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DOES $\lambda_{2,2}$ VARY?

C. A. WAGNER

NOVEMBER 1971

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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ABSTRACT

An attempt has been made to find a secular drift in $\lambda_{2,2}$, or the phase of the low order and degree portion of the geogravity field. This portion may be associated with mass anomalies near the core-mantle boundary. From the geomagnetic evidence, such anomalies might have westward drifts on the order of 0.5 degrees/year. Tracking data on 8 synchronous satellites over a period of 6 years were examined for residual accelerations which might be explained by a drift of the $\lambda_{2,2}$ gravity phase angle. No conclusive movement of $\lambda_{2,2}$ was detected. But a measured upper bound on the drift of less than 0.05 degrees/year is still compatible with possible slow moving irregularities in the region of the core-mantle boundary.

CONTENTS

DOES $\lambda_{2,2}$ VARY?

INTRODUCTION

Until fairly recently, the gravitational field associated with the earth's mass has been assumed to be fixed. It is obvious however, that there must be small but important changes of the field, in time, due to a variety of causes.

Since 1965, measurements of satellite orbit deviations have confirmed the existence of the solar tide induced in the earth's body.[1], [2] These observations have generally agreed with previous assumptions about the overall elasticity of the earth. In addition, recent satellite orbit studies have revealed changes in the earth's oblateness associated with known rotation rate changes and seasonal effects perhaps due to solar heating.[3], [4]

But so far there have been no unambiguous measurements of field changes due to the dynamics of the earth's own mass redistribution. The well known secular variations of the magnetic field (on the order of 0.5°/yr.), the almost certain existence of continental drift as well as the more speculative internal convection currents in the mantle, all imply significant changes in the 'fixed' gravitational field.[5] Some of these changes (if they exist) are now capable of being measured by satellites.

The only known ground based observations of geogravity changes, from long term sea level monitoring, are inconclusive.[5] The data itself (determination of mean sea level from tidal records) is highly variable. The estimated trends yield formal standard errors an order of magnitude greater than the estimations themselves. However, predicted sea level trends on the basis of a 0.2°/yr. westward drift of only a small fraction (~20%) of the low degree and order harmonics are at the level of accuracy of the observations (2 to 3 cm/yr.). The implication is that no more than a small fraction of the low order anomalies are associated with the core or the core-mantle boundary. However, the interpretation of the observations is ambiguous since changes in sea level may be due to a number of causes not associated with gravity drift. For example, continental plate motions (vertical and horizontal) on the earth's surface are now thought to be of the order of 0.1 to 10 cm/yr.[5], [6]

The use of satellite tracking data over long periods of time offers the possibility of monitoring these gravity changes unambiguously.

THE EXPERIMENT MODEL AND DATA ANALYSIS

The Model

In theory the determination of gravity changes by satellite tracking could be made from any well and long observed orbit. In this study the record of the synchronous (24 hour) orbits were examined because they have almost ideal geometric and dynamic properties for monitoring changes in the low order field.

Geometrically the orbits are nearly geostationary. For long periods of time they have remained over about the same geographic longitudes. In principal, such an orbit is in effect a gravity meter set up at a single longitude for long periods of time. The determination of the anomalous acceleration by tracking spacecraft in these orbits gives repetitive 'gravity' measurements at the same locations; the ideal experimental situation for monitoring long term changes.

Dynamically, the synchronous orbits are ideal because, being stationary, they are extremely sensitive to the very small anomalous accelerations from the gravity field. On stationary orbits these accelerations build up cumulatively. Being amplified, they are easily detected and therefore well measured. The additional benefit of the high synchronous orbit is that it is sensitive to only the lowest degree and order gravity anomalies. These are the only anomalies which, because of their long wavelengths, might originate near the core-mantle boundary in spite of the great overburden pressures.[5]

Considering the geogravity field developed into an infinite series of spherical harmonics, the geostationary orbits are especially sensitive to (or resonant with) the longitude dependent terms. In fact all but about 15% of the resonant acceleration on these satellites is accounted for by the (2,2) harmonic term.[7] The experiment in this study was to try and detect a steady shift or drift of the (2,2) harmonic, in particular it's phase angle ($\lambda_{2,2}$), examining 6 years of tracking data. The amplitude of the (2,2) harmonic ($J_{2,2}$) might also be considered variable. But it's variability is more difficult to justify physically since it's change implies stress changes inside the earth. Significant steady variation in $\lambda_{2,2}$, on the other hand, would imply no stress changes but merely lateral flow of anomalies in response to existing stresses (for example, at the core mantle boundary).

This experiment was designed with two objectives:

1. To measure directly any long term acceleration changes on geostationary orbits in the same longitude region and
2. To measure indirectly the same long term acceleration changes over many longitudes with respect to a drifting $\lambda_{2,2}$ model.

Consider the dominant resonant acceleration in longitude on a 24 hour orbit: [8]

$$\ddot{\lambda} = (A_{2,2}) \sin 2(\lambda - \lambda_{2,2}) \text{ radians/day}^2 , \tag{1}$$

where;

$$A_{2,2} = 12\pi^2 J_{2,2} \left[\frac{6}{a^2} (1 + \cos I)^2 /4\right],$$

a is the orbit's semimajor axis in earth radii, I is it's inclination and $J_{2,2}$ and $\lambda_{2,2}$ are related to the conventional unnormalized gravity coefficients [9] ($C_{2,2}$ and $S_{2,2}$) by:

$$C_{2,2} = J_{2,2} \cos 2 \lambda_{2,2},$$

and

$$S_{2,2} = J_{2,2} \sin 2 \lambda_{2,2}.$$

The longitude λ is defined as $M + \omega + N - \theta_e$, in terms of the conventional Kepler elements; mean anomaly, argument of perigee, right ascension of the ascending node and the Greenwich hour angle. For the 24 hour circular orbit λ is the actual mean geographic longitude of the satellite. If we allow only $\lambda_{2,2}$ to change in time, the velocity of this change can be found from the velocity of the change in $\ddot{\lambda}$. Namely, differentiating (1) with respect to time:

$$\frac{d\ddot{\lambda}}{dt} = -2A_{2,2}\,[\cos 2(\lambda - \lambda_{2,2})]\,\frac{d\lambda_{2,2}}{dt},$$

or

$$\frac{d\lambda_{2,2}}{dt} = -\frac{d\ddot{\lambda}}{dt}\;\frac{\sec 2\,(\lambda - \lambda_{2,2})}{2\,A_{2,2}}\,. \tag{2}$$

Stated another way, if $\Delta\lambda_{2,2}$ and $\Delta\ddot{\lambda}$ are small changes in $\lambda_{2,2}$ and $\ddot{\lambda}$ from an arbitrary time when the phase angle was $\lambda_{2,2}$ and the acceleration was $\ddot{\lambda}$, then (2) implies that:

$$\Delta\lambda_{2,2} = -\frac{\Delta\ddot{\lambda}}{2\,A_{2,2}}\;\sec 2\,(\lambda - \lambda_{2,2})\,. \tag{3}$$

From equation (3) we can convert measurements of residual accelerations to measurements of phase changes. This inferred measurement of phase change is the basis of the experiment.

Data Analysis

In the first experiment (the repetitive measurement of the acceleration at about the same longitude), the data consists of 28 sets of mean Kepler elements for Intelsat 2F3 (as determined by the Communications Satellite Corporation between 1967 and 1971). These are shown in Table 1. The mean longitudes show that the satellite was within only a few degrees of 11° west during this time. The elements were determined by COMSAT from radar range, elevation and azimuth data over periods of about a month, taken from tracking stations in Maine and Europe. They are separated into 3 free drift arcs by station keeping maneuvers performed about a year apart. Values of $C_{2,2}$ and $S_{2,2}$ were determined from the accelerated drift in each arc by fitting the λ data (essentially) in each arc to numerical ephemerides generated by the Rapid Orbit Analysis and Determination program (ROAD).[10] The accelerated drifts themselves are shown in Figure 1.

From the determined values of $C_{2,2}$ and $S_{2,2}$ in each arc, best determined values of $\ddot{\lambda}$ were found for an orbit whose elements were the average of the elements in the 3 arcs. The results of this measurement are given in Table 2. Considering the first arc in time as the base measurement, residual accelerations (Arc 2 and 3 values minus the Arc 1 value) were computed for the 2nd and 3rd arcs of Intelsat 2F3. From these, referred phase changes were calculated from (3) using a value of $A_{2,2}$ ($I = 0°$) of 3×10^{-5} radians/day^2 based on a $J_{2,2}$ of 1.8×10^{-5}, and a value of $\lambda_{2,2}$ of $-15°$. This data is also listed in Table 2. Evidently, from the overlap of the acceleration residuals themselves, there is no clear secular drift at this longitude over the 4 year period.

To test the precision of the hypothesis that $\lambda_{2,2}$ does have a simple linear drift in time, we can fit the data in Table 2 to the function:

$$\Delta\lambda_{2,2} = X_1 + X_2 t, \tag{4}$$

where t is the time from some convenient reference. The parameter X_2 in this function is the drift rate $\dot{\lambda}_{2,2}$.

A weighted least squares solution for $\dot{\lambda}_{2,2}$ with the data in Table 2 weighted according to their formal standard deviations is also given in Table 2. The judgement above is confirmed that no clear secular trend is seen in this data. But the observations are limited in time span and longitude. As a result, the precision and reliability of this test is not sufficient to decide whether or not low order and degree geogravity drift exists worldwide at the same level as geomagnetic drift ($\sim$0.5°/yr).

To broaden the scope of this measurement, and to make it more precise, 12 additional arcs of 24 hour satellites were examined with a worldwide distribution of longitudes and extending back to 1965. (See Table 3.) The basic data here (mean Kepler elements) are the same kind as in the first (direct) experiment.[11], [12] In this indirect experiment, $C_{2,2}$ and $S_{2,2}$ were best fitted (by ROAD) to the mean elements in all these arcs and the mean longitude residuals (observed minus computed values) were further analyzed for acceleration trends. The fixed geopotential in this determination included harmonics to (5,5) found from non-synchronous satellites. This analysis consisted of fitting quadratic polynomials (in the time) to the residuals. The $\Delta\ddot{\lambda}$ quantities in Table 3 are the residual accelerations from this analysis. The $\Delta\lambda_{2,2}$ 'measurements' are reductions of the $\Delta\ddot{\lambda}$'s according to (3). Once again the $\Delta\lambda_{2,2}$ 'measurements' in the 15 arcs were tested for a linear time drift according to the model of (4). The results of the global drift measurements are shown in Figure 2 and compared to a westward drift line of 0.5°/yr. through this time period.

Clearly if any low degree and order geogravity drift occurs, it is at least an order of magnitude slower than the equivalent wavelengths of geomagnetic drift.

While the previous test was quite convincing, by covering more than two longitude regions the acceleration residuals were subject to systematic errors from poorly determined resonant terms beyond the 2nd order. Therefore, in an attempt to sharpen the result, a more limited data set was chosen of the best arcs in only the two regions 165° - 195° east, and 10° - 15° west (see Table 4).

This final test, while conceptually cleaner than the last, gave a somewhat poorer estimate because the time span was slightly less and fewer arcs were used. In addition, the 'best' (most precisely determined) arcs were not in this limited set. The results; residual accelerations, residual phase shifts, and estimated phase drift rate in this experiment are given in Table 4 and summarized in Figure 3.

SUMMARY AND CONCLUSIONS

An examination of mean elements in 15 arcs of eight 24-hour satellites over a 6 year period shows no significant geopotential changes. In particular, the data was tested for a possible drift of $\lambda_{2,2}$, the phase angle of the 2nd order tesseral gravity harmonic. Over the 6 year period, the data was only compatible with a westward drift more than an order of magnitude slower than the westward drift of similar components of the geomagnetic field. However, the precision of the data and the short time span was not sufficient to measure rates less than about 0.02°/year.

It is possible that the earth's core drifts (or slips) westward with respect to the mantle at about 0.5°/yr. (as implied by the geomagnetic field drift). If so, then this satellite result is only compatible with an origin for the lower order and degree geogravity anomalies in the slower drifting portions of core-mantle boundary or the mantle itself. In any case there is considerable doubt that any part of the highly compressed lower mantle or viscous core can support even the mild shear stresses implied by the low order geogravity anomalies.[13] These satellite measurements are certainly compatible with this view also. They only definitely rule out an origin for these anomalies in a significantly drifting core.

ACKNOWLEDGMENT

The author is grateful to Mary Boumila of Computer and Software Inc., Hyattsville, Md., for setting up and executing the computer runs in this study.

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Table 1: Mean Element Observations for Intelsat 2F3

INTELSAT 2F3-1

NO.	TIME (MJD)	A(E.R.)	E	INCL (°)	OMEGA (°)	NODE(°)	MEAN (°)	LAMBDA (° EAST)
1	39607.02430560	6.61098299	.0002459	1.3270	286.7815	287.0465	357.8790	348.6381
2	39621.58680560	6.61109304	.0001358	1.2890	313.8516	287.6700	187.1493	348.7486
3	39648.66666670	6.61099090	.0002560	1.2350	284.5675	288.1574	271.4931	348.8544
4	39648.66666670	6.61079889	.0002833	1.2190	282.4352	287.9384	275.8003	348.8104
5	39684.05027780	6.61090899	.0002826	1.1000	272.3587	288.0572	111.5111	349.1482
6	39719.00000000	6.61100906	.0004205	1.0160	264.8617	287.4722	121.6912	349.3375
7	39781.16666670	6.61071119	.0005701	0.8940	290.2674	287.1439	218.5057	349.9551
8	39814.04166670	6.61090826	.0005517	0.8260	292.8240	289.9911	200.9500	350.3999
9	39845.00000000	6.61070524	.0005678	0.7290	302.5942	292.7730	204.4508	350.9390
10	39864.00000000	6.61071218	.0004933	0.6730	313.0696	293.4055	212.4459	351.3147
11	39881.00000000	6.61071707	.0003222	0.6260	308.2134	294.5369	233.3107	351.6987
12	39905.00000000	6.61050405	.0003249	0.5570	319.7986	294.1462	246.4002	352.3271

INTELSAT 2F3-2

NO.	TIME (MJD)	A(E.R.)	E	INCL (°)	OMEGA (°)	NODE(°)	MEAN (°)	LAMBDA (° EAST)
1	40405.77083330	6.61111706	.0001313	0.7980	352.9549	72.7948	116.0613	348.7097
2	40412.00000000	6.61112195	.0001561	0.8040	285.2277	71.9092	279.2271	348.6229
3	40439.00000000	6.61103397	.0000652	0.8730	236.5199	74.0109	352.2141	348.3314
4	40468.00000000	6.61106504	.0001303	0.9350	181.3160	75.6925	74.0376	348.1087
5	40503.00000000	6.61126005	.0001941	1.0030	201.5150	76.6686	86.9993	347.7480
6	40530.00000000	6.61117705	.0003596	1.0640	206.0583	75.5901	109.8416	347.4425
7	40573.00000000	6.61114406	.0004427	1.1960	220.2291	75.0781	138.1758	346.8527
8	40598.25000000	6.61122118	.0003828	1.2800	230.4830	75.9267	241.3523	346.4761
9	40630.00000000	6.61114226	.0004725	1.3590	242.2044	76.9230	169.3286	345.8439
10	40641.66666670	6.61128587	.0003921	1.4410	273.5268	69.7090	36.4511	345.5756

INTELSAT 2F3-3

NO.	TIME (MJD)	A(E.R.)	E	INCL (°)	OMEGA (°)	NODE(°)	MEAN (°)	LAMBDA (° EAST)
1	40648.61111110	6.61100000	.0002830	1.4720	269.7282	69.7749	26.9189	345.4658
2	40679.00000000	6.61095930	.0002295	1.5170	296.9030	70.0766	169.9434	346.0141
3	40760.25347220	6.61098120	.0001124	1.7420	330.4586	69.3754	309.4620	347.0498
4	40797.00000000	6.61105500	.0000327	1.8470	166.8539	70.1119	57.4652	347.2557
5	40896.00000000	6.61087000	.0002818	2.0470	201.5231	70.3980	120.4539	347.5706
6	41065.00000000	6.61110290	.0003444	2.5240	265.7846	69.7280	242.1275	346.5583

Table 2: Results of Repeated Measurement of the Acceleration on Intelsat 2F3

Time Period: 39600 - 41100 MJD

Longitude Range: 345° - 353° East

Average Orbit Elements: a = 6.61 E.R., e = 0.004, I = 1°, λ = 348.2° East

Acceleration and Phase Measurements					
Arc	$\ddot{\lambda}$ (10^{-5} Rad./Day^2)	Mid Arc Time (MJD)	Longitude (°East)	$\Delta\ddot{\lambda}$ (10^{-5} Rad./Day^2)	$\Delta\lambda_{2,2}$ (10^{-2} Rad.)
1	-0.015 ± 0.010	39750	348.2	0.000 ± 0.010	0.00 ± 0.17
2	-0.019 ± 0.006	40500	348.2	-0.004 ± 0.006	+0.07 ± 0.10
3	-0.020 ± 0.011	40900	348.2	-0.005 ± 0.011	+0.08 ± 0.18

$$\langle \dot{\lambda}_{2,2} \rangle = +0.004 \pm 0.029\ °/\text{Yr.}$$

Table 3: Results of Measurements of Residual Accelerations in 15 Arcs of 24-Hour Satellites

Arc	Time Span (MJD)	Mid Arc Time (MJD)	Average Inclination (°)	Longitude Span (° East)	Average Longitude (° East)	$\Delta\ddot{\lambda}$ (10^{-5} Rad./Day^2)	$\Delta\lambda_{2,2}$ (10^{-2} Rad.)
Syncom 2,8	38815-38918	38850	32	65-68	66	-0.023±0.025	-0.48 ± 0.50
Syncom 3, 11	39075-39263	39150	0	165-172	168	+0.021±0.020	-0.35 ± 0.34
Syncom 3, 13	39376-39531	39450	1	159-161	160	+0.053±0.015	-0.88 ± 0.25
Intelsat 2F3-1	39607-39905	39750	1	349-352	350	+0.008±0.010	-0.14 ± 0.17
Intelsat 2F4-1	40323-40608	40450	1	179-194	184	+0.002±0.010	-0.07 ± 0.21
Early Bird 1	38897-39080	39000	0	324-332	331	+0.000±0.015	0.00 ± 0.28
Intelsat 2F4-2	40617-40782	40700	1	188-195	191	+0.000±0.005	0.00 ± 0.14
ATS 3-2	40267-40337	40300	0	287-288	387	-0.003±0.002	-0.12 ± 0.08
Intelsat 2F3-2	40406-40642	40500	1	346-349	348	+0.001±0.010	-0.02 ± 0.17
Skynet 1,5	41049-41069	41050	1	50	50	-0.015±0.003	-0.39 ± 0.08
Skynet 1, 1	40652-40672	40650	2	39-40	39	-0.006±0.003	-0.32 ± 0.16
Early Bird 2	39096-39219	39150	1	321-331	327	-0.006±0.004	+0.12 ± 0.08
Intelsat 2F3-3	40650-41100	40900	1	345-348	347	+0.009±0.010	-0.18 ± 0.17
ATS 5-2	40742-41200	40950	1	255	255	+0.007±0.003	+0.10 ± 0.05
Intelsat 2F4-3	40817-41065	40950	2	189-215	194	-0.008±0.004	+0.22 ± 0.13

$$\langle \dot{\lambda}_{2,2} \rangle = -0.002 \pm 0.016 \ °/\text{Yr.}$$

Table 4: Results of Measurements of Residual Accelerations in Satellite Arcs in 2 Longitude Regions

Arc	$\Delta\ddot{\lambda}$ (10^{-5} Rad./Day^2)	$\Delta\lambda_{2,2}$ (10^{-2} Rad.)	Mid Arc Time (MJD)	Average Longitude (°East)
Syncom 3, 11	+0.008 ± 0.020	+0.14 ± 0.34	39150	168
Intelsat 2F4-2	-0.002 ± 0.005	+0.06 ± 0.14	40700	191
Intelsat 2F4-3	-0.007 ± 0.004	+0.18 ± 0.13	40950	194
Intelsat 2F3-2	-0.003 ± 0.010	+0.05 ± 0.17	40500	348
Intelsat 2F3-3	0.000 ± 0.010	0.00 ± 0.17	40900	347
Intelsat 2F4-1	0.014 ± 0.010	-0.30 ± 0.21	40450	184
Intelsat 2F3-1	-0.003 ± 0.010	0.04 ± 0.17	39750	350

$$\langle \dot{\lambda}_{2,2} \rangle = +0.010 \pm 0.027\ °/\text{Yr.}$$

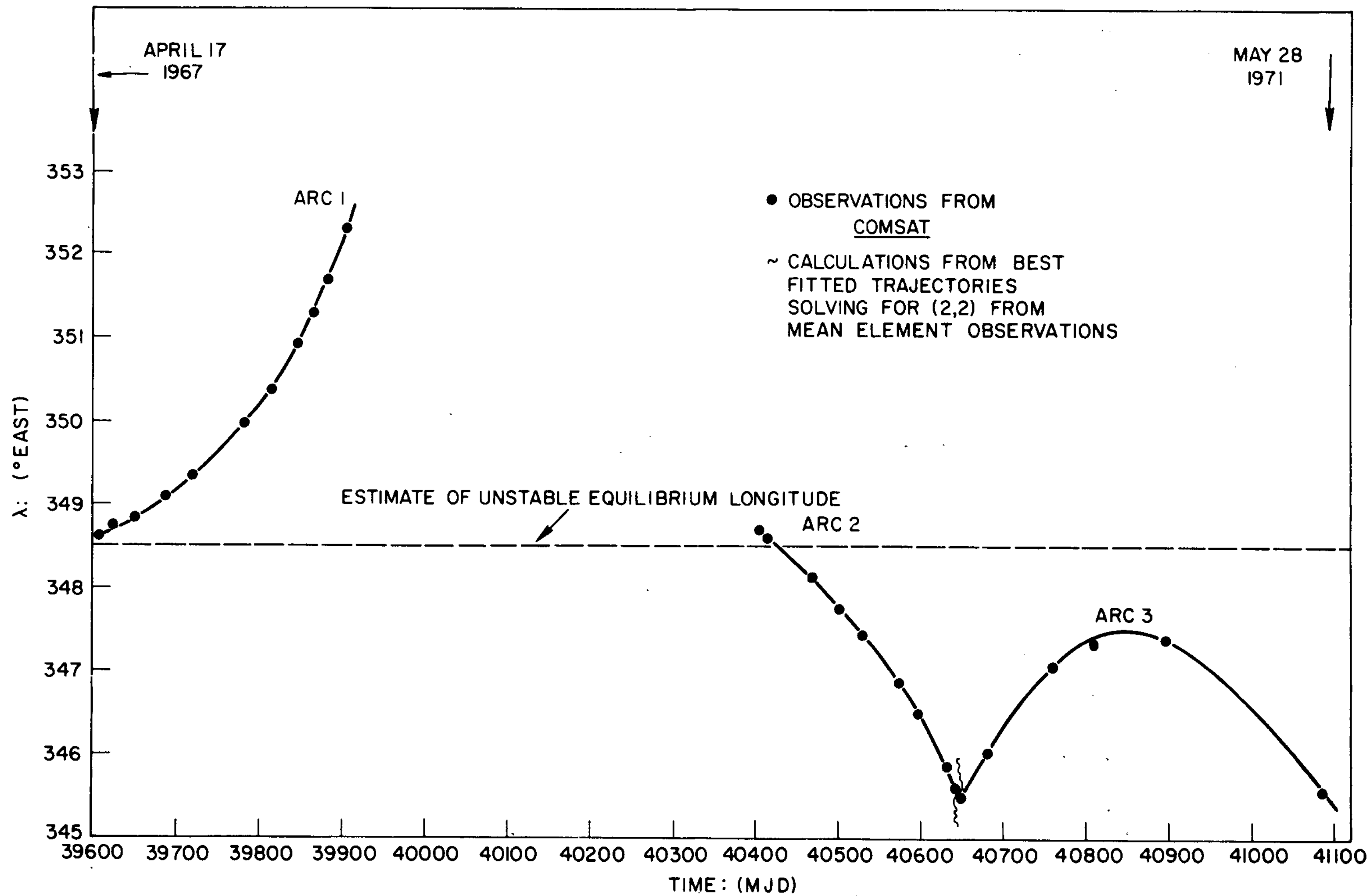

Figure 1. Accelerated Drifts of Intelsat 2F3 Over a 4 Year Period

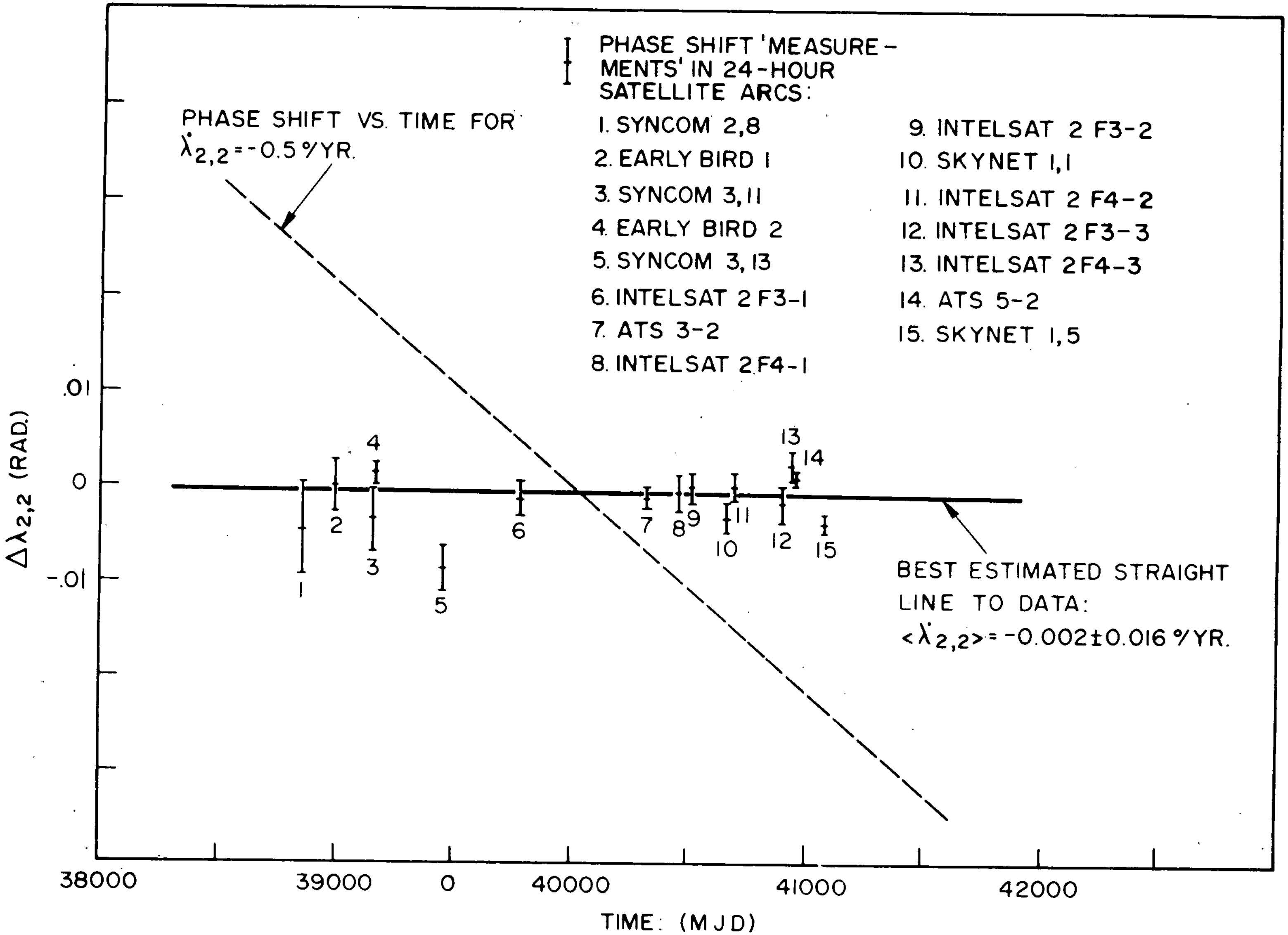

Figure 2. Phase Shift Measurements in 15 Arcs of 24-Hour Satellites

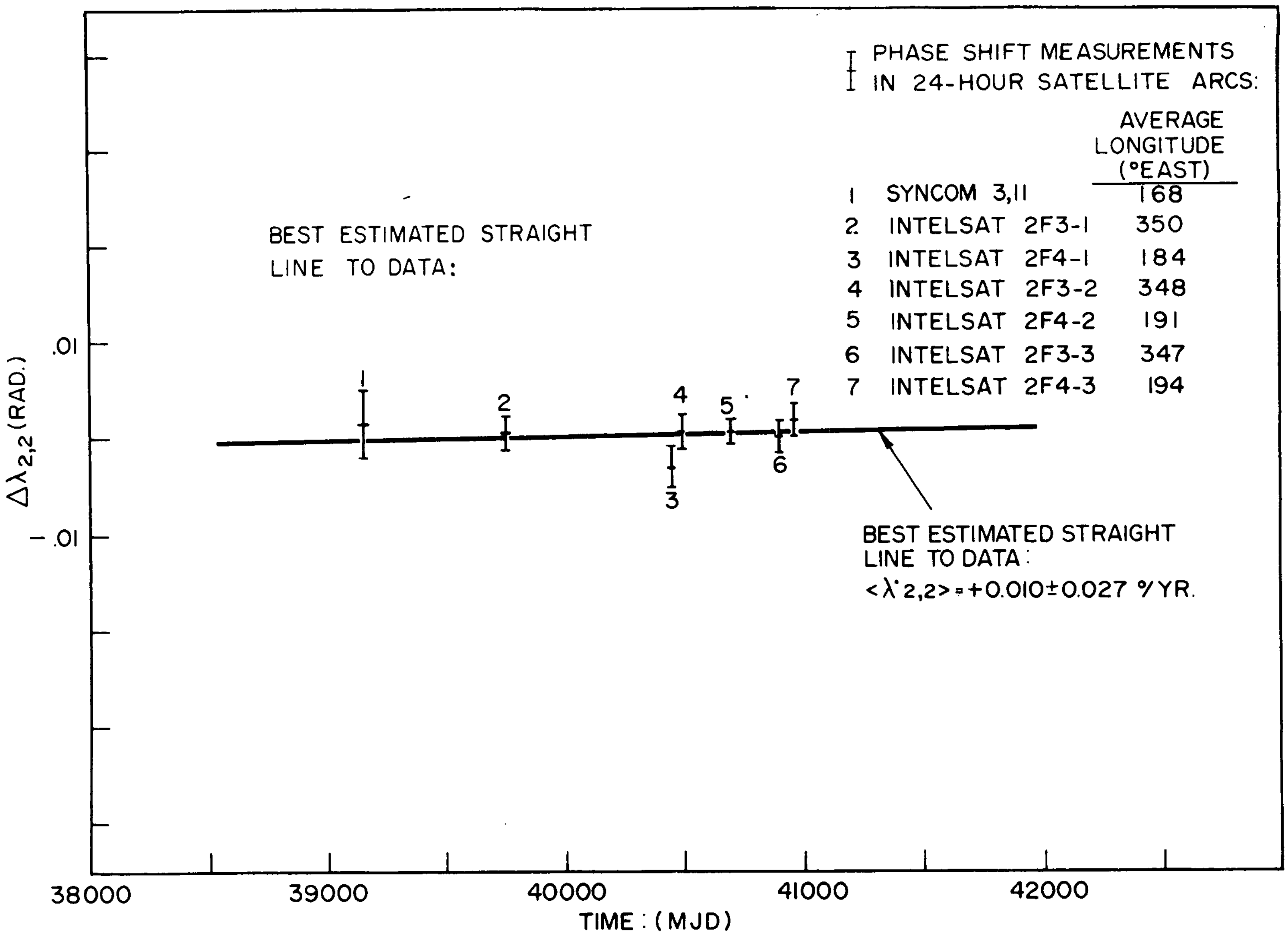

Figure 3. Phase Shift Measurements For 24 Hour Satellite Arcs in 2 Longitude Regions