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AN EXACT CLOSED-FORM SOLUTION FOR CONSTANT-AREA COMPRESSIBLE FLOW WITH FRICTION AND HEAT TRANSFER

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16. Abstract

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SUMMARY

The well-known differential equation for the one-dimensional flow of a compressible fluid with heat transfer and wall friction has no known solution in closed form for the general case. This report presents a closed form solution for the special case of constant heat flux per unit length and constant specific heat. The solution was obtained by choosing the square of a dimensionless flow parameter as one of the independent variables to describe the flow. From this exact solution, an approximate simplified form is derived that is applicable for predicting low Mach number flow performance characteristics for many types of constant area passages in internal flow. The data included in this report are considered sufficiently accurate for use as a guide in analyzing and designing internal gas flow systems.

INTRODUCTION

In this report one-dimensional flow characteristics of a compressible fluid under simultaneous action of friction and heat transfer in a constant area passage are investigated. The pressure drop in a constant area passage results from the simultaneous action of friction and heat transfer. In the technical literature, the various equations describing the flow characteristics of a compressible fluid under the simultaneous influence of friction and arbitrary heat transfer are not expressed in closed form (see, e.g., section 8.9 of ref. 1). Presently, no generalized integrating factor has been found that would permit closed-form integration for any fluid temperature variation.

In the present analysis, a completely integrated solution is presented for the case in which the total temperature of the fluid varies linearly with distance through a constant flow area passage. For constant specific heat, this solution constitutes flow at constant heat flux. From this exact solution, an approximate simplified form is derived

that is applicable for predicting one-dimensional flow performance for most types of operation in constant area passages in one-dimensional flow.

This analysis develops a working chart and relations that simplify the determination of the Mach number change in cooling and heating passages and, consequently, simplifies the calculation of the pressure drop required across these passages to obtain the necessary mass flow of gas. The method developed applies to the flow of gas through straight ducts having any constant shape, cross-sectional area, and roughness.

The solution is presented in the form of working charts constructed for a ratio of specific heats equal to 1.40 and 1.30. The variables cover ranges sufficiently large to include many applications of engineering interest. The data included in this report are considered to be sufficiently accurate for use as a guide in analyzing and designing internal gas flow systems for aircraft.

SYMBOLS

A duct flow area, m^2; ft^2

a speed of sound, m/sec; ft/sec

c_p specific heat at constant pressure, J/kg-K; Btu/lbm-$^\circ$R

c_v specific heat at constant volume, J/kg-K; Btu/(lb/$^\circ$R)

D equivalent hydraulic diameter, m; ft

F_f friction force, N; lbf

f local friction factor = $\tau_w / \rho v^2 / 2$, dimensionless

g_c conversion factor, (kg-m)/(N-sec^2); (lbm-ft)/(lbf-sec^2)

J Joule's constant, N-m/J; lbf-ft/Btu

k heat flux parameter, $q/c_p T_0$ (see eq. (2)), dimensionless

M Mach number in duct at distance x, dimensionless

P total pressure at distance x, N/m^2; lbf/ft^2

p static pressure at distance x, N/m^2; lbf/ft^2

Q heat input to stream, J/kg; Btu/lbm

q constant heat flux, J/kg; Btu/lbm

R gas constant, N-m/kg-K; (lbf ft)/(lbm-$^\circ$R)

T total temperature at distance x, K ($^\circ$R)

t static temperature at distance x, K ($^\circ$R)

V   dimensionless flow parameter at distance x, $(v/\sqrt{g_c R T_0})$, dimensionless

v   axial velocity in flow passage distance x, m/sec; ft/sec

$\dot{w}$ weight flow rate of fluid, kg/sec; lbm/sec

x   axial distance through flow passage, m; ft

γ ratio of specific heats, c_p/c_v, dimensionless

Δ increment of a dimensionless parameter

μ integrating factor, dimensionless

ξ friction-distance parameter $= \left(4\bar{f}\frac{x}{D} = \int_0^x \frac{4f}{D} dx \right)$, dimensionless

ρ mass density, kg/m^3; lbm/ft^3

τ defined by eq. (2) as ratio (T/T_0), dimensionless

τ_w frictional shearing stress at the wall, N/m^2; lbf/ft^2

Subscripts:

D   duct

l limiting condition

0   at x = 0

METHOD OF ANALYSIS

For one-dimensional flow, the pressure at any point in a fluid is uniquely defined by the mass velocity $(G = \dot{w}/A_D = \rho v)$, the temperature, and the Mach number at that point. The following development deals with the variation of Mach number along a constant area duct. If the variation of Mach number is known, the corresponding pressure distribution along the duct is readily determined from the continuity equation for the one-dimensional flow of a perfect fluid.

The investigation of frictional effects will be confined to steady one-dimensional flow of a compressible fluid in a constant cross-sectional area duct. For these conditions a flow model and the forces acting on a fluid element in the x-direction are illustrated in the following sketch:

Mass of fluid element
$d_m = (\rho/g_c)A_D \, dx$

(a)
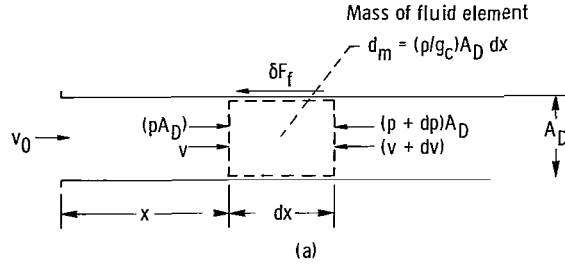
$$\sum F_x = 0$$

or

$$F_x = pA_D - (p + dp)A_D - \delta F_f \tag{1}$$

The mechanisms of friction and heat transfer are so similar that one cannot be had without the other (ref. 1, p. 242). Therefore, in order to calculate the pressure drop and other flow characteristics more accurately, it is necessary to take account of friction as well as of heat transfer. In this analysis we shall assume the flow area, specific heat, the weight rate flow, and the heat flux all to be constant. Therefore, the rate of heat transfer may be expressed by means of the energy equation in terms of the increase in stagnation enthalpy. Thus,

$$dQ = c_p \, dT$$

$$\left(\frac{dQ}{d\xi}\right)d\xi = c_p \, dT$$

$$\frac{q\xi}{c_p} = T - T_0$$

$$\frac{T}{T_0} = 1 + \frac{q}{c_p T_0}\,\xi$$

or

$$\tau = 1 + k\xi \tag{2}$$

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where

$$\tau = \frac{T}{T_0}$$

and

$$k = \frac{q}{c_p T_0}$$

Therefore, the value of k is primarily determined by the heat flux q. Also, from equation (2) it can be seen that the assumptions of constant specific heat and constant heat flux are directly analogous to a linear variation in stagnation temperature T along the duct.

If the dimensionless parameters V, ξ, and t/T_0 are defined by the following expressions

$$V \equiv \frac{v}{\sqrt{g_c R T_0}} \tag{3}$$

$$\xi \equiv \frac{4}{D} \int_0^x f \, dx = \frac{4\bar{f}x}{D} \tag{4}$$

$$\frac{t}{T_0} = \frac{T}{T_0} - \frac{\gamma - 1}{2\gamma} \left(\frac{v^2}{g_c R T_0} \right)$$

$$= \tau - \frac{\gamma - 1}{2\gamma} V^2 \tag{5}$$

where $\tau = T/T_0$, then the relation between the standard definition of Mach number (M = v/a) and the dimensionless flow parameter V results:

$$M^2 = \frac{V^2}{\gamma \tau - \left(\frac{\gamma - 1}{2} \right) V^2} \tag{6}$$

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or

$$V^2 = \frac{\gamma M^2 \tau}{1 + \left(\frac{\gamma - 1}{2}\right)M^2} \tag{7}$$

For assumed conditions, the one-dimensional steady-state momentum equation for a compressible flow given in reference 1 (eq. (8.21)) can be rearranged to this form

$$dv^2 + 2g_c R\left(dt + t\,\frac{d\rho}{\rho}\right) + v^2\left(\frac{4f\,dx}{D}\right) = 0$$

Making proper substitutions for friction-distance parameters ξ and axial velocity v, the momentum equation becomes

$$dV^2 + 2\left(\frac{dt}{T_0} + \frac{t}{T_0}\cdot\frac{d\rho}{\rho}\right) + V^2\,d\xi = 0$$

Logarithmic differentiation of the continuity equation for constant flow area gives

$$\frac{d\rho}{\rho} = -\frac{dv}{v} = -\frac{dV}{V}$$

Eliminating the density term results in a new expression of the momentum equation:

$$dV^2 + 2\left(\frac{dt}{T_0} - \frac{t}{T_0}\frac{dV}{V}\right) + V^2\,d\xi = 0 \tag{8}$$

A combination of equations (5) and (8) yields

$$\left(V^2 + 2\frac{d\tau}{d\xi}\right)d\xi = \left(\frac{\tau}{V^2} - \frac{\gamma + 1}{2\gamma}\right)dV^2 \tag{9}$$

Substitution of equation (2) into equation (9) results in

$$d\xi - \frac{k\xi}{V^2(V^2 + 2k)}\,dV^2 = \left(\frac{1 - \frac{\gamma + 1}{2\gamma}V^2}{V^2 + 2k}\right)\frac{dV^2}{V^2} \tag{10}$$

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Equation (10) is linear in ξ, and, by multiplying through with the integrating factor,

$$\mu = \left(\frac{V^2 + 2k}{V^2}\right)^{1/2} \tag{11}$$

which satisfies the necessary requirements that

$$\frac{\partial}{\partial V^2}(\mu M) \equiv \frac{\partial}{\partial \xi}(\mu N)$$

where

$$M = 1$$

$$N = \frac{-\left(k\xi + 1 - \frac{\gamma + 1}{2\gamma}V^2\right)}{V^2(V^2 + 2k)}$$

(see ref. 2), equation (10) becomes an exact differential equation, which in mathematical notation can be written as

$$f(V^2)d\xi + \xi g(V^2)dV^2 + h(V^2)dV^2 = 0 \tag{12}$$

Integration of equation (12) gives

$$\xi = \left(\frac{V}{V_0}\right)\left(\frac{V_0^2 + 2k}{V^2 + 2k}\right)^{1/2}\left(\frac{1}{k}\right) - \frac{1}{k} + \frac{V}{2} \cdot \frac{\frac{\gamma + 1}{\gamma}}{(V^2 + 2k)^{1/2}} \ln\left[\frac{V_0^2 + k + V_0\left(V_0^2 + 2k\right)^{1/2}}{V^2 + k + V(V^2 + 2k)^{1/2}}\right] \tag{13}$$

In order to solve for ξ for adiabatic conditions, the following steps were taken to eliminate the heat flux parameter k from equation (13):

$$\xi = \frac{\left(1 + k\frac{2}{v_0^2}\right)^{1/2} - \left(1 + k\frac{2}{v^2}\right)^{1/2}}{k\left(1 + k\frac{2}{v^2}\right)^{1/2}} - \left(\frac{\frac{\gamma + 1}{2\gamma}}{1 + k\frac{2}{v^2}}\right)\ln\left\{\frac{v^2}{v_0^2}\left[\frac{1 + \frac{k}{v^2} + \left(1 + k\frac{2}{v^2}\right)^{1/2}}{1 + \frac{k}{v_0^2} + \left(1 + k\frac{2}{v_0^2}\right)^{1/2}}\right]\right\}$$

then

$$\left(1 + \frac{2k}{v_0^2}\right)^{1/2} = 1 + \frac{k}{v_0^2} - \frac{k^2}{2v_0^4} + \ldots$$

$$\left(1 + \frac{2k}{v^2}\right)^{1/2} = 1 + \frac{k}{v^2} - \frac{k^2}{2v^4} + \ldots$$

Therefore,

$$\xi = \frac{\left(\frac{1}{v_0^2} - \frac{1}{v^2}\right) + \frac{k}{2}\left(\frac{1}{v^4} - \frac{1}{v_0^4}\right)}{\left(1 + k\frac{2}{v^2}\right)^{1/2}} - \frac{\frac{\gamma + 1}{2\gamma}}{1 + k\frac{2}{v^2}}\ln\left\{\frac{v^2}{v_0^2}\left[\frac{1 + \frac{k}{v^2} + \left(1 + k\frac{2}{v^2}\right)^{1/2}}{1 + \frac{k}{v_0^2} + \left(1 + k\frac{2}{v_0^2}\right)^{1/2}}\right]\right\} + \ldots$$

When $k \rightarrow 0$,

$$\xi = \left(\frac{1}{v_0^2} - \frac{1}{v^2}\right) + \frac{\gamma + 1}{2\gamma}\ln\left(\frac{v_0^2}{v^2}\right) \tag{14}$$

Substitution of equation (7) into equation (14) with $k = 0$, gives

$$\xi = \frac{1}{\gamma}\left(\frac{1}{M_0} + \frac{1}{M}\right)\left(\frac{1}{M_0} - \frac{1}{M}\right) + \left(\frac{\gamma + 1}{\gamma}\right)\ln\left[\frac{M_0}{M}\left(\frac{1 + \frac{\gamma + 1}{2}M^2}{1 + \frac{\gamma + 1}{2}M_0^2}\right)^{1/2}\right] \tag{15}$$

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It is interesting to note that equation (15) is of the same form as a combination of equations (6.20) and (6.21) given on page 167 of reference 1 for adiabatic flow with friction.

Useful relations of dimensionless weight rate flow parameters can be written as follows:

(1) In terms of static pressure: Substitution for M from equation (6) into the following equation taken from reference 1 (p. 82),

$$\frac{\dot{w}\sqrt{T}}{A_D p} = \left(\frac{g_c \gamma}{R}\right)^{1/2} (M) \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/2}$$

which expresses the weight rate flow parameters in the classical form, results in a dimensionless weight rate flow parameter for diabatic conditions

$$\frac{\dot{w}\sqrt{g_c R T}}{A_D g_c p} = \frac{V}{\tau^{1/2}} \left[1 - \left(\frac{\gamma - 1}{2\gamma}\right)\frac{V^2}{\tau}\right]^{-1} \tag{16}$$

(2) In terms of total pressure: Again, substitution for M from equation (6) into the following equation, taken from reference 1 (p. 84)

$$\frac{\dot{w}\sqrt{T}}{A_D P} = M\left(\frac{\gamma g_c}{R}\right)^{1/2} \left(\frac{1}{1 + \frac{\gamma - 1}{2} M^2}\right)^{(\gamma+1)/[2(\gamma-1)]}$$

results in a dimensionless weight rate flow parameters for diabatic conditions

$$\frac{\dot{w}\sqrt{g_c R T}}{A_D g_c P} = \frac{V}{\tau^{1/2}} \left[1 - \left(\frac{\gamma - 1}{2\gamma}\right)\frac{V^2}{\tau}\right]^{1/(\gamma-1)} \tag{17}$$

From equations (16) and (17), it follows that the static and total pressure ratios for diabatic conditions will be, respectively,

$$\frac{p}{p_0} = \left(\frac{M_0}{M}\right)\left[\left(\frac{1 + \frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M^2}\right)(1 + k\xi)\right]^{1/2} \tag{18}$$

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and

$$\frac{P}{P_0} = \left(\frac{M_0}{M}\right)(1 + k\xi)^{1/2}\left(\frac{1 + \frac{\gamma - 1}{2}M^2}{1 + \frac{\gamma - 1}{2}M_0^2}\right)^{(\gamma+1)/[2(\gamma-1)]} \tag{19}$$

Dividing equation (18) or (19), gives

$$\frac{\dfrac{p}{p_0}}{\dfrac{P}{P_0}} = \left(\frac{1 + \frac{\gamma - 1}{2}M^2}{1 + \frac{\gamma - 1}{2}M_0^2}\right)^{-\gamma/(\gamma-1)} \tag{20}$$

Equation (20) indicates for subsonic flow that total pressure drop always exceeds static pressure drop in a constant area compressible flow because of fluid acceleration.

An approximate explicit expression for V may be obtained by use of a Maclaurin expansion of the logarithmic quantity that appears in equation (13). Using the first two terms of such an expansion eventually leads to the result

$$V = (1 + k\xi)\left(V_0^2 + 0.0001\,k\xi\right)^{1/2}\left\{1 - V_0^2\xi\left[\left(\frac{\gamma + 1}{12\gamma}\right)\left(20V_0^2 + 15k\right) + 2 + k\xi\right]\right\}^{-1/2} \tag{21}$$

LIMITING CONDITIONS FOR DIABATIC FLOW

In compressible subsonic flow, as the gas flows along the duct, its static pressure decreases while its velocity increases. As shown in reference 1 (pp. 255 to 256), the effect of the wall friction, for diabatic (nonadiabatic) conditions, is to accelerate the flow velocity.

Equation (9) can be rearranged to the following form

$$\frac{d\xi}{dV^2} = \frac{k\xi + 1 - \left(\frac{\gamma + 1}{2\gamma}\right)V^2}{V^2(V^2 + 2k)}$$

When $d\xi = 0$, there can be no change in Mach number; that is,

$$\frac{d\xi}{dv^2} = \frac{k\xi + 1 - \left(\frac{\gamma+1}{2\gamma}\right)v^2}{v^2(v^2 + 2k)} = 0$$

from which, the limiting parameter V_l is

$$V_l^2 = \left(\frac{2\gamma}{\gamma+1}\right)(1 + k\xi_l) \tag{22}$$

Substituting V_l^2 into equation (6), gives

$$M_l^2 = \frac{V_l^2}{\gamma(1 + k\xi_l) - V_l^2\left(\frac{\gamma-1}{2}\right)}$$

$$= \frac{\left(\frac{2\gamma}{\gamma+1}\right)(1 + k\xi_l)}{\gamma(1 + k\xi_l) - \left(\frac{2\gamma}{\gamma+1}\right)(1 + k\xi_l)\left(\frac{\gamma-1}{2}\right)}$$

$$= 1.0$$

Therefore,

$$M_l = 1.0 \tag{23}$$

It is interesting to note that, for a constant heat flux, as is the case assumed in this analysis, the same result (that is, $M_l = 1.0$) could be obtained using equation (8.71a) of reference 1.

The relation between the limiting or choking static pressure ratio p_l/p_o and the friction-distance parameter ξ can be obtained by substituting $M_l = 1.0$ into equation (18). Thus,

$$\left(\frac{p_l}{p_0}\right) = (M_0)\left[\left(\frac{1 + \frac{\gamma-1}{2}M_0^2}{1 + \frac{\gamma-1}{2}}\right)(1 + k\xi)\right]^{1/2} \tag{24}$$

and for adiabatic conditions when $k = 0$ equation (24) reduces to equation (6.22) of reference 1. Also, the relation between the limiting or choking total pressure ratio P_l/P_0 and the friction distance parameter ξ can be obtained by substituting $M = 1.0$ into equation (19):

$$\left(\frac{P_l}{P_0}\right) = (M_0)(1 + k\xi)^{1/2}\left(\frac{1 + \frac{\gamma - 1}{2}M_0^2}{1 + \frac{\gamma - 1}{2}}\right)^{(\gamma+1)/[2(\gamma-1)]} \tag{25}$$

Again, for adiabatic conditions equation (25) reduces to equation (6.26) of reference 1.

As can be seen from equations (25) and (24), the heat addition to the stream tends to increase choking pressure ratio p_l/p_0 and P_l/P_0, respectively, by the factor of $(1 + k\xi)^{1/2}$.

RESULTS AND DISCUSSION

Diabatic Flow with Friction

The mechanisms of friction and heat transfer are similar because friction and heat transfer cannot exist independently of each other. Therefore, in order to calculate the pressure drop and other flow parameters of interest, for example, such as bled hot or cold gas flow from a turbofan engine, it is necessary to consider both friction and heat transfer together.

In this analysis the heat transfer is accounted for by assuming a constant heat flux q. As has been proved under the method of analysis (see eq. (2) and fig. 1), a constant heat flux is directly analogous to the case of a linear variation of total temperature along the duct.

For one-dimensional flow, the pressure at any point in a fluid is uniquely defined by the mass velocity ($\dot{w}/A_D = \rho v$), the temperature T, and the Mach number M at that point. Once the local Mach number along the duct is known, the other flow parameters such as total or static pressure drop, weight rate flow per unit area, and the temperature distribution along the duct are readily determined. Figure 2 represents the variation of local Mach number M against the friction-distance parameter ξ for an inlet Mach number of 0.15, $\gamma = 1.3$ and 1.4, and a range of heat flux parameters $0 \le k \le 0.3$.

As can be seen from this plot, for a given friction-distance parameter ξ, the Mach number is very sensitive to an increase in heat flux parameter k.
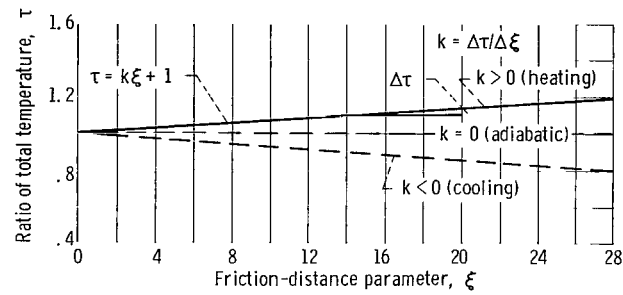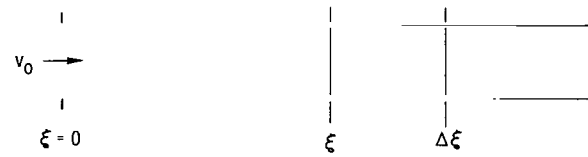
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$v_0 \longrightarrow$

$\xi = 0$ ξ $\Delta\xi$

$k = \Delta\tau/\Delta\xi$

$\tau = k\xi + 1$ $\Delta\tau$ $k > 0$ (heating)

$k = 0$ (adiabatic)

$k < 0$ (cooling)

Ratio of total temperature, τ

Friction-distance parameter, ξ

Figure 1. - Linear variation of total temperature with friction-distance parameter.

Limiting Mach number, $M_L = 1.0$

Heat flux parameter, k

0.3   .25   .20   .15   .10

Ratio of specific heats, γ

1.3
1.4

.05

0

Local Mach number in duct, M

Friction-distance parameter,
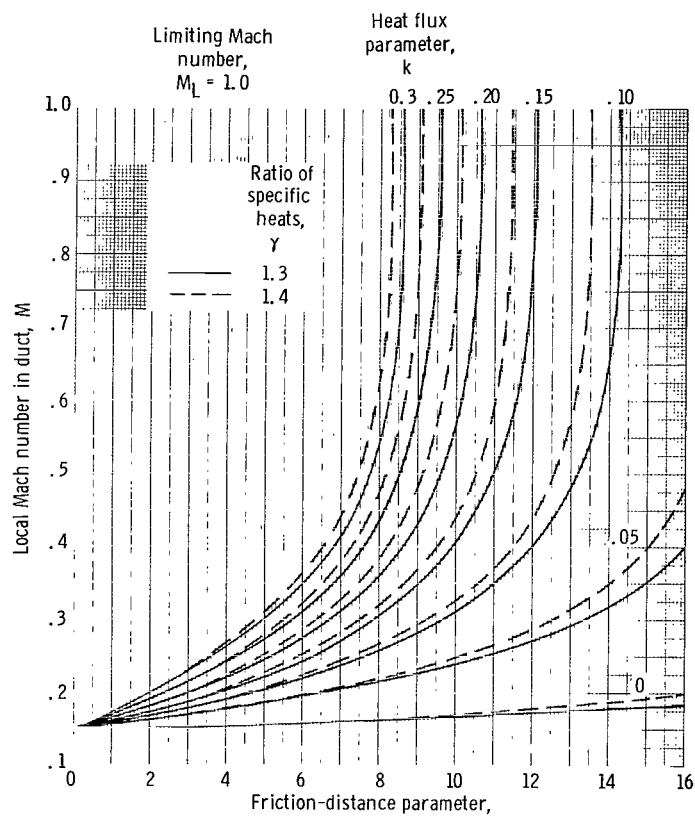
Figure 2. - Variation of local Mach number in the duct with friction-distance parameter. Inlet Mach number, 0.15.
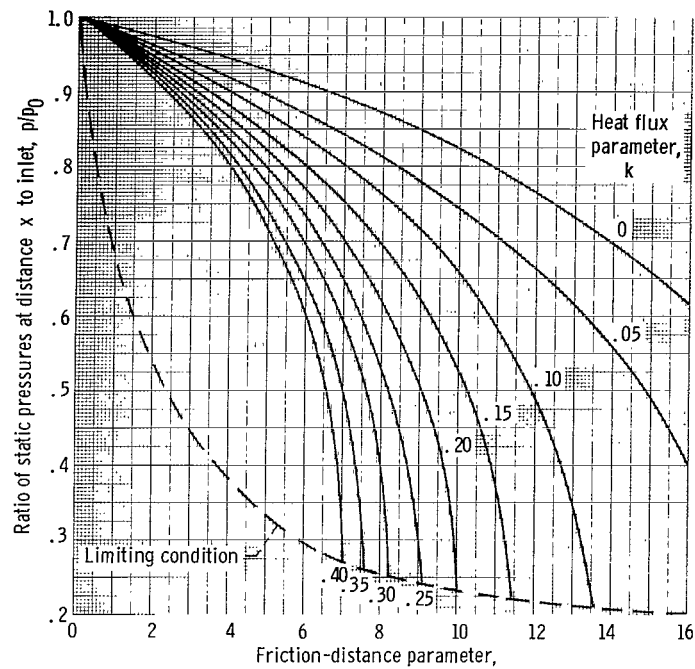
Figure 3. - Static pressure ratio as function of friction-distance parameter. Local Mach number, 0.15; ratio of specific heats, 1.4. Limiting condition is determined by choked Mach number for diabatic conditions.
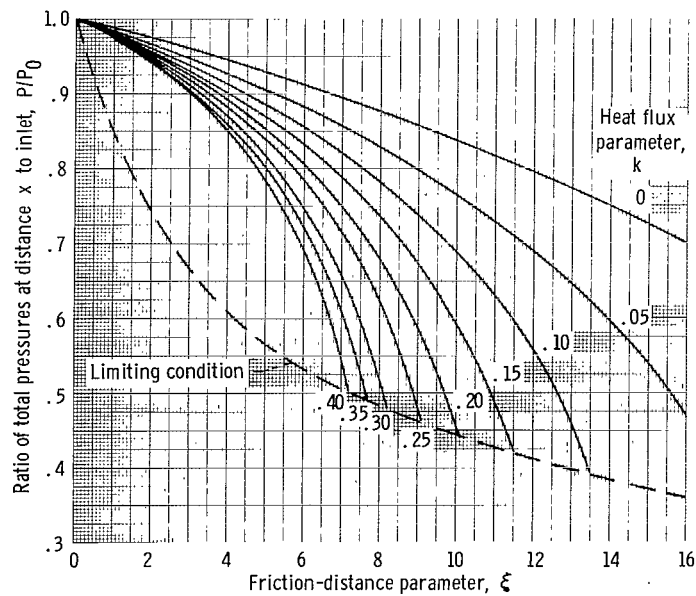

Figure 4. - Total pressure ratio as function of friction-distance parameter. Local Mach number, 0.15; ratio of specific heats, 1.4. Limiting condition is determined by choked Mach number for diabatic conditions.

Figure 3 presents the ratio of the local to inlet static pressures as a function of friction-distance parameter ξ, and figure 4 presents the ratio of the local to inlet total pressure as a function of the friction-distance parameter ξ. In figure 5, the weight flow parameter $\dot{w}\sqrt{T}/A_D P$ is plotted against friction-distance parameter ξ. Figures 3 to 5 are presented for a range of heat flux parameters $0 \le k \le 0.4$ that were calculated using equation (2) and from the test data presented in reference 3. In all of the figures, the inlet Mach number equals 0.15 and $\gamma = 1.4$. The dashed line in the figures represents the limiting or choking conditions, defined by equations (24) and (25), respectively.

If the variation of local Mach number is known, the corresponding corrected weight flow rate for diabatic conditions is readily determined from equation (16) in terms of local static pressure or from equation (17) in terms of the local total pressure along the duct.
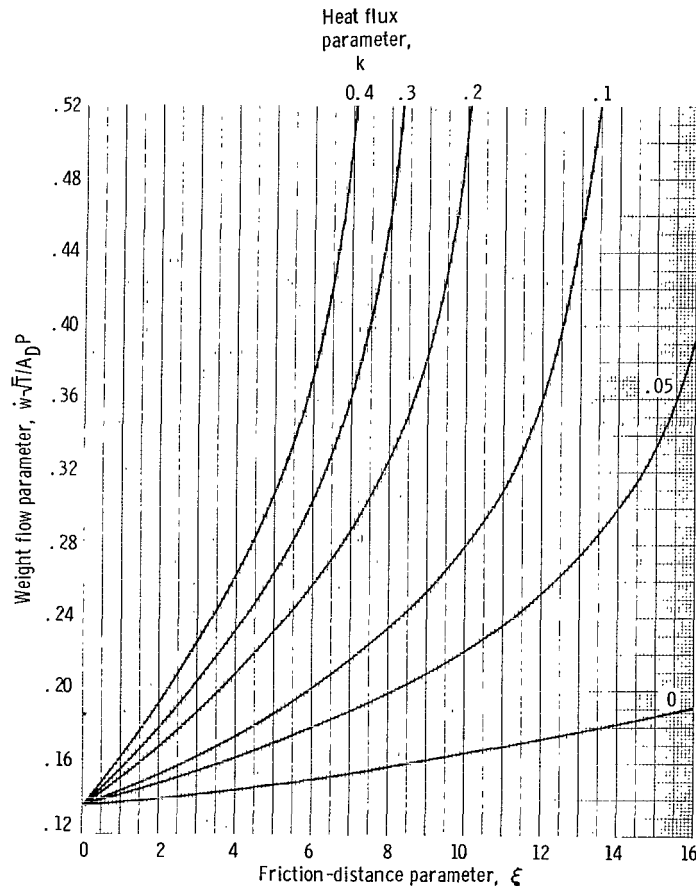

Figure 5. – Variation of weight flow parameter with friction distance. Inlet Mach number, 0. 15; ratio of specific heats, 1. 4.
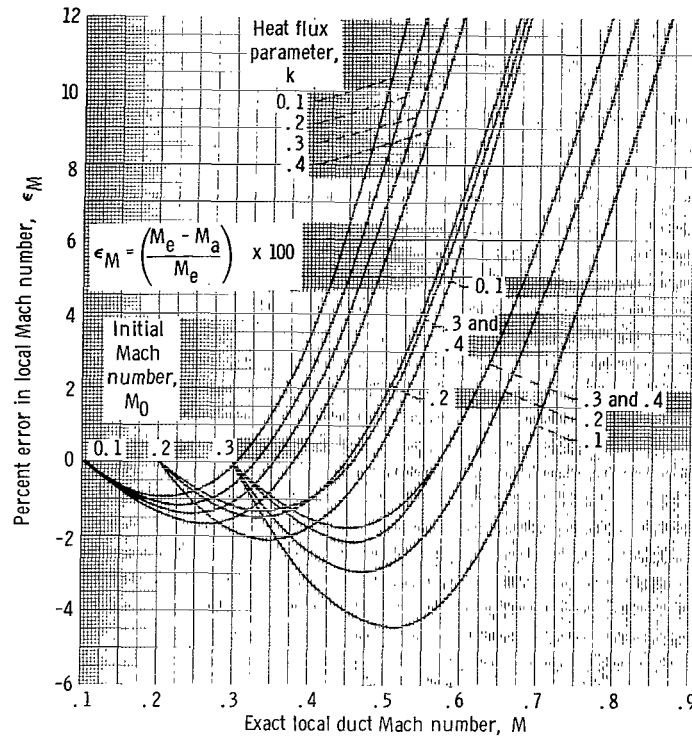
Figure 6. - Percent error introduced in local Mach number by using approximate solution (eq. (21)). Ratio of specific heat, 1.4.

Accuracy of Approximate Analysis

Figure 6 presents the relative error of the local Mach number calculated using an approximate equation (21) and an exact equation (13). It can be seen that, for an inlet Mach number equal to 0. 1, an approximate equation (21) is accurate to within ±1. 5 percent relative to an exact equation (13) up to a local Mach number of 0. 3 for all values of heat flux parameters k. However, for an inlet Mach number of 0. 2, an accuracy of approximately ±2 percent is maintained up to a local Mach number of 0. 5. Although it is not shown, the error in the local Mach number is insensitive to a decrease in γ from 1. 4 to 1. 3.

In general, it can be seen from figure 6 that, for all values of heat flux parameters k, the relative error of the local Mach number calculated using an approximate equation (21) and an exact equation (13) progressively increases with an increase in inlet Mach number M_0.

SUMMARY OF RESULTS

For constant area flow, in which the heat flux and specific heat remain constant, a

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method is developed for calculating the local flow Mach number, the pressure drop, and, therefore, the weight flow rate of a compressible fluid under the simultaneous action of friction and heat transfer. The results are presented in the form of relations and working charts.

A closed form solution to the problem is achieved by choosing the square of a dimensionless flow parameter as one of the independent variables to describe the flow. From this exact solution, an approximate simplified solution is presented which is in good agreement with the exact solution for low subsonic Mach numbers.

Lewis Research Center,
    National Aeronautics and Space Administration,
        Cleveland, Ohio, September 21, 1971,
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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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