1/2}\ddot{\lambda},\ \text{m/sec/yr.,} \qquad (21)$$

If a is given in km and $\ddot{\lambda}$ in radians/sidereal day^2.

For example, the maximum altitude 12 hour orbit has the specifications $a = 26{,}550$ km, and $e = .725$. If $\ddot{\lambda} = 5 \times 10^{-5}$ rad/days, ΔV (minimum), (from (21)) is found to be 1.51 m/sec/yr or 4.95 ft/sec/yr. This calculation establishes the scale of the velocity requirement in Figure 15. Positive station keeping ΔV's are to be made along the track, while negative ΔV's are to be made against the track of the orbit.

As another example, consider the orbit proposed in Figure 1 for the small applications technology satellite (SATS) program. This orbit (of 50° inclination) has an argument of perigee of about 270° and ascending equator crossing (AEC) longitudes at 136° west and 45° east.

From Figure 14, the 'stable' ascending equator crossing (AEC) longitudes for this (maximum altitude) orbit are at 60° east and 120° west. Thus the mean longitudes (λ) of this orbit are displaced 15° west of the 'stable' mean longitudes. From Figure 15 (or Figure 11) the 'stable' mean longitudes of this orbit are at 27° and 207° east. Therefore, the mean longitudes of the given orbit are at 27 − 15 = 12° east and 207 − 15 = 192° east.

A simpler calculation of the mean longitude (λ) of this orbit can be made with the use of equation (15):

$$\lambda = \lambda_{AEC} - \frac{1}{s} \Delta\omega.$$

For this 12 hour orbit; $s = 2$, $e = 0.725$, $\omega = 270°$ and $\Delta\omega$ (from Figure 13) = 66°. Thus, $\lambda = 45 - 66/2 = 12°$ or 192° as found previously. From Figure 15, the resonant acceleration on this orbit is 5×10^{-5} rad./day^2 and the station keeping requirements are 1.5 m/sec/yr.

SUMMARY AND CONCLUSIONS

The eccentric 12 hour orbit has already been one of the most useful orbits for earth oriented applications. The United States intends to make further use of these orbits for many applications in the decade of the 1970's and beyond. Since these orbits are strongly perturbed by resonant terms in the geopotential, it is of interest to estimate these effects, and, if possible minimize them for a particular mission.

In general it is found that:

1. The longitude acceleration on these quasi-stationary satellites is highly dependent on the argument of perigee.
2. For each (equatorial and critically inclined) orbit specification (e, ω) only two longitudes for the placements of the equator crossing result in orbits which are in equilibrium with the resonant geopotential forces. One of these orbits is in stable equilibrium with these forces and the other is in unstable equilibrium with them.
3. For intermediate inclinations, the 'stable' longitudes change slowly with time (as the argument of perigee changes) as long as the orbit inclination is within about 15 degrees of critical. The stability of the equatorial orbit rapidly degenerates for inclinations above 10 degrees.

Specifically it is found that for the maximum altitude 12 hour orbit of intermediate inclination (50° → 70°):

1. 'Stable' orbits exist for ascending equator crossing longitudes between 55° and 85° east, as long as the argument of perigee is within ±100°.
2. The maximum station keeping requirements to keep the geographic trace constant against resonant geopotential forces are in the range of .3 to 3 m/sec/yr.

REFERENCES

1. E. W. Hymowitz, J. H. Conn, W. A. Hovis and others, "Small Applications Technology Satellite, Program and Spacecraft Study Report," NASA GSFC-X-730-70-350, Greenbelt, Md., Sept., 1970

2. B. E. Shute and J. Cheville, "The Lunar-Solar Effect on the Orbital Lifetimes of Artificial Satellites with Highly Eccentric Orbits," Planet. Space Sci. 14, 361 (1966)

3. G. E. Cook and Diana W. Scott, "Lifetimes of Satellites in Large Eccentricity Orbits," Planet. Space Sci. 15, 1549, (1967)

4. Eileen Quinn, D. G. King Hele and J. A. Pilkington, "Table of Artificial Earth Satellites," Royal Aircraft Establishment, Farnborough, Hants., England, April 1968

5. G. S. Gedeon, B. C. Douglas and M. T. Palmiter, "Resonance Effects on Eccentric Satellite Orbits," The Journal of the Astronautical Sciences 14, 147, July-August 1967

6. R. R. Allen, "Satellite Resonance with Longitude-Dependent Gravity-II, Effects Involving the Eccentricity," Planet. Space Sci. 15, 1829, (1967)

7. C. A. Wagner, "Determination of Low Order Resonant Gravity Harmonics From the Drift of Two Russian 12-Hour Satellites," Journal of Geophysical Research, 73, 4661, (1968)

8. C. A. Wagner, "Combined Solution for Low Degree Longitude Harmonics of Gravity from 12- and 24-Hour Satellites," Journal of Geophysical Research, 73, 7651, (1968)

9. W. M. Kaula, Theory of Satellite Geodesy, Blaisdell Pub.; Waltham, Mass. (1966)

10. B. C. Douglas, M. T. Palmiter and G. S. Gedeon, "Resonant Satellite Geodesy Study – Final Report," for NASA-GSFC under contract NAS-5-10469 (C. A. Wagner, Contract Monitor), Greenbelt, Md. (1968)

11. E. M. Gaposchkin, "Tesseral Harmonic Coefficients and Station Coordinates from the Dynamic Method," In: Smithsonian Astrophysical Observatory Special Report No. 200, Vol. 2; Cambridge, Mass. (1966)

12. C. A. Wagner, "The Earth's Longitude Gravity Field as Sensed by the Drift of Three Synchronous Satellites," NASA TN-D 3557, Wash. D.C. (1966)

13. C. A. Wagner, "The Use of Resonant Librating Orbits in Satellite Geodesy," NASA TN D-4081, Wash. D.C. (1967)

Table 1

Harmonic Amplitude Ratio (X) as a Function of Inclination

for High Altitude (High e) 12 Hour Satellites

	I	X
	0°	0
	10	0.123
	20	0.509
	30	1.16
	40	2.13
	50	3.54
	60	5.34
(Crit. Incl.)	63.4°	6.20
	70°	9.90

Data Derived from
Reference 10.

FIGURE CAPTIONS

Figure 1. Alternate Ground Trace for 12-Hour Orbit – 50° Inclination

Figure 2. Mean Longitude (λ) History for 1966 96 A (Intelsat 2F1)

Figure 3. Mean Longitude (λ) History for 1964 49E (Cosmos 41 Rocket)

Figure 4. Actual Ascending Crossing Longitudes (λ_{AEC}) for 1969 61A (Molniya 12)

Figure 5. Stability Conditions With Respect to the Longitude (λ) Drift

Figure 6. Equatorial Orbit Reference Points

Figure 7. Stable Equilibrium Longitudes (λ) for High Altitude 12 Hour Satellites

Figure 8. Period of Apsidal Rotation (T_p) for a High Eccentricity 12 Hour Satellite

Figure 9. Composite Harmonic Functions for Intermediate Inclination 12 Hour Satellites

Figure 10. Equilibrium Mean Longitude Change as a Function of Argument of Perigee for a 12 Hour Satellite

Figure 11. Stable Equilibrium Mean Longitudes (λ_E) for Maximum Altitude 12 Hour Satellites of Intermediate Inclination

Figure 12. Reference Points of Orbit and Equatorial Planes on the Celestial Sphere

Figure 13. Perigee Excess ($\Delta\omega$) for Eccentric Orbit Satellites

Figure 14. 'Stable' Ascending Equator Crossing Longitudes (λ_{AEC}) for 12 Hour Satellites

Figure 15. Acceleration and Station-Keeping Requirements on a 12 Hour Satellite

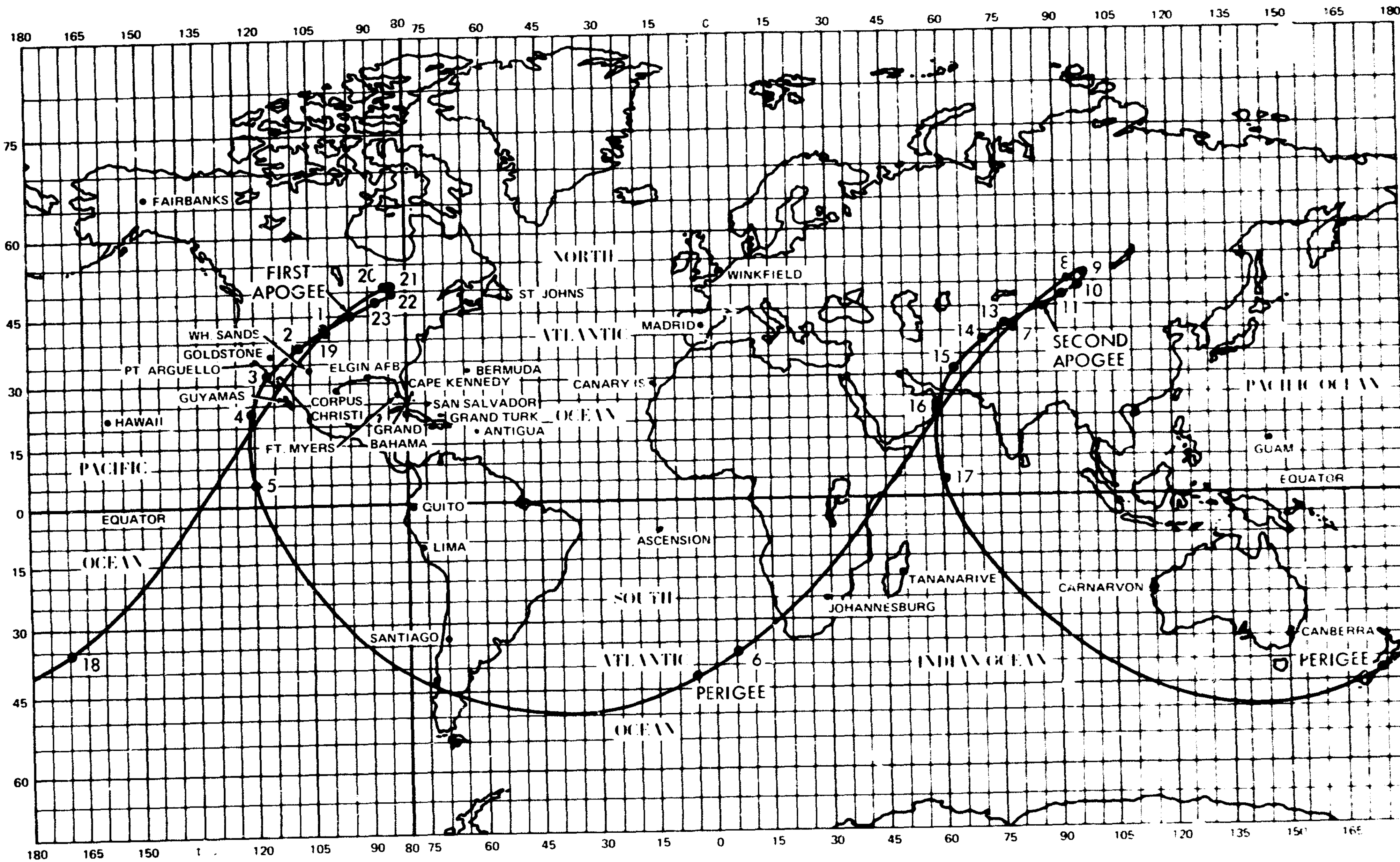


Figure 1. Alternate Ground Trace for 12-Hour Orbit – 50° Inclination

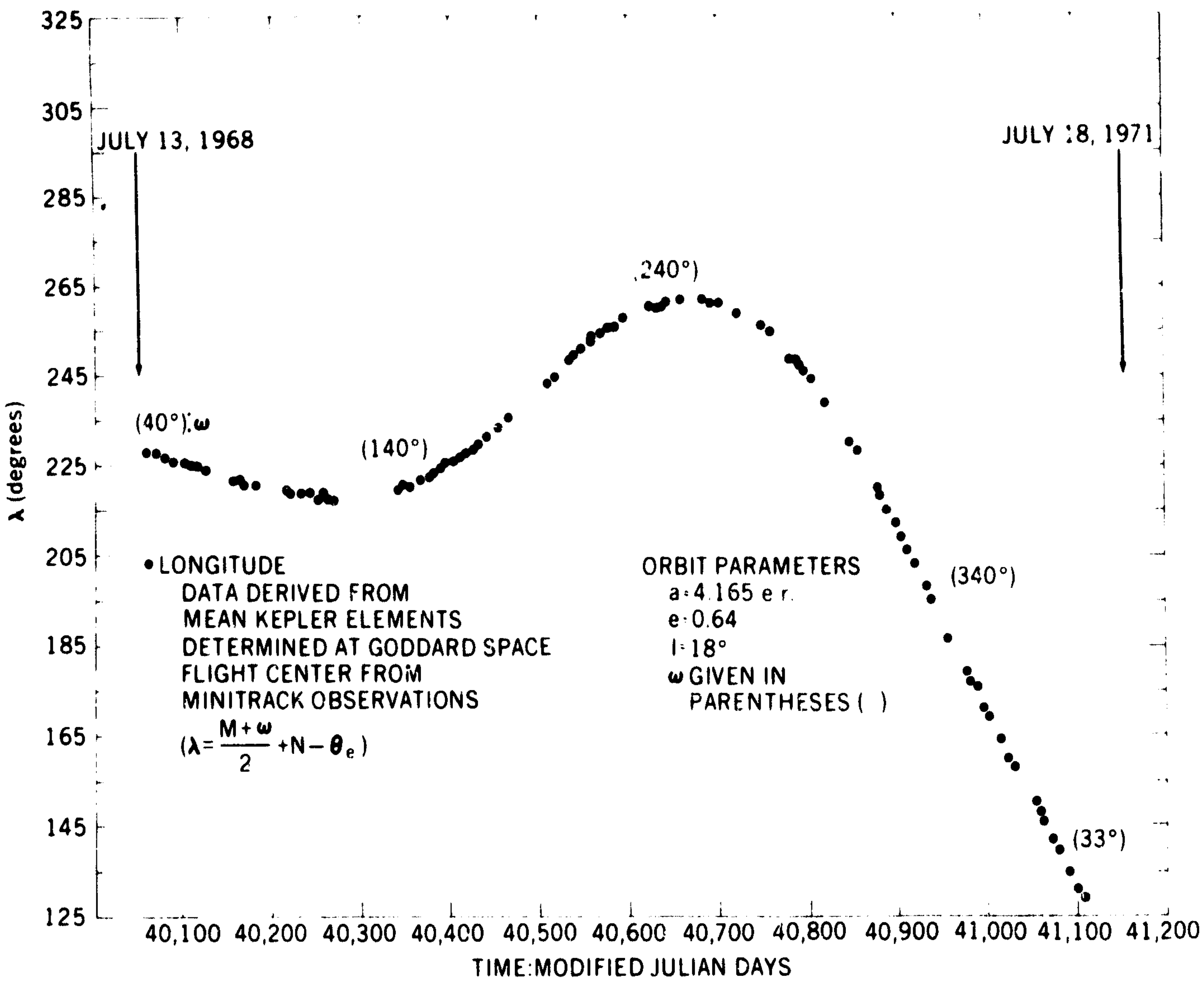


Figure 2. Mean Longitude (λ) History for 1966 96 A (Intelsat 2 F1)

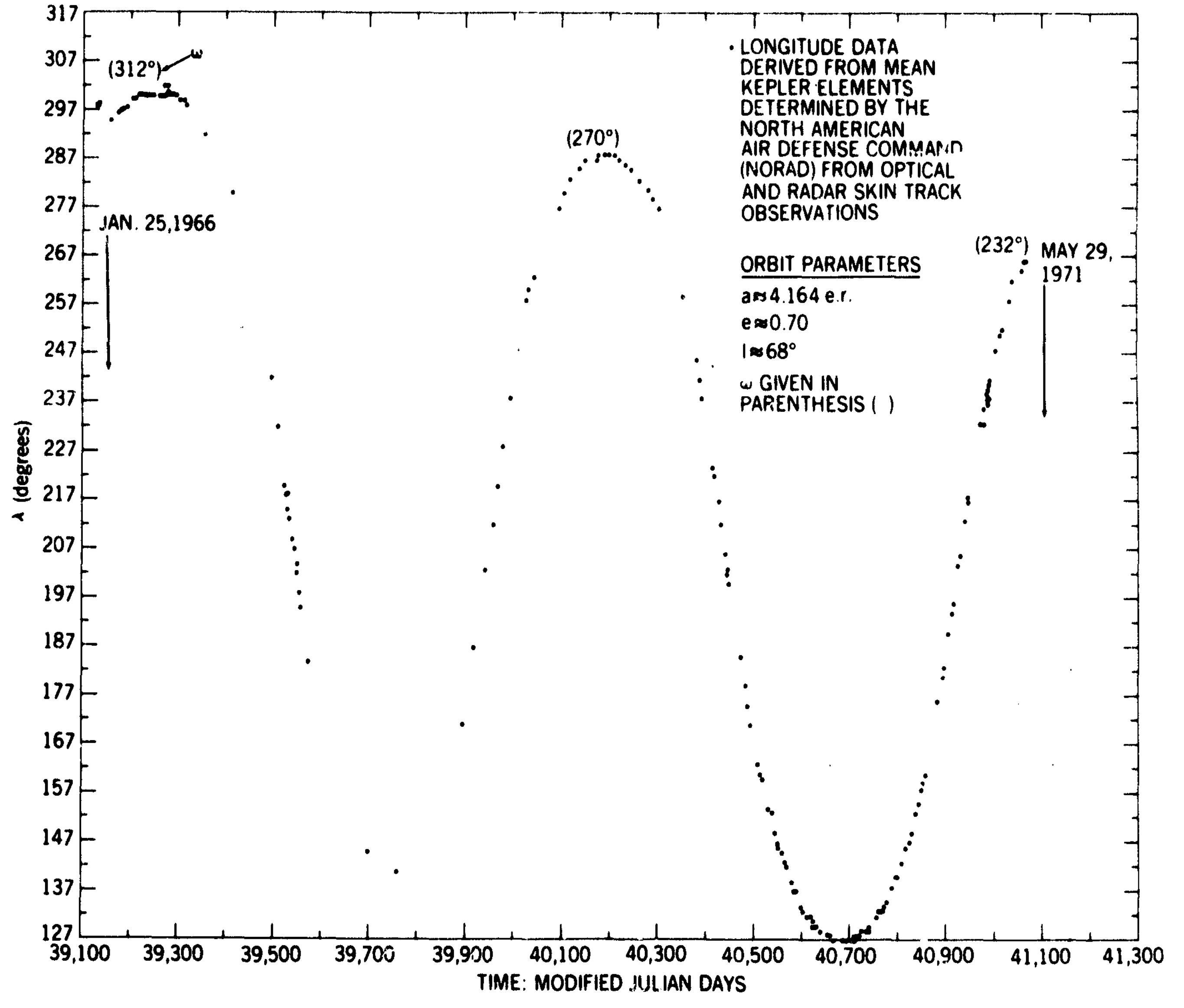


Figure 3. Mean Longitude (λ) History for 1964 49E (Cosmos 41 Rocket)

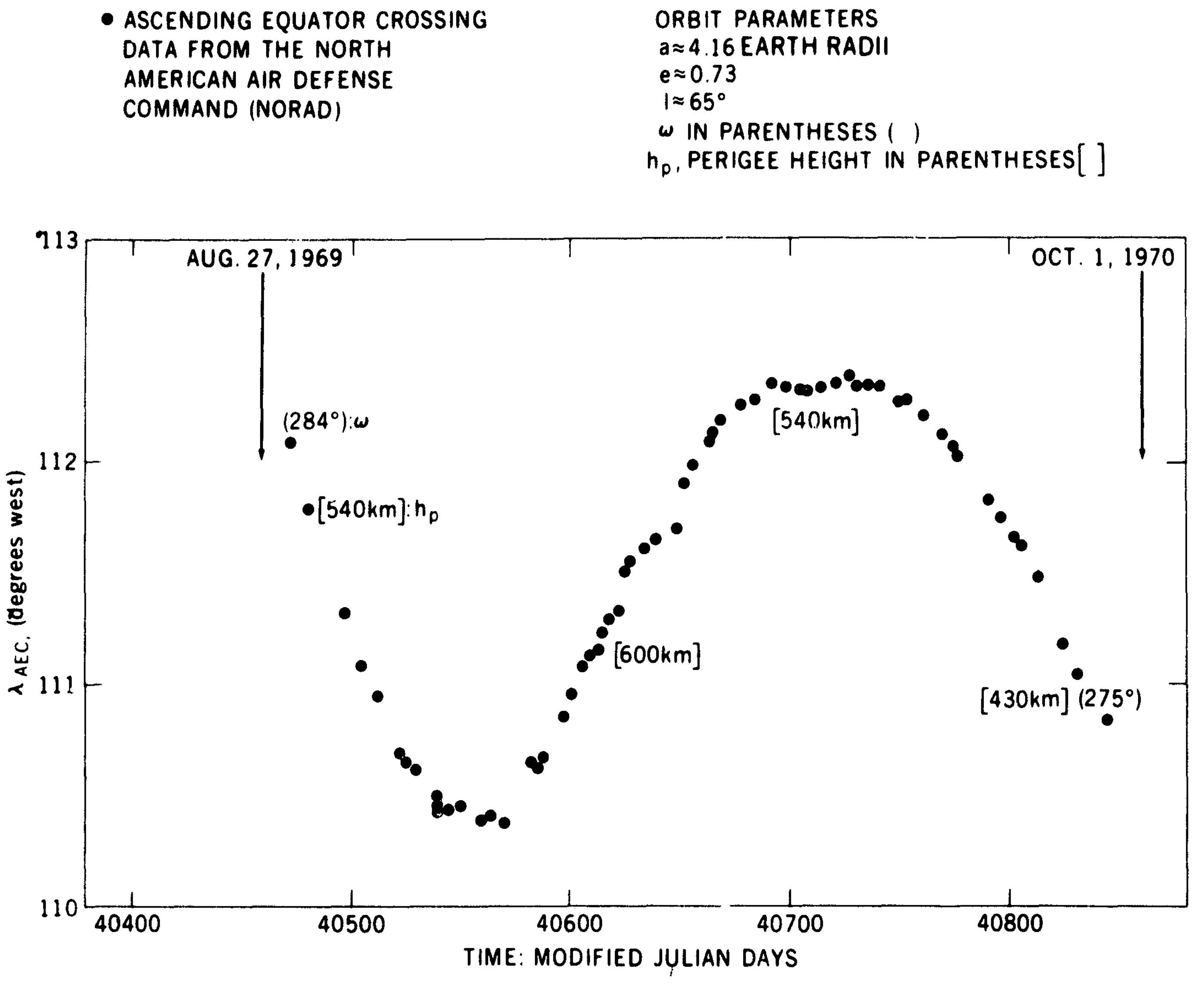


Figure 4. Actual Ascending Crossing Longitudes (λ_{AEC}) for 1969 61A (Molniya 12)

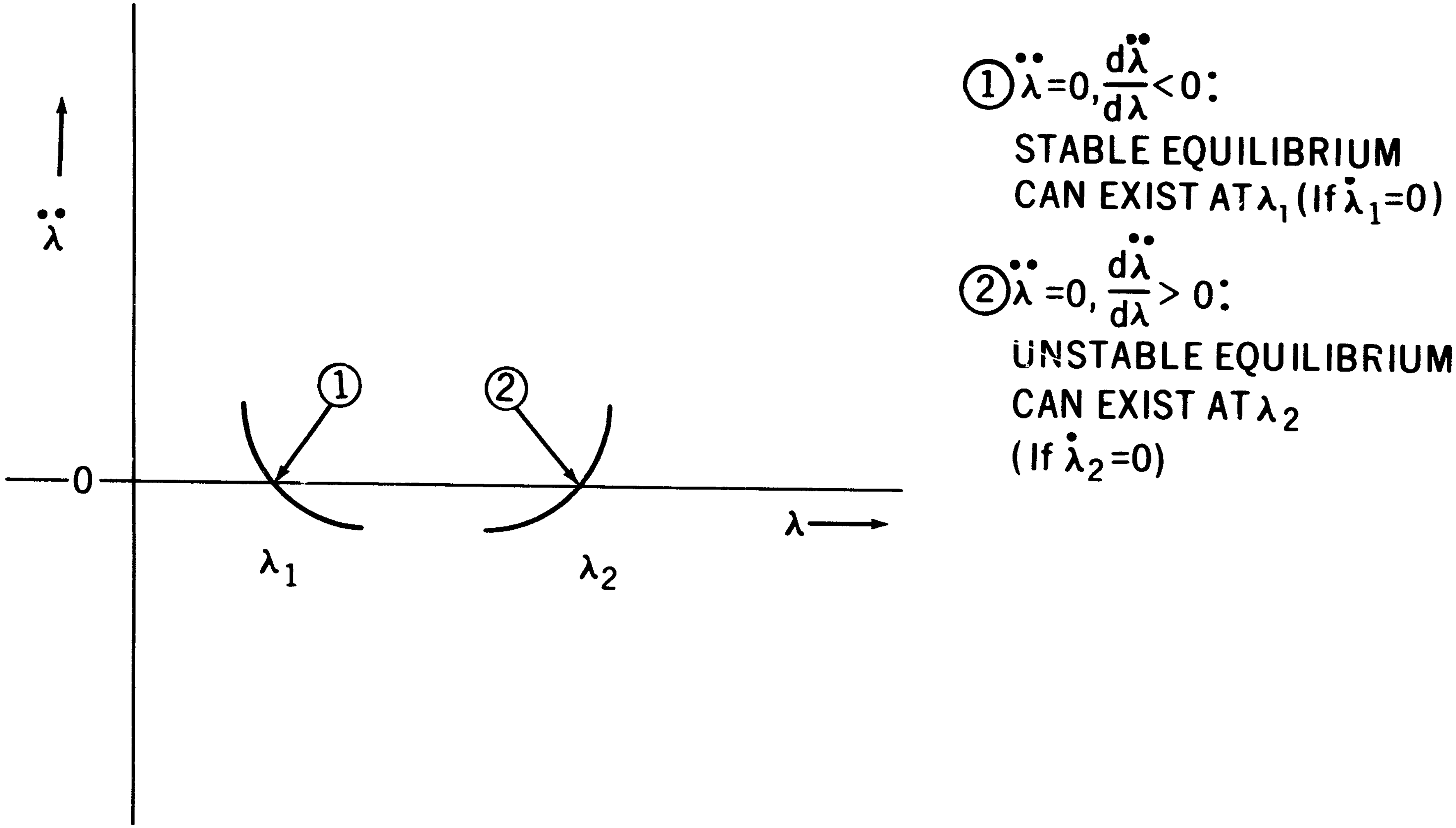
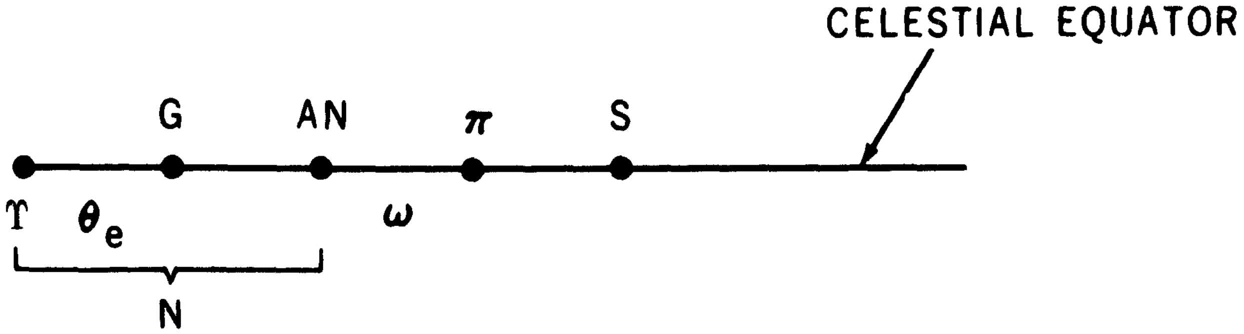


Figure 5. Stability Conditions With Respect to the Longitude (λ) Drift



♈ : VERNAL EQUINOX

θ_e : HOUR ANGLE OF GREENWICH (G) WHEN SATELLITE IS AT S

N: RIGHT ASCENSION OF THE ASCENDING NODE (AN); ARBITRARY FOR EQUATORIAL ORBIT

ω: ARGUMENT OF PERIGEE POINT (π) FROM AN.

Figure 6. Equatorial Orbit Reference Points

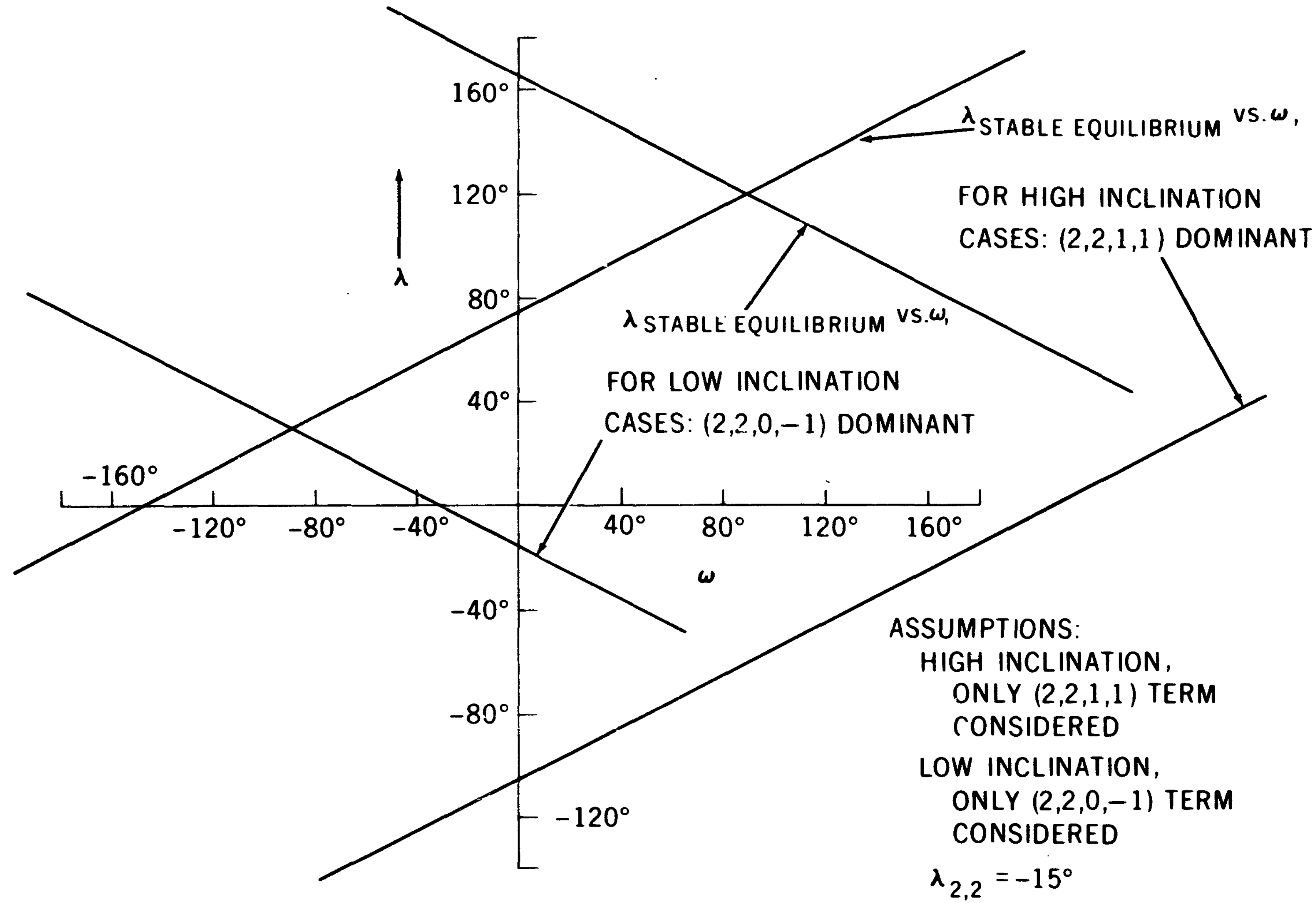


Figure 7. Stable Equilibrium Longitudes (λ) for High Altitude 12 Hour Satellites

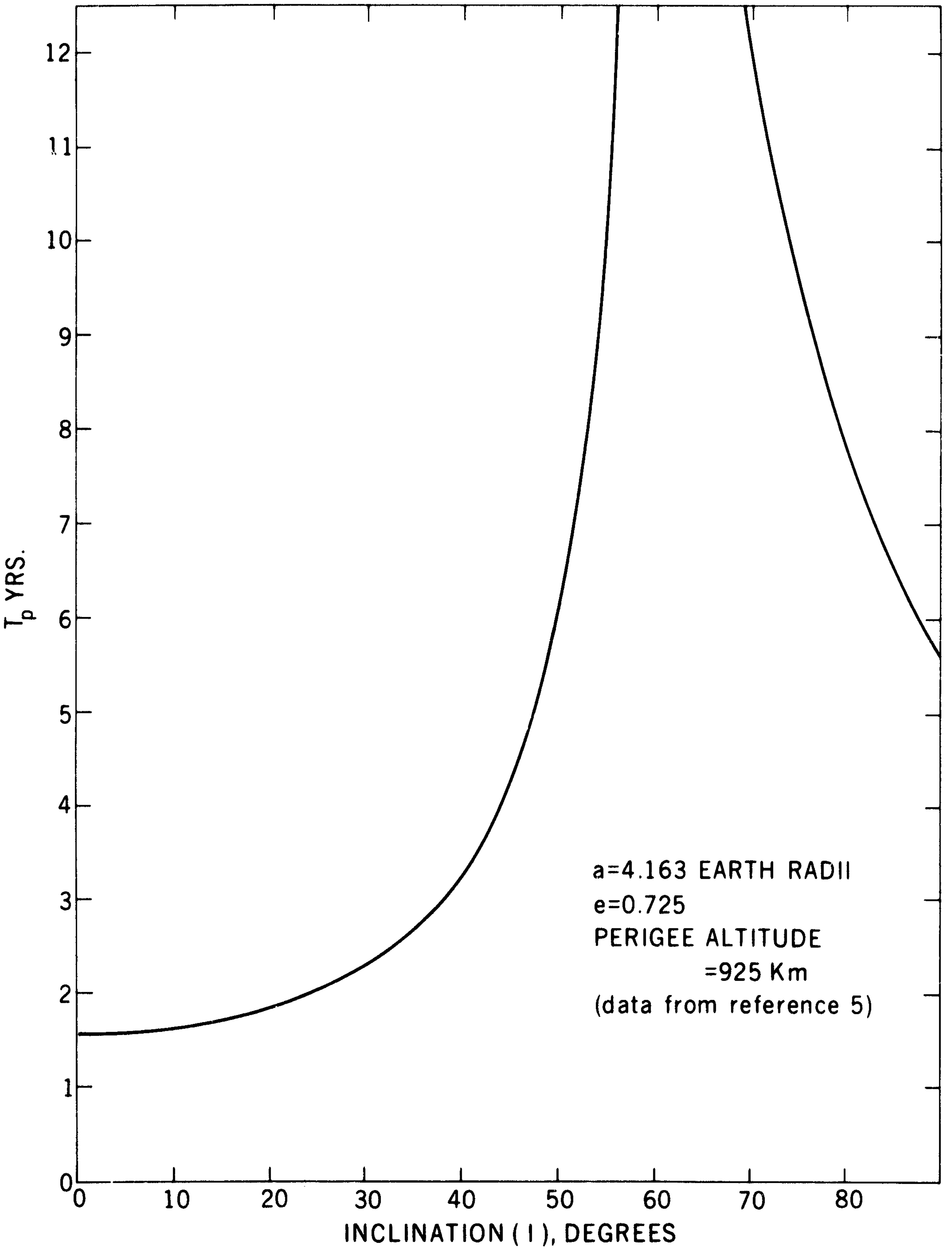


Figure 8. Period of Apsidal Rotation (T_p) for a High Eccentricity 12 Hour Satellite

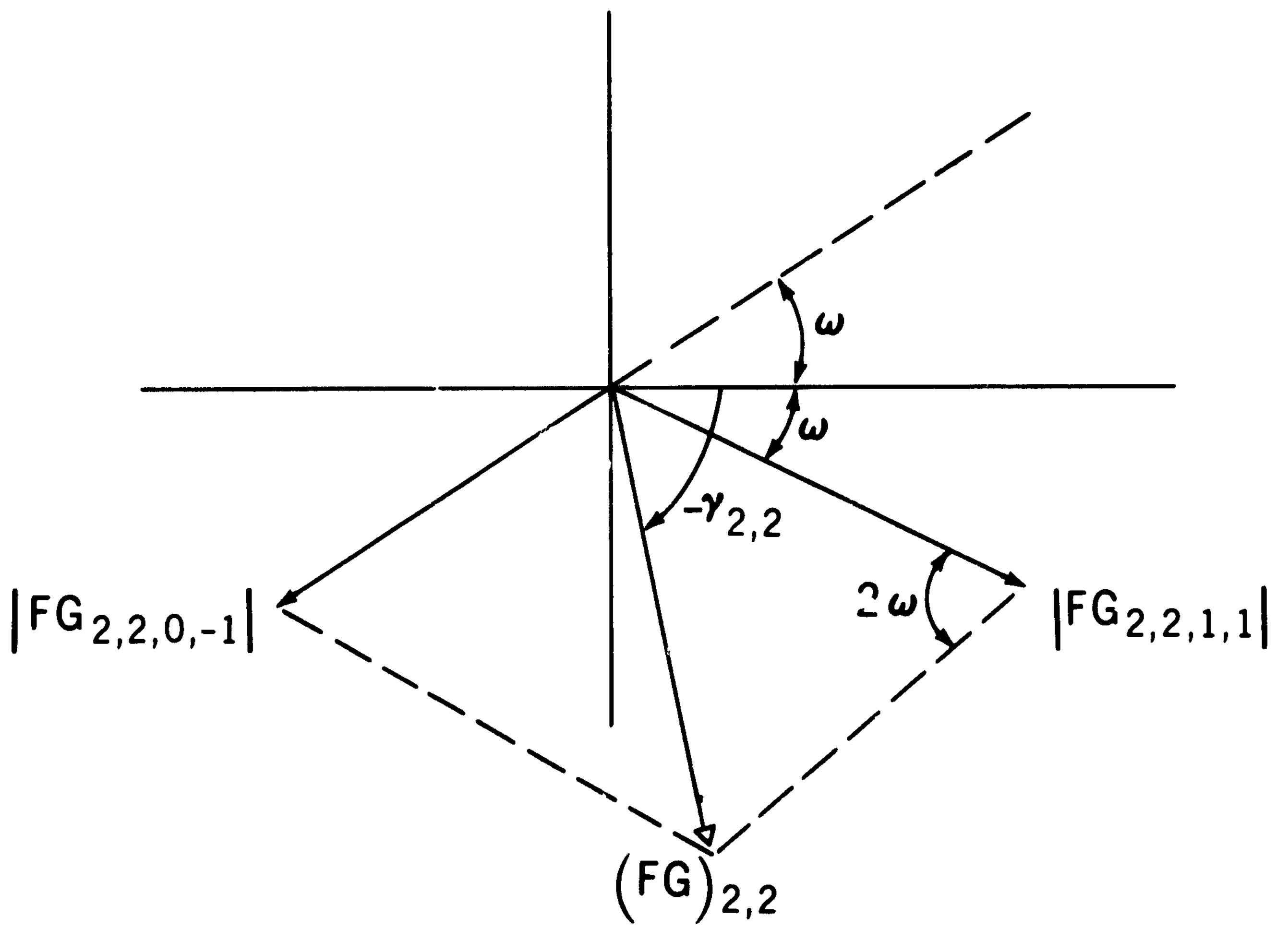


Figure 9. Composite Harmonic Functions for Intermediate Inclination 12 Hour Satellites

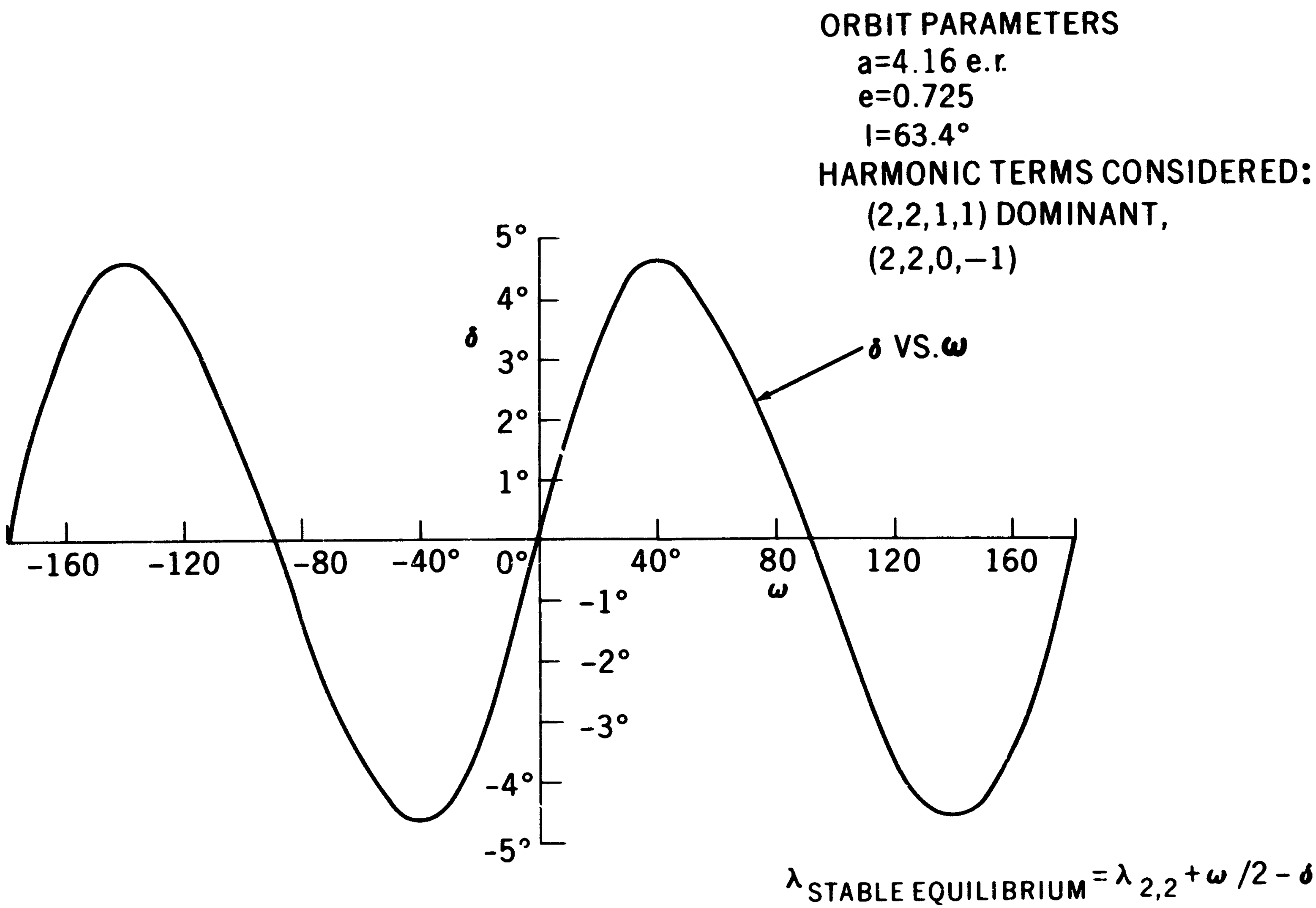


Figure 10. Equilibrium Mean Longitude Change as a Function of Argument of Perigee for a 12 Hour Satellite

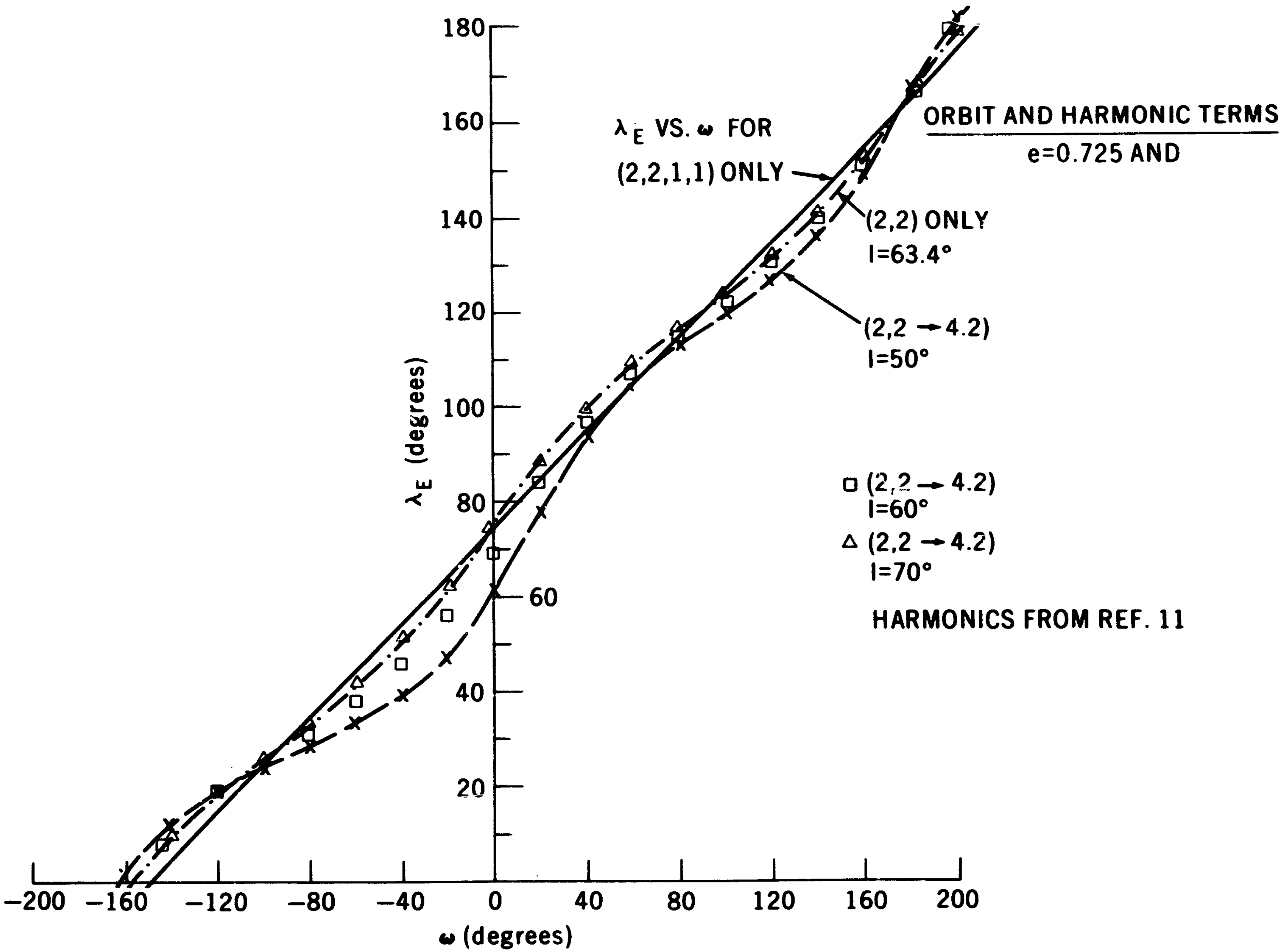


Figure 11. Stable Equilibrium Mean Longitudes (λ_E) for Maximum Altitude 12 Hour Satellites of Intermediate Inclination

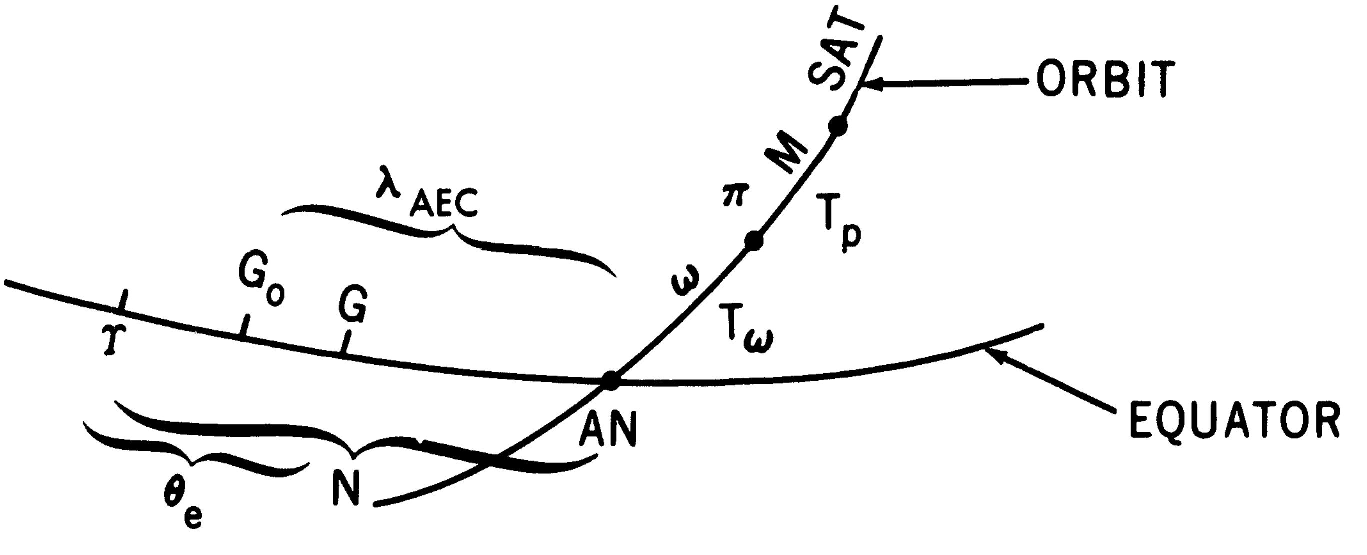


Figure 12. Reference Points of Orbit and Equatorial Planes on the Celestial Sphere

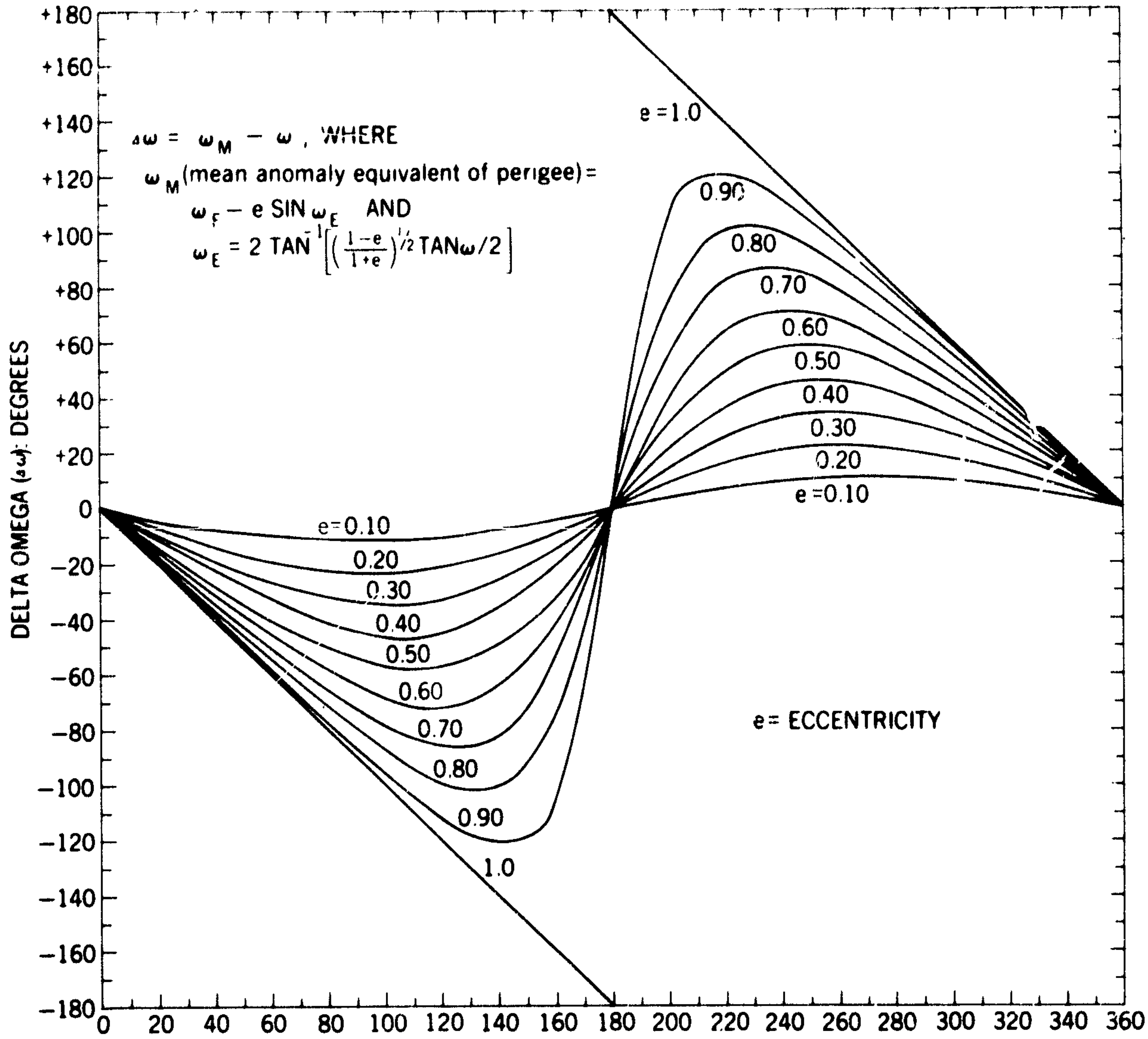


Figure 13. Perigee Excess ($\Delta\omega$) for Eccentric Orbit Satellites

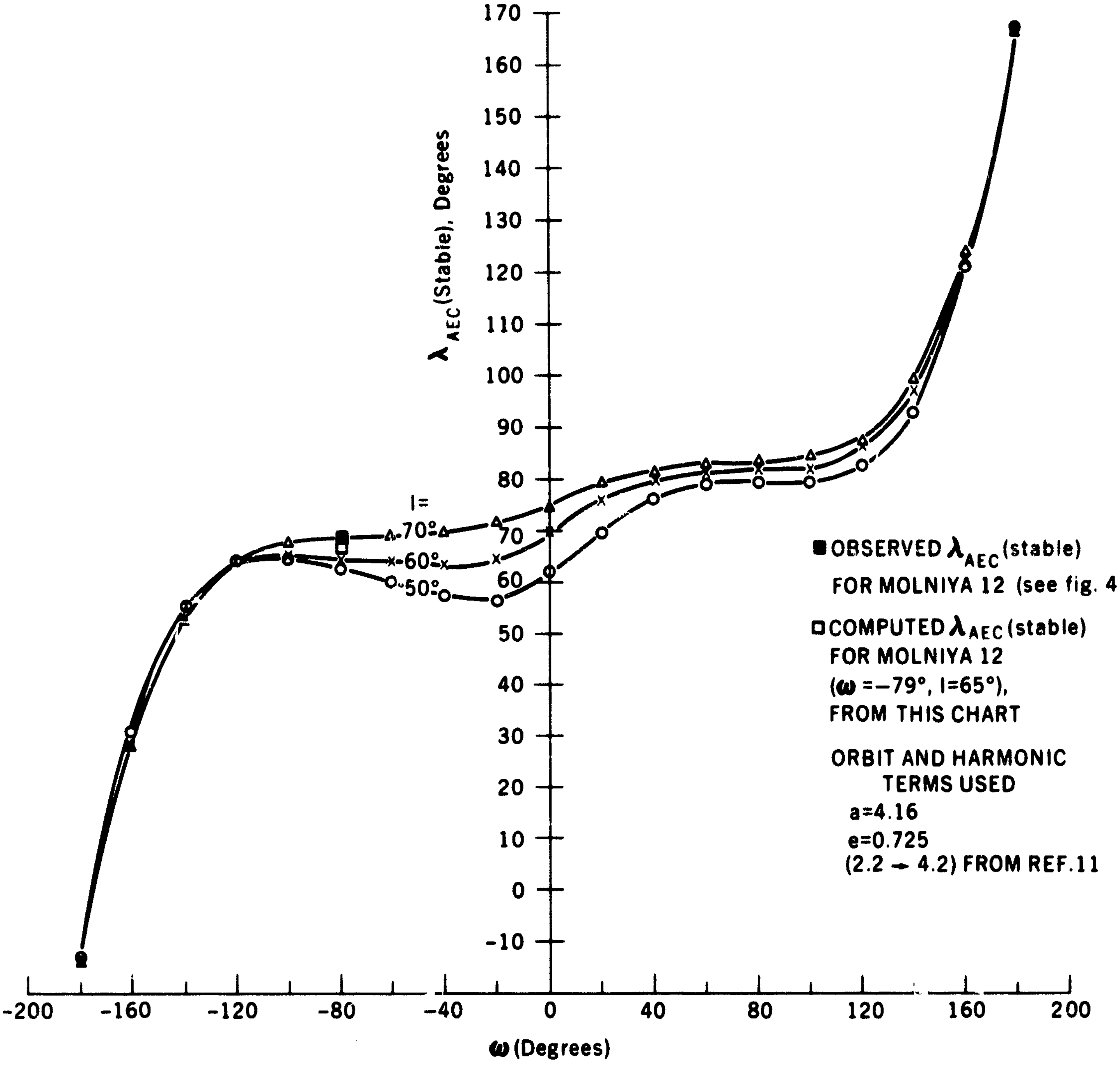


Figure 14. 'Stable' Ascending Equator Crossing Longitudes (λ_{AEC}) for 12 Hour Satellites

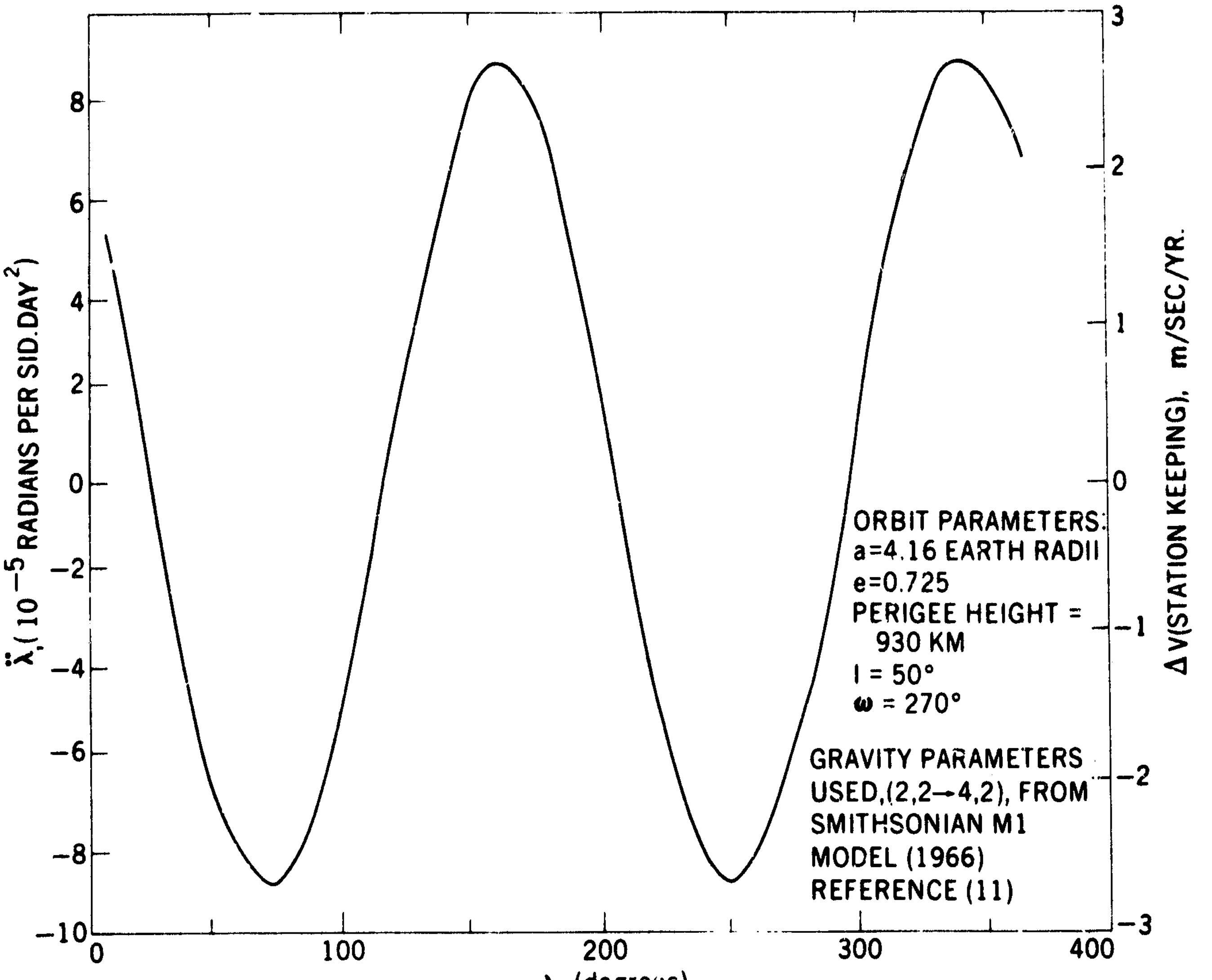


Figure 15 Acceleration and Station-Keeping Requirements on a 12 Hour Satellite