Final Report

A SEISMIC SURVEY OF THE MANSON DISTURBED AREA

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ABSTRACT

The region in north-central Iowa referred to as the Manson disturbed area was investigated with the seismic refraction method and the bedrock configuration mapped. The area is approximately 30 km in diameter and is not detectable from the surface topography; however, water wells that penetrate the bedrock indicate that the bedrock is composed of disturbed Cretaceous sediments with a central region approximately 6 km in diameter composed of Precambrian crystalline rock.

Seismic velocity differences between the overlying glacial till and the Cretaceous sediments were so small that a statistical program was developed to analyze the data. The program developed utilizes existing 2 segment regression analyses and extends the method to fit 3 or more regression lines to seismic data.

The seismic data was collected on two-mile centers as reversed profiles. The data was analyzed and depths and velocities were determined and plotted as two separate maps. The resulting bedrock topography includes closed depressions, but the circular pattern observed in the subsurface data is not reflected in the bedrock topography. Bedrock seismic velocity patterns appear to correlate with the Bouguer gravity map developed in an earlier study. The Precambrian crystalline rock displayed velocities ranging between 3000 m/sec to 5000 m/sec whereas the disturbed sediments ranged between 2000 m/sec to 3000 m/sec.

The study was not able to provide the necessary data to establish the origin of the crater. The bedrock configuration map and the velocity distribution map do point to another crystalline subcrop west and south of the present outcrop.

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INTRODUCTION

The identification of granitic well cuttings by the Iowa Geological Survey from water well samples taken near the town of Manson, Iowa, suggested that an unusual geologic feature was buried at relatively shallow depths beneath glacial drift that blankets most of Iowa. A number of water wells drilled into bedrock near the wells from which the granitic cuttings were taken, revealed that the normal sequence of bedrock formations did not exist in the region. Moreover, the cuttings from these wells were highly deformed. The Iowa Geological Survey established a boundary enclosing all of these wells and labeled the region the Manson disturbed area.

In an endeavor to explain the occurrence of the granitic rock and anomalously soft well-water in this region, the Iowa Geological Survey and U.S. Geological Survey conducted a drilling program near Manson in 1952. From a bore hole, drilled approximately 6 km north of Manson, 118 m of crystalline core was recovered below 28 m (93 ft.) of glacial drift.

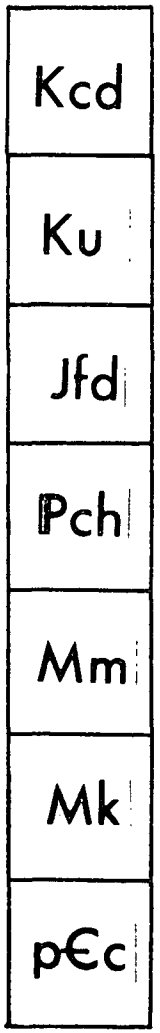
Petrologic analysis showed that it consisted of highly deformed gneiss, gneissoid granite, granite, and diabase, all Precambrian in age (Hoppin and Dryden, 1958). The dimensions of the major components of each rock type were also varied. Breccias, with rock fragments as long as 45 cm grading down to chloritized microbreccia, made up a large part of the core. Weathering alterations occurred as deep as 60 m into the crystalline material (Dryden, 1955). Lidiak et al. (1966) have dated the crystalline rock by the Rb-Sr method to be between 1.13 and 1.44 x 10^9 years. Bunch (1968) found that the minerals in the crystalline mass had numerous unusual microstructures. Quartz had well-defined multiple kink bands, and deformation bands

in plagioclase were observed.

After an examination of available well logs, both Dryden (1955) and Hoppin and Dryden (1958) suggested that the crystalline rock surface is nearly flat-topped and covers an area approximately 4 km^2 directly beneath the glacial drift. Extending beyond this area, the surface of the crystalline rock falls away in all directions with steeper slopes to the southeast. They concluded that the Manson disturbed area has many similarities with the characteristics of cryptovolcanic features as outlined by Bucher (1936). Bunch (1968) concluded that the numerous structural features in the minerals indicated that the Manson disturbed area had undergone violent disruption. Irrefutable evidence that the area had been subjected to shock pressures that only a meteorite could produce, such as shock-formed glass and/or high pressure silica polymorphs, had not been discovered in the thin sections.

A compilation of elevations of the basement rock of Iowa by Yoho (1967) shows that at two locations in Ft. Dodge the top of the basement rock is 315 and 399 m below mean sea level and is typical of north central Iowa. The elevation at which the crystalline rock was found north of Manson by Hoppin and Dryden (1958) was 351 m above mean sea level. Therefore, the relief of the crystalline rock in the Manson disturbed area must be near 700 m.

A ground water resource study of Webster County by Hale (1955) suggested that the regional structure in the Manson area was abruptly broken by faulting, forming a roughly circular basin. Figure 1 is a geologic map of the Manson area as presented by the Iowa Geological Survey (1969). The Precambrian subcrop is shown surrounded by highly deformed lower Cretaceous shales. Mississippian limestones and dolomites border the disturbed area on



Kcd — Colorado and Dakota Groups. Sandstone and shale with minor limestone.

Ku — Cretaceous Undifferentiated. Variably calcareous, severely deformed shales, minor sandstone and carbonate rock.

Jfd — Ft. Dodge Beds. Gypsum and red and green shale.

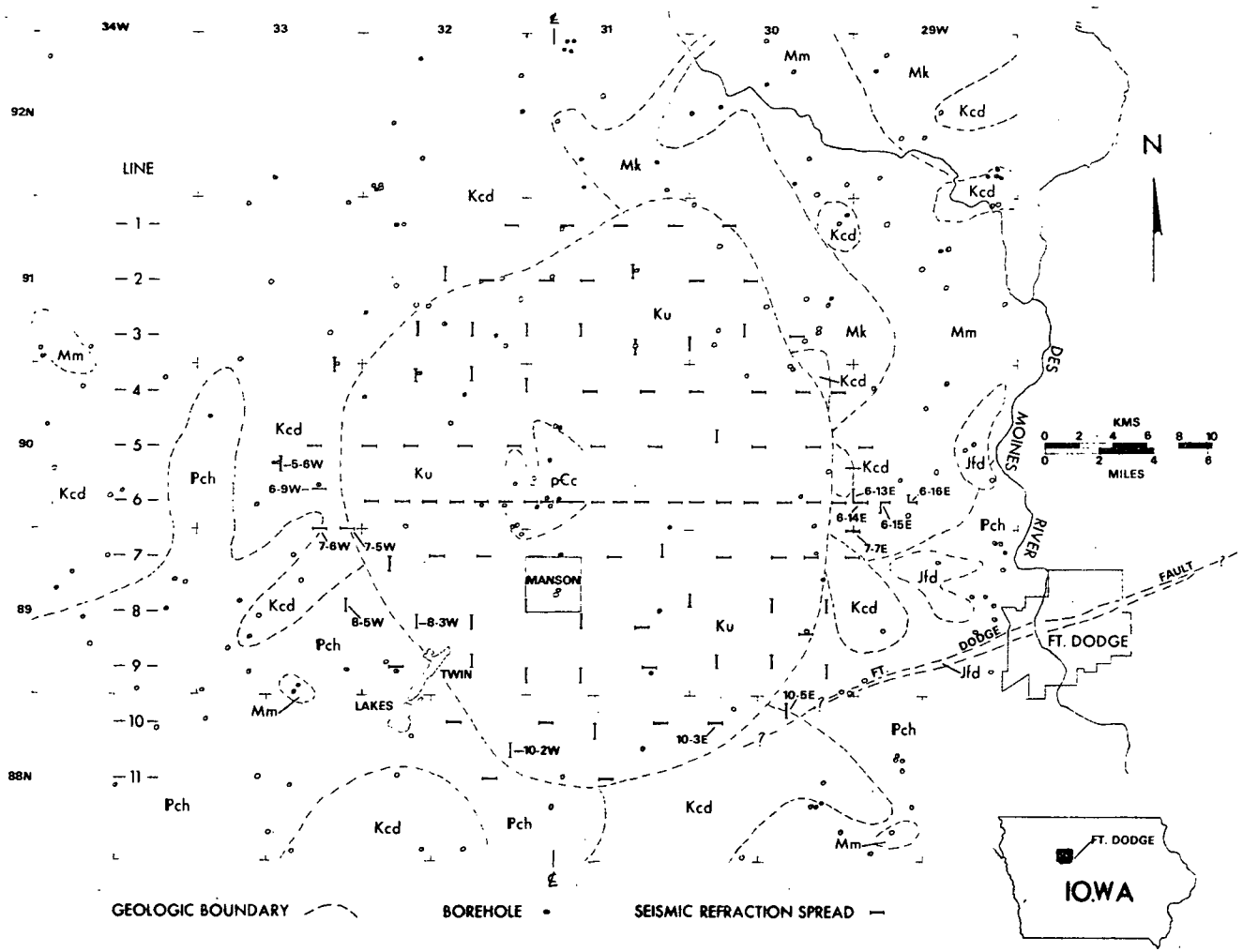
Pch — Cherokee Group. Cyclic deposits with carbonaceous shale, clay, siltstone, sandstone, and thick coal beds; minor, but persistent limestone beds.

Mm — Meramec Series. Mainly St. Louis limestone: limestone and dolomite, sandstone locally predominant; locally contains chert.

Mk — Kinderhook Series. Mainly Gilmore City Limestone: light gray, fossiliferous limestone, commonly oolitic; Hampton Formation: limestone and dolomite; fossiliferous gray chert in lower portion; and Prospect Hill Formation: greenish-gray siltstone.

pEc — Crystalline Rocks. Includes granite, gneiss, and gneissoid granite; Rb-Sr dates 1.13-1.44 B.Y.

Figure 1. General geologic map of the Manson disturbed area with seismic refraction and borehole locations


34W
33
32
31
30
29W
92N
91
90
89
88N
LINE
1
2
3
4
5
6
7
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9
10
11
Mm
Mk
Kcd
Ku
pCc
Pch
Jfd
N
KMS
0 2 4 6 8 10
0 2 4 6
MILES
DES MOINES RIVER
FAULT
DODGE
FT. DODGE
MANSON
TWIN LAKES
6-5W
6-6W
6-9W
7-6W
7-5W
8-5W
8-3W
10-2W
6-13E
6-16E
6-14E
6-15E
7-7E
10-5E
10-3E
GEOLOGIC BOUNDARY
BOREHOLE
SEISMIC REFRACTION SPREAD
FT. DODGE
IOWA

the east, and Cretaceous shales and sandstones border on the north, west and south.

In 1953 an aeromagnetic survey was flown over the disturbed area (Henderson et al., 1963), and the resultant map was incorporated into a regional aeromagnetic map of central and southeast Iowa (Henderson and Vargo, 1965). Figure 2 is a reproduction of a portion of the 1953 survey. A magnetic high with a peak value of 4470 gammas roughly coincides with the outline of the crystalline rock as established from borehole information. Also prominent on the aeromagnetic map are several other anomalies. An elongate anomaly, trending NW-SE and having an intensity of 4605 gammas, is centered at a point 6 km northwest of Manson. Smaller anomalies (4115 and 3620 gammas) are located further from the center of the structure.

Holtzman (1970) recently completed a gravity survey of the disturbed region. He found that the regional effect of the mid-continent gravity high, which lies to the southeast of the Manson disturbed area, could be minimized with a trend surface technique that was successfully applied by Coons, Woollard and Hershey (1967) to similar gravity data. The Bouguer gravity map is shown on Figure 3. Holtzman found that a negative Bouguer anomaly in the interior of the disturbed area reflected, in part, low density brecciated sedimentary rock. Residual maps, calculated from as high as sixth degree trend surfaces, suggested to Holtzman that a peripheral graben bounds the eastern and southern edge of the Manson disturbed area, and that an inner arcuate fault paralleling the graben might also be located roughly equidistant between the center of the structure and the graben. Holtzman concluded that the gravity data from Manson had an appearance similar to gravity data from impact features. With this information at

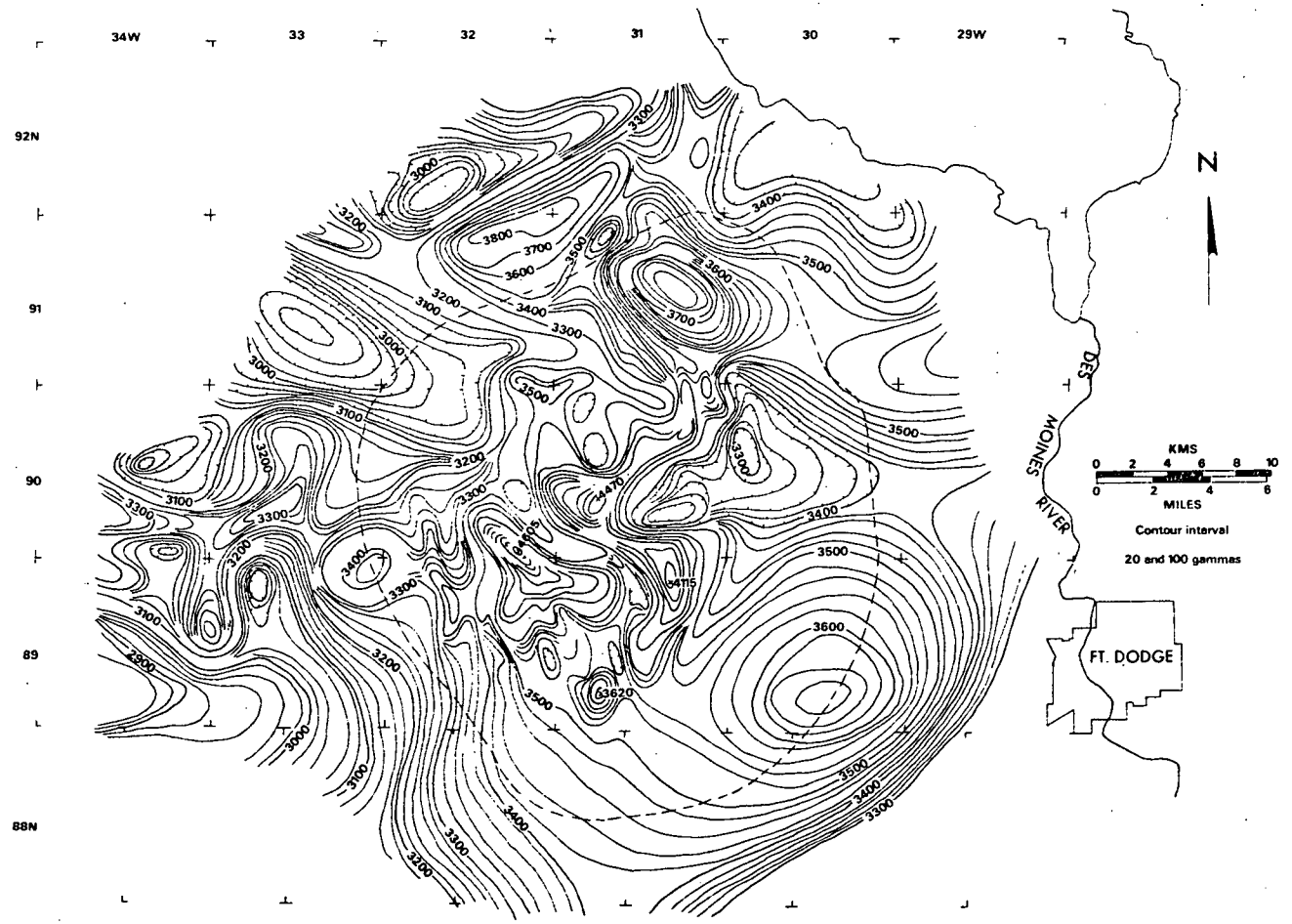


Figure 2. Aeromagnetic map of the Manson disturbed area

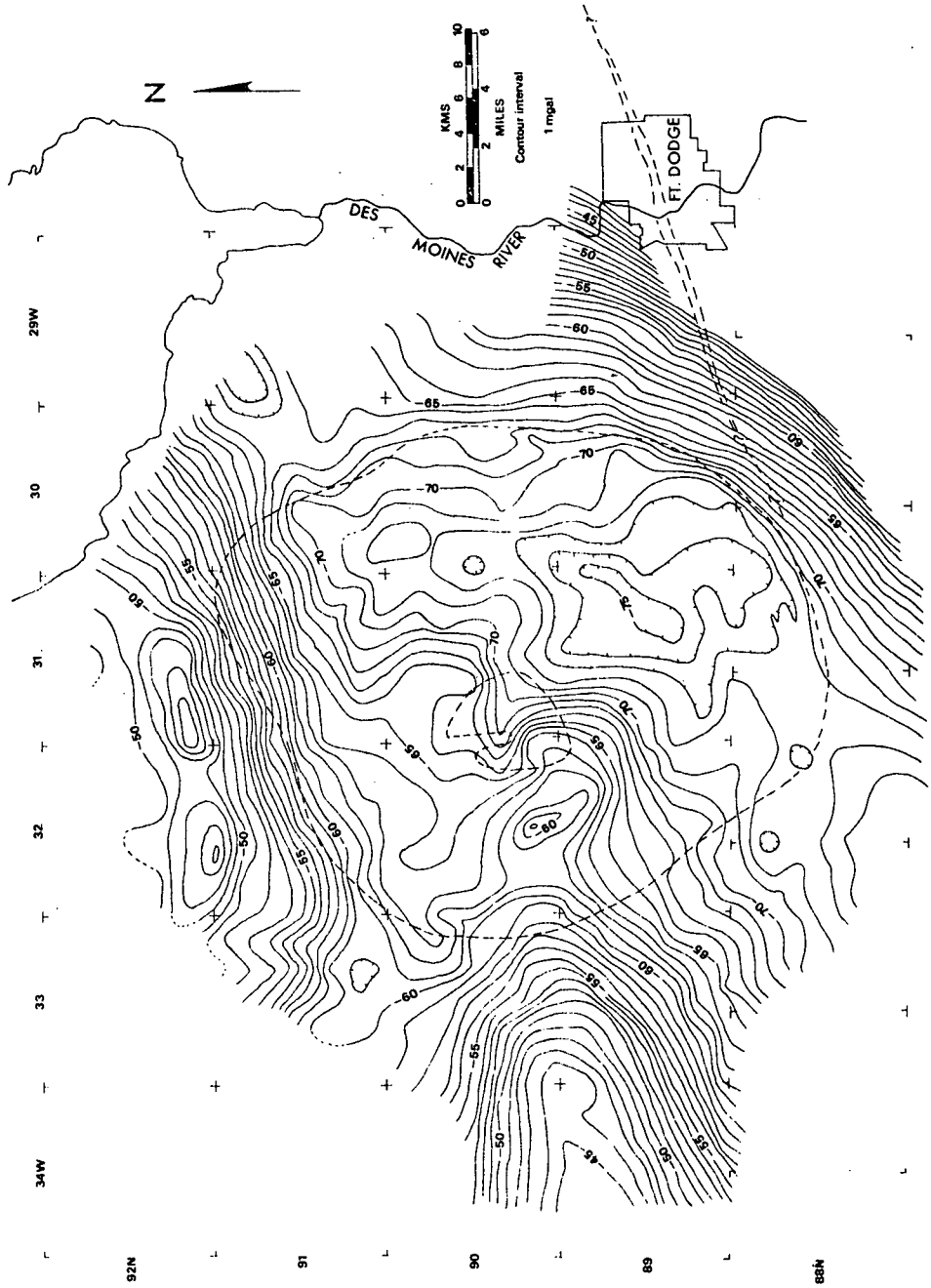


Figure 3. Bouguer gravity anomaly map of the Manson disturbed area

hand the seismic refraction study was initiated to investigate the nature and configuration of the upper portion of the Manson disturbed area.

COLLECTION OF THE SEISMIC DATA

A total of 107 seismic locations were occupied in the course of the investigation. The data were collected at sites loosely fitting a 3.2 x 3.2 km (2 x 2 mi.) grid to take advantage of the spacing of the county roads in the area. In many cases the seismic locations could not be exactly located at the grid points because it was necessary to avoid primary roads, farm drainage tiles, and electric hi-lines. For identification purposes, the seismic sites were numbered along profile lines east or west of a north-south trending center line. The center line was drawn 1.6 km (1 mi.) east of the west edge of the townships in range 31W, and 11 profile lines perpendicular to the center line were drawn with a spacing every 3.2 km (2 mi.). See Figure 1. An example of the identification of a station would be the seismic site marked 3-2E. It would be the second seismic site east of the center line on line 3. Where a seismic site lies between two profile lines, the number is marked on the figure. All grid locations were occupied except 8-4W and 10-4E.

The seismic data were collected with a truck-mounted, 24-channel, analog, seismic recorder. Single 14 Hz geophones were spaced 32 m apart along two 12-channel seismic lines. The low-pass filters on the amplifier banks were locked out so that the lower effective frequency was the 14 Hz of the geophone response. A high-pass filter of 120 Hz was used throughout the field operation. Amplifier gains were adjusted to enhance the first arrival breaks. Timing on the seismic records was marked by timing lines spaced 0.010 seconds apart. Two independent 100 Hz oscillators were simultaneously recorded and later checked against each other, giving a precision of the

lines better than 0.001 second in five seconds.

Two basic geophone spread configurations were employed. At 29 locations, most of which were along line 6, a split spread geometry was used, and a charge was detonated between them. This geometry produced two sets of 12 arrival times for each shot. In several locations on the eastern side of the study area in the undisturbed region, the two cables were strung perpendicular to each other; in this fashion the presence of a marker horizon below the bedrock surface might be used to detect the true bedrock dip. Also at 13 other sites, a second charge was detonated at one end of the seismic line so that true reversed profiling might be accomplished on 12 traces. At the 78 other sites, the spread geometry consisted of stringing the cables along a straight line, and charges were detonated at each end of the seismic line. This geometry produced 24 arrival times for each shot.

Five to six pound (about 25 NT) charges of 40% strength stick dynamite were buried at a depth of approximately 3 m in the glacial drift. The charge was placed beyond the farthest geophone at a distance in line with the seismic line of from 20 to 60 m and at a distance perpendicular to the line of from 3 to 15 m.

From the seismic recordings the first arrival times and time breaks were picked with a 3X power binocular microscope mounted with an eyepiece reticule. Three trained observers analyzed several records, and a comparison showed that the arrival times could be picked with a precision of ± 0.002 seconds. Because the mechanical alignment of the galvanometer and timing line optics in the recording oscillograph cannot be accurately assured below this level, the overall accuracy of the arrival time determinations was sufficiently low to warrant the use of one observer for the

bulk of the data.

The raw time values and distances were encoded and digitized. Each coding form was sight verified with the original seismic record, and the digital records were then mechanically verified with the coding forms. Several consistency checks were made on the digitized data. As an example, on 24 trace records the time shift and offset distances on traces 1-12 were compared to those of traces 13-24, and the difference in ground distance between trace 12 and 13 was checked against the standard takeout spacing.

Several corrections were applied to the time and distance data. A relay closure time of 0.001 second was added to each of the raw first arrival times. The horizontal distance from shot point to geophone for each arrival time was corrected for the distance the shot point was offset from the seismic line. The relief along the geophone spread was usually less than 5 m; only in one instance was the relief as great at 15 m. Because of this low relief, time corrections due to topography were not calculated.

A reconnaissance altimeter survey of many of the gravity stations established by Holtzman (1970) confirmed his published ± 1.2 m elevation accuracy. For each seismic location an altimeter loop was tied from the seismic shot points to the nearest gravity station. The horizontal datum plane for each shot, from which the depth calculations were made, was set at an elevation, E, such that $E = E_{shot} - C/2$, where E_{shot} is the elevation of the top of the shot hole and C is the depth to the center of charge. Therefore the accuracy of the elevations of the refracting horizons is within ± 2.5 m.

STATISTICAL METHOD

In the development of the refraction method, it was immediately realized that there was some reproduceable linear relationship between the source-to-receiver surface distance and the time of arrival of the seismic energy. The exact linear relationship, as inferred from geometrical optics, is often expressed as a line segment on a time-distance plot of the data, and the least squares method has been used to 'best fit' these line segments to the data.

Background

The statistical implications of the 'best fit' regression analysis have not, until recently, been appreciated. Steinhart and Meyer (1961) have reviewed the use of statistical uncertainty in seismic refraction, and they have concluded that the results of seismic refraction investigations have seldom been presented with estimates of uncertainty.

Because the error of measurement of distance is normally several orders of magnitude less than the error of the measurement of time, as a reasonable approximation, the error associated with the data may be considered to arise only from time. The measurement of distance is then taken to be without error. From this, all deviation from the time value predicted by this linear relationship are thought to arise from the error of the measurement of time only.

The principle of least squares states that the best estimate of the true time value at a distance is one that minimizes the sum of the squares of the time deviations from their estimates. The wave slowness and time intercept, determined with least squares regression, are used as

coefficients of the regression line of best fit. But by the admission of error, there cannot be considered an 'exact' functional relationship between time and distance, and so, the measurement of the coefficients is also subject to error.

No assumption as to the distribution of time deviations at a distance need be made. The principle of least squares does not imply the normal distribution of errors, only that the estimated parameters are unbiased and have minimum variance. Borcherdt and Healy (1968), in fact, suggest that for crustal refraction data, the distribution is not normal but is more peaked than normal. Regardless of whether or not it is, the central limit theory states that the distribution of the sum of a number of small errors will approach a normal distribution when the number of measurements grows large. If no assumption as to the distribution of time error at a particular distance is made, or no measure of the distribution is attempted, only the determination of the slope and time intercept parameters with the method of least squares may be made. Ultimately the time deviations at a distance are not as important as the ability of the model to fit the situation.

The method of least squares was first used to choose the line that 'best fit' the data over a range of distances chosen by the seismologist. Steinhart and Meyer (1961) presented the uncertainty of the wave slowness and time intercept for a line segment under the assumption that the error of time was normally distributed. Statistics may further be used to choose the 'best fit' combination of line segments for the entire range of distances over which the refraction data was collected. This problem has not been attacked in the field of seismology. The problem has only recently been investigated by those in the field of statistics.

Sprent (1961) considered the determination of the critical value of the independent variable when there were known to be two connected regression lines. Quandt (1958b) considered the estimation of parameters when the regression system obeys two separate regimes. He concluded that some previous determination must be made as to the number of regressions present in the system. Several difficulties were encountered by Quandt (1958a) when he attempted to establish statistical tests to determine this number. Robinson (1964) investigated the change-over value for polynomical regressions. Hudson (1966) presented the general solution to the estimation of parameters and change-over values for both linear and higher order polynonomial regressions.

Two computational schemes, following Hudson's guidelines, have been developed to estimate the number of regressions which will fit a set of data. Bellmen and Roth (1969) have used a dynamic programming approach, and McGee and Carleton (1970) have applied the method of cluster analysis.

Recently seismologists have approached the determination of join point (or critical distance) by using selective data rejection. Iyer et al. (1969) used an iterative scheme which rejects data with high time deviations. Segments of the data are selected to be fitted, and at each successive iteration, data points which deviate from the calculated regression line by some specified level are rejected from subsequent calculations. The curve fitting method formulated by Kaila and Narain (1970) differs from the method used by Iyer et al. in that weights are assigned to each data point. The weights are assigned on the basis of the normal probability function for the time error. Then the regression for each line segment is calculated using all the data points weighted for that particular regression. Because of slow

convergence, however, Kaila and Narain found it necessary to revert to data rejection in the vicinity of the critical distance at higher iteration levels.

A method is presented here that makes no supposition as to the quality of the data and takes advantage of several characteristics of seismic refraction data. Because shallow seismic refraction data may be modeled by a series of line segments, many aspects of the general solution of the multisegmented regression problem, as presented by Hudson, may be omitted. The parameters for each line segment, wave slowness and time intercept, are calculated, and under the assumption that the distribution of time values at a distance are normally and independently distributed, the standard deviations of these parameters are found. An algorithm, utilizing Hudson's 2 segment regression analysis, is presented here which extends his method to fit 3 or more regression lines to seismic refraction data. The statistical uncertainties of the parameters for each line segment are used to estimate the uncertainty in the determination of depth.

Unconstrained Join Type

Let the n time and distance pairs form a series in ascending order, and let there be only one time value for each distance, as $[(t_1, d_1),\ldots, (t_n,d_n)]$. Let the t_i be the dependent variable, and let the d_i be the independent variable. Now the set of time-distance pairs may be partitioned into m sections, where $j = 1,\ldots,m$, and a simple linear regression equation may be applied to each section of data. Let one of these sections be composed of a series of s pairs of values $[(t_g,d_g),\ldots,(t_h,d_h)]$. Here $s = h-g+1$. The model equation for the jth section of the data is

$$t_i = \alpha_j + \beta_j d_i + \varepsilon_{ij} \qquad i = g,\ldots,h \tag{1}$$

The line parameters α_j and β_j are the time-intercept and wave slowness, respectively. The random variables ε_{ij} are normally and independently distributed with a mean of zero and variance σ_j^2. Let the sample estimate for the time intercepts be $E(\alpha_j) = a_j$ and the sample estimate for the wave slowness by $E(\beta_j) = b_j$. The estimator of the variance is $E(\sigma_j^2) = s_j^2$. The estimator of each ε_{ij} is zero, so the m regression equations are determined by calculating $(a_1, b_1,\ldots, a_m, b_m)$. Statistical weights could easily be included here, but for the present discussion they have been omitted.

The line segments should be continuously connected at points D_j which are positioned between the end elements of the partitions. There must also be at least 2 points of data on each segment, then the time estimates are

$$\begin{aligned}
\hat{t}_i &= a_1 + b_1 d_i && \text{for} \quad 0<d_1<d_2<D_1 \\
\hat{t}_i &= a_2 + b_2 d_i && \text{for} \quad D_1<d_i<d_{i+1}<D_2 \\
&\cdot \quad \cdot \quad \cdot && \cdot \quad \cdot \quad \cdot \\
\hat{t}_i &= a_j + b_j d_i && \text{for} \quad D_{j-1}<d_i<d_{i+1}<D_j \\
&\cdot \quad \cdot \quad \cdot && \cdot \quad \cdot \quad \cdot \\
\hat{t}_i &= a_m + b_m d_i && \text{for} \quad D_m<d_{n-1}<d_n
\end{aligned} \tag{2}$$

For the jth section of the time-distance series, the deviation of each time value, t_i, from it's estimation, $\hat{t}_i$, is

$$e_{ij} = t_i - a_j - b_j d_i \tag{3}$$

Temporarily let the jth section of the data be considered separately, as $[(t_i,d_i)\ i = g,\ldots,h]$. The residual sum of squares from Equation 3 is

$$RSS_j = \sum_{k=1}^{s} e_k^2 \tag{4}$$

where the length of the jth section is s. The following sums must also be calculated.

$$Sd_j = \sum_{k=1}^{s} d_k \qquad St_j = \sum_{k=1}^{s} t_k$$

$$Sdd_j = \sum_{k=1}^{s} d_k^2 \qquad Stt_j = \sum_{k=1}^{s} t_k^2 \tag{5}$$

$$Sdt_j = \sum_{k=1}^{s} d_k t_k$$

Steinhart and Meyer (1961, Eq. 1 and 2) have presented the solution of the line parameters, a_j and b_j. They are

$$a_j = \frac{Sdd_j St_j - Sd_j Sdt_j}{sSdd_j - (Sd_j)^2} \tag{6}$$

and

$$b_j = \frac{sSdt_j - Sd_j St_j}{sSdd_j - (Sd_j)^2} \tag{7}$$

Now the variance of the wave slowness is

$$\sigma_j^2(b) = RSS_j/[(s-2)SSd_j] \tag{8}$$

where

$$SSd_j = \sum_{k=1}^{s} (d_k - \langle d \rangle)^2$$

The angle brackets indicate average. Steinhart and Meyer (Eq. 13) have shown that the variance of the mean value of the regression line at any distance, d, is

$$\sigma_j^2(d) = \frac{RSS_j}{s-2}\left[\frac{1}{s} + \frac{(d-\langle d\rangle)^2}{SSd_j}\right] \tag{9}$$

At the time intercept, d = 0, the variance is

$$\sigma_j^2(a) = (RSS_j/s-2)\ (1/s + \langle d\rangle^2/SSd_j) \tag{10}$$

Substituting Equation 8 into Equation 10 and using the identity

$$SSd_j = \sum_{k=1}^{s} d_k^2 - (\sum_{k=1}^{s} d_k)^2/s, \qquad \text{then}$$

$$\sigma_j^2(a) = [\sigma_j^2(b)\ Sdd_j]/s \tag{11}$$

Equations 4, 8, and 11 may be applied to any section of the data.

The join points $D_1,\ldots,D_m$ represented on the model given by Equation 2 are not constrained to a particular value but may be located anywhere between the uppermost distance d_i of one segment and the lowermost distance d_{i+1} of the next segment. Let this type of join, where the data are separated, be called s-join. The total residual sum of squares is

$$RSS_T = \sum_{j=1}^{m} RSS_j \tag{12}$$

Consider a 2-segment regression (r = 2). The computation necessary to find the best statistical fit of 2 lines fitted to the data proceeds by passing a line through $[(t_1,d_1), (t_2,d_2)]$ without error and regressing a line through $[(t_3,d_3),\ldots, (t_n,d_n)]$. If the intersection of these 2 lines

is between d_2 and d_3, then these are a valid pair of regression lines because the regression lines join where the data have been separated. The total regression sum of squares $RSS_T = RSS_1 + RSS_2$ for this combination of lines. If instead the intersection is not between d_2 and d_3 this combination of lines is rejected as a possible 2-segment regression candidate. Next the first (3) pairs and the last (n-3) pairs are fitted. Again, if the intersection is found to be between d_3 and d_4 we have an acceptable candidate. The procedure is continued, testing each possible combination of data, to the last combination which is $[(t_1,d_1),\ldots(t_{n-2},d_{n-2})]$ and $[(t_{n-1},d_{n-1}), (t_n,d_n)]$. Of these candidates the best statistical fit is the candidate which has the smallest RSS_T. For r = 2 the total number of combinations that must be tested is (n-2).

Hudson (1966, p. 1104) has shown that for those candidates that have not been rejected, the estimates of the regression parameters are valid. After the best statistical fit has been found, the coefficients (a_1,a_2,b_1, b_2) are calculated from Equations 6 and 7 and their variances [$\sigma_1^2(a)$,$\sigma_1^2(b)$, $\sigma_2^2(a)$,$\sigma_2^2(b)$] are calculated from Equations 8 and 11.

If r>2 then the same basic computational plan would follow, and if all the join points were s-join, the overall least squares solution would be the best fit of r line segments found as local least squares solutions from all possible combinations of the data points.

Constrained Join Type

Although the data have been searched for all s-join candidates, the set of all possible linear regression fits is still not complete. The model regression equations in Equation 2 must also be tested for the case where

the join points are located on time ordinate fixed at the data point distances. This type of join is here called c-join because, for each pair of regression lines, one data point is common to the solution of each line. The system of model equations, with the constraint that the lines join somewhere on ordinate corresponding to d_i, has been solved with the use of a Lagrange multiplier.

As before, the details of the procedure will be presented for r = 2. The appropriate linear constraint is that the lines join at some D.

$$a_1 + b_1 D = a_2 + b_2 D \tag{13}$$

Again let the time-distance pairs be $[(t_1,d_1),\ldots,(t_n,d_n)]$, and let the c-join be located at $D = d_\gamma$. Before the calculations of the residual sum of squares and the coefficients of the line segments are made, a number of sums must be found.

$$\begin{aligned} Sd_1 &= \sum_{k=1}^{\gamma} d_k & Sd_2 &= \sum_{k=\gamma+1}^{n} d_k \\ Sdd_1 &= \sum_{k=1}^{\gamma} d_k^2 & Sdd_2 &= \sum_{k=\gamma+1}^{n} d_k^2 \end{aligned} \tag{14}$$

Following Hudson (p. 1126) the terms ϕ and ψ are

$$\phi = a_1 - a_2 + (b_1 - b_2)D \tag{15}$$

and

$$\psi = \frac{Sdd_1 - 2DSd_1 + \gamma D^2}{\omega_1} + \frac{Sdd_2 - 2DSd_2 + (n-\gamma)D^2}{\omega_2} \tag{16}$$

where

$$\omega_1 = \gamma Sdd_1 - (Sd_1)^2$$

$$\omega_2 = (n-\gamma)\, Sdd_2 - (Sd_2)^2$$

Let RSS_T^* be the total residual sum of squares for two line segments connected with a c-join, and from before, let RSS_1 and RSS_2 be the unconstrained residual sum of squares, then

$$RSS_T^* = RSS_1 + RSS_2 + \phi^2/\psi \tag{17}$$

Finally, let $(a_1^*, b_1^*, a_2^*, b_2^*)$ be the coefficients of the line segments joined by c-join, and let (a_1, b_1, a_2, b_2) be the unconstrained coefficients as determined from before. The new coefficients are:

$$\begin{aligned} a_1^* &= a_1 - (\phi/\psi\omega_1)[Sdd_1 - DSd_1] \\ b_1^* &= b_1 + (\phi/\psi\omega_1)[Sd_1 - \gamma D] \\ a_2^* &= a_2 + (\phi/\psi\omega_2)[Sdd_2 - DSd_2] \\ b_2^* &= b_2 - (\phi/\psi\omega_2)[Sd_2 - (n-\gamma)D] \end{aligned} \tag{18}$$

A line segment with these coefficients does not pass through its point of mean time and distance.

The computational scheme could proceed from this point by testing the data from $\gamma = 2,\ldots,n-1$ and finding the minimum RSS_T^*, as had been done for the s-join candidates, but several short cuts may be taken. The complete set of combinations with s-joins may be used to reduce the number of combinations with c-joins. Hudson (1966, Theorem 3) has investigated the case for $r = 2$. Each of these combinations may be rejected and not calculated if they pass one of the following 5 tests:

Test 1. At each s-join candidate, say at $i = \xi$, the c-join candidate need not be calculated at $\gamma = \xi, \xi + 1$.

Test 2. If at any i, $RSS_1 + RSS_2 > \text{Min } RSS_T$ (Minimum RSS_T), even if the join point was not between the end members of the 2 sets of data, the c-join candidate need not be calculated at $\gamma = i, i+1$.

At this point the static tests for the elimination of possible c-join candidates have been exhausted, and the calculation of RSS_T^* must be made at c-join locations from $\gamma = 3,\ldots,n-2$. After each calculation of RSS_T^* through, the next 2 dynamic tests should be made to further reduce the number of possible c-join locations.

Test 3. Let a RSS_T^* be calculated and from these let a Min RSS_T^* be found. If at any i, $RSS_1 + RSS_2 > \text{Min } RSS_T^*$, even if the join was not located between the end members, then the c-join candidate need not be calculated for $\gamma = i, i+1$.

Test 4. Let a RSS_T^* be calculated at some γ and that the mean distances for the left and right line segments are found as $\langle d_l \rangle = Sd_1/\gamma$ and $\langle d_r \rangle = Sd_2/(n-\gamma)$. If the join point D from the s-join calculations is $\langle d_l \rangle < D < \langle d_r \rangle$ then RSS_T^* need not be calculated at any γ such that $\langle d_l \rangle \leq d_\gamma \leq \langle d_r \rangle$.

Finally a test at $\gamma=2$ and $\gamma = n-1$ is presented.

Test 5. At $\gamma = 2$ and $\gamma = n-1$, regression lines may be fitted and residual sum of squares, RSS_2 and RSS_{n-1}, may be calculated on the data sets $[(t_2,d_2),\ldots,(t_n,d_n)]$ and $[(t_1,d_1),\ldots,(t_{n-1},d_{n-1})]$, respectively. Then a second regression line may be drawn through (t_1,d_1) and the intersection of the first regression line with d_2 (without error). Similarly a second regression may be drawn through (t_n,d_n) and the intersection of the second regression line with d_{n-1}. If $RSS_2 > \text{Min } RSS_T$, then RSS_T^* at $\gamma = 2$ need not be calculated. And if $RSS_{n-1} > \text{Min } RSS_T$, then RSS_T^* at $\gamma = n-1$

need not be calculated.

At this point for r = 2, all (n-3) calculations have been completed for the s-join candidates, and of these, the 2-segment candidate with Min RSS_T has been retained. There are (n-2) possible candidates with c-join, and hopefully, many of these have been rejected by the tests. From those that were calculated, the c-join candidate with Min RSS_T^* has also been retained. There are then a total of (2n-5) possibilities for r = 2. The final least squares solution is either the minimum s-join candidate or the minimum c-join candidate, whichever has the smaller residual sum of squares.

As before when r>2, the same basic computational scheme would be followed for the search for c-join candidates. It is not surprising that the total number of possible candidates grows rapidly for problems involving many segment models or large sets of data, although many may be rejected by the tests. When n = 24 and r = 2 the number of possible candidates is 43, and when n = 24 and r = 3 the total is 841. For greater n or r the number of possible candidates is extremely large. For example, when n = 100 and r = 6 the total number of possible candidates is near 1.9×10^9.

An Algorithm

Seismic refraction measurements provide data which are sufficiently linear to allow certain liberties in the curve-fitting process. The first of these is to suppose that some reasonable estimate may be made of the maximum number of permissible line segments present in the investigation. The estimate does not need to be known, but that only some upper limiting number may be chosen.

This decision though must be tempered with the inherent tendency for

higher-numbered regression systems to fit the data better than lower-numbered systems. To be guided in this direction, one must choose a value high enough so that no line segment is missed, but not so high that the curve is 'over-fitted' and the solution is physically impossible. This would be the case when the fitted solution contains velocity reversals. The second criterion then is that starting from the origin, the velocity of each line segment must monotonically increase. In practice, this is a valuable guide to the selection of the best least squares solution.

In a 2 segment linear regression model, the residual sum of squares may be plotted against join point distance. In the limit as $n \to \infty$ the residual sum of squares form a curve, and the minimum point on this curve corresponds to the distance at which the two regression lines intersect. For a 3 segment linear regression as $n \to \infty$ the best least squares solution is the minimum point on a surface of residual sum of squares. This surface is formed on 2 axes which correspond to the D_1 and D_2 join point values and a third axis which is the residual sum of squares. The anology is similar for more line segments. If this r-dimension surface is reasonably simple, the minimum point may be reached by making a series of traverses across the surface, each of which is accomplished by varying the values of only one join point. Each traverse is equivalent to the 2 segment regression problem considered earlier.

To evaluate an $r>2$ regression system, a number of points along the data must be chosen to effectively lower the number of possible candidates to be searched at any one time to $(2n-5)$. These test points should be chosen as near as possible to the suspected join points, but it is preferable to choose them slightly below the suspected join point. In this way no

data points from the next higher line segment enter the calculation. The procedure advances by searching for 2 segment joins in successive sets of data. Each successive set of data has an upper bound which has been set by a test point and a lower bound which has been set at the join point found in a preceding join search.

As an example, suppose there are 3 suspected line segments to be fitted to time-distance pairs where $[(t_i,d_i)\ i = 1,\ldots,n]$. There must be 1 test point chosen at or slightly below the suspected join between the second and third line segments, say at $i = T$. The pairs $[(t_i,d_i)\ i = 1,\ldots,T]$ are first searched for the join between the first and second line segment. Let the solution for this set of data be an s-join candidate between $i = \zeta$ and $i = \zeta + 1$, as $d_\zeta < D_1 < d_{\zeta+1}$. The data $[(t_i,d_i)\ i = \zeta+1,\ldots,n]$ is now searched for the join between the second and third line segment. Suppose the solution here is a c-join candidate such that $d_\eta < D_2 < d_{\eta+1}$. The final solution is composed of 3 line segments joined at D_1 and D_2, and the total residual sum of squares is equal to the sum of the residual sum of squares from each line segment.

If the solution in the first search had been a c-join candidate the data in the second search would be scanned from $[(t_i,d_i)\ i = \zeta,\ldots,n]$. Or if a join point had not been found in the first search the data in the second search would have been scanned from $[(t_i,d_i)\ i = 1,\ldots,n]$, and only a 2 segment solution would have been found. The computation for each search would proceed accordingly.

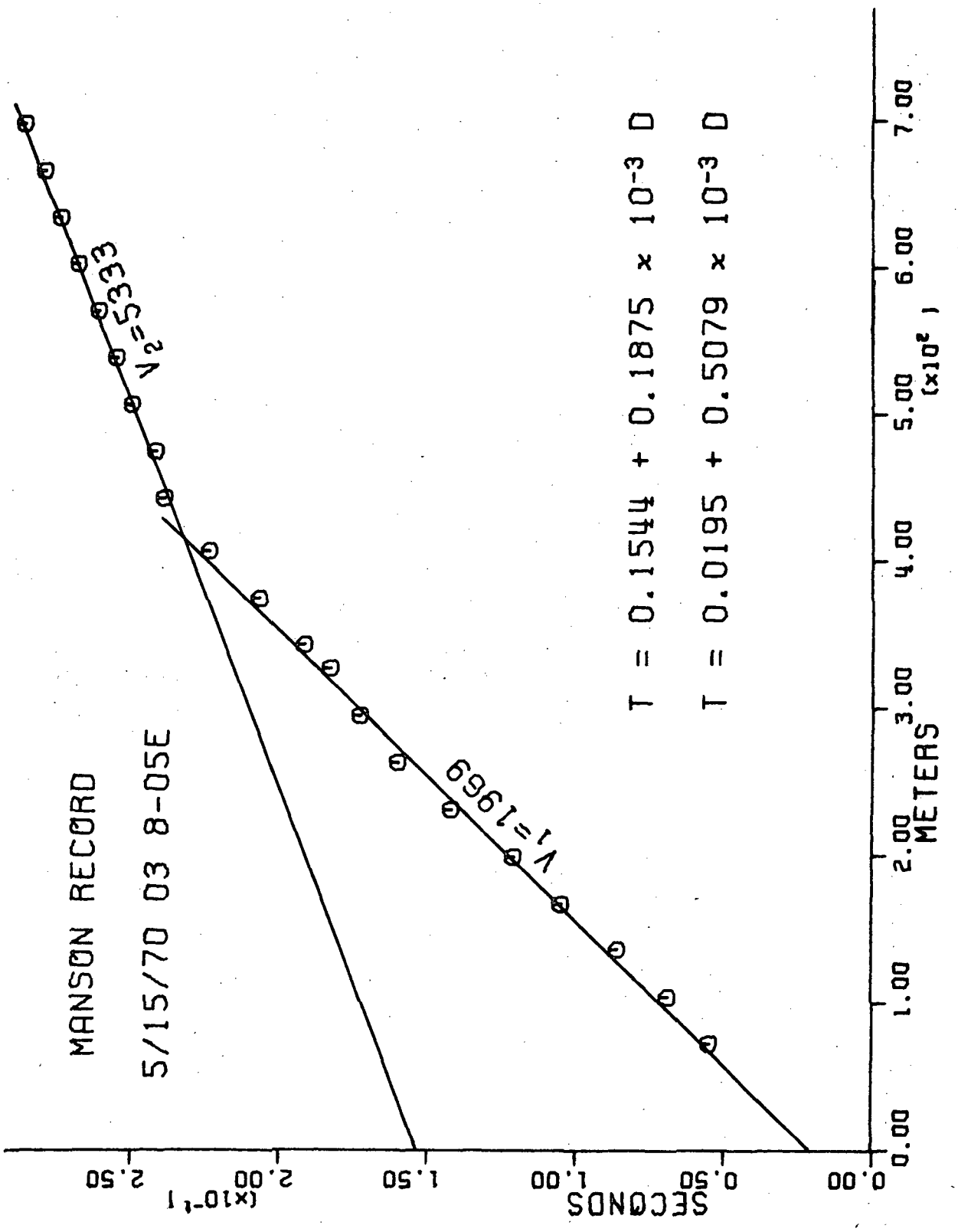
For an n-segment linear regression a vector of test points must be chosen as $(T_1,\ldots,T_{n-2})$ and the algorithm proceeds with (n-1) levels of 2 segment join search. Because the number of possible candidates to be

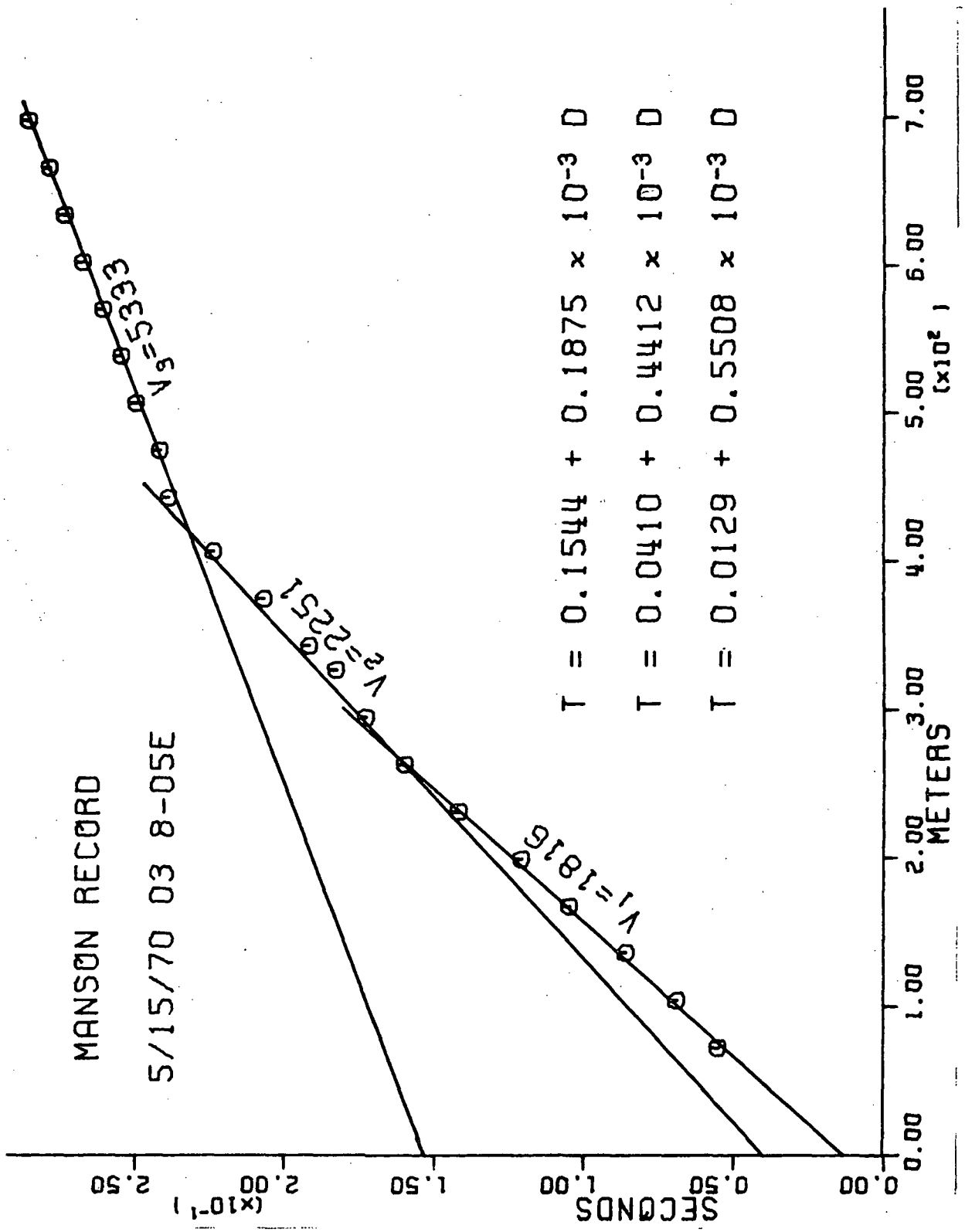
searched at each level of join point selection is restricted to the search for a 2 segment fit, the actual computation is greatly reduced.

Examples

An example of a set of data from a typical shallow refraction shot is presented in Figure 4. Here $[(t_i,d_i)\ i = 1,\ldots,21]$. A linear regression with all 21 data points yields a line with a velocity of 2682 m/sec and a residual sum of squares of 0.47325×10^{-2} sec^2. The best s-join candidate from a 2 segment regression analysis is superimposed on the data in the figure. The join point is located at a distance of 421 m and the total residual sum of squares is 0.13222×10^{-3} sec^2. All possible c-join combinations were eliminated by the 5 tests, so the s-join candidate is the best overall 2 segment regression fit.

The best 3 segment regression is presented in Figure 5. A test point at d_8 was chosen, and a 2 segment analysis of $[(t_i,d_i)\ i = 1,\ldots,8]$ revealed that the best fit was a c-join candidate joined on the time ordinate corresponding to d_2. The second level of data then consisted of $[(t_i,d_i)\ i = 2,\ldots,21]$. The best statistical fit here was an s-join candidate with a join between d_{12} and d_{13}. To be sure the test point was correctly chosen, the set $[(t_i,d_i)\ i = 1,\ldots,12]$ was analyzed. This time the best fit was a c-join candidate with join point at d_7. This third scan of the data protected the overall solution from an inaccurate choice of test point. Here the test point of d_8 was too low and should have perhaps been d_{12}. The overall 3 segment regression fit has a total residual sum of squares of 0.44170×10^{-4} sec^2. A borehole adjacent to the shot showed that the 3 segment fit compared favorably in velocity and depth with glacial drift,


MANSON RECORD
5/15/70 03 8-05E
V_1=1969
V_2=5333
T = 0.1544 + 0.1875 x 10^{-3} D
T = 0.0195 + 0.5079 x 10^{-3} D
SECONDS
METERS
(x10^2)
(x10^{-1})
0.00
0.50
1.00
1.50
2.00
2.50
3.00
4.00
5.00
6.00
7.00


MANSON RECORD
5/15/70 03 8-05E
V_3=5333
V_2=2251
V_1=1819
T = 0.1544 + 0.1875 x 10^{-3} D
T = 0.0410 + 0.4412 x 10^{-3} D
T = 0.0129 + 0.5508 x 10^{-3} D
SECONDS
(x10^{-1})
METERS
(x10^{2})
0.00
0.50
1.00
1.50
2.00
2.50
3.00
4.00
5.00
6.00
7.00

shale, and limestone.

These two examples show that two valid regression models may be fitted to the shallow seismic refraction data. At other times only a 2 segment regression model may be acceptable. This situation might arise if the 3 segment regression model contained a velocity reversal between two adjoining line segments. In the first case additional information must be used to discriminate between the two cases. This information is acquired from a knowledge of the geological conditions in the area.

ANALYSIS OF NEAR-SURFACE VELOCITIES

Multi-layered earth models of any dimension, whether they are based on bedrock structures measured in meters or crustal structures measured in kilometers, may be developed from seismic refraction data only when velocity measurement of the uppermost layer has been made. On many scales the earth may be approximated by a model bounded by a series of planar surfaces. The compressional wave velocity of this uppermost earth material may, in theory, be calculated from the reciprocal of the slope of a line segment that passes through the origin of the time-distance graph of the data.

Selection of Data

Many seismic refraction investigations are hampered by a lack of knowledge of the near-surface geologic environment near the shot and under each geophone. In shallow refraction investigations the plot of some records reveals that perhaps one or two data points nearest the shot fall markedly below the first fitted line segment. They may represent random scatter about the first line segment, or they may represent travel times from direct waves passing through a shallower layer. In either case, the first layer is not detected because there is a lack of data from each record due to the fixed sampling intervals along the distance axis at which time was measured. By combining the data from all records the lack of data at near distances on individual records is overcome, and the maximum times in each interval may be used to estimate the minimum velocity of the first detected layer of a generalized model for the entire study area.

In the Manson area, line segments drawn through the first several time distance points on each shot record did not pass through the origin, so the

entire set of first arrival time-distances pairs was plotted. The field of points is shown on the scatter diagram (Figure 6). The locus of points near the time axis clearly demonstrates that the first layer line segments do not pass through the origin. Although the lower border of data scatter is not well marked, the upper border is strikingly smooth.

In an endeavor to better understand the nature of this upper border, the distances of all time-distances pairs were partitioned with a class interval of 5 m, as 0-5 m, 5-10 m, etc., and the corresponding time values in each interval were scanned for the maximum time in each interval. A plot of these maximum time values was found to be quite regular even to a distance of 500 m from the origin. The maximum time at this distance was 0.275 sec. The scatter of the maximum times beyond 500 m is probably due to the smaller sample sizes in each interval at these farther distances.

It was found that the locations of these time-distance pairs which had a maximum time were uniformly scattered over the entire area. The original records, from which these time values were taken, were then reexamined to see if these times might have been taken from traces where the refracted wave was obsured or where the arrival time was incorrectly measured. Because of the large size of the shot charges, first arrival onsets were quite strong even at the farthest recording distances. It is unlikely that the maximum time values could be attributed to poor record quality or to second arrivals from later phases, particularly for distances less than 500 m.

Statistical Analysis

Once that it had been established that the maximum time values were valid refraction arrival times, these maximum time-distance pairs were

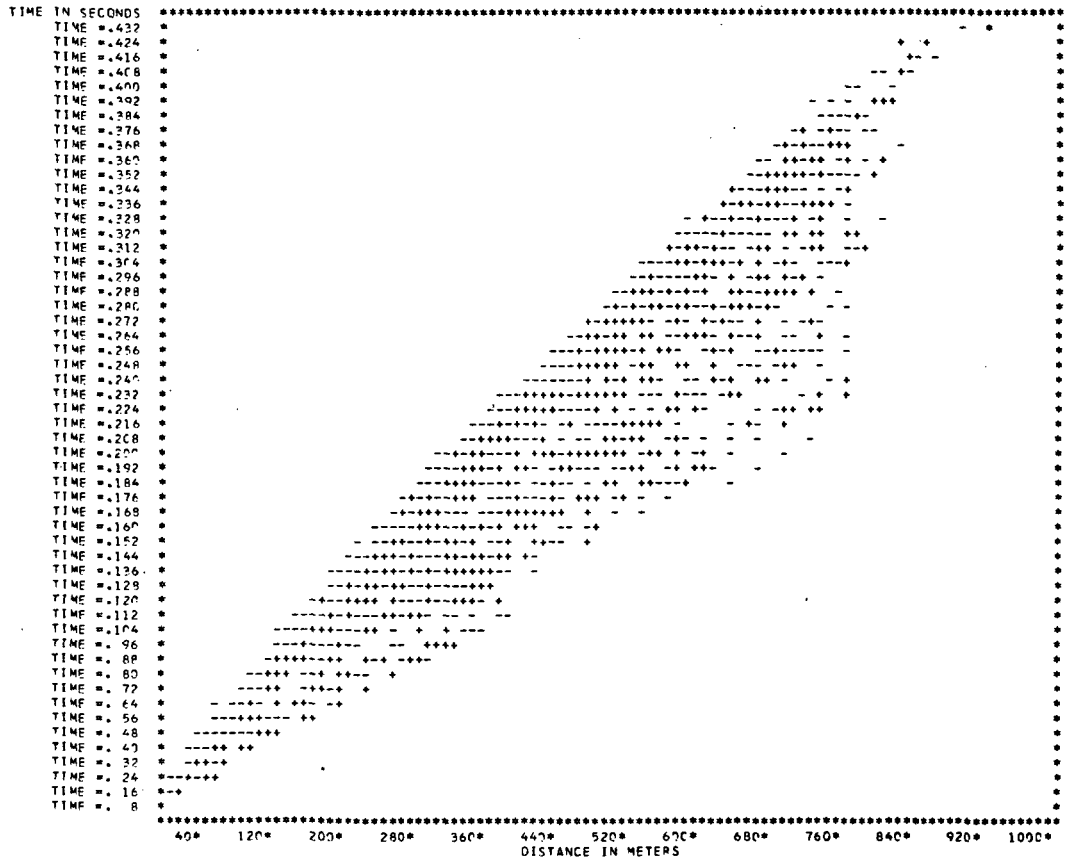


Figure 6. Scatter diagram of all time-distance measurements

statistically fitted with line segments as if they were data from a single seismic record as described previously. By this method a portion of the set of all possible combinations of linearly regressed line segments was searched for the one fit which had the minimum residual sum of squares. The first 100 time-distance pairs were fitted with 1, 2, 3, and 4 line segments. The results are tabulated in Table 1. The first line segment from the 4 segment model best fits the maximum time values at the nearest distances. Models with a greater number of line segments suffered from velocity reversals in the first and second line segments, probably due to random deviation of the maximum time values and accentuated by the fewer number of points represented on the first fitted line segment.

Table 1. Statistical fits of first 100 maximum times

Model	Points	Velocity[a]	Residual sum of squares[b]
1 layer	100	1919	0.24530
2 layer	53	1771	
	47	2138	0.12464
3 layer	12	1462	
	50	1844	
	38	2203	0.10938
4 layer	7	1258	
	20	1749	
	40	1863	
	35	2200	0.10772

[a] In m/sec.

[b] In 10^{-2} sec^2.

From the analysis of the data collected at Manson, it is inferred that the minimum velocity of the first detected layer is near 1250 m/sec. A line with a slope corresponding to a layer with velocity of 1250 m/sec may not be made to pass through the origin and still remain within the data scatter. Therefore, the first layer has not been detected, for if it were, the line with a velocity of 1250 m/sec would pass through the origin.

Several investigators (Hobson, 1969; Staub, 1969) have suggested that at very short distances, the first detected energy arrives at the geophone from a ground-coupled air wave when the velocity of the surficial layer is less than the velocity of sound. Although the detection of this air wave was not an object of the present study, all depth calculations in this study include a first line segment which is attributed to a ground-coupled air wave. To avoid confusion the first statistically fitted layer has been called the first layer. The depth to the '"air" - first layer' interface has been found to range from 1 to 2.5 m.

ANALYSIS OF DEEPER VELOCITIES

The velocity determinations from the statistical line-fitting analysis of individual seismic records may be combined to indicate the variability of the velocities of various seismic refractors. The utility of the shallow seismic refraction technique rests not on the analysis of each seismic record, but on how well seismic refractors may be traced below many seismic stations, and how well the depths to the seismic refractors compare with lithologic changes at depth, as indicated from borehole data.

Statistical Analysis

Nineteen seismic spreads were placed as near as possible to boreholes where adequate well logs were available, so that a correlation between seismic refractor layers and lithologic units could be established. These records were plotted and analyzed with the statistical line fitting method mentioned previously. Test points were chosen as near as possible to the join between suspected second and third line segments. Each seismic record was fitted with 2 and 3 line segments.

The uniqueness of the statistical fit depends on the choice of test points. In an attempt to improve the 3 line segment fits, each record was analyzed with a variety of test points. The final statistical fit of the data for the 3-layer model does not seem to be very sensitive to the location of the test point. Experience on artificially generated 3-layer data has shown that if the test point is chosen between the middle of the second line segment and the join point between the second and third lines segments, a correct solution is found.

The statistical fitting method was also measured by how well it

eliminated the number of calculations of 2 segment regressions. For any particular set of n time-distance pairs, the search for the best fit of 2 line segments joined between data points (s-join) was necessarily applied to all (n-3) combination of data points. The robustness of the 5 tests used to eliminate the calculation of the regressions of 2 line segments connected with a c-join was measured by the percentage of calculations that had been eliminated. Of the (n-2) possible combinations to be tested for c-join, approximately 90% could be eliminated from consideration for any one of the 5 tests. The rejection rate for each of the 5 tests was approximately: test 1 - 0%, test 2 - 50%, test 3 - 40%, test 4 - 2%, and test 5 - 8%.

After the validity of the statistical method had been tested, rough correlations were made between model velocities and the borehole lithologies. Test points based on those correlations were then picked on plots of seismic profiles without well log control. These seismic records were then analyzed for both 2 and 3 line segments. In no instance could 4 line segments be satisfactorily fitted to the data because no more than 24 data points were available for each statistical analysis. After the best statistical fits had been made it was necessary to decide, for each record, whether the 2 layer or 3 layer model best fit the geologic conditions.

Velocity Distributions

The final results are plotted on Figure 7. The frequency polygons for the first layer velocities, second layer velocities, and a combination of first, second, and third layer velocities are plotted with the distance axis divided by a class interval of 50 m/sec. The results are plotted at the

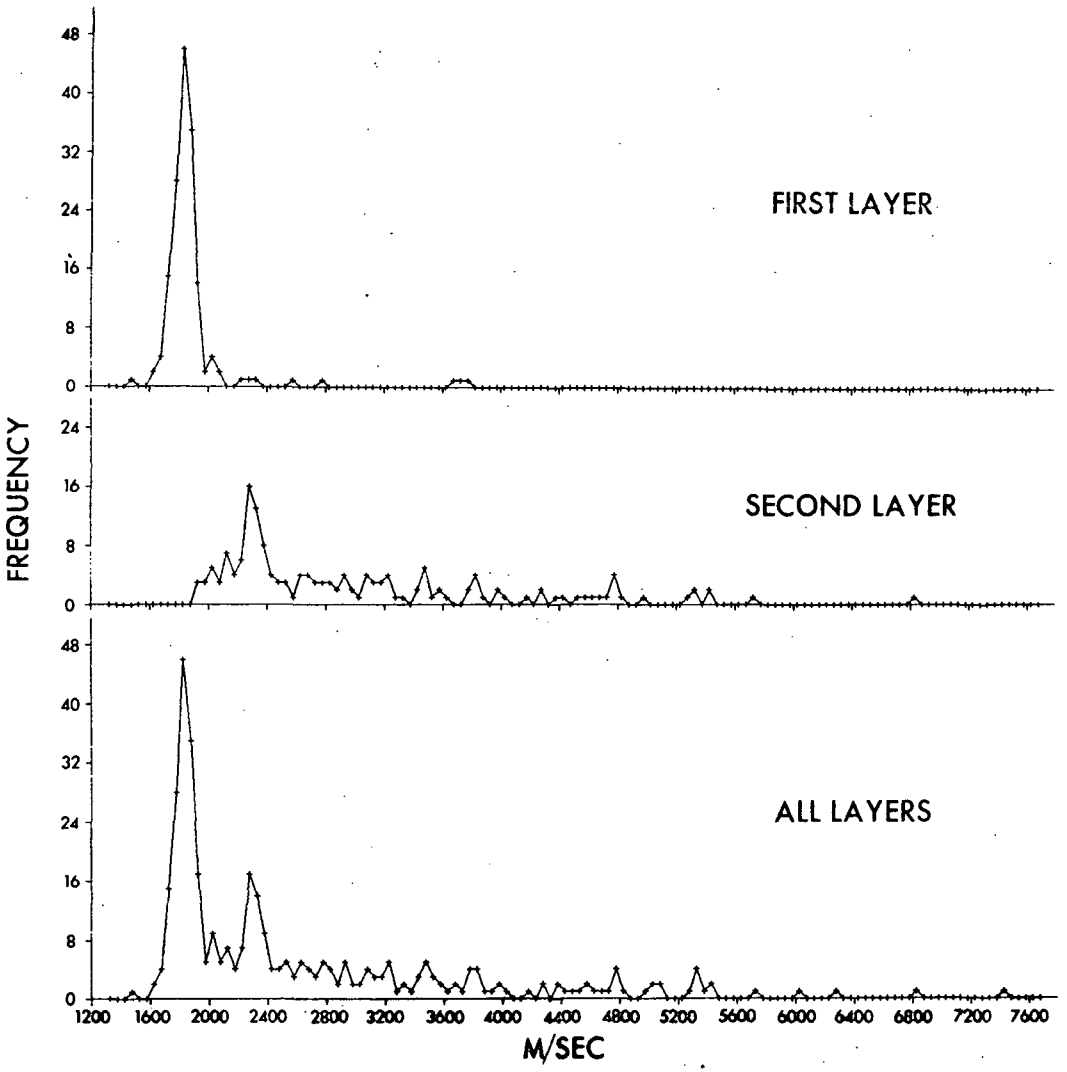


Figure 7. Distribution of seismic velocities by layer

mid-point between the ends of the class intervals.

The narrow distribution of first layer velocities is attributed to glacial drift. Much of the velocity of the glacial drift is between 1800 and 1850 m/sec. It is significant to note that the distribution about the main peak is symmetric. This indicates that there is only one type of material measured in the distribution. The presence of a second material with a slightly higher velocity, for example the disturbed shale, would skew the distribution to the right. The presence of a second material with a slightly lower velocity would accordingly skew the distribution to the left. The shape of the main first layer distribution compares favorably with the velocity distribution of other glacial drifts as presented by Hobson (1969), Lennox and Carlson (1969) and Staub (1969). The several first layer velocities above 3600 m/sec were from seismic stations on the eastern portion of the area in the undisturbed area where the glacial drift was known to be quite thin.

The frequency of velocities between 2250 and 2300 m/sec on the second graph is the highest on the plot. The distribution is not nearly as sharp as the glacial drift distribution in the first layer but it represents a portion of the disturbed shale velocity distribution. Superimposed on this graph are the velocities of the undisturbed bedrock layers which include undisturbed limestones and dolomites ranging in velocity from 3000 to 7000 m/sec and the velocity of the crystalline bedrock which ranges from 3000 to 5000 m/sec.

As an example of the irregularity of the bedrock velocity, at station 6-13E, situated parallel to the rim of the Manson disturbed area, a 24 trace record shooting north yielded first, second, and third layer velocities of

1761, 3982, and 5360 m/sec. The second shot detonated between the cables produced records in both the north and south direction. For the north spread, layers of 2061 and 5417 m/sec were detected and for the south, layers of 1762 and 4540 were found. The west end of station 6-14E was adjacent to the south end of station 6-13E. Here, the west spread detected layers of 1595 and 4764 m/sec, and the east spread detected layers of 1679, 3077, and 6275 m/sec. It may be seen that neither the second or third layer may be easily correlated with any other layer on any other record.

The graph of the combined distributions from the first, second, and third layer reflects all the features mentioned previously. The few points added at higher velocities are from deeper refracting horizons in the carbonate sequence of bedrocks in the eastern undisturbed area. Both the disturbed and undisturbed velocity distributions are suspect of being quite scattered.

VELOCITY CONSIDERATIONS

The velocity distributions are not a true representation of the velocities of the seismic refractors. The presence of dipping refractors, velocity anisotropy, velocity reversals, hidden layers, and inadequate velocity contrast will affect, in varying degrees, the measurement of seismic velocity. Each will be considered in this order.

Dipping Refractors

At nearly all locations the shooting pattern was such that travel times in opposite directions were recorded. In the split spread configuration, where the shot was detonated between the two geophone cables, the travel times represent samples of data collected from separate geologic settings, and in the reverse profile configuration, where shots were detonated near the far ends of the cables, the travel times represent samples of data collected from one geologic setting. The split spread configuration will then produce 2 seismic records based on 2 separate sections of material, and the 2 velocities, taken from the first line segments in each record, may be compared. Similarly, the velocities, taken from the second line segments in each record, may be compared, and so on. The reverse profile configuration also allows these velocity comparisons to be made and further allows a check to be made on travel time reciprocity.

These two tests must be applied to each pair of line segments from corresponding records before the dip calculations may be made. These tests have not, to the authors knowledge, been suitably framed in statistical analysis as they must, if the validity of the ultimate results are to be considered in the light of error of measurement. The statistical tests of

coefficients for several regression lines is a topic of analysis of covariance, and may be found in many books on linear regression, for example Wine (1964).

First consideration must be given to the question whether the slopes, b_1 and b_2, of each pair of corresponding line segments are statistically different. If there is no statistical difference between the slope estimates, they may be combined, and one slope value may be computed for that pair. If they are different, then a valid pair of slopes whose reciprocals are apparent dip velocities may be analyzed.

Following Wine (1964, p. 554), 2 true model equations, based on separate sets of time-distance data, may be written as

$$\begin{aligned} t_{1i} &= \alpha_1 + \beta_1 d_{1i} + \varepsilon_{1i} \qquad \text{where} \quad i = 1,\ldots,n_1 \\ t_{2i} &= \alpha_2 + \beta_2 d_{2i} + \varepsilon_{2i} \qquad \text{where} \quad i = 1,\ldots,n_2 \end{aligned} \tag{19}$$

The least-squares estimators of α_1, α_2, are a_1, a_2, and the estimators of β_1, β_2 are b_1, b_2. Now under the assumption that ε_{1i} and ε_{2i} are normally and independently distributed with mean zero and constant variance σ_1^2 and σ_2^2 at any point on each regression line, the estimated regression of time on distance is

$$\hat{t}_{1i} = a_1 + b_1 d_{1i}$$
$$\hat{t}_{2i} = a_2 + b_2 d_{2i} \qquad (20)$$

because the expectation of the random component of each equation is zero.

Let the terms e_1, e_2 be the measure of deviation at each d_i for each regression line in Equation 19 and are given as

$$e_{1i} = t_{1i} - a_1 - b_1 d_{1i}$$
$$e_{2i} = t_{2i} - a_2 - b_2 d_{2i} \qquad (21)$$

These are, in a sense, a measure of randomness at each d_i for each regression line. The residual sum of squares of Equations 21 are

$$RSS_1 = \sum_{i=1}^{n_1} e_{1i}^2$$
$$RSS_2 = \sum_{i=1}^{n_2} e_{2i}^2 \qquad (22)$$

The test of the equivalence of 2 slope parameters is stated as the hypothesis H_o: $\beta_1 = \beta_2 = \beta_0$. Under the null hypothesis that $\beta_1 - \beta_2$ is normally distributed and the common population variance is unknown, the statistic τ is given by

$$\tau = \frac{b_1 - b_2}{s[(1/SSd_1) + (1/SSd_2)]^{1/2}} \qquad (23)$$

where

$$SSd_1 = \sum_{i=1}^{n_1} (d_{1i} - \langle d_1 \rangle)^2$$

$$SSd_2 = \sum_{i=1}^{n_2} (d_{2i} - \langle d_2 \rangle)^2 \qquad (24)$$

and the pooled estimate of variance s^2 in Equation 23 is

$$s^2 = \frac{RSS_1 + RSS_2}{n_1 + n_2 - 4} \qquad (25)$$

Now instead of applying Equation 8 to segments of one regression analysis, let it be applied to these 2 regression lines from separate records.

$$SSd_1 = RSS_1 / [(n_1-2)\ \sigma_1^2\ (b)]$$

$$SSd_2 = RSS_2 / [(n_2-2)\ \sigma_2^2\ (b)] \qquad (26)$$

The statistic τ is

$$\tau = \frac{b_1 - b_2}{\left\{\left[\frac{RSS_1 + RSS_2}{n_1 + n_2 - 4}\right]\left[\frac{(n_1-2)\ \sigma_1^2(b)}{RSS} + \frac{(n_2-2)\ \sigma_2^2(b)}{RSS_2}\right]\right\}^{1/2}} \qquad (27)$$

The τ is distributed with $n_1 + n_2 - 4$ degrees of freedom.

After τ has been calculated it can be compared with the Student's t, here labeled τ, with the same degree of freedom and an appropriate level of significance. If τ is greater than Student's τ the hypothesis is rejected, and it is concluded that b_1 and b_2 are indeed different.

If the hypothesis cannot be rejected, the pooled estimate of β is given as

$$\langle b \rangle = \frac{SSd_1 - b_1 + SSd_2 - b_2}{SSd_1 + SSd_2} \tag{28}$$

and $\langle a_1 \rangle$ and $\langle a_2 \rangle$ are

$$\begin{aligned} \langle a_1 \rangle &= \langle t_1 \rangle - \langle b \rangle \langle d_1 \rangle \\ \langle a_2 \rangle &= \langle t_2 \rangle - \langle b \rangle \langle d_2 \rangle \end{aligned} \tag{29}$$

In this case the analysis may not proceed to the second test with this particular pair of line segments.

As an example of an analysis of slope differences, consider the case where two records from a reverse profile configuration had been best fitted with 3 line segments. The 2 first line segments would be tested to see whether or not there was a significant difference between the slopes. Then the 2 second line segments would be tested to see whether or not there was a significant difference between these slopes. And finally, the 2 third line segments would be tested.

A fundamental criteria in seismic refraction is that opposite ended shots and receivers may be interchanged without affecting the travel times, in other words, they are reciprocal. From this, each pair of line segments crossing the opposite-intercept must have the same time value. To reduce this to a statistically convenient question, the line segments from a hypothetical reversed profile are replotted on one set of axes. Then in an n-layer case, the first line segments should superimpose, and the second to the nth matched pair of line segments should each intersect at a distance S from the origin, which is equal to the shot point to shot

point distance.

Now by shifting the time axis to a new time axis at distance S, the second to the nth matched line intersect on this axis. The regression equations then become

$$t_{1i} = a_1 + b_1 S + b_1 d_{1i}$$
$$t_{2i} = a_2 + b_2 S + b_2 d_{2i} \qquad (30)$$

By shifting axis, the data are simply coded, and no statistical changes have been made. The statistical test that the two regression lines intersect at this new axis is H_0: $a_1 + b_1 S = a_2 + b_2 S$. The statistic τ is then stated as

$$\tau = \frac{a_1 - a_2 + S(b_1 - b_2)}{s\,[(Sdd_1/n_1 SSd_1) + (Sdd_2/n_2 Sdd_2)]^{1/2}} \qquad (31)$$

where $Sdd_1 = \sum_{i=1}^{n_1} d_{1i}^2$ and $Sdd_2 = \sum_{i=1}^{n_2} d_{2i}^2$. As was done with Equation 8, Equation 11 may be rewritten in terms of 2 regression lines from separate records as

$$Sdd_1 = n_1 \sigma_1^2(a)/\sigma_1^2(b)$$
$$Sdd_2 = n_2 \sigma_2^2(a)/\sigma_2^2(b) \qquad (32)$$

Then by substituting the Equations 32 into Equation 31, the statistical τ is

$$\tau = \frac{a_1 - a_2 + S(b_1 - b_2)}{\left\{\left[\frac{RSS_1 + RSS_2}{n_1 + n_2 - 4}\right]\left[\frac{(n_1-2)\sigma_1^2(a)}{RSS_1} + \frac{(n_2-2)\sigma_2^2(a)}{RSS_2}\right]\right\}^{1/2}} \quad (33)$$

As before, the calculated τ is compared with Student's τ with $n_1 + n_2 - 4$ degrees of freedom at an appropriate level of significance. If the τ is greater than Student's τ, the hypothesis that the two regression lines intersect at distance D is rejected, and therefore, there is a significant difference in the opposite intercept times. If the hypothesis cannot be rejected, then there is no significant difference in the opposite intercept times.

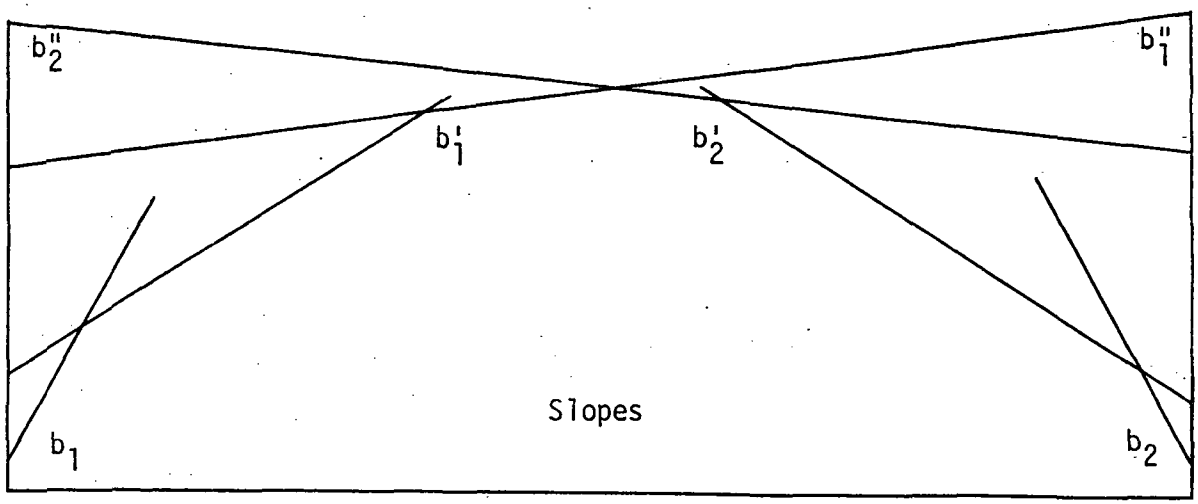
Because the data from split spread profiles were collected from different geologic settings, only the test of slope may be applied. And where reverse profile coverage is available, dip may be calculated on the top of the second layer only when (1) there is no significant difference in the slopes of the first layer, (2) there is a significant difference in the slopes of the second layer, and (3) there is not a significant difference in the opposite-intercept times of the second line segments.

The line segment pairs for each seismic location were tested for slope and opposite-intercept criteria. Unfortunately when the least square fits were calculated only the combined residual sum of squares was retained for those line segments connected by a c-join. Therefore no estimate of the residual sum of squares for the left and right hand line segments was retained. Because of this, neither test of covariance could be applied to any combination of lines where one of the lines did not have a calculated

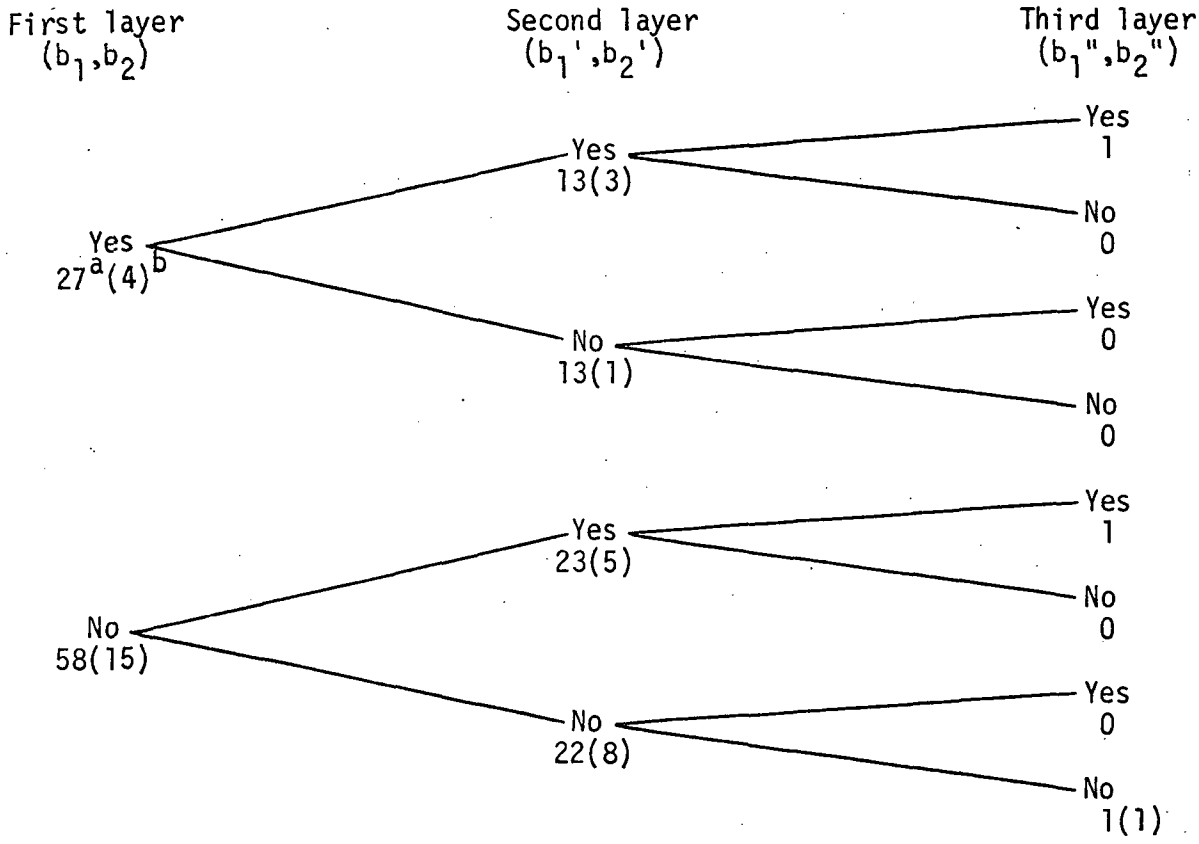
residual sum of squares. Also rejected from consideration were those line segments which had been fitted through only two points.

The results of the slope test are shown on Figure 8. A significant difference in slope was inferred at the 90% confidence level, or simply that there was at least a 90% chance that there was a true difference in slope. Of the stations tested for slope difference, there were approximately 30% (27) where there was a significant difference in the first velocity and approximately 70% (58) where there was not. The results have been drawn in the form of branches to show the different possible cases. Dip calculations on a 2-layer model could only be made on those sites where there was not a sufficient difference in the slopes of the first layer, but where there was a sufficient difference in the second layer. Where no differences were found in either the first or second layer slopes, a horizontal interface model was assumed. This does not imply that the interface is truly horizontal, but that the magnitude of the subsurface undulations of the interface are of such a sufficiently low relief that the time deviations due to this surface have been combined with the error of measurement of time so that the model is simply a plane passing through the average depth to this surface. It is interesting to note that only 23 stations passed the slope criteria for the 2-layer dipping model, and of these, only 1 case could be tested for dip of both the second and third layers.

The results of the opposite-intercept test of the 23 sites that had passed both first and second layer slope tests is presented in Table 2. As in the slope test, a significant difference in opposite-intercept times was inferred as there was at least a 90% chance that there was a true



Significant difference in slopes



[a]Total of all stations.

[b]Station with good borehole control.

Figure 8. Slope test of all seismic sites by line segment

Table 2. Opposite-intercept test of 23 sites that had passed 2-layer slope test for a 2-layer dipping layer model

Seismic site	Shot no.	Significant difference	Relation to Manson disturbed area		
			Outside	Edge	Inside
2-2W	1+2	Yes			
3-3E	3+4	No			x
5-5E	1+2	No		x	
5-1W	2+4	Yes			
5-6W[a]	6+5	No	x		
6-11E	1E+1W	--[b]			
6-6E	1+2	--[b]			
6-1E	6+5	--[b]			
6-4W[a]	1+2	--[b]			
6-7W[a]	3E+3W	--[b]			
6-7W[a]	3E+4	--[b]			
7-6E	4E+4W	--[b]			
7-2W	4+3	No			x
7-3W	1+2	Yes			
7-4W	1+2	No		x	
8-2W	5+6	Yes			
8-5W	2+1	No	x		
9-4E	1+2	No		x	
9-1W	1+2	Yes			
9-3W[a]	6+5	No	x		
10-3E	3+4	No		x	
0-1E	7+8	No			x
10-2W[a]	9+10	No		x	
11-2W	8+7	No	x	—	—
		12	4	5	3

[a]Sites with well control.

[b]Not calculated.

difference in time. Only 3 seismic locations within the Manson disturbed area passed all criteria on which the 2-layer dipping refractor model is based.

Velocity Anisotropy

Of the 46 reverse profile stations which were within the disturbed area, there were 27 stations at which there was no significant difference in the first layer; 12 stations where there was a significant difference in the first layer; and 7 stations where no determination could be made.

At the 12 locations where there was a statistical difference in the first layer, there were 6 locations where there was a significant difference in the second layer and 6 locations where there was not a significant difference. Significant velocity differences in the first layer are explained by the great variability of the unconsolidated drift. Lenticular sand bodies within the drift, variability of both grain size and lithology, and degree of water saturation may have an effect on glacial drift velocities.

At the other 27 stations where the slope test had been made, there were 7 stations where tests of significance in the second layer could not be made and 12 stations where there was not a significant difference in velocity. At the remaining 8 stations, the slope tests for dip calculation were successful. Finally, of these 8 stations, 5 stations did not pass the opposite-intercept test and 3 stations did pass. Only at these 3 sites may a simple 2-layer model with a dipping interface be applied. This is approximately 9% of the total number of stations within the Manson disturbed area. Those stations where significant velocity differences in the second layer were found, indicate that the first bedrock layer was anisotropic to seismic waves.

Velocity Reversals and Hidden Layers

Before the areal distribution of the first bedrock velocity may be considered, the presence of a layer between the glacial drift and the

bedrock must be considered. If the velocity of this material is lower than the velocity of the glacial drift, the layer will be totally undetected by the horizontal refraction method and will only be revealed by borehole seismic measurements. The effect of a velocity reversal is to reduce the velocity of the upper layer. In other words, an average velocity is obtained for the first 2 layers. Because the first layer for this area of study is glacial drift, the presence of an appreciable number of cases where a second layer with a velocity lower than that of the glacial drift would skew the distribution to the left. From the distributions of the first layer velocity in Figure 7, this is not the case, with the possible exception of one occurrence of a first layer between 1450 and 1500 m/sec.

Green (1962) has investigated the case where a layer has a velocity between the velocity of an overlying layer and the velocity of an underlying layer, but whose thickness is below a certain critical thickness. In such a case the layer will not be detected; therefore both the velocity and the thickness will not be measurable. Hence it is a hidden layer. There are also cases where a refractor is not detected due to the spacing of the geophones. In the former case, the refractor is a true hidden layer because it will never be recorded regardless of the geophone spacing. In the later case, the refractor is an apparent hidden layer.

A true hidden layer between the first and second layer, whose velocity was intermediate between the two, was suspected at six locations (3-3E, 4-1E, 5-1E, 5-4E, 8-6E, and 11-1W)(Figure 1). Unfortunately well logs from boreholes near these sites did not reveal the presence of either a recognizable basal lithologic unit in the glacial drift or an unusual first bedrock unit.

At locations 3-3E, 4-1E, and 8-6E a hidden layer velocity of 2300, 2200, and 2250 m/sec, respectively, was assumed. These were chosen by averaging the second layer velocities from stations east and west of these sites. Calculations based on Green's method were performed on the velocities and intercepts at these sites, and the maximum thickness of a true hidden layer for each record was determined.

At sites 5-1E and 5-4E results from shots in one direction were inconsistent with the rest of the records and were discarded. At site 11-1W no hidden layer velocity was assigned because the velocity of the Pennsylvanian shales and sandstones in the area have not been measured accurately.

Velocity Contrast

Once a reasonably accurate representation of the first bedrock layer has been established, the areal distribution of the velocities will reflect, in some fashion, the lateral variations of the bedrock material. These results are presented on the left side of Figure 9. The graphs were computed by averaging the data by refraction line. There appears to be no trend in the averaged first layer (glacial drift) velocities. The averaged glacial drift velocity of 2143 m/sec on line 4 is unexplainably high. The second layer velocity for individual velocities below 3000 m/sec is presented to illustrate the variations in the disturbed shale bedrock. Rather prominent regions of higher velocity are present on line 2 and 9. The utility of the algorithm developed previously is demonstrated on line 4 where there is a lack of velocity contrast between the first and second layer velocities. In such cases the statistical determinations of the best fit line segments may be the only method by which consistent velocity data may be obtained.

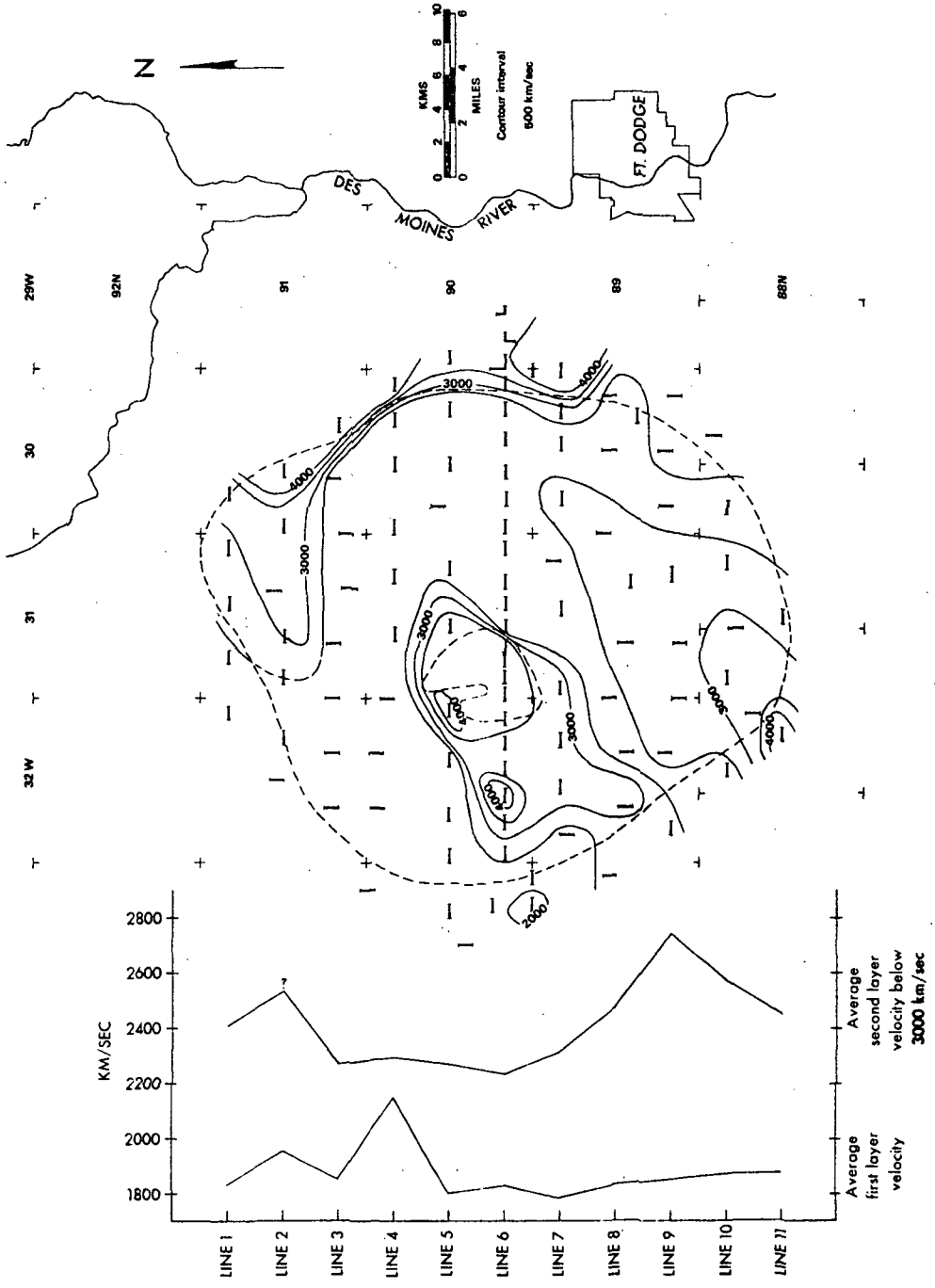


Figure 9. Bedrock velocity graph and contour map of the Manson disturbed area

BEDROCK VELOCITY AND TOPOGRAPHY

The mean velocity of the two first bedrock velocities, measured at each station, was chosen to approximate the true bedrock velocity. Here, the first line segment is chosen to represent the velocity of glacial drift, and the second line segment is chosen to represent the velocity of the first bedrock layer. This calculation is always higher than the true velocity of the bedrock, if all requirements for dip calculations have been met on the seismic plots. The equation of the true refractor velocity, as presented by Nettleton (1940, p. 269), may be rewritten in terms of mean velocity. Let V be the true refractor velocity, θ be the apparent dip angle, and $\langle V \rangle$ be the mean velocity of the refractor as measured from opposite directions at a seismic station, then $V = [4R/(1+R)^2]\cos\theta\langle V \rangle$, where R is the ratio of the lower refractor layer velocity to the higher refractor layer velocity. The measurement $\langle V \rangle$ is less than 5% greater than V for $R>0.68$ and $\theta<16^{o}$. For all stations in the Manson area $R>0.65$. Therefore, the mean velocity of the first bedrock layer is less than 5% higher than the true first bedrock velocity.

The areal distribution of the first bedrock layer is shown on Figure 9. The area in the center of figure bounded by the 3500 m/sec contour roughly coincides with the area of crystalline rock, as determined by borehole methods. Another area of high bedrock velocity is located 10 km due west of the center of the Manson structure and has a diameter of 2 km. It is reasonable to assume that the crystalline rock also subcrops in this area. The limestones and dolomites which comprise the bedrock on the eastern flank of the undisturbed area are delineated quite well by the 4000 m/sec

contour, but velocity data alone may not be used to delineate the edge of the disturbed area elsewhere. Borehole information near location 11-1W reveals that the first bedrock layer, Pennsylvania shales and sandstones, is approximately 3 m thick in this area. The bedrock velocity at station 11-1W reflects the velocity of the underlaying Mississippian limestone and sandstones, and the Pennsylvanian shales and sandstones have been averaged in with the glacial drift.

The determination of the depth to the seismic refractors may be calculated by a variety of methods. A method employing the concept of delay time, introduced by Gardner (1939), has been successfully applied [for example the study by Pakiser and Black (1957)]. A similar method employing the concept known as time-depth was developed by Hagedoorn (1959) and Hawkins (1961). Both of these methods make some assumption as to the ray-path. Because the bedrock is suspected of having velocity anisotropy and an irregular surface, depth determinations were made with the velocity-time intercept method, presented by Nettleton (1940), Dobrin (1960), and Steinhart and Meyer (1961). Because very few seismic records could be subjected to dip calculations, depth calculations were based on a simple horizontal model.

In view of the obvious simplicity of this model, error bounds on the depth determinations were made purposefully high. Let the first layer correspond to the first line segment, j=1. The error bounds of depth to the top of say the jth layer (j>1) were calculated with error bounds on the parameters of the jth line segment. Confidence limits were chosen so that there was a .99 probability that the true time intercept and wave slowness were within the calculated bounds. The time intercept and wave slowness

for the first to the jth line segment were first used to calculate the depth to the top of the jth layer. The lower depth bound was found by substituting the value of the upper confidence limit of the time intercept and the value of the lower confidence limit of the wave slowness of the jth regression line and then by recalculating the depth. The upper depth bound was found by substituting the value of the lower confidence limit of the time intercept and the value of the upper confidence limit of the wave slowness and then by recalculating the depth. The upper and lower equations for the jth line segment with n data points are

$$t_i^u = a_j + \tau\sigma_j(a) + [b_j + \tau\sigma_j(b)]d_i$$
$$t_i^l = a_j - \tau\sigma_j(a) + [b_j + \tau\sigma_j(b)]d_i \qquad (34)$$

where the superscripts u and l indicate the times for the upper and lower line segments and where τ is Student's $\tau_{0.01}$ with (n-2) degrees of freedom.

Figure 10 is based on an interpretation of a combination of the areal distribution of the depth determinations and bedrock elevations from boreholes. The irregular surface within the Manson disturbed area is sharply contrasted by the gentle slopes outside the area. The average elevation within the boundaries of the disturbed area is about 300 m above sea level, whereas the 320 m contour line roughly traces the average elevation of the undisturbed bedrock surface outside the boundaries. Within the disturbed area a number of bedrock highs and lows are prominent. The central area of crystalline bedrock is seen to be a topographic high. Hills of disturbed shale in the northeast and southwest portion of the disturbed area are present. Six closed depressions surround these bedrock highs. The

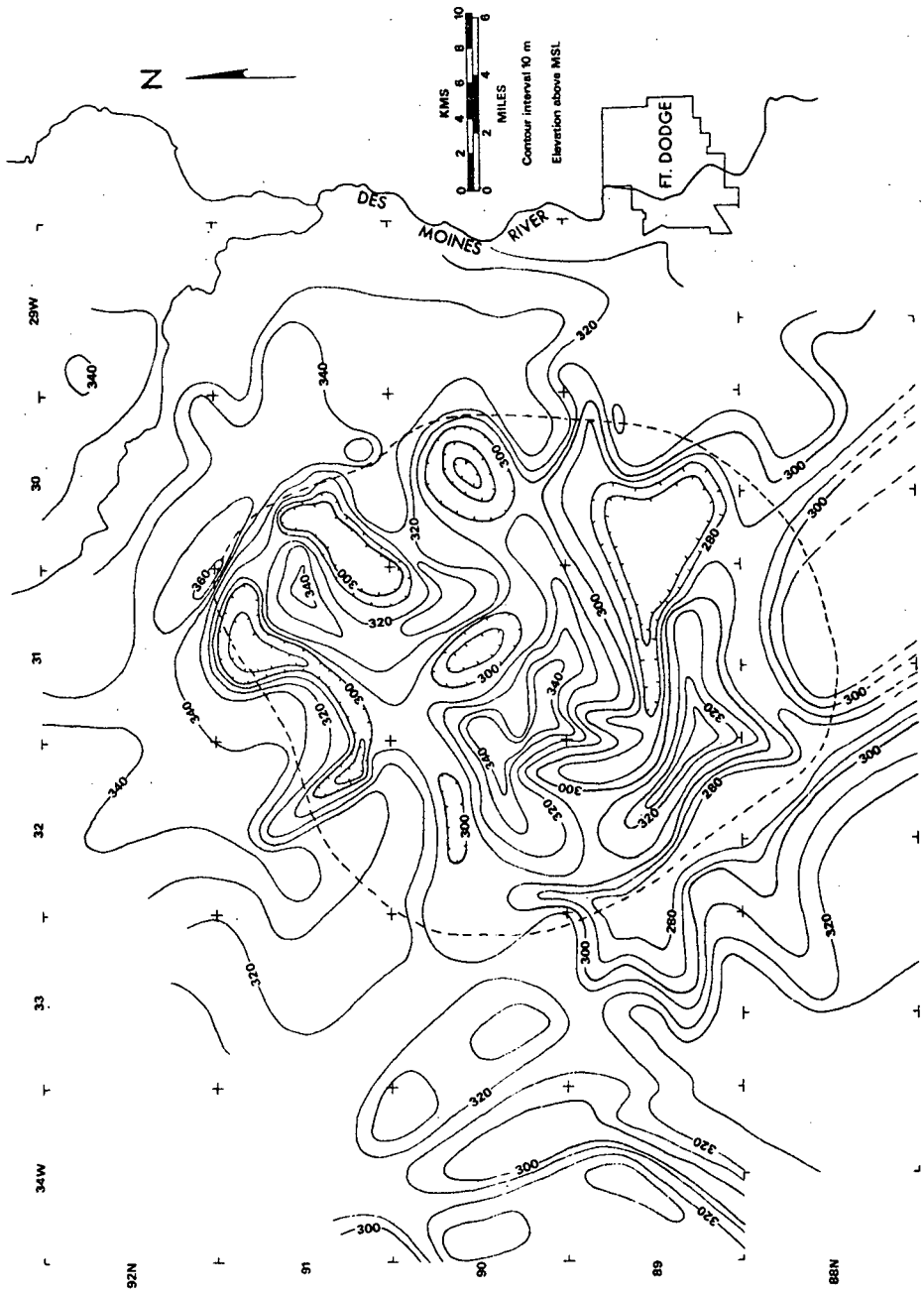


Figure 10. Bedrock topographic contour map of the Manson disturbed area

drainage ways of the disturbed area are not well developed. Several undisturbed bedrock valleys terminate in the depressed area. Two bedrock valleys in the southern portion trend northwest-southeast and their gradient is toward the southeast, away from the disturbed area. The eastern bedrock valley is a headward extension of the bedrock valley shown by Hale (1955, p. 151) to extend southeast across the southwest corner of Webster county. The major drainage network in north-central Iowa is known to drain to the southeast.

DISCUSSION

A number of features have come to light from the seismic investigation. The Manson disturbed area has been shown to have the appearance of an impact crater. A central uplift of extremely deformed crystalline rock, which was lifted some 700 m from the basement, now extends through a shallow circular basin of highly deformed Cretaceous shales. At the time of burial by Pleistocene drift, the area surrounding the uplift had not developed an integrated drainage network, but occurred as a number of rises and shallow closed depressions. The lack of an integrated drainage network within the disturbed area may indicate that the structure is relatively young.

The relative erosional resistance of the Mississippian limestones and dolomites beyond the northeast rim of the area contrasts with the lack of resistance to erosion of the Pennsylvanian shales and sandstones beyond the southwest rim. This is expressed by the very steep slope along the rim in the northeast and the more gentle slope along the south and southwest rims. Two rather extensive headward erosion channels had developed in the Pennsylvanian shales and sandstones prior to burial. The western bedrock valley follows the edge of the disturbed area for some distance. The zone between the disturbed and undisturbed bedrock must have been less resistant to erosion than either the disturbed or undisturbed bedrock. The shape of the western bedrock valley suggests that the disturbed bedrock was comparatively more erosionally resistant than the undisturbed Pennsylvanian shales.

The second crystalline subcrop, as suggested by the bedrock velocity map, is also seen to be a western extension of the center crystalline

bedrock high. Indeed, the gravity high (Figure 3) in the area coincides with this suspected subcrop of crystalline rock. The elongate 4605 gamma magnetic anomaly (Figure 2) is situated parallel to the crystalline bedrock high and shifted 1 km to the east.

Only 20% of the seismic records taken at sites well within the Manson disturbed area could be represented by a 3-layer seismic model. At many of these sites the third layer detected by a shot in one direction was not detected in the shot record in the other direction. A third layer with velocity about 3400 m/sec may be present below sites 3-2E and 3-1E, but the depths to this layer are quite irregular and dip calculations could not be made. For the most part, there is no mappable seismic refractor, below that of the drift-shale interface, to a depth of 250 m.

The rim structure appears to have considerable complexity. The line segments fitted to the records at stations 6-13E to 6-16E showed that there is a large variation of velocity in the undisturbed bedrock east of the Manson disturbed area (seen earlier). No consistent structural model, no matter how general, could be made of the geometric arrangement of the bedrock layers. Simple seismic models may not be applied in these areas, and it points to considerable geologic complexity, probably due to complex folding of the bedrock units adjacent to the Manson disturbed area.

Another significant feature is the seismic velocity of the crystalline rock in the center and western portion of the structure. Press (1966) has reported that granite velocities may range from 4.8 to 5.9 m/sec. Velocity histograms of granite and gneiss were presented by Hasselstrom (1969) for shallow seismic refraction measurements. In each case at least 75% of the sample had a velocity greater than 5000 m/sec. At Manson the velocity of

the mixture of granite, gneiss, and diabase is between 3000 and 5000 m/sec and averages near 4000 m/sec. The unusually low velocity of this crystalline mass may indicate that this rock body has an abnormally high porosity. This may be a result of the large scale brecciation of the crystalline mass during the implacement event.

The unusual deformation characteristics of the minerals of the central crystalline mass have shown that this structure could have been created by one of only two 'catastrophic' events. Several similarities of the Manson structure to the Canadian craters are noted. This crystalline rock has a close similarity to rocks taken from central uplifts in larger Canadian craters, and the age of this crystalline mass is near the approximate age of the basement rock as observed in some Canadian craters (Michael Dence, personal communication, 1970).

Regardless of which mechanism had created the structure, the basement rock would be involved in the final disposition of the materials, because the thickness of the sedimentary column is considerably less than the diameter of the structure (roughly 1:400). Therefore gravity and magnetic anomalies from the upthrown basement rock may not be used to distinguish between the two mechanisms.

The questions raised by the occurrence of this complex structure must be answered by sifting and evaluating a wealth of data, some of which is contradictory or coincidental. The nature of the origin of the Manson disturbed area must await the collection and interpretation of additional data.

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APPENDIX A. LIST OF THE COMPUTER PROGRAM

1. Purpose:

The program fits joined regression lines to an ordered series of measurements on the basis of residual sum of squares. For each line segment, the parameters and their standard deviations are calculated. The measurements are taken to be seismic refraction time-distance pairs, and from the results of the line fitting analysis, depths and depth error bounds are determined.

2. Method:

The set of up to 24 time-distance measurements are sequentially partitioned into subsets of data, each of which is searched for the best 2 segment fit. Each subset has an upper bound, which is chosen prior to the analysis, and a lower bound, which is determined by the position of the join point from a preceding 2 segment fit. The final solution is derived from a combination of the 2 segment fits.

For a subset of data, pairs of regression lines are fitted to all combinations of the data, and the pair of regression lines which have the least combined residual sum of squares are taken to be the best fit.

Least squares regression is used to best fit the line segments. The intercept and wave slowness (parameters a and b) of each regression line are calculated, and under the assumption that the variable, time, is independently and normally distributed with constant variance, the standard deviations of intercept and wave slowness are found. The depth calculations are based on the standard time intercept-wave slowness equations. Error bounds in depth have been treated previously.

3. Accuracy:

The accuracy of the program was measured from test data presented in Table 3. These data were analyzed by (1) the OMNITAB program available at the Iowa State Computation Center, (2) a least squares program written by Bevington (1969), and (3) a subroutine from this program, SJOIN. All programs were run on the Computation Center's IBM 360/65. An evaluation of widely used regression programs by Wampler (1970) found that the single precision OMNITAB program is correct to 4 and 6 significant figures for the first two coefficients of a fifth degree polynomial. From the test data, the calculations of a and b by subroutine SJOIN agree with those from the OMNITAB program to 5 and 6 figures (Table 4). The standard deviations of a and b agree to 7 and 6 figures. The calculations of residual sum of squares from the subroutine SJOIN and program OMNITAB are included. The results of the Bevington program are presented as another comparison.

Because the calculation of residual sum of squares is fundamental to the validity of the program, the time values, t (Table 3), were adjusted for a range of values. The results are presented in Table 5. The approximate range of values used in the analysis of the data from this study were $0<d<800$ and $0.005<t<0.450$, so the residual sum of squares calculations are valid to approximately 5 significant figures. Because any 2 calculations of the residual sum of squares were equal to at most 4 digits, the program is considered to be valid for the statistical decisions undertaken.

Table 3. Test data of distance, d, and time, t

d	t
15.	12.
90.	28.
195.	45.
300.	63.
405.	82.

Table 4. Comparison of three computer programs

	Program		
Coefficient	SJOIN[a]	OMNITAB[b]	Bevington[a]
a[c]	10.53826	10.538361	10.538420
$\sigma(a)$	0.91706004	0.91706008	0.916853
b	0.17642574	0.17642599	0.176426
$\sigma(b) \times 10^{-2}$	0.37415442	0.37415458	0.3741
RSS[d]	4.1312900	4.1345215	--

[a]Double precision.

[b]Single precision.

[c]The form of the equation is $t = a + bd$.

[d]Residual sum of squares.

Table 5. Significant figures of the calculation of residual sum of squares for various values of time

Data	Approximate velocity[a]	Significant figures[b]
$(d_i,t_i)^c$	5.833	$\geq$13
$(d_i,0.1t_i)$	58.33	5
$(d_i,0.01t_i)$	583.3	6
$(d_i,0.001t_i)$	5833.	4

[a]Velocity is the reciprocal of b.

[b]Based on the manual calculation RSS = 4.131290027447.

[c]Where i = 1,...,5.

4. Usage:

a. Input:

A standardized punched card record format is described in the comment cards of subroutine RREC.

b. Output:

Although the length of the output is a function of the number of fitted line segments, the number of data, and the number of acceptable regression candidates, a typical 2 segment analysis of 24 data points will yield about 1 page of printed output.

c. Operating Procedures:

The first card of the data deck must specify the mode of operation

of the program (see the comments in the main program). The next 2 cards must be the first 24 Student's values with a format of D6.3. The input records then follow. The end of data is specified by placing a blank card and a card with 99 punched in cols. 47-48 after the final record.

d. Constants:

A total of 4560 bytes has been allocated to unlabeled COMMON and 116 bytes to COMMON area /LABEL/.

5. Subroutines Required:

The subroutines required are RREC, SETS, SJOIN, CJOIN, OUTPUT, and ABYSS. The main program calls subroutines RREC, SETS, and ABYSS. Subroutines SETS calls subroutines SJOIN, CJOIN, and OUTPUT.

A list of the program follows.

```
C                                                                       001
C     SET UP TO APPLY SETS AND ABYSS TO REDUCED RECORDS                 002
C                                                                       003
C     SUBROUTINES REQUIRED:  RREC,SETS,ABYSS                            004
C                                                                       005
      REAL*8 BETA(12,23),STD(8,23),GAMMA(5,22),DELTA(6,6),AMIN          006
      REAL*8 ZETA(6,5),S(24)                                            007
      INTEGER*4 NREAD/1/,NPUNCH/2/,NPRINT/3/                            008
      INTEGER*4 ID(5)                                                   009
      REAL T(49),D(49)                                                  010
      INTEGER IU(10),II(24),NPTS(6)                                     011
      COMMON BETA,STD,GAMMA                                             012
      COMMON/LABEL/ID,II                                                013
C                                                                       014
C     THE FIRST CARD PROVIDES THE USER WITH SEVERAL PROGRAMMING OPTIONS. 015
C     COL. 1 = 0  DATA ANALYZED AS PER TEST POINT VECTOR.               016
C            = 1  DATA ANALYZED AS PER TEST POINT VECTOR AND 2-LAYER MODEL. 017
C            = 2  DATA ANALYZED AS 2-LAYER MODEL ONLY.                  018
C     COL. 2 = 0  VELOCITY CARDS NOT PUNCHED.                           019
C            = 1  VELOCITY CARDS PUNCHED.                               020
C     COL. 3 = 0  DEPTH DETERMINATIONS ARE NOT CALCULATED.              021
C            = 1  DEPTH DETERMINATIONS ARE CALCULATED.                  022
C                                                                       023
      READ (NREAD,8) KK,KL,KM                                           024
    8 FORMAT (3I1)                                                      025
C                                                                       026
C     THE SECOND AND THIRD CARDS ARE THE STUDENT'S T CONSTANTS.         027
C                                                                       028
      READ(NREAD,9) (S(I),I = 1,24)                                     029
    9 FORMAT (12D6.3)                                                   030
C                                                                       031
      DO 10 I = 1,24                                                    032
   10 II(I) = I                                                         033
    1 CALL RREC (L,LL,IEND,T,D,10,IU)                                   034
      IF (L.EQ.99) GO TO 99                                             035
      KA = 0                                                            036
      DO 20 ITEST = 1,10                                                037
```

```
      IF (IU(ITEST).EQ.0) GO TO 3                                       038
   20 CONTINUE                                                          039
      ITEST = 11                                                        040
    3 KA = KA + 1                                                       041
      IF (KK.EQ.2) ITEST = 1                                            042
      AMIN = 0.D0                                                       043
      WRITE (NPRINT,5)                                                  044
    5 FORMAT ('1')                                                      045
      CALL SETS (IEND,ITEST,IU,T,D,IDEL,DELTA,NPTS,AMIN)                046
      WRITE (NPRINT,29)                                                 047
   29 FORMAT ('0',132('*')  /' LAYER',4X,'INTERCEPT',4X,'STD OF INT',7X, 048
     X 'SLOPE',7X,'STD OF S',4X,'VELOCITY',4X,'RESID SS',6X,'N PTS'/     049
     X ' ',132('_'))                                                    050
      DO 30 I = 1,IDEL                                                  051
   30 WRITE (NPRINT,31) I,(DELTA(J,I),J = 1,6),NPTS(I)                  052
   31 FORMAT (' ',3X,I2,3X,F10.4,3(2X,D12.5),5X,F5.0,2X,D12.5,7X,I2)    053
      WRITE (NPRINT,32) AMIN                                            054
   32 FORMAT ('0TOTAL RESIDUAL SUM OF SQUARES = ',D20.8)                055
C                                                                       056
C     PUNCH UP LAYER DATA                                               057
C                                                                       058
      IF (KL.EQ.0) GO TO 33                                             059
      WRITE (NPUNCH,50) ID,IDEL                                         060
   50 FORMAT (5A4,2X,I2)                                                061
      DO 60 I = 1,IDEL                                                  062
   60 WRITE (NPUNCH,61) NPTS(I),(DELTA(J,I),J = 1,6)                    063
   61 FORMAT (4X,I2,2X,6D12.5)                                          064
C                                                                       065
C     FIND DEPTHS BOTH WITH AND WITHOUT THE LVL                         066
C                                                                       067
   33 IF ((IDEL.EQ.1).OR.(KM.EQ.0)) GO TO 70                            068
      WRITE (NPRINT,34)                                                 069
   34 FORMAT ('1'/3('-'/)/' ',132('='))                                 070
      KKK = 0                                                           071
   35 KKK = KKK + 1                                                     072
      CALL ABYSS (IDEL,NPTS,S,DELTA,ZETA)                               073
      IF (KKK.EQ.2) WRITE (NPRINT,36)                                   074
```

```
   36 FORMAT (' WITH LOW VELOCITY LAYER')                                  075
      WRITE (NPRINT,37) ID,IDEL                                            076
   37 FORMAT ('0',10X,'RECORD  ',5A4,' NUMBER OF LAYERS ',I2/'ON PTS',     077
     X 4X,'INTERCEPT',3X,'STD OF INT',4X,'SLOPE',5X,'STD OF S',4X,         078
     X 'VELOCITY',4X,'RESID MS')                                          079
      DO 6 I = 1,IDEL                                                      080
    6 WRITE (NPRINT,38) NPTS(I),(DELTA(J,I),J = 1,6)                       081
   38 FORMAT (4X,I2,2X,6D12.5)                                             082
      WRITE (NPRINT,39)                                                    083
   39 FORMAT ('0',9X,'THICKNESS',2X,'UPPER LIMIT',2X,'LOWER LIMIT',4X,     084
     X 'DEPTH',4X,'LOWER LIMIT',2X, 'UPPER LIMIT')                        085
      IF = IDEL - 1                                                        086
      DO 40 I = 1,IF                                                       087
   40 WRITE (NPRINT,41) (ZETA(J,I),J = 1,6)                                088
   41 FORMAT (11X,F7.2,5X,F7.2,6X,F7.2,4X,F7.2,5X,F7.2,6X,F7.2,           089
     X 10X,'IN METERS')                                                   090
      WRITE (NPRINT,39)                                                    091
      DO 43 J = 1,IF                                                       092
      DO 42 I = 1,6                                                        093
   42 ZETA(I,J) = ZETA(I,J) * 3.2803D0                                     094
   43 WRITE (NPRINT,44)  (ZETA(I,J),I = 1,6)                               095
   44 FORMAT (11X,F7.2,5X,F7.2,6X,F7.2,4X,F7.2,5X,F7.2,6X,F7.2,           096
     X 10X,'IN FEET')                                                     097
      WRITE (NPRINT,45)                                                    098
   45 FORMAT ('0',132('='))                                                099
      IF (KKK.EQ.2) GO TO 70                                               100
C                                                                          101
C     SET FIRST LAYER TO HAVE AIR-WAVE CHARACTERISTICS                     102
C                                                                          103
      IA = IDEL + 1                                                        104
      IB = IDEL + 2                                                        105
      DO 46 I = 1,IDEL                                                     106
      NPTS(IB - I) = NPTS(IA - I)                                          107
      DO 46 J = 1,6                                                        108
   46 DELTA(J,IB - I) = DELTA(J,IA - I)                                    109
      IDEL = IDEL + 1                                                      110
      NPTS(1) = 0                                                          111
```

```
      DELTA(1,1) = 0.D0                                                    112
      DELTA(2,1) = 0.D0                                                    113
      DELTA(3,1) = 3.01705D-03                                             114
      DELTA(4,1) = 0.D0                                                    115
      DELTA(5,1) = 0.D0                                                    116
      DELTA(6,1) = 0.D0                                                    117
      GO TO 35                                                             118
C                                                                          119
   70 IF (ITEST.EQ.1) GO TO 75                                             120
      ITEST = 1                                                            121
      IF ((KK.EQ.1).AND.(KA.EQ.1)) GO TO 3                                 122
   75 GO TO 1                                                              123
   99 IF (KL.EQ.1) WRITE (NPRINT,100)                                      124
  100 FORMAT ('-',10X,'VELOCITY CARDS HAVE BEEN PUNCHED')                  125
      STOP                                                                 126
      END                                                                  127
```

```
      SUBROUTINE RREC (L,LL,IEND,T,D,LIU,IU)                          RREC001
C
C ***************************************************************************
C *                                                                         *
C *   SUBROUTINE RREC READS REDUCED SEISMIC RECORDS.  THE COMMAND CARD PREFAC- *
C *   ING EACH RECORD IS READ FOR UP TO 10 TEST POINTS AS WELL.             *
C *   ARGUMENTS                                                             *
C *   L =                    END OF DATA SET CODE. WHEN 99 IS READ, IT SIGNALS *
C *   L =                    THE END OF THE DATA SET.                       *
C *   LL =                   SPREAD GEOMETRY CODE                           *
C *   IEND =                 LENGTH OF T AND D                              *
C *   T =                    TIME VECTOR                                    *
C *   D =                    DISTANCE VECTOR                                *
C *   LIU =                  LENGTH OF TEST POINT VECTOR                    *
C *   IU =                   TEST POINT VECTOR                              *
C *                                                                         *
C *   REDUCED RECORD FORMAT                                                 *
C *   CARD COLUMN            REMARK                                         *
C *                                                                         *
C *   ------------------------------------------------------------------    *
C *        COMMAND CARD                                                     *
C *   1-2                    M = BATCHING CODE                              *
C *   3-4                    ISEL = SELECTION CODE                          *
C *  ------                                                                 *
C * | 6-7 |                  PAIRS OF VALUES FOR TEST POINT VECTOR.         *
C *  ------                                                                 *
C *   CONTINUES TO COLUMN 25 TOTAL OF 10 VALUES.                            *
C *                                                                         *
C *   ------------------------------------------------------------------    *
C *        IDENTIFICATION CARD                                              *
C *   1-20                   IDENTIFICATION                                 *
C *   21-22                  SPREAD GEOMETRY CODE                           *
C *   47-48                  END OF DATA SET FIELD                          *
C *                                                                         *
C *   ------------------------------------------------------------------    *
C *        TIME CARDS                                                       *
C *   1-5                    LENGTH OF TIME VECTOR                          *
```

```
C *  -------                                                                *
C * | 11-15 |              A TIME IN SECONDS.       0.000                  *
C *  -------                                                                *
C *   CONTINUES TO COLUMN 70 TOTAL OF 12 TIMES.                            *
C *                                                                        *
C *  ______________________________________________________________________*
C *       DISTANCE CARDS                                                   *
C *   1-5                LENGTH OF DISTANCE VECTOR (MATCHING TIME VECTOR)  *
C *  -------                                                               *
C * | 11-15 |              DISTANCE IN METERS.      000.0                  *
C *  -------                                                               *
C *   CONTINUES TO COLUMN 70 TOTAL OF 12 DISTANCES                         *
C *                                                                        *
C *       DATA STACK                                                       *
C *   LL = 0: COMMAND,IDENTIFICATION,TIME,DISTANCE,COMMAND,IDENTIFICATION, *
C *              TIME,DISTANCE                                             *
C *   LL = 1: COMMAND,IDENTIFICATION,TIME,TIME,DISTANCE,DISTANCE           *
C *                                                                        *
C ************************************************************************
C
      INTEGER*4 NREAD/1/                                                RREC002
      INTEGER*4 ID(5)                                                   RREC003
      REAL T(1),D(1)                                                    RREC004
      INTEGER IU(1),II(24)                                              RREC005
      COMMON/LABEL/ID,II                                                RREC006
      READ (NREAD,9) M,ISEL,(IU(I),I = 1,LIU)                           RREC007
    9 FORMAT (2I2,1X,10I2)                                              RREC008
      READ (NREAD,11) ID,LL,L                                           RREC009
   11 FORMAT (5A4,I2,24X,I2)                                            RREC010
      IF (L.EQ.99) GO TO 99                                             RREC011
      READ (NREAD,12) IEND,(T(I),I = 1,12)                              RREC012
   12 FORMAT (I5,5X,12F5.3)                                             RREC013
      IF (IEND.GT.12) READ(NREAD,13)(T(I),I = 13,IEND)                  RREC014
   13 FORMAT (10X,12F5.3)                                               RREC015
      READ (NREAD,14) (D(I),I = 1,IEND)                                 RREC016
   99 RETURN                                                            RREC017
      END                                                               RREC018
```

```
      SUBROUTINE SETS (IEND,ITEST,IU,T,D,IDEL,DELTA,NPTS,AMIN)          SETS001
C
C ***************************************************************************
C *                                                                         *
C *   SUBROUTINE SETS PARTITIONS VECTORS T AND D, AND TESTS THE SUBSET PAIRS  *
C *   FOR A JOIN POINT.  THE LOWER BOUND (LB) OF THE PARTITION IS CHOSEN TO BE *
C *   THE LOWER JOIN POINT; THE UPPER BOUND (UB) IS THE T-D ELEMENT PAIR WHOSE *
C *   SUBSCRIPTS ARE SELECTED FROM VECTOR IU.  THE COMPUTATIONAL SCHEME IS TO  *
C *   FIRST SET THE LB AT T(1),D(1) AND THE UB AT T(IU(1)),D(IU(1)).  THE LB OF*
C *   THE I-TH PARTITION OF T AND D IS T(JOIN),D(JOIN), IF ONE EXISTS, AND THE *
C *   UB IS T(IU(I)),D(IU(I)).  IF NO LEAST RSS CANDIDATE HAS BEEN FOUND, THE  *
C *   LB REMAINS AT T(1),D(1).  THE COEFFICIENTS AND STANDARD DEVIATIONS OF THE*
C *   LINEAR SEGMENTS ARE STORED IN ARRAY DELTA.                              *
C *                                                                           *
C *                IEND = LENGTH OF T AND D                                   *
C *                ITEST = LENGTH OF IU + 1                                   *
C *                IU = TEST POINT VECTOR                                     *
C *                T,D = TIME AND DISTANCE VECTORS                            *
C *                IDEL = DIMENSION OF DELTA, (4 X IDEL)                      *
C *                DELTA(1,I) = INTERCEPT OF I-TH LAYER                       *
C *                DELTA(2,I) = STANDARD DEVIATION OF INTERCEPT               *
C *                DELTA(3,I) = SLOPE OF I-TH LAYER                           *
C *                DELTA(4,I) = STANDARD DEVIATION OF SLOPE                   *
C *                DELTA(5,I) = VELOCITY OF I-TH LAYER                        *
C *                DELTA(6,I) = RESIDUAL SUM OF SQUARES FOR I-TH LAYER        *
C *                NPTS = VECTOR OF LENGTH IDEL CONTAINING THE NUMBER OF DATA *
C *                       POINTS IN EACH LAYER                                *
C *                AMIN = TOTAL RESIDUAL SUM OF SQUARES                       *
C *                                                                           *
C *   SUBROUTINES REQUIRED:  SJOIN,CJOIN,OUTPUT                               *
C *                                                                           *
C ***************************************************************************
C
      REAL*8 BETA(12,23),STD(8,23),GAMMA(5,22),DELTA(6,6)               SETS002
      REAL*8 TMIN,SMIN,AMIN,SAVE(6)/6*0.D0/                             SETS003
      INTEGER*4 NREAD/1/,NPUNCH/2/,NPRINT/3/                            SETS004
      REAL T(24),D(24),ST(24),SD(24)                                    SETS005
```

```
      INTEGER IU(10),INDEX(22),NPTS(1)                                  SETS006
      COMMON BETA,STD,GAMMA                                             SETS007
      IDEL = 0                                                          SETS008
      ISHIFT = 0                                                        SETS009
      ISAVE = 0                                                         SETS010
      MARK = -1                                                         SETS011
      IU(ITEST) = IEND                                                  SETS012
      IM = 0                                                            SETS013
      I1 = 1                                                            SETS014
      I2 = 1                                                            SETS015
C                                                                       SETS016
      DO 50 I = 1,ITEST                                                 SETS017
      L = IU(I) - ISHIFT                                                SETS018
      IF (L.LT.4) GO TO 50                                              SETS019
      K = 0                                                             SETS020
      DO 8 J = 1,L                                                      SETS021
      ST(J) = T(J + ISHIFT)                                             SETS022
    8 SD(J) = D(J + ISHIFT)                                             SETS023
      IS = ISHIFT                                                       SETS024
    9 CALL SJOIN (L,ST,SD)                                              SETS025
      CALL CJOIN (L,0,SD,INDEX,TMIN,ITMIN,SMIN,ISMIN)                   SETS026
      CALL OUTPUT (L,ST,SD,INDEX,I,IS)                                  SETS027
      WRITE (NPRINT,1) ITMIN,TMIN,ISMIN,SMIN                            SETS028
    1 FORMAT (' T INDEX=',I3,' MINIMUM T=',D13.5,10X,'S INDEX=',I3,     SETS029
     X ' MINIMUM S=',D13.5)                                             SETS030
      IF ((TMIN.EQ.1.D70).AND.(SMIN.EQ.1.D70)) GO TO 50                 SETS031
      IF (TMIN.EQ.1.D70) GO TO 20                                       SETS032
      IF (SMIN.EQ.1.D70) GO TO 10                                       SETS033
      IF (TMIN.GT.SMIN) GO TO 20                                        SETS034
C     T CANDIDATE                                                       SETS035
   10 ISHIFT = ITMIN + 1 + ISHIFT                                       SETS036
      IF (.NOT.(((MARK + 1).EQ.I).AND.(K.EQ.0))) GO TO 12               SETS037
      NPTS(IDEL + 3) = L - ITMIN - 1                                    SETS038
      DELTA(1,IDEL + 3) = BETA(4,ITMIN)                                 SETS039
      DELTA(2,IDEL + 3) = STD(5,ITMIN)                                  SETS040
      DELTA(3,IDEL + 3) = BETA(5,ITMIN)                                 SETS041
      DELTA(4,IDEL + 3) = STD(6,ITMIN)                                  SETS042
```

```
      DELTA(5,IDEL + 3) = STD(8,ITMIN)                                  SETS043
      DELTA(6,IDEL + 3) = BETA(6,ITMIN)                                 SETS044
      L = ISHIFT - ISAVE                                                SETS045
      K = K + 1                                                         SETS046
      IS = ISAVE                                                        SETS047
      DO 11 J = 1,L                                                     SETS048
      ST(J) = T(J + ISAVE)                                              SETS049
   11 SD(J) = D(J + ISAVE)                                              SETS050
      GO TO 9                                                           SETS051
   12 IF ((K.EQ.1).AND.((BETA(5,ITMIN).GT.BETA(2,ITMIN)).OR.            SETS052
     X (DELTA(3,IDEL + 3).GT.BETA(5,ITMIN)))) GO TO 18                  SETS053
      IF (IDEL.EQ.0) GO TO 13                                           SETS054
      IF (DELTA(3,IDEL).LT.BETA(I1 + 1,ITMIN)) GO TO 50                 SETS055
   13 IDEL = IDEL + 1                                                   SETS056
      NPTS(IDEL) = ITMIN + 1                                            SETS057
      IF (IM.EQ.1) NPTS(IDEL) = L - ITMIN - 1                           SETS058
      DELTA(1,IDEL) = BETA(I1,ITMIN)                                    SETS059
      DELTA(2,IDEL) = STD(I2,ITMIN)                                     SETS060
      DELTA(3,IDEL) = BETA(I1 + 1,ITMIN)                                SETS061
      DELTA(4,IDEL) = STD(I2 + 1,ITMIN)                                 SETS062
      DELTA(5,IDEL) = STD(I2 + 3,ITMIN)                                 SETS063
      DELTA(6,IDEL) = BETA(I1 + 2,ITMIN)                                SETS064
      IF (K.EQ.1) GO TO 14                                              SETS065
      IF (I.LT.ITEST) GO TO 50                                          SETS066
   14 IM = IM + 1                                                       SETS067
      I1 = 4                                                            SETS068
      I2 = 5                                                            SETS069
      GO TO (13),IM                                                     SETS070
      IM = 0                                                            SETS071
      IF (K.EQ.1) IDEL = IDEL + 1                                       SETS072
      GO TO 50                                                          SETS073
C     S CANDIDATE                                                       SETS074
   20 IF ((MARK + 1).NE.I) GO TO 31                                     SETS075
      IF ((K.EQ.1).AND.((GAMMA(4,ISMIN).GT.GAMMA(2,ISMIN)).OR.          SETS076
     X (DELTA(3,IDEL + 3).GT.GAMMA(4,ISMIN)))) GO TO 18                 SETS077
      IF (K.EQ.1) GO TO 23                                              SETS078
      IDEL = IDEL + 1                                                   SETS079
```

```
      NPTS(IDEL) = ISE                                                  SETS080
      DO 30 J = 1,6                                                     SETS081
   30 DELTA(J,IDEL) = SAVE(J)                                           SETS082
   31 IF (I.EQ.ITEST) GO TO 21                                          SETS083
      MARK = I                                                          SETS084
      ISAVE = ISHIFT                                                    SETS085
      ISHIFT = ISMIN + ISHIFT                                           SETS086
      ISE = ISMIN + 1                                                   SETS087
      SAVE(1) = GAMMA(1,ISMIN)                                          SETS088
      SAVE(2) = STD(1,ISMIN)                                            SETS089
      SAVE(3) = GAMMA(2,ISMIN)                                          SETS090
      SAVE(4) = STD(2,ISMIN)                                            SETS091
      SAVE(5) = 1.D0 / GAMMA(2,ISMIN)                                   SETS092
      SAVE(6) = BETA(3,ISMIN)                                           SETS093
      GO TO 50                                                          SETS094
   18 NPTS(IDEL + 3) = 0                                                SETS095
      DO 19 J = 1,6                                                     SETS096
   19 DELTA(J,IDEL + 3) = 0.D0                                          SETS097
      GO TO 50                                                          SETS098
   21 IF (GAMMA(4,ISMIN).GT.GAMMA(2,ISMIN)) GO TO 50                    SETS099
      IF (IDEL.EQ.0) GO TO 23                                           SETS100
      IF (GAMMA(2,ISMIN).GT.DELTA(3,IDEL)) GO TO 50                     SETS101
   23 KK = 0                                                            SETS102
      I1 = 1                                                            SETS103
      I2 = 1                                                            SETS104
   22 IDEL = IDEL + 1                                                   SETS105
      NPTS(IDEL) = ISMIN + 1                                            SETS106
      IF (KK.EQ.1) NPTS(IDEL) = L - ISMIN                               SETS107
C     NOTE: THE STANDARD DEVIATIONS OF THE S CANDIDATE COEFFICIENTS ARE  SETS108
C     TAKEN FROM STD AND ARE APPROXIMATIONS OF THE TRUE STANDARD        SETS109
C     DEVIATIONS.                                                       SETS110
      DELTA(1,IDEL) = GAMMA(I1,ISMIN)                                   SETS111
      DELTA(2,IDEL) = STD(I2,ISMIN)                                     SETS112
      DELTA(3,IDEL) = GAMMA(I1 + 1,ISMIN)                               SETS113
      DELTA(4,IDEL) = STD(I2 + 1,ISMIN)                                 SETS114
      DELTA(5,IDEL) = 1.D0 / GAMMA(I1 + 1,ISMIN)                        SETS115
      KK = KK + 1                                                       SETS116
```

```
      I1 = 3                                                          SETS117
      I2 = 5                                                          SETS118
      GO TO (22),KK                                                   SETS119
      DELTA(6,IDEL - 1) = 0.DO                                        SETS120
      DELTA(6,IDEL) = GAMMA(5,ISMIN)                                  SETS121
      IF (K.EQ.1) IDEL = IDEL + 1                                     SETS122
   50 CONTINUE                                                        SETS123
C                                                                     SETS124
      IF (IDEL.EQ.0) GO TO 80                                         SETS125
      AMIN = 0.DO                                                     SETS126
      DO 75 I = 1,IDEL                                                SETS127
   75 AMIN = AMIN + DELTA(6,I)                                        SETS128
      RETURN                                                          SETS129
C     IF NO CANDIDATE HAS BEEN FOUND, RUN THE COMPLETE SET.           SETS130
   80 CALL SJOIN (IEND,T,D)                                           SETS131
      CALL CJOIN (IEND,0,D,INDEX,TMIN,ITMIN,SMIN,ISMIN)               SETS132
      CALL OUTPUT (IEND,T,D,INDEX,0,0)                                SETS133
      IDEL = 1                                                        SETS134
      NPTS(1) = IEND                                                  SETS135
      AMIN = BETA(8,IEND - 1)                                         SETS136
      DELTA(1,1) = BETA(1,IEND - 1)                                   SETS137
      DELTA(2,1) = STD(1,IEND - 1)                                    SETS138
      DELTA(3,1) = BETA(2,IEND - 1)                                   SETS139
      DELTA(4,1) = STD(2,IEND - 1)                                    SETS140
      DELTA(5,1) = STD(4,IEND - 1)                                    SETS141
      DELTA(6,1) = BETA(8,IEND - 1)                                   SETS142
      RETURN                                                          SETS143
C                                                                     SETS144
C     DELTA CONSTRAINTS                                               SETS145
C        1.  L MUST BE GREATER THAN 3.                                SETS146
C        2.  IF MINIMUM T = MINIMUM S, THE MINIMUM T IS CHOSEN.       SETS147
C        3.  VELOCITIES MUST MONOTONICALLY INCREASE.                  SETS148
C                                                                     SETS149
      END                                                             SETS150
```

```
      SUBROUTINE SJOIN (N,Y,X)                                          SJOIN001
C
C ***************************************************************************
C *                                                                         *
C *    SUBROUTINE SJOIN CALCULATES A SERIES OF COMBINATIONS OF BEST FIT     *
C *    ESTIMATES OF Y UPON X, WHICH ARE BASED ON A SIMPLE LINEAR MODEL      *
C *                  Y = A1 + B1 X  FOR X(1) < X < X(I)                     *
C *                  Y = A2 + B2 X  FOR X(I+1) < X < X(N)                   *
C *    THE SERIES COMPRISES THOSE COMBINATIONS OF LINES FROM I = 2 TO I = N - 2 *
C *    AND A SIMPLE LINEAR MODEL Y = A1 + B1 X FOR ALL X.                   *
C *                  N = LENGTH OF X AND Y                                  *
C *                  Y = DEPENDENT VARIABLE VECTOR                          *
C *                  X = INDEPENDENT VARIABLE VECTOR                        *
C *    BETA MUST BE DIMENSIONED 12 X (N-1)                                  *
C *    STD MUST BE DIMENSIONED 8 X (N-1)                                    *
C *                                                                         *
C *    THE OUTPUT DATA ARE STORED AS:                                       *
C *                  BETA(1,I)=A1, BETA(2,I)=B1, BETA(3,I)= RESID SUM OF SQUARES *
C *                  BETA(4,I)=A2, BETA(5,I)=B2, BETA(6,I)= RESID SUM OF SQUARES *
C *                  BETA(7,I) = X VALUE OF THE INTERSECTION OF THE TWO LINES *
C *                  BETA(8,I) = COMBINED RESIDUAL SUM OF SQUARES           *
C *                  STD(1,I) = STANDARD DEVIATION OF A1                    *
C *                  STD(2,I) = STANDARD DEVIATION OF B1                    *
C *                  STD(3,I) = CORRELATION COEFFICIENT                     *
C *                  STD(4,I) = RECIPROCAL OF B1                            *
C *                  STD(5,I) = STANDARD DEVIATION OF A2                    *
C *                  STD(6,I) = STANDARD DEVIATION OF B2                    *
C *                  STD(7,I) = CORRELATION COEFFICIENT                     *
C *                  STD(8,I) = RECIPROCAL OF B2                            *
C *                                                                         *
C ***************************************************************************
C
      IMPLICIT REAL*8 (A-H,O-Z)                                         SJOIN002
      REAL*8 NUM,NUMER                                                  SJOIN003
      REAL X(1),Y(1)                                                    SJOIN004
      COMMON BETA(12,23),STD(8,23),GAMMA(5,22)                          SJOIN005
      JJ = N - 1                                                        SJOIN006
```

```
      DO 28 J = 1,JJ                                                    SJOIN007
      I1 = -2                                                           SJOIN008
      J1 = 7                                                            SJOIN009
      K1 = -3                                                           SJOIN010
      JM = 1                                                            SJOIN011
      JN = J + 1                                                        SJOIN012
    9 I1 = I1 + 3                                                       SJOIN013
      I2 = I1 + 1                                                       SJOIN014
      I3 = I1 + 2                                                       SJOIN015
      J1 = J1 + 2                                                       SJOIN016
      J2 = J1 + 1                                                       SJOIN017
      K1 = K1 + 4                                                       SJOIN018
      K2 = K1 + 1                                                       SJOIN019
      K3 = K1 + 2                                                       SJOIN020
      K4 = K1 + 3                                                       SJOIN021
      SX = 0.D0                                                         SJOIN022
      SY = 0.D0                                                         SJOIN023
      SXX = 0.D0                                                        SJOIN024
      SYY = 0.D0                                                        SJOIN025
      SXY = 0.D0                                                        SJOIN026
      DO 10 I = JM,JN                                                   SJOIN027
      XI = X(I)                                                         SJOIN028
      YI = Y(I)                                                         SJOIN029
      SX = SX + XI                                                      SJOIN030
      SY = SY + YI                                                      SJOIN031
      SXX = SXX + XI * XI                                               SJOIN032
      SYY = SYY + YI * YI                                               SJOIN033
   10 SXY = SXY + XI * YI                                               SJOIN034
      G = JN - JM + 1                                                   SJOIN035
      NUM = G * SXY - SX * SY                                           SJOIN036
      NUMER = SXX * SY - SX * SXY                                       SJOIN037
      DENOM = G * SXX - SX * SX                                         SJOIN038
      DENO = G * SYY - SY * SY                                          SJOIN039
      BETA(I1,J) = NUMER / DENOM                                        SJOIN040
      BETA(I2,J) = NUM / DENOM                                          SJOIN041
      IF (G.EQ.2.D0) GO TO 11                                           SJOIN042
      A = DENOM * DENO - NUM * NUM                                      SJOIN043
```

```
      B = A / ((G-2.D0) * DENOM * DENOM)                                  SJOIN044
      BETA(I3,J) = A / (G * DENOM)                                        SJOIN045
      GO TO 12                                                            SJOIN046
   11 BETA(I3,J) = 0.D0                                                   SJOIN047
   12 BETA(J1,J) = SX                                                     SJOIN048
      BETA(J2,J) = SXX                                                    SJOIN049
      IF (BETA(I3,J).GT.0.D0) GO TO 13                                    SJOIN050
      STD(K1,J) = 0.D0                                                    SJOIN051
      STD(K2,J) = 0.D0                                                    SJOIN052
      GO TO 14                                                            SJOIN053
   13 STD(K1,J) = DSQRT((B/(G*G))*(DENOM+SX*SX))                          SJOIN054
      STD(K2,J) = DSQRT(B)                                                SJOIN055
   14 IF ((DENOM.LE.0.D0).OR.(DENO.LE.0.D0)) GO TO 15                     SJOIN056
      STD(K3,J) = NUM/(DSQRT(DENOM)*DSQRT(DENO))                          SJOIN057
      GO TO 16                                                            SJOIN058
   15 STD(K3,J) = 0.1D2                                                   SJOIN059
   16 IF (DABS(BETA(I2,J)).LT.1.D-50) GO TO 17                            SJOIN060
      STD(K4,J) = 1.D0/BETA(I2,J)                                         SJOIN061
      GO TO 18                                                            SJOIN062
   17 STD(K4,J) = 1.D70                                                   SJOIN063
   18 IF (J.LT.(JJ - 1)) GO TO 23                                         SJOIN064
      IF (J.EQ.JJ) GO TO 20                                               SJOIN065
      BETA(5,J)=(BETA(1,J)+BETA(2,J)*X(N-1)-Y(N))/(X(N-1)-X(N))           SJOIN066
      BETA(4,J) = Y(N) - BETA(5,J)*X(N)                                   SJOIN067
      BETA(6,J) = 0.D0                                                    SJOIN068
      BETA(8,J) = BETA(3,J)                                               SJOIN069
      BETA(11,J) = X(N)                                                   SJOIN070
      BETA(12,J) = X(N) * X(N)                                            SJOIN071
      DO 19 I = 5,7                                                       SJOIN072
   19 STD(I,J) = 0.D0                                                     SJOIN073
      STD(8,J) = 1.D0 / BETA(5,J)                                         SJOIN074
      JM = N                                                              SJOIN075
      GO TO 25                                                            SJOIN076
   20 DO 21 I = 4,7                                                       SJOIN077
   21 BETA(I,J) = 0.D0                                                    SJOIN078
      BETA(8,J) = BETA(3,J)                                               SJOIN079
      BETA(11,J) = 0.D0                                                   SJOIN080
```

```
      BETA(12,J) = 0.D0                                                 SJOIN081
      DO 22 I = 5,8                                                     SJOIN082
   22 STD(I,J) = 0.D0                                                   SJOIN083
      GO TO 28                                                          SJOIN084
   23 IF (I1.NE.1) GO TO 24                                             SJOIN085
      JM = JN + 1                                                       SJOIN086
      JN = N                                                            SJOIN087
      GO TO 9                                                           SJOIN088
   24 BETA(8,J) = BETA(3,J) + BETA(6,J)                                 SJOIN089
   25 IF ((BETA(2,J) - BETA(5,J)).GT.0.D0) GO TO 26                     SJOIN090
      BETA(7,J) = 1.D70                                                 SJOIN091
      GO TO 27                                                          SJOIN092
   26 BETA(7,J) = (BETA(4,J)-BETA(1,J))/(BETA(2,J)-BETA(5,J))           SJOIN093
   27 IF(((BETA(7,J).LT.X(JM-1)).OR.(BETA(7,J).GT.X(JM))).AND.          SJOIN094
     X (J.LT.(JJ-1)))  BETA(8,J) = 1.D70                                SJOIN095
   28 CONTINUE                                                          SJOIN096
      RETURN                                                            SJOIN097
      END                                                               SJOIN098
```

```
      SUBROUTINE CJOIN (N,LOCK,X,INDEX,TMIN,ITMIN,SMIN,ISMIN)          CJOIN001
C
C ***************************************************************************
C *                                                                         *
C *    SUBROUTINE CJOIN CALCULATES A SERIES OF COMBINATIONS OF BEST FIT     *
C *    ESTIMATES OF Y UPON X, WHICH ARE BASED ON A SIMPLE LINEAR MODEL      *
C *                  Y = A1 + B1 X  FOR X(1) < X < X(I)                     *
C *                  Y = A2 + B2 X  FOR X(I) < X < X(N)                     *
C *    THE INDEPENDENT VECTOR X HAS LENGTH N.  SEVERAL TESTS ARE APPLIED TO *
C *    MINIMIZE THE NUMBER OF MODELS ON WHICH THE CALCULATIONS MUST BE MADE. *
C *    EACH ELEMENT OF VECTOR INDEX (LENGTH = N - 2) IS KEYED TO INDICATE   *
C *    WHETHER ANY TEST HAS SUCESSFULLY ELIMINATED THE CALCULATION OF THAT  *
C *    MODEL, AND IF SO, WHICH TEST.  IF LOCK = 1, ALL TESTS ARE LOCKED OUT, AND*
C *    THE RSS FOR ALL MODELS IS CALCULATED.  THE MINIMUM RSS FROM SJOIN (TMIN) *
C *    AND ITS INDEX (ITMIN) AND THE RSS FROM CJOIN (SMIN) AND ITS INDEX (ISMIN)*
C *    ARE FOUND.  GAMMA MUST BE DIMENSIONED 5 X (N - 2).                   *
C *                                                                         *
C *    THE OUTPUT DATA ARE STORED AS:                                       *
C *                  GAMMA(1,I) = A1, GAMMA(2,I) = B1                       *
C *                  GAMMA(3,I) = A2, GAMMA(4,I) = B2                       *
C *                  GAMMA(5,I) = COMBINED RESIDUAL SUM OF SQUARES          *
C *                                                                         *
C ***************************************************************************
C
      IMPLICIT REAL*8 (A-H,O-Z)                                        CJOIN002
      REAL X(1)                                                        CJOIN003
      INTEGER INDEX(1)                                                 CJOIN004
      COMMON BETA(12,23),STD(8,23),GAMMA(5,22)                         CJOING05
      NN = N - 3                                                       CJOIN006
      NNN = N - 2                                                      CJOIN007
      IJK = -1                                                         CJOIN008
      SMIN = 1.D70                                                     CJOIN009
      TMIN = 1.D70                                                     CJOIN010
      DO 4 J = 1,NNN                                                   CJOIN011
      DO 4 I = 1,5                                                     CJOIN012
    4 GAMMA(I,J) = 0.D0                                                CJOIN013
      INDEX(1) = 1                                                     CJOIN014
```

```
      DO 5 I = 2,NNN                                                    CJOIN015
      INDEX(I) = I                                                      CJOIN016
      IF (LOCK.EQ.1) GO TO 5                                            CJOIN017
C     BY TEST 1                                                         CJOIN018
      IF (BETA(8,I-1).GE.1.D70) GO TO 5                                 CJOIN019
      INDEX(I) = 101                                                    CJOIN020
      INDEX(I - 1) = 100                                                CJOIN021
    5 CONTINUE                                                          CJOIN022
    6 DO 7 I = 1,NN                                                     CJOIN023
      IF (BETA(8,I).GE.TMIN) GO TO 7                                    CJOIN024
      TMIN = BETA(8,I)                                                  CJOIN025
      ITMIN = I                                                         CJOIN026
    7 CONTINUE                                                          CJOIN027
      IF (TMIN.EQ.1.D70) ITMIN = N/2                                    CJOIN028
      IF (LOCK.EQ.1) GO TO 20                                           CJOIN029
C     BY TEST 2                                                         CJOIN030
      DO 9 I = 2,NN                                                     CJOIN031
      IF ((BETA(3,I-1)+BETA(6,I-1)).LT.TMIN) GO TO 9                    CJOIN032
      INDEX(I) = 201                                                    CJOIN033
      INDEX(I - 1) = 200                                                CJOIN034
    9 CONTINUE                                                          CJOIN035
C                                                                       CJOIN036
   10 J = ITMIN                                                         CJOIN037
      GO TO 13                                                          CJOIN038
   20 IJK = 0                                                           CJOIN039
   11 J = 1                                                             CJOIN040
      GO TO 13                                                          CJOIN041
   12 J = J + 1                                                         CJOIN042
   13 IF (J.GE.NNN) GO TO 48                                            CJOIN043
      IF ((INDEX(J).EQ.0).OR.(INDEX(J).GE.100)) GO TO 16                CJOIN044
C                                                                       CJOIN045
C     SELECTION OF POINT - TRY  X(J+1)                                  CJOIN046
C                                                                       CJOIN047
      INDEX(J) = 0                                                      CJOIN048
      E = J + 1                                                         CJOIN049
      F = N - J - 1                                                     CJOIN050
      XX = X(J+1)                                                       CJOIN051
```

```
      D1 = E * BETA(10,J) - BETA(9,J)**2                                CJOIN052
      D2 = F * BETA(12,J) - BETA(11,J)**2                               CJOIN053
      G = BETA(1,J) - BETA(4,J) + XX*(BETA(2,J)-BETA(5,J))              CJOIN054
      H = (BETA(10,J)-2.D0*XX*BETA(9,J)+XX**2*E)/D1 + (BETA(12,J)-2.D0* CJOIN055
     X XX*BETA(11,J)+XX**2*F)/D2                                        CJOIN056
      Z = G / H                                                         CJOIN057
      GAMMA(1,J) = BETA(1,J) - G/(H*D1) * (BETA(10,J) - XX*BETA(9,J))   CJOIN058
      GAMMA(2,J) = BETA(2,J) + G/(H*D1) * (BETA(9,J) - XX*E)            CJOIN059
      GAMMA(3,J) = BETA(4,J) + G/(H*D2) * (BETA(12,J) - XX*BETA(11,J))  CJOIN060
      GAMMA(4,J) = BETA(5,J) - G/(H*D2) * (BETA(11,J) - XX*F)           CJOIN061
      GAMMA(5,J) = BETA(3,J) + BETA(6,J) + G**2/H                       CJOIN062
      IF (GAMMA(5,J).GE.SMIN) GO TO 17                                  CJOIN063
      SMIN = GAMMA(5,J)                                                 CJOIN064
      ISMIN = J                                                         CJOIN065
   17 IF (LOCK.EQ.1) GO TO 16                                           CJOIN066
C     BY TEST 3                                                         CJOIN067
      DO 14 I = 2,NNN                                                   CJOIN068
      IF ((BETA(3,I-1)+BETA(6,I-1)).LT.SMIN) GO TO 14                   CJOIN069
      INDEX(I) = 301                                                    CJOIN070
      INDEX(I-1) = 300                                                  CJOIN071
   14 CONTINUE                                                          CJOIN072
      IF (BETA(8,J).EQ.1.D70) GO TO 16                                  CJOIN073
      X1 = BETA(9,J)/E                                                  CJOIN074
      X2 = BETA(11,J)/F                                                 CJOIN075
C     BY TEST 4                                                         CJOIN076
      IF ((BETA(7,J).LT.X1).OR.(BETA(7,J).GT.X2)) GO TO 16              CJOIN077
      DO 15 I = 1,NNN                                                   CJOIN078
      IF ((X(I + 1).GT.X2).OR.(X(I + 1).LT.X1)) GO TO 15                CJOIN079
      INDEX(I) = 400                                                    CJOIN080
   15 CONTINUE                                                          CJOIN081
   16 IJK = IJK + 1                                                     CJOIN082
      IF (IJK) 11,11,12                                                 CJOIN083
   48 SMIN = 1.D70                                                      CJOIN084
      DO 49 I = 1,NN                                                    CJOIN085
      IF (INDEX(I).NE.0) GO TO 49                                       CJOIN086
      IF (GAMMA(5,I).GT.SMIN) GO TO 49                                  CJOIN087
      SMIN = GAMMA(5,I)                                                 CJOIN088
```

```
      ISMIN = I                                                    CJOIN089
   49 CONTINUE                                                     CJOIN090
      IF (TMIN.EQ.1.D70) ITMIN = NN                                CJOIN091
      IF (BETA(8,NNN).GE.TMIN) GO TO 52                            CJOIN092
      IF (INDEX(NNN).GE.100) GO TO 53                              CJOIN093
   50 GAMMA(1,NNN) = BETA(1,NNN)                                   CJOIN094
      GAMMA(2,NNN) = BETA(2,NNN)                                   CJOIN095
      GAMMA(3,NNN) = BETA(4,NNN)                                   CJOIN096
      GAMMA(4,NNN) = BETA(5,NNN)                                   CJOIN097
      GAMMA(5,NNN) = BETA(8,NNN)                                   CJOIN098
      INDEX(NNN) = 0                                               CJOIN099
      IF (GAMMA(5,NNN).GT.SMIN) GO TO 51                           CJOIN100
      SMIN = GAMMA(5,NNN)                                          CJOIN101
      ISMIN = NNN                                                  CJOIN102
   51 RETURN                                                       CJOIN103
C     BY TEST 5                                                    CJOIN104
   52 INDEX(NNN) = 500                                             CJOIN105
   53 IF (SMIN.EQ.1.D70) ISMIN = NNN                               CJOIN106
      RETURN                                                       CJOIN107
      END                                                          CJOIN108
```

```
      SUBROUTINE OUTPUT (L,T,D,INDEX,IJ,ISHIFT)                          OUT001
C
C ***************************************************************************
C *                                                                         *
C *   SUBROUTINE OUTPUT PRINTS THE COMPLETE STATISTICAL ANALYSIS OF SJOIN AND  *
C *   CJOIN OF A SUBSET T AND D, EACH OF LENGTH L. INDEX IS THE VECTOR CONTAIN-*
C *   ING THE TEST CODES FROM CJOIN. IJ IS THE RUN NUMBER.  ISHIFT IS THE NUM- *
C *   BER OF DISCARDED T AND D ELEMENTS PRECEEDING THE SUBSET.                *
C *                                                                         *
C ***************************************************************************
C
      REAL*8 BETA(12,23),STD(8,23),GAMMA(5,22)                           OUT002
      INTEGER*4 NREAD/1/,NPUNCH/2/,NPRINT/3/                             OUT003
      INTEGER*4 ID(5)                                                    OUT004
      REAL T(1),D(1)                                                     OUT005
      INTEGER INDEX(1),II(24),IK(24)                                     OUT006
      COMMON BETA,STD,GAMMA                                              OUT007
      COMMON/LABEL/ID,II                                                 OUT008
      I1 = 1 + ISHIFT                                                    OUT009
      I2 = L + ISHIFT                                                    OUT010
      WRITE (NPRINT,2) IJ,I1,I2,ID                                       OUT011
    2 FORMAT ('ORUN NUMBER  ',I2,2X,'INTERVAL ',I2,' - ',I2,10X,5A4//    OUT012
     X 6('   DISTANCE - TIME  '))                                        OUT013
      DO 10 I = 1,L                                                      OUT014
   10 IK(I) = II(I) + ISHIFT                                             OUT015
      WRITE (NPRINT,3) (IK(I),D(I),T(I),I = 1,L)                         OUT016
    3 FORMAT (6(' ',I2,F8.1,F7.3,2X))                                    OUT017
      WRITE (NPRINT,102)                                                 OUT018
  102 FORMAT ('0',14X,'A1',12X,'B1',11X,'RSS1',11X,'A2',12X,'B2',11X,'RS  OUT019
     XS2',11X,'JOIN',13X,'T')                                            OUT020
      K = L - 1                                                          OUT021
      DO 4 J = 1,K                                                       OUT022
      N1 = J + 1 + ISHIFT                                                OUT023
      N2 = J + 2 + ISHIFT                                                OUT024
      IF (J.GE.(K-1)) N2 = 0                                             OUT025
      IF (BETA(8,J).EQ.1.D70) GO TO 4                                    OUT026
      WRITE (NPRINT,5) N1,N2,(BETA(I,J),I = 1,8),N1,N2                   OUT027
```

```
  5 FORMAT (' ',2I2,1X,8D15.5,1X,2I2)                                          OUT028
  4 CONTINUE                                                                   OUT029
    WRITE (NPRINT,103)                                                         OUT030
103 FORMAT ('0',11X,'STD(A1)',8X,'STD(B1)',9X,'CORR',9X,'VELO(B1)',8X,         OUT031
   X 'STD(A2)',8X,'STD(B2)',9X,'CORR',9X,'VELO(B2)')                           OUT032
    DO 7 J = 1,K                                                               OUT033
    N1 = J + 1 + ISHIFT                                                        OUT034
    N2 = J + 2 + ISHIFT                                                        OUT035
    IF (J.GE.(K-1)) N2 = 0                                                     OUT036
    IF (BETA(8,J).EQ.1.D70) GO TO 7                                            OUT037
    WRITE (NPRINT,5) N1,N2,(STD(I,J),I = 1,8),N1,N2                            OUT038
  7 CONTINUE                                                                   OUT039
    WRITE (NPRINT,104)                                                         OUT040
104 FORMAT ('0',13X,'A1',13X,'B1',13X,'A2',13X,'B2',13X,'S')                   OUT041
    KK = L - 2                                                                 OUT042
    DO 12 J = 1,KK                                                             OUT043
    N3 = J + 1 + ISHIFT                                                        OUT044
    IF (GAMMA(5,J).EQ.0.D0) GO TO 12                                           OUT045
    WRITE (NPRINT,13) N3,(GAMMA(I,J),I = 1,5),N3,INDEX(J)                      OUT046
 13 FORMAT (' ',2X,I2,1X,5D15.5,1X,I2,I10)                                     OUT047
 12 CONTINUE                                                                   OUT048
    RETURN                                                                     OUT049
    END                                                                        OUT050
```

```
      SUBROUTINE ABYSS (N,NPTS,S,DELTA,ZETA)                          ABYSS001
C
C ***************************************************************************
C *                                                                         *
C *   SUBROUTINE ABYSS CALCULATES THE DEPTHS TO THE INTERFACES WITH THE ARRAY  *
C *   DELTA AND VECTOR NPTS.                                                *
C *                                                                         *
C *                N = NUMBER OF LAYERS -- DELTA(6,N)                       *
C *                S = STUDENTS-T VECTOR                                    *
C *                ZETA(1,I) = THICKNESS OF I-TH LAYER BASED ON I-TH MEAN COEFS*
C *                ZETA(2,I) = THICKNESS OF I-TH LAYER BASED ON THE LOWER CON- *
C *                            FIDENCE LIMIT                                *
C *                ZETA(3,I) = THICKNESS OF I-TH LAYER BASED ON THE UPPER CON- *
C *                            FIDENCE LIMIT                                *
C *                ZETA(4,I) = DEPTH OF I-TH LAYER BASED ON I-TH MEAN COEFS    *
C *                ZETA(5,I) = DEPTH OF I-TH LAYER BASED ON LOWER CONFIDENCE   *
C *                ZETA(6,I) = DEPTH OF I-TH LAYER BASED ON UPPER CONFIDENCE   *
C *                                                                         *
C ***************************************************************************
C
      IMPLICIT REAL*8 (A-H,O-Z)                                       ABYSS002
      REAL*8 DELTA(6,1),ZETA(6,1),S(24),DSQRT                         ABYSS003
      INTEGER*4 NPRINT/3/                                             ABYSS004
      INTEGER NPTS(1)                                                 ABYSS005
      IF (N.EQ.1) RETURN                                              ABYSS006
      NN = N - 1                                                      ABYSS007
      D = 0.D0                                                        ABYSS008
      DO 25 I = 1,NN                                                  ABYSS009
      DO 25 J = 1,6                                                   ABYSS010
   25 ZETA(J,I) = 0.D0                                                ABYSS011
      DO 30 M = 1,NN                                                  ABYSS012
      J = 0                                                           ABYSS013
      MM = M - 1                                                      ABYSS014
      JJ = NPTS(M + 1) - 2                                            ABYSS015
      IF (JJ.GE.1) GO TO 2                                            ABYSS016
      MMM = M + 1                                                     ABYSS017
      WRITE (NPRINT,1) MMM                                            ABYSS018
```

```
 1 FORMAT (' LAYER ',I2,' HAS FEWER THAN 3 DATA POINTS')          ABYSS019
   JJ = 1                                                         ABYSS020
 2 A = DELTA(1,M + 1)                                             ABYSS021
   B = DELTA(3,M + 1)                                             ABYSS022
 3 J = J + 1                                                      ABYSS023
   IF ((DELTA(3,M)**2 - B**2).LE.0.D0)    GO TO 6                 ABYSS024
   R = DSQRT(DELTA(3,M)**2 - B**2)                                ABYSS025
   IF (M.GE.2) GO TO 4                                            ABYSS026
   ZETA(J,1) = A/(2.D0*R)                                         ABYSS027
   GO TO 6                                                        ABYSS028
 4 T = 0.D0                                                       ABYSS029
   DO 5 I = 1,MM                                                  ABYSS030
   IF ((DELTA(3,I)**2 - B**2).LE.0.D0)  GO TO 6                   ABYSS031
   P = DSQRT(DELTA(3,I)**2 - B**2)                                ABYSS032
 5 T = T + P * ZETA(1,I)                                          ABYSS033
   ZETA(J,M) = (A/2.D0-T)/R                                       ABYSS034
 6 IF (J - 2) 10,20,30                                            ABYSS035
10 A = DELTA(1,M + 1) + S(JJ) * DELTA(2,M + 1)                    ABYSS036
   B = DELTA(3,M + 1) + S(JJ) * DELTA(4,M + 1)                    ABYSS037
   D = D + ZETA(1,M)                                              ABYSS038
   ZETA(4,M) = D                                                  ABYSS039
   GO TO 3                                                        ABYSS040
20 A = DELTA(1,M + 1) - S(JJ) * DELTA(2,M + 1)                    ABYSS041
   B = DELTA(3,M + 1) - S(JJ) * DELTA(4,M + 1)                    ABYSS042
   ZETA(5,M) = D - ZETA(1,M) + ZETA(2,M)                          ABYSS043
   GO TO 3                                                        ABYSS044
30 ZETA(6,M) = D - ZETA(1,M) + ZETA(3,M)                          ABYSS045
   RETURN                                                         ABYSS046
   END                                                            ABYSS047
```

APPENDIX B. TABULATION OF PREFERRED REGRESSION FITS

Table 6. Tabulation of preferred regression fits

Shot no.	Location	Shot elev. m	1st vel. m/sec	2nd vel. m/sec	Depth m	Depth$^-$ m	Depth$^+$ m	3rd vel. m/sec	Depth m	Depth$^-$ m	Depth$^+$ m
07	1-04E	357.3	3365(18)[a]								
08	1-04E	356.6	3131(09)	4189(14)	61.7	81.5	44.7				
10	1-03E E[b]	354.2	1859(05)	2212(07)	30.5	37.5	24.3	3761(11)	120.2	189.7	68.4
09	1-03E E	354.8	1887(06)	2268(07)	35.7	70.2	13.8	4263(07)	138.3	255.7	61.5
11	1-03E W	356.8	1886(04)	2810(21)	30.4	41.7	20.0				
12	1-02E	363.0	1805(11)	2534(12)	74.7	81.9	67.9				
13	1-02E	359.1	1890(12)	2526(11)	81.3	112.3	55.7				
02	1-01E	359.6	1871(08)	2253(09)	41.7	103.9	8.1	3657(08)	159.4	211.1	117.8
01	1-01W	364.5	1796(08)	4423(15)	77.2	86.5	68.2				
02	1-01W	363.5	1679(02)	3292(22)	22.6	26.1	19.2				
01	2-04E	358.9	1817(09)	4699(15)	98.6	111.9	85.9				
01	2-04E	358.9	2339(06)	3433(19)	45.9	76.1	21.0				
04	2-03E	368.4	2202(05)	3515(16)	46.4	57.8	35.7				
03	2-03E	368.2	1735(04)	3140(19)	32.9	36.3	29.5				
06	2-02E	361.2	1823(03)	2909(07)	26.7	43.4	13.2	3218(12)	61.9	96.3	38.4
05	2-02E	360.7	1883(04)	3117(14)	30.9	40.9	21.6				
04	2-01E	361.1	2576(06)	3159(13)	36.5	58.8	18.2				
03	2-01E	358.9	1752(08)	3850(16)	68.1	78.7	58.1				
03	2-01W	364.2	1881(12)	2655(12)	81.2	107.3	59.0				
04	2-01W	362.5	1890(11)	2562(13)	73.2	93.6	55.4				
01	2-02W	361.5	1809(08)	2730(14)	80.1	91.2	69.7				
02	2-02W	361.1	1843(09)	2080(06)	35.8	74.5	12.5	2800(08)	114.8	179.4	68.7
04	2-03W	370.3	1927(11)	3103(13)	90.2	101.3	79.8				
03	2-03W	370.3	1898(14)	2960(10)	95.1	148.5	54.5				
02	3-05E	354.7	4370(04)								
01	3-05E	356.2	3000(02)	4839(04)	25.4	-[c]	-				
02	3-04E	361.2	1855(04)	2164(07)	22.9	39.0	11.1	3087(12)	91.1	113.6	71.4
01	3-04E	361.0	1886(09)	3132(14)	78.1	85.9	70.7				
03	3-03E	360.8	1878(14)	3058(08)	105.2	120.9	90.7				
04	3-03E	363.1	1867(11)	2939(11)	99.0	113.1	86.0				

07	3-02E	369.2	1867(08)	2423(09)	58.2	83.2	38.3	3877(05)	169.0	307.7	86.3
08	3-02E	364.9	1823(07)	2322(08)	44.2	63.2	28.8	3108(09)	124.4	162.3	93.8
06	3-01E	367.5	1838(10)	2205(05)	52.5	-	-	3525(09)	152.9	177.4	131.1
05	3-01E	364.7	1849(09)	2428(07)	64.5	99.6	37.9	3442(09)	160.6	238.6	104.8
08	3-01W	363.4	1707(02)	1916(11)	10.1	20.1	2.4	2457(12)	74.9	89.8	61.4
07	3-01W	368.8	1829(03)	2259(09)	46.0	58.9	34.9	2467(12)	92.5	168.6	49.0
06	3-02W	369.5	1792(06)	2218(17)	35.9	44.4	28.1				
05	3-02W	369.0	1905(08)	2238(04)	58.1	171.4	11.9	2544(10)	105.2	153.5	71.2
06	3-03W	377.1	1847(12)	2845(10)	95.8	122.6	72.6				
05	3-03W	378.5	1853(07)	2133(02)	41.9	-	-	2934(15)	95.6	115.1	78.4
02	4-06E	358.9	1781(02)	4776(10)	16.5	20.3	12.8				
02	4-06E	358.9	4619(11)								
02	4-05E	358.1	1747(07)	2309(15)	45.4	56.3	35.5				
01	4-05E	355.2	1638(06)	2326(18)	40.1	68.3	17.1				
03	4-04E	362.2	1845(07)	2252(13)	46.0	57.7	35.5	3203(04)	166.6	169.4	163.9
04	4-04E	359.0	1828(09)	2373(10)	55.2	72.1	40.5	2642(05)	116.2	-	-
06	4-03E	366.6	1864(08)	2290(06)	47.8	75.7	27.6	3931(10)	148.3	169.0	129.5
05	4-03E	367.1	1855(05)	2355(05)	37.5	70.7	16.3	2847(12)	77.6	118.2	48.5
07	4-02E	368.7	1903(06)	2113(12)	32.5	69.3	9.5	3797(06)	184.1	260.2	125.5
08	4-02E	370.9	1915(14)	2831(08)	111.1	162.8	70.9				
02	4-01E	367.0	1825(10)	3977(14)	102.5	110.4	94.7				
01	4-01E	368.4	1897(10)	4019(14)	108.5	122.4	95.3				
04	4-01W	372.7	1820(10)	2350(13)	64.0	74.1	54.7				
03	4-01W	370.3	2032(02)	2315(18)	45.9	90.7	15.7				
07	4-02W	376.8	1862(11)	2226(11)	54.4	80.1	34.0				
08	4-02W	375.2	1812(09)	2225(14)	50.6	57.0	44.7				
01	4-03W	372.9	1815(10)	2279(13)	67.2	92.3	46.1				

[a]Number of points on line segment.

[b]Where not indicated, the shot with cable strung east or north is listed first at each station.

[c]Indicates wide statistical limits.

Table 6. (Continued)

Shot no.	Location	Shot elev. m	1st vel. m/sec	2nd vel. m/sec	Depth m	Depth$^-$ m	Depth$^+$ m	3rd vel. m/sec	Depth m	Depth$^-$ m	Depth$^+$ m
02	4-03W	375.6	1775(10)	2228(11)	59.1	75.8	44.4				
05	4-04W N	378.6	1836(13)	4840(10)	137.7	153.1	123.2				
03	4-04W S	379.1	1997(07)	3596(05)	100.2	120.5	82.2				
04	4-04W N	378.5	1837(12)	4371(07)	128.4	168.0	94.8				
01	5-06E	352.3	3756(08)	5701(05)	60.2	-	-				
01	5-06E	352.3	3682(04)	4747(08)	32.3	47.7	19.9				
01	5-05E	367.1	1839(06)	2020(17)	19.1	64.0	-				
02	5-05E	363.8	1861(12)	2251(11)	63.4	97.3	37.3				
03	5-04E	350.9	1893(11)	2468(13)	62.8	78.4	49.0				
04	5-04E	357.1	1765(11)	2672(13)	82.8	91.5	74.5				
06	5-03E	359.7	1820(08)	2296(16)	48.7	56.1	41.7				
05	5-03E	361.2	1829(09)	2133(02)	50.2	-	-	2586(13)	102.9	129.5	80.9
07	5-02E	369.7	1801(06)	2130(16)	42.8	58.8	29.3	2909(02)	172.9	-	-
08	5-02E	369.6	1742(04)	1926(13)	15.6	23.9	8.6	2545(07)	115.4	158.1	88.6
09	5-01E	371.9	1816(11)	3484(12)	81.8	89.3	74.7				
10	5-01E	374.3	1852(09)	3587(15)	88.7	97.1	80.6				
02	5-01W	371.2	1748(04)	4790(07)	43.2	50.5	36.3				
04	5-01W	372.1	1683(03)	3788(15)	27.1	31.7	22.6				
02	5-02W	368.1	1757(07)	2148(12)	43.3	80.1	18.2	7442(04)	248.3	302.9	201.3
01	5-02W	373.0	1770(12)	2290(10)	62.1	106.8	30.3				
04	5-03W	371.9	1797(11)	2279(13)	61.7	75.8	49.0				
03	5-03W	368.5	1780(11)	2254(12)	63.0	94.4	38.0				
05	5-04W	376.2	1820(11)	2260(10)	62.2	88.2	41.0				
06	5-04W	377.6	1486(05)	2190(17)	35.5	45.7	26.0				
07	5-05W	379.0	1793(11)	2344(11)	68.5	95.1	46.8				
08	5-05W	377.1	1826(11)	2342(07)	68.5	110.1	37.7				
06	5-06W	381.6	1887(15)	6379(09)	185.6	220.7	153.9				
05	5-06W	379.5	1730(04)	3219(16)	60.3	74.7	46.7				
04	6-16E E	342.6	1756(02)	3207(06)	12.1	36.9	-	4462(05)	49.7	83.4	26.8
04	6-16E N	342.6	1870(02)	3012(06)	10.8	23.6	1.1	4562(05)	52.9	-	-

03	6-15E E	347.4	2770(04)	3800(03)	25.4	-	-	6006(05)	64.0	82.1	49.1
03	6-15E S	347.4	1626(02)	4574(09)	17.1	20.6	13.7				
01	6-14E	347.5	1679(02)	3077(05)	23.4	59.0	1.0	6275(05)	87.2	178.3	34.4
01	6-14E	347.5	1595(02)	4764(09)	23.1	39.6	24.9				
02	6-13E N	349.0	1761(02)	3982(10)	25.2	33.6	17.3	5360(09)	100.6	187.8	46.3
01	6-13E N	359.4	2061(03)	5417(06)	49.3	55.2	43.7				
01	6-13E S	359.4	1762(04)	4540(07)	37.8	51.4	25.6				
01	6-12E	347.0	1791(05)	5271(07)	51.6	62.1	41.9				
01	6-12E	347.0	2253(08)								
01	6-11E E	351.1	1736(05)	2115(06)	27.4	60.9	7.1				
01	6-11E W	351.1	1727(03)	1912(08)	13.7	25.0	5.4				
02	6-11E W	351.7	1682(02)	2058(08)	21.4	33.4	11.5	2357(12)	66.9	141.0	24.8
01	6-10E	349.0	1781(04)	2384(07)	27.3	48.2	11.5				
01	6-10E	349.0	2127(02)	2347(08)	13.7	35.2	1.3				
01	6-09E	355.8	1799(07)	2000(04)	31.3	-	-				
01	6-09E	355.8	1869(07)	2406(04)	50.1	101.2	20.6				
02	6-08E	342.5	1752(07)	2269(04)	43.7	-	-				
02	6-08E	342.5	1860(04)	2133(05)	30.1	71.8	8.5	2909(02)	89.5	-	-
03	6-07E	359.3	1846(11)								
03	6-07E	359.3	1684(05)								
01	6-06E E	366.4	1822(08)	2000(03)	33.7	33.8	33.7				
01	6-06E W	366.4	1813(07)	2148(04)	42.3	-	-				
02	6-06E E	365.5	1843(11)	2346(11)	60.4	68.2	53.2				
03	6-05E E	361.0	1842(08)	2043(04)	34.4	-	-				
03	6-05E W	361.0	1858(05)	2717(04)	33.7	-	-				
04	6-05E E	365.9	1894(08)	2384(11)	67.2	83.7	52.7				
05	6-04E E	365.1	1514(02)	1907(09)	12.6	17.1	8.0				
05	6-04E W	365.1	1831(06)	2034(05)	31.3	-	-				
06	6-04E E	366.9	1854(11)	2284(11)	56.2	73.8	41.0				
07	6-03E E	372.1	1752(07)	2158(04)	35.4	-	-				
07	6-03E W	372.1	1852(07)	2015(05)	28.9	-	-				
08	6-03E E	373.0	1764(06)	1957(07)	26.5	49.3	11.1	3377(07)	141.5	214.4	87.6
03	6-02E E	374.0	1744(06)	3247(05)	49.5	107.0	12.5				
03	6-02E W	374.0	1727(03)	4258(06)	47.9	63.7	33.9				
04	6-02E E	374.6	1745(04)	3811(18)	36.0	69.6	6.9				
06	6-01E	371.8	2059(03)	3889(18)	33.3	45.9	21.6				
05	6-01E	370.0	1826(03)	3464(07)	32.9	36.1	29.8	4380(10)	89.2	121.5	63.4
06	6-01W W	373.3	1806(03)	3961(17)	35.3	39.6	31.0	8000(02)	236.4	-	-

Table 6. (Continued)

Shot no.	Location	Shot elev. m	1st vel. m/sec	2nd vel. m/sec	Depth m	Depth$^-$ m	Depth$^+$ m	3rd vel. m/sec	Depth m	Depth$^-$ m	Depth$^+$ m
05	6-01W E	373.3	1317(03)	4066(08)	31.6	36.6	26.7				
05	6-01W W	373.3	1455(02)	4103(09)	36.1	44.2	28.4				
07	6-02W E	368.2	1697(03)	3682(07)	31.2	39.1	23.8				
07	6-02W W	368.2	1772(02)	4507(04)	34.9	51.3	21.5				
08	6-02W W	373.3	1585(02)	3467(09)	26.5	31.4	21.8	4788(09)	89.5	161.1	44.0
04	6-03W	371.7	1831(07)	3979(13)	80.9	89.6	72.5				
03	6-03W	372.5	1888(04)	3069(07)	36.8	50.1	25.5	3414(12)	71.2	131.3	36.6
01	6-04W	374.6	1853(08)	2601(08)	55.1	132.8	12.0	4007(06)	160.3	231.5	107.7
02	6-04W	372.7	1815(07)	3779(14)	72.6	90.8	56.0				
05	6-05W E	375.7	1842(11)								
05	6-05W W	375.7	1844(11)								
06	6-05W E	373.9	1919(14)	3812(05)	143.9	162.5	126.7				
01	6-06W E	372.4	1672(05)	4814(07)	50.9	59.2	42.9				
01	6-06W W	372.4	1957(04)	3829(06)	46.5	54.7	38.8				
02	6-06W E	372.4	1735(04)	4388(07)	53.2	70.1	37.8				
03	6-07W	369.1	1780(05)	3794(06)	60.3	75.6	46.8				
03	6-07W	369.1	1822(07)	3368(05)	69.6	82.7	57.9				
04	6-07W W	369.8	1764(05)	3214(10)	45.2	61.8	30.3				
01	6-08W	369.2	1856(10)								
01	6-08W	369.2	1870(10)								
02	6-08W	373.8	1835(09)	2611(11)	86.3	131.2	51.4				
08	6-09W	375.5	1785(10)	2279(14)	63.8	90.0	42.1				
07	6-09W	375.5	1857(13)	2623(10)	95.9	118.5	75.5				
03	7-07E	346.6	1724(03)	4295(09)	34.5	40.5	28.8				
03	7-07E	346.6	3746(06)	6809(05)	80.7	-	-				
04	7-06E	349.5	1707(05)	4647(06)	55.5	71.2	41.6				
04	7-06E	349.5	1775(03)	4167(08)	45.4	53.2	38.0				
05	7-05E	350.8	1728(04)	1920(08)	21.1	50.1	4.6				
05	7-05E	350.8	1793(07)	2777(05)	41.0	-	-				
02	7-04E	352.6	1779(10)	2441(14)	70.6	89.0	54.4				

01	7-04E	349.8	1717(11)	2396(12)	67.4	90.4	47.8					
04	7-03E	355.4	1838(10)	2686(14)	75.5	85.5	66.0					
03	7-03E	356.5	1835(12)	2692(12)	77.2	95.9	60.7					
06	7-02E	362.7	1794(10)	2321(13)	64.6	80.7	50.3					
05	7-02E	364.4	1628(10)	2343(14)	60.6	68.1	53.4					
08	7-01E	371.6	1858(11)	2237(13)	58.3	72.5	45.8					
07	7-01E	369.3	1925(10)	2271(14)	61.3	78.5	46.4					
06	7-01W	372.4	1826(05)	2134(08)	25.2	56.8	6.4	3693(12)	113.0	130.9	96.9	
05	7-01W	375.9	1825(09)	3212(11)	87.9	101.8	75.0					
04	7-02W	375.4	1806(09)	3194(10)	75.9	113.2	45.5					
03	7-02W	375.8	1808(09)	3575(11)	90.1	114.0	68.8					
01	7-03W	367.2	1781(06)	3455(14)	60.2	75.2	46.5					
02	7-03W	369.5	1786(06)	2858(06)	60.6	119.5	22.2					
01	7-04W	370.9	1760(10)	2315(12)	59.4	69.9	40.8					
02	7-04W	365.4	1780(09)	2376(11)	67.2	84.2	54.1					
03	7-05W	375.8	1813(06)	2192(04)	38.4	-	-					
03	7-05W	375.8	1762(04)	1955(07)	26.2	40.2	15.4					
04	7-06W	375.1	1801(07)	2667(02)	81.5	-	-					
04	7-06W	375.1	1662(05)	1998(06)	27.5	37.7	18.9					
01	8-06E	348.2	1868(07)	5335(16)	75.8	79.4	72.3					
02	8-06E	346.8	1800(07)	5338(18)	74.6	77.4	71.7					
03	8-05E E	352.1	1816(07)	2251(06)	45.1	-	-	5333(09)	165.4	177.6	153.8	
02	8-04E	355.0	1839(12)	2366(11)	69.1	112.3	36.9					
01	8-04E	348.3	1653(06)	2287(12)	49.1	61.7	37.8					
06	8-03E	364.2	1855(13)	2736(10)	92.2	126.3	63.9					
05	8-03E	364.8	1910(15)	3213(07)	131.5	242.7	58.4					
07	8-02E	365.7	1828(12)	2188(11)	54.5	103.5	20.9					
08	8-02E	365.4	1898(14)	2471(08)	86.2	112.5	64.0					
03	8-01E	367.3	1826(10)	2588(12)	70.1	78.2	62.5					
04	8-01E	367.7	1872(12)	2902(10)	94.5	110.6	79.7					
02	8-01W	369.1	1869(15)	2372(07)	83.1	161.2	33.9					
01	8-01W	372.7	1926(15)	2314(07)	78.0	150.2	32.9					
05	8-02W	372.7	1788(09)	2299(08)	56.1	68.1	45.4	3544(06)	164.6	208.9	128.2	
06	8-02W	370.1	1746(07)	2090(08)	35.7	66.6	15.1	3038(09)	122.2	157.9	92.5	
03	8-03W	368.7	1822(12)	3617(09)	102.2	116.1	89.1					
04	8-03W	370.1	1739(08)	3088(13)	64.9	68.9	61.0					
02	8-05W	373.0	1866(14)	2701(09)	100.7	125.6	78.9					

Table 6. (Continued)

Shot no.	Location	Shot elev. m	1st vel. m/sec	2nd vel. m/sec	Depth m	Depth$^-$ m	Depth$^+$ m	3rd vel. m/sec	Depth m	Depth$^-$ m	Depth$^+$ m
01	8-05W	371.9	1874(16)	3324(07)	143.6	181.0	111.4				
01	9-05E	359.2	1881(07)	3200(02)	66.6	-	-	5017(12)	124.4	148.8	103.3
02	9-05E	358.5	1825(07)	2942(02)	59.1	-	-	5086(12)	125.1	147.6	105.0
01	9-04E	354.0	1918(10)	2607(08)	74.4	111.9	45.3				
02	9-04E	357.9	1933(10)	2771(09)	90.2	114.6	69.2				
04	9-03E	356.2	1914(08)	2403(10)	75.3	91.3	61.0				
03	9-03E	361.3	1979(15)	2538(07)	102.5	249.9	28.7				
05	9-02E	360.2	1646(09)	3051(13)	91.7	120.9	66.3				
06	9-02E	357.8	1783(07)	2876(16)	59.2	73.1	46.4				
07	9-01E	367.4	2020(10)	2799(14)	77.6	83.7	71.8				
08	9-01E	364.9	1836(08)	2631(16)	60.3	67.1	53.8				
01	9-01W	373.0	1758(05)	3071(16)	41.1	49.7	32.9				
02	9-01W	376.2	1839(07)	2471(08)	52.7	67.1	40.3	5091(07)	181.4	214.2	152.6
04	9-02W	368.0	1841(10)	2831(13)	83.5	93.4	74.1				
03	9-02W	363.7	1873(11)	2886(12)	93.0	109.9	77.5				
06	9-03W	372.9	1802(08)	2716(15)	70.4	80.9	60.6				
05	9-03W	370.6	1790(11)	2807(13)	81.3	96.2	67.6				
02	10-5E	347.6	1867(07)	2667(03)	49.8	-	-	5035(10)	127.1	140.5	114.4
01	10-5E	350.1	1842(09)	4005(02)	91.1	-	-	5345(11)	141.5	170.5	117.5
03	10-3E	351.3	1937(13)	3404(04)	153.1	306.2	63.1				
04	10-3E	353.9	1901(08)	2422(12)	67.7	85.7	52.0				
05	10-2E	356.9	1868(09)	2903(13)	63.9	69.5	58.6				
06	10-2E	356.7	1855(08)	2901(14)	62.3	71.6	53.6				
07	10-1E	356.6	1789(05)	2973(18)	42.5	66.0	21.9				
08	10-1E	359.6	1806(05)	3167(15)	62.9	71.3	54.8				
03	10-1W	374.7	1868(10)	3462(13)	88.3	100.7	76.7				
04	10-1W	369.4	2006(09)	3470(14)	84.8	91.9	77.9				
09	10-2W	364.7	1856(11)	3281(12)	99.7	113.9	86.4				
10	10-2W	364.6	1848(08)	2795(16)	62.1	80.8	45.4				
08	10-3W	373.0	1887(16)	2377(08)	92.1	141.4	55.6				

07	10-3W	370.1	1879(11)	2267(08)	76.5	215.2	13.0				
06	11-1E	357.7	1936(06)	2133(02)	14.4	-	-	5312(14)	94.5	99.7	89.4
05	11-1E	356.6	2003(07)	5433(15)	89.7	95.3	84.2				
08	11-1W	367.7	1804(06)	4776(15)	68.6	73.1	64.2				
07	11-1W	367.7	1753(12)	4965(10)	73.9	75.9	72.0				