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# SPACECRAFT ORBIT/EARTH SCAN DERIVATIONS, ASSOCIATED APL PROGRAM, AND APPLICATION TO IMP-6

GENE A. SMITH



OCTOBER 1971

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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## ABSTRACT

Given the orbital elements of an earth satellite, the right ascension of its position, and certain S/C design characteristics, a number of other parameters can be computed which are useful in analyzing earth-S/C relations and in checking telemetered attitude parameters. For the case where an earth sensor is mounted at an angle to the S/C spin axis, the determination of whether or not the earth is in view by the sensor is of interest.

For the program described in this paper, known orbital elements, a starting S/C position right ascension, S/C spin axis right ascension and declination, sensor-to-spin-axis mounting angle, angular width of earth sensor, and sun position are used to determine:

- declination of S/C
- radius vector of S/C
- apparent angular radius of earth seen from S/C
- angle from S/C spin axis to S/C nadir (earth center)
- angle subtended by S/C sensor scan across earth
- time to or from perigee
- earth shadow relation to S/C position (when earth seen by S/C).

The computations are made for a series of right ascensions at increments of a chosen angle. Earth shadow calculations require information on sun position at time of S/C position. This is updated from initial sun position data at time of S/C perigee.

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## 1. INTRODUCTION

### 1.1 Aim

The aim of this paper is to present a time shared, remote site, demand processed computer program, its derivation, and test results from it for analysis of selected orbit, attitude, and S/C parameter effects on earth sensor detections of the earth. The S/C is assumed to be spin stabilized with an earth sensor mounted at an angle $\gamma$ to the spin axis and having a field of view or scan angle of $S_c$ degrees.

For pre-launch analysis, the program may be used to simulate effects for nominal parameters which then would be useful in preparing attitude data processing programs. After launch, comparison of results from a simulation using estimated parameters and from computations on actual satellite data would produce deviations helpful in isolating problems.

### 1.2 Initial Parameters

The aspect — position and orientation — of a satellite can be uniquely determined by the specification of six parameters — three for position and three for orientation or attitude. This determination is essential for correct analysis of S/C telemetry data. In addition, certain S/C design characteristics affect the S/C sensor readings and subsequent telemetry readout. Finally, knowledge of the sun's position allows prediction of the S/C's view of the shadowed earth.

#### 1.2.1 Orbit

S/C position is specified by reference to its orbit at a certain time. This reduces to three spatial coordinates — usually inertial geocentric Cartesian or spherical. Cartesian and spherical coordinates are related as shown in Figure 1-1. The Z axis is coincident with the earth's polar or spin axis, the X axis is in the equatorial plane intersecting the celestial sphere at the First Point of Aries, ♈ , and the Y axis is defined to make a right handed orthogonal coordinate system. An arbitrary vector $\vec{R}$ can be expressed in terms of the X, Y, Z components such that

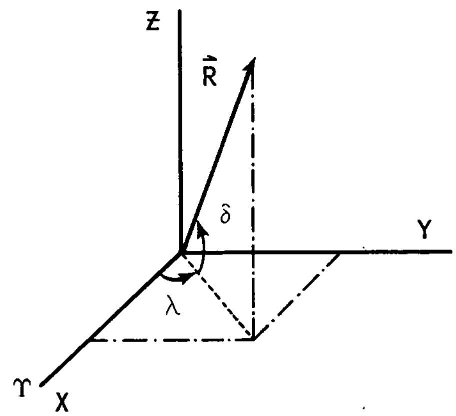



Figure 1-1. Cartesian and Spherical Coordinates

$$X_R^2 + Y_R^2 + Z_R^2 = R^2 .$$

The vector $\vec{R}$ is also specified by the magnitude R, right ascension $\lambda$, and declination $\delta$ where $\lambda$ is the counter-clockwise rotation of ϒ to the R projection on the X-Y plane and $\delta$ is the angle from there to the $\vec{R}$ vector. The transformation is:

$$X = R \cos \delta \cos \lambda$$

$$Y = R \cos \delta \sin \lambda$$

$$Z = R \sin \delta$$

The orbit is characterized by six independent orbital elements. The most common is the set of Keplerian elements consisting of:

- $\epsilon$ — eccentricity or shape of orbit ellipse
- a — semi-major axis or size of orbit ellipse
- i — inclination of orbit plane to equatorial plane
- $\Omega$ — right ascension of ascending node
- $\theta_p$ — argument of perigee or tilt of orbit
- $t_p$ — epoch or time of perigee.

Often included to define a particular point on the orbit at arbitrary time t is:

$$\text{M} - \text{mean anomaly} = \frac{2\pi}{\text{period}} \times (t - t_p)$$

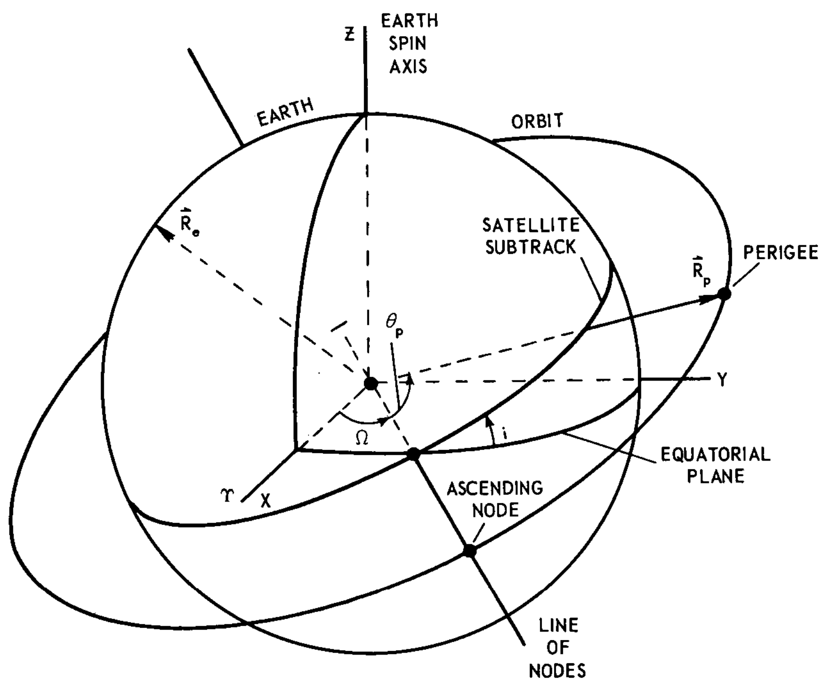



Figure 1-2. Orbit Referenced to Inertial Coordinates

Derived quantities such as semi-minor axis $b = \sqrt{a^2 - \epsilon^2 a^2}$, perigee distance $R_p = a(1 - \epsilon)$, apogee distance $R_a = a(1 + \epsilon)$, and, from Kepler's third law, period $T = \left(\frac{4\pi^2}{g} a^3\right)^{1/2}$ (where g is earth's gravitational constant) are formed to simplify computations and visualization.

Because a satellite's orbit is not constant in time due to perturbations by the moon, sun, and non-spherical nature of earth's gravitational field, the time rates of change of $\Omega$, $\theta_p$, i, and T or frequent updating of elements are needed to provide accurate solutions of desired parameters.

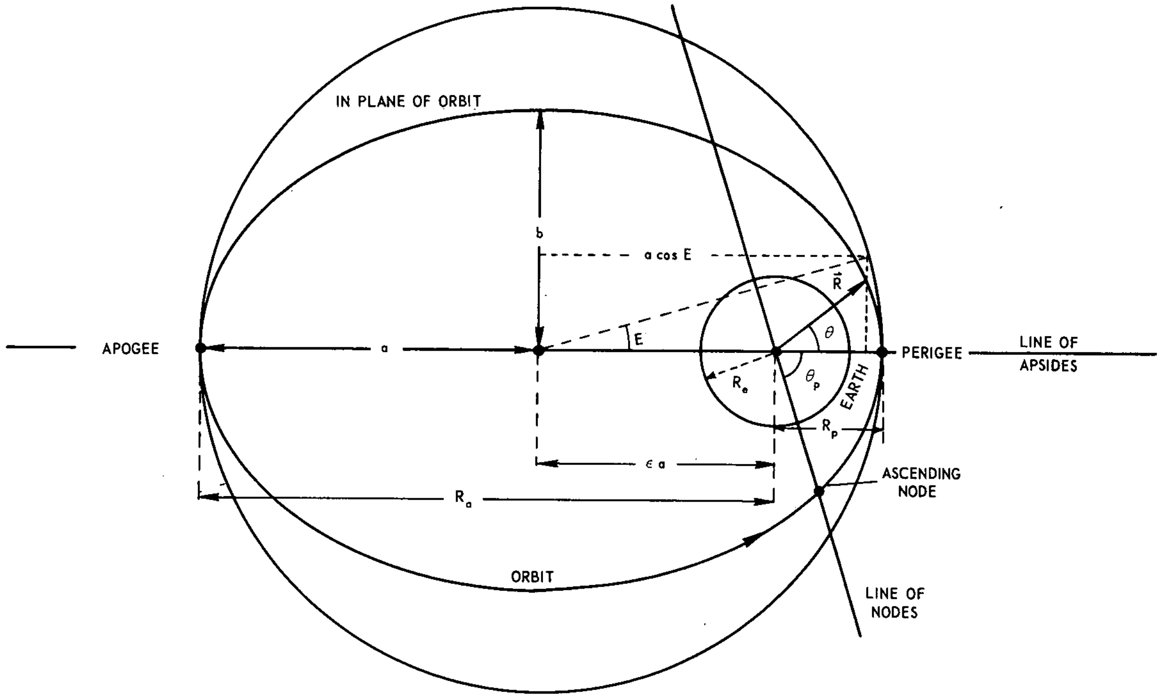



Figure 1-3. Orbit Ellipse Geometry

### 1.2.2 Attitude

Attitude is the relationship of the S/C coordinate system to the inertial (or other intermediate) coordinate system. This relationship is often expressed in terms of Eulerian angles such as roll, pitch, and azimuth (Figure 1-4): that is, a set of three rotations which transform one coordinate system of arbitrary orientation into a parallel orientation with the respective axes of a second coordinate system.

Let:

X, Y, Z = S/C coordinates

$X'''$, $Y'''$, $Z'''$ = inertial coordinates

$X'$, $Y'$, $Z'$ and $X''$, $Y''$, $Z''$ = intermediate coordinates

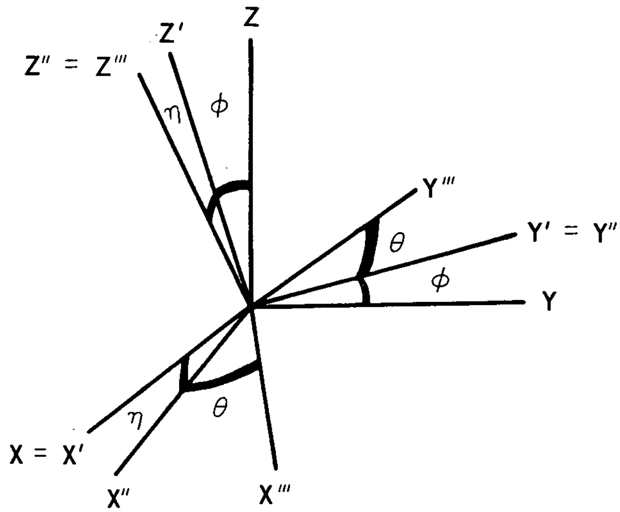



Figure 1-4. Eulerian Angles

roll angle $\phi$ = rotation about X axis

pitch angle $\eta$ = rotation about new Y' axis

azimuth angle $\theta$ = rotation about final Z''' axis

The S/C coordinates can be expressed as a function of Euler angles and inertial coordinates in matrix notation by:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R(\phi)\, R(\eta)\, R(\theta) \begin{bmatrix} X''' \\ Y''' \\ Z''' \end{bmatrix}$$

where

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R(\eta) = \begin{bmatrix} \cos\eta & 0 & -\sin\eta \\ 0 & 1 & 0 \\ \sin\eta & 0 & \cos\eta \end{bmatrix}, \quad R(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}.$$

The inverse relation, solving for inertial coordinates, is:

$$\begin{bmatrix} X''' \\ Y''' \\ Z''' \end{bmatrix} = R'(\theta)\ R'(\eta)\ R'(\phi) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

where the R′ matrices are inverses of the respective R's. Since these matrices are orthogonal, the inverses are equal to their transposes e.g.

$$R'(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = [R(\theta)]^T = R(-\theta)$$

In many cases, the primary method is not by this approach to attitude solution but instead by the determination of the S/C spin axis in terms of inertial coordinates. Thence, evaluation of the S/C spin rate and knowledge of the S/C spin axis orientation effectively characterize the S/C attitude. This is equivalent to another set of Eulerian angles with respect to inertial and S/C coordinates:

right ascension $\lambda$ = rotation about $Z'''$ axis

co-declination $(90 - \delta)$ = rotation about new $Y''$ axis.

These two rotations transform $Z'''$ axis in inertial coordinates to the Z axis of S/C coordinates. A final rotation derived from spin rate and a time since reference complete the attitude determination.

### 1.2.3 Spacecraft

For a spin stabilized S/C, the axis of rotation is coincident to the largest moment of inertia. These are often designated as the Z axis and $I_3$, respectively. If the other principal moments of inertia $I_1$ and $I_2$ are equal, then the S/C is dynamically balanced and the Z axis will tend to align with the S/C momentum vector which is fixed in inertial space (when disturbing torques are absent). The effect of disturbance torques is slow such that orientation of the spin axis is approximately constant over the S/C period. A rapid reorientation may be commanded from the ground; in which case, the exact nature of the effect on the S/C must be known for accurate following of spin axis motion.

To determine satisfactorily the Z direction, it is necessary to equip the S/C with attitude sensors capable of measuring the direction of known objects from the S/C at selected times. These known objects may be the sun, the stars, the earth, and the earth's magnetic field. All of these are reasonably well defined as a function of time. For the S/C discussed in this paper, two sensors are needed — an earth sensor and a digital solar sensor.

The earth sensor is a narrow field of view instrument sensitive to the difference between the sunlit earth horizon and space and to the contrast between the earth terminator and the shadowed earth, the earth terminator being the boundary between the sunlit earth and the shadowed earth. When the S/C rotates, the sensor sweeps out an annulus on the celestial sphere with a radius the same as the sensor-to-spin-axis mounting angle. This angle $\gamma$ for particular spin orientation and orbit position is the major factor in whether the earth is scanned or not. Another factor is the field of view or scan angle $S_c$. This is effectively the width of the annulus on the celestial sphere and determines the time the earth is seen. Pulses are initiated by photoelectric cell action whenever an horizon terminator is sensed thus providing time of earth and earth width measurements.

For an earth horizon sensor for which the solution of spin axis equations have been based on detection of both horizons, it is important to determine if a terminator rather than an horizon is sensed. This is done analytically in most cases rather than through S/C electronics.

The digital solar sensor provides a readout of the angle between the sun vector and the spin axis. With a wide field of view of about 180° centered at right angles to the spin axis, it scans the entire celestial sphere every spin period, hence senses the sun each rotation (except when eclipsed by the earth).

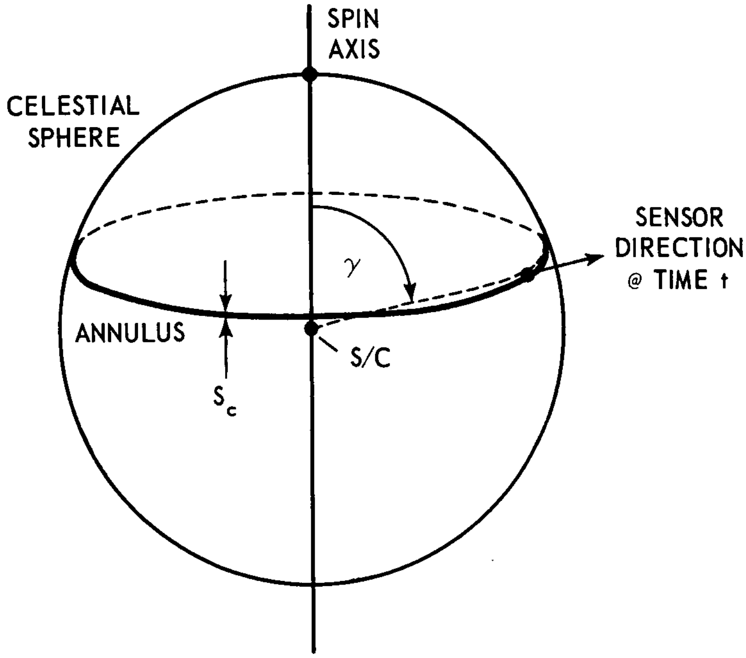



Figure 1-5. Spin Axis/Earth Sensor Mounting Angle

The angle is determined by the response of a bank of photocells onto which a narrow slit allows sunlight to fall — the slit being perpendicular to the spin axis. The particular photocells excited represent a binary coding of the sun angle with the resolution of the sensor depending on the total number of unmasked cells. For instance, if the bank is a rectangular array of cells in rows and columns with the slit parallel to the rows (the columns parallel to the spin axis) and with the columns composed of different numbers of alternating masked and unmasked cells, then the most significant digit is determined by the column with just two large cells — one of which is masked and cannot respond to the sunlight. In effect, this column determines which hemisphere the sun is in with respect to the spin axis. The next column has four cells each half the length of the ones in the first column. These two columns limit the sun angle value to a certain 45° sector. Each additional column further narrows the resolution on the sun angle until with eight columns the resolution is about 3/4°. This is due to the eighth column having $2^8$ = 256 cells alternating 128 masked and 128 unmasked such that 180° is divided into 256 bands as in Figure 1-6.

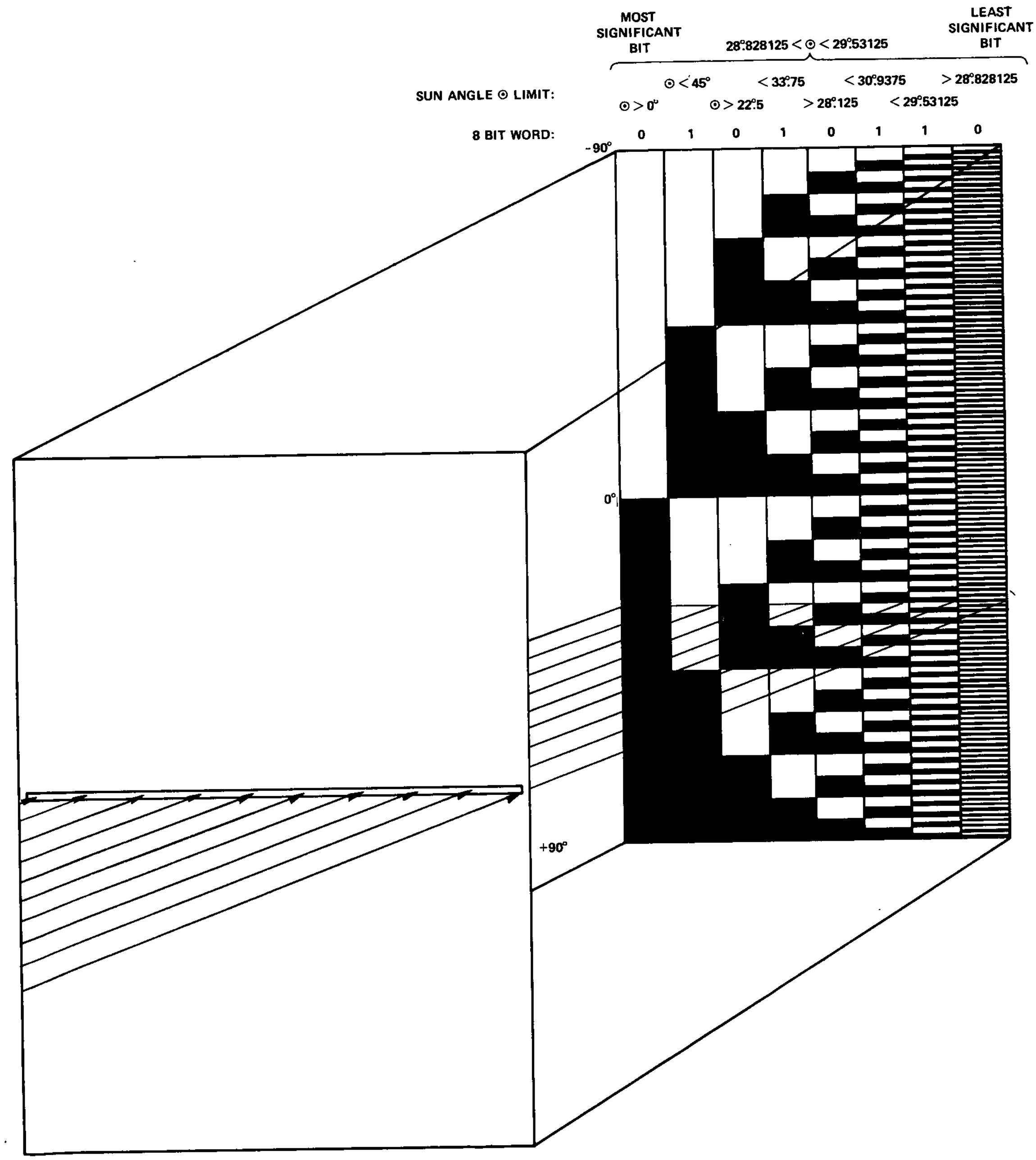



Figure 1-6. Digital Solar Sensor Schematic

In addition to the narrow slit perpendicular to the spin axis, another narrow slit — the command slit — is parallel to the spin axis and supplies azimuth angle information. This slit has a 180° field of view and excites at least one photocell in an additional column of unmasked cells whenever the sun is sensed. The photocell signal is amplified and a centered sun pulse is generated from it.

To complete the electronics necessary for interpreting pulse relations into attitude parameters, constant frequency counters are initiated or terminated by one of the various pulses supplied by the sensors. Thus a pulse from the digital solar sensor turns on a counter which is later turned off by a pulse from the earth sensor. The counter is read into the telemetry stream at the appropriate time and is converted on the ground into a relative time by:

$$C_{SE}/f_{SE} = t_{SE}$$

where $C_{SE}$ = counts between solar and earth sensor pulses, $f_{SE}$ = frequency of counter (counts/sec), and $t_{SE}$ = time between sun and earth sensing.

Other relations are determined by counters which (1) count from sun pulse to sun pulse which provides spin period by:

$$C_{SP}/f_{SP} = t_{SP},$$

(2) count from first earth sensor pulse from leading edge of earth's horizon to next earth sensor pulse initiated by trailing edge of earth's horizon which leads to scanned earth width by:

$$(C_{EW}/f_{EW}) \times (360°/t_{SP}) = \text{earth width angle},$$

and (3) count from a certain bit of a certain word of the telemetry sequence to the sun pulse which gives a time relating the telemetry to the attitude parameters by:

$$C_{TS}/f_{TS} = t_{TS}.$$

The sun sensor provides values for the spin axis which are located on an annulus around the sun direction with a radius equal to the sun angle. Similarly, the earth sensor horizon detections lead to the angle subtended by the S/C spin axis and the S/C nadir or S/C to earth center vector. This angle in turn is the radius of another annulus centered at the earth to which the spin axis must point. The intersection of the two annuluses is the ambiguous solution of the spin axis when provided an earth sensor and a sun sensor. Previous knowledge of the attitude is usually sufficient to eliminate one solution.

### 1.2.4 Sun

The initial sun parameters are included in order to derive earth shadow and S/C relations. It is sufficient to know the right ascension of the sun at the time of perigee, inclination (obliquity) of the ecliptic plane to the equatorial plane, and apparent angular diameter of the sun as seen from earth orbit. Specification of right ascension and inclination allow computation of declination and interpolation of sun position to time of S/C. Both computations are discussed in Section 2.

For a particular day and hour of the year, the American Ephemeris and Nautical Almanac supplies the right ascension of the sun, the inclination angle or mean obliquity of the ecliptic plane (which changes slowly and is approximately 23°27'), and the apparent angular diameter of the sun as seen from earth orbit (also slowly changing — near 32 minutes of arc).

Since the sun has a finite distension and is not a point source, rays from different areas of the sun are tangent to the earth at slightly different angles. Those rays from the apparent edge of the sun directed tangent to the earth meet at a point producing a cone of shadow or umbra behind the earth. The points where the sun is only partially obscured by the earth is the penumbra. The umbra extends along the anti-sun line 1 348 640 km (Figure 1-7).

When the S/C is in eclipse, its earth sensor cannot detect an earth horizon as it isn't sensitive to the differential between the shadowed earth and space. Location of the S/C with respect to the umbra is thus critical in knowing if attitude solutions from the sun and earth sensors will be possible at certain times.

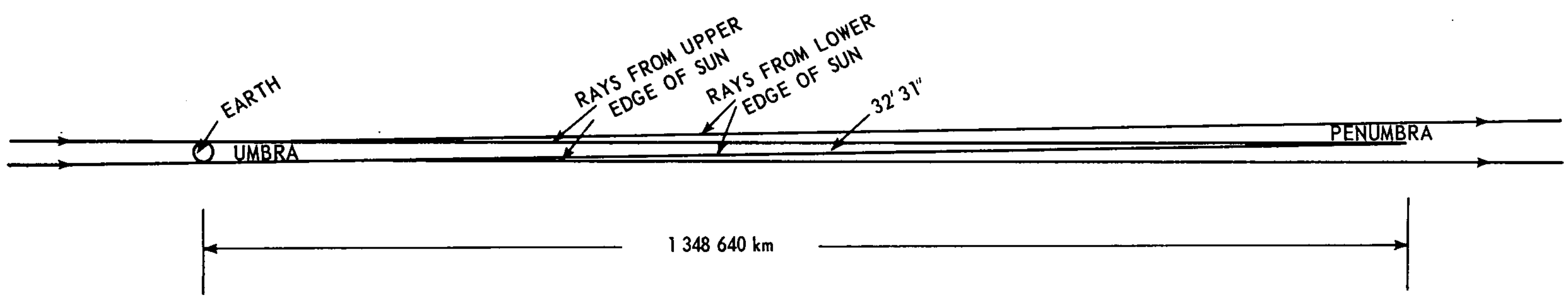



Figure 1-7. Cross Section of Earth Shadow

## 2. DERIVATIONS

From the few initial parameters given in the previous section, a number of others can be derived which fully characterize the S/C's changing relationships to the earth and sun. Several relations are derived which could have been included as initial parameters; but where possible, derivations are made to simplify the input requirements.

### 2.1 S/C Position

In one orbit of the earth, a S/C revolves through 360° of right ascension. For each value in right ascension, a unique value of declination is found. For a series of right ascensions, a corresponding series of declinations is obtained, and for each S/C position pair of right ascension and declination other parameters are derived.

#### 2.1.1 Right Ascension of S/C

The series of right ascension inputs is determined by recognition of orbit sectors of interest. Thus, a preliminary general view of the orbit would require computations based on right ascensions 10° apart over the entire 360° range. Brief analysis of results would indicate specific regions for further study. This could involve computations from RA's 1° apart over any 20° range, for example. To enable selected computation, specification of a starting RA, final or limiting RA, and RA increment is made when program is called.

#### 2.1.2 Declination of S/C

For a particular S/C right ascension $\lambda$, inclination of orbit j, and right ascension of ascending node $\Omega$, the S/C declination $\delta$ and other angles of interest are found from spherical trigonometric laws of sines and cosines. Thus, from Figure 2-1, the intermediate interior angle $\zeta$ is calculated by:

$$\cos \zeta = -\cos j \cos 90^\circ + \sin j \sin 90^\circ \cos(\lambda - \Omega)$$

$$\cos \zeta = \sin j \cos(\lambda - \Omega) . \tag{1}$$

The declination angle $\delta$ is then given by:

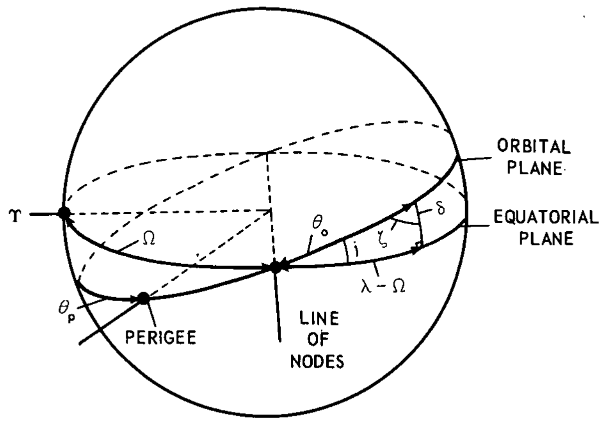



Figure 2-1. S/C Declination and Angle from Perigee

$$\sin \delta = \frac{\sin (\lambda - \Omega)}{\sin \zeta} \sin j . \tag{2}$$

## 2.1.3 Radius Vector of S/C

Also referring to Figure 2-1, the angular distance $\theta_0$ from the ascending node to the S/C position follows from the derivation of $\delta$ by:

$$\cos \theta_0 = \cos \delta \cos (\lambda - \Omega) + \sin \delta \sin (\lambda - \Omega) \cos 90°$$

$$\cos \theta_0 = \cos \delta \cos (\lambda - \Omega) . \tag{3}$$

The question of "which quadrant?" arises as $\theta_0$ can take on values from 0° to 360° while Eq. (3) limit results to 0°-180°. The desired results are obtained by setting:

$$\theta_0 = 360° - \theta_{0_e} \quad \text{when} \quad \lambda - \Omega > 180°$$

$$\theta_0 = \theta_{0_e} \quad \text{when} \quad \lambda - \Omega \leq 180°$$

where $\theta_{0_e}$ is derived from Eq. (3).

The derivation of $\theta_0$ is important in the determination of the angle from perigee $\theta$:

$$\theta = \theta_0 - \theta_p \tag{4}$$

where $\theta_p$ is the argument of perigee. This angle, in turn, is necessary to compute the radius vector to the S/C by an equivalent statement of Kepler's first law:

$$R = \frac{R_p (1 + \epsilon)}{1 + \epsilon \cos \theta} \tag{5a}$$

where $R_p$ is the distance from earth center to perigee and $\epsilon$ is the orbit eccentricity.

## 2.2 Half Earth Angle

The apparent angular radius $\rho$ of the earth as seen from the S/C at R just depends on the earth radius $R_e$ and R.

$$\sin \rho = \frac{R_e}{R} \tag{6}$$

Thus as the S/C orbits the earth, the angle subtended by the earth changes continuously, reaching a maximum at perigee — $2 \sin^{-1} R_e/R_p$ — and a minimum at apogee — $2 \sin^{-1} R_e/R_a$.

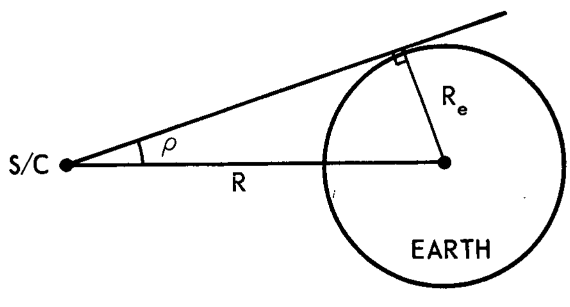



Figure 2-2. Half Earth Angle

## 2.3 Angle from S/C Spin Axis to S/C Nadir

The vector from the S/C to earth center is oppositely directed to the spacecraft zenith vector and as such is the same as the S/C nadir vector.

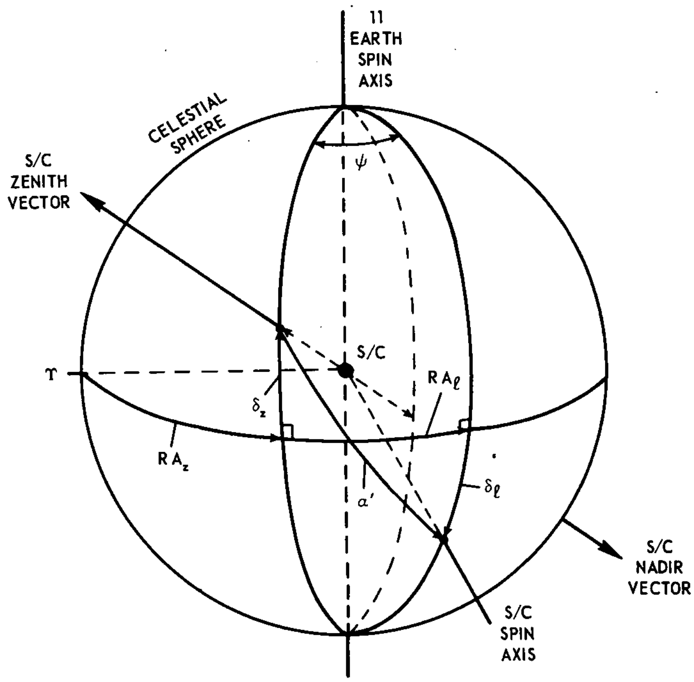



Figure 2-3. Angle from Spin Axis to Zenith Vector

The angle from the spin axis to the S/C nadir can be determined if given the coordinates of the spin axis and the S/C nadir. This angle is necessary for comparison with the spin axis-to-earth sensor mounting angle $\gamma$ to indicate if the earth is viewed by the earth sensor. If the spin axis right ascension and declination are $\lambda_\ell$ and $\delta_\ell$ respectively and at time t the S/C position coordinates are $\lambda_z$ and $\delta_z$, then the angle $\alpha'$ between the two axes is found by:

$$\psi = \cos^{-1} \cos (\lambda_\ell - \lambda_z) \tag{7}$$

where the cosine functions serve to limit the value of $\psi$ to less than 180° as required. Thence,

$$\cos \alpha' = \cos (90° - \delta_\ell) \cos (90° - \delta_z) + \sin (90° - \delta_\ell) \sin (90° - \delta_z) \cos \psi$$

$$\cos \alpha' = \sin \delta_\ell \sin \delta_z + \cos \delta_\ell \cos \delta_z \cos \psi \tag{8}$$

Since $\alpha'$ is the angle between the spin axis and the zenith vector, the angle from the spin axis to the nadir vector is:

$$\alpha = 180° - \alpha' \tag{9}$$

## 2.4 Earth Crossing Angle

By the previous developments, the parameters are made available which are necessary for calculating the angle of the horizon-to-horizon crossing of the earth sensor scan. This angle is the same as the one derived from S/C telemetry from counts between the first earth horizon pulse and the second earth horizon pulse (assuming both horizons and not an horizon and a terminator are viewed).

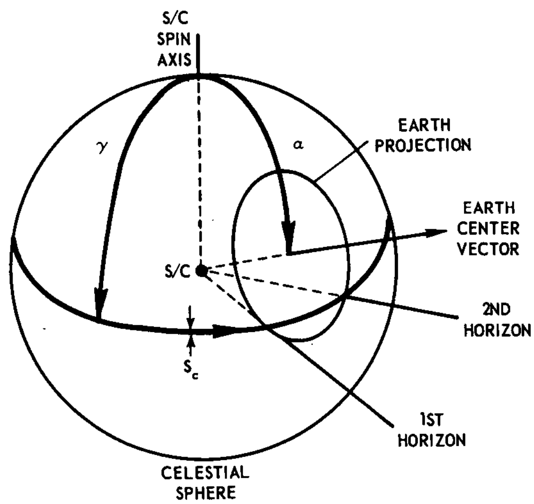



Figure 2-4. Scan Annulus Intersection of Earth

### 2.4.1 Scan Range

Depending on whether $\alpha$ is less than $\gamma$ or $\alpha$ is greater than $\gamma$, the intermediate angle $\nu$ and subsequent angle comparisons differ.

For $\alpha > \gamma$, define:

$$\nu = \gamma + \frac{1}{2} S_c .$$

If

$$\eta \leq \alpha - \rho, \tag{10a}$$

then earth is not in scan range.

For $\alpha < \gamma$, define:

$$\nu = \gamma - \frac{1}{2} S_c .$$

If

$$\nu \geq \alpha + \rho, \tag{10b}$$

then earth is not in scan range.

Where $S_c$ is the earth sensor scan angle, $\gamma$ is the mounting angle, and $\rho$ and $\alpha$ are as developed above. If $\alpha = \gamma$, then the earth must be in scan range such that $\nu = \gamma$ in following development.

.4.2 Spherical Triangle

When the proper selection of intermediate steps leading to the crossing angle is made, the algorithm is simplified by holding true independently of the relation between $\alpha$ and $\gamma$.

Define a spherical triangle by the S/C spin axis intersection of the celestial sphere (CS), the earth center vector (S/C nadir) intersection of the CS, the second horizon vector extended to CS, and the great circle arcs connecting these CS intersections. These three arcs are $\alpha$, $\nu$, and $\rho$ respectively as in Figures 2-5. By the law of cosines, the interior angle X is found from:

$$\cos \rho = \cos \nu \cos \alpha + \sin \nu \sin \alpha \cos X$$

or

$$\cos X = \frac{\cos \rho - \cos \nu \cos \alpha}{\sin \nu \sin \alpha}. \tag{11}$$

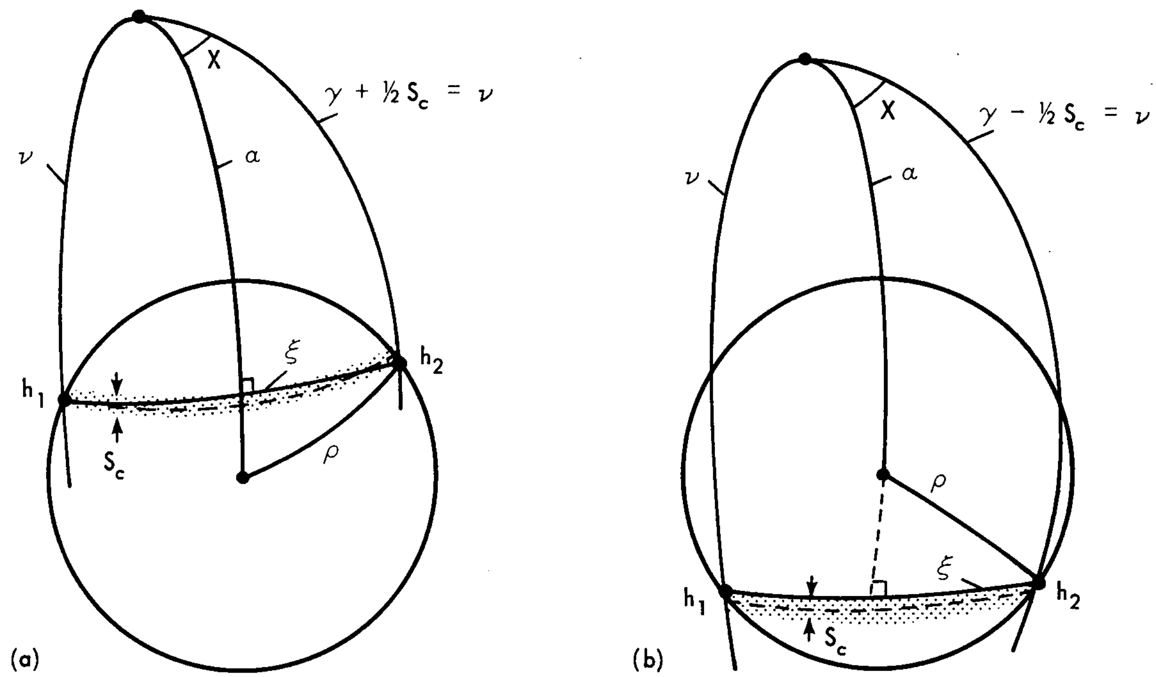



Figure 2-5. Spherical Triangles for Determination of Earth Crossing Angle

### 2.4.3 Half Crossing Angle

Finally, by the law of sines applied to the right spherical triangle containing $\xi$:

$$\frac{\sin \xi}{\sin X} = \frac{\sin \nu}{\sin 90^\circ}$$

or

$$\sin \xi = \sin X \sin \nu . \tag{12}$$

Twice $\xi$ is the crossing angle from horizon to horizon of the earth sensor scan.

## 2.5 S/C Time to/from Perigee

Two methods, one angular and the other areal, can be used to solve for the time a S/C takes to move from perigee to any position in the orbit or to move from any position to perigee depending on which time is shorter.

### 2.5.1 Angular Approach

This time is related to the area of a sector of the orbit ellipse by Kepler's second law of planetary motion: the radius vector sweeps out equal areas in equal times. In addition, an angular relation to time is represented by Kepler's equation:

$$M = E - \epsilon \sin E \tag{13a}$$

where E is the eccentric anomaly, $\epsilon$ is the orbit eccentricity, and the mean anomaly M is the product of the mean angular motion or average orbital rate $n = 2\pi/T$ and the time t from perigee $M = nt$. Also, T is the period of the orbit. Thus,

$$n = \frac{(E - \epsilon \sin E)}{t}. \tag{13b}$$

From the geometry of Figure 1-3, E is related to the radius vector by:

$$\sin E = \frac{R \sin \theta}{a\sqrt{1 - \epsilon^2}} \tag{14a}$$

where

$$R = \frac{a(1 - \epsilon^2)}{1 + \epsilon \cos \theta}, \tag{5b}$$

such that

$$\sin E = \frac{\sqrt{1 - \epsilon^2} \sin \theta}{1 + \epsilon \cos \theta}. \tag{14b}$$

Substituting (14b) into (13b) and solving for t gives:

$$t = \frac{T}{2\pi}\left(\sin^{-1}\left(\frac{\sqrt{1 - \epsilon^2} \sin \theta}{1 + \epsilon \cos \theta}\right) - \epsilon\left(\frac{\sqrt{1 - \epsilon^2} \sin \theta}{1 + \epsilon \cos \theta}\right)\right) \tag{15}$$

2.5.2 Areal Approach

The form based on Kepler's second law is derived from the formula:

$$\frac{t}{A} = \frac{T}{\pi a b} = \text{constant} \tag{16}$$

where t is the time to/from perigee, A is the area of the sector of the ellipse bounded by the perigee and position vectors, T is the orbit period and $\pi$ a b is the total area of the ellipse.

The sector area is determined by integrating from perigee to position the differential area. This also leads to the differential statement of Kepler's second law — the areal velocity is constant. This is true since the angular momentum of a S/C in orbit is a constant,

$$h = R^2 \dot{\theta} = \text{constant}, \tag{17}$$

and the differential area is

$$d A = \frac{1}{2} R \cdot R d \theta \tag{18}$$

as in Figure 2-6 which leads to the areal velocity

$$\frac{d A}{d t} = \frac{1}{2} R^2 \frac{d \theta}{d t} = \text{constant}. \tag{17'}$$

The sector area is obtained on integrating (18):

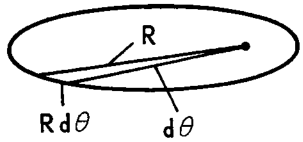



Figure 2-6. Differential Area of an Ellipse

$$\int_0^A dA = \int_0^\theta \frac{1}{2} R^2 \, d\theta = \frac{a^2 (1 - \epsilon^2)^2}{2} \int_0^\theta \frac{d\theta}{(1 + \epsilon \cos \theta)^2} \qquad (19a)$$

where again

$$R = \frac{a (1 - \epsilon^2)}{1 + \epsilon \cos \theta}. \qquad (5b)$$

The integration is obtained from tables to give

$$A = \frac{a^2 (1 - \epsilon^2)^2}{2} - \left[\frac{1}{(1 - \epsilon^2)} \left\{\frac{\epsilon \sin \theta}{1 + \epsilon \cos \theta} - \frac{2}{(1 - \epsilon^2)^{1/2}} \tan^{-1}\left(\sqrt{\frac{1 - \epsilon}{1 + \epsilon}} \tan \frac{\theta}{2}\right)\right\}\right] (19b)$$

Then from (16), the solution for t is:

$$t = \frac{T}{\pi ab} A = \frac{T}{\pi ab} \frac{a^2 (1 - \epsilon^2)}{2} \left[- \frac{\epsilon \sin \theta}{1 + \epsilon \cos \theta} + \frac{2}{(1 - \epsilon^2)^{1/2}} \tan^{-1}\left(\sqrt{\frac{1 - \epsilon}{1 + \epsilon}} \tan \frac{\theta}{2}\right)\right]$$

$$t = \frac{T}{2\pi} \frac{a}{b} (1 - \epsilon^2) \left[- \frac{\epsilon \sin \theta}{1 + \epsilon \cos \theta} + \frac{2}{(1 - \epsilon^2)^{1/2}} \tan^{-2}\left(\sqrt{\frac{1 - \epsilon}{1 + \epsilon}} \tan \frac{\theta}{2}\right)\right] \qquad (20)$$

Either Equation (15) or (20) for t relates the time to or from perigee of a S/C to the angle between the radius vector at perigee and the radius vector to the S/C position. For purposes of this paper, (20) is the preferred form as it can be evaluated for $\theta$ angles up to 180° without requiring quadrant considerations.

2.6 Interpolation of Anti-Sun Line to Time Since Perigee

Due to the possibility of interpolating sun position information to the time desired from an initial sun position at a reference time, entry of a lengthy table of sun positions vs. time is avoided. The reference time is the time of the S/C at perigee just prior to the series to be computed

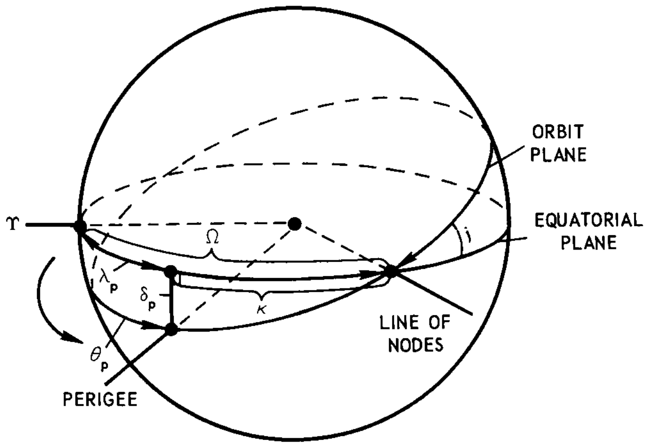



Figure 2-7. Perigee Right Ascension and Declination

and the reference position is the right ascension coordinate of the anti-sun line at reference time, where the anti-sun line is oppositely directed to the sun position. For any series of computations, the initial action is to determine the right ascension of the perigee $\lambda_p$. This angle is a function of the argument of perigee $\theta_p$, right ascension of ascending node $\Omega$, inclination of orbit plane j, and perigee declination found from:

$$\sin \delta_p = \sin \theta_p \sin j . \tag{21}$$

The equatorial plane angle $\kappa$ from ascending node to perigee right ascension is:

$$\cos \kappa = \cos \theta_p / \cos \delta_p . \tag{22}$$

Thence the R A of perigee is given by:

$$\lambda_p = \Omega - \kappa \qquad (\theta_p < \Omega) \tag{23a}$$

Again, quadrants must be considered to provide correct solution in all cases. Whence

$$\lambda_p = \Omega + \kappa \qquad (\theta_p > \Omega) \tag{23b}$$

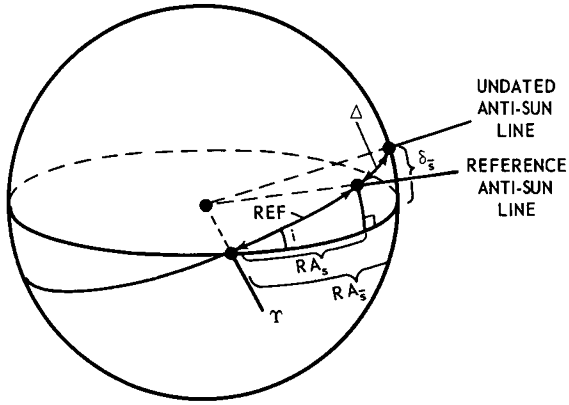



Figure 2-8. Update of Anti-Sun Line

Additionally, the reference arc to the anti-sun line from the first point of Aries is determined by:

$$\cos \text{REF} = \cos \text{RA}_S \cos \delta_S \qquad (24a)$$

with

$$\text{REF} = 360° - \text{REF of Eq. (24a)} \qquad (24b)$$

for $\text{RA}_S > 180°$.

After the S/C declination through time computations have been made, the time $\tau$ from reference perigee to specified orbit position is compared to the time $t_y$ the sun takes to apparently move through 360° of arc — the tropical year. This leads to the angle $\Delta$ through which the sun's radius vector moved when the S/C was at reference perigee to the time the S/C was at the specified orbit position. This angle $\Delta$ is the increment to the reference anti-sun line position:

$$\frac{\Delta}{360°} = \frac{\tau}{t_y}$$

or

$$\Delta = (\tau \times 360°)/t_y \qquad (25)$$

where $t_y$ is the tropical year,

$$t_y = 365.24219879 - .00000614 \times DT \tag{26}$$

and D T is the time since Jan. 1, $0^{hr}\, 0^{min}\, 0^{sec}$, 1900 in centuries e.g. D T = .71 for Jan. 1, 1971. The angle $\Delta$ in the ecliptic plane is added to the reference arc REF to define the new anti-sun arc. The components of this arc $RA_{\bar{S}}$ and $\delta_{\bar{S}}$ of the new sun position are evaluated from the laws of sines and cosines:

$$\frac{\sin \delta_{\bar{S}}}{\sin i} = \frac{\sin (REF + \Delta)}{\sin 90°}$$

or

$$\sin \delta_{\bar{S}} = \sin i \, \sin (REF + \Delta) . \tag{27}$$

Thence,

$$\cos (REF + \Delta) = \cos RA_{\bar{S}} \cos \delta_{\bar{S}} + \sin RA_{\bar{S}} \sin \delta_{\bar{S}} \cos 90°$$

or

$$\cos RA_{\bar{S}} = \frac{\cos (REF + \Delta)}{\cos \delta_{\bar{S}}} \quad (\text{when } REF + \Delta \leq 180°) \tag{28a}$$

Again, with quadrant considerations being made,

$$\cos RA_{\bar{S}} = \frac{360 - \cos (REF + \Delta)}{\cos \delta_{\bar{S}}} \quad (\text{when } REF + \Delta > 180°) \tag{28b}$$

## 2.7 Earth Shadow — S/C Relations

To determine if the S/C is shadowed by the earth or if it senses a terminator rather than a shadowed horizon, it is necessary to compute the earth shadow or umbra and S/C intersection.

2.7.1 Shadow/Terminator

Let $\hat{\bar{S}}$ be the anti-sun direction and $\hat{z}$ be the unit vector of the S/C radius vector; then the arc $\beta$ of the great circle connecting the two vectors is found from:

$$\cos \beta = \cos (90° - \delta_z) \cos (90° - \delta_{\bar{S}}) + \sin (90° - \delta_z) \sin (90° - \delta_{\bar{S}}) \cos \mu \quad (29)$$

where

$$\cos \mu = \cos (RA_z - RA_{\bar{S}}) . \quad (30)$$

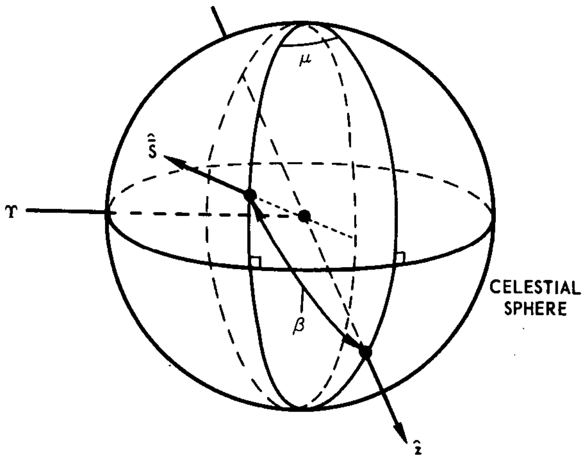



Figure 2-9. Anti-Sun Line and S/C Angle

Let vector $\hat{\bar{t}}$ be the terminator tangent intersecting the orbit at $P_0$. Also, $S_E$ is the angular diameter of the sun from earth orbit. Because of $S_E$, $\hat{\bar{t}}$ will not be parallel to $\hat{\bar{S}}$ but will be displaced toward $\hat{S}$ by $1/2\ S_E$ (Figure 2-11).

Let the radius vector to the intersection of the orbit with the terminator tangent be $\vec{R}_0$ and the arc between $\hat{z}$ ($\vec{R}_0$) and $\hat{\bar{S}}$ be $\beta_0$. The angle $\sigma_0$ between $\vec{R}_0$ and the earth radius $R_e$ at terminator tangent is then given by:

$$\cos \sigma_0 = \frac{R_e}{R_0} = (\sin \rho_0 \text{ of Eq. } (6)) \quad (31a)$$

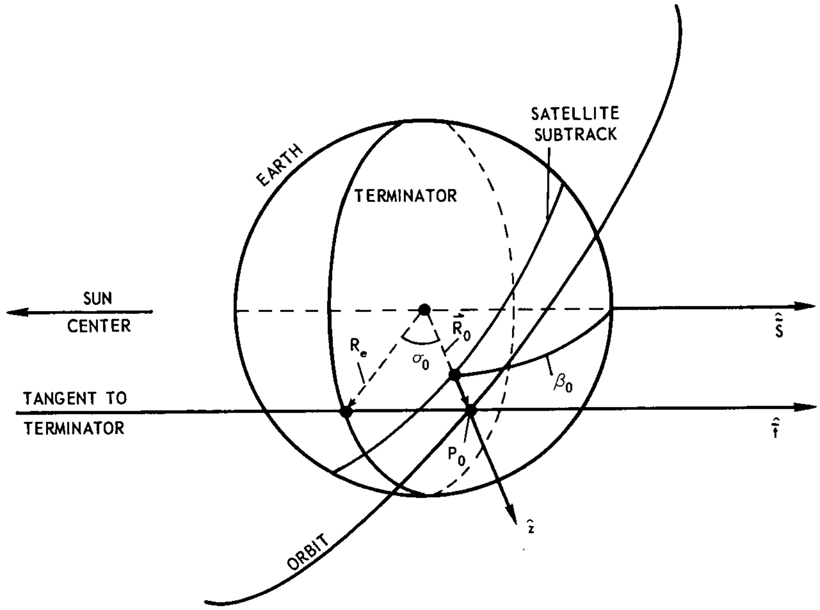



Figure 2-10. Terminator Tangent and Anti-Sun Line

The angle between $\hat{\hat{S}}$ and $\vec{R}_e$ is $90° - 1/2\ S_E$ since the angle between $\hat{t}$ and $\hat{\hat{S}}$ is $1/2\ S_E$ and $\hat{t}$ is perpendicular to $\vec{R}_e$. If general angle $\sigma$ is defined as:

$$\cos\sigma = \frac{R_e}{R} \quad (= \sin\rho \text{ of Eq. (6)}) \tag{31b}$$

for any orbit locus R and $\beta$ is as defined in Eq. (29), then the conditions for S/C and shadow are:

$$\sigma + \beta + \frac{1}{2} S_E \;\stackrel{\geq}{=}\; 90° \tag{32a}$$

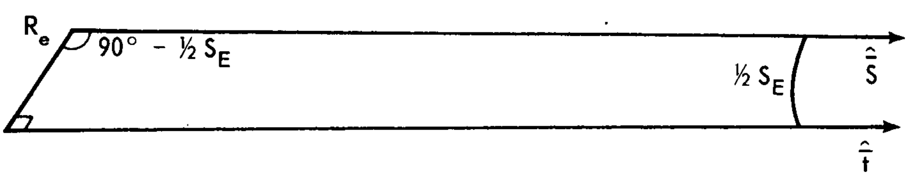



Figure 2-11. Half Angular Diameter of Sun

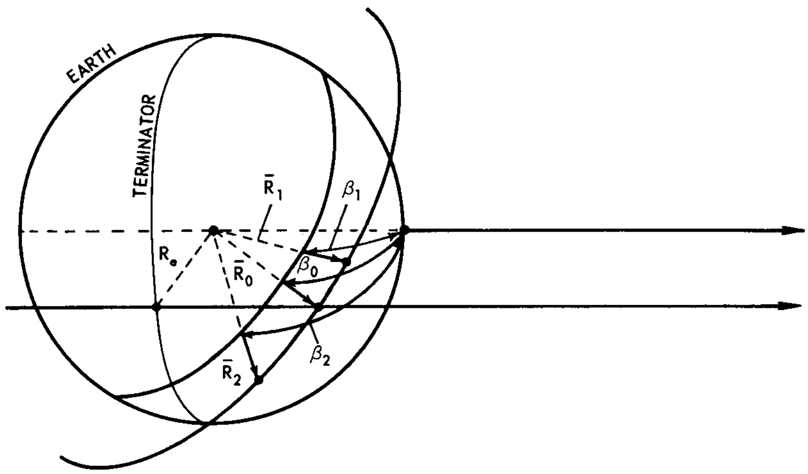



Figure 2-12. S/C in Shadow, at Terminator, and Sunlit

for S/C entering the earth shadow at terminator,

$$\sigma + \beta + \frac{1}{2} S_E < 90° \tag{32b}$$

for S/C completely in shadow,

$$\sigma + \beta + \frac{1}{2} S_E > 90° \tag{32c}$$

for S/C to scan at least one sunlit horizon.

2.7.2 Horizon/Terminator

Finally, for the case of Eq. (32c), to determine if the S/C senses two horizons rather than an horizon and a terminator, compare the angle $\beta$ between $\hat{\bar{S}}$ and $\vec{R}$ with the angle $\sigma$ between $R_e$ and R where $R_e$ is earth radius to the terminator tangent, R is the radius vector to the S/C, and $\hat{\bar{S}}$ is the anti-sun line direction. If a terminator and an horizon is sensed,

$$\beta - \sigma < 90° - \frac{1}{2} S_E \tag{33a}$$

and (32c) holds.

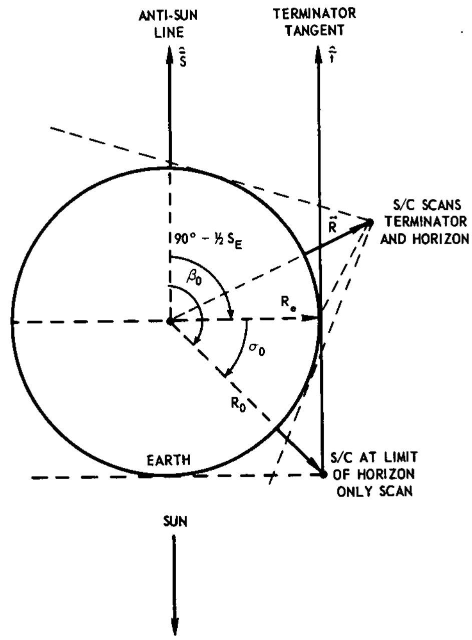



Figure 2-13. Geometry of Horizon or Terminator Detection

If two horizons are viewed,

$$\beta - \sigma \geq 90° - \frac{1}{2} S_E \tag{33b}$$

and (32c) holds.

# 3. PROGRAM

## 3.1 APL

### 3.1.1 Description

APL is a programming language used in conjunction with remote terminals connected by dataphones to a digital computer. The characters used for APL require a special "type ball" on the typewriter terminal. These characters are illustrated and defined in APPENDIX A.

Many separate terminals can be connected to the computer at one time through time sharing. All individual work spaces are kept separate so that, in general, no interference from other users is felt. Use of the computer is through a demand processor rather than a batch processor of conventional card/tape programs. This means access to the computer is in real time with no delay between coding and response.

The following is a discussion of only those aspects of APL utilized in the program of Section 3.2 and is not a comprehensive examination of all of APL's facets. For that, see REFERENCES.

Control of the computer from the terminal is afforded by a set of system commands initializing an individual's use of the computer, controlling and monitoring workspaces, and setting output and printout limits. Thus, after turning on the terminal, phoning computer, getting tone, cradling phone, and, depending on terminal, setting switches and awaiting lights, the "sign-on" command is given:

```
)nnnn :LLLLL
```

where nnnn is an assigned, unique user number and LLLLL is a user-chosen lock word. The return key is struck on the 2741 terminal (return key, then ALTN key and 5 key struck simultaneously on the 1052 terminal) to let computer respond with a message. Note, all computer messages start on far left of paper, but the starting column for the user is indented six columns.

After the computer message is printed, the workspace containing the program is loaded:

```
)LOAD  WWWW
```

where WWWW is the workspace ID and the program can be run, modified, or erased.

When finished with the desired modifications and the corrected workspace is preferred for future use over the old, the newly altered workspace can be saved:

```
)SAVE
```

with the old workspace ID retained. If initial creation of the program had been performed after sign-on and no load command made, then the new programming effort can be saved and assigned a workspace name (or new workspace ID assigned to current workspace) by:

```
)SAVE   WWWW
```

where WWWW becomes the new workspace ID and can be loaded in future terminal sessions.

To keep aware of the workspaces and their contents assigned to the user's number, several commands are available:

```
)LIB
```

lists names of workspaces in user's library.

```
)FNS
```

lists user created function in the workspace currently loaded.

```
)VARS
```

lists global variable names in that workspace, where global refers to all variable names except those used as arguments of functions or localized to function (as described later).

When a different workspace is desired and it is necessary to remove current contents without losing it for future use, use:

```
)CLEAR
```

If any workspace is no longer necessary, it can be deleted by:

```
)DROP   WWWW
```

where WWWW is the workspace ID. If particular variable or function names of an active workspace are superfluous, use:

```
)ERASE   JJJ  KKKK  LL  MMMMMM  . . .
```

where the assorted letters represent a list of variable and function names.

When all activities at the terminal have been completed, sign-off is by:

```
)OFF
```

or

```
)OFF:LLLL
```

where LLL is a word lock which must be entered with the user number when signing on.

Several other system commands exist and should be familiar to anyone wishing to use the full scope of APL, but the preceding are sufficient for many purposes. See REFERENCES for sources with more detail and fuller coverage.

The characters of APPENDIX A supply the basic tools for operating on values both singly (monadically) and jointly (dyadically). Using just these operators and numerical constants provides much more versatility and usefullness than any desk calculator. But the addition of the special symbols and their rules of association makes possible another order of activity. Definition of variables and programming of functions is thence possible.

One APL convention is especially important to keep in mind. Whether in desk calculator mode or in programming mode, there is no hierarchy of character importance, e.g. × does not get priority over +. Instead, all computations are performed from the right of a line of symbols toward the left. Each operation made in turn. With judicious use of parenthesis, the exactly desired algorithm is created e.g.

$$21-5+3\star2\times6 \quad (1)$$

is interpreted as

$$21-(5+(3\star(2\times6))) \quad (2)$$

in APL, rather than as

$$(21-5)+((3\star2)\times6) \quad (3)$$

in an hierarchial scheme. For results of (1) to be as for (3), the parenthesis must be supplied.

To associate a value with a variable name, the special symbol ←, the specification arrow, is used e.g.

```
A←180
B←90
```

or more involved,

```
C←A+B
D←A*B
```

At any time the value of a variable name may be checked by printing the name with no associated operations.

```
      C
270
```

where the 270 is the computer response. If a new variable is being created and its value is desired, the quad symbol ⎕ is used.

```
      ⎕←E←(C÷B)×A÷4
135
```

where 135 is the computer response and would be the response if E was requested.

When a mistake of a character or characters on a line is made but before the return key is hit, correction of the error can be effected by backspacing to error and striking the ATTN key on the 2741 or the LINE FEED key on the 1052. The carriage skips a line (prints a caret ^ on the 2741) and deletes the error character and all characters to the right. Correct characters are typed in to finish line as desired.

Using just variables and constants with the operators of APPENDIX A, the terminal can be used conveniently in desk calculator mode. A more powerful set of operators and the flexibility of variables makes it more useful than a desk calculator. However, the full impact of the versatility of APL comes in programming functions.

Programming mode is entered by typing a del ∇ followed by one of six different function header forms:

| No Explicit Result | Explicit Result With Dummy Variable A | |
|---|---|---|
| ∇FTN | ∇A←FTN | No Arguments |
| ∇FTN Y | ∇A←FTN X | One Argument |
| ∇X FTN Y | ∇A←X FTN Y | Two Arguments |

The arguments and result are local to the function, which means the calling sequence arguments and explicit result global variable have names independent of the function header arguments and explicit result dummy variable names. Additional local variables can be designated for use in the function (but not available for use after each execution of the function) by including in the function header each additional local variable separated by semicolons; e.g.

```
∇FTN X;U;V;W
```

where ∇FTN X is one of the header forms and ;U;V;W are extra local variables.

Simple examples of calling forms are given below for the six header forms:

```
FTN
FTN THETA
RANGE FTN ALPHA
FORCE←FTN×MASS
D←T×(FTN A)
TEMP←PRES FTN VOL
```

After the function header, the next lines may include coding of mathematical expressions, definitions, function control, or formats for output. If a mathematical expression is formed and variables used outside the function are needed, it is not necessary to bring them into the function through arguments or statements analogous to FORTRAN's COMMON. If not local to another function, they are global and thus available to any function or expression as last defined or evaluated. In addition, the result of any expression in a function becomes global unless it is the same as the dummy variable of or localized in the function header. The benefit of localizing names is in being able to use the same names in different functions without interference.

After the function header is typed and the return key hit, the computer types

```
[1]
```

on the next line, skips three spaces, and awaits typed input by the user. The next lines are typed in similar fashion, building the desired algorithm.

If branching or iterations are required, the branch arrow → is used at the first of a line followed by the destination of the branch. This destination may be the number of a line, an APL expression resulting in a line number, a line label, the number zero which causes branches out of the function, a blank space which branches out of the function and all levels of functions being executed, a null vector such as ⍳0 which causes sequence to drop to next instruction, and various combinations depending on values of variables. The following illustrate the branch:

```
→5
→MARK

→(AD=2)/MARK
```

where the 0 compression of MARK produces the null vector so sequence falls to next line; while for AD = 2, the 1 compression of MARK results in MARK so next sequence is to label MARK.

The label is attached to the first of a line and separated from the expression by a colon, e.g.

```
[5]   MARK:XI←A+B÷C
```

The purpose of the label is in not having to change the line number each time a change in the program adds or deletes line(s) previous to the given line.

The use of alphabetic statements for output edit and other purposes is permitted by the use of a pair of single quote marks separated by any number of APL characters, e.g.

```
[6]   'SOLUTION OF KEPLER''S EQUATION:  '
```

For each execution of the function through statement [6], the computer prints out:

```
SOLUTION OF KEPLER'S EQUATION:
```

If an evaluated expression is wanted on the same line, add a semicolon after the last quote mark and include the expression:

```
[10]   'DISTANCE TRAVELED:   ';D←VEL×T
```

This results in the expression being evaluated, associated with variable D, and printed without requiring a quad symbol:

```
DISTANCE TRAVELED:    320
```

where VEL = 32 and T = 10.

The appearance of double quotes alone on a line will cause a line to be skipped at execution time.

The last character of the function must be the del — either at the end of an expression or by itself on the last line. This is done in order to leave programming mode and to return to desk calculator mode where the function can be tested.

If errors in the function are detected or changes in it are desired, a very convenient set of function editing procedures is available. To produce an entire listing of the function and leave the terminal in programming mode ready for entry of another line, type:

> del, function name (without arguments), open bracket, quad, close bracket, e.g.

```
∇FTN[⎕]
```

This produces a function listing and next line number then awaits a new line to be typed. If addition of new lines is all that is required, type them and strike a final del. If a previously typed line is seen to need correction, then type:

> open bracket, line number to be corrected, quad, approximate correction column, close bracket, e.g.

$\underbrace{\texttt{[19]}}_{\text{computer types}}$ $\underbrace{\texttt{[12⎕9]}}_{\text{user types}}$

When return key is struck after last bracket,

```
[12]  B2←XI*(A÷C)
       @
```

is printed by computer, line is skipped, and ball is aligned at position designated by @. If BI was the variable name desired, the user back spaces one (or any number of spaces forward or backward necessary to align ball beneath mistake), types a slash /

to delete the two, types a one (or any number 1-9 or A, B, C, . . . for 5, 10, 15, . . .) to cause that number of spaces to be inserted in front of the ←, and strikes the return key. The computer responds by retyping the line, deleting all characters with a slash beneath, inserting spaces in front of all characters with a number or letter beneath, and then backspaces to the first empty space to be filled.

```
[12]  B ←XI*(A÷C)
       @
```

User now types I and RETURN. The computer types next line number and awaits user action.

If nothing more needs to be corrected, ∇ is typed leaving the rest of the function intact. However, if a line needs to be deleted, type:

[line number] e.g.

$\underbrace{[13]}_{\text{COMPUTER}}$ $\underbrace{[6]}_{\text{USER}}$

[6]

On 2741, hit ATTN key. On 1052, hit LINE FEED key. Computer appropriately skips to next line, erasing the line from the function.

```
[7]
```

If a line needs to be seen before deciding if it is to be deleted, type:

[line number ⎕ ] e.g.

```
[7]    [8⎕]
[8]    PICK←AFT×(UP<100)
[8]    @
```

If a new line is not desired, either go to another line or type ∇ to leave programming mode. Otherwise type new line.

To insert a line in place of another without printing old line, type:

[line number], expression e.g.

```
[8]    [10] VALUE←PICK÷PRICE
```

To insert a new line or lines between original lines, type:

[decimal number between two successive line numbers] e.g.

```
[11]  [13.5]
[13.5] 'NEW LINE ONE'
[13.6] 'NEW LINE TWO'
```

New decimal numbers are supplied in this case up to `[13.9]` after which `[14]` is typed by computer. To provide more new lines for insertion, a smaller decimal line number should be supplied such as:

```
[13.11]
```

The sequence new line numbers are formed is thence:

```
[13.11],[13.12],[13.13],[13.14],[13.15],[13.16]
[13.17],[13.18],[13.19],[13.2],[13.3],[13.4],
[13.5],...
```

After all corrections, deletions, and insertions have been made and the final ∇ typed to leave programming mode, the computer renumbers all lines as integers; however, before the ∇ is typed all line numbers are as they were last printed.

These rules can best be understood by using them in various situations and with an aim to minimum effort and efficient reprogramming.

### 3.1.2 Advantages

The choice of APL as the coding language for the derivations of this paper is based on several factors:

- the APL characters and coding rules lend themselves to mathematical applications.
- the development of the program is facilitated by use of a computer from a terminal in the manner of a desk calculator.
- the capability of writing and preserving functions for performing various algorithms is available and simple to use.
- the modification of preserved functions and variables is simple and complete.

- extensive knowledge of all of the language or of the computer actions is not necessary.

- the making, detection, and correction of mistakes is immediate, and in general, nondistructive of further activity at the terminal.

- initializing functions for repeated loops is simple.

- output from running a program is immediately received, checked, and rerun if necessary.

- an algorithm can be debugged before being coded in another language.

- arrays are handled easily in many complex applications by the use of the powerful operation set.

### 3.1.3 Disadvantages

Due to the relatively slow print rate of the terminal typewriters, a user encounters a lengthy delay in getting on a terminal whenever the previous user has extensive printout.

Whenever the computer goes down during program testing, much tedious retyping or program checkouts are required to get back to the pre-failure status. This is an especially frustrating problem when corrections have not yet been saved.

All input parameters must be entered by hand on most terminals and/or tape input facilities are not easily utilized and are not generally available from the terminals. If reruns are contemplated, semi-permanent storage of the parameters must be prepared with resultant loss of available storage.

Formatting of output with column editing is to some extent inconvenient in that columns of figures must be carefully arranged rather than occurring as a natural result of the format designators.

## 3.2 Coding

### 3.2.1 Program Design

To correctly write programs is the obvious aim of any user of the APL programming mode. In addition, several other factors should be kept in mind while coding. The effect of these factors is to have an overall program design.

If a number of similar conversions are made, it is more efficient and clearer to define a function which is initiated whenever the conversion is needed. A single function can often be rewritten as several functions in order to allow certain algorithms to be used independently of the others. In this case, control or driver functions are written which call various functions and supply intermediate and relating steps.

The design of the spacecraft/earth scan program resulted in several types of functions being developed to perform the various actions:

> parameter initialization routines, program drivers, utility (or trigonometric conversion) routines, and computation routines.

For analysis of a desired orbit, the orbital elements, spin axis right ascension and declination, sun elements at time of S/C perigee, and fraction of century since 1900 are related to the proper variable names in a parameter initialization function. This function can be named at later times allowing recomputation through the program without retyping the parameters. Several of these initialization routines are formed for the orbits of interest. There are two drivers: one references the entire set of derivations of Section 2, the other is for S/C position through earth crossing angle computations only. The drivers set iterations through the routines for a series of S/C position right ascensions. The utility routines are designed to convert from APL character representation of trigonometric relations to a more recognizable and simple form. The computation routines are particular algorithms to compute one or several dependent parameters.

The block diagram of Figure 3-1 illustrates the relationship of the functions to one another for the main driver routine TRUN.

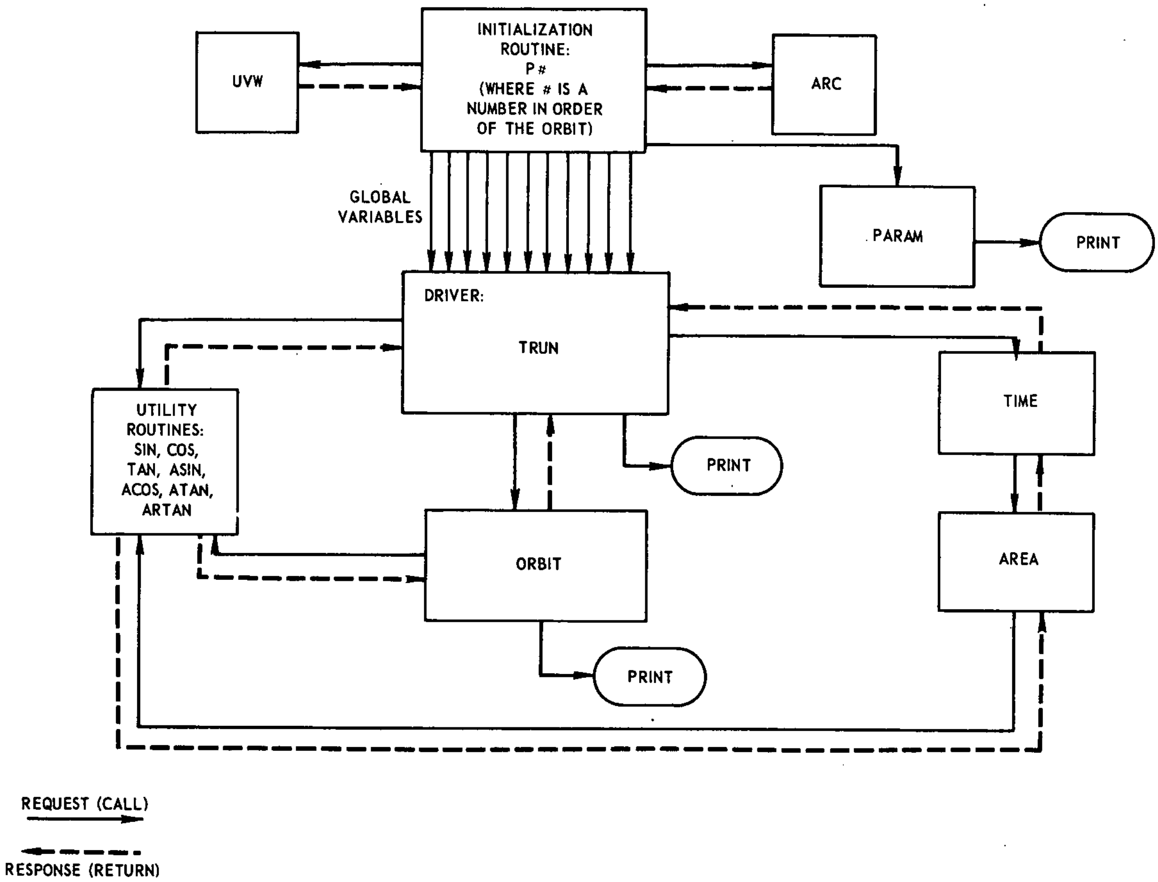



Figure 3-1. Block Diagram of Limited Driver Routine

The simpler structure of the block diagram for the driver routine RUN is shown in Figure 3-2.

### 3.2.2 Initialization Routines

By simply typing an initialization function name, a set of orbit, S/C, and sun parameters are fed into appropriate global variable names which can thence be referenced by any other function. Replacement by another set is a matter of typing another initialization function name. The general form of the function is as follows:

∇ P#  where P# is the program name and # is the number of the orbit order

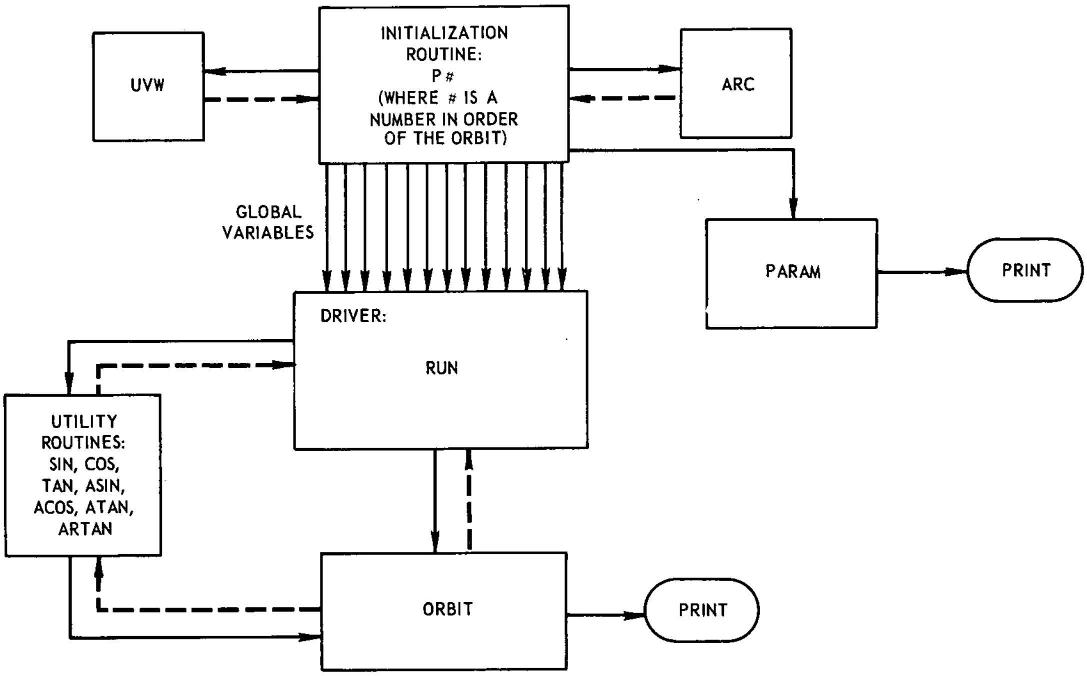



Figure 3-2. Block Diagram of Limited Driver Routine

J ← orbit inclination
EPSLON ← orbit eccentricity
THETAP ← argument of perigee
OMEGA ← right ascension of ascending node
PER ← period of orbit
RA ← radius vector at apogee
RP ← radius vector at perigee
AS ← (RA + RP) × .5 (computes semi-major axis)
BS ← (RA × RP) * .5 (computes semi-minor axis)
RAL ← right ascension of S/C spin axis
DAL ← declination of S/C spin axis
UVW (call to function to compute eccentricity relations)
RAS ← right ascension of anti-sun line at time of perigee
DAS ← ARC RAS (computes anti-sun line declination)
DT ← fraction of century since 1900.0 at perigee
SR ← semi-diameter of sun for orbit
I ← inclination of ecliptic for orbit

A routine PARAM to produce a partial listing of the parameters of P# is included to provide a check of the current parameters available for computation.

### 3.2.3 Program Driver Routines

The drivers RUN and TRUN are used for repeated passes through function ORBIT for a series of input S/C right ascensions. Each successive RA is a constant increment of the preceding one. The increment, starting RA, and limiting RA are the arguments of the function. This requires three function arguments whereas the allowed function formats seem to allow but two. Actually, since the input arguments may be n dimensional vectors, each vector component may be a changing, function input.

Thus for RUN and TRUN, the first argument on the left of the function name is the incremental value applied to successive RA's and the second argument is a vector with two components — the first is the beginning RA and the second is the limiting RA. For example,

```
10 RUN 20 50
```

produces three passes through function ORBIT for RA's of 20°, 30°, and 40°. Note, 50° is not entered.

The difference in RUN and TRUN is in the presence of the computation algorithms for time, sun, and shadow parameters in TRUN but not in RUN. Thus, RUN output parameters are limited to those from the function ORBIT.

### 3.2.4 Utility Routines

The utility routines for this program are all trigonometric. They were created to eliminate extra coding necessary to express in degrees the trigonometric relations of APL which are simplest for radian measure. The functional coding also resulted in more understandable reading of the functions. One radian measure trig function is also included for this reason.

### 3.2.5 Computation Routines

In initializing parameters, the function P# calls two computation functions. The first, UVW, produces three values dependent on the orbit eccentricity:

$$U = 1 - \epsilon^2$$

$$V = 1 + \epsilon$$

$$W = 1 - \epsilon$$

for simplification of formulae used in other functions. The function ARC computes for a given RA of the ecliptic plane the corresponding declination.

Function ORBIT determines S/C declination and radius vector, half earth (radius) angle, S/C spin axis to earth center (S/C nadir) angle, and whether or not earth is in scan range and if so the crossing angle. A print statement of each parameter is also produced.

Having explicit results are two functions for computation of time to or from perigee. TIME is the function for the basic equation:

$$T = \frac{\text{area of sector}}{\text{total area}} \times \text{period.}$$

The area of sector depends on the angle $\theta$ from perigee and is computed from function AREA. The total area of orbit ellipse is a variable, set as

$$\pi \times AS \times BS.$$

TRUN also performs computations as well as controlling program flow. It determines RA and $\delta$ of S/C perigee, sets the sun reference angle, and for each cycle through ORBIT computes RA and $\delta$ of anti-sun line, angles between $\hat{S}$, $\vec{R}$, and $\vec{R}_e$, and terminator/shadow bounds.

# 4. APPLICATION

Use of the described program for computation of S/C orbit/earth scan relations has been made for several satellites, though chiefly designed and analyzed for IMP-6 (IMP-I). A description of that spacecraft and its early orbit parameters follows

## 4.1 IMP-6 S/C Parameters

IMP-6 was engineered with a digital solar sensor and an earth horizon detector. Combined with the aspect system electronics, the sensors form an optical aspect system which provides:

- the angle between the S/C spin axis and the sun direction,
- the angle between the S/C spin axis and the local vertical,
- the fraction of S/C spin period between sun sensing and earth sensing.

The IMP-6 S/C is a sixteen-sided drum, 72 inches in length, 53 inches across, with antennae and booms extending from the main body. It spins about the drum's axis of symmetry with its attitude sensors aligned on the same side of the S/C such that there is no angular bias to measurements relating the two sensor references.

The digital solar sensor is as described in Section 1.2.3 except it has a nine bit encoder for the sun angle allowing more resolution — to about a third of a degree.

A tenth bit supplies azimuth data from the command slit. New digital solar sensor information is read into the telemetry stream along with ten additional remainder bits every 81.92 seconds as the twenty bit OA scan. Repeat OA scan data is read out three times before new sun angle data is supplied.

The earth horizon sensor is a small (1-1/2°) optical spectrum instrument mounted 90° from the spin axis. Thus, the mounting angle $\gamma$ for IMP-6 is 90° and the SCAN angle is 1-1/2°. Detection of horizons or horizon and terminator produces signals which are shaped into pulses.

The aspect system electronics produce a sun centered pulse whenever the azimuth bit is set by detection of the sun in the command slit. This pulse initiates 1.6 kc counters which are on until

- the next sun pulse
- the earth pulse produced by the next earth horizon sensing.

These counters represent spin period and earth-in respectively. The spin period count relates to time by

$$T_{SP} = C_{SP} \text{ (counts)} \times .000625 \text{ (secs/count)}$$

where 1/1.6 kc = .000625 secs/count. The earth-in count gives the time from sun to first earth horizon in a similar fashion.

In addition, two successive earth horizon pulses trigger and stop a counter which thereby supplies an earth-width count to the telemetry. This earth-width count provides the time the sensor takes to scan from an earth horizon to the next. When coupled with spin period, the scanned earth-width angle is produced.

$$\frac{T_{EW}}{T_{SP}} \times 360° = \text{Earth-width angle (in degrees)}$$

Finally, a pulse generated by the S/C initiates a 1.6 kc counter which is then stopped by the next sun pulse. The resultant count is the sun-time count. The sun-time relates all of the sensor times to the S/C telemetry word times.

The optical aspect counts are provided to the S/C telemetry every 81.92 seconds and 20.48 apart. The order in which the counts are produced and are placed in the telemetry stream is

- OAST — sun time count
- OASP — spin period count
- OAEI — earth-in count
- OAEW — earth-width count.

For IMP-6, the nominal spin period is about five seconds.

## 4.2 Orbital Elements

The IMP-6 pre-launch prediction orbital elements were as follows:

| | | |
|---|---|---|
| semi-major axis | — | 114 606.63 km |
| eccentricity | — | .942 |
| inclination | — | 28.3° |
| argument of perigee | — | 293.8° |
| RA of ascending node | — | 171.9° |

The orbit actually achieved shortly after launch on 3/13/71 1616 hours had these elements:

| | | |
|---|---|---|
| semi-major axis | — | 109 529.835 |
| eccentricity | — | .9381564 |
| inclination | — | 28.597879° |
| argument of perigee | — | 301.97723° |
| RA of ascending node | — | 233.6023°. |

## 4.3 Sun Parameters

The sun parameters on 3/13/71 at 1700 hours were:

| | | |
|---|---|---|
| apparent RA | — | $23^h\ 32^m\ 20.07^s$ |
| apparent declination | — | -3° 32'49.5" |
| semi-diameter | — | 16'06.98" |
| obliquity | — | 23° 26'42."775. |

The anti-sun line is 180° or oppositely directed to the sun line. Thus, the declination is of opposite sign and the right ascension is 180° less.

## 4.4 Nominal Attitude Values

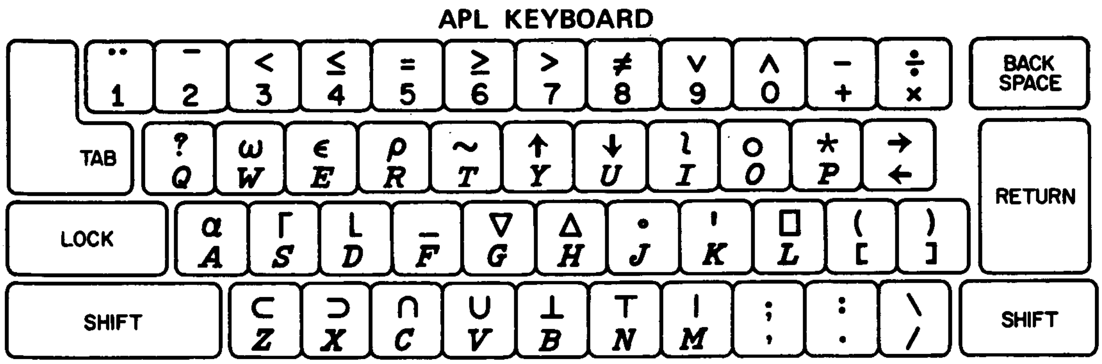
The nominal orientation of the IMP-6 spin axis is perpendicular to the ecliptic plane. Specifically, the +Z axis is aligned with the south ecliptic pole. Thus, RA $\approx$ 90° and $\delta \approx$ -66.5° for spin axis.

Looking "down" on the +Z axis, the S/C turns or spins in a counter-clockwise sense. The spin rate is about 11 rpm.

## 4.5 Comments on Results from IMP-6

For the first few weeks after IMP-6 was launched, two periods each orbit were favorable for scanning the earth. However, the scans near orbit perigee were almost entirely in the earth's shadow. Thus, only one period each orbit provided earth scan data to the telemetry. The simulation run of APPENDIX C reflects this situation as well as other range, duration, and angle characteristics of the earth scans. Improved correspondence between telemetry and simulation is possible by adjustment of the mounting angle, field of view, and spin axis parameters.

# APPENDIX A — APL CHARACTERS AND CONVENTIONS




| SPECIAL SYMBOLS | |
|---|---|
| A←⎕ | Request evaluated input. Assign value to A. |
| 'X7∗' | Quote marks delimit character data constants. |
| A←⍞ | Request character data input; assign it to A. |
| ⎕←A | Print value in A and pass it to left. |
| ⍝ | Lamp symbol; first character of comment statement. |
| T∆F←V | Set trace vector of statement numbers for function F. |
| S∆F←V | Set stop vector of statement numbers for function F. |
| →X | Branch to X; X may be a label or statement number. If X is a vector, →X[1]. If X is empty vector, there is no branch; the next line is taken. If X is not a statement number in the function, terminate execution of the function. |
| → | Terminate entire execution sequence of functions. |
| : | Separates label from statement in functions. |
| ; | Separates character and numeric data in same output statement: |

Overstruck symbols: A̲B̲C̲D̲E̲F̲G̲H̲I̲J̲K̲L̲M̲N̲O̲P̲Q̲R̲S̲T̲U̲V̲W̲X̲Y̲Z̲∆̲⍋⍒⍱⍲⌽⊖⍉!⍕⍎⍞⍝⍀⌿

| SYSTEM DEPENDENT FUNCTIONS | |
|---|---|
| I | Definition of ⌶I |
| 19 | Accumulated keying time (time during which the keyboard has been unlocked awaiting entries) during this session. |
| 20 | The time of day. |
| 21 | The central computer time used in this session. |
| 22 | The amount of available space (in bytes). |
| 23 | The number of terminals currently connected. |
| 24 | The time at the beginning of this session. |
| 25 | The date. |
| 26 | The first element of the vector ⌶27. |
| 27 | The vector of statement numbers in the state indicator. |
| 28 | A code indicating the terminal device being used. |
| 29 | User sign-on number. |

NOTES
1. All times in 1÷60 seconds
2. Date is represented by a 6-digit integer; successive digit pairs represent month, day, and year.
3. ⌶27 yields a vector; all other results are scalars.

| FUNCTION EDITING | |
|---|---|
| FORM | RESULT |
| [⎕] | LIST ENTIRE FUNCTION. |
| [⎕N] | LIST BEGINNING WITH LINE N. |
| [N] | INSERT OR REPLACE LINE N; TO DELETE, FOLLOW WITH INTERRUPT AND CARRIAGE RETURN. |
| [N⎕] | LIST LINE N AND INSERT REPLACE OR DELETE. |
| [N⎕M] | LIST LINE N AND PLACE TYPE BALL UNDER COLUMN M FOR EDITING. |

WHEN EDITING A LINE OF A FUNCTION, SPACES TO BE REMOVED ARE INDICATED BY '/', SPACES TO BE ADDED BY SINGLE DIGITS OR LETTERS; THE SPACES ARE INSERTED BEFORE THE INDICATED SPOT; WHEN LETTERS ARE USED, THE NUMBER OF SPACES INSERTED IS EQUAL TO 5×'ABCDEFGHI...'⍳LETTER.

<table>
<tr><th colspan="2">PRIMITIVE SCALAR DYADIC FUNCTIONS A ⍟ B</th><th>⍟<br>⍟</th><th colspan="2">PRIMITIVE SCALAR MONADIC FUNCTIONS ⍟ B</th></tr>
<tr><th>Definition or Example</th><th>Name</th><th></th><th>Name</th><th>Definition or Example</th></tr>
<tr><td>1.5 ↔ ¯2+3.5 5.5 ↔ 2+3.5 ¯1.5 ↔ 2+¯3.5</td><td>Add</td><td>+</td><td>No Change</td><td>0+B ↔ +B 3.5 ↔ +3.5 ¯3.5 ↔ +¯3.5</td></tr>
<tr><td>¯1.5 ↔ 2-3.5 1.5 ↔ 3.5-2 5.5 ↔ 2-¯3,5</td><td>Subtract</td><td>-</td><td>Change Sign</td><td>0-B ↔ -B ¯3.5 ↔ -3.5 3.5 ↔ -¯3.5</td></tr>
<tr><td>5 ↔ 4×1.25 ¯3 ↔ 6×¯.5 0 ↔ 0×¯.09</td><td>Multiply</td><td>×</td><td>Signum</td><td>Sign of B: 1 ↔ ×7.2 0 ↔ ×0 ¯1 ↔ ×¯3</td></tr>
<tr><td>1.75 ↔ 3.5÷2 ¯5 ↔ 10÷¯2 4 ↔ 12÷3</td><td>Divide</td><td>÷</td><td>Reciprocal</td><td>1÷B ↔ ÷B .5 ↔ ÷2 ¯2 ↔ ÷¯.5</td></tr>
<tr><td>A raised to the power B ↔ A*B<br>8 ↔ 2*3 2 ↔ 4*.5 ¯3 ↔ ¯27*(1÷3)</td><td>Power</td><td>*</td><td>Exponential</td><td>(2.71828...)*B 2.71828... ↔ *1<br>4 ↔ *1.386294361... 20.0855... ↔ *3</td></tr>
<tr><td>(⍟B)÷⍟A ↔ logarithm of B for base A ↔ A⍟B<br>1.87506... ↔ 10⍟75 3 ↔ 2⍟8</td><td>Logarithm</td><td>⍟</td><td>Natural Logarithm</td><td>(2.71828...)⍟B N ↔ *⍟N ↔ ⍟*N<br>1.386294361... ↔ ⍟4 ¯0.693147... ↔ ⍟.5</td></tr>
<tr><td>Larger of A and B ↔ A⌈B<br>7 ↔ 3⌈7 6.01 ↔ 6.01⌈6.01 ¯3 ↔ ¯3⌈¯7</td><td>Maximum</td><td>⌈</td><td>Ceiling</td><td>Smallest integer ≥ B ↔ ⌈B<br>4 ↔ ⌈3.141 ¯3 ↔ ⌈¯3.141 101 ↔ ⌈101</td></tr>
<tr><td>Smaller of A and B ↔ A⌊B<br>3 ↔ 3⌊7 6.01 ↔ 6.01⌊6.01 ¯7 ↔ ¯3⌊¯7</td><td>Minimum</td><td>⌊</td><td>Floor</td><td>Largest integer ≤ B ↔ ⌊B<br>3 ↔ ⌊3.141 ¯4 ↔ ⌊¯3.141 101 ↔ ⌊101</td></tr>
<tr><td>3 ↔ 5|13 B-(|A)×⌊B÷|A ↔ A|B For A≠0<br>5 ↔ 0|5 B ↔ A|B For A=0, B≥0<br>5 ↔ 14|5 Domain Error ↔ A|B For A=0, B<0<br>.14 ↔ 1|3.14 4 ↔ 5|¯11 D.E. ↔ 0|¯4</td><td>Residue</td><td>|</td><td>Magnitude</td><td>Absolute value of B ↔ |B<br>9.5 ↔ |9.5 9.5 ↔ |¯9.5 0 ↔ |0</td></tr>
<tr><td>6 ↔ 2!4 (!B)÷(!A)×!B-A ↔ A!B For A≤B<br>0 ↔ 9!3 1 ↔ 5!5 0 ↔ A!B For A>B<br>Complete Beta Function for A and B<br>¯10 ↔ 3!¯3 4.9346... ↔ 1.1!4.5</td><td>Combinations or Binomial Coefficient or Beta</td><td>!</td><td>Factorial or Gamma</td><td>B×!B-1 ↔ !B For B≥1, B an integer; 1 ↔ !0<br>Gamma(B+1) ↔ !B for non-integer B 6 ↔ !3<br>39916800 ↔ !11 2.68344... ↔ !2.3<br>3.3283... ↔ !¯2.3 D.E. ↔ !¯2</td></tr>
<tr><td>See Table of Mixed Functions</td><td></td><td>?</td><td>Roll</td><td>Random choice from ⍳B ↔ ?B (Origin dependent)<br>Random integer from 1 2 3 4 5 6 7 8 ↔ ?8</td></tr>
<tr><td>See Table at right</td><td>Circular</td><td>○</td><td>Pi times</td><td>B×3.14159... ↔ ○B 6.283185... ↔ ○2</td></tr>
<tr><td>A B A∧B A∨B A⍲B ∼(A∧B) A⍱B ∼(A∨B)<br>0 0 0 0 1 1 1 1<br>0 1 0 1 1 1 0 0<br>1 0 0 1 1 1 0 0<br>1 1 1 1 0 0 0 0</td><td><br>And<br>Or<br>Nand<br>Nor</td><td>∼<br>∧<br>∨<br>⍲<br>⍱</td><td>Not</td><td>Logical Negation 0 ↔ ∼1 1 ↔ ∼0</td></tr>
<tr><td>Relational Tests: Result is 1 if the relation is true; result is 0 if the relation is false.<br>1 ↔ 3<4 0 ↔ 4<3<br>1 ↔ 4≤4 0 ↔ 6≤4<br>1 ↔ 9=9 0 ↔ 2=9<br>1 ↔ 7≥7 0 ↔ 3≥8<br>1 ↔ 7>0 0 ↔ 2>8<br>1 ↔ 3≠6 0 ↔ 9≠9</td><td>Less<br>Not Greater<br>Equal<br>Not Less<br>Greater<br>Not Equal</td><td><<br>≤<br>=<br>≥<br>><br>≠</td><td></td><td></td></tr>
</table>

<table>
<tr><th colspan="2">Table of Dyadic ○ Functions</th></tr>
<tr><td>(1-B*2)*.5 ↔ 0○B</td><td>(1-B*2)*.5 ↔ 0○B</td></tr>
<tr><td>Arcsin B ↔ (-1)○B</td><td>Sine B ↔ 1○B</td></tr>
<tr><td>Arccos B ↔ (-2)○B</td><td>Cosine B ↔ 2○B</td></tr>
<tr><td>Arctan B ↔ (-3)○B</td><td>Tangent B ↔ 3○B</td></tr>
<tr><td>(¯1+B*2)*.5 ↔ (-4)○B</td><td>(1+B*2)*.5 ↔ 4○B</td></tr>
<tr><td>Arcsinh B ↔ (-5)○B</td><td>Sinh B ↔ 5○B</td></tr>
<tr><td>Arccosh B ↔ (-6)○B</td><td>Cosh B ↔ 6○B</td></tr>
<tr><td>Arctanh B ↔ (-7)○B</td><td>Tanh B ↔ 7○B</td></tr>
<tr><td colspan="2">B in radians</td></tr>
</table>

<table>
<tr><th colspan="4">DETERMINING AND CHANGING TERMINAL INPUT MODE</th></tr>
<tr><th>APL Typed:</th><th>Input Mode</th><th>APL Expects:</th><th>Escape by Typing:</th></tr>
<tr><td>6 space indent</td><td>Immediate Execution</td><td>Any Input</td><td>- - -</td></tr>
<tr><td>⎕:</td><td>Evaluated Input</td><td>Any APL expression</td><td>→ or )CLEAR or )LOAD wsid</td></tr>
<tr><td>Keyboard unlocked at left margin</td><td>⍞ Input<br>Unmatched quote mark</td><td>1 line of characters<br>Any characters</td><td>⍝ OBSUBST BS=backspace<br>' (quote mark)</td></tr>
<tr><td>[n]<br>[n] line of function indent to some position<br>[n] line of function no carrier return</td><td>Function Definition<br>Line-edit Phase 1<br>Line-edit Phase 2</td><td>Line of function or editing command<br>/, 0-9, A-Z<br>Line modification by "visual fidelity"</td><td>∇ or )CLEAR or )LOAD wsid<br>RETURN<br>∇ as rightmost character</td></tr>
</table>

## Primitive Mixed Functions

| Name | Sign<1> | Definition or example<2> |
|---|---|---|
| Shape | ρA | Dimension vector of A    ρS ↔ Empty vector    ρρA ↔ Rank of A<br>ρP ↔ 4    ρE ↔ 3 4    ρ7 ↔ ⍳0    ρρE ↔ 2    ρρρA ↔ 1 |
| Reshape | VρA | Reshape A to dimension V    E ↔ 3 4ρ⍳12    12ρE ↔ ⍳12    2 5ρE ↔ 1 2 3 4 5<br>7ρP ↔ 2 3 5 7 2 3 5    0ρP ↔ ⍳0    (⍳0)ρP ↔ 2    6 7 8 9 10 |
| Ravel | ,A | Make A into a vector ↔ (×/ρA)ρA    P ↔ ,P    ,E ↔ ⍳12    ρ,7 ↔ 1 |
| Catenate <3 4> | A,B<br>A,[I]B | Join two arrays along an existing coordinate. A and B conform if either ρA ↔ ρB or (I≠⍳ρρA)/ρA ↔ ρB or ρA ↔ (I≠⍳ρρB)/ρB or if A or B is scalar. A scalar argument is extended (reshaped) to conform to the other argument.<br>'T','HIS' ↔ 'THIS'    ABCDABCD    4 1 1 1<br>2 3 5 7,1 2 ↔ 2 3 5 7 1 2    X,X ↔ EFGHEFGH    4,MN,[1] 5 5 5 ↔ 4 2 2 2 / 4 3 3 3 / 4 5 5 5    where MN ↔ 1 1 1 / 2 2 2 / 3 3 3<br>IJKLIJKL |
| Laminate <3 4> | A,[I]B<br>S,S | Join two arrays along a new coordinate. Two scalars laminate to a length-2 vector. Two non-scalar arguments must have identical shapes. If one argument is scalar, it is extended to match the shape of the non-scalar argument. I is non-integral and shows what existing coordinate(s) the new coordinate lies next to.<br>'O','K' ↔ 'OK'    C,[.5]D ↔ ABC / DEF    C,[1.5]D ↔ AD / BE / CF    C,[0.5] '*' ↔ ABC / ***    where C ↔ 'ABC'<br>100,5 ↔ 100 5    D ↔ 'DEF' |
| Index <4 6> | V[A]<br>M[A;A]<br>A[A;...;A] | P[2] ↔ 3    E[;1] ↔ 1 5 9    ABCD<br>E[1;] ↔ 1 2 3 4    E[1 3;3 2 1] ↔ 3 2 1 / 11 10 9    'ABCDEFGHIJKL'[E] ↔ EFGH<br>P[4 3 2 1 1 3] ↔ 7 5 3 2 2 5    IJKL |
| Index <3> generator | ⍳S | First S integers    ⍳5 ↔ 1 2 3 4 5    ⍳0 ↔ An empty vector |
| Index of <3> | V⍳A | First occurrence of A in V, or 1+ρV if no occurrence    5 1 2 5 / 3 5 4 5 / 5 5 5 5 ↔ P⍳E<br>P⍳3 ↔ 2    5 6 5⍳5 ↔ 1    P[P⍳5 2] ↔ 5 2 |
| Take<br>Drop | V↑A<br>V↓A | Take or drop the (\|V[I]) first (if V[I]≥0) or last (if V[I]<0) elements along coordinate I of A    ABC<br>¯2↑P ↔ 5 7    2↓'THIS' ↔ 'IS'    6↓'THIS' ↔ Empty vector    ¯6↑P ↔ 0 0 2 3 5 7    2 3↑X ↔ EFG |
| Grade up<3><br>Grade down | ⍋V<br>⍒V | The permutation which would order V (ascending -- ⍋; or descending -- ⍒)<br>⍋3.7 10 3.7 1.2 ↔ 4 1 3 2    ⍒3.7 10 3.7 1.2 ↔ 2 1 3 4    V[⍋V] gives ascending sort |
| Compress <5><br>Expand | V/A<br>V\A | 1 0 1 0/P ↔ 2 5    1 0 1 0/E ↔ 1 3 / 5 7 / 9 11    1 2 3 4 ↔ 1 0 1/[1]E ↔ 1 0 1⌿E / 9 10 11 12    1 0 1 1 1\X ↔ A BCD / E FGH / I JKL<br>1 0 1\⍳2 ↔ 1 0 2 |
| Reverse <5><br>Rotate | ⌽A<br>B⌽A | Reflect A    ⌽⍳4 ↔ 4 3 2 1    ⌽X ↔ DCBA / HGFE / LKJI    ⊖X ↔ ⌽[1]X ↔ IJKL / EFGH / ABCD    1 0 ¯1⌽X ↔ BCDA / EFGH / LIJK<br>Circulate A left B places |
| Transpose <6> | V⍉A<br><br>⍉A | Coordinate I of A becomes coordinate V[I] of result    2 1⍉X ↔ ⍉X ↔ AEI / BFJ / CGK / DHL<br>Transpose last two coordinates    1 6 11 ↔ 1 1⍉E |
| Membership | A∊A | ρW∊Y ↔ ρW    E∊P ↔ 0 1 1 0 / 1 0 1 0 / 0 0 0 0<br>P∊⍳3 ↔ 1 1 0 0 |
| Deal <3> | S?S | W?Y ↔ Random selection of W elements from ⍳Y without replacement    52?52 ↔ card deck shuffle<br>(⍳6)∊2?6 ↔ vector with exactly two 1's |
| Decode, Base value | B⊥A | Polynomial evaluation    10⊥1 7 7 6 ↔ 1776    X⊥C ↔ Value of polynomial with coefficients C<br>16⊥4 0 10 ↔ 1034    (for several values of X if X is a matrix) |
| Encode, Representation | B⊤A | Radix conversion of A to base B    10 10 10 10⊤1776 ↔ 1 7 7 6    0 1⊤3.141592 ↔ 3 0.141592 (Split integer and fraction)<br>16 16 16 16⊤1034 ↔ 0 4 0 10 |
| Matrix divide | ⌹A<br>B⌹A | Generalized inverse of A    A has (ρρA)=2 and ≥/ρA    ⌹ minimizes Euclidean norm of error term<br>B has (ρρB)∊1 2<br>and 1↑ρB ↔ 1↑ρA    105 97 114 ⌹ 4 8 5 / 3 9 2 / 7 10 2 ↔ 2 9 5    V⌹(ρV)∘.*0 1 2 ↔ Quadratic least-squares fit of V |

<1> Restrictions on argument ranks are indicated by: S for scalar, V for vector, M for matrix, A or B for any. Except as the first argument of S⍳A or S[A], a scalar may be used instead of a vector. A one-element array may replace any scalar.

<2> Arrays used in examples:    P ↔ 2 3 5 7    E ↔ 1 2 3 4 / 5 6 7 8 / 9 10 11 12    X ↔ ABCD / EFGH / IJKL

<3> Function result depends on index origin.

<4> Elision of any index selects all along that coordinate.

<5> The function is applied along the last coordinate; the symbols ⌿, ⍀, and ⊖ are equivalent to /, \, and ⌽, respectively, except that the function is applied along the first coordinate. If [S] appears after any of the symbols, the relevant coordinate is determined by the scalar S.

<6> Index origin affects left argument of ⍉ and index list of [A;...;A].

## Composite Functions

| | |
|---|---|
| ⊙/A | Reduction across last coordinate of A. |
| ⊙/[S]A | Reduction across coordinate S of A. |
| A⊙.⊙B | Generalized Inner Product of A and B. |
| A∘.⊙B | Generalized Outer Product of A and B. |

where ⊙ can be any of +-×÷*⍟⌈⌊|!○<≤=≥>≠∨∧⍱⍲
E.G. +/V sums all elements of vector V.
A+.×B is ordinary matrix product of A and B.

# APPENDIX B — APL CODED PROGRAM ROUTINES

## INITIALIZATION ROUTINES

```
      ∇P1[⎕]∇
    ∇ P1
[1]   J←28.597879
[2]   EPSLON←0.9381564
[3]   THETAP←301.97723
[4]   OMEGA←233.6023
[5]   PER←6012.525
[6]   RA←212439.76
[7]   RP←6619.9106
[8]   AS←(RA+RP)÷2
[9]   BS←(RA×RP)*0.5
[10]  RAL←87.2
[11]  DAL←¯66.5
[12]  UVN
[13]  I←23.445215
[14]  RAS←173.125
[15]  DAS←ARC RAS
[16]  DT←0.712
[17]  SR←((16÷60)+7÷3600)
    ∇

      ∇P2[⎕]∇
    ∇ P2
[1]   J←28.825
[2]   EPSLON←0.939560665
[3]   THETAP←302.533
[4]   OMEGA←233.196
[5]   PER←5956.1
[6]   RA←210945.7
[7]   RP←6619.9106
[8]   AS←(RA+RP)×0.5
[9]   BS←(RA×RP)*0.5
[10]  RAL←87.9
[11]  DAL←¯64.8
[12]  UVN
[13]  I←23.44517333
[14]  RAS←176.87
[15]  DAS←ARC RAS
[16]  DT←0.7124
[17]  SR←(16÷60)+5.9÷3600
    ∇
```

```
      ∇P3[⎕]∇
    ∇ P3
[1]   J←28.8181
[2]   EPSLON←0.93745
[3]   THETAP←302.7863
[4]   OMEGA←233.0821
[5]   PER←5961.713
[6]   RA←211012.6
[7]   RP←6811.451
[8]   AS←(RA+RP)×0.5
[9]   BS←(RA×RP)*0.5
[10]  RAL←89.3
[11]  DAL←¯68.6
[12]  UVW
[13]  I←23.45515694
[14]  RAS←178
[15]  DAS←ARC RAS
[16]  DT←0.7125
[17]  SR←(16÷60)+5.28÷3600
    ∇

      ∇P5[⎕]∇
    ∇ P5
[1]   J←29.8448
[2]   EPSLON←0.93106
[3]   THETAP←306.509
[4]   OMEGA←229.419
[5]   PER←5973.362
[6]   RA←210590.3
[7]   RP←7517.35
[8]   AS←(RA+RP)×0.5
[9]   BS←(RA×RP)*0.5
[10]  RAL←85.6
[11]  DAL←¯69.8
[12]  UVW
[13]  I←23.44512222
[14]  RAS←190.8
[15]  DAS←ARC RAS
[16]  DT←0.7126
[17]  SR←(16÷60)+1.6÷3600
    ∇
```

```
      ∇P3[⎕]∇
    ∇ P3
[1]   J←28.8181
[2]   EPSLON←0.93745
[3]   THETAP←302.7863
[4]   OMEGA←233.0821
[5]   PER←5961.713
[6]   RA←211012.6
[7]   RP←6811.451
[8]   AS←(RA+RP)×0.5
[9]   BS←(RA×RP)*0.5
[10]  RAL←89.3
[11]  DAL←¯68.6
[12]  UVW
[13]  I←23.45515694
[14]  RAS←178
[15]  DAS←ARC RAS
[16]  DT←0.7125
[17]  SR←(16÷60)+5.28÷3600
    ∇

      ∇P4[⎕]∇
    ∇ P4
[1]   J←29.9656
[2]   EPSLON←0.9349181
[3]   THETAP←306.5338
[4]   OMEGA←229.3509
[5]   PER←9585.09
[6]   RA←211286.4
[7]   RP←7106.718
[8]   AS←(RA+RP)×0.5
[9]   BS←(RA×RP)*0.5
[10]  RAL←88.2
[11]  DAL←¯70.4
[12]  UVW
[13]  I←23.4551825
[14]  RAS←186.125
[15]  DAS←ARC RAS
[16]  DT←0.7125
[17]  SR←(16÷60)+3÷3600
    ∇
```

```
      ∇P5[⎕]∇
    ∇ P5
[1]   J←29.8448
[2]   EPSLON←0.93106
[3]   THETAP←306.509
[4]   OMEGA←229.419
[5]   PER←5973.362
[6]   RA←210590.3
[7]   RP←7517.35
[8]   AS←(RA+RP)×0.5
[9]   BS←(RA×RP)*0.5
[10]  RAL←85.6
[11]  DAL←¯69.8
[12]  UVW
[13]  I←23.44512222
[14]  RAS←190.8
[15]  DAS←ARC RAS
[16]  DT←0.7126
[17]  SP←(16÷60)+1.6÷3600
    ∇

      ∇PARAM[⎕]∇
    ∇ PARAM
[1]   'ORBIT INCLINATION:                    ';J
[2]   'ECCENTRICITY:                         ';EPSLON
[3]   'RT.ASC. OF ASC. NODE:                 ';OMEGA
[4]   'ARGUMENT OF PERIGEE:                  ';THETAP
[5]   'PERIGEE:                              ';RP
[6]   'APOGEE:                               ';RA
[7]   'DECLINATION OF Z AXIS:                ';DAL
[8]   'RT.ASC. OF Z AXIS:                    ';RAL
[9]   'SEMIMAJOR AXIS:                       ';AS
[10]  'SEMIMINOR AXIS:                       ';BS
[11]  'R.A. OF ANTI-SUN LINE AT PERIGEE TIME:   ';RAS
[12]  'DEC. OF ANTI-SUN LINE AT PERIGEE TIME:   ';DAS
    ∇
```

*PROGRAM DRIVER ROUTINES*

```
      ∇RUN[⎕]∇
    ∇ M RUN N;Y
[1]   Y←N[1]
[2]  START:ORBIT Y
[3]   Y←Y+M
[4]   →(Y<N[2])/START
[5]   Y
    ∇

      ∇TRUN[⎕]∇
    ∇ M TRUN N;Y;LARP;SRD;SIB
[1]   DEP←ASIN((SIN THETAP)×SIN J)
[2]   NRAP←ACOS((COS THETAP)÷COS DEP)
[3]   RAP←((360|OMEGA+NRAP)×(THETAP≤180))+(360|OMEGA-NRAP)×(THETAP>180)
[4]   Y←N[1]
[5]   REF←ACOS((COS RAS)×COS DAS)
[6]   REF←(REF×RAS≤180)+(360-REF)×RAS>180
[7]   ''
[8]   ''
[9]  START:ORBIT Y
[10]  Y←Y+M
[11]  RAZ←360|LAMB
[12]  LAR←((RAZ<RAP)×360)+RAZ-RAP
[13]  LARP←((LAR≤180)×LAR)+(LAR>180)×(360-LAR)
[14]  'TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                    '; 365 24 60 ⊤TIME LARP
[15]  →(NOT=1)/CON
[16]  TRP←TIME LARP
[17]  TRP←(((RAZ-RAP)≤0)×(PER-TRP))+((RAZ-RAP)>0)×(PER+TRP)
[18]  TM←TRP
[19]  TY←(365.24219879-6.14E¯6×DT)×24×60
[20]  DRS←(TM÷TY)×360
[21]  RDR←REF+DRS
[22]  DASB←ASIN((SIN RDR)×SIN I)
[23]  RASB←ACOS((COS RDR)÷COS DASB)
[24]  RASB←((RDR≤180)×RASB)+(180<RDR)×360-RASB
[25]  SRD←RASB,DASB
[26]  'RA AND DEC OF ANTI-SUN LINE:                          ';SRD
[27]  MU←ACOS(COS(LAMB-RASB))
[28]  NDAZ←90-DAZB
[29]  NDAS←90-DASB
[30]  BETA←ACOS(((COS NDAZ)×COS NDAS)+(SIN NDAZ)×(SIN NDAS)×COS MU)
[31]  SIG←ACOS(RE÷R)
[32]  SIB←BETA,SIG
[33]  →(90<SIG+BETA+SP)/OUTT
[34]  'S/C IN SHADOW -- ANGLES FROM ANTI-SUN LINE, R, AND RE:       ';SIB
[35]  →CON
[36] OUTT:→(90>BETA+SP-SIG)/TERM
[37]  'ANGLES FROM ANTI-SUN LINE, R, AND RE:                       ';SIB
[38]  →CON
[39] TERM:'TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  ';SIB
[40]  →CON
[41] CON:''
[42]  ''
[43]  →(Y<N[2])/START
[44]  ''
[45]  ''
    ∇
```

```
      ∇SIN[⎕]∇
    ∇ Y←SIN A
[1]   Y←1○A÷P
    ∇

      ∇COS[⎕]∇
    ∇ Y←COS A
[1]   Y←2○A÷P
    ∇

      ∇TAN[⎕]∇
    ∇ X←TAN Y
[1]   X←3○Y÷180
    ∇

      ∇ASIN[⎕]∇
    ∇ Y←ASIN A
[1]   Y←(¯1○A)×180÷○1
    ∇

      ∇ACOS[⎕]∇
    ∇ Y←ACOS A
[1]   Y←(¯2○A)×180÷○1
    ∇

      ∇ATAN[⎕]∇
    ∇ Y←ATAN X
[1]   Y←(¯3○X)×180÷○1
    ∇

      ∇ARTAN[⎕]∇
    ∇ X←ARTAN Y
[1]   X←¯3○Y
    ∇
```

COMPUTATION ROUTINES

```
      ∇ARC[⎕]∇
    ∇ X←ARC RA;Y
[1]   Y←ACOS((COS RA)×SIN I)
[2]   X←(ACOS((COS I)÷SIN Y))×(×(180-RA))
    ∇
      ∇UVW[⎕]∇
    ∇ UVW
[1]   U←1-EPSLON*2
[2]   V←1+EPSLON
[3]   W←1-EPSLON
    ∇
      ∇ORBIT[⎕]∇
    ∇ ORBIT RAZB;CZ;RDR;RALPH;CX;CNU;SNU
[1]   NOT←0
[2]   ADD←360×(RAZB<OMEGA)
[3]   RAZB←RAZB+ADD
[4]   LAMB←RAZB
[5]   LMOM←LAMB-OMEGA
[6]   CZ←((SIN J)×COS LMOM)
[7]   Z←ACOS CZ
[8]   DAZB←ASIN(((SIN J)×SIN LMOM)÷SIN Z)
[9]   THNOT←ACOS((COS DAZB)×COS LMOM)
[10]  THNOT←(THNOT×(LMOM<180))+(360-THNOT)×(LMOM>180)
[11]  THETA←THNOT-THETAP
[12]  R←(RP×(1+EPSLON))÷(1+EPSLON×COS THETA)
[13]  RDR←RAZB,DAZB,R
[14]  'RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  ';RDR
[15]  RHO←ASIN RE÷R
[16]  PSI←ACOS(COS(|RAL-RAZB))
[17]  ALPHA←180-ACOS(((SIN(90-DAL))×(SIN(90-DAZB))×(COS(PSI)))+(COS(90-DAL))×(COS(90-DAZB)))
[18]  RALPH←RHO,ALPHA
[19]  'HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:  ';RALPH
[20]  →(ALPHA>GAMMA,ALPHA<GAMMA)/ALPGTGAM,ALPLTGAM
[21]  'EARTH CROSSING ANGLE IS:                              ';2×RHO
[22]  ''
[23]  NU←GAMMA
[24]  →COMXI
[25] ALPGTGAM:NU←GAMMA+0.5×SCAN
[26]  →((ALPHA-RHO)≥NU)/OUT
[27]  →COMXI
[28] ALPLTGAM:NU←GAMMA-0.5×SCAN
[29]  →((ALPHA+RHO)≤NU)/OUT
[30] COMXI:CNU←COS NU
[31]  SNU←SIN NU
[32]  CX←((COS RHO)-CNU×COS ALPHA)÷SNU×SIN ALPHA
[33]  X←ACOS CX
[34]  XI←ASIN(SNU×SIN X)
[35]  'EARTH CROSSING ANGLE:                                   ';2×XI
[36]  ''
[37]  →0
[38] OUT:'EARTH NOT IN SCAN RANGE'
[39]  NOT←1
[40]  ''
    ∇
      ∇TIME[⎕]∇
    ∇ T←TIME TH
[1]   TOTAR←○AS×BS
[2]   T←(AREA TH)×PER÷TOTAR
    ∇
      ∇AREA[⎕]∇
    ∇ A←AREA THE;X
[1]   X←1+EPSLON×COS THE
[2]   A←0.5×(AS*2)×(U*2)×(((2÷U*1.5)×(ARTAN(((W÷V)*0.5)×TAN(THE÷2))))-((EPSLON÷U)×(SIN THE)÷X))
    ∇
```

## APPENDIX C — IMP-6: SELECTED RUN RESULTS

```
APPENDIX C -- IMP-6:  SELECTED RUN RESULTS


SELECT INITIAL PARAMETERS:

      P5

PRINT SELECTED PARAMETERS:

      PARAM
ORBIT INCLINATION:              29.8448
ECCENTRICITY:                   0.93106
RT.ASC. OF ASC. NODE:           229.419
ARGUMENT OF PERIGEE:            306.509
PERIGEE:                        7517.35
APOGEE:                         210590.3
DECLINATION OF Z AXIS:          ¯69.8
RT.ASC. OF Z AXIS:              85.6
SEMIMAJOR AXIS:                 109053.825
SEMIMINOR AXIS:                 39787.95033
R.A. OF ANTI-SUN LINE AT PERIGEE TIME:   190.8
DEC. OF ANTI-SUN LINE AT PERIGEE TIME:   ¯4.64578413

ACOMPUTE FOR S/C RIGHT ASCENSIONS 0 THROUGH 350 DEGREES
      A IN INCREMENTS OF 10 DEGREES:

      10 TRUN 0 360

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  0  23.54522847  210562.0841
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   1.735819845  69.47493863
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                        2  1  29.17891525


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  10  20.01769243  174535.3662
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   2.094265077  76.0797296
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                        1  0  35.13592133


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  20  15.73884165  113689.3857
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   3.21607777  83.26526337
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                        0  11  10.65838321


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  30  10.79971136  70790.66005
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   5.169300334  90.90390251
EARTH CROSSING ANGLE:                                    10.33421005

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                        0  5  31.81984186
RA AND DEC OF ANTI-SUN LINE:                               194.3745392  ¯6.145003105
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE: 163.8646718  84.83069967


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  40  5.364062118  46032.79237
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   7.964353889  98.78925892
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                        0  3  5.513999278
```

```
RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  50  ¯0.3333359251  31914.59401
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   11.52824875  106.6316334
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  1  55.32072163


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  60  ¯6.014154107  23564.20714
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   15.70424333  114.0803272
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  1  17.78837369


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  70  ¯11.40295327  18399.4049
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   20.28258797  120.7678623
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  55.68119562


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  80  ¯16.27230278  15065.19639
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   25.04758701  126.3596694
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  41.61265283

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  90  ¯20.4672577  12827.37668
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   29.81695059  130.5952763
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  32.02912672


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  100  ¯23.90438852  11272.51772
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   34.45893871  133.3182512
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  25.11509423


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  110  ¯26.55407095  10159.46509
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   38.88837906  134.4938085
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  19.87162173


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  120  ¯28.41805652  9343.344212
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   43.05049366  134.2034511
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  15.71535082


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  130  ¯29.5102508  8735.216665
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   46.9003774  132.6082967
EARTH CROSSING ANGLE:                                    47.18668561

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  12.28699651
RA AND DEC OF ANTI-SUN LINE:                                194.5775538  ¯6.229259842
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  64.85954108  43.0996226


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  140  ¯29.84352844  8279.999004
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   50.38172354  129.8944486
EARTH CROSSING ANGLE:                                    69.77085659
```

```
TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                   0  0  9.353978813
RA AND DEC OF ANTI-SUN LINE:                                          194.5794176  ¯6.230032883
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:            56.37535809  39.61827646

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:            150  ¯29.42249642  7944.144221
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:             53.40565186  126.2269883
EARTH CROSSING ANGLE:                                              86.3047586

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                   0  0  6.758035295
RA AND DEC OF ANTI-SUN LINE:                                          194.5810672  ¯6.230717078
S/C IN SHADOW -- ANGLES FROM ANTI-SUN LINE, R, AND RE:                47.92961343  36.59434814

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:            160  ¯28.24138698  7708.7764
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:             55.83156054  121.727966
EARTH CROSSING ANGLE:                                              98.56258356

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                   0  0  4.385471208
RA AND DEC OF ANTI-SUN LINE:                                          194.5825749  ¯6.231342394
S/C IN SHADOW -- ANGLES FROM ANTI-SUN LINE, R, AND RE:                39.4319214  34.16843946

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:            170  ¯26.28672807  7566.026542
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:             57.45834651  116.4767786
EARTH CROSSING ANGLE:                                              107.0364784

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                   0  0  2.149263481
RA AND DEC OF ANTI-SUN LINE:                                          194.5839959  ¯6.231931768
S/C IN SHADOW -- ANGLES FROM ANTI-SUN LINE, R, AND RE:                30.84341252  32.54165349

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:            180  ¯23.54522847  7517.355617
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:             58.04441161  110.5250614
EARTH CROSSING ANGLE:                                              111.8345017

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                   0  0  0.02250769569
RA AND DEC OF ANTI-SUN LINE:                                          194.5853759  ¯6.232504155
S/C IN SHADOW -- ANGLES FROM ANTI-SUN LINE, R, AND RE:                22.28245889  31.95558839

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:            190  ¯20.01769243  7573.164531
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:             57.37381169  103.9202704
EARTH CROSSING ANGLE:                                              112.9391145

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                   0  0  2.19494566
RA AND DEC OF ANTI-SUN LINE:                                          194.5867564  ¯6.233076715
S/C IN SHADOW -- ANGLES FROM ANTI-SUN LINE, R, AND RE:                14.48642163  32.62618831

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:            200  ¯15.73884165  7753.212239
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:             55.35075994  96.73473663
EARTH CROSSING ANGLE:                                              110.3452748

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                   0  0  4.433223217
RA AND DEC OF ANTI-SUN LINE:                                          194.5881788  ¯6.233666623
S/C IN SHADOW -- ANGLES FROM ANTI-SUN LINE, R, AND RE:                10.8857525  34.64924006

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:            210  ¯10.79971136  8087.438984
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:             52.05960223  89.09609749
EARTH CROSSING ANGLE:                                              104.117846

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                   0  0  6.809482583
RA AND DEC OF ANTI-SUN LINE:                                          194.5896888  ¯6.234292892
S/C IN SHADOW -- ANGLES FROM ANTI-SUN LINE, R, AND RE:                15.9043864  37.94039777
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RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  220  ¯5.364062118  8616.897451
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   47.7478373  81.21074108
EARTH CROSSING ANGLE:                                    94.54896853

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  9.411168757
RA AND DEC OF ANTI-SUN LINE:                                194.5913421  ¯6.234978569
S/C IN SHADOW -- ANGLES FROM ANTI-SUN LINE, R, AND RE:      25.29119606  42.2521627


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  230  0.3333359251  9394.87012
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   42.75764248  73.36836664
EARTH CROSSING ANGLE:                                    80.63119201

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  12.35269294
RA AND DEC OF ANTI-SUN LINE:                                194.5932113  ¯6.235753804
S/C IN SHADOW -- ANGLES FROM ANTI-SUN LINE, R, AND RE:      35.9439982  47.24235752


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  240  6.014154107  10489.05746
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   37.45065866  65.91967278
EARTH CROSSING ANGLE:                                    60.5081148

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  15.79350703
RA AND DEC OF ANTI-SUN LINE:                                194.5953978  ¯6.236660618
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  46.94367487  52.54934134


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  250  11.40295327  11986.79861
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   32.14739397  59.23213773
EARTH CROSSING ANGLE:                                    24.26103098

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  19.96818935
RA AND DEC OF ANTI-SUN LINE:                                194.5980507  ¯6.237760828
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  57.87078839  57.85260603


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  260  16.27230278  14006.28184
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   27.08931497  53.64033065
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  25.23948341


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  270  20.4672577  16717.8143
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   22.42780927  49.40472368
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  32.19698918


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  280  23.90438852  20381.805
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   18.23615171  46.68174876
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  41.85150603


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  290  26.55407095  25416.63233
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   14.53341198  45.50619154
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  0  56.04256214


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  300  28.41805652  32523.8843
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   11.30940521  45.79654889
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  1  18.35600985
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RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  310  29.5102508  42926.62579
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   8.544815779  47.3917033
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  1  56.36190792

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  320  29.84352844  58817.53312
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   6.225388085  50.10555138
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  3  7.555267506

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  330  29.42249642  84063.20077
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   4.35141027  53.77301167
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  5  36.29487557

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  340  28.24138698  124186.2532
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   2.943987588  58.27203398
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  11  21.38627963

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  350  26.28672807  178414.5863
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   2.048710405  63.52322143
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         1  0  59.45282956

 SELECT NEW INITIAL PARAMETERS
      R2

 PRINT NEW PARAMETERS
      PARAM
ORBIT INCLINATION:          28.825
ECCENTRICITY:               0.939560665
RT.ASC. OF ASC. NODE:       233.196
ARGUMENT OF PERIGEE:        302.533
PERIGEE:                    6619.9106
APOGEE:                     210945.7
DECLINATION OF Z AXIS:      ¯64.8
RT.ASC. OF Z AXIS:          87.9
SEMIMAJOR AXIS:             108782.8053
SEMIMINOR AXIS:             37368.9935
R.A. OF ANTI-SUN LINE AT PERIGEE TIME:  176.87
DEC. OF ANTI-SUN LINE AT PERIGEE TIME:  1.356372094

∩COMPUTE FOR RA 20 THROUGH 40 DEGREES BY 1 DEGREE:

      1 TRUN 20 40

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  20  16.76778967  105959.973
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   3.450953903  83.81960339
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  9  22.71690996

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  21  16.34227441  100778.7751
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   3.628604704  84.58921916
EARTH NOT IN SCAN RANGE
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TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                              0  8  39.86207636

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  22  15.91016496  95849.9314
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   3.81546617  85.36306335
EARTH NOT IN SCAN RANGE

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                              0  8  0.9080598329

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  23  15.47156886  91171.10519
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   4.01158548  86.14099344
EARTH CROSSING ANGLE:                                    5.074560081

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                              0  7  25.48327796
RA AND DEC OF ANTI-SUN LINE:                                     180.3311163  ¯0.143584952
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  152.87018  85.98841452

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  24  15.02660031  86737.42785
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   4.217004001  86.92285925
EARTH CROSSING ANGLE:                                    7.03730979

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                              0  6  53.24765124
RA AND DEC OF ANTI-SUN LINE:                                     180.3513596  ¯0.1523630937
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  152.2990574  85.782996

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  25  14.57538033  82542.11343
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   4.431756966  87.70850276
EARTH CROSSING ANGLE:                                    8.312522715

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                              0  6  23.89147269
RA AND DEC OF ANTI-SUN LINE:                                     180.3697947  ¯0.1603571112
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  151.6982505  85.56824303

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  26  14.11803678  78576.97534
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   4.655873141  88.49775797
EARTH CROSSING ANGLE:                                    9.190457175

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                              0  5  57.13381319
RA AND DEC OF ANTI-SUN LINE:                                     180.386598  ¯0.1676435083
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  151.069227  85.34412686

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  27  13.65470451  74832.85369
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   4.889374522  89.29045065
EARTH CROSSING ANGLE:                                    9.778368451

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                              0  5  32.72064878
RA AND DEC OF ANTI-SUN LINE:                                     180.401929  ¯0.1742914617
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  150.4134259  85.11062548

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  28  13.18552537  71299.96334
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   5.132276031  90.08639827
EARTH CROSSING ANGLE:                                    10.17784079

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                              0  5  10.42284905
RA AND DEC OF ANTI-SUN LINE:                                     180.4159316  ¯0.1803633689
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  149.7322482  84.86772397

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  29  12.71064827  67968.17304
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   5.384585236  90.88540985
EARTH CROSSING ANGLE:                                    10.76593934
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TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                  0  4  50.03412732
RA AND DEC OF ANTI-SUN LINE:                                         180.4287353  ¯0.1859154062
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:           149.0270501  84.61541476

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:           30  12.23022917  64827.22585
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:            5.646302078  91.68728584
EARTH CROSSING ANGLE:                                             11.1376163

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                  0  4  31.36902349
RA AND DEC OF ANTI-SUN LINE:                                         180.4404567  ¯0.1909980783
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:           148.2991379  84.35369792

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:           31  11.74443111  61866.90995
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:            5.917418628  92.49181806
EARTH CROSSING ANGLE:                                             11.31450627

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                  0  4  14.26096682
RA AND DEC OF ANTI-SUN LINE:                                         180.4512002  ¯0.1956567457
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:           147.5497652  84.08258137

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:           32  11.25342417  59077.18862
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:            6.197918851  93.29878963
EARTH CROSSING ANGLE:                                             11.30620679

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                  0  3  58.56044869
RA AND DEC OF ANTI-SUN LINE:                                         180.4610599  ¯0.1999321226
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:           146.7801314  83.80208115

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:           33  10.75738541  56448.29638
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:            6.487778409  94.10797491
EARTH CROSSING ANGLE:                                             11.11293778

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                  0  3  44.13332221
RA AND DEC OF ANTI-SUN LINE:                                         180.4701199  ¯0.2038607394
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:           145.9913816  83.51222159

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:           34  10.25649881  53970.80802
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:            6.786964472  94.91913953
EARTH CROSSING ANGLE:                                             10.72555301

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                  0  3  30.85923635
RA AND DEC OF ANTI-SUN LINE:                                         180.4784558  ¯0.20747537
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:           145.1846069  83.21303553

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:           35  9.750955129  51635.68568
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:            7.095435567  95.73204036
EARTH CROSSING ANGLE:                                             10.12290117

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                  0  3  18.63020563
RA AND DEC OF ANTI-SUN LINE:                                         180.4861354  ¯0.2108054205
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:           144.3608461  82.90456443

RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:           36  9.24095185  49434.30864
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:            7.41314145  96.54642557
EARTH CROSSING ANGLE:                                             9.264519515

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                                  0  3  7.34931201
RA AND DEC OF ANTI-SUN LINE:                                         180.4932196  ¯0.2138772834
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:           143.5210876  82.58685855
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RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  37  8.726692967  47358.48956
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   7.740023003  97.36203466
EARTH CROSSING ANGLE:                                    8.071170407

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  2  56.92953265
RA AND DEC OF ANTI-SUN LINE:                                180.4997631  ¯0.2167146561
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  142.6662713  82.259977


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  38  8.208388835  45400.48023
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   8.076012174  98.17859861
EARTH CROSSING ANGLE:                                    6.35980999

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  2  47.29268563
RA AND DEC OF ANTI-SUN LINE:                                180.5058149  ¯0.2193388289
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  141.7972914  81.92398783


RIGHT ASCENSION, DECLINATION, AND RADIUS VECTOR OF S/C:  39  7.686255962  43552.96934
HALF EARTH ANGLE AND S/C Z AXIS TO EARTH CENTER ANGLE:   8.421031934  98.9958399
EARTH CROSSING ANGLE:                                    3.432678659

TIME(DYS,HRS,MINS) TO/FROM PERIGEE:                         0  2  38.3684847
RA AND DEC OF ANTI-SUN LINE:                                180.5114191  ¯0.2217689415
TERMINATOR SENSED -- ANGLES FROM ANTI-SUN LINE, R, AND RE:  140.9149989  81.57896807
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