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RESEARCH MEMORANDUM

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A FLIGHT AND ANALOG STUDY OF THE EFFECT OF ELEVATING THE RADAR-BORESIGHT AXIS UPON STABILITY AND TRACKING PERFORMANCE OF AN AUTOMATICALLY CONTROLLED INTERCEPTOR

By Donald C. Cheatham and Charles W. Mathews

Langley Aeronautical Laboratory
Langley Field, Va.

[redacted]

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A FLIGHT AND ANALOG STUDY OF THE EFFECT OF ELEVATING THE RADAR-BORESIGHT AXIS UPON STABILITY AND TRACKING PERFORMANCE OF AN AUTOMATICALLY CONTROLLED INTERCEPTOR

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SUMMARY

Flight and analog-simulator tests have been made with a prototype automatic interceptor in order to study the effects of elevating the radar-boresight axis upon the stability and tracking performance of the system. The interceptor system was one that was designed to perform the lead-pursuit type of attacks, and test runs were made in flight and on the analog simulator both with and without lead-angle computation.

The results of the tests showed that elevating the radar-boresight axis had a marked stabilizing effect upon the system and greatly improved the tracking performance of the system. Elevating the radar-boresight axis had the effect of generating a geometric feedback that was equivalent (from a stability standpoint) to the use of bank-attitude feedback. The advantage of elevating the radar-boresight axis of the test system was limited to elevations of the order of 5° above the roll axis because higher elevations excited a 1-cycle-per-second lateral oscillation. Elevations of the order of $3\frac{1}{2}^{\circ}$ were, however, sufficient to allow elimination of the electrical bank-attitude feedback that was necessary for stable operation of the basic system, and as a result the system could track a turning target with small transient errors and zero steady-state errors.

INTRODUCTION

One of the more difficult problems associated with current automatic interceptor systems is the achievement of adequate dynamic stability. This problem results from the high automatic-control gain levels

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required for good tracking performance under conditions of high acceleration or changing acceleration such as occur in pursuit attacks from large angles off the target tail or as a result of target maneuvers. While progress has been made with prototype systems, more knowledge is needed concerning the effects that variations in certain basic-system parameters have upon the stability and performance of the system. One such parameter is the elevation of the radar-boresight axis. The radar-boresight axis is defined as the position of the radar-antenna axis in the plane of symmetry which produces no tracking-error signal. Since the radar-boresight axis is oriented approximately parallel to the gun line of an interceptor, changes in the elevation of one of these axes would necessitate corresponding changes in the other.

Studies of the effect of this parameter upon the tracking performance of an interceptor controlled by a human pilot (ref. 1) has shown that marked improvements were obtained as the gun line was elevated. In addition, reference 2 presents analog-computer results which indicate that elevating the radar-boresight axis should help to stabilize an automatically controlled interceptor. Briefly, this benefit is realized because rolling the interceptor can directly correct for azimuth errors without waiting for the interceptor to turn. It was desirable to see if these advantages could be realized in the case of an actual automatic interceptor where untoward effects of radar-antenna dynamics may exist.

This paper presents the results of flight tests of a prototype automatic interceptor in which the elevation of the radar-boresight axis was varied. In addition, analog-computer studies of the same automatic interceptor system are correlated with the flight tests.

SYMBOLS

b wing span, ft

$\bar{c}$ mean aerodynamic chord, in.

μ^* elevation of radar-boresight axis with respect to interceptor armament-datum line, deg (see fig. 6)

μ elevation of radar-boresight axis with respect to interceptor roll axis, deg

σ steering error (for zero lead-angle case, angular displacement of interceptor radar-antenna axis from radar-boresight axis), mils

ω_{LS} angular rate of line of sight, radians/sec

T_f time of flight of projectile fired from interceptor to target, sec

a acceleration, ft/sec^2

R range from interceptor to target, ft

Λ kinematic lead angle, radians

K constant

θ pitch angle, radians

ϕ bank angle, radians

δ_e elevator deflection, radians

δ_a aileron deflection, radians

g acceleration due to gravity, 32.2 ft/sec^2

V velocity, ft/sec

τ time constant, sec

ϵ error voltage

p Laplace operator, per sec

A dot above a quantity denotes differentiation with respect to time.

A prime above a quantity denotes that the quantity has been modified by feedbacks or a shaping network.

Subscripts:

F interceptor

B target

E elevation measurement in interceptor coordinates

D deflection measurement in interceptor coordinates

XZ vertical measurement in spacial coordinates

XY horizontal measurement in spacial coordinates

C commanded

A response produced solely by elevator deflection (no gravity effects)

I integrator

o initial position

1 position after interceptor maneuver (such as rolling to a new attitude)

(o) initial condition

Subscript associated with K denotes automatic-control-system gain on the signal symbolized by the subscript.

APPARATUS

Flight-Test System

The automatic interceptor system consisted of a radar fire-control system, a tie-in computer, and an automatic pilot installed in a subsonic jet fighter airplane. A photograph of the airplane is presented in figure 1, and its dimensional and mass characteristics are presented in table I. Reference 3 is a report covering the stability characteristics of this airplane. The complete system has been previously described in detail in references 4 and 5 and is described herein only in terms of generalized block diagrams (except for a more detailed description of modifications that were made to the system). The lead-angle information from the fire-control computer was used on only one flight during the flight tests covered by this paper, and thus the flight-test system (with this one exception) was one that attempted to perform pure pursuit tracking.

The elevation channel is shown schematically by the block diagram presented in figure 2. The operation and the automatic-control gains are unchanged from those described in reference 4.

The deflection channel is shown schematically by the block diagram of figure 3. Of particular note is the use of a bank-angle signal for stabilization of the tracking loop. This signal causes the system to establish a bank angle proportional to the deflection tracking error. This mode of operation produces an undesirable effect during maneuvers which requires a banked attitude. Under these conditions and with no integral signal present, a tracking error must be generated to command the desired bank angle. The integrator shown in the diagram provides a means for eliminating this "bias" tracking error by cancelling the bank-angle signal over long time periods. As discussed in reference 5, this means of compensation is not entirely satisfactory.

The detailed operation of the tie-in was somewhat different from that described in reference 4. The deflection tracking-error gain was the equivalent of 20° of aileron deflection per degree of error, and the bank-attitude gain was 1.0° of aileron per degree of bank angle. The tie-in was modified, however, so that the deflection tracking error and the bank-attitude feedback signals were made to vary inversely with the absolute value of the elevation tracking error plus a constant. Figure 4 presents a curve showing the modification that was effected in these signals as the elevation tracking error was varied. This modification was made in connection with a phase of the investigation not reported herein. The error-integration circuit described in reference 4 was used for only a few runs. All other automatic-control gains are the same as those tabulated in reference 4.

An additional modification involved the use in the aileron channel of an autopilot servo actuator which had an increased stall torque. This servo is restrained primarily by the control-system feel springs, and the increase in stall torque enabled the maximum aileron deflection to be increased from about ±4° to about ±8° (as measured on the ground). The frequency response of this servo, as measured on the ground, is presented in figure 5. During the flight tests the ailerons were limiting at values of less than 5°. Thus the servo response, under actual test conditions, may be significantly different from the response measured on the ground. This reduction in aileron travel is attributed to the low temperature at operating altitude which produced a stiffening of some flexible vapor seals attached to the control-system linkages and which produced greater loads for the servo to overcome.

The relationship between the various axes associated with the tracking problem is presented in the diagram in figure 6 for the case of zero deflection error and no lead angles. The armament-datum line is a line fixed by the designer within the airframe in the plane of symmetry and is not necessarily coincident with the gun line. The radar-boresight axis is also in the plane of symmetry and is normally oriented with respect to the armament-datum line at an angle determined by tactical considerations. The gun line (not shown in the figure) would ordinarily be set approximately parallel to the radar-boresight axis. Location of the radar-boresight axis during the present tests was varied from +1/2° to +5° above the armament-datum line. The radar-antenna axis establishes the approximate line of sight to the target (within the tracking accuracy of the radar), and the angle between this axis and the radar-boresight axis establishes the interceptor tracking error in elevation. The interceptor roll axis also has an important relationship to the tracking problem. As is discussed in a later section, this axis does not necessarily remain fixed with respect to the airframe.

Analog Simulator System

The analog studies were based upon the representation of the different phases of the interceptor problem as expressed by the equations presented in the appendix. A functional diagram of the simulated problem in which these equations were incorporated is presented in figure 7. The transfer functions used in the representation of the tie-in dynamics were obtained from reference 4 and from bench tests. The servo-system dynamics are based upon a first-order approximation of the flight-test-system servo-response characteristics as determined from bench tests. The deflection limits of the aileron servo were set at $\pm 5^{\circ}$, but runs were also made with this limit at $\pm 10^{\circ}$. There were only slight differences in the performance between the two settings. The transfer functions of the airplane dynamics were obtained from reference 3. The simulation does not include coupling between interceptor pitch, roll, or yawing motions. In the simulation, the interceptor was constrained so that there would be no sideslip angles produced and so that rolling took place about an axis fixed in the interceptor. The attack-geometry equations were obtained from reference 6.

The radar dynamics were assumed to be perfect; that is, the radar exactly established the line of sight to the target at all times. In addition, in some cases a simplified simulation of a kinematic lead-angle computer was included which utilized a constant for the projectile time of flight T_f of 1.5 seconds. No radar noise was included, but, in order to approximate the noise filtering used in the lead-angle computer of the actual system, a first-order lag function $\frac{1}{1 + \tau p}$ was employed. The value of τ used during the tests was varied from 0 to 2.0 seconds.

The physical relationship between the axes relating to the tracking problem, as set up on the analog simulator, are presented in the diagram in figure 8. This diagram differs from the one for the flight-test system (fig. 6) in that the interceptor roll axis is assumed to be fixed with respect to the airframe, and the radar-boresight axis is referenced to this roll axis rather than to an armament-datum line. The tracking reference axis is introduced in order to account for the addition of lead angles. This axis is displaced from the radar-antenna axis by the elevation and deflection lead angles. (Fig. 8 does not show a deflection lead angle.) The tracking error, when lead angles are included, becomes the angle between the radar-boresight axis and the tracking reference axis. With no lead angles, the tracking reference axis and the radar-antenna axis are coincident.

TESTS

Flight

All flights were made at an altitude of 20,000 feet at a speed corresponding to an indicated Mach number of 0.76. A range of about 1,000 yards and a zero closing rate were established between the interceptor and the target aircraft (a single-place jet fighter) before each run, and an attempt was made to maintain these conditions during the runs. The test runs all began in a straight and level tail chase and were of two general types as follows:

1. Runs in which the automatic interceptor system was engaged with an initial tracking error in deflection. The runs included the transient response as the system attempted to establish steady tracking on a nonmaneuvering target.

2. Runs in which the target executed a steady turning maneuver after the interceptor had established steady tail-chase tracking.

Runs generally were made with the automatic-control-system gains set at the values considered basic for the flight tests (see table II). In addition, runs were made on some of the flights in which variations of the bank-attitude-feedback gain were made.

Flights were made with the radar-boresight axis elevated $+\frac{1}{2}^\circ$, $+2^\circ$, $+3\frac{1}{2}^\circ$, and $+5^\circ$ above the armament-datum line. An elevation of 0° was not used because this elevation caused the interceptor to be in the wake of the target.

The one flight in which lead angles were included was made with the radar-boresight axis $3\frac{1}{2}^\circ$ above the armament-datum line.

Analog Simulator

Tests on the analog simulator involved runs which were similar to those made in the flight tests. An entry into a steady turning maneuver by the target was approximated by a step increase in horizontal acceleration to the target applied perpendicular to the line of sight. In addition, some runs were made which simulated an interceptor attack originating from positions which were at moderate angles off the target tail where the interceptor was initially pointed at the target (zero tracking error). These runs were accomplished by setting in initial conditions on the angular velocity of the line of sight $\omega_{LS_{XY}}$.

Runs were made with the angularity between the radar-boresight axis and the interceptor roll axis varied from -2° to $+10^{\circ}$. The automatic-control-system gains were approximately the same as for those considered basic for the flight-test system (see table II) except when the effects of specific deflection-channel gains were being studied. Runs were made both with and without the lead-angle computer.

RESULTS AND DISCUSSION

Effects of Interceptor Rolling Motion Upon Tracking Errors

In order to understand the stability effects of elevating the radar-boresight axis (as is discussed subsequently), it is desirable to consider how rolling motions of the interceptor may affect the tracking errors for varied elevations of the radar-boresight axis. The effects may be visualized in a qualitative sense by examining the diagrams presented in figure 9. The diagrams represent an oversimplified case in which the interceptor is assumed to roll about a fixed axis in the airplane. The diagrams present the projection of the radar-boresight axis and the interceptor roll axis upon a plane that is perpendicular to the roll axis and contains the target. Three different elevations of the radar-boresight axis with respect to the interceptor roll axis are shown as follows: In case (a) the radar-boresight axis is alined with the roll axis; in case (b) the radar-boresight axis is above the roll axis; and in case (c) the radar-boresight axis is below the roll axis. In each of these three cases the target is located at the same place relative to the radar coordinate system before the interceptor banks to the angle ϕ. After the bank the radar coordinate system is shown by the dashed lines, and in each case the target is in a different relative location. In case (a) the deflection error is considerably reduced and the elevation error is somewhat increased. In case (b) the interceptor bank in effect translates the radar coordinate system toward the target with the result that the deflection error is decreased by a considerably larger amount than in case (a). The increase in elevation error is less in case (b) than in case (a). In case (c) the interceptor bank translates the radar coordinate system away from the target with the result that the deflection error is actually increased even though the interceptor banks toward the target. The increase in elevation error is greater in case (c) than in the other two cases.

The effects on the tracking errors illustrated by the diagrams in figure 9 may be shown more explicitly in equation form. For either figure 9(b) or 9(c) the tracking errors which exist after the interceptor banks to the new attitude may be written as follows:

$$\sigma_{E_1} = \left(\sigma_{E_o} + \mu\right)\cos\Delta\phi + \sigma_{D_o}\sin\Delta\phi - \mu \qquad (1)$$

$$\sigma_{D_1} = \sigma_{D_o}\cos\Delta\phi - \left(\sigma_{E_o} + \mu\right)\sin\Delta\phi \qquad (2)$$

These equations show the interdependence of elevation error and deflection error and also show the effect of elevation of the radar-boresight axis on these errors as the interceptor rolls. Because the effects of small disturbances in bank angle are of importance when system stability is considered, it is desirable to know just which of the terms of equations (1) and (2) most influence the changes in the tracking error components under such conditions. This influence can be determined by assuming that the change in bank angle $\Delta\phi$ is sufficiently small that $\cos\Delta\phi$ can be assumed to be 1.0 and $\sin\Delta\phi$ can be assumed to be $\Delta\phi$ in radians. Thus, approximate equations for the change in tracking error due to rolling through a bank angle $\Delta\phi$ can be written as follows:

$$\Delta\sigma_E = \sigma_{E_1} - \sigma_{E_o} = +\sigma_{D_o}\Delta\phi \qquad (3)$$

$$\Delta\sigma_D = \sigma_{D_1} - \sigma_{D_o} = -\left(\sigma_{E_o} + \mu\right)\Delta\phi \qquad (4)$$

Equation (3) shows that for a given change in bank angle the change in elevation tracking error is proportional to the deflection error. Equation (4) shows that for a given change in bank angle the change in deflection tracking error is proportional to the elevation error and the elevation of the radar-boresight axis. Of particular importance is the fact that whenever an elevation error or a boresight elevation exists, there is a geometric proportion between the deflection error and the bank angle. This relationship is similar to that achieved electrically in the test interceptor system through use of bank-angle feedback in the deflection channel. Reference 5 discusses this electrical bank-angle feedback in some detail. In the case of the geometric feedback, positive values of σ_E and μ result in contributions of those terms to a change in σ_D in a sense opposite to the direction of bank (stable feedback configuration). Conversely, negative values of σ_E and μ cause these terms to contribute to σ_D in the same sense as the direction of bank; that is, a positive bank causes a positive increase in deflection error (unstable feedback configuration).

As mentioned previously, this description of the relationship between bank angle and tracking-error components is actually an oversimplification of the problem because of the fact that yawing and pitching motions generally are coupled with rolling motion. The important parameter is the instantaneous location of the axis about which the angular motion exists. This axis is defined herein as the resultant "roll" axis. Variations in the location of the axis about which the resultant rolling occurs are dependent upon such factors as the stability of the airplane and the moments produced by control inputs. An analysis was made of the orientation of this resultant roll axis during selected flight-test runs where fairly smooth oscillatory lateral motions existed. The orientation of this axis was determined by summing vectors representing roll and yaw angular rates. Pitch rate was found to be relatively small and was not considered because the component of the resultant vector in the plane of symmetry was felt to be the important factor. Figure 10 shows a typical variation of the position of the resultant roll axis in the plane of symmetry during one cycle of a lateral oscillation. Also included in figure 10 are the time histories of roll rate and yaw rate. The average position of the resultant roll axis was determined by integrating the area under the curve representing the resultant roll axis and averaging the values obtained for several cycles of oscillation. The determination of the average resultant roll-axis position by summing the "in-phase" component of the yaw rate with the roll rate was found to be practical. For the run shown in figure 10 the average position of the resultant roll axis was about coincident with the armament-datum line. Apparently the pitch-rate and yaw-rate loops of the automatic control system of the test interceptor to an appreciable extent constrained the average resultant roll axis of the interceptor close to the armament-datum line. Evidence of this constraint was shown by the large (over 20 to 1) ratio of roll to yaw that was maintained by the system.

Similarity of Flight Results and Analog-Computer Results

In a problem as complex as an automatic interceptor attack it is difficult to establish how complete a simulation is necessary where analog studies are to be made. The results obtained with the simulation as described in the section on "Apparatus" gave close agreement with flight results without any adjustments in parameter settings. An example of this agreement is given in figure 11 which presents the time histories of deflection tracking error and bank angle following engagements with a deflection error of about 100 mils. Both flight-test and analog-simulator runs are shown. The automatic-control-system gains were approximately the same, and the radar-boresight axis was elevated $1/2^{\circ}$ above the armament-datum line for the flight-test run and zero degrees above the roll axis for the analog-simulator run. Although slightly different elevations of the radar-boresight axis relative to the average roll axis exist (of the order of $1/2^{\circ}$), the two runs are felt to be roughly comparable and do show practically the same frequency and damping characteristics. The steady-state portion of the flight-test run shows more variations than the analog simulator, but primarily these variations are probably due to the effects of radar noise which was not included in the analog simulation.

The similarity of flight-test and analog-simulator results noted in figure 11 was apparent to a large degree in all phases of the tests which were covered by both methods. Caution is advised, however, in using as simplified a representation of the overall problem for other interceptor studies as was used herein, especially where appreciably higher interceptor-roll and yaw rates may be encountered or where radar-antenna dynamics may be less favorable.

Effect of Elevating the Radar-Boresight Axis Upon Stability

Flight test.- The effect of elevating the radar-boresight axis of the test automatic interceptor system is illustrated by the results presented in figure 12. This figure presents time histories of the deflection tracking error, the interceptor bank angle and the interceptor aileron deflection following engagement with an initial tracking error of about 25 mils in deflection. Automatic-control-system gains were the same for all runs. For the case where the elevation of the radar-boresight axis above the armament-datum line μ^* was $1/2^{\circ}$, the response shows a long-period oscillation that was lightly damped. Increasing μ^* to 2° and then to $3\frac{1}{2}^{\circ}$ increased the damping of the long-period mode of motion and also decreased the time required for the error response initially to reach zero (rise time). Increasing μ^* to 5° did not appreciably change the damping of the long-period mode or the rise time but did have a tendency to excite a 1-cycle-per-second oscillation that increased the tracking errors slightly during the tail-chase portion of the run. The source of this short-period oscillation is discussed in a subsequent section. A survey of the runs presented in figure 12, and also of other runs that were made, indicate that the optimum value of μ^* for the system tested (without lead-angle computation) was about $3\frac{1}{2}^{\circ}$.

Analog simulator without lead angles.- Elevating the tracking reference axis on the analog simulator gave results that were in good agreement with the flight-test results. Figure 13 presents analog time histories of deflection tracking error, bank angle, and aileron deflection response following engagements of the system with 50 mils initial-deflection tracking error. The automatic-control-system gains were the same as those used in the flight-test runs presented in figure 12, and runs are presented for elevations of the radar-boresight axis above the roll axis of -2°, 0°, and $+2^{\circ}$. The stabilizing effect of positive elevations is readily apparent, and conversely, the case of μ equal to -2° shows a destabilizing effect. This stabilizing effect of elevating the radar-boresight axis was also present during runs in which the interceptor began the attack from a position off the target tail. Figure 14 shows time histories of two runs in which the interceptor was initially at a position 30° off the target tail. On one run μ was equal to 0°, and on the other run μ was equal to $+3^{\circ}$. For the case where μ was equal to 0°, the response shows a lightly damped oscillation of about 1/6 cycle per second. The deflection tracking error was approximately proportional to the bank attitude as the interceptor turned onto a path directly behind the target because bank-attitude feedback was used to stabilize the system. In comparison, the case where μ was equal to $+3^{\circ}$ shows that the system was very stable; however, the tracking error was still approximately proportional to the bank attitude because bank-attitude feedback was still present in the system.

Analog simulator with lead angles.- The inclusion of the lead angle in the analog problem produced a destabilizing effect upon the lateral response of the system. The severity of the destabilizing effect was dependent upon the magnitude of the filter time constant τ_{Λ} used in the lead-angle approximation. For runs that consisted of engaging the system with an initial deflection error of 100 mils, there was no perceptible difference between the time histories of the response without lead angles and with lead angles but with no lead-angle filtering. For actual systems, considerations of radar noise dictate that filtering be used in the lead-angle computation; however (as shown in figure 15), inclusion of the lead angle with a filter time constant of 1.0 second causes a decrease in system damping. Also included in figure 15 is a case with lead-angle computation and with τ_{Λ} equal to 1.0 second but with the radar-boresight axis raised to $+2^{\circ}$ above the roll axis. This latter case shows that increasing the elevation of the radar-boresight axis is also effective in increasing system stability where lead angles are involved. The time histories of the interceptor response presented in figure 16 are for cases where the interceptor is tracking a target entering a steady turn. The inclusion of lead angles in the system for this type of run had a similar effect on system stability, as was noted previously for the runs consisting of an engagement with an initial deflection error (see fig. 15), although the destabilizing effects of lead-angle filtering are more pronounced during the turn. This fact is true primarily because a steady value of lead angle is generated in the steady turning maneuver. Again, there was practically no difference between the time histories describing the interceptor response to a 1 g target turn for the case of no lead angles and for the case including lead angles without filtering. As can be seen in figure 16, including lead angles with a filter time constant of 1.0 second caused the system to be unstable in response to a target turn. This figure includes a no-lead-angle run for comparison purposes. Also included is a run in which the system included lead angles with a filter time constant of 1.0 second, and the radar-boresight axis was elevated 2° in order to illustrate again the stabilizing influence of this factor. Instead of approaching zero, the deflection tracking error approaches a steady-state value of about 40 mils. This "bias" error exists because bank-attitude feedback is used to stabilize the lateral motion of the interceptor. This problem is discussed further in the next section.

The reason that filtering on the lead-angle computation has a destabilizing effect upon the system tracking is that this filtering detrimentally affects the ability of the lead-angle computer to resolve correctly the elevation and deflection lead angles as the interceptor banks. This fact may be seen by examining a typical situation that could exist when tracking a target in a steady turn, such as is shown in figure 17. In this case the interceptor is banked to the right in order to turn with the target and is leading the target. Initially, the radar-boresight axis is alined right on the predicted future position of the target so that no tracking error exists in the system. Consider, however, that some spurious signal causes the interceptor to bank through the angle $\Delta\phi$ (the roll axis is coincident with the radar-boresight axis in this example). If the lead-angle computer instantly resolves the lead angle into its correct components, the predicted target position stays fixed and no tracking error is introduced into the system (except that which might develop from the interceptor pitch and yaw response). If filtering exists in the lead-angle computer, however, the elevation and deflection components of the lead angle do not change instantly. If there is no change in these components of lead angle (as in the case of heavy computer filtering), the predicted position of the target would be translated to the position indicated on the figure, and there would exist a deflection tracking error in the same direction as the incremental bank angle which obviously would be a destabilizing influence on the system.

The deflection tracking error generated from this source will always be in a direction that will tend to destabilize the system. For a given change in bank angle, the required change in deflection lead angle due to interceptor rolling is almost in direct proportion to the elevation lead angle ($\Delta_{\Lambda_D} = \Lambda_E \sin \Delta\phi$); therefore, the destabilizing influence is almost in direct proportion to the magnitude of the elevation lead angle as well as the filter time constant. The elevation lead angle may change quite radically during a lead-pursuit attack, and it is therefore expected that the lateral stability of the system may also change quite radically. In order to obtain a satisfactory degree of stability throughout an attack, it is evident that some variation of parameters with the magnitude of lead angles may be required. Another approach, as discussed in references 7 and 8, would be to provide cross-roll correction signals to compensate the system for the lead-angle errors that result from the filtering.

Effect of Elevating the Radar-Boresight Axis Upon the Required Bank-Angle Feedback

Flight tests without lead angles.- Runs with the test automatic interceptor system with μ^* equal to $+1/2^{\circ}$ required the use of bank-angle feedback in order to prevent unstable oscillations of the system. With this feedback the basic system is unable to track a turning target with zero tracking error unless further compensation is provided, because a tracking error is required to command the bank angle needed to turn with the target. Normally this deficiency is compensated for in the test system by the use of the deflection-error-integrator circuit, but because of the destabilizing effect of such a circuit insufficient gain can be used to effect a rapid solution without reducing the system damping to too low a value. This problem and other means of compensating the system are more fully discussed in reference 5.

The similarity of the stabilizing effects of electrical bank-angle feedback and radar-boresight elevation has already been indicated. Thus, elevation of the radar-boresight axis would appear to offer another solution to this "bias" error problem. The need for bank-angle feedback is reduced, and thereby the steady-state tracking error during a target turn can be reduced. For the case of μ equal to $3\frac{1}{2}^{\circ}$, runs were made with the bank-angle feedback eliminated. Figure 18(a) presents the time histories of the response to an initial deflection tracking error at engagement for this case, and it can be seen that stable operation exists. The beneficial effects of eliminating K_{ϕ} are illustrated in figure 18(b) which shows time histories for a case of the target entering a steady turn (bank angle = 30°). The same configuration and gains were used as for the run presented in figure 18(a). A comparative case where normal K_{ϕ} was used also is included in the figure 18(b). When normal K_{ϕ} is used, the tracking error gradually builds up to a steady state of about 25 mils, but when K_{ϕ} is eliminated the tracking error shows only a small transient as the target enters the turn and quickly settles down to small excursions about zero error during the steady turn. Thus, if the radar-boresight axis is elevated sufficiently to eliminate the need for bank-angle feedback for stability, the system is able to track a turning target without any added compensation.

Flight tests with lead angles.- On the single flight that was made with lead-angle computation included (radar-boresight axis elevated $3\frac{1}{2}^{\circ}$

for this flight), runs were made without the electrical bank-angle feedback. The system was stable during runs which consisted of engaging the automatic control system with initial deflection tracking errors and also during runs which consisted of the target entering and holding steady turns. The damping of the system was, however, noticeably less on these runs than on corresponding runs without lead-angle computation included. The filter time constant of the lead-angle computer was approximately 1.4 seconds. In order to offset this decrease in damping which occurs when lead angles are included, either the radar-boresight axis has to be elevated an additional amount or some electrical bank-angle feedback signal is required.

Analog simulator without lead angles.- Analog-simulator results showed that with μ equal to $+2^{\circ}$, stable operation could be obtained with the roll-angle feedback eliminated. Figure 19(a) presents time histories of the response of the system following an engagement with an initial deflection error for this case and also for the case where normal bank-angle feedback was used. In addition, a time history is presented for the case of μ equal to 0° and no bank-angle feedback to illustrate the severe instability which occurs. The case with μ equal to $+2^{\circ}$ and with the bank-angle feedback eliminated is not as stable as the case with roll-angle feedback but does settle down on target after one oscillation. Figure 19(b) presents time histories for these same configurations for cases in which the target performs a 1 g turn. The case with μ equal to $+2^{\circ}$ and without bank-angle feedback shows only a slight transient and settles down to about zero error shortly after the turning maneuver starts, whereas the case with roll-angle feedback exhibits a significant steady-state error. The advantage of eliminating the bank-attitude feedback was also obvious from the results of runs in which the interceptor began the attack from a position 30° off the target tail. Figure 20 shows time histories of two such runs where μ was equal to $+3^{\circ}$. On one run bank-attitude feedback was included and on the other run it was eliminated. A comparison of these two runs shows that the case without bank-attitude feedback had a smaller peak value of deflection tracking error, and this error was eliminated in a much shorter time than in the case with bank-attitude feedback $\left(6\frac{1}{2}\right.$ seconds compared with more than 30 seconds$\left.\right)$.

Other runs that were made with higher values of μ and without bank-attitude feedback showed even tighter tracking of turning target than that shown in figure 19(b). At higher elevations of μ however, it was found that an oscillation with a frequency of about 1 cycle per second was excited when engagements were made with initial deflection errors. This type of oscillation was encountered in the flight tests, as was pointed out in an earlier section. This oscillation also occurred when the electrical bank-angle signals were used at high gain levels; in fact, the damping of the mode of motion associated with this 1-cycle-per-second frequency was determined largely by the gain on the bank-attitude feedback (for a given value of μ). When the radar-boresight axis was elevated with respect to the roll axis, the deflection tracking error was directly affected by banking the interceptor, as discussed in a previous section. Thus, elevating the radar-boresight axis in a sense adjusted the gain on this geometric bank-angle feedback. When either or a combination of the gains on these feedbacks became large, the damping of this mode was decreased and the system displayed a tendency to oscillate at the approximately 1-cycle-per-second frequency. Figure 21 shows time histories for two cases where this 1-cycle-per-second oscillation was noted (electrical bank-attitude feedback was eliminated in these cases). For the case where μ was equal to $+5^{\circ}$, the 1-cycle-per-second oscillation was fairly well damped; however, for the case where μ was equal to $+10^{\circ}$, the oscillation was neutrally stable. If radar noise had been included in the simulation, it is believed that the case where μ was equal to $+5^{\circ}$ would have shown the 1-cycle-per-second oscillation to be almost continuously excited. From a consideration of stability then, the advantages derived from elevating the radar-boresight axis would probably be limited to moderate values of μ.

When the bank-angle feedback is obtained by radar-boresight elevation (geometric feedback), the feedback path includes the dynamics associated with the radar-antenna drive system. The agreement obtained between flight results (with antenna dynamics included) and analog results (with perfect antenna dynamics) indicates that the performance of the antenna drive of the flight-test radar was sufficiently good to eliminate antenna dynamics as a factor in these tests. The tracking characteristics of this radar system are described in reference 9. Other investigations using other fire-control systems have shown the antenna dynamics to be an important factor (see ref. 10).

Application of the Principle of Elevated Radar-Boresight Axis

The beneficial effects of elevating the radar-boresight axis in the test automatic interceptor have been shown by both flight and analog-simulator tests. The principle appears to have direct application to interceptors bearing guns or launching guided missiles. It is also applicable to the bank-to-turn missile. The stability implications with respect to the relation between the roll axis and the radar-boresight axis warrant consideration in the design of all weapons systems, even though elevation of the radar-boresight axis (and launcher line) may not be feasible. Consideration of these effects should enable the designer to select an autopilot configuration providing desirable turn coordination during rolling maneuvers. This coordination should constrain the interceptor to roll about an axis which will not produce serious destabilizing inputs. Consideration of these effects should also aid the designer in the determination of yaw-channel requirements in the fire- and flight-control system.

CONCLUSIONS

Flight and analog-simulator studies of a prototype interceptor system have led to the following conclusions:

1. Elevating the radar-boresight axis of an automatic interceptor has a marked stabilizing effect upon the tracking performance. Conversely, depressing this axis has a destabilizing effect.

2. The stabilizing effect of elevating the radar-boresight axis existed because a geometric feedback was generated when the interceptor banked. This stabilizing effect was equivalent to that obtained by the use of electrical feedback of bank attitude in the automatic control system.

3. The advantage of elevating the radar-boresight axis of the test system was limited to elevations of the order of 5° above the roll axis because higher elevations excited a lightly damped 1-cycle-per-second oscillation; however, high electrical bank-attitude-feedback gains also excited this oscillation.

4. Elevating the radar-boresight axis of the flight-test system $3\frac{1}{2}^{\circ}$ above the armament-datum line enabled stable operation without the use of the electrical bank-attitude feedback which was necessary for the basic system stability. This enabled the system to track a turning target with small transient errors and zero steady-state errors.

5. Including lead angles in the automatic interceptor system presented a destabilizing influence that was noted in the lateral motion of the interceptor. The severity of the destabilizing influence was related to the amount of filtering used in the lead-angle computation and to the magnitude of the elevation lead angle required by the run.

6. Generally good correlation was obtained between flight tests and analog-simulator tests on all phases mutually covered.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 5, 1957.

APPENDIX

EQUATIONS USED IN ANALOG-SIMULATOR REPRESENTATION OF AUTOMATICALLY CONTROLLED INTERCEPTOR PROBLEM

The equations used in the analog studies are as follows:

Radar:

$$\sigma_E(\text{no lead angle}) = \sigma_{F_{XZ}} \cos \phi_F + \sigma_{F_{XY}} \sin \phi_F - \mu \qquad \text{(A1)}$$

$$\sigma_D(\text{no lead angle}) = \sigma_{F_{XY}} \cos \phi_F - \sigma_{F_{XZ}} \sin \phi_F \qquad \text{(A2)}$$

Lead-angle computer:

$$\Lambda_E = T_f\left(\frac{\omega_{LS_E}}{1 + \tau p}\right) \qquad \text{(A3)}$$

$$\Lambda_D = T_f\left(\frac{\omega_{LS_D}}{1 + \tau p}\right) \qquad \text{(A4)}$$

Tie-in:

$$\delta_{e_C} = \left(K_{\sigma_E}\sigma_E - K_{\dot{\theta}}\dot{\theta}\right)\left(\frac{1 + 0.3p + 0.02p^2}{1 + 1.02p + 0.02p^2}\right) \qquad \text{(A5)}$$

$$\delta_{a_C} = \frac{K_{\sigma_D}\sigma_D + \frac{K_I}{p}\sigma_D - K_{\phi'}\phi'}{\left(\left|\sigma_E\right| + K\right)(1 + 0.1p)} - K_{\dot{\phi}}\dot{\phi} \qquad \text{(A6)}$$

$$\phi' = \frac{1 + 2p}{1 + 4p}\phi \qquad \text{(A7)}$$

Servo:

$$\delta_e = \frac{1}{1 + 0.15p}\,\delta_{e_C} + \delta_e(\text{trim}) \tag{A8}$$

$$\delta_a = \frac{1}{1 + 0.15p}\,\delta_{a_C} \tag{A9}$$

Airplane:

$$a_A = \frac{V}{g}\left(\frac{0.138p^2 + 0.205p + 62.2}{p^2 + 4.42p + 21.2}\right)\delta_e \tag{A10}$$

$$\dot{\theta}_A = \left(\frac{35.4p + 62.2}{p^2 + 4.42p + 21.2}\right)\delta_e \tag{A11}$$

$$\dot{\phi} = \left(\frac{11}{1 + 0.2p}\right)\delta_a \tag{A12}$$

Geometry:

$$\sigma_{F_{XZ}} = \frac{1}{p}\left(\frac{a_{B_{XZ}}}{R_p} - \frac{a_{F_{XZ}}}{R_p} - \dot{\theta}_{F_{XZ}}\right) + \mu + \sigma_{F_{XZ}}(o) \tag{A13}$$

$$\omega_{LS_{XZ}} = \frac{1}{R}\left(\frac{a_{B_{XZ}}}{R_p} - \frac{a_{F_{XZ}}}{R_p}\right) + \omega_{LS_{XZ}}(o) \tag{A14}$$

$$\sigma_{F_{XY}} = \frac{1}{p}\left(\frac{a_{B_{XY}}}{R_p} - \frac{a_{F_{XY}}}{R_p} - \dot{\theta}_{F_{XY}}\right) + \sigma_{F_{XY}}(o) \tag{A15}$$

$$\omega_{LS_{XY}} = \frac{1}{R}\left(\frac{a_{B_{XY}}}{R_p} - \frac{a_{F_{XY}}}{R_p}\right) + \omega_{LS_{XY}}(o) \tag{A16}$$

$$\dot{\theta}_{F_{XZ}} = \dot{\theta}_A \cos \phi_F - \frac{g}{V} \qquad \text{(A17)}$$

$$a_{F_{XZ}} = a_A \cos \phi_F - 1 \qquad \text{(A18)}$$

$$\dot{\theta}_{F_{XY}} = \dot{\theta}_A \sin \phi_F \qquad \text{(A19)}$$

$$a_{F_{XY}} = a_A \sin \phi_F \qquad \text{(A20)}$$

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TABLE I

DIMENSIONAL AND MASS CHARACTERISTICS OF FLIGHT-TEST VEHICLE

Overall length, ft	48.04
Wing:	
Span, ft	41.70
Area, sq ft	294.0
Section, wing-fold	NACA 65_1-212
Incidence, deg	-0.5
Aspect ratio	5.9
Dihedral, deg	3.0
Mean aerodynamic chord, in.	88.4
Leading-edge sweepback, deg	0
Ailerons:	
Mean chord rearward of hinge line, ft	1.24
Span, percent b/2	32.8
Horizontal-tail surfaces:	
Total area, sq ft	70.1
Span, ft	17.8
Elevator area rearward of hinge line, sq ft	18.7
Distance from $0.256\bar{c}$ to elevator hinge line, ft	24.0
Dihedral, deg	10.0
Vertical-tail surfaces:	
Total area, sq ft	39.9
Rudder area rearward of hinge line, sq ft	9.6
Distance from $0.256\bar{c}$ to rudder hinge line, ft	22.2
Approximate weight at flight-test conditions, lb	20,700
Relative density (at 20,000 ft)	41.6
Center-of-gravity station, percent mean aerodynamic chord	25.7
Moment of inertia about X-axis, slug-ft^2	15,145
Moment of inertia about Y-axis, slug-ft^2	41,677
Moment of inertia about Z-axis, slug-ft^2	54,616

TABLE II

AUTOMATIC-CONTROL GAINS CONSIDERED NORMAL FOR THE FLIGHT TESTS AND ANALOG-SIMULATOR TESTS OF THE AUTOMATIC-INTERCEPTOR PROBLEM

Deflection error gain $K_{\sigma_{F_D}}$, $\frac{\text{deg aileron}}{\text{deg deflection error}}$	20.0
Deflection-error integrator gain K_I, $\frac{\text{(deg aileron)/sec}}{\text{deg deflection error}}$ . .	0
Bank-attitude feedback gain K_{ϕ}, $\frac{\text{deg aileron}}{\text{deg bank attitude}}$	1.0
Roll-rate feedback gain $K_{\dot{\phi}}$, $\frac{\text{deg aileron}}{\text{deg/sec roll rate}}$	0.25
Elevation-error gain $K_{\sigma_{F_E}}$, $\frac{\text{deg elevator}}{\text{deg elevation error}}$	6.5
Pitch-rate feedback gain $K_{\dot{\theta}}$, $\frac{\text{deg elevator}}{\text{deg/sec pitch rate}}$	1.5

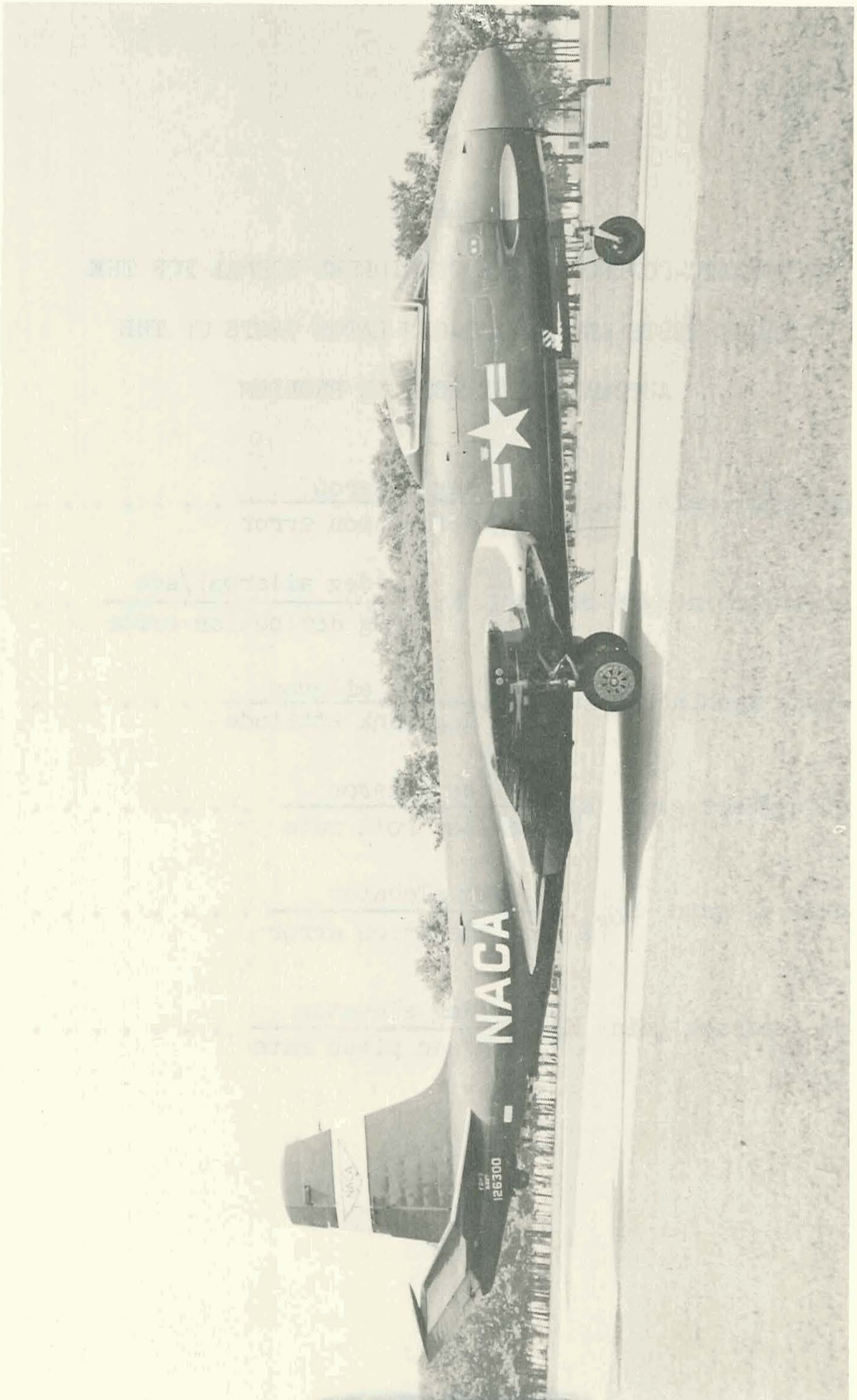


Figure 1.- Side view of flight-test airplane. L-57-2329

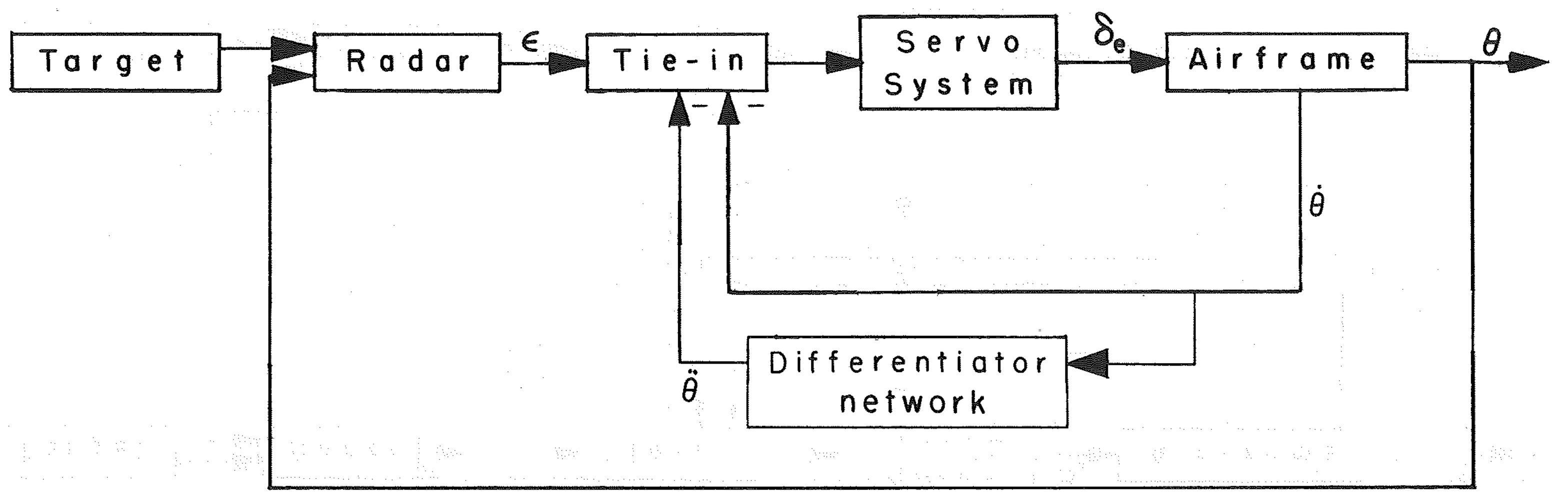


Figure 2.- Schematic diagram of elevation channel of automatic interceptor system.

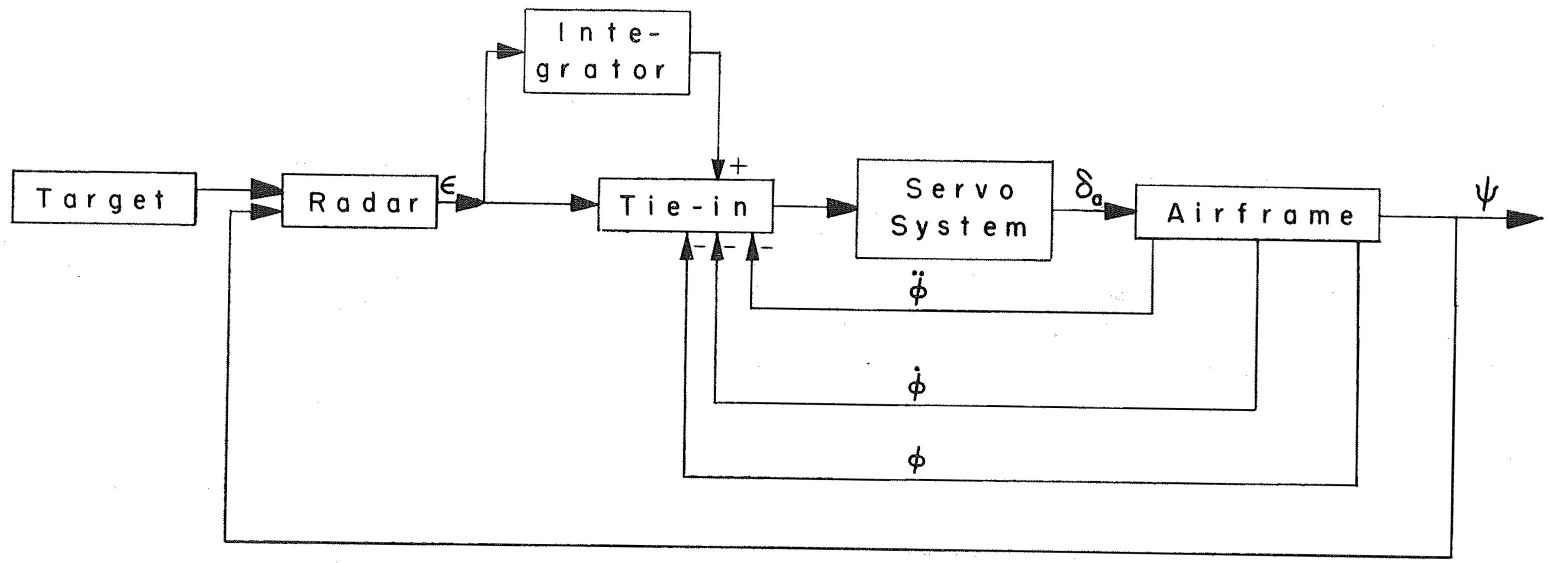


Figure 3.- Schematic diagram of deflection channel of automatic interceptor system.

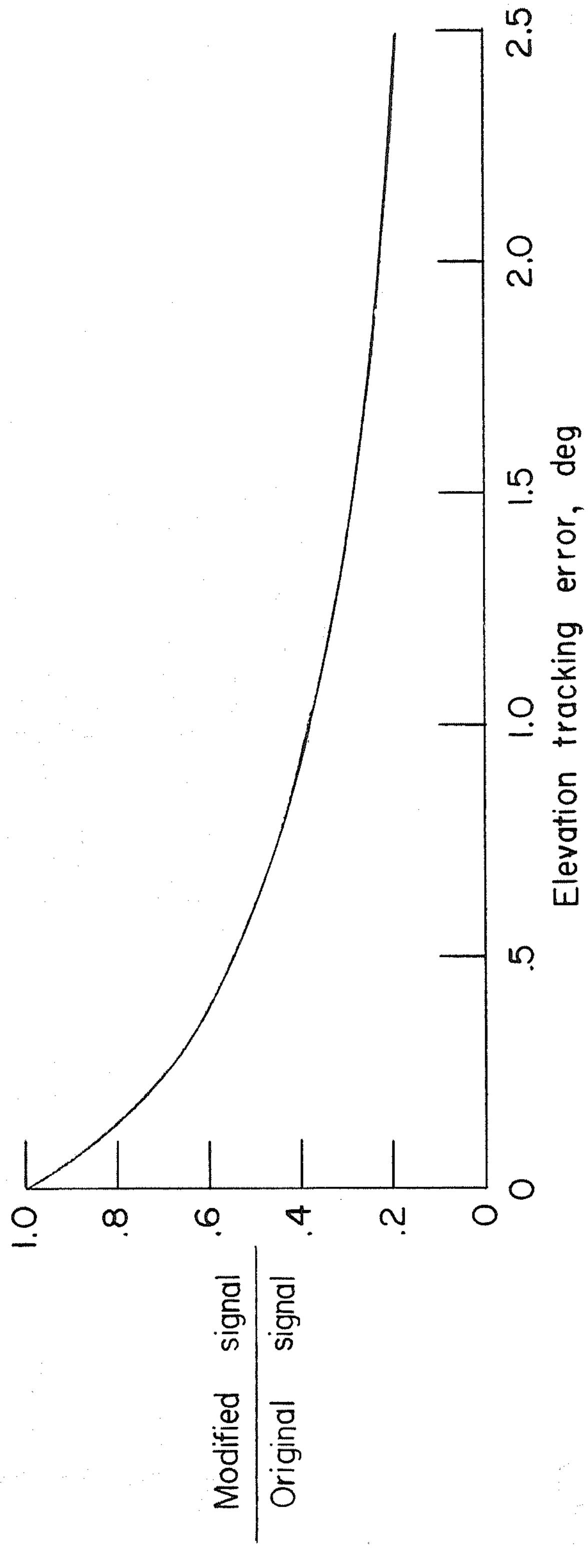


Figure 4.- Modification of deflection tracking-error signal and roll-attitude feedback signal effected in tie-in.

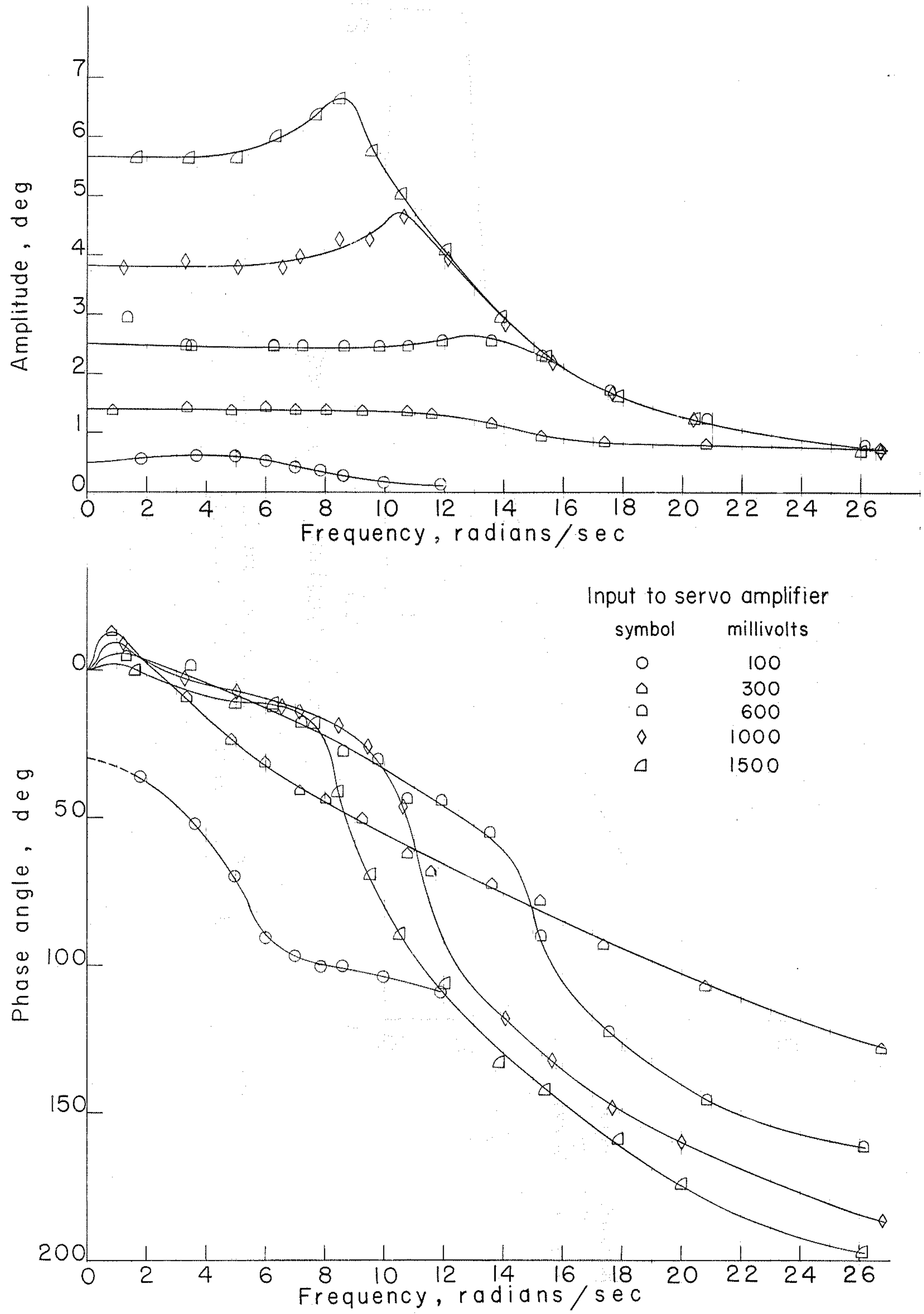


Figure 5.- Frequency response of aileron-control-system servo actuator.

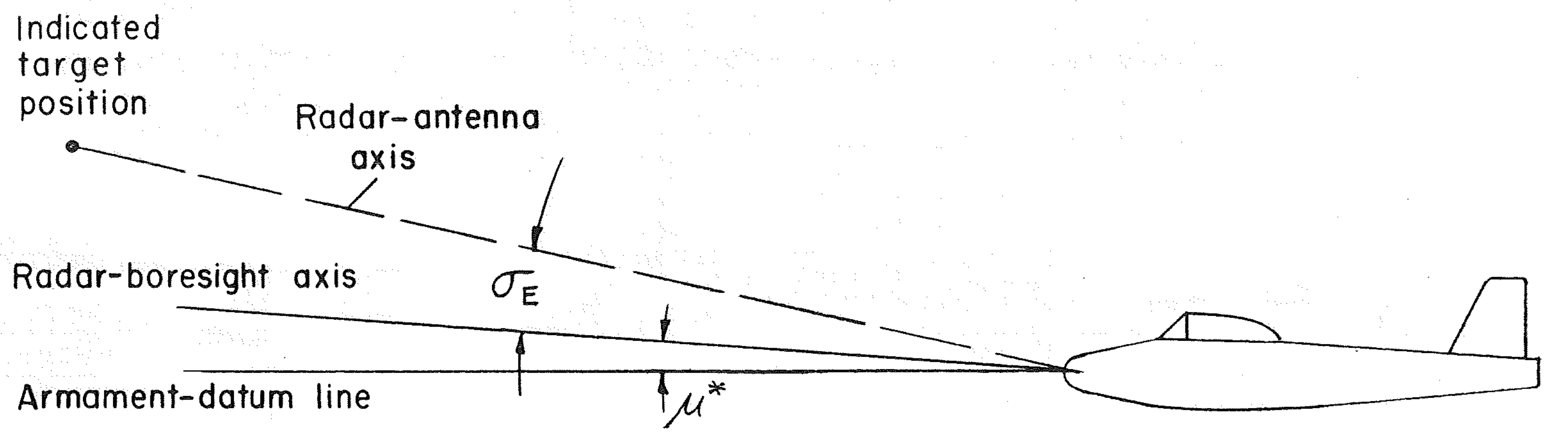


Figure 6.- Diagram of relationship between automatically controlled interceptor axis system where no deflection tracking error exists. (Applies to flight tests without lead angles.)

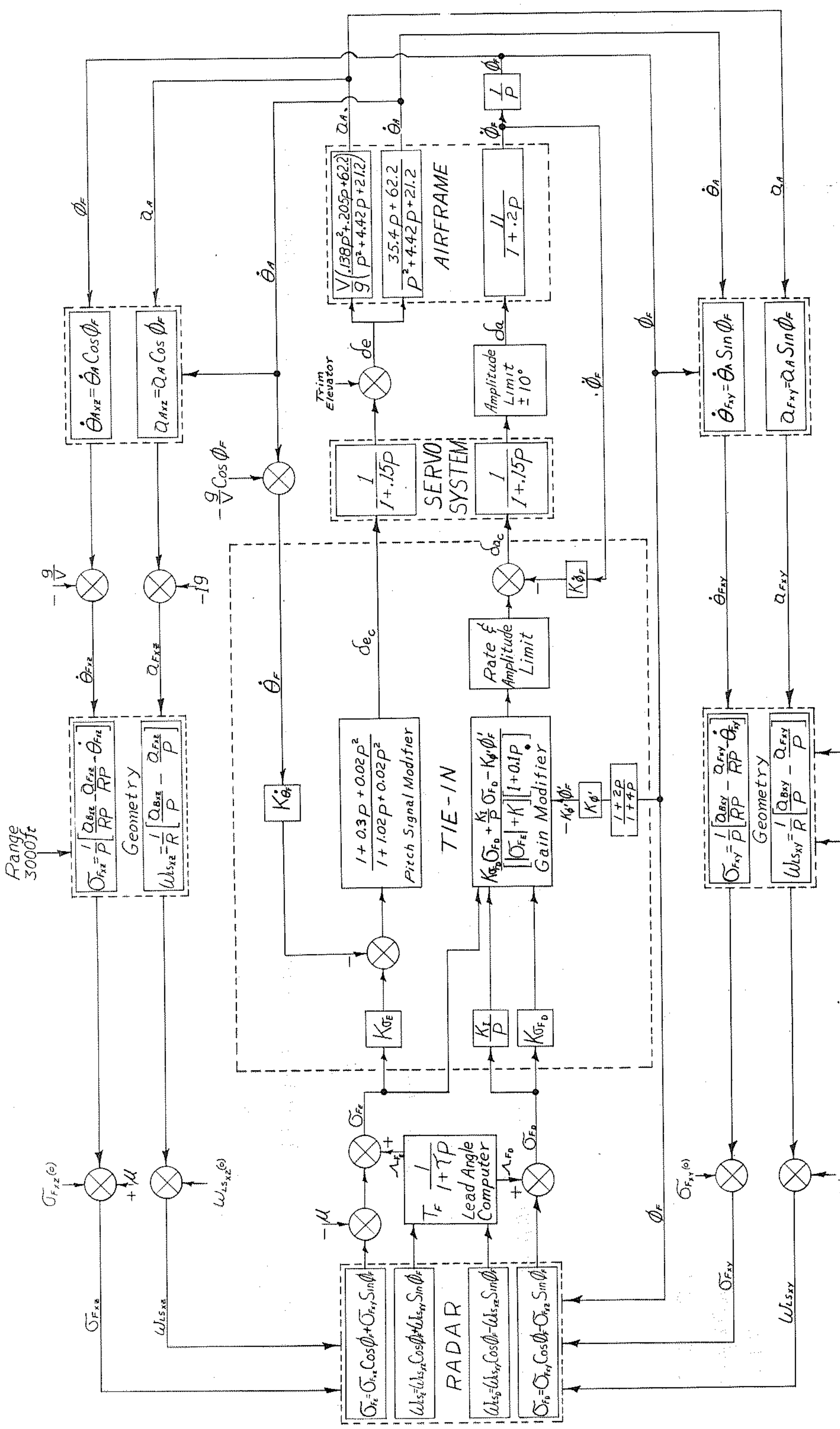


Figure 7.- Schematic of automatic-interceptor problem as set up on analog simulator.

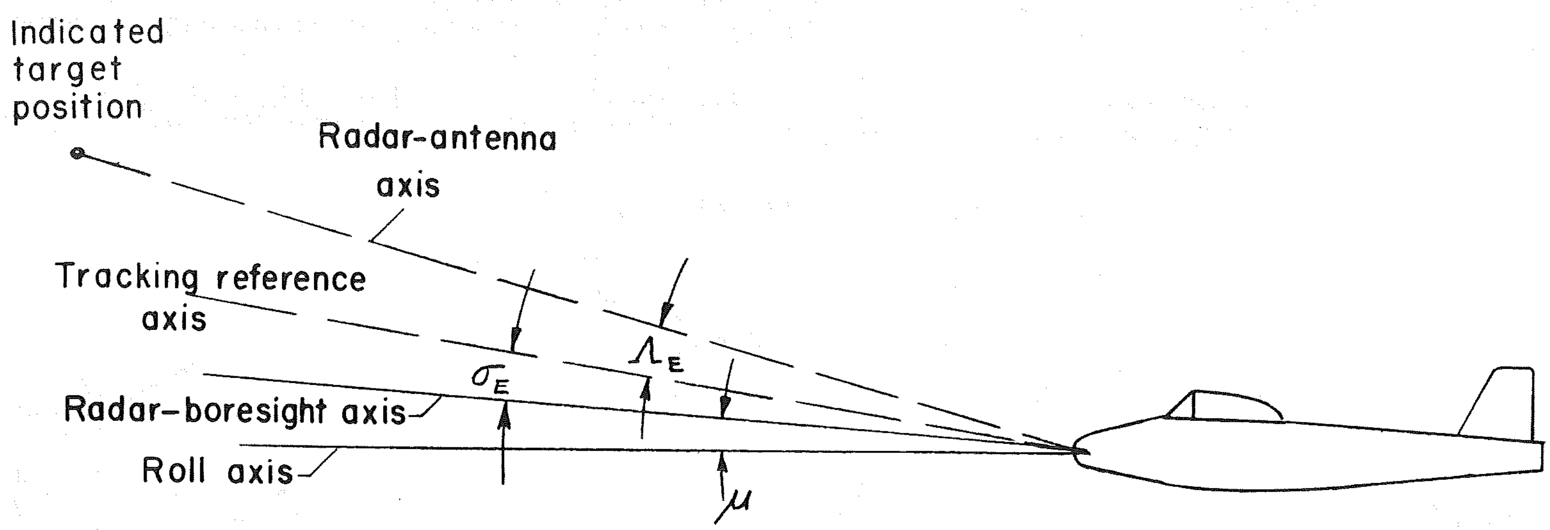
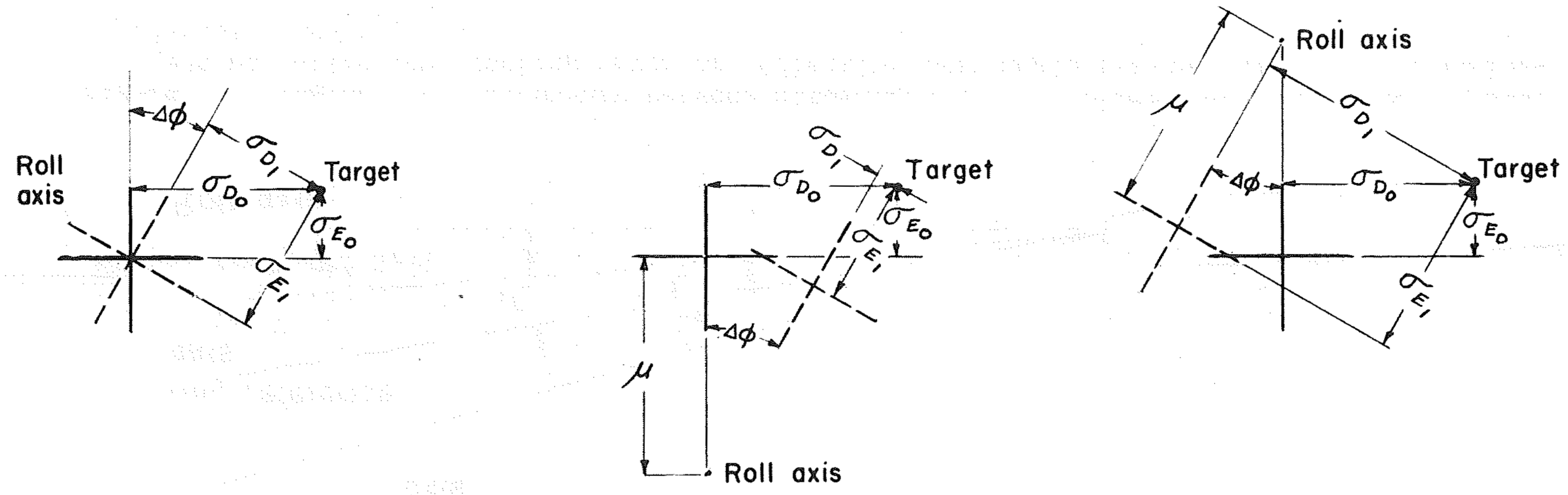


Figure 8.- Diagram of relationship between automatically controlled interceptor-axis system where no deflection tracking error or deflection lead angle exists. (Applies to analog-simulator tests.)

Note: Dashed lines denote radar coordinates after roll to right



(a) Radar-boresight axis colinear with roll axis.

(b) Radar-boresight axis elevated above roll axis.

(c) Radar-boresight axis depressed below roll axis.

Figure 9.- Two-dimensional diagrams showing effects on tracking errors caused by an interceptor rolling through a moderate bank angle where different orientations of the radar-boresight axis with respect to the roll axis exists.

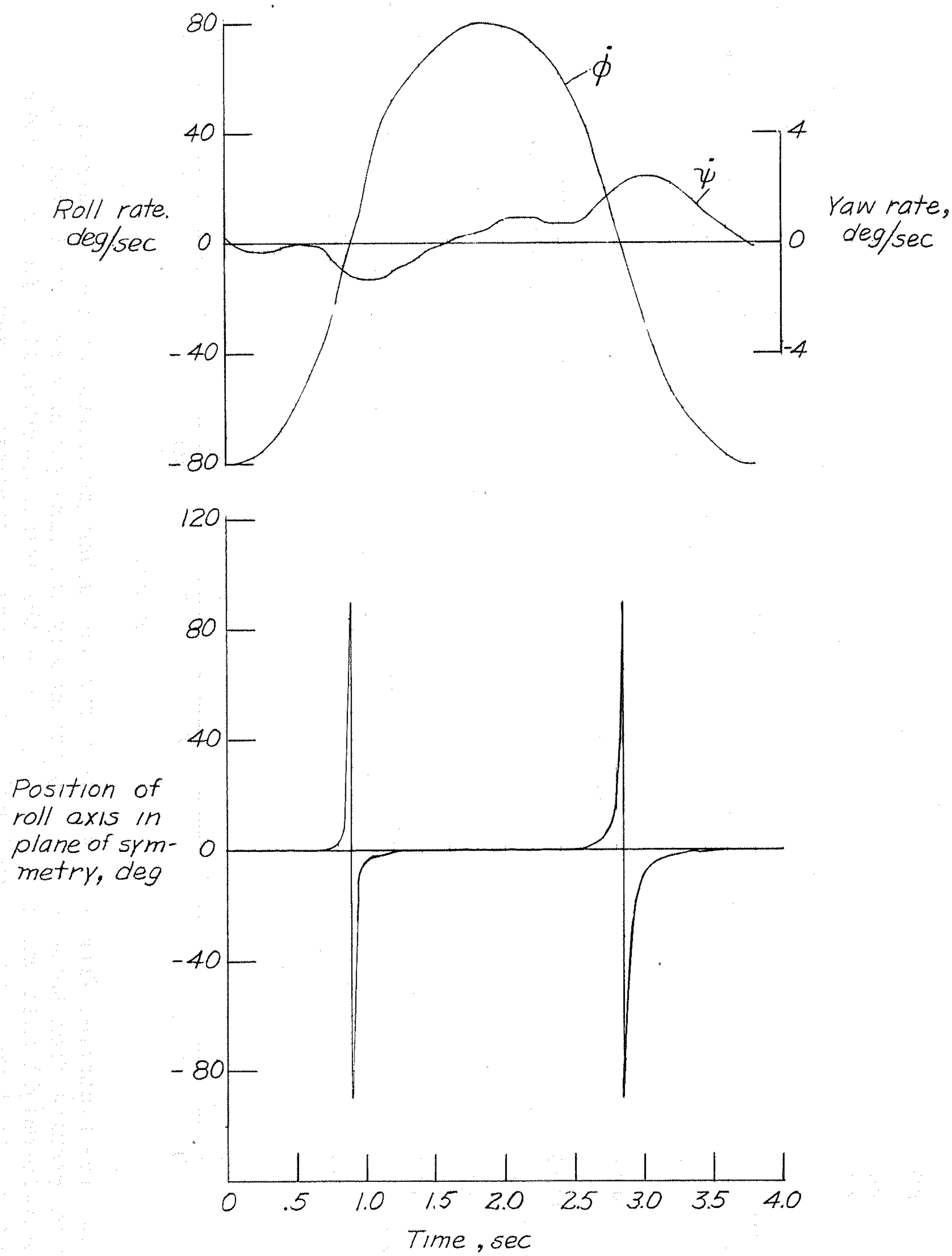


Figure 10.- Time histories of one cycle of a typical interceptor lateral oscillation used in determining average position of roll axis in the plane of symmetry.

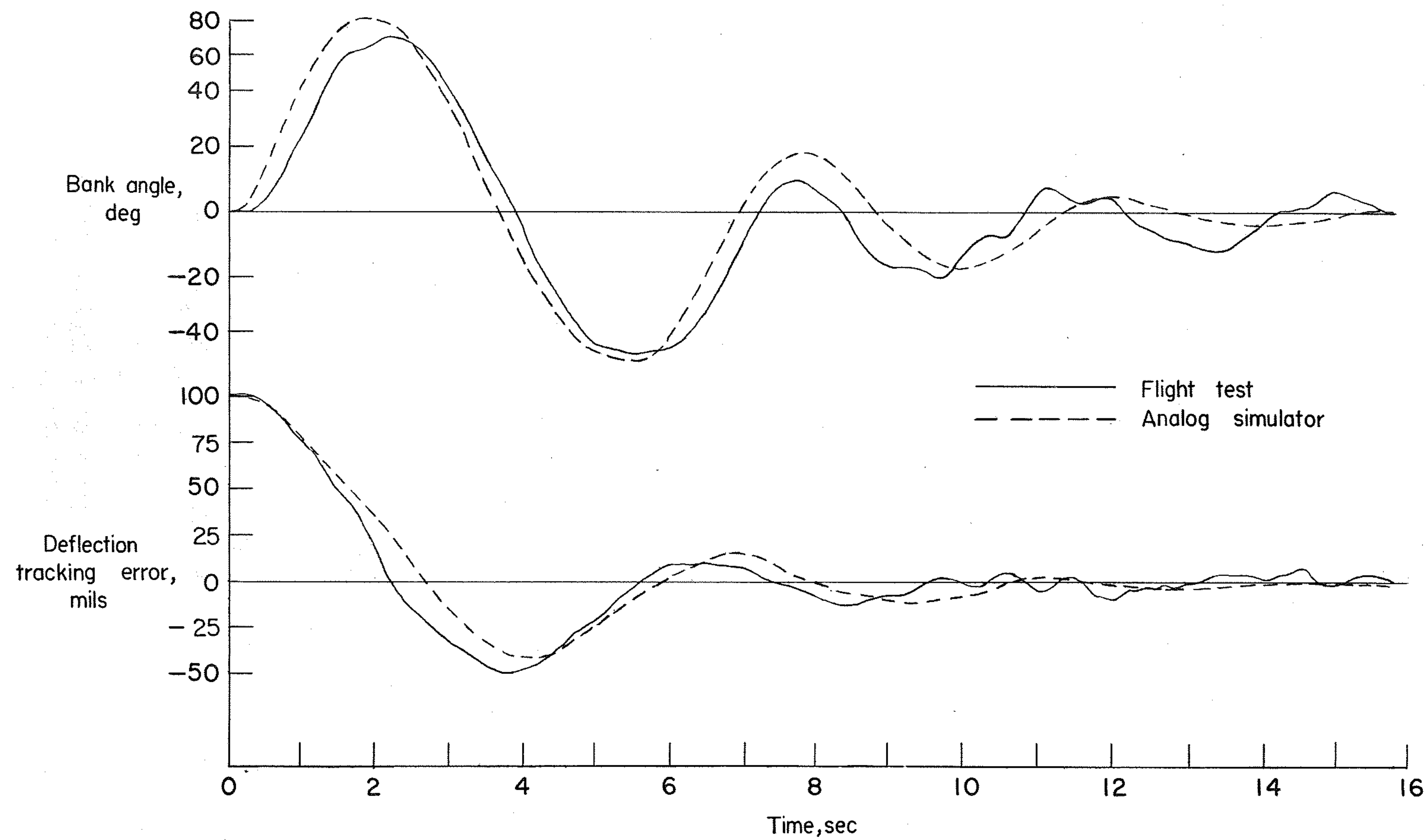


Figure 11.- Time histories from a flight-test run and an analog-simulator run illustrating similarity of results from these tests. Approximately the same gains were used in both runs. The radar-boresight axis was oriented 1/2° above the armament-datum line for the flight-test system and coincident with the roll axis for the analog-simulator system.

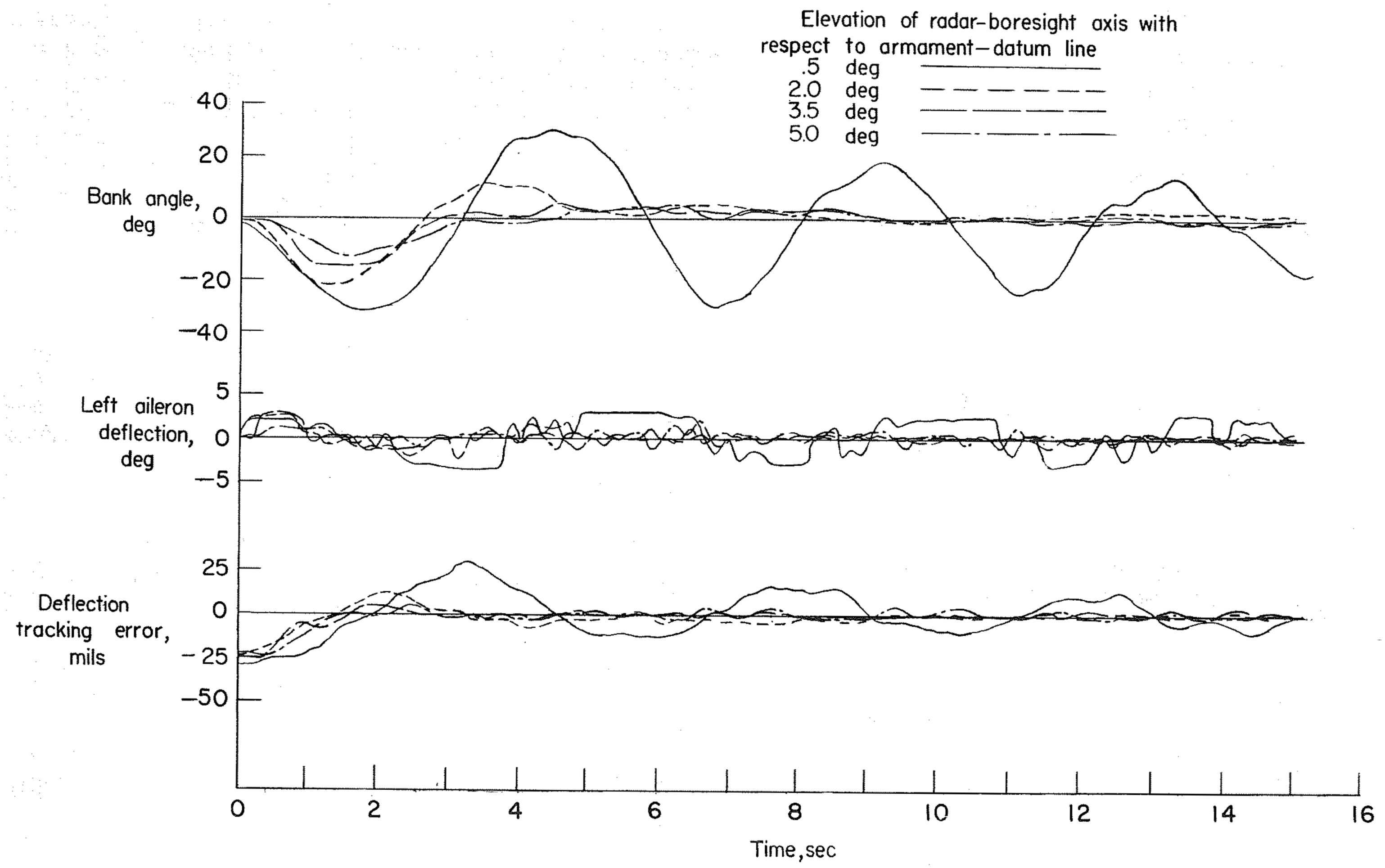


Figure 12.- Flight-test time histories of interceptor response to an initial deflection error showing the stabilizing effect of elevating the radar-boresight axis. Normal gains were used in each case. Target is nonmaneuvering.

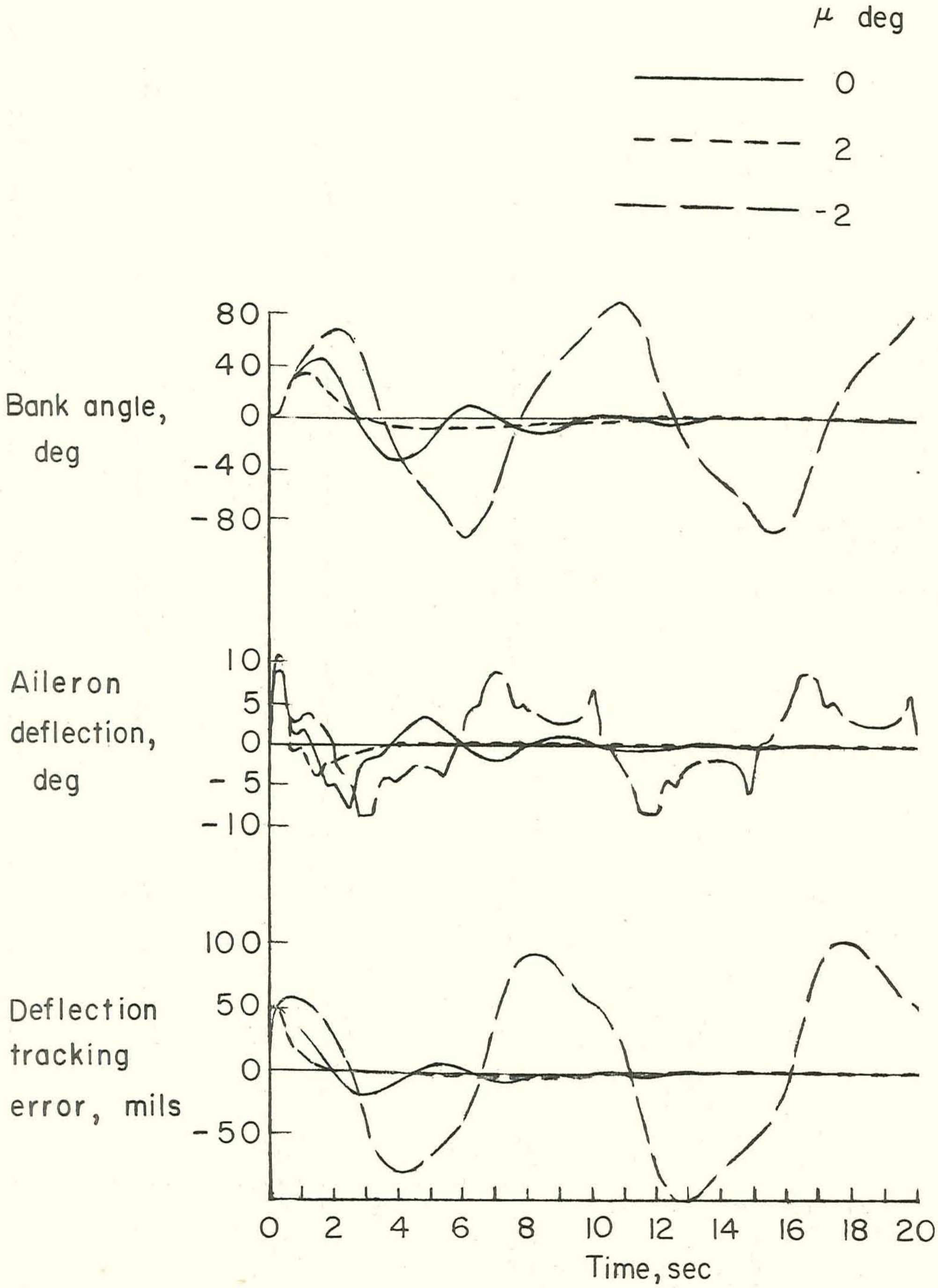


Figure 13.- Analog-simulator time histories of interceptor response to an initial deflection error showing the stabilizing effect of elevating the radar-boresight axis. Normal gains were used in each case, and the target is nonmaneuvering. No lead angles.

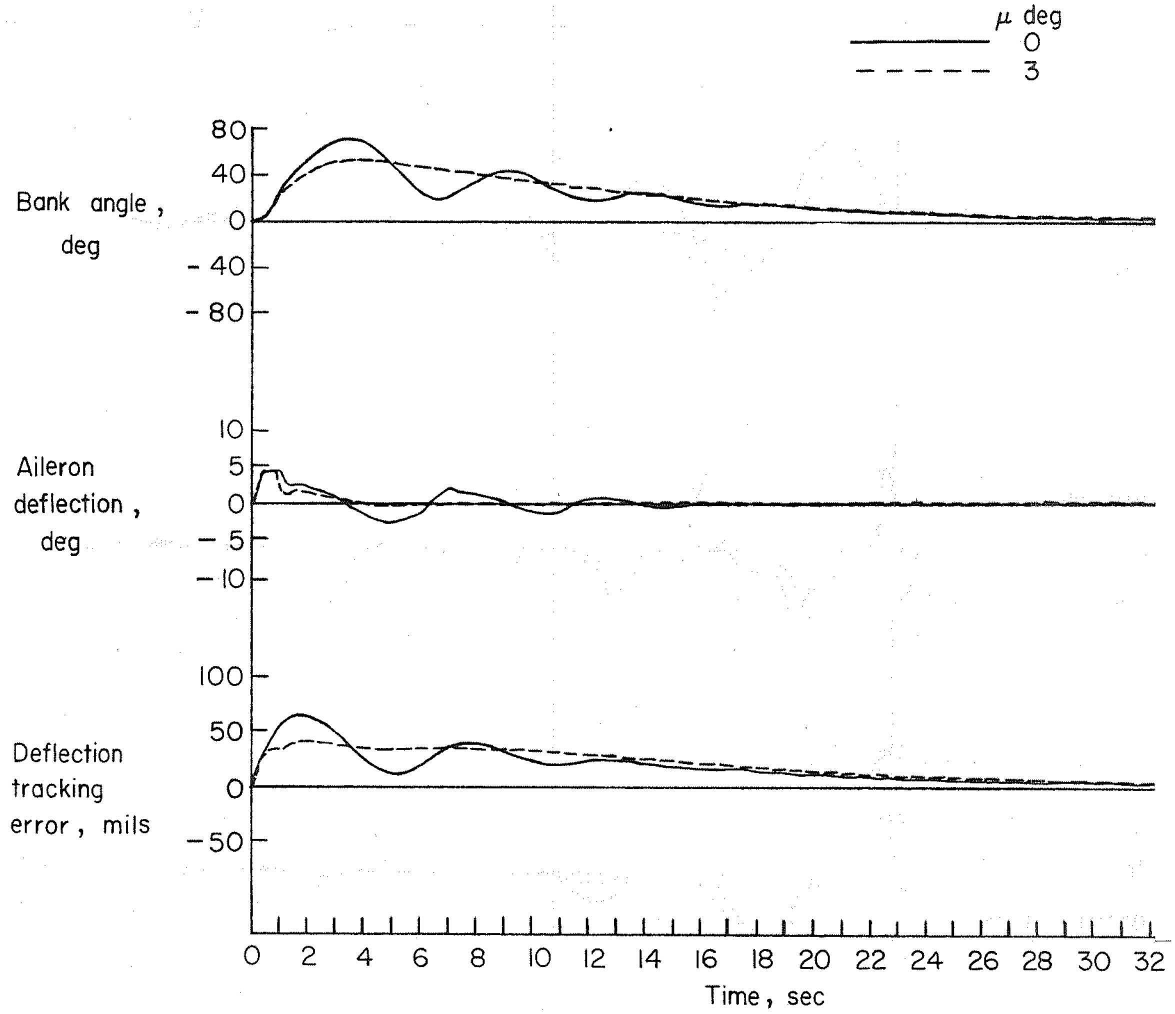


Figure 14.- Time histories from analog-simulator run where interceptor began attack from 30° off target tail showing stabilizing influence of elevating the radar-boresight axis. Range equals 7,000 feet.

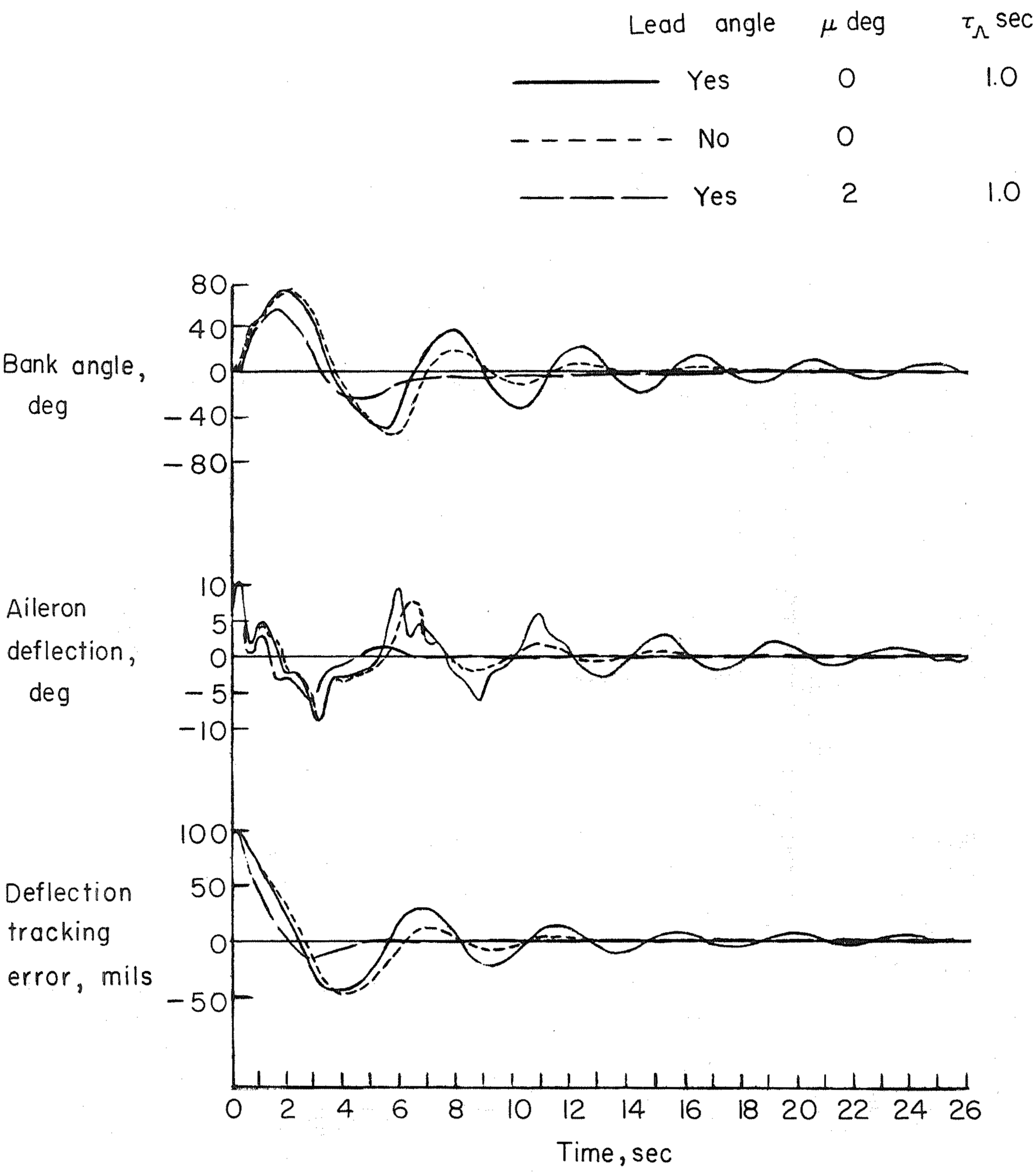


Figure 15.- Analog-simulator time histories of interceptor response to initial deflection error showing destabilizing effect of lead-angle filtering and stabilizing effect of elevating radar-boresight axis. Normal gains were used in each case. Target is nonmaneuvering.

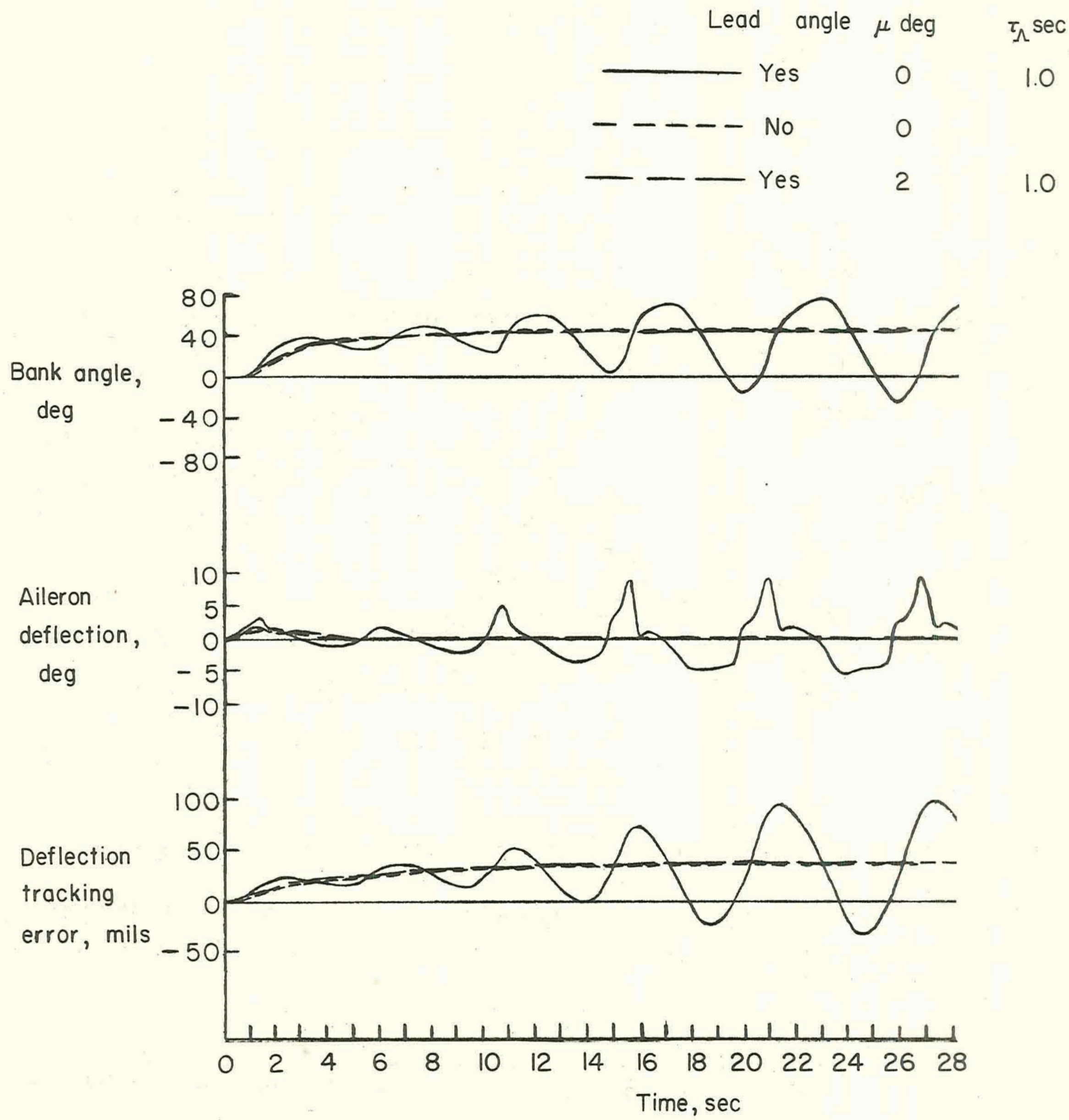
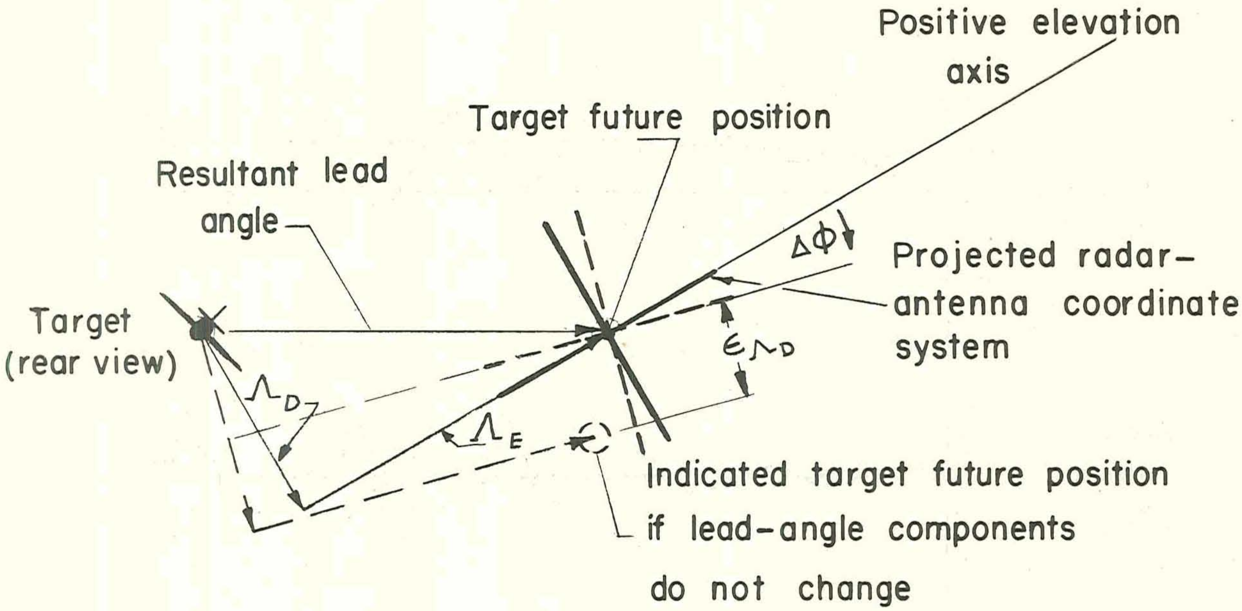
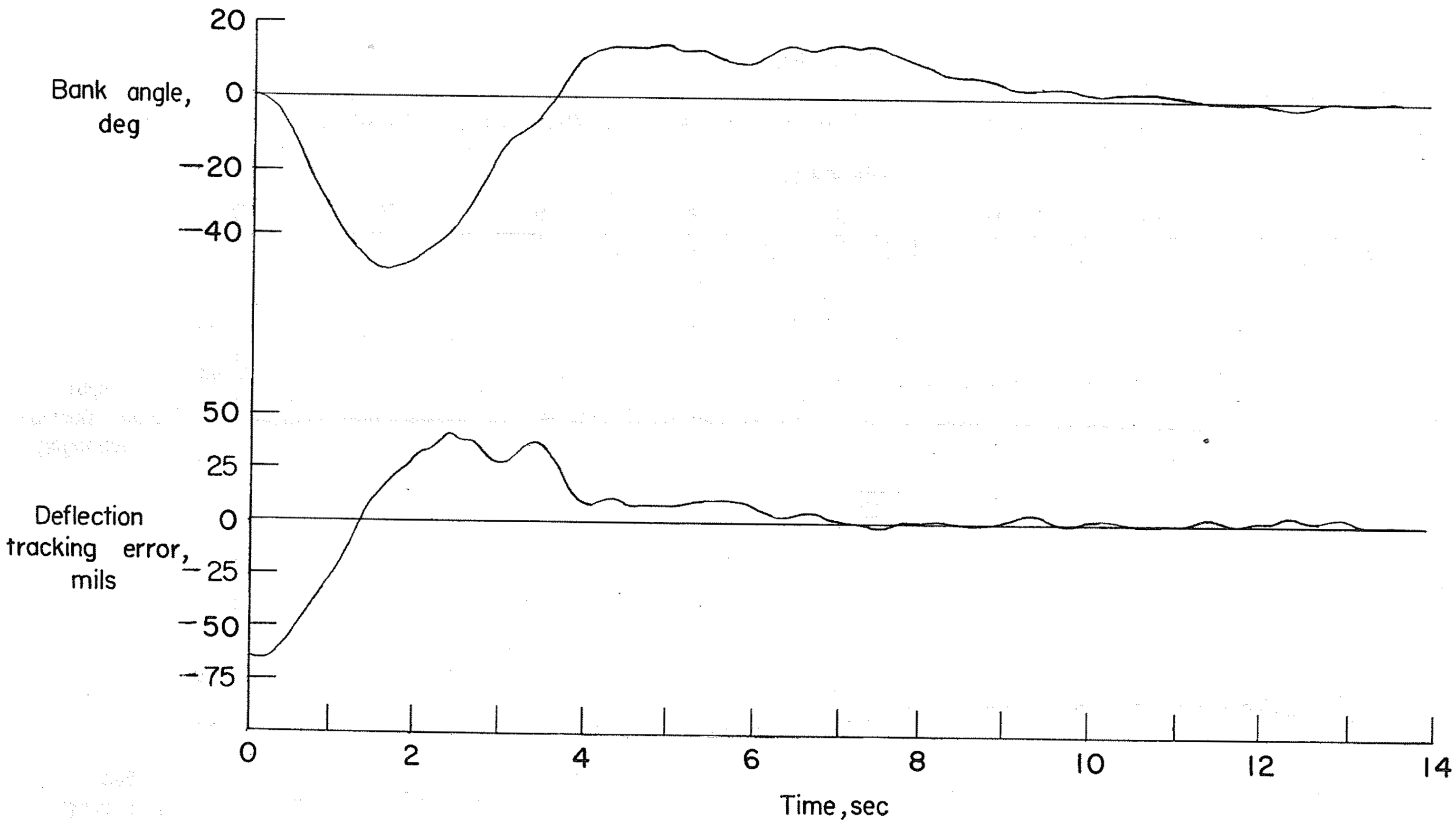


Figure 16.- Analog-simulator time histories of interceptor response to target turning maneuver showing destabilizing effect of lead-angle filtering and stabilizing effect of elevating radar-boresight axis. Normal gains were used in each case.



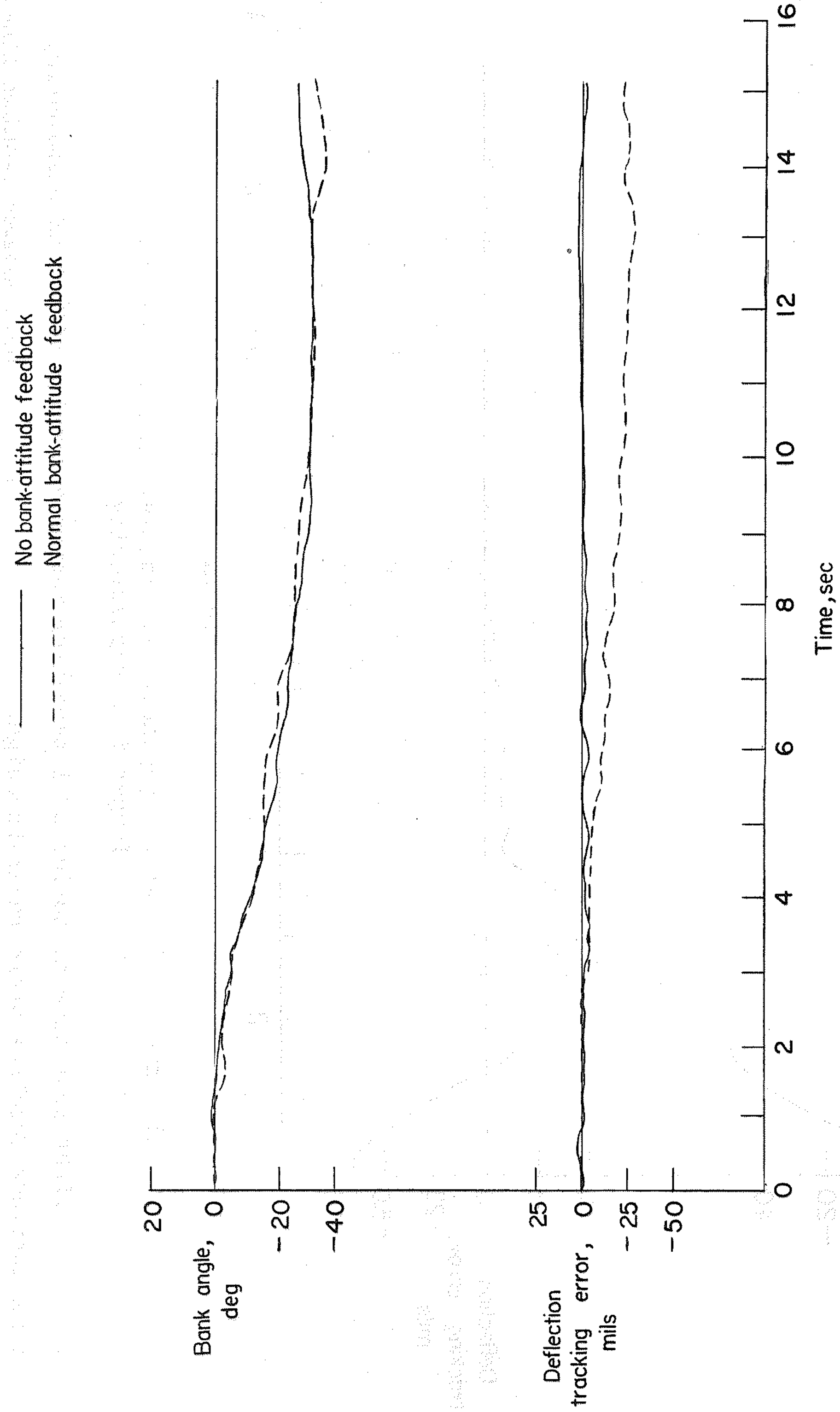
Note: Dashed lines denote position after interceptor bank

Figure 17.- Two-dimensional diagram illustrating how lead-angle filtering creates a destabilizing influence upon interceptor tracking performance.



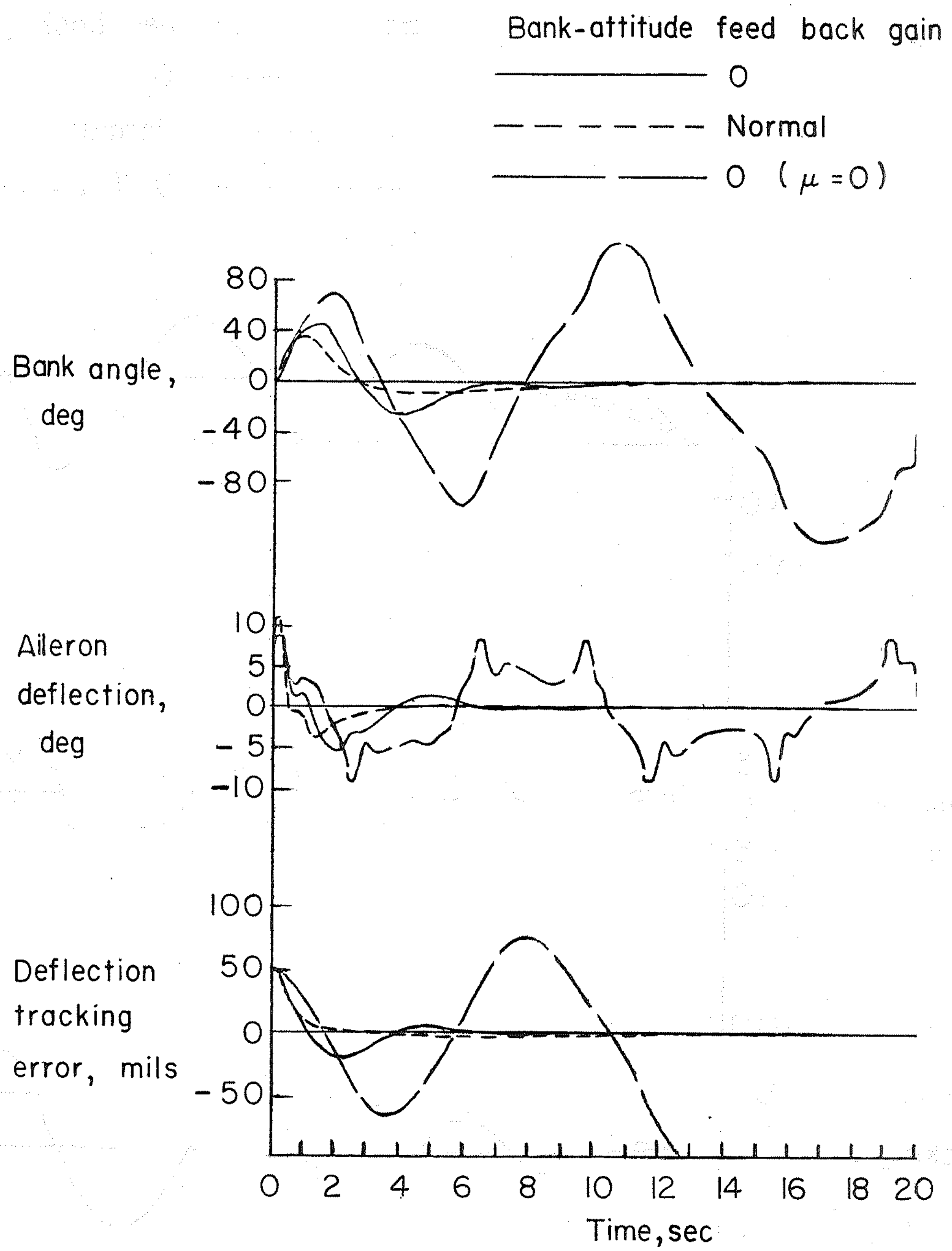
(a) Response following engagement with initial deflection error. Target is nonmaneuvering.

Figure 18.- Flight-test time histories of interceptor response showing effect of eliminating bank-attitude feedback where radar-boresight axis is elevated $3\frac{1}{2}^{\circ}$ above armament-datum line.



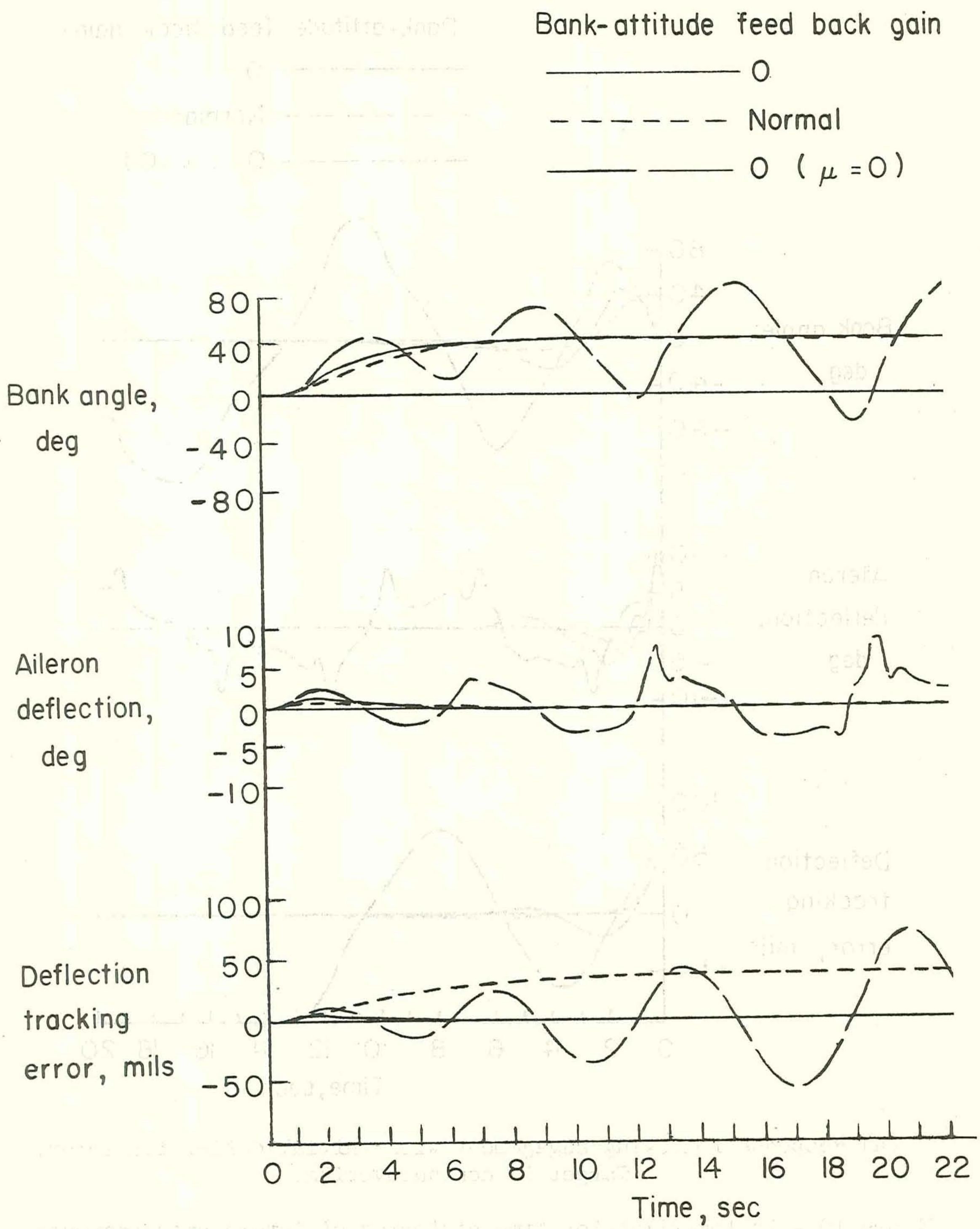
(b) Response following target entry into a steady turn (30° bank).

Figure 18.- Concluded.



(a) Response following engagement with initial deflection error. Target is nonmaneuvering.

Figure 19.- Analog-simulator time histories of interceptor response showing effect of eliminating bank-attitude feedback where radar-boresight axis is elevated 2° above roll axis.



(b) Response following target entry into steady 1 g turn.

Figure 19.- Concluded.

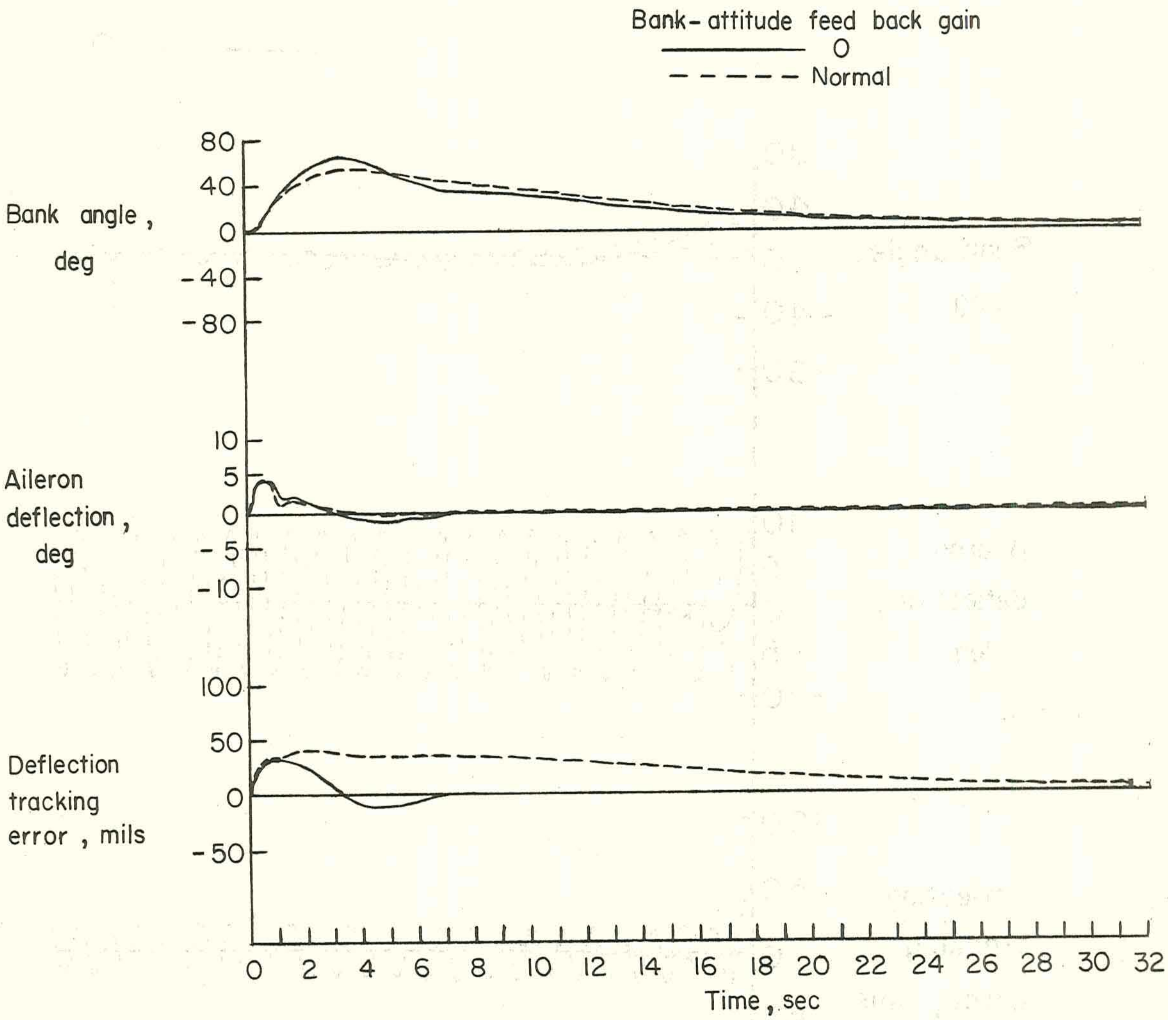


Figure 20.- Time histories from analog-simulator run where interceptor began attack from 30° off target tail showing beneficial effect of eliminating bank-attitude feedback. Range equals 7,000 feet.

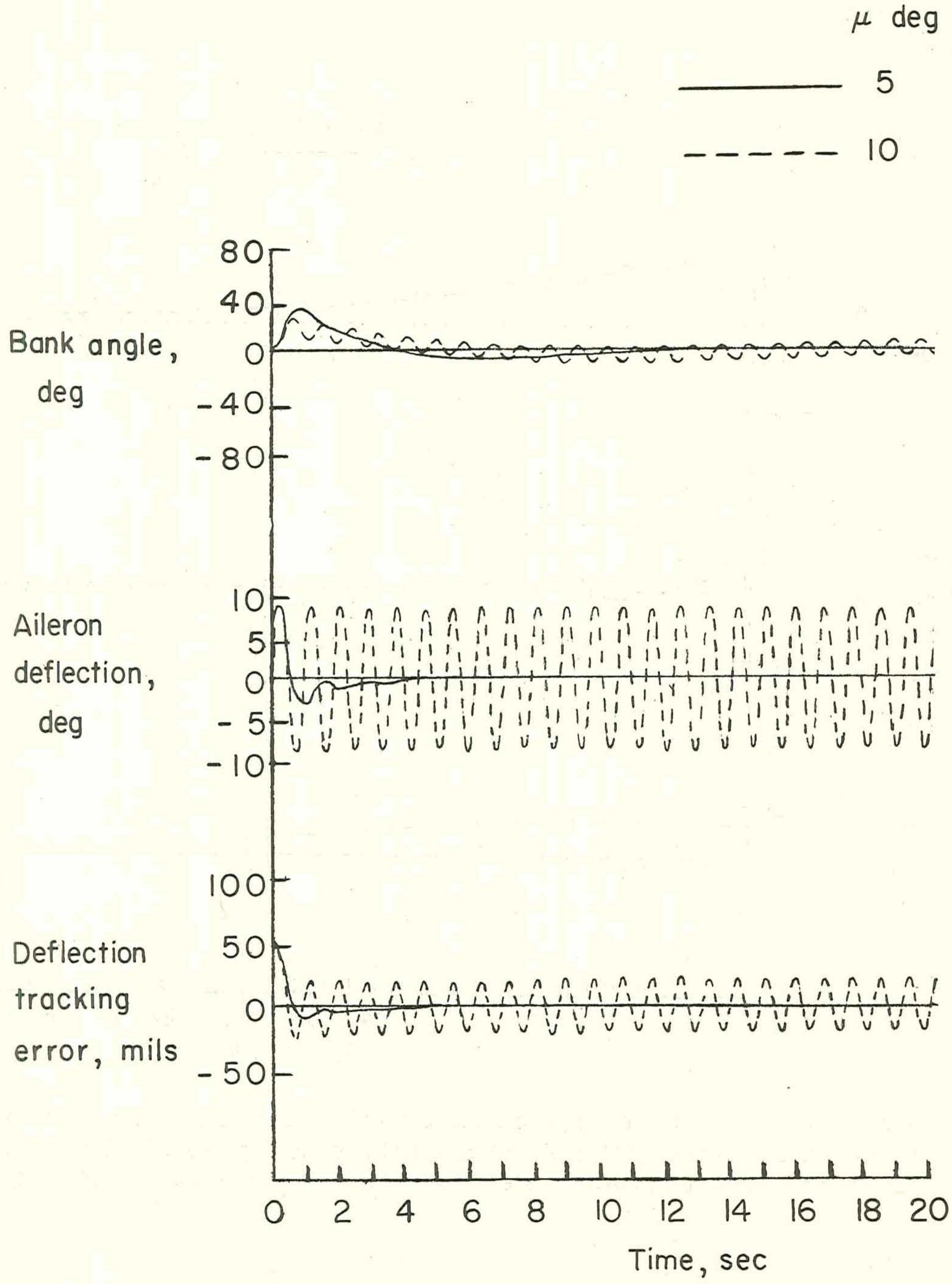


Figure 21.- Analog-simulator time histories showing the one-cycle-per-second oscillation that results when radar-boresight axis is elevated to large angles. Bank-attitude feedback is eliminated. No lead angles. Target is nonmaneuvering.

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[redacted]