

65736

A NEUTRAL PARTICLE WAKE METHOD FOR MEASURING THE ATMOSPHERIC TEMPERATURE FROM A SATELLITE

L. H. BRACE
W. R. HOEGY
R. F. THEIS
L. E. WHARTON

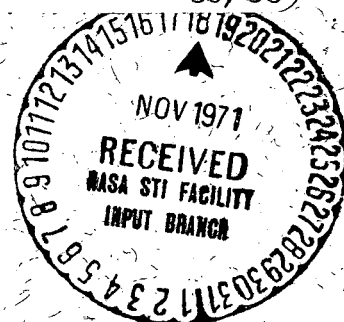
N72-10973 (NASA-TM-X-65736) A NEUTRAL PARTICLE WAKE
METHOD FOR MEASURING THE ATMOSPHERIC
TEMPERATURE FROM A SATELLITE L.H. Brace,
et al (NASA) Jul. 1971 36 p CSCL 20M

Unclas
08690

G3/33

JULY 1971

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
Springfield, Va. 22151

**GSFC**

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

FACILITY FORM 602

(ACCESSION NUMBER)

36

(PAGES)

TMX-65736

(NASA CR OR TMX OR AD NUMBER)

(THRU)

B3

(CODE)

33

(CATEGORY)

A NEUTRAL PARTICLE WAKE METHOD
FOR MEASURING THE
ATMOSPHERIC TEMPERATURE FROM A SATELLITE

L. H. Brace
W. R. Hoegy
R. F. Theis
L. E. Wharton*

Laboratory for Planetary Atmospheres

July 1971

*Physics Department, University of Maryland

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

PRECEDING PAGE BLANK NOT FILMED

A NEUTRAL PARTICLE WAKE METHOD
FOR MEASURING THE
ATMOSPHERIC TEMPERATURE FROM A SATELLITE

L. H. Brace
W. R. Hoegy
R. F. Theis
L. E. Wharton

Laboratory for Planetary Atmospheres
Goddard Space Flight Center
Greenbelt, Maryland 20771

ABSTRACT

A method is described that would permit a satellite-borne neutral mass spectrometer to perform measurements of the atmospheric temperature. The method employs the spectrometer to examine the partial pressure variations that occur as the wake of a small rectangular baffle is swept across the entrance orifice of the spectrometer. For a given baffle size and mounting distance from the orifice the depth of the resulting pressure minimum depends only upon the thermal velocity, or temperature, of the observed species. A check upon the validity of the method can be obtained by measuring the wake characteristics of more than one species and/or by employing each of several baffle sizes.

The theory employed includes the effect of a finite orifice size, finite baffle length and the backscattering of particles from the baffle into the orifice. It is found that a suitable baffle arrangement can be achieved that will permit the temperature to be measured over at least the range normally encountered in the thermosphere (200°-2000°K) and, depending upon the sensitivity and background pressure of the spectrometer, over an altitude range of about 140 to 600 kilometers.

CONTENTS

| | Page |
|---|------|
| ABSTRACT | iii |
| INTRODUCTION | 1 |
| DESCRIPTION OF THE NEUTRAL WAKE METHOD..... | 2 |
| The Pressure Characteristics..... | 6 |
| Temperature Resolution | 6 |
| SOURCES OF ERROR..... | 11 |
| Scattering..... | 11 |
| Finite Orifice Size | 13 |
| The Finite Length of the Baffle..... | 13 |
| Spin Axis Attitude Errors..... | 13 |
| Background Pressure | 17 |
| SELECTION OF THE BAFFLE SIZE..... | 17 |
| THE LOW ALTITUDE LIMIT..... | 19 |
| THE USE OF MULTIPLE BAFFLES AND MASSES..... | 20 |
| THE NEUTRAL WAKE EXPERIMENT ON A NON-SPINNING SPACECRAFT | 20 |
| APPENDIX — THEORY OF THE METHOD..... | 23 |
| REFERENCES..... | 32 |

A NEUTRAL PARTICLE WAKE METHOD
FOR MEASURING THE
ATMOSPHERIC TEMPERATURE FROM A SATELLITE

INTRODUCTION

The gas kinetic temperature, T , is a parameter of intrinsic interest in the study of the thermosphere, the region between about 100 and 300 kilometers above the Earth's surface. Its large spatial and temporal variations reflect the complex character of the sources of atmospheric heating, and any attempt to understand this region requires a knowledge of its temperature behavior.

Most of our knowledge about the global behavior of temperature has been inferred from satellite drag measurements of the atmospheric density (Jacchia, 1964), a procedure that involves tenuous assumptions about the static nature of the atmosphere at lower altitudes and provides little information about the small scale spatial and temporal variations in temperature.

Incoherent backscatter observations of ion temperature have been employed to infer the behavior of the neutral gas temperature (McClure, 1971). These observations provide high resolution of the diurnal and altitudinal variations but are limited to a few fixed locations about the globe.

Satellite measurements of the neutral particle scale height by pressure gages and neutral particle mass spectrometers have not been reliable for deriving T because of the presence of horizontal structure in the atmosphere that tends to obscure the altitudinal variations. This problem is largely overcome by rocketborne mass spectrometers (Spencer et al., 1965, 1969) (Nier, 1964) but

these measurements are available in very limited numbers and at only a few locations.

Satellite measurements of 6300 Å airglow appear to provide good resolution of the global distribution of the exospheric temperature (Blamont, 1971) but provide little resolution of the altitudinal variations in temperature.

The neutral particle wake method described in this paper permits essentially instantaneous measurements of temperature along the satellite path from as high as perhaps 600 km down to approximately 140 kilometers, nearly the lowest altitudes envisioned for the atmosphere Explorer satellites for which a version of the experiment is being developed by Spencer (1971).

The method is related to the velocity scan technique employed on rockets (Spencer et al., 1965) and on the San Marco satellite (Spencer, 1971), but introduces a baffle to amplify the effect of the thermal velocity of the atmospheric particles. This method was first reported by Brace and Wu (1968) (1970) and is in this paper extended to include the effects of particle scattering, a finite baffle length, and other effects not considered earlier.

DESCRIPTION OF THE NEUTRAL WAKE METHOD

A major difficulty in performing direct measurements of atmospheric temperature from a satellite arises from its large translational velocity and the correspondingly large energy of the particles relative to the satellite. The method discussed here overcomes this difficulty by interrupting the directed

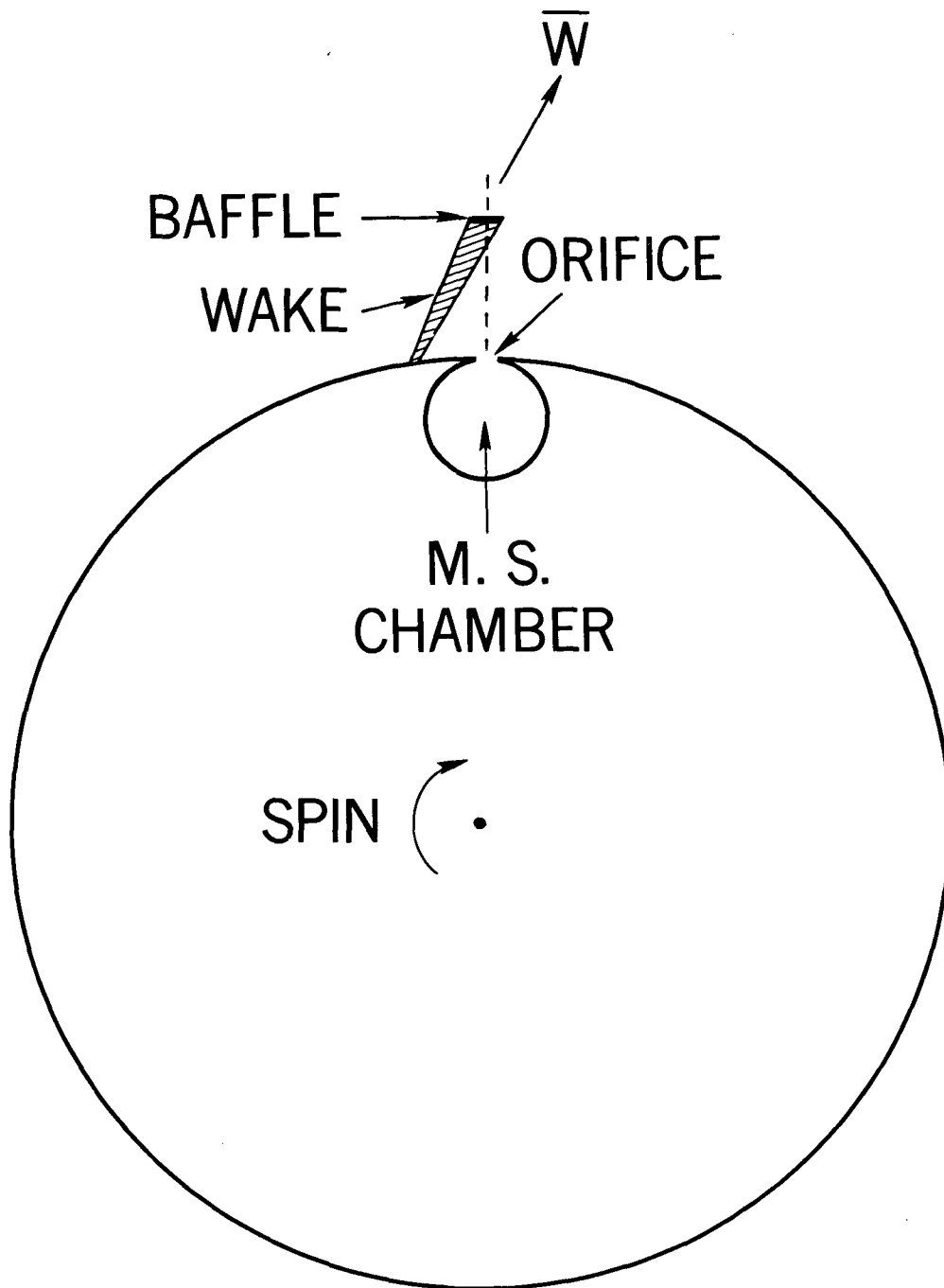


Figure 1a. Experimental arrangement. As the satellite spins, the wake behind the long rectangular baffle sweeps across the entrance orifice of a neutral particle mass spectrometer causing reductions in the partial pressures that are highly sensitive to the atmospheric temperature. The satellite spin axis is maintained perpendicular to the orbital plane so that the center of the wake passes over the orifice.

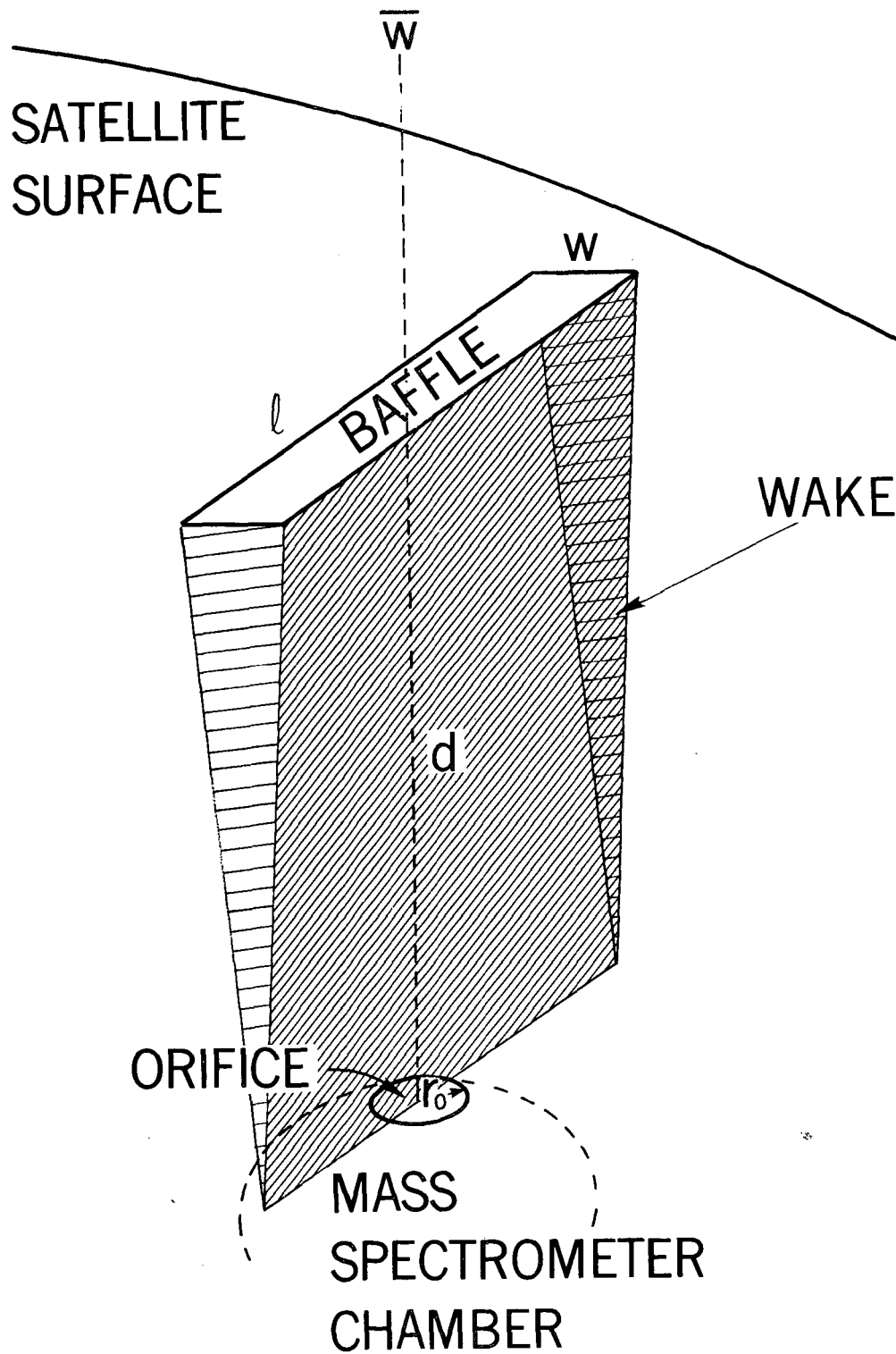


Figure 1b. Definition of the baffle parameters that control the temperature resolution obtained. A conceptual drawing of the wake at zero angle of attack is also shown.

flux of particles by a baffle and then sensing those particles that have moved into its wake owing to their thermal motions. The flux within the wake is very sensitive to temperature because the particles that populate this region do so only because of their thermal motion.

The experimental arrangement is shown in Figures 1a and 1b. In this arrangement, the sampling chamber of the mass spectrometer is mounted such that its orifice is perpendicular to the spin axis of the satellite. The orifice is circular with radius, r_o , that is small compared to the baffle width, w . The baffle length, ℓ , is several times its width and is mounted a distance, d , out along the normal to the orifice. The satellite is assumed to spin about an axis that is perpendicular to the orbital plane so that the velocity vector, \bar{w} , passes through the orifice normal once per satellite rotation and sweeps the wake of the baffle across the orifice. The resulting reduction in flux entering the orifice causes a pressure reduction within the chamber that for a given baffle depends only upon the species sampled and its temperature in the atmosphere. Because of their higher thermal velocities only H and He can populate the near wake, while O and N_2 can only populate the far wake. By selecting the species to be observed and the baffle dimensions, a desired decrease in partial pressure can be achieved for any given temperature. Therefore the temperature resolution is somewhat adjustable by selection of dimensions, in particular the w/d ratio.

The spin pressure variations shown in this paper were calculated using the theory outlined in the Appendix. A basic assumption has been that the particles suffer no collisions until they enter the sampling chamber where they thermalize to the wall temperature before passing back out through the orifice. We also assume equilibrium flow such that the influx and outflux of a given species are

equal, that is that the flux variations with angle of attack do not occur too rapidly for the chamber pressure to follow, and that chemical reactions with the surface do not generate or remove particles at a rate that is significant when compared with the net influx and outflux.

The Pressure Characteristics

To place the wake effect in perspective relative to the entire spin variation, Figure 2 shows the normalized N_2 partial pressure characteristics for two baffle ratios. The dashed line, $w/d = 0$, is the usual pressure characteristic for an orificed chamber with no baffle (Spencer et al., 1965). The larger baffle $w/d = 0.2$ causes a deeper and wider wake than the smaller baffle $w/d = 0.05$. The temperature was assumed to be 1200°K for this calculation. In all calculations the velocity is assumed to be 8 km/sec.

Temperature Resolution

To illustrate the temperature resolution that can be attained we have used the theoretical formulations outlined in the Appendix to calculate the N_2 partial pressure characteristics for two baffle ratios $w/d = 0.05$ and 0.1 , and for a family of temperatures from 300° to 1800°K. Figures 3 and 4 show these wake pressure characteristics for small angles of attack where the baffle is effective. Changes in temperature affect both the width and depth of the wake characteristics. Although there is no simple relationship between the baffle size and the temperature resolution, it can be seen from these curves that the temperature resolution varies with the size of the baffle. This is evident more clearly in Figure 5 which shows the percentage decrease in pressure at zero angle of attack for a family of baffles and range of temperatures. The slope of the curves is in a sense a

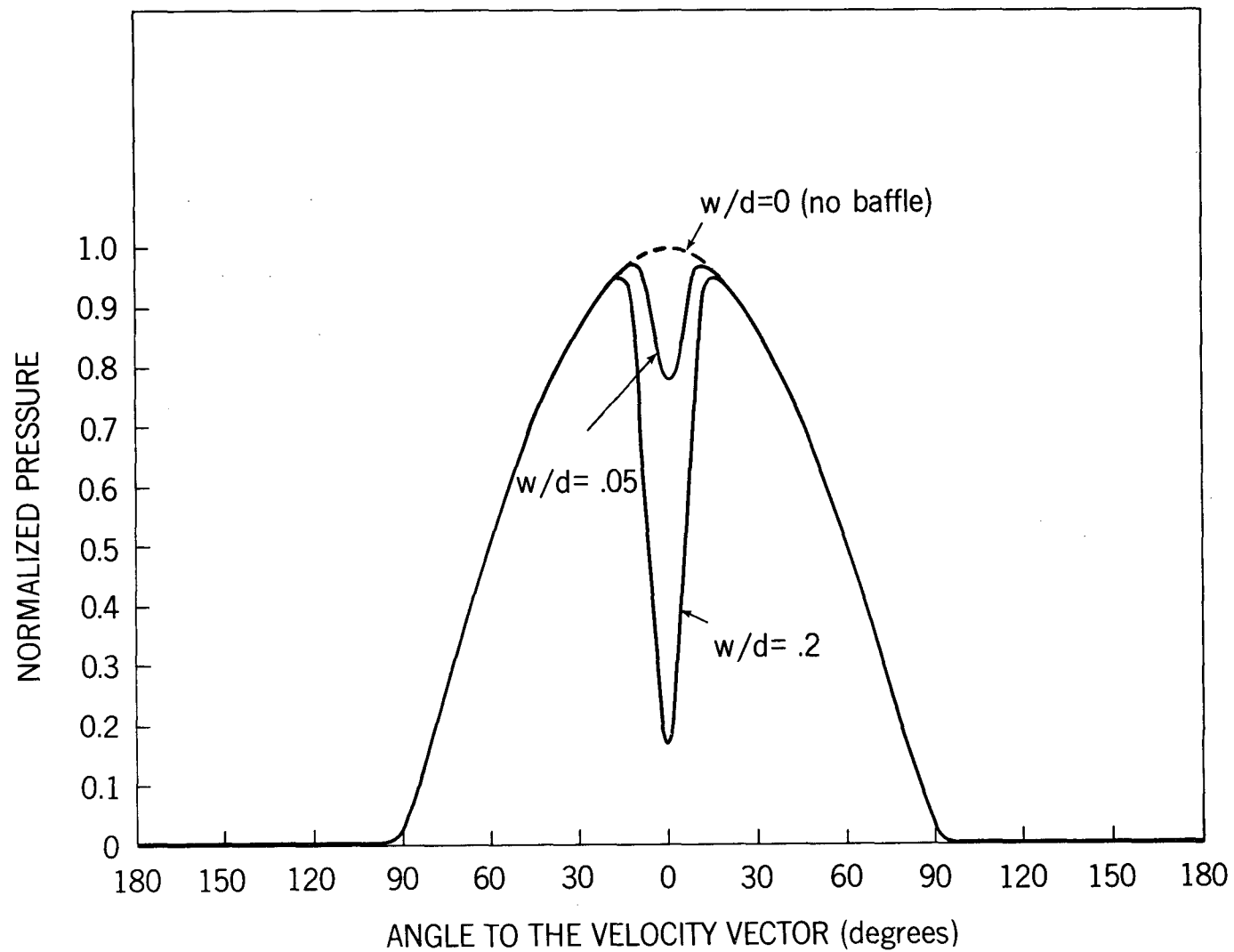


Figure 2. Normalized Pressure Characteristics for N_2 at $1200^\circ K$ and Baffles of $w/d = 0$ (No Baffle), 0.05 and 0.2

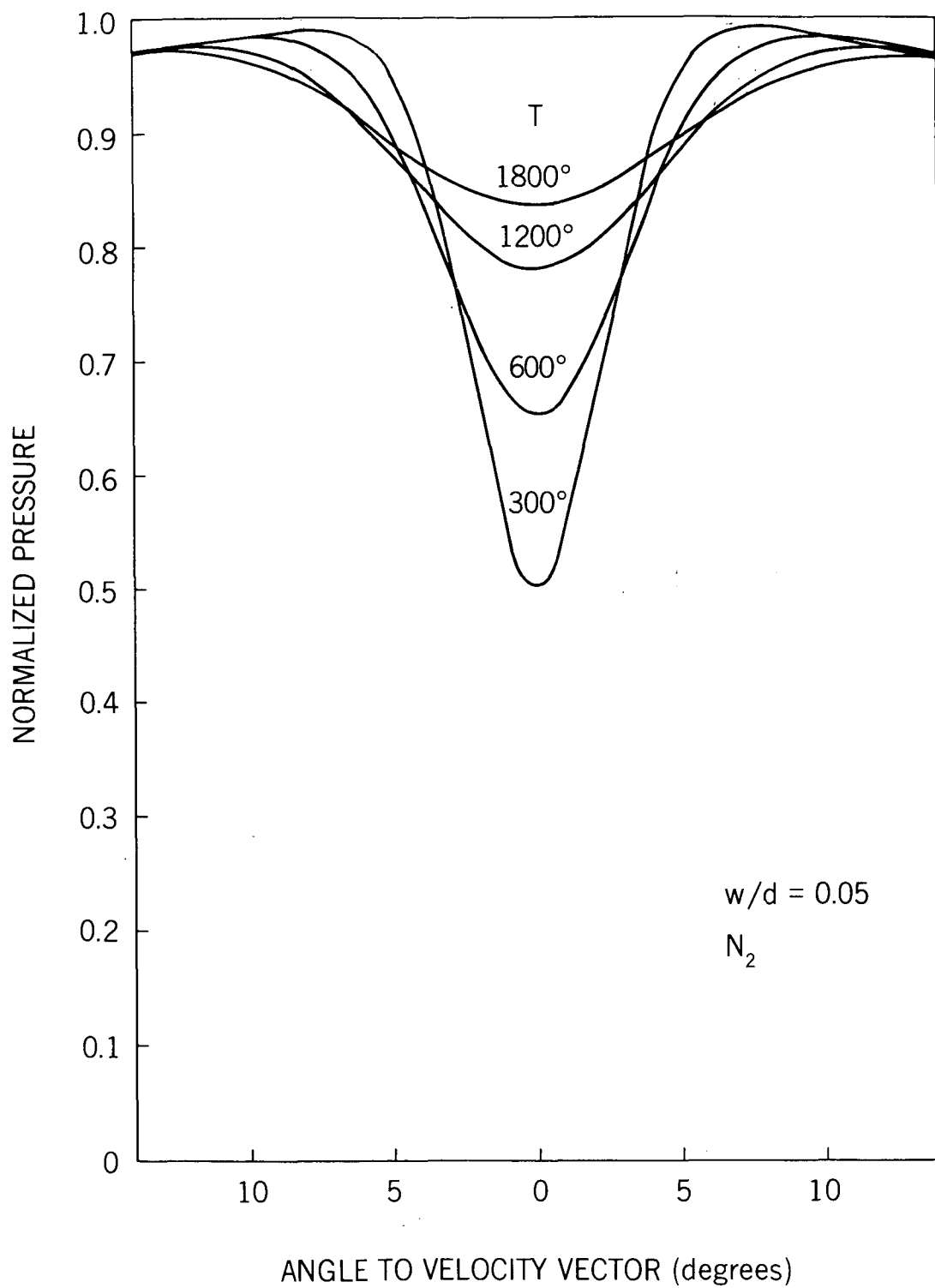


Figure 3. Wake Characteristics of N_2 for a Baffle of $w/d = 0.05$ at Various Atmospheric Temperatures

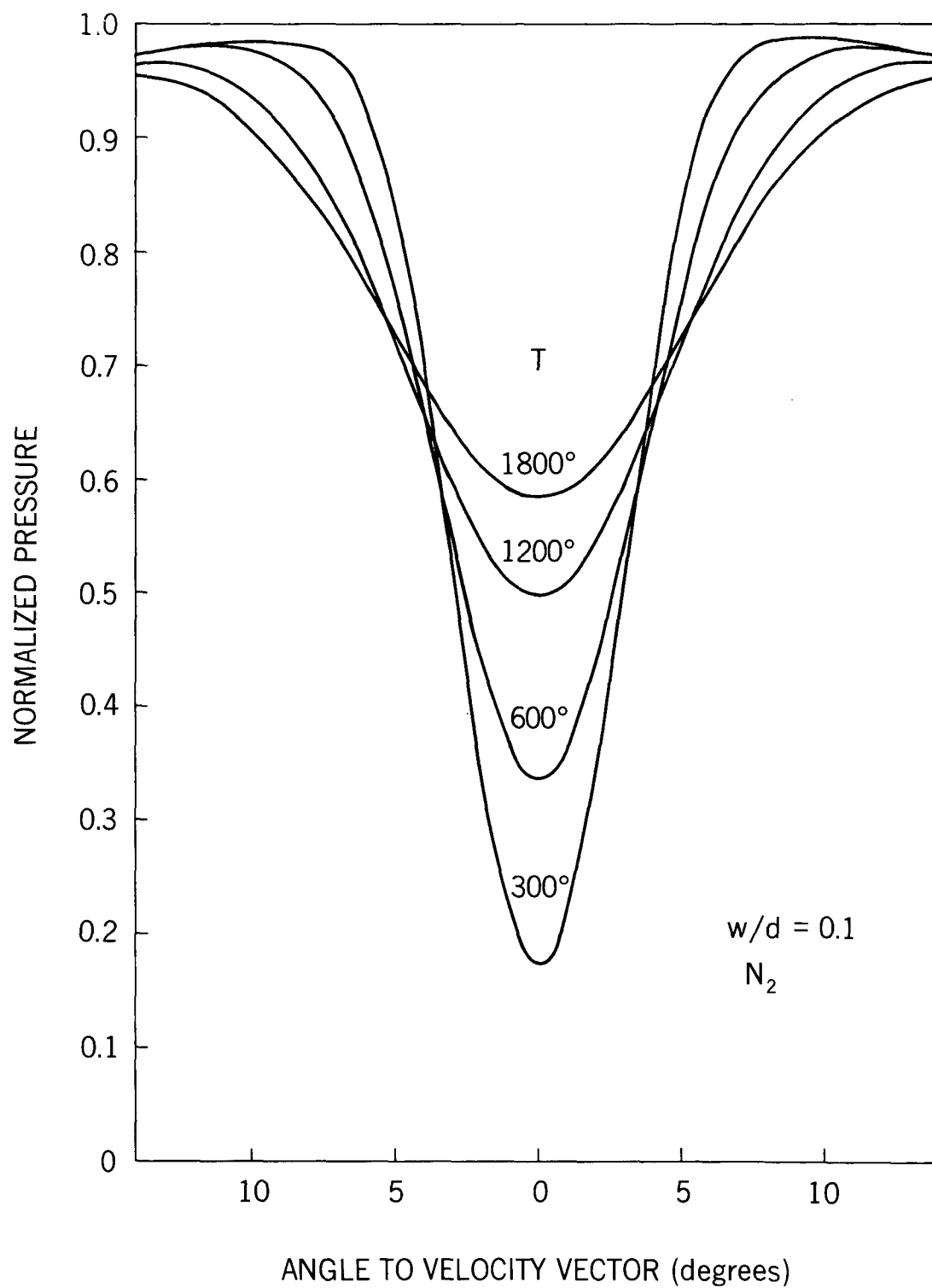


Figure 4. Wake Characteristics of N_2 for a Baffle of $w/d = 0.1$ at Various Temperatures.

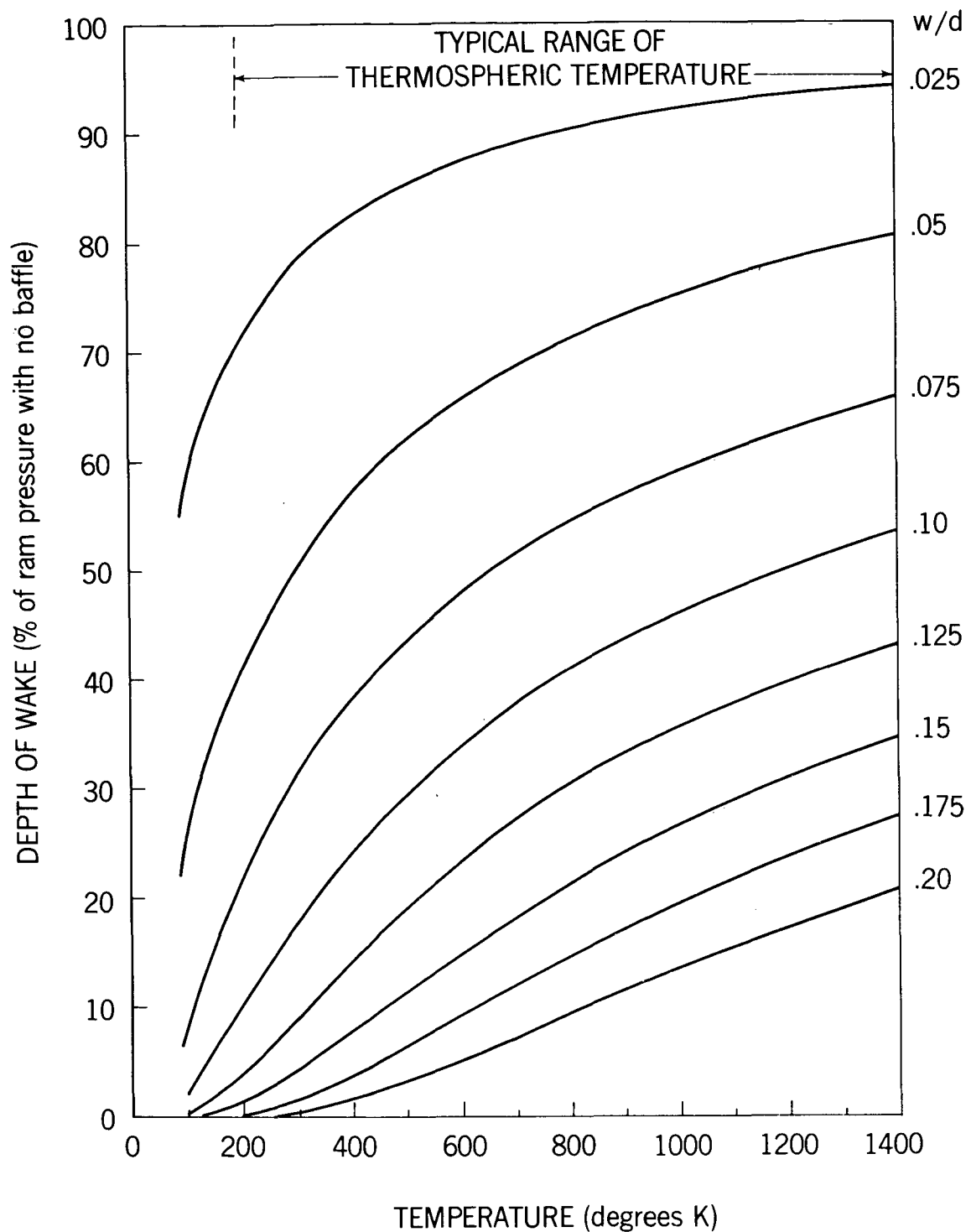


Figure 5. The depth of the wake in N_2 , at zero angle of attack, for a family of baffles. The slope of each curve is a measure of the temperature resolution afforded by that particular baffle. The smaller baffles have better temperature resolution of low temperatures, while larger baffles have more uniform resolution over the entire range of temperatures.

measure of the temperature resolution, with steeper slopes corresponding to higher resolution. The larger baffles (0.15 to 0.2) have more linear response and thus about the same temperature resolution throughout the entire range. Smaller baffles (0.025 to 0.1) have enhanced resolution at lower temperatures.

SOURCES OF ERROR

A number of sources of error in this method have been examined and included in the theoretical treatment of the Appendix. In the remainder of this paper we present the results of a study of these effects.

Scattering

The scattering of particles from the spacecraft to the baffle and then into the orifice produces a flux that adds to the normal pressure characteristic, enhancing the pressure at all angles of attack. This flux is approximately constant over the range of angles containing the baffle wake and, for large baffles, produces a non-negligible increase in the entire pressure characteristic. Figure 6 shows the effect of scattering for a baffle of $w/d = 0.15$. The scattering component rapidly becomes important for larger baffle sizes because both the length and width of the baffle must be increased, for reasons outlined later, and the scattering flux is proportional to the baffle area ($\ell \times w$). Furthermore, the scattered flux is competing with an ever-decreasing wake flux associated with the larger baffles. Conversely, the scattering can be considered negligible for baffles smaller than $w/d = 0.1$. The effect on the pressure characteristics would not be visible in a plot like Figure 6.

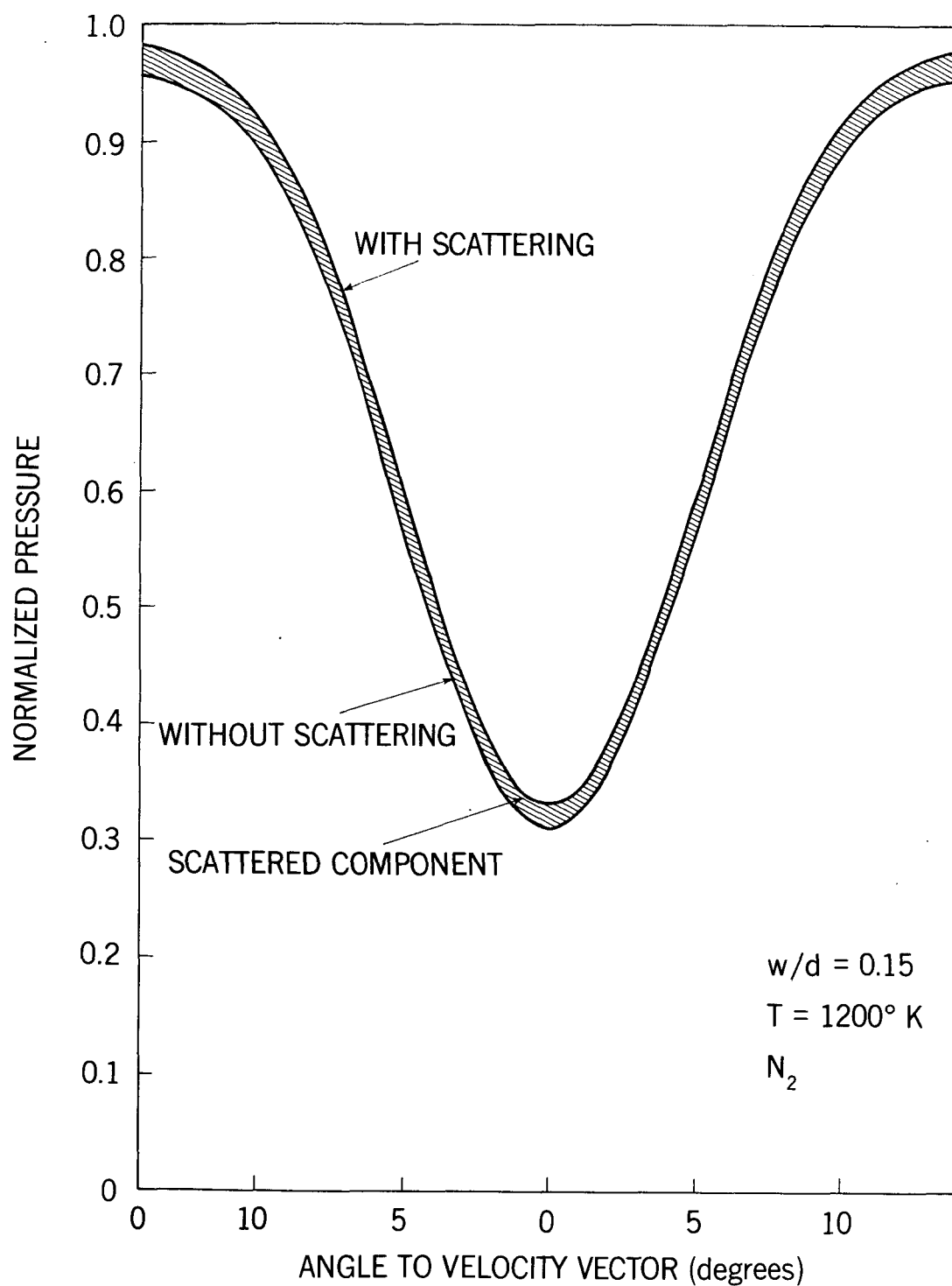


Figure 6. The effect of scattering from the inner surface of a large baffle upon the wake characteristic of N_2 at 1200°K . Scattering can normally be neglected for baffles, smaller than $w/d = 0.1$.

Finite Orifice Size

Inherent in the previous discussion has been the assumption that the orifice was small compared to the dimensions of the wake, and although the theory includes the effect of a finite orifice size, computing time associated with the data analysis can be greatly reduced by selecting orifice and baffle dimensions such that r_o/w is small enough to be unimportant. To illustrate this effect 1200° N₂ pressure characteristics were calculated for r_o/w ratios of 0, 1/2 and 1. As shown in Figure 7, the wake characteristic is essentially unchanged until r_o/w approaches 1/2, that is until the orifice diameter approaches the baffle width.

The Finite Length of The Baffle

If the length of the baffle is great enough to eliminate any significant flux from the ends of the baffle, then the wake characteristic will be determined only by w . Figure 8 shows the N₂ pressure characteristics for $w/d = 0.1$ and $\ell/w = 1, 2, 3$ and 4. There is insignificant change in the characteristic beyond a ratio of about 4 for this size baffle. Although we have shown the finite length effect in terms of ℓ/w , it happens that this effect can be defined more conveniently in terms of ℓ/d , as shown in Figure 9. Here we see that ℓ should exceed 0.3 d for smaller baffles and 0.4 d for larger baffles if the wake characteristic is to be independent of ℓ . The dashed line indicates the ℓ/d ratio required to approach the infinite length case within 1/2 %. Longer baffles could be employed, but only at the expense of increased backscattered flux.

Spin Axis Attitude Errors

As noted earlier, we have assumed that the spin axis of the satellite will be maintained normal to the orbit plane to assure scanning of the middle of the

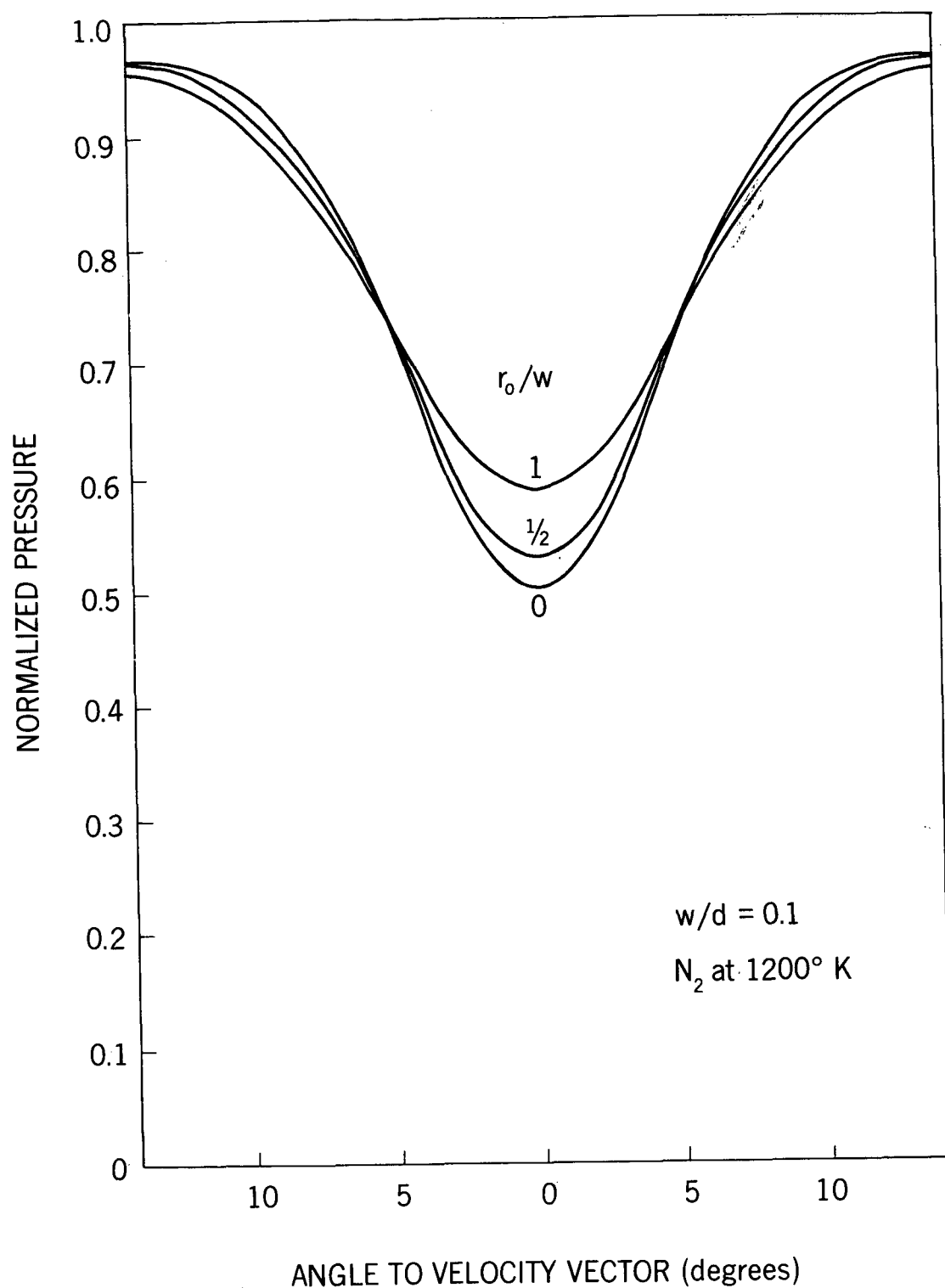


Figure 7. The effect of finite orifice size upon the wake characteristics. The orifice radius, r_o , should be kept much smaller than the baffle width, w , to avoid the need for corrections.

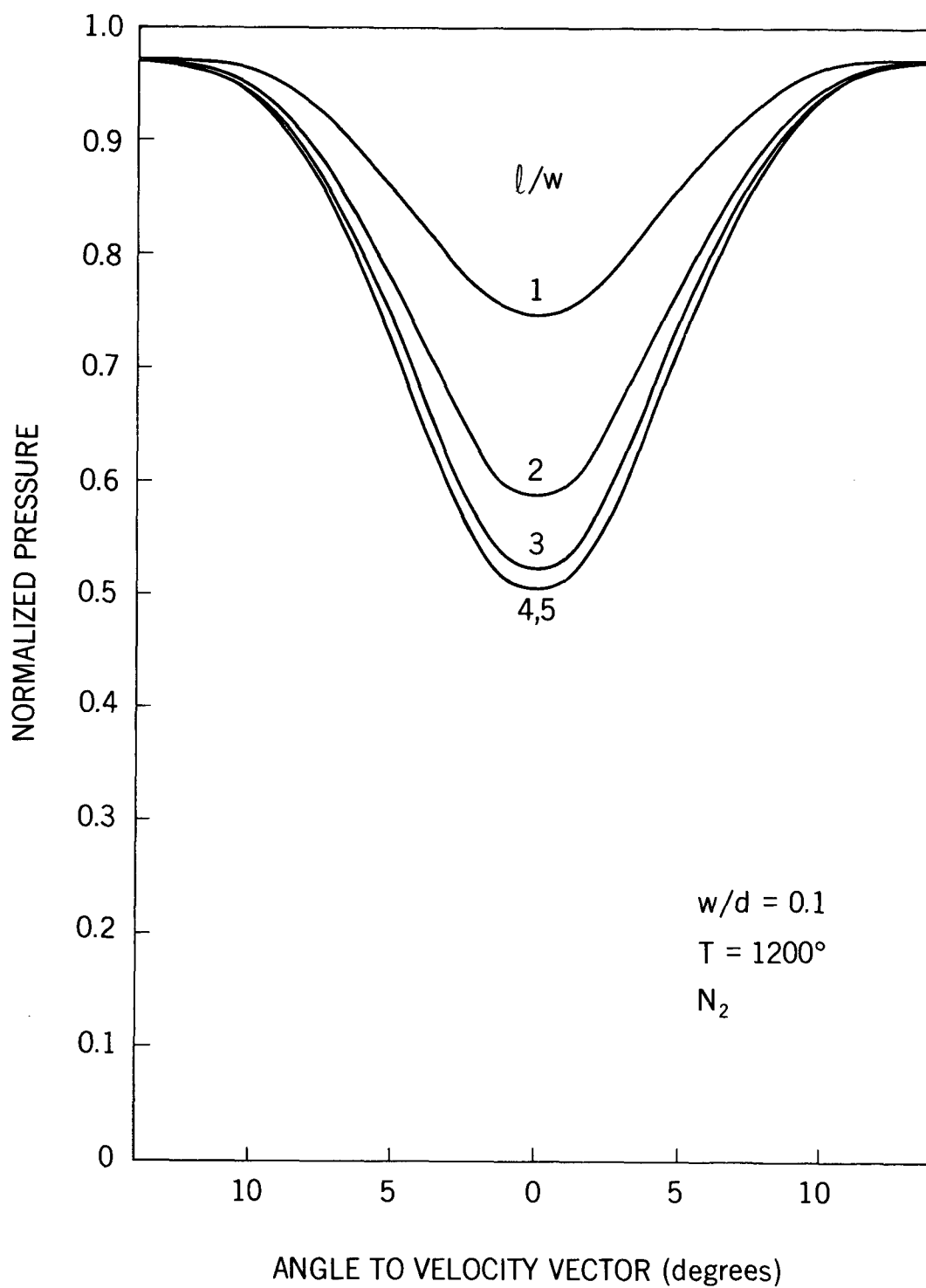


Figure 8. The effect of baffle length on the wake pressure characteristic for a baffle of $w/d = 0.1$.

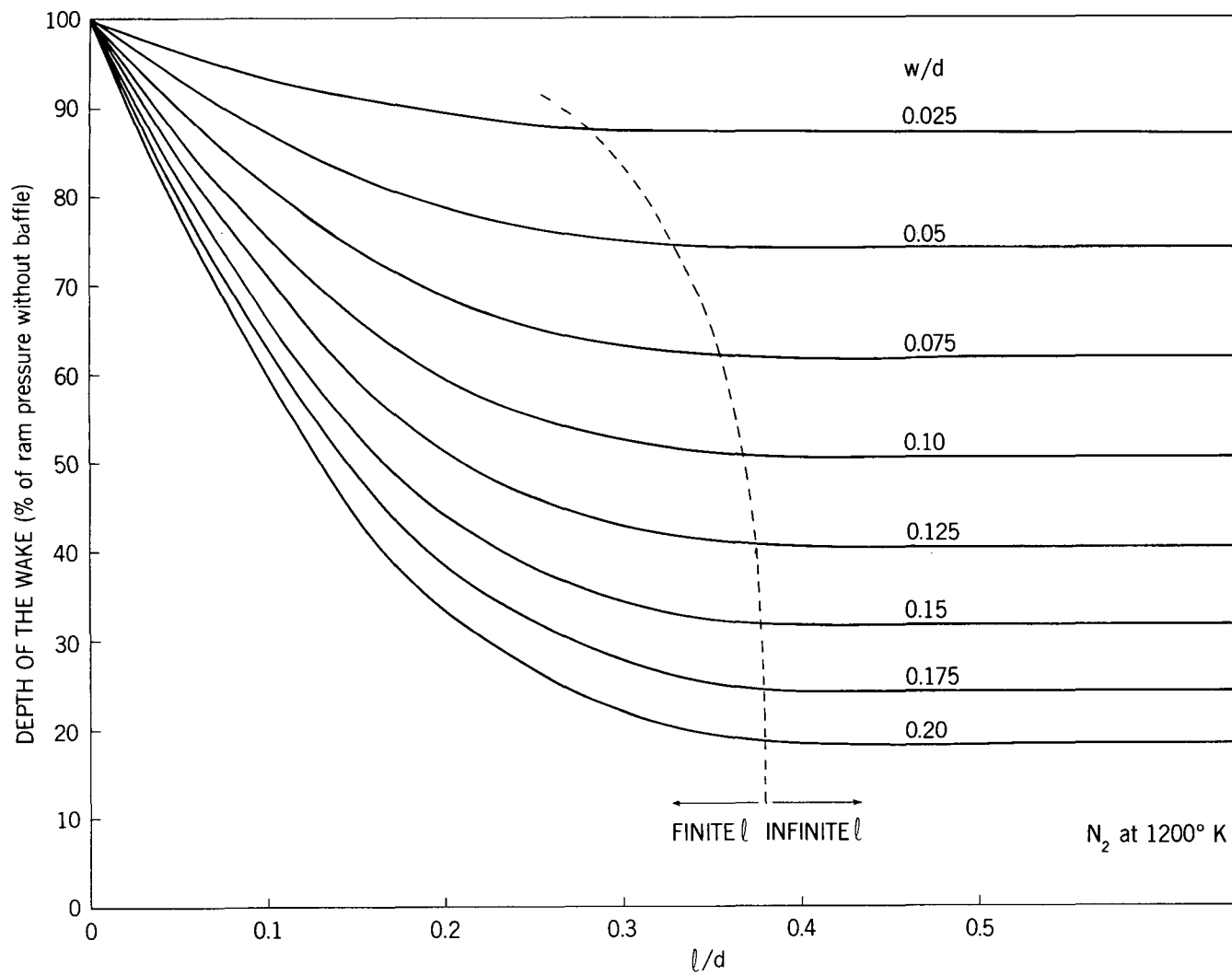


Figure 9. Depth of the wake in N_2 at 0° angle of attack as a function of l/d for a family of baffle w/d ratios. The dashed line represents the l/d ratio needed to essentially eliminate the flux from the ends of the baffle.

wake directly across the spectrometer orifice. Spin axis errors permit additional flux from one end of the baffle that tends to reduce the depth of the wake characteristic. Figure 10 demonstrates this for a baffle $\ell/w = 4$, $w/d = 0.1$ for 0° , 5° , 10° , and 15° . The errors are negligible until the spin axis error exceeds 10° , a degree of control that can be achieved routinely. In applications where larger attitude errors are expected a slightly longer baffle would be required.

Background Pressure

The background pressure within the spectrometer chamber caused by surface outgassing ultimately limits the highest altitudes to which each species can be resolved. The background differs from species to species and may vary with time in a way that distorts the spin pressure characteristics. Errors in the temperature can be minimized by avoiding very large baffles whose wake pressures approach the background pressure. Baffles smaller than $w/d = 0.1$ are preferable as they produce wake pressure variation of less than a factor of five, where background pressures normally will be a factor of 10^2 to 10^3 below the ram pressure.

SELECTION OF THE BAFFLE SIZE

From the discussion in previous sections it appears that large baffles, $w/d > 0.1$, should be avoided to reduce the scattered flux component and to avoid errors due to uncertainties in the background pressure behavior. When used with N_2 , Ar and O, baffles in the range $0.05 < w/d < 0.1$ provide excellent temperature resolution and need little or no correction for scattering and background. Even smaller baffles provide good resolution in N_2 and Ar.

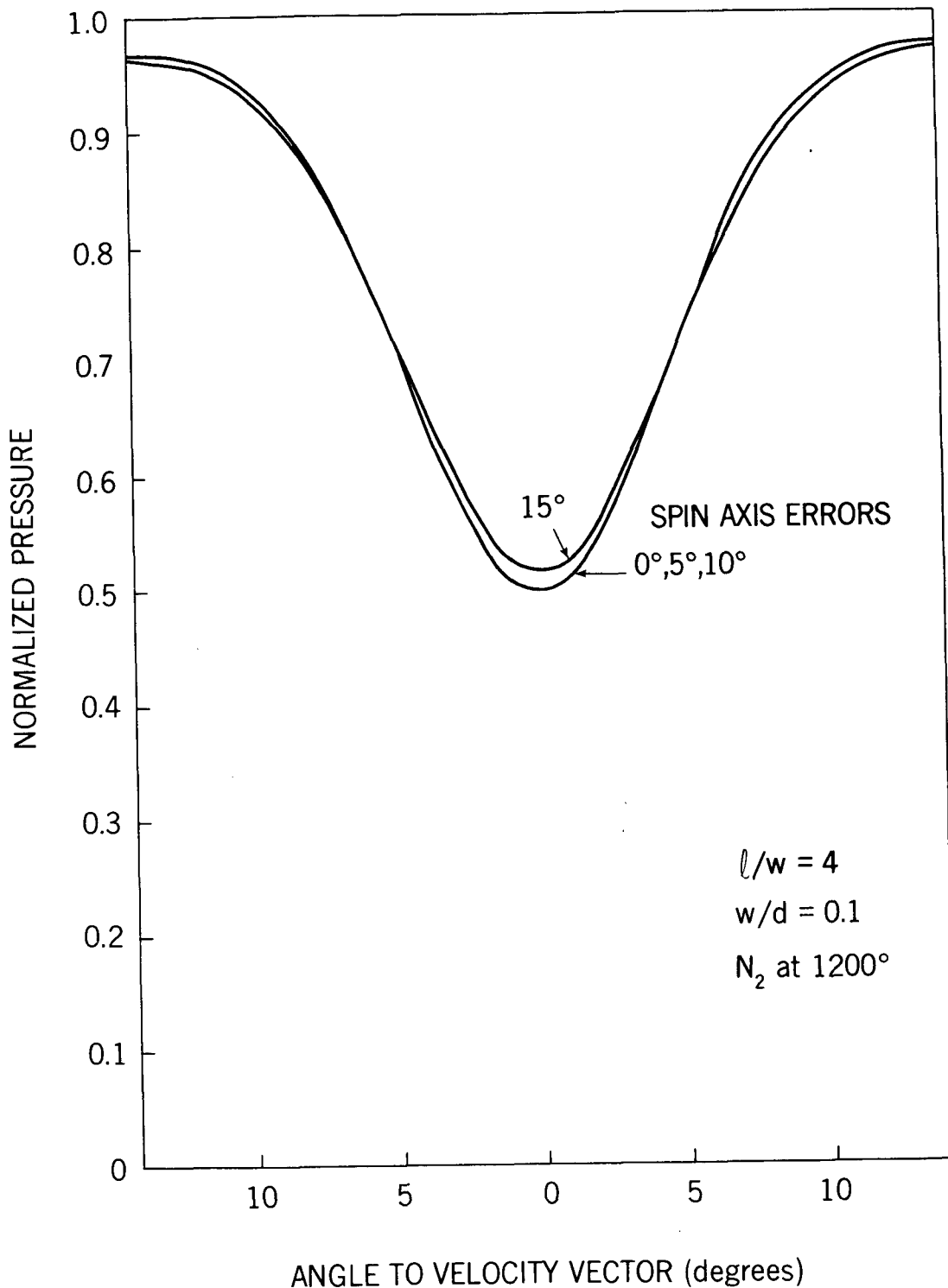


Figure 10. The effect of errors in maintaining the satellite spin axis attitude perpendicular to the orbit plane. Spin axis errors up to 10° produce negligible changes in the characteristics for a baffle ratio $\ell/w = 4$. Spin attitude is easily maintained within $\pm 5^\circ$ in current satellites.

From the previous discussion it is clear that in general the dimensions w , d , ℓ and r_o all have an effect upon the pressure characteristics and thus the temperature resolution of the experiment. However, the effects of a finite ℓ and r_o can be removed almost completely by selecting dimensions such that $\ell/d > 0.4$ and $r_o/w < 0.25$ thus leaving the wake characteristics dependent only on w/d and T . From a data analysis point of view this is especially desirable for a satellite-borne experiment, as the vast amounts of data require simple, fast running analysis programs.

In practice, gas conductance requirements will usually determine the orifice size and thus the minimum baffle width. The mounting distance is then determined by the w/d ratio corresponding to the desired temperature resolution. Finally the baffle length is selected to exceed 0.3 to 0.4 d to avoid fluxes from the ends of the baffle.

Assuming an orifice radius of 0.2 cm and a desired $r_o/w = 0.25$, a $w/d = 0.1$ baffle would have the dimensions,

$$w = 0.8 \text{ cm}, d = 8 \text{ cm and } \ell = 3.2 \text{ cm}.$$

If for any reason a larger baffle or greater mounting distance is desired, these dimensions can be scaled up proportionally without changing the pressure characteristics.

THE LOW ALTITUDE LIMIT

The low altitude limit of this method is determined by the high pressure characteristics of the spectrometer that is used and the mean free path of the particles in the vicinity of the baffle-orifice arrangement. Owing to the high

velocity of the satellite the pressure within the orifice and immediately ahead of the vehicle will be enhanced by a factor of up to 50. Typical spectrometers exhibit non-linear response at pressures above about 1×10^{-4} torr with the result that distortion of the pressure characteristics can be expected at atmospheric pressures of about 2×10^{-6} torr, or at altitudes below about 140 kilometers. Collisions in the vicinity of the spacecraft also can be expected to modify the wake characteristics below this altitude. Thus one can expect that the method will provide temperature measurements down to about 140 kilometers without additional analysis to correct for collisions.

THE USE OF MULTIPLE BAFFLES AND MASSES

The useful altitude range, accuracy, and credibility of the measurements could be greatly enhanced by providing the option to select from two or more baffle widths and to resolve the pressure characteristics of two or more species. For example, the use of O and H₂ with a larger baffle would extend the temperature measurements to higher altitudes than is feasible with N₂ and Ar which would be used at lower altitudes. Furthermore the confidence in the method would be enhanced if the experimental pressure characteristics of the various species for various baffle sizes are found consistent with the theoretically derived characteristics. Deviations from the expected pressure characteristics will permit identification of the bounds of valid measurements and provide data for improved understanding of the factors affecting the measurement accuracy.

THE NEUTRAL WAKE EXPERIMENT ON A NON-SPINNING SPACECRAFT

In this paper we have limited our discussion to a baffle mounted rigidly in front of an orificed-chamber on a spinning spacecraft. We then rely on the

spinning motion to scan the wake of the baffle across the orifice. Essentially the same effect can be obtained on a stabilized satellite by passing the baffle in front of the orifice in some suitable manner.

One approach which offers promise is a propeller type arrangement in which one or more baffles are mounted on a spinning shaft such as shown in Figure 11. The advantages of this system are that (1) multiple w/d ratios could be employed to optimize the pressure characteristics for different masses or temperatures and (2) optimum occultation rates could be achieved by changing the drive shaft speed. It would, of course, be necessary to arrange the drive shaft and baffle supports so as to minimize backscattering of particles into the orifice.

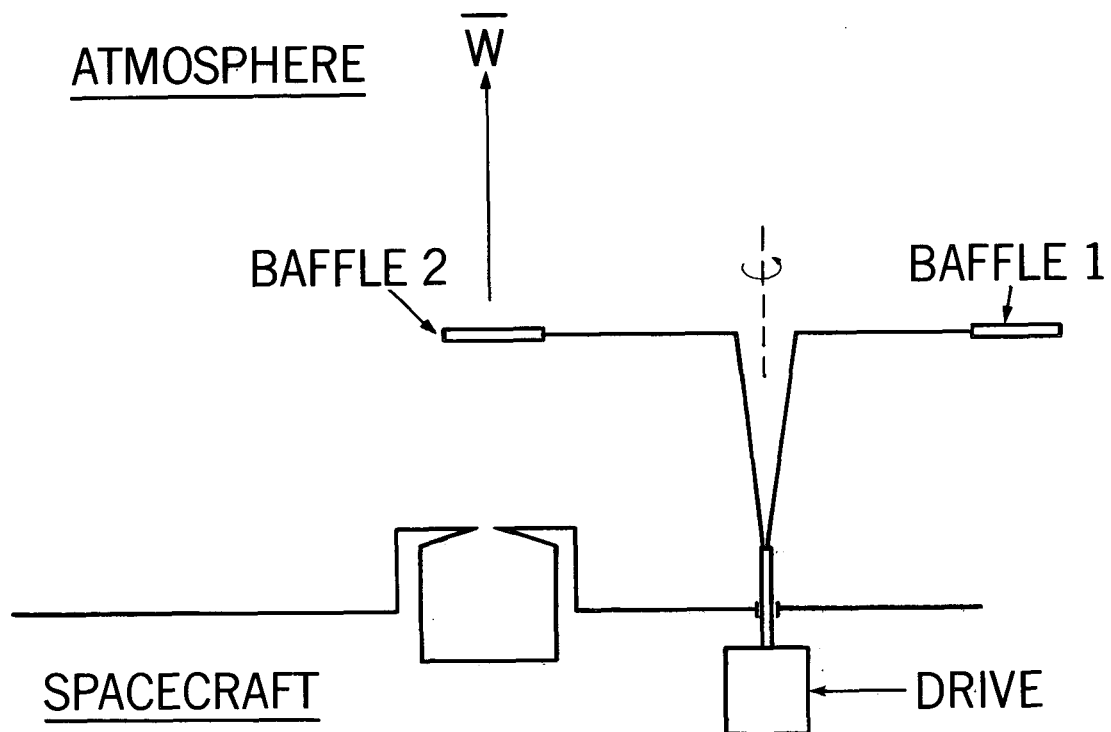


Figure 11. Baffle arrangement for a non-spinning spacecraft in which the orifice looks forward along the orbit. In a propeller-type arrangement multiple baffles are swept across the spectrometer orifice at a controllable rate. By varying the sweep rate and viewing the wake characteristics in various atmospheric species, the internal consistency of the method can be checked rigorously and the various sources of error evaluated.

APPENDIX

THEORY OF THE METHOD

In this appendix expressions are derived for the flux of neutral particles into an orifice located on a satellite surface in the presence of a baffle such as shown in Figure A-1. It is assumed that no collisions among the particles take place near the satellite-baffle configuration. Particles moving toward the orifice therefore may enter it only if their rectilinear paths do not intercept the baffle. Thus the baffle reduces the flux by an amount, F_{wake} , from the amount, F_{direct} , that would reach the orifice in the absence of the baffle. Because of scattering off the satellite and the underside of the baffle, the orifice flux is also enhanced by an amount F_{scatter} . The net flux through the orifice, F_{net} , is then the sum of these three components

$$F_{\text{net}} = F_{\text{direct}} - F_{\text{wake}} + F_{\text{scatter}}. \quad (\text{A-1})$$

The net flux is the number of particles per second passing through the orifice area, $A = \pi r_o^2$, where r_o is the orifice radius. To calculate the chamber pressure, we assume that the particles suffer many thermalizing collisions within the spectrometer and escape through the orifice at the chamber temperature. The internal pressure is then directly proportional to F_{net} . The pressure plots in this paper were calculated in this way and normalized to unity for convenient display. The actual magnitude of the pressure of course depends on the abundance of that particular species at the time of the measurement.

The velocity distribution of the neutral particles near the satellite is assumed to be Maxwellian with a superimposed drift, \vec{w} .

$$f(\vec{v}) = n (m/2\pi kT)^{3/2} \exp \left[-\frac{m}{2kT} (\vec{v} - \vec{w})^2 \right], \quad (A-2)$$

where n is the ambient density of the particles of mass, m , and temperature, T . The flux is derived for a single species, while the total flux is the sum of the individual fluxes of all species present. Since a mass spectrometer is employed in the experiment only the individual fluxes are of interest.

Determination of F_{direct}

The direct flux is given by the integral

$$F_{\text{direct}} = \int dA_o \int d\vec{v} \hat{n}_o \cdot \vec{v} f(\vec{v}), \quad (A-3)$$

where \hat{n}_o is a unit vector directed perpendicular and into an orifice area element, dA_o . Evaluation of the integral leads to the well known expression,

$$F_{\text{direct}} = n (kT/2\pi m)^{1/2} A_o f(\vec{s} \cdot \hat{n}_o), \quad (A-4)$$

where

$$\vec{s} = (m/2kT)^{1/2} \vec{w}$$

and

$$f(s) = \pi^{1/2} s \operatorname{erfc}(-s) + e^{-s^2}. \quad (A-5)$$

$f(s)$ is the standard function used deriving concentration and temperature from mass spectrometers and pressure gages on rockets and satellites (Spencer et al., 1965, 1969).

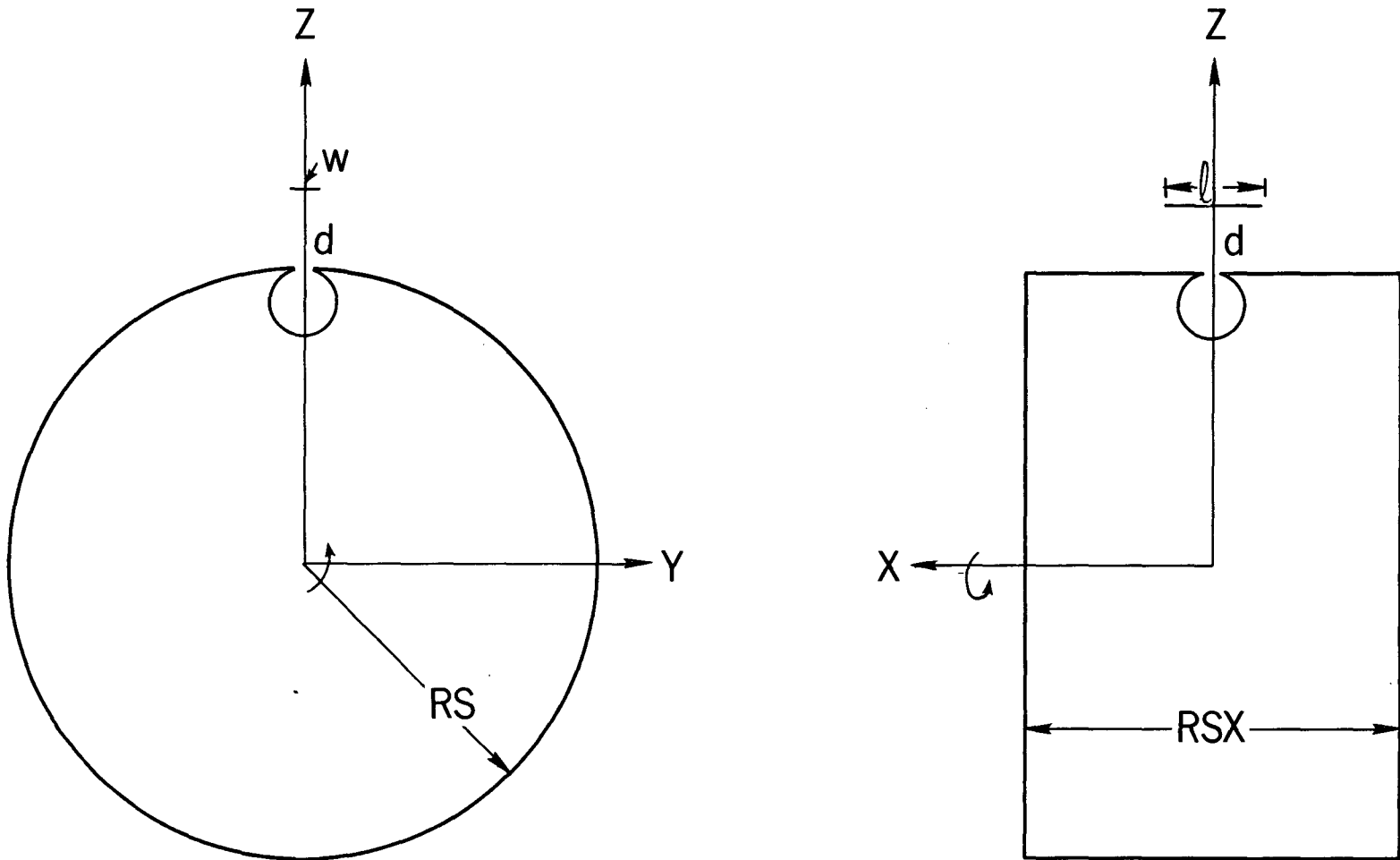


Figure A-1. Coordinate System and Dimensions Used in Formulating the Theory of the Neutral Wake Method

Determination of F_{wake}

The flux intercepted by the baffle, F_{wake} , is obtained by integrating (A-3) over all rectilinear paths that intersect the baffle surface. The rectangular coordinate system employed is shown in Figure A-1. The coordinate system is fixed in the satellite with its origin at the center of the orifice, the X axis along the axis of the satellite (a right circular cylinder), and the Z axis perpendicular to the orifice (\hat{n}_0 along the negative Z axis). The baffle is a rectangular surface of length, ℓ , width, w , lying in a plane parallel to the orifice (X-Y plane) and mounted a distance, d , from the orifice.

Introducing the symbols $BA(X, Y)$ and $OA(X, Y)$;

$$BA(X, Y) = \begin{cases} 1 & ; X, Y \text{ in baffle area} \\ 0 & ; \text{otherwise} \end{cases}$$

$$OA(X, Y) = \begin{cases} 1 & ; X, Y \text{ in orifice area} \\ 0 & ; \text{otherwise} \end{cases},$$

the condition that the particle path intersect the baffle is given by,

$$BA\left(X + \frac{v_x}{v_z} d, Y + \frac{v_y}{v_z} d\right),$$

where (X, Y) is a point on the orifice surface. Therefore F_{wake} is given by

$$F_{\text{wake}} = \int dA_0 \int d\vec{v} \hat{n}_0 \cdot \vec{v} f(\vec{v}) OA(X, Y) BA\left(X + \frac{dv_x}{v_z}, Y + \frac{dv_y}{v_z}\right). \quad (A-6)$$

Transforming to the normalized velocity, $\vec{c} = \vec{v} (m/2kT)^{1/2}$ and performing the integrals over c_x and c_y , we find

$$\begin{aligned}
F_{\text{wake}} = & n (kT/2\pi m)^{1/2} \frac{1}{2} \int dA_0 \int_0^\infty c_n dc_n \exp [-(c_n - s_n)^2] O A(X, Y) \\
& \cdot \left[\operatorname{erf} \left(\frac{\ell/2 - X}{d} c_n + s_n \right) + \operatorname{erf} \left(\frac{\ell/2 + X}{d} c_n - s_n \right) \right] \\
& \cdot \left[\operatorname{erf} \left(\frac{w/2 - Y}{d} c_n + s_n \right) + \operatorname{erf} \left(\frac{w/2 + Y}{d} c_n - s_n \right) \right], \quad (\text{A-7})
\end{aligned}$$

where $\bar{s} = \bar{w} (m/2kT)^{1/2}$ and $s_n = s \cos \theta$. s_n is the component of the speed ratio directed into the orifice. The remaining three dimensional integral is evaluated in two different ways: (1) as a small orifice approximation when this is suitable, or (2) as a triple power series in $\ell/2d$, $w/2d$ and $r_o/2d$ when the orifice radius is not small compared to w . In the small orifice approximation, it is assumed that all of the flux passes through the orifice center. Then F_{wake} is evaluated by a single numerical integral over c_n .

$$\begin{aligned}
F_{\text{wake}} = & n (kT/2\pi m)^{1/2} A_0 \frac{1}{2} \int_0^\infty c_n dc_n \exp [-(c_n - s_n)^2] \\
& \cdot \left[\operatorname{erf} \left(\frac{\ell}{2d} c_n + s_x \right) + \operatorname{erf} \left(\frac{\ell}{2d} c_n - s_x \right) \right] \\
& \cdot \left[\operatorname{erf} \left(\frac{w}{2d} c_n + s_y \right) + \operatorname{erf} \left(\frac{w}{2d} c_n - s_y \right) \right]. \quad (\text{A-8})
\end{aligned}$$

This form is valid for a baffle of arbitrary size and has been used in all of the calculations presented in this paper, except for illustrating the orifice size effect.

The formula for F_{wake} for a finite size orifice will not be presented here, but will appear in a paper to be published later. It is obtained by expanding the error functions and performing the integral over c_n .

Determination of $F_{scatter}$

The scattering contributions to the orifice flux are computed assuming diffusive scattering with the satellite and baffle surfaces. Therefore the velocity distribution of neutrals scattered from a surface element dA is,

$$F_{scatter}(\vec{v}) = \frac{dF_{striking}}{dA} \frac{2}{\pi} \left(\frac{m}{2kT_s} \right)^2 \exp \left(-\frac{mv^2}{2kT_s} \right), \quad (A-9)$$

$$\hat{n}_{out} \cdot \vec{v} > 0$$

where T_s is the temperature of the surface from which the particles scattered and $\frac{dF_{striking}}{dA}$ is the flux per unit area impinging on the surface element dA .

The flux $F_{scatter}$ is evaluated in three steps:

- (1) evaluate the total flux to the baffle underside from neutrals that come from the free stream, F_{sc1} . This component becomes important at angles beyond 90° where the free stream particles impinge directly on the underside of the baffle.
- (2) evaluate the total flux to the baffle underside from neutrals that have first scattered from the satellite surface, F_{sc2} , and
- (3) evaluate the geometrical factor determining what fraction of the flux striking the baffle underside, actually reaches the orifice.

$$G = \frac{F_{scatter}}{F_{sc1} + F_{sc2}}.$$

This calculation uses the simplifying assumption that the flux striking the baffle underside is uniform over the entire baffle, an assumption that is valid for a small baffle.

The evaluation of F_{sc1} is similar to that for $F_{direct} - F_{wake}$, where the satellite plays the role of the screening baffle. The shadow of the satellite (a right circular cylinder) is approximated by a rectangle of sides $RSX / \left(1 + \frac{RS}{RS + d}\right)$ and $2dRS / ((RS + d)^2 - RS^2)^{1/2}$, respectively. This boundary approximates the outline of the satellite observed at the baffle center. Expressing the direct and wake fluxes in a single integral gives,

$$F_{sc1} = n (kT / 2\pi m)^{1/2} A_b \frac{1}{2} \int_0^\infty c dc \exp [-(c + s_n)^2] \left\{ 4 - \left[\operatorname{erf} \left(\frac{RSX c}{2d (1 + RS/(RS + d))} - s_x \right) + \operatorname{erf} \left(\frac{RSX c}{2d (1 + RS/(RS + d))} + s_x \right) \right] \right. \\ \left. \left[\operatorname{erf} \left(\frac{RS c}{((RS + d)^2 - RS^2)^{1/2}} - s_y \right) + \operatorname{erf} \left(\frac{RS c}{((RS + d)^2 - RS^2)^{1/2}} \right) \right] \right\} \quad (A-10)$$

where A_b is the baffle area, RSX is the length of the satellite and RS is its radius.

In the evaluation of F_{sc2} , the flux of particles striking an area element of the satellite surface is required. This flux is approximated by

$$\frac{dF}{dA} = n (kT / 2\pi m)^{1/2} f(\vec{s} \cdot \hat{n}(\beta)), \quad (A-11)$$

where $\hat{n}(\beta) = (0, \sin \beta, \cos \beta)$ is the unit normal vector directed into the surface element $RS \, d\beta \, dX$. This approximation neglects the shadow of the baffle on the satellite and therefore slightly overestimates F_{sc2} . Using A-9 and A-11, F_{sc2} is given by,

$$F_{sc2} = RS \, n \, (kT/2\pi m)^{1/2} \int d\beta \int_{-\frac{RSX}{2}}^{+\frac{RSX}{2}} dX \, f(\vec{s} \cdot \hat{n}(\beta)) \frac{2}{\pi} \int d\vec{c} \quad (A-12)$$

$$\cdot |\vec{c} \cdot \bar{n}(\beta)| \exp(-c^2) B A \left(X + \frac{Dc_x}{c_z}, RS \sin \beta + \frac{Dc_y}{c_z} \right),$$

where

$$D = d + RS(1 - \cos \beta)$$

and

$$\vec{c} \cdot \hat{n}(\beta) < 0.$$

The result is expressed as a single numerical integral over β , to be presented in the later paper.

Finally, the geometrical factor, G , is evaluated assuming that the flux density reaching the baffle is also uniform,

$$\frac{dF_{striking}}{dA} = \frac{F_{sc1} + F_{sc2}}{A_b} \quad (A-13)$$

Using Equation A-13 and the distribution function of Equation A-9, $F_{scatter}$ becomes,

$$F_{\text{scatter}} = (F_{\text{sc1}} + F_{\text{sc2}}) G, \quad (\text{A-14})$$

where

$$G = \frac{2}{\pi A_b} \int_0^\infty c \, dc \int \int dX \, dY \, B A(X, Y) \quad (\text{A-15})$$

$$\cdot \int \int d c_X \, d c_Y \, O A \left(X + \frac{c_X}{c} d, Y + \frac{c_Y}{c} d \right) \exp - (c^2 + c_X^2 + c_Y^2) ,$$

On comparing (A-15) with F_{wake} , (A-6), it is found that

$$G = \frac{F_{\text{wake}} \big|_{\vec{s}=0}}{n (k T / 2\pi m)^{1/2} A_b}, \quad (\text{A-16})$$

Thus G may be computed from either the power series expansion of F_{wake} , or in closed form from the small orifice approximation,

$$G = \frac{A_0}{A_b} \frac{2}{\pi} \sum_{\ell \leftrightarrow w} \frac{\ell / 2d}{(1 + (\ell / 2d)^2)^{1/2}} \tan^{-1} \left(\frac{wd / 2}{(1 + (\ell / 2d)^2)^{1/2}} \right). \quad (\text{A-17})$$

REFERENCES

- Blamont, J. E. and J. M. Luton, "OGO-VI neutral temperature measurements — comparison with temperature models", presented at COSPAR meeting, Seattle, June 1971.
- Brace, L. H. and W. C. Wu, "A neutral particle wake experiment for measuring upper atmosphere temperature from a moving vehicle", Goddard Space Flight Center report X-621-68-397, Nov. 1968.
- Brace, L. H. and W. C. Wu, "A neutral particle wake method for measuring thermosphere temperature", Trans. Amer. Geophys. Union, 51, 4, 380, 1970.
- Jacchia, L. G., "Static diffusion models of the upper atmosphere with empirical temperature profiles", Smithsonian Astrophys. Obs. Spec. Rpt. 170, 1964.
- McClure, J. P., "Thermospheric temperature variations inferred from incoherent scatter observations", J. Geophys. Res., 76, 3106, 1971.
- Nier, A. O., J. H. Hoffman, C. Y. Johnson, and J. C. Holmes, "Neutral composition of the atmosphere in the 100 to 200 kilometer range", J. Geophys. Res., 69, 979, 1964.
- Spencer, N. W., L. H. Brace, G. R. Carignan, D. R. Taeusch and H. Niemann, "Electron and molecular nitrogen temperature and density in the thermosphere", J. Geophys. Res., 70, 2665, 1965.

Spencer, N. W., G. R. Newton, G. R. Carignan and D. R. Taeusch, "Thermospheric temperature and density variations with increasing solar activity", Space Res. X, North-Holland Publish. Co., Amsterdam, 389, 1970.

Spencer, N. W., private communication, 1971.