COMPUTER DISCRIMINATION PROCEDURES APPLICABLE TO AERIAL AND ERTS MULTISPECTRAL DATA

by

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ABSTRACT

Two statistical models are compared in the classification of crops recorded on color aerial photographs. A theory of error ellipses is applied to the pattern recognition problem. An elliptical boundary condition classification model (EBC), useful for recognition of candidate patterns, evolves out of error ellipse theory. The EBC model is compared with the minimum distance to the mean (MDM) classification model in terms of pattern recognition ability. The pattern recognition results of both models are interpreted graphically using scatter diagrams to represent measurement space. Measurement space, for this report, is determined by optical density measurements collected from Kodak Ektachrome Infrared Aero Film 8443 (EIR). The EBC model is shown to be a significant improvement over the MDM model.

INTRODUCTION

Earth ground patterns, represented by optical density feature measurement vectors were collected from Kodak Ektachrome Infrared Aero Film 8443 (EIR). Two statistical approximations of the distribution of these measurement vectors were developed. A feature measurement vector was constructed from EIR film optical density measurements to red, green, and blue light. Each optical density measurement was considered as an element (feature) of the feature measurement vector. Each element of the vector is a dimension of measurement space. A measurement vector with red, green, and blue light optical density measurements gives three dimensions to measurement space[1].

Specific earth ground patterns most often investigated in agriculture are crops, water bodies, and bare soil (Fig. 1). These different kinds of earth ground patterns can be considered as categories to be recognized by a classification model. The recognition ability for a classification model will be dependent on its approximation, usually statistical, of the feature measurement vector distribution for each category.

The set of all possible ground pattern feature measurement vectors is called measurement space[1]. The distribution of these measurement vectors in measurement space result in a scattering of points. Each vector is represented by a point in measurement space. The natural groupings or clusters of optical density measurement vectors (points) for EIR film in measurement space can be associated with specific ground pattern categories. Proper classification of a candidate pattern can be regarded as identifying a point in measurement space belonging to a particular cluster (category) of points.

Part of the pattern recognition problem is to design classification models that partition measurement space into cells that correspond closely to the clusters associated with each ground pattern category. The other part of the pattern recognition problem is the selection of characteristic features used to construct measurement space which contains the ground pattern categories.

The elliptical boundary condition (EBC) and minimum distance to the mean (MDM) statistical classification models are compared in this report. The EBC model partitions measurement space into elliptical cells while the MDM model partitions measurement space into irregular polygonal cells. Recognition results of ground pattern categories on EIR film were determined for each model. Scattering diagrams, representing measurement space, were developed to explain recognition results obtained for each classification model.

DATA MANAGEMENT PROCEDURES

Ground patterns investigated in this report are shown in Figure 1. Water is shown in three modes of sun glinting defined as low reflecting (W), medium reflecting (M), and high reflecting (H). A cotton field (C) is shown with 90 to 100% vegetation cover. Bare soil is shown in dry (B) and disked (D) conditions. These ground patterns were randomly sampled for flight lines 1, 11, and 12 established by the United States Department of Agriculture, Weslaco, Texas to include as many agricultural conditions in the Rio Grande Valley as possible.

A Hasselblad camera with a 50 mm lens was used to obtain aerial photographs on EIR (8443) film (70 mm) from these three flight lines. The camera was mounted in a plane so that photographs could be taken directly above (1800 to 3000 feet) ground pattern areas. Each flight line was photographed with 60 percent forward overlap. Six rolls of film were used between July 14 and July 27 of 1968 between the hours of 1052 and 1457 CDT. A Tiffen 15G filter was used on all cameras. A typical exposure was 1/250 at f/10. All film processing followed the recommended times, temperatures, and solution concentrations recommended by Eastman Kodak's E-3 process.

Positive transparencies of selected ground pattern areas were mounted in a Joyce-Lobl microdensitometer. Three scans were made of transects running through ground pattern areas appearing in selected frames. A scan was made using a red filter (Wratten 92), green filter (Wratten 93), and a blue filter (Wratten 94) in both beams of the microdensitometer. Each scan produced 20 optical density readings for each filter along each transect. This is equivalent to 20 three-dimensional feature measurement vectors per transect. One transect was established for each ground pattern area appearing in a frame. The mean of the 20 readings per transect for each filter was determined and used in pattern recognition procedures. These mean transects are listed in Appendix I.

A total of 160 mean transects for seven ground pattern categories were sampled as follows, 52 for cotton, 46 for dry bare soil, 20 for sorghum, 19 for low reflecting water, 9 for disked bare soil, 8 for high reflecting water, and 6 for medium reflecting water. All mean transects from each ground pattern category were used to establish the reference mean of the MDM classification model and the reference operation constants for the EBC classification model. Recognition tests for both models were conducted using all transects.

Ground pattern areas on transparencies were selected on the basis of uniformity in visual appearance of their images. A grease pencil was used to mark the location of the transect on the transparency for microdensitometer measurements. This procedure gave ground pattern areas with homogeneous optical density characteristics. Other criteria used for selection of ground pattern areas were derived from ground truth considerations for each category as follows:

a. Cotton selected was in the boll stage with 90 to 100% vegetation cover.
b. Sorghum selected was near harvest stage with 90 to 100% vegetation cover.
c. Bare soil (dry) represented a variety of dry soil surface conditions with 0 - 15% vegetation cover.
d. Bare soil (disked) selected was wet or freshly tilled soil with 0 - 15% vegetation cover.
e. Water selected was visually categorized into three levels of reflectance (low, medium, and high) depending on sun glinting (sun angle).

ERROR ELLIPSE THEORY

The mathematics associated with error ellipses evolves from the elliptical law of error[2]. This law provides a method for describing the distribution of accidental errors for experimental measurements involving many variables (features or dimensions) with elliptical curves.

In two dimensions (x, y) the law of error is expressed by the general equations:

$$\phi = R \exp -(ax^2+2bxy+cy^2) \tag{1}$$

where

$$ax^2 + 2bxy + cy^2 - H = 0. \tag{2}$$

Equation (1) is a probability function that expresses a two-dimensional distribution. Equation (2) are curves of like probability that are assumed not to diverge to infinity. There are an infinite number of values for H, and thus an infinite number of probability curves exist. Since equation (2) must be an ellipse, then

$$b^2 - ac < 0 \quad . \tag{3}$$

By rotation of the x, y plane about the origin, the equation of these ellipses may be written as

$$a' x'^2 + c' y'^2 - H = 0 \quad , \tag{4}$$

where the equations of translation are

$$\tan 2\theta = -2b/(c-a) \quad , \tag{5}$$

$$c' = \tfrac{1}{2}(a+c) \pm b \csc 2\theta \quad , \tag{6}$$

$$a' = \tfrac{1}{2}(a+c) \mp b \csc 2\theta \quad . \tag{7}$$

The major and minor axes of the ellipse, 2α and 2β, respectively, may be determined as follows:

$$x'^2/\alpha^2 + y'^2/\beta^2 = 1 \quad , \tag{8}$$

where

$$\alpha^2 = H/a' \text{ and } \beta^2 = H/c' \quad . \tag{9}$$

The probability of a measurement falling outside an ellipse with $H = H_o$ is

$$\int \exp (-H)\, dH = \exp (-H_o) \quad , \tag{10}$$

where the integration limits are ∞ and H_o. If it is desired that 5 percent of the experimental points should fall outside the error ellipse, then

$$\exp(-H_o) = 0.05 \quad , \tag{11}$$

$$H_o = 2.99573227 \quad . \tag{12}$$

To determine the coefficients, a, b, and c, of equation (2) the following equations are used:

$$a = \frac{\Sigma\varepsilon_i^2\ (n-1)}{2[\Sigma\delta_i^2\ \Sigma\varepsilon_i^2 - (\Sigma\delta_i\varepsilon_i)^2]} \quad , \tag{13}$$

$$b = \frac{-\Sigma\delta_i\varepsilon_i\ (n-1)}{2[\Sigma\delta_i^2\ \Sigma\varepsilon_i^2 - (\Sigma\delta_i\varepsilon_i)^2]} \quad , \tag{14}$$

$$c = \frac{\Sigma\delta_i^2\ (n-1)}{2[\Sigma\delta_i^2\ \Sigma\varepsilon_i^2 - (\Sigma\delta_i\varepsilon_i)^2]} \quad , \tag{15}$$

where

$$\Sigma\delta^2 = \Sigma(x-\bar{x})^2 = \Sigma x^2 - (\Sigma x)^2/n \quad , \tag{16}$$

$$\Sigma\varepsilon^2 = \Sigma(y-\bar{y})^2 = \Sigma y^2 - (\Sigma y)^2/n \quad , \tag{17}$$

$$\Sigma\delta\varepsilon = \Sigma(x-\bar{x})\ (y-\bar{y}) = \Sigma xy - \Sigma x\Sigma y/n \quad . \tag{18}$$

The mathematics needed for the n-dimensional error ellipse is given below:

$$P_{ij} = \Sigma\ \frac{\delta_i\delta_j}{n_{ij}-1} \quad , \tag{19}$$

$$a_{ij} = 1/2[P_{ij}{}^T]^{-1} \quad , \tag{20}$$

$$\delta_i \delta_j = (x_i - \bar{x}_i)(x_j - \bar{x}_j) \quad , \tag{21}$$

where

$$P_{ij} = P_{ij}^{T} \quad , \tag{22}$$

for a symmetrical matrix. The (T) notation indicates the transpose matrix operation.

The (a_{ij}) matrix contains the coefficients for the general error ellipse for any dimension. The elements for the (a_{ij}) matrix which correspond to the coefficients in the two dimensional general equation (2) are:

$$\begin{aligned} a_{11} &= a \quad , \\ a_{12} &= b \quad , \\ a_{22} &= c \quad . \end{aligned} \tag{23}$$

Expanding the (a_{ij}) matrix equation (20) and combining with the feature measurement vector X produces the following general equation:

$$\begin{aligned} a_{11}x_1' &+ 2a_{12}x_1'x_2' + \cdots + 2a_{1n}x_1'x_n' \\ &+ a_{22}x_2'x_2' + \cdots + 2a_{2n}x_2'x_n' \\ &\qquad + \cdots + a_{nn}x_n'x_n' \\ &\qquad - H = 0 \quad . \end{aligned} \tag{24}$$

The primes on the x terms above mean that the feature measurement vector elements $x_1, x_2, \cdots, x_n$ must be translated before they can be used by equation (24) as follows:

$$x_1' = x_1 - \bar{x}_1$$

$$x_2' = x_2 - \bar{x}_2 \qquad (25)$$

$$\cdots$$

$$x_n' = x_n - \bar{x}_n \quad .$$

EBC CLASSIFICATION MODEL FOR n-DIMENSIONS

For m ground cover pattern classes $\omega_1, \omega_2, \cdots, \omega_m$, m error hyper-ellipses in n-dimensional space can be generated using equations (19) through (25).

The classification rule for identification of a measurement vector X for m ground cover pattern classes may be formed as follows:

Classify candidate measurement vector X as belonging to pattern class

(ground cover category) ω_L if:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ijL} x'_{iL} x'_{jL} < \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ijk} x'_{ik} x'_{jk} \qquad (26)$$

for all $k \neq L$ where the index k varies from 1 to m and excludes L.

A rejection class (threshold) for "everything else" may be constructed for the n-dimensional error hyper-ellipse. A candidate measurement vector X is classified as from class ω_L if it meets the criterion in (26) and fulfills the following condition

$$\sum_{k=1}^{n} \sum_{j=1}^{n} a_{ijL} x'_{iL} x'_{jL} \leq H \quad . \qquad (27)$$

MDM CLASSIFICATION MODEL

Using the EBC optical density measurement vectors m mean and standard deviation vectors ($\bar{X}_i$ and S_i) can be developed for m ground pattern classes (categories) ($\omega_1, \omega_2, \cdots, \omega_m$). If red (R), green (G), and blue (B), light density readings are used as characteristic features $X_{1\,ik}$, $X_{2\,ik}$, and $X_{3\,ik}$, respectively, constituting a sample from the ith ground pattern class and kth observation, then $\bar{X}_i$ and S_i are defined as:

$$\bar{X}_i = \frac{1}{N_i} \sum_{k=1}^{N_i} X_{ik} = \begin{bmatrix} \frac{1}{N_i} \sum_{k=1}^{N_i} X_{1ik} \\ \cdots \\ \frac{1}{N_i} \sum_{k=1}^{N_i} X_{3ik} \end{bmatrix} = \begin{bmatrix} \bar{X}_{1i} \\ \cdots \\ \bar{X}_{3i} \end{bmatrix}, \tag{28}$$

$$S_i = \frac{\sum_{k=1}^{N_i} (X_{ik} - \bar{X}_i)^2}{N_i - 1} = \begin{bmatrix} S_{1i} \\ \cdots \\ S_{3i} \end{bmatrix}, \tag{29}$$

where $k = 1, \cdots, N_i$ number of observations for the ith ground cover pattern class.

The MDM classification rule for identification of a candidate measurement vector X for m ground cover pattern classes may be formed as follows:[3]

Classify measurement vector X as belonging to pattern class (category) ω_L if

$$\sum_{j=1}^{n} [X(j) - \bar{X}_L(j)]^2 < \sum_{j=1}^{n} [X(j) - \bar{X}_i(j)]^2 \tag{30}$$

for all $i \neq L$ where $\bar{X}_i$ is m-1 mean measurement vectors calculated for X from m-1 pattern classes, excluding pattern class ω_L, and X_L is the Lth mean measurement vector calculated for X from pattern class ω_L. The index $j = 1, \cdots, n$ represents the number of elements (using optical density measurements to red, green, and blue light) formed X, $\bar{X}_L$, and $\bar{X}_i$ by each pattern recognition model.

A rejection class for "everything else" may be developed by employing standard deviations as threshold values. Mathematically a candidate measurement vector is classified as from class ω_L if it meets the criterion from equation (30) and fulfills the following condition

$$\sum_{j=1}^{n} [X(j) - \bar{X}_L (j)]^2 \leq T_L \quad , \tag{31}$$

where T_L is the threshold for class ω_L and is given by

$$T_L = \sum_{j=1}^{n} S_L (j)^2 \quad . \tag{32}$$

EXPERIMENTAL RESULTS AND DISCUSSION

Scattering diagrams were developed to approximate two dimensional measurement space for all ground pattern categories used in this report. Scatter plots using the 160 mean transects of red versus green (Figure 2 and Figure 3), red versus blue (Figure 4), and blue versus green (Figure 5) light optical density measurements were constructed. For all four figures each of the 160 mean transect points are identified with a letter as cotton (C), dry bare soil (B), sorghum (S), low reflecting water (W), medium reflecting water (M), high reflecting water (H), and disked bare soil (D). In Figure 2 the partitioning of measurement space with cells for the MDM model is indicated with dashed lines that form irregular polygons. Partitioning with cells for the EBC model is indicated with solid lines that form ellipses in Figure 3. A circle with the appropriate letter identifies the mean for each circle. Table 1 contains the elliptical coefficients and means for all ground pattern categories used to construct Figures 2 and 3.

From Figure 5 it can be seen that the blue versus green scatter plot does not yield very good separation among ground pattern categories. The scatter plot in Figure 4 indicates better separation of ground pattern categories than Figure 5 but there is some confusion between the cotton and sorghum categories. The scatter plots in Figure 2 and Figure 3 show good separation among all ground pattern categories. For this reason the red versus green combination was used for tests comparing MDM and EBC classification models.

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From casual inspection of Figure 2 and Figure 3 it should be immediately obvious that the EBC classification model will yield the best approximation of each ground pattern category. The clusters representing all ground pattern categories, except cotton, cross over MDM partitioning polygonal cells (Figure 2). The points that cross over represent those candidate measurement vectors that will be incorrectly classified by the MDM classification model. The elliptical cells can be tailored for close correspondence to each ground pattern category cluster with very few points of any cluster falling outside an elliptical cell (Figure 3).

A recognition test, using the mathematics developed for the MDM and EBC models, was conducted using the red and green optical density measurements in Appendix I. The results of this test is shown in Tables 2 and 3. Overall recognition by the EBC model (94.5%) was higher than the MDM model (80.6%).

CONCLUSIONS

The theory of error ellipses, developed by Coolidge, was applied to the pattern recognition problem. The EBC classification model evolved from error ellipse theory yields better pattern recognition results (94.5%) than the MDM classification model (80.6%). Scatter diagrams show graphically (Figures 2 and 3) that the EBC model provides a much better description of ground cover category clusters than the MDM model does. These results indicate that the EBC model is more efficient than the MDM model for the pattern recognition problem.

Scatter diagrams provide a graphical method for determining those characteristic features that yield optimum discrimination results. For this report the scatter diagrams in Figure 2 through 5 indicated that optimum discrimination results could be obtained with the red versus green (Figures 2 and 3) optical density measurements. Ground pattern categories seemed to have better separation using red versus green density measurements as compared to using either the red versus green or the green versus blue density measurements.

REFERENCES

1. Haralick, R. M., and Kelly, G. L., "Pattern Recognition with Measurement Space and Spatial Clustering for Multiple Images," Proceedings of the IEEE, Vol. 57, No. 4, April 1969.

2. Coolidge, Julian L., "An Introduction to Mathematical Probability," Dover Publications, Inc., New York, N. Y. 1962.

3. Sebestyen, G. S., and Edie, J., "Pattern Recognition Research," Air Force Cambridge Research Lab., Bedford, Mass., Rept. 64-821 (AD 603 692), 1964.

Table I. Reference ground pattern mean vectors and covariance matrixes for optical density readings of red and green filters used for MDM and EBC classification models.

Ground pattern category	Mean vector elements		(a_{ij}) matrix elements		
	$\bar{X}_1$ Red Optical density	$\bar{X}_2$ Green Optical density	a_{11}	a_{12} & a_{21}	a_{22}
Cotton	0.2740	1.4907	276.21	-108.20	59.11
Bare Soil (Disked)	1.3171	1.4400	272.19	-394.00	582.10
Water (L.R.)	1.1678	0.8373	105.53	-116.90	137.70
Water (H.R.)	0.3851	0.3084	8374.50	-15730.00	29628.00
Bare Soil (Dry)	0.4366	0.5512	205.45	-182.90	219.05
Water (M.R.)	1.1375	1.0758	994.55	-1337.00	1919.70
Sorghum	0.5351	1.2910	93.45	-70.34	61.45

L. R. - Low Reflectance
M. R. - Medium Reflectance
H. R. - High Reflectance

Table II. Comparison of recognition results for the MDM classification models using the indicated ground pattern categories with red and green light EIR optical density measurements as characteristic features. A threshold class (T) for "every thing else" was used.

Ground pattern category	Number of observation	Ground Pattern classified as								Percent recognition
		C	B	S	W	D	H	M	T	
Cotton [C]	52	52								100.0
Bare Soil [B]	46		35	1			7	3		76.3
Sorghum [S]	20		1	17		2				85.0
Water (L.R.) [W]	19		4		9	4		2		47.3
Bare Soil (Disked) [D]	9					5		4		55.6
Water (H.R.) [H]	8		2				6			75.0
Water (M.R.) [M]	6							6		100.0

Overall percent recognition 80.6%
L. R. - Low Reflectance
H. R. - High Reflectance
M. R. - Medium Reflectance

Table III. Comparison of recognition results for the EBC classification model using the indicated ground pattern categories with red and green light EIR optical density measurement as characteristic feature. A threshold class (T) for "everything else" was used.

Ground pattern category	Number of observation	Ground Pattern classified as								Percent recognition
		C	B	S	W	D	H	M	T	
Cotton [C]	52	51							1	98.1
Bare Soil [B]	46		41			2			3	89.2
Sorghum [S]	20			19					1	95.0
Water (L.R.) [W]	19				19					100.0
Bare Soil (Disked) [D]	9					8			1	89.0
Water (H.R.) [H]	8				1		7			87.5
Water (M.R.) [M]	6							6		100.0

Overall percent recognition 94.5
L. R. - Low Reflectance
H. R. - High Reflectance
M. R. - Medium Reflectance

W - Water (low reflecting)
M - Water (medium reflecting)
H - Water (high reflecting)
C - Cotton
D - Bare Soil (disked)
B - Bare Soil (dry)

Positive reproduction, of a 70mm positive infrared transparency (Ektachrome 8443), showing bare soil, cotton, and water conditions in the vicinity of the Rio Grande River.

Fig. 1

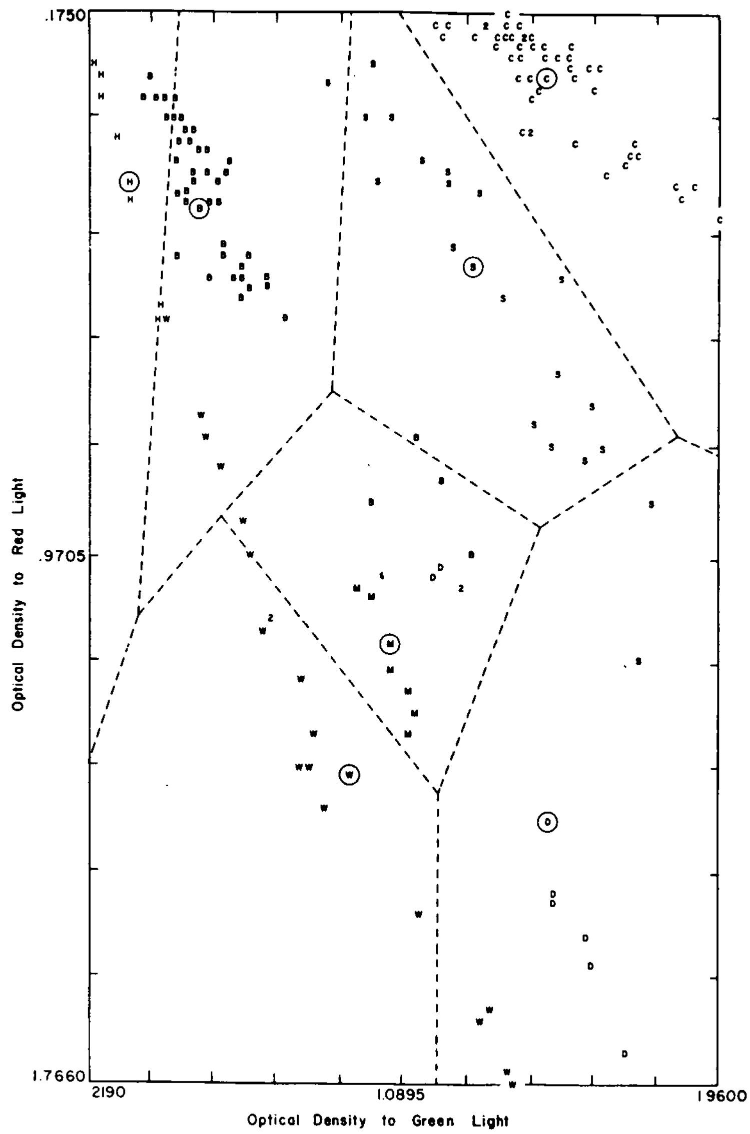

Figure 2.- Scatter diagram of red versus green optical density measurements for 160 mean transect readings. Decision regions for the MDM classification model appear as dashed lines. Appropriate letters indicate identity of points as cotton (C), bare soil (B), sorghum (S), low reflecting water (W), medium reflecting water (M), high reflecting water (H), and disked bare soil (D). The seven ground cover means appear as a circle with the appropriate letter.

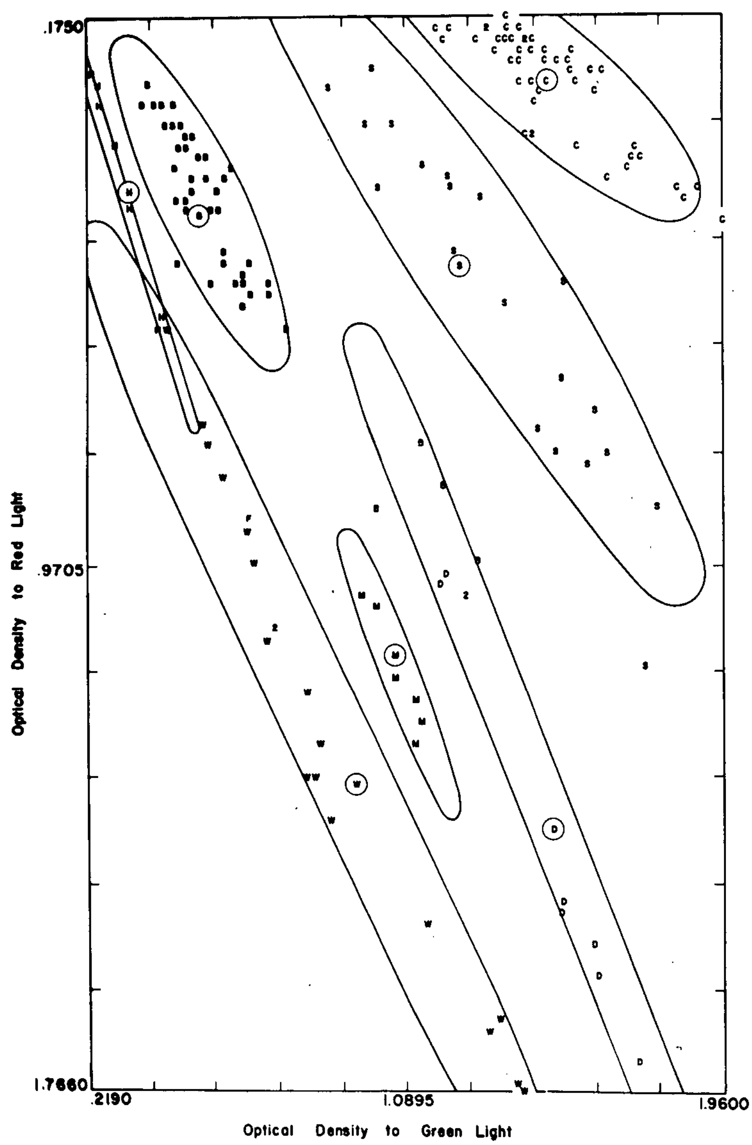

Figure 3.- Scatter diagram of red versus green light optical density measurements showing decision regions for the EBC classification model. Letter identification of points is the same as Fig. 2.

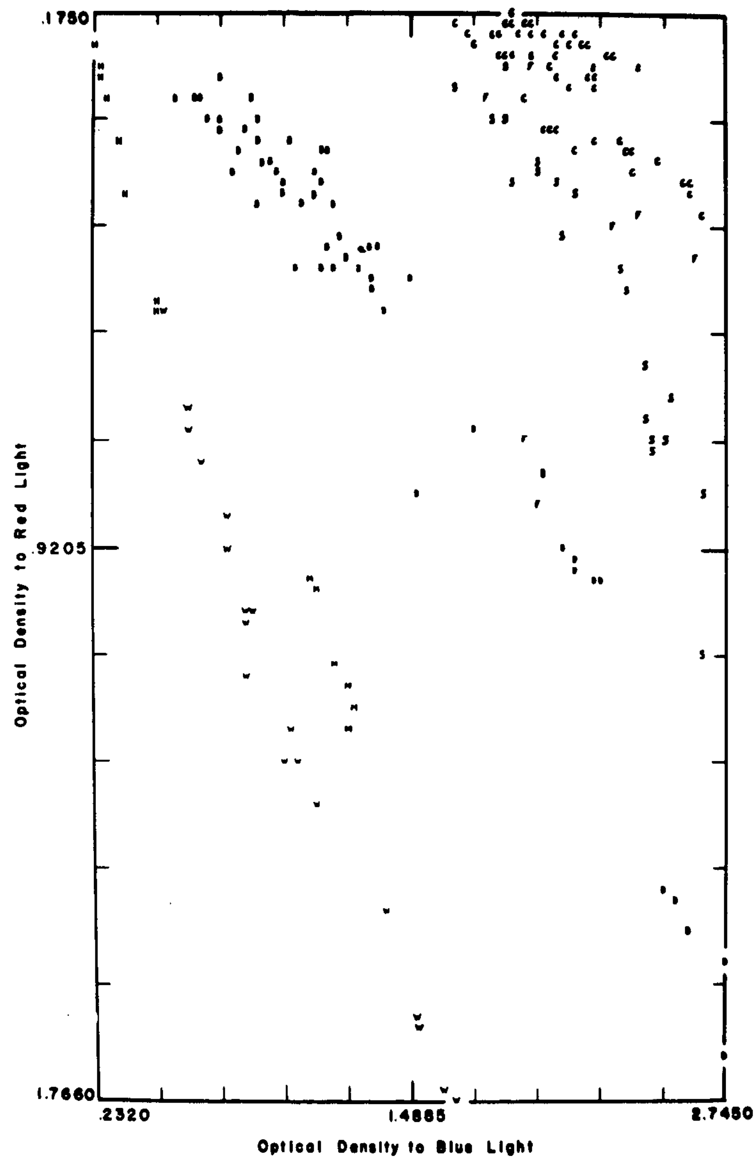

Figure 4.- Scatter diagram of red versus blue light optical density measurements. Letter identification of points is the same as Fig. 2.

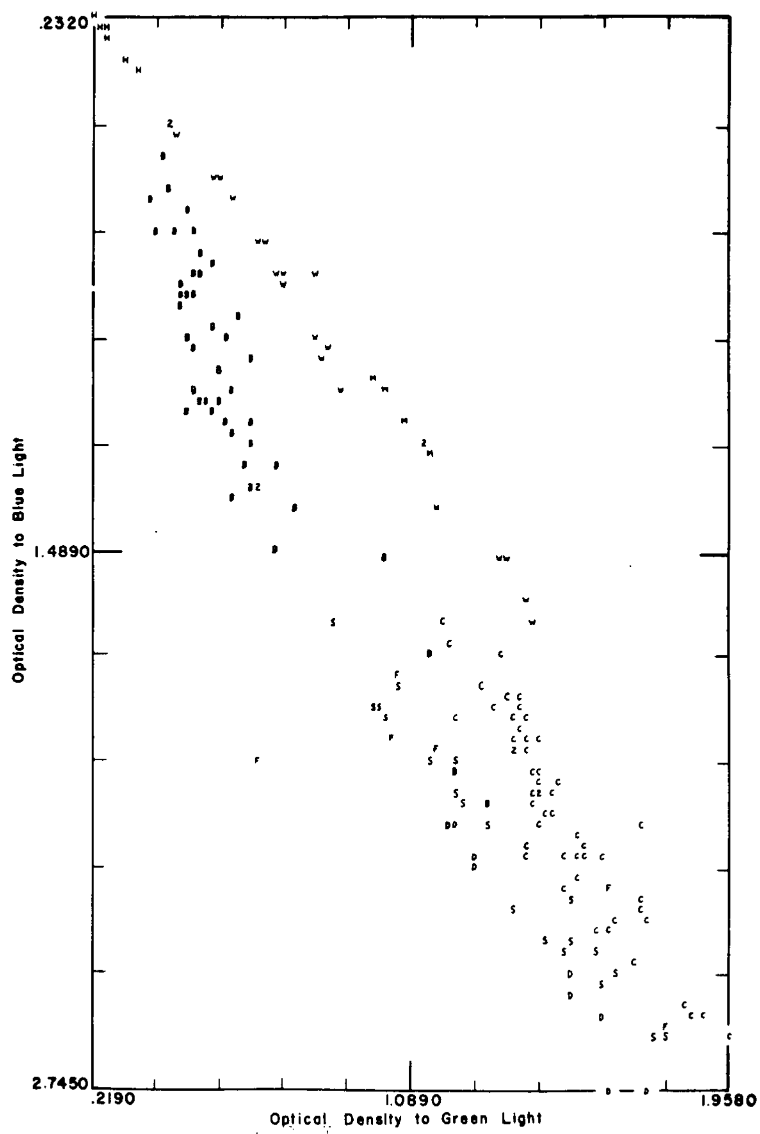

Figure 5.- Scatter diagram of blue versus green light optical density measurements. Letter identification of points is the same as Fig. 2.

APPENDIX I

Computer printout of 160 mean transect optical density measurements for Red, Green, Blue, and White Light.

A Optical density measurements to red light
B Optical density measurements to green light
C Optical density measurements to blue light
D Optical density measurements to white light
E The average of red, green, and blue light optical density measurements
F Identification number for each ground pattern category as follows:

3 - Cotton

2 - Bare soil (dry)

19 - Sorghum

23 - Water (low reflecting)

4 - Bare soil (disked)

8 - Water (high reflecting)

13 - Water (medium reflecting)

A	B	C	D	E	F
0.236	1.377	1.848	1.136	1.154	3
0.235	1.398	1.886	1.157	1.173	3
0.235	1.381	1.869	1.159	1.161	3
0.236	1.405	1.974	1.174	1.205	3
0.252	1.480	2.044	1.212	1.259	3
0.261	1.620	2.399	1.294	1.427	3
0.211	1.362	1.964	1.150	1.179	3
0.208	1.341	1.848	1.133	1.132	3
0.230	1.535	2.159	1.246	1.308	3
0.209	1.425	2.010	1.173	1.215	3
0.214	1.435	2.134	1.186	1.261	3
0.212	1.420	2.082	1.174	1.238	3
0.223	1.428	2.066	1.181	1.239	3
0.245	1.537	2.256	1.250	1.346	3
0.235	1.506	2.295	1.210	1.345	3
0.258	1.508	2.226	1.224	1.331	3
0.277	1.400	2.229	1.185	1.302	3
0.228	1.400	2.182	1.162	1.270	3
0.259	1.596	2.400	1.270	1.418	3
0.407	1.651	2.367	1.364	1.475	3
0.397	1.695	2.474	1.383	1.522	3
0.366	1.561	2.205	1.299	1.378	3
0.201	1.199	1.719	1.023	1.040	3
0.198	1.167	1.673	1.003	1.013	3
0.211	1.380	1.914	1.121	1.168	3
0.206	1.280	1.816	1.075	1.101	3
0.195	1.205	1.903	1.029	1.101	3
0.265	1.551	2.198	1.241	1.338	3
0.375	1.708	2.136	1.376	1.406	3
0.474	1.958	2.657	1.544	1.697	3
0.433	1.882	2.598	1.499	1.638	3
0.424	1.837	2.581	1.470	1.614	3
0.442	1.855	2.595	1.490	1.631	3
0.376	1.730	2.375	1.381	1.494	3
0.382	1.716	2.337	1.384	1.478	3
0.373	1.722	2.311	1.381	1.469	3
0.296	1.439	1.929	1.216	1.222	3
0.233	1.477	2.067	1.216	1.259	3
0.225	1.469	2.129	1.206	1.275	3
0.282	1.453	2.119	1.217	1.284	3
0.259	1.541	2.210	1.252	1.336	3
0.272	1.434	2.060	1.205	1.255	3
0.282	1.612	2.210	1.317	1.368	3
0.230	1.339	1.743	1.140	1.104	3
0.357	1.435	2.044	1.247	1.279	3
0.355	1.416	2.058	1.250	1.276	3
0.346	1.431	2.009	1.246	1.262	3
0.175	1.374	1.896	1.121	1.149	3
0.186	1.365	1.946	1.130	1.166	3
0.186	1.374	1.972	1.127	1.177	3
0.190	1.400	1.950	1.148	1.180	3
0.187	1.308	1.861	1.072	1.119	3

A	B	C	D	E	F
0.557	0.653	1.046	0.706	0.752	2
0.558	0.662	1.179	0.727	0.799	2
0.409	0.520	0.781	0.564	0.570	2
0.628	0.769	1.394	0.834	0.930	2
0.521	0.679	1.334	0.758	0.845	2
0.556	0.716	1.277	0.763	0.850	2
0.415	0.555	0.973	0.589	0.648	2
0.593	0.660	1.337	0.750	0.863	2
0.468	0.574	1.059	0.661	0.700	2
0.542	0.659	1.229	0.737	0.810	2
0.888	1.027	1.515	1.099	1.143	2
0.443	0.493	1.106	0.586	0.681	2
0.522	0.486	1.170	0.612	0.726	2
0.462	0.496	0.885	0.568	0.615	2
0.552	0.560	1.127	0.646	0.747	2
0.399	0.470	0.899	0.547	0.589	2
0.519	0.598	1.356	0.698	0.824	2
0.577	0.724	1.501	0.803	0.934	2
0.514	0.600	1.202	0.676	0.772	2
0.565	0.666	1.344	0.747	0.859	2
0.559	0.635	1.284	0.722	0.826	2
0.446	0.477	0.977	0.563	0.633	2
0.790	1.132	1.746	1.144	1.223	2
0.856	1.208	2.013	1.218	1.359	2
0.965	1.293	2.084	1.307	1.447	2
0.468	0.582	1.181	0.653	0.744	2
0.377	0.539	1.148	0.608	0.688	2
0.388	0.552	1.156	0.615	0.699	2
0.338	0.471	0.873	0.522	0.561	2
0.361	0.500	0.999	0.565	0.620	2
0.359	0.482	0.890	0.547	0.577	2
0.310	0.465	0.859	0.520	0.545	2
0.269	0.387	0.723	0.448	0.460	2
0.348	0.489	0.736	0.527	0.524	2
0.306	0.403	0.566	0.434	0.425	2
0.416	0.606	1.105	0.665	0.709	2
0.378	0.549	0.812	0.578	0.579	2
0.354	0.514	0.829	0.554	0.566	2
0.332	0.472	0.682	0.511	0.495	2
0.296	0.430	0.633	0.456	0.453	2
0.396	0.624	0.929	0.651	0.650	2
0.332	0.448	0.743	0.512	0.508	2
0.423	0.581	0.998	0.632	0.667	2
0.299	0.384	0.649	0.445	0.444	2
0.424	0.515	1.126	0.607	0.688	2
0.356	0.505	0.847	0.560	0.569	2

A	B	C	D	E	F
0.416	1.213	1.995	1.102	1.208	19
0.341	0.989	1.877	0.982	1.069	19
0.248	1.003	1.860	0.947	1.037	19
0.825	1.586	2.433	1.477	1.615	19
0.425	1.026	1.884	1.000	1.112	19
0.279	0.882	1.668	0.858	0.943	19
0.559	1.526	2.315	1.353	1.467	19
0.694	1.528	2.426	1.400	1.550	19
0.804	1.639	2.494	1.511	1.646	19
0.894	1.791	2.637	1.650	1.774	19
0.747	1.603	2.516	1.482	1.622	19
0.586	1.360	2.344	1.313	1.430	19
1.133	1.745	2.654	1.656	1.844	19
0.779	1.451	2.425	1.397	1.552	19
0.811	1.505	2.434	1.441	1.583	19
0.517	1.230	2.088	1.145	1.278	19
0.330	1.051	1.804	0.980	1.061	19
0.439	1.291	2.138	1.171	1.289	19
0.398	1.139	1.988	1.060	1.175	19
0.433	1.216	2.062	1.128	1.237	19
1.086	0.730	0.825	0.752	0.880	23
1.284	0.831	0.981	0.864	1.032	23
1.281	0.843	1.026	0.881	1.050	23
1.354	0.905	1.102	0.944	1.120	23
0.849	0.601	0.653	0.621	0.701	23
0.972	0.693	0.756	0.708	0.807	23
1.068	0.742	0.823	0.758	0.878	23
1.248	0.861	1.011	0.887	1.040	23
0.791	0.559	0.617	0.582	0.656	23
0.769	0.550	0.600	0.566	0.639	23
1.070	0.740	0.848	0.764	0.886	23
1.660	1.349	1.520	1.343	1.510	23
1.744	1.397	1.610	1.405	1.584	23
1.667	1.339	1.518	1.341	1.508	23
1.766	1.413	1.666	1.423	1.615	23
1.160	0.832	0.824	0.798	0.939	23
1.504	1.159	1.377	1.156	1.347	23
0.615	0.453	0.515	0.456	0.528	23
0.922	0.663	0.751	0.666	0.779	23

A	B	C	D	E	F
0.979	1.212	2.130	1.264	1.440	4
1.009	1.188	2.140	1.244	1.446	4
1.020	1.264	2.215	1.306	1.499	4
1.017	1.269	2.236	1.313	1.507	4
1.720	1.732	2.742	1.808	2.065	4
1.543	1.614	2.600	1.676	1.919	4
1.487	1.530	2.485	1.605	1.854	4
1.494	1.518	2.543	1.595	1.852	4
1.585	1.633	2.745	1.700	1.968	4
0.304	0.261	0.279	0.271	0.281	8
0.443	0.335	0.358	0.345	0.379	8
0.617	0.431	0.481	0.443	0.510	8
0.607	0.427	0.484	0.442	0.506	8
0.217	0.219	0.232	0.227	0.223	8
0.247	0.238	0.247	0.247	0.244	8
0.273	0.246	0.259	0.258	0.259	8
0.373	0.310	0.324	0.311	0.335	8
0.304	1.050	1.798	1.001	1.050	6
0.248	1.152	1.961	1.041	1.120	6
0.536	1.779	2.626	1.489	1.648	6
0.491	1.624	2.283	1.394	1.466	6
0.472	1.596	2.384	1.372	1.484	6
0.816	1.032	1.953	1.107	1.267	6
0.909	0.672	1.996	1.171	1.192	6
1.020	0.986	[illegible]	0.993	1.029	13
1.139	1.074	1.193	1.061	1.135	13
1.237	1.121	1.241	1.112	1.200	13
1.211	1.137	1.265	1.120	1.204	13
1.042	1.011	1.112	0.997	1.055	13
1.176	1.126	1.248	1.114	1.183	13