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# NASA TECH BRIEF
*Marshall Space Flight Center*



## Numerical Solution of Potential Flow Problems in Terms of Flux Components

Flux components in vapor flow are numerically determined by solution of a potential flow problem. In a two-dimensional Cartesian coordinate system, this flow is described by Laplace's equation:

$$\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad (1)$$

where T is the potential and y and z are coordinates. The flux components are then defined in terms of the potential by

$$v = -k\frac{\partial T}{\partial y} \qquad (2)$$

$$w = -k\frac{\partial T}{\partial z} \qquad (3)$$

where v and w are the flux components in the y and z direction, respectively, and k is the transport coefficient. There are several excellent numerical methods of solution of equation (1); however, this solution yields values of the velocity potential, and the velocity components must be obtained from these values by numerical differentiation. Consequently, it is desirable to develop a numerical solution for this problem in terms of the velocity components.

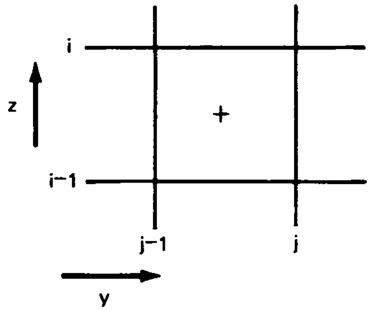


Figure 1.

Flux components can be determined by solving the differential equations which define them. These are the continuity principle

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (4)$$

and an irrotationality condition

$$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0 \qquad (5)$$

Equation (4) is equivalent to equation (1), and equation (5) is obtained by equating the cross-partial derivatives of (2) and (3). Both equations contain only first derivatives, and therefore, a centered difference approach is the appropriate one.

The method of solution must be somewhat different, however, as the potential flow problem has split boundary conditions in both directions. Typical boundary conditions define either v or w at all boundaries. The region in which these equations apply is divided in both directions by grid lines. The first approach to this problem is to determine both v and w at all intersections in the grid. A typical section of the grid is shown in Figure 1. Finite difference analogs for the continuity equation and the condition for irrotational flow equation are centered about the center of the square unit (point marked by the cross), and each finite difference equation contains four values of v and four of w. The analogs obtained are second-order correct. Several iterative methods have been developed for the solution of the resulting equations. This method provides an alternate numerical solution of potential flow problems in two dimensions. However, it has no particular advantages over the methods for solving in terms of the potential.

A simple innovation in the method of defining the grid leads to a number of advantages which makes the method very promising. The location for the points of this checkerboard method are shown in Figure 2. The

(continued overleaf)

dependent variable v is determined as the points designated by the squares (□) (Figure 2), and w is determined as those designated by the triangles (△). The analog to the continuity equation is centered about the point marked by the open circle (○); and that to the irrotationality condition equation, by the filled circle (●). As a result, each finite difference equation contains only two values of v and two of w. These equations are much simpler, and the equations from adjacent rows can be combined so that all variables can be eliminated except the boundary conditions and values of v and w from the center rows of points. The matrix of coefficients for these equations is of the band form and can be solved by an algorithm. The solution has negligible round-off error for as many as twenty rows of each variable. In this manner, a direct solution is obtained for the potential flow problem with no iteration. Furthermore, even though the checkerboard method is also second-order correct, it has been found to have much less truncation error than the conventional method on several problems for which the exact solution is known.

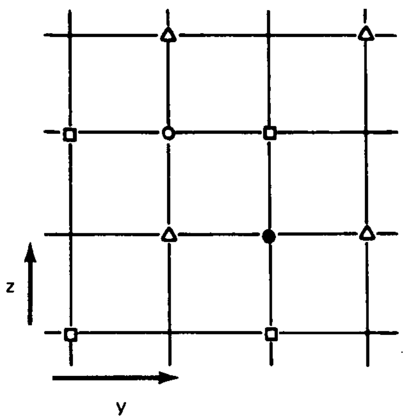


Figure 2.

## Notes:

1. Information concerning this innovation may be useful in many areas of physics, e.g., conduction, heat transfer, etc.
2. Requests for further information may be directed to:
   Technology Utilization Officer
   Marshall Space Flight Center
   Code A&PS-TU
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