X-552-71-143
PREPRINT

NASA TM X- 65762

PRELIMINARY GODDARD GEOPOTENTIAL USING OPTICAL TRACKING DATA AND A COMPARISON WITH SAO MODELS

F. J. LERCH
C. A. WAGNER
B. H. PUTNEY
K. G. NICKERSON

OCTOBER 1971

GSFC — GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

N72-12887 (NASA-TM-X-65762) PRELIMINARY GODDARD GEOPOTENTIAL USING OPTICAL TRACKING DATA AND A COMPARISON WITH SAO MODELS F.J. Lerch, et al (NASA) Oct. 1971 25 p CSCL 22B

Unclas
09751

G3/31

X-552-71-143

PRELIMINARY GODDARD GEOPOTENTIAL USING OPTICAL TRACKING DATA AND A COMPARISON WITH SAO MODELS

F. J. Lerch
C. A. Wagner
B. H. Putney
Geodynamics Branch
Trajectory Analysis and Geodynamics Division

and

K. G. Nickerson
Computer Science Corporation
Silver Spring, Maryland

October 1971

Goddard Space Flight Center
Greenbelt, Maryland

PRECEDING PAGE BLANK NOT FILMED

PRELIMINARY GODDARD GEOPOTENTIAL USING OPTICAL TRACKING DATA AND A COMPARISON WITH SAO MODELS*

ABSTRACT

A preliminary Goddard Space Flight Center (GSFC) geopotential and center of mass station coordinate solution has been obtained from satellite orbital data using numerical integration theory. This geodetic solution is a prelude to a more general solution which will combine the 1971 International Satellite Geodesy Experiment (ISAGEX) laser data with the present data being employed. The present GSFC geopotential solution consists of the spherical harmonic coefficients through degree and order eight with higher order satellite resonant coefficients. The solution represents a first iteration result from 17 satellites with approximately 150 weekly orbital arcs containing some 40,000 optical observations. The GSFC preliminary result is compared with final results from the Smithsonian Astrophysical Observatory (SAO) solutions including the 1969 SAO Standard Earth II solution. One aspect of interest for the comparison is that SAO uses an analytic theory for the orbital solution whereas GSFC uses a numerical integration theory. The comparison of geopotential results shows that good agreement exists in general but that there are some areas of minor differences. Remarkable agreement exists with the zonal coefficients. This is particularly noteworthy since the zonal coefficients in the GSFC solution have been obtained from their short term effects simultaneously with the rest of the geopotential. On the other hand the zonals in the SAO solution were obtained separately from their long term effects.

The data presently being employed in the GSFC solution is largely similar to the satellite data employed in the 1969 SAO solution. This data is primarily Baker Nunn optical, although some Minitrack Optical Tracking System and Minitrack Interferometer data were employed in the GSFC solution. Twelve of the principal Baker-Nunn sites were used for the center of mass station coordinates. In the comparison between the GSFC and the 1969 SAO solution, the root mean square of differences between corresponding values of geodetic parameters gave approximately 10 meters for station coordinates and 9×10^{-8} for the 8×8 set of normalized geopotential coefficients. The first iteration result of the present GSFC solution is being extended to provide for additional iterations on the data and to include additional satellites.

*Paper presented at the DOD Geodesy/Gravity Technical Exchange Meetings, National Bureau of Standards, Gaithersburg, Maryland, April 21, 1971.

PRELIMINARY GODDARD GEOPOTENTIAL USING OPTICAL TRACKING DATA AND A COMPARISON WITH SAO MODELS

DISCUSSION

A preliminary satellite solution of the geopotential field and geocentric station coordinates is presented and compared with recent SAO solutions. This solution is part of a more general development at GSFC that will combine surface gravity data, geometrical data, and electronic, laser, and optical satellite data into a unified geodetic solution.

The discussion is organized into the following sections:

1. Description of Solution and Data Employed.

2. General Summary of Comparison Results.

3. Direct Comparison of GSFC and SAO Solutions.

4. Satellite Model for Geodetic Solution Using Numerical Integration.

5. Characteristics of Present Solution and Modified Approach for Extended Geodetic Solution.

1. Description of Solution and Data Employed

The description of the geodetic satellite solution and data employed is presented in Figure 1. The geopotential consists of a field of spherical harmonic coefficients complete to degree and order eight plus higher order satellite resonant coefficients. The data employed are similar to the satellite data employed in the 1969 SAO solution. These are primarily Baker-Nunn optical observations. Also included are Minitrack Optical Tracking System and Minitrack Interferometer data. Twelve of the principal Baker-Nunn sites were used for the determination of the center of mass station coordinates.

A list of satellites and orbital arcs employed in the solution are presented in Figure 2. The preliminary values for the geopotential coefficients and center of mass station coordinates are listed in Table XI including a distribution of observations by station. A more complete account of the satellite arc solutions including an RMS of observational residuals per satellite is presented in Table X.

Geodetic Solution Parameters

- Geopotential – 8 × 8 field of spherical harmonic coefficients plus resonant coefficients
- Stations – 12 Principal Baker-Nunn Optical Tracking Sites

Satellite Data

- 17 close earth satellites, 154 arcs
- satellite orbital arc lengths 7 to 14 days
- 47,000 Baker-Nunn observations
- 6,500 MOTS observations
- 1,300 Minitrack Direction Cosines

Figure 1. Description of Solution and Data.

	Satellite	Inclination	# of Arcs		Satellite	Inclination	# of Arcs
1.	ANNA 1B ('62)	50°	22	10.	DI-C ('67)	40°	4
2.	GRS ('63)	50	4	11.	BE-B ('64)	80	4
3.	TRANSIT 4A ('61)	67	14	12.	BE-C ('65)	41	12
4.	TELSTAR 1 ('62)	44	5	13.	GEOS I ('65)	59	38
5.	SECOR 5 ('65)	69	1	14.	OSCAR 7 ('66)	89	4
6.	GEOS-II ('68)	106	15	15.	ECHO R.B. ('60)	47	2
7.	COURIER 1B ('60)	28	13	16.	TIROS 9 ('65)	96	3
8.	OVI-2 ('65)	144	4	17.	ALOUETTE 2 ('65)	80	3
9.	OGO-2 ('65)	87	7				

Figure 2. Satellites.

2. General Summary of Comparison Results

The first iteration for the 8 × 8 geopotential and station coordinate solution, in general, gave reasonable results when compared with recent SAO solutions (SAO '69, SAO '66, SAO B13.1). Zonal coefficients up to degree 8 and the set of coefficients for degree n = 2 to 8 and order m less than 4 compared very well with final results of the SAO solutions. The coefficients for $m \geq 4$ when grouped as a set appeared relatively poorer in comparison with similar results from the SAO solutions. This was considered significant and as a result a second iteration solution has been set in process. Station coordinate solutions when compared with the 1969 SAO Standard Earth II values gave an RMS value for coordinate (Cartesian)

(1) Geopotential – 8 × 8 Set of Normalized Coefficients

Type	RMS of Coefficient Differences: RMS (10^6)	No. of Coefficients
Zonals	.03	7
Total Set	.09	75
Subsets by Order (m)		
$m \leq 3$	.06	45
$m \geq 4$	.14	30

(2) Geocentric Station Coordinates (12 Stations)

RMS ~ 10 meters for station coordinate differences.

Figure 3. Summary of Comparison Results Between GSFC and SAO ('69) Standard Earth II Solutions.

differences of 10 meters. The total RMS (root mean square) of the coefficient differences between our 8 × 8 field and corresponding values of the SAO '69 field is 9×10^{-8} (for normalized coefficients). These results are displayed in Figure 3.

3. Direct Comparison of GSFC and SAO Solutions

The SAO solutions used in the comparison with the GSFC preliminary result are identified below, together with references where more complete information may be found.

- SAO M1 — Smithsonian 1966 Standard Earth I 8 × 8 field of spherical harmonic coefficients plus resonance and some other tesseral coefficients. Zonals complete to degree 14. Satellite data solution. (Reference 1.)
- SAO '69 – Smithsonian 1969 Standard Earth II 16 × 16 field of coefficients plus resonance. Zonals complete to degree 22. (Reference 2.) Combined satellite and surface gravity data solution.
- SAO B13.1 – Earlier 1969 solution similar to SAO '69 above.
- GSFC Solution 8 × 8 field plus higher degree resonance. Satellite data solution. The SAO M1 (1966) solution was used as a starting solution.

Some general characteristics of these solutions are presented in Figure 4. One distinguishing aspect of the solution techniques is that SAO employs an analytic theory and GSFC employs a numerical integration theory. A Cowell type of technique is employed for the numerical integration with an automatic selection feature of variable order and variable stepsize to control a desired orbital ephemeris accuracy of modeled forces.

Solution	Tesseral Field		Satellites (Tesserals)		Maximum Degree of Zonals (Satellite Contributions)	
	From Satellite Data	From Surface Gravity Data	Number of Satellites	Length of Arcs (days)	Short Term Effects	Long Term Effects
SAO ('69)(1)	12 × 12	16 × 16	21	30, 14	–	21
SAO (M1)(1)	8 × 8	–	13	14	–	14
GSFC(2)	8 × 8	–	15	7 to 14	8	–
B13.1	Similar to SAO ('69)					

(1) Higher degree satellite resonant coefficients among orders 9-14 are also included. Zonals are obtained separately in the solution.

(2) Higher order satellite resonant coefficients are included. Zonals are obtained simultaneously in the solution from short term satellite zonal effects.

Figure 4. General Characteristics of Solutions.

Tables I through IX compare the GSFC and SAO solution parameters for geopotential coefficients out to degree and order eight and for the geocentric station coordinates. GSFC solution values are listed in Table XI. The original purpose of comparing corresponding numerical values between the solutions was to assess the validity of the preliminary values of the GSFC solution. In the process, there turned out to be some very good agreement and interesting results. A list of the comparison tables is given in Figure 5 for reference.

3.1 Geopotential Coefficient Comparison

$\bar{C}_{n,m}$ and $\bar{S}_{n,m}$ represent the normalized geopotential coefficients for degree n and order m. The coefficients $\bar{C}_{1,0}$, $\bar{C}_{1,1}$, $\bar{S}_{1,1}$ are zero because the origin of the coordinate system is at the center of mass of the Earth. $\bar{C}_{2,1}$ and $\bar{S}_{2,1}$ are zero because the axis of maximum inertia is assumed coincident with the rotation axis. A special solution is presented in Table VI in which these axes are not constrained and $C_{2,1}$, $S_{2,1}$ are not zero.

Table	Title
I	Comparison of Normalized Zonal Coefficients
II	RMS Coefficient Variation
III	RMS of Coefficient Differences by Degree
IV	RMS of Coefficient Differences by Order
V	RMS of Coefficient Differences by Sets for Orders $m \leq 3$ and $m \geq 4$.
VI	Polar Tilt Coefficients
VII	Comparison of Low Degree and Order Coefficients for Synchronous and Close Earth Satellite Solutions
VIII	RMS of Station Coordinate Differences
IX	Adjustment for Scale, Translation, and Differential Rotation Between GSFC and SAO ('69) Station Coordinate Solutions

Figure 5. Tables for Direct Comparison of GSFC and SAO Solutions.

Tables I through VII are presented for the geopotential comparison. Notes have been added to the tables for discussion and explanation of all quantities presented. Comparison of selected sets of geopotential coefficients are summarized statistically in the tables. All geopotential coefficients and related quantities are scaled by 10^6.

3.2 Station Coordinate Comparison

A comparison of station coordinate differences between the GSFC and SAO solutions are presented in Tables VIII and IX. GSFC solution values for the 12 fundamental Baker-Nunn tracking station coordinates are presented in Table XI. Geocentric coordinates (X, Y, Z) are employed in the GSFC solution. The Z coordinate is along the polar axis ($\hat{z}$), the X coordinate is the intersection ($\hat{x}$) of the Earth's true equator and the Greenwich meridian, and Y is along $\hat{y} \equiv \hat{z} \times \hat{x}$. SAO employs (References 1 and 2) a (terrestial) Earth fixed reference system with use of the BIH values of UT1-A1 (UT1 - UTC) and IPMS polar motion values for a mean pole of 1900-1905.[1] GSFC employed USNO values of UT1-A1 (UT1-UTC) for the rotation of the Earth.

---

[1] BIH – Bureau International de l'Heure
IPMS – International Polar Motion Service
UTC – Universal Time Transmitted (National Bureau of Standards)
UTI – Universal Time, for rotation of the Earth about its axis
AI – Atomic Time, Naval Observatory (uniform time)
A.S. – SAO Atomic time system, coordinated with own master clock.

TABLE I

Comparison of Normalized Zonal Coefficients $\bar{C}_{n,0}$

(Scaled by 10^6)

Degree	GSFC	SAO '69	SAO M1	$\Delta 1^{(1)}$	$\Delta 2^{(1)}$	Standard$^{(2)}$ Deviation
2	-484.177	-484.166	-484.173	-.01	-.01	.0002
3	.956	.959	.962	+.00	+.00	.0002
4	.556	.531	.550	.02	.02	.0003
5	.066	.069	.063	.00	-.01	.0002
6	-.176	-.139	-.179	-.04	-.04	.0003
7	.091	.093	.086	.00	-.01	.0002
8	.089	.029	.065	.06	.04	.0004
				RMS$^{(3)}$ .028	.022	.0003

(1) $\Delta 1$ = GSFC − SAO '69 value.

$\Delta 2$ = SAO M1 − SAO '69 value.

$\Delta 1 - \Delta 2$ = GSFC − SAO M1 value is very small for the zonals.

The zonal coefficients of degree 9 to 14 in the M1 model were held fixed in the GSFC 8 × 8 geopotential solution.

(2) Standard deviations are obtained from the variance covariance matrix of the solution. See Note (2) Table III.

(3) RMS is the root mean square of the coefficient differences and of the standard deviations.

TABLE II

$\sigma_n^{(1)}$ RMS Coefficient Variation

(Scaled by 10^6)

n	GSFC	SAO '69	SAO M1	Pellinen[2]	Kaula[2]
3	1.12	1.11	1.10	1.14	1.11
4	.53	.53	.50	.57	.63
5	.35	.33	.33	.34	.40
6	.25	.22	.25	.23	.28
7	.22	.17	.16	.16	.20
8	.15	.09	.12	.12	.16

(1) σ_n represents the average size coefficient in each degree and is the RMS value of the 2n + 1 coefficients in each degree.

$$\bar{\sigma}_n \equiv \left[\sum_{m=0}^{n} (\bar{C}^2_{n,m} + \bar{S}^2_{n,m})/(2n+1)\right]^{1/2}$$

(2) Pellinen's values of σ_n show more consistent agreement with the SAO values, whereas Kaula's values are slightly larger for the higher degrees but which agree better with the GSFC values.

A recent formula by Pellinen (1969 Reference 3) and an earlier "rule of thumb" formula by Kaula (Reference 4), based upon analysis of gravimetry data, were used to obtain these values for σ_n. Kaula obtained the simple formula $\sigma_n = 10/n^2\ (10^{-6})$. Using the basic formula by Pellinen associated with the degree variances of gravity anomaly, values for σ_n may be derived by the formula below and are referred to as Pellinen's values:

$$\sigma_n = (11.2)/(n-1)\ [(2n+1)\ n^{1.13}]^{1/2}\ (10^{-6}).$$

TABLE III

RMS of Coefficient Differences[1] by Degree

Degree n	GSFC	SAO M1	SAO B13.1	Standard[2] Deviation	σ_n [3] Pellinen
2	.01	.02	.03	.005	
3	.03	.11	.08	.005	1.14
4	.08	.06	.03	.003	.57
5	.14[4]	.06	.07	.005	.34
6	.08	.07	.04	.004	.23
7	.10	.09	.05	.006	.16
8	.10	.07	.06	.005	.12
TOTAL	.09	.08	.05	.005	

(1)Coefficient differences are obtained by subtracting coefficient values for each of the solutions listed from corresponding values of the SAO '69 solution. The RMS is recorded for each of the 2n + 1 differences in each degree n for each of the solutions listed in the table. The total RMS is taken over all differences. (In degree 2 only 3 values are used.)

$$\mathrm{RMS} \equiv \left[\sum_{m=0}^{n} (\Delta\overline{C}^2_{n,m} + \Delta\overline{S}^2_{n,m})/2n + 1 \right]^{1/2}$$

$\Delta\overline{C}$ and $\Delta\overline{S}$ represent the numerical differences between corresponding values in the solutions.

(2)Standard deviations for each of the coefficients in the GSFC solution varied approximately for each degree n from .002 to .007, increasing with the order m from m = 1 to n. The RMS of the standard deviations is recorded for each degree n over the 2n values. The standard deviation of the zonal coefficients is given in Table I which can be seen to be an order of magnitude smaller than the nonzonal values.

(3)Pellinen's value of σ_n is listed so that one can compare the average size (RMS) difference for each degree with the average size coefficient in each degree.

(4)This value for degree 5 is noteworthy large and it exceeds the average size coefficient for degree 8. Two coefficient differences, one of order 4 and the other of order 5 in the set of degree 5 coefficients, were quite larger than the others. Values of .23 and .33 appeared for these differences.

TABLE IV

RMS of Coefficient Differences[1] by Order

Order m	Number of Coefficients	GSFC	SAO M1	SAO B13.1
0	7	.03	.02	same zonals as SAO '69
1	12	.05	.05	.05
2	14	.04	.07	.03
3	12	.08	.12[2]	.07
4	10	.13[3]	.06	.03
5	8	.16[3]	.06	.06
6	6	.09[3]	.07	.05
7	4	.12[3]	.10	.04
8	2	.18[3]	.12	.13
TOTAL	75	.09	.075	.05

(1)RMS of coefficient differences are defined as in Table III except that the set of coefficients are defined by order in place of degree. For each m the set of coefficients run in degree from n = m to 8.

(2)This value for order 3 in the M1 solution is slightly high. See also note (2) below Table VII in the comparison with Carl Wagner's values.

(3)These values are all larger than the corresponding values in the other solutions particularly for orders 4 and 5. See Table V and Figure 6 below. Some of the individual differences in each of the sets were quite small but too many larger ones existed.

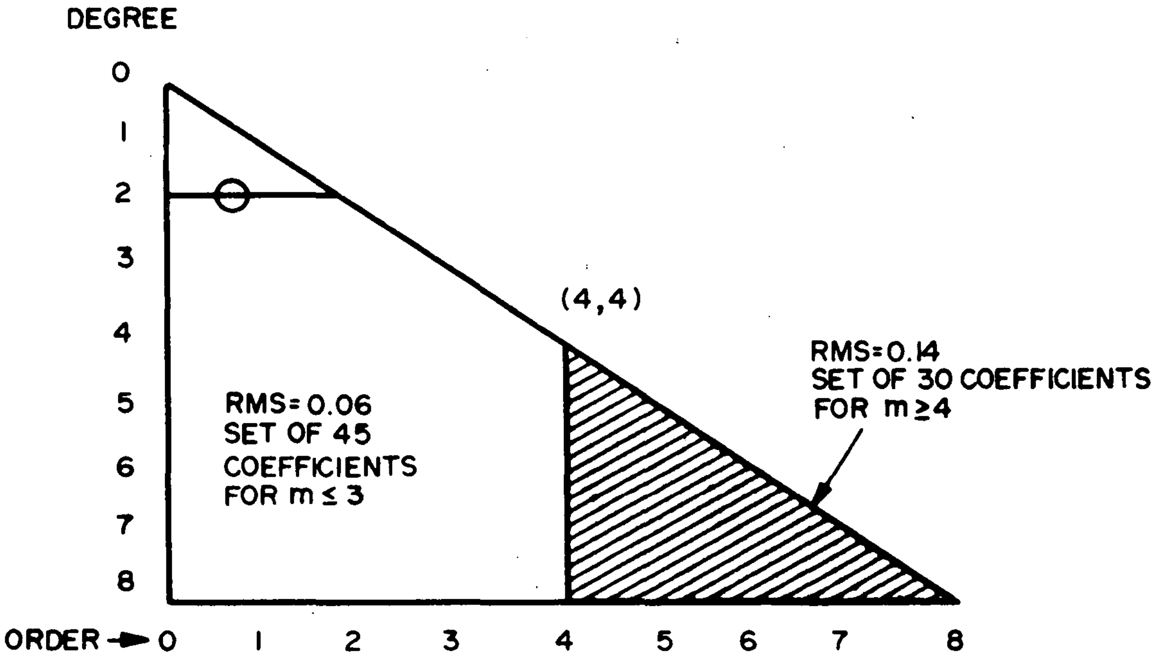

Figure 6. Pictorial View of the Division of Coefficients by Order m for Sets $m \leq 3$ and $m \geq 4$ and Associated RMS Differences.

TABLE V

RMS of Coefficient Differences in Sets $m \leq 3$ and $m \geq 4$

m	GSFC	SAO M1	SAO B13.1
≤ 3	.06	.08	.05 (38 coefficients)
≥ 4	.14(1)	.08	.06

(1)Satellite perturbations corresponding to the set of coefficients for $m \geq 4$ have relatively shorter wavelengths and smaller amplitudes and thus require relatively greater accuracy in the orbital residuals for good recovery.

(2)The fact that similar differences for $m \geq 4$ do not appear in any of the other two SAO solutions, it is not expected that a significantly different picture would result if either the M1 solution or the B13.1 solution was used as a reference solution in forming the basic differences for the RMS values.

TABLE VI

Polar Tilt Coefficients

Values	Standard Deviation
$C_{21} = .012$	.001
$S_{21} = -.008$	.001

These values, theoretically, are expected to be of the order of .001 (10^{-6}). The values are well within the total RMS coefficient difference of .09 from Table III. Again more accuracy is needed for the observational residuals to ascertain values that correspond to the recovery capability exhibited by the standard deviations in the above table.

TABLE VII

Comparison of Low Degree and Order Coefficients for Synchronous and Close Earth Satellite Solutions

	Synchronous [1]	GSFC	SAO '69	SAO M1	SAO B13.1
C_{22}	2.44	2.40	2.41	2.38	2.41
S_{22}	-1.41	-1.36	-1.36	-1.35	-1.41
C_{33}	.70	.70	.69	.56 [2]	.79 [2]
S_{33}	1.42	1.49	1.43	1.62 [2]	1.29 [2]

(1) These values, obtained by Carl Wagner (Reference 5), support the good agreement found for the low order coefficients in the GSFC and the SAO '69 solutions.

(2) These values appear to be in less agreement with the others. The disagreement of order 3 coefficients noted in Table IV for the M1 field shows up again in comparison with values presented here from satellite deep resonant analysis (24 hour synchronous type satellites). In the case of the B13.1 values, it turns out that these two particular coefficients have about the largest disagreement of the entire solution with the SAO '69 solution. The general good agreement between these latter two solutions may be observed in Tables III, IV, and V.

Table VIII lists several cases for comparison of station coordinates for reference. Only case 1 will be discussed.

In the preliminary GSFC solution, Earth model parameters of $a_e = 6{,}378{,}144$ and $GM = 3.986009 \times 10^{14}$ m^3 sec^{-2} were adopted from a previous SAO C-7 Earth model. These values correspond to a slightly different scale than that employed in recent SAO solutions. Also no provision was made in the present GSFC solution for a polar motion adjustment to a terrestial mean pole. In view of this and the fact that SAO uses a basic reference frame associated with the mean equator and equinox of 1950 and GSFC uses a reference frame of true equator and equinox of date for an epoch associated with each satellite arc of data, it seemed appropriate to examine the effects of a transformation adjustment on station coordinates for additional comparison. The adjustment consisted of a scale, translation, and differential rotation for the station coordinate comparison, and this resulted in only a mild reduction in the above RMS value of station coordinate differences. The parameter adjustments compared very favorably with modeling differences associated with longitudinal and polar orientation, scale and center of mass shift and results are presented in Table IX.

TABLE VIII

RMS of Geocentric Station Coordinate Differences[1]

(for twelve Baker-Nunn Stations)

Case	ΔX	ΔY	ΔZ	TOTAL
1. GSFC vs. SAO '69	7.7	9.0	13.2	17.7 (meters)
2. GSFC 1 vs. SAO '69	10.3	11.6	15.6	22.0
3. GSFC 1 vs. GSFC	7.2	5.8	10.1	13.7
4. GSFC vs. SAO C-7	12.5	11.7	15.2	22.9
5. SAO C-7 vs. SAO '69	11.3	11.0	16.5	22.8[2]

Case 1 – Differences on station coordinates formed between the GSFC and the SAO '69 solutions. Station coordinates expressed in a geocentric earth reference frame.

Case 2 – Differences formed between the GSFC 1 solution and the SAO '69 solution. The GSFC 1 subset solution held the starting geopotential fixed (M1).

Case 3 – Differences formed between the above two GSFC solutions.

Case 4 – Differences formed between the GSFC (as in case 1) solution and the SAO C-7 station coordinates. The SAO C-7 coordinates were used as the starting values of the GSFC solution.

Case 5 – Differences formed between SAO C-7 and SAO '69.

(1)Twelve of the thirteen SAO Baker-Nunn stations were used. WOOMERA was omitted as the data available for it was not completely used. WOOMERA Baker-Nunn station site had been moved in 1964 to AUSBAK. The distribution of observations per Baker-Nunn station site is given in Table XI-B.

(2)From this comparison it appears that the SAO C-7 station coordinates are in worst agreement. C-7 coordinates were used as starting values in the GSFC solution.

TABLE IX
Adjustment for Scale, Translation, and Differential Rotation Between the GSFC and SAO ('69) Station Coordinates

A. Center of Mass Shift Between GSFC and SAO Solution by Taking the Average of the Coordinate Differences for the 12 Baker-Nunn Stations

Shift	ΔX	ΔY	ΔZ	TOTAL
SAO '69 – GSFC	2.8	1.4	-0.5	3.3 meters

B. Adjusted RMS[1] of Geocentric Station Coordinates for a Scale Translation, and Orientation Adjustment[2]

Adjusted RMS	Scale	Translation $\overline{\Delta R}$ (meters)			Differential Rotation (radians 10^6) about axes		
	s	ΔX	ΔY	ΔZ	$n(\hat{x})$	$m(\hat{y})$	$l(\hat{z})$
17.1 (meters)[3]	$-3(10^{-7})$	3.6	1.0	-0.4	.14	-.56	-.08

(1)Least squares adjustment formula: $\vec{R}_A = \vec{R} + s\vec{R} + \Delta\vec{R} + \vec{V} \times \vec{R}$. $\vec{R} \equiv (x, y, z)$ GSFC geocentric coordinates, $\vec{V} \equiv (n, m, \ell)$ differential rotation, $\vec{R}_s \equiv (x, y, z)$ SAO coordinates. The adjustment is made by treating $\vec{R}_s$ as a fixed reference system of geocentric coordinates. The adjusted

$$\mathrm{RMS} = \left[\sum_{12\ \mathrm{stations}} (\vec{R}_s - \vec{R}_A)\cdot(\vec{R}_s - \vec{R}_A)\right]^{1/2} \Big/ 12^{1/2}.$$

(2)The value of s corresponds to -2 meters ($a_e s$) for scale adjustment. Values of (n, m) correspond to a polar offset ~ 3.6 meters. Value of ℓ corresponds to a longitudinal offset ~ .5 meters.

(3)The previous unadjusted value of RMS, 17.7 meters, given in Table VIII indicates very little reduction from the adjustment. The observational residuals in the second iteration result are expected to be smaller and thus may produce better agreement on station coordinate values. See Table X for RMS of observational residuals by satellite.

4. Satellite Model for Geodetic Solution Using Numerical Integration

The satellite geodesy mode of data reduction employed in the GSFC solution is presented in Figure 7. A Geodetic and Station Recovery (GEOSTAR) computer program system is presented in References 6 and 7. The numerical integration models for the computational solution of the systems of differential equations listed in Figure 8 are described in these references. The numerical integration technique, with special additional self starting techniques, is basically a Cowell type and provides for a variable order and variable stepsize feature which may be selected to obtain the desired accuracy in the ephemeris for the modeled set of differential equations. GSFC models the gravitational forces of the moon, sun, and the geopotential (spherical harmonics), and employs a solar radiation pressure model and an atmospheric drag model using a recent Jacchia representation for the density. Satellite orbital position was modeled for better than a meter of accuracy in the preliminary solution.

- Preliminary solutions for orbital parameters, providing for a drag and/or a solar radiation pressure parameter, on individual satellite data arcs.
- Contributions of geodetic parameters are obtained in terms of least squares normal equations on individual satellite arcs.
- These normal equations are then combined into a simultaneous solution for the geodetic parameters in a linear adjustment.
- Arc lengths of 7 to 14 days chosen to cover the beat periods associated with satellite resonant effects and to minimize long term effects of modeling errors. Data spans of consecutive data over longer periods are connected successively through the 7 to 14 day arcs.

Figure 7. Satellite Geodesy Mode of Data Reduction.

Numerical integration of the systems of differential equations employ geocentric satellite coordinates in an inertial reference frame of the true equator and equinox of epoch. The epoch is associated with each individual satellite arc and is chosen at the beginning of the day of the occurrence of the first observation in the arc. The time system is A1 atomic time. All processing in connection with the motion of the Earth's true equator and instantaneous rotation is modeled through the luni-solar precession and nutation formulas and the use of the USNO bulletins of UT1-A1 (UT1-UTC) values. Polar motion has not been applied in the present solution. Observational processing and corrections are not presented here for this discussion.

(Each system is reduced to a first order system of 6 simultaneous equations)

System of Differential Equations	Number of Systems	Computation Time (Seconds/day)[1]
Geocentric Inertial Force Equations	1	6
Geocentric Variational Equations of Geocentric Epoch Parameters (State Transition Matrix)	6	2 (6 systems)
Geocentric Variational Equations of Geopotential Parameters	One for Each Coefficient	1/2 for each coefficient
Geocentric Variational Equations for One Drag Parameter and One Solar Radiation Pressure Parameter (parameters represent scale factors)	2	1/2 (for solar) 1 (for drag)

[1]These figures are based on average execution times on the GSFC 360/95 computer. They amounted to about ten minutes of processing per weekly arc after the initial orbits have been obtained for the six geocentric state elements at epoch.

Figure 8. Systems of Simultaneous Differential Equations Solved Through Numerical Integration.

5. Characteristics of Present Solution and Modified Approach for Extended Solution

In all, however, even though the comparisons are reasonably favorable for a first iteration result, certain limitations existed which are felt to be significant and have been adjusted in the current processing toward a second iteration result. A logistics type problem, providing for the availability (such as orbital starting elements) and computer program handling of a larger collection of satellite data arcs, has been resolved. And a problem, associated with the reduction of relatively large observational residuals on certain satellite arcs particularly those associated with relatively strong drag perturbations, has been resolved. The latter effects may be easily seen in the RMS values of satellite observational residuals associated with the lower altitude satellites in Figure 10. Figure 9 is presented to compare limitations in present solutions and modified approach in the new solutions.

Limitations – Present Solution	Remedy – New Solution
1. Data distribution – optical satellite data only. Limited geodetic parameter solution.	1. Satellite data (electronic, laser, and optical), geometrical data, and surface gravity data. More complete and unified geodetic solution.
2. Relatively large observational residuals for low and middle altitude satellites.	2. Improved modeling of drag parameters.
3. One iteration result for geodetic parameters. Preliminary iteration on orbital parameters only, including a limited drag parameter.	3. Preliminary iteration on orbital parameters (including item 2) and satellite families of resonant coefficients before solution of basic geodetic parameters.
4. Spherical harmonic representation of the geopotential.	4. Consideration will be given toward an extended representation of the geopotential – spherical harmonics for the global gravity variations supplemented with a residual potential to represent more directly the local gravity variations.

Figure 9. Preliminary Solution Characteristics and Modified Approach in New Solution.

A summary is given in Figure 10 of the RMS of satellite observation residuals. RMS values have been obtained from the initial satellite arcs which used the SAO M1 geopotential field as the starting solution. Some results have been obtained by substituting the GSFC solution for the 8×8 geopotential field into the orbital arc solutions but no marked improvement was observed.

Solutions are in process, with use of an extended drag model, for a determination of satellite families of resonant coefficients. It is expected that this technique will sufficiently reduce the size of the observation residuals to a linear region where a fine resolution of the short period terms of the tesseral field may then be determined in a simultaneous solution over all the satellite arcs.

Satellites	Perigee Height (km)	RMS[1]	# of Arcs
1. OVI-2	410	10"	4
2. OGO-2	420	24"[2]	7
3. GRS	420	12"	4
4. DI-C	580	9"	4
5. TRANSIT 4A	880	11"	14
6. OSCAR 7	870	5"	4
7. BE-B	910	12"	4
8. BE-C	940	17"[2]	12
9. TELSTAR 1	960	11"	5
10. COURIER 1B	970	5"	13
11. ANNA 1B	1030	5"	22
12. GEOS-II	1100	5"	15
13. SECOR 5	1140	7"	1
14. GEOS-I	1200	3"	38
15. ECHO R.B.	1510	7"	2
16. TIROS 9	700	Minitrack	
17. ALOUETTE 2	500	Data	
TOTAL: OPT. OBS. $\sim$ 54,000		RMS $\simeq$ 7.7" (seconds of arc)	

(1)RMS values are based upon the initial satellite solutions where the SAO M1 (1966) model was used as a starting solution. Satellites are ordered by perigee height and they tend to show larger RMS values for the lower altitude satellites.

(2)These satellites are strongly affected by satellite resonance and this accounts for some of the large RMS value. These satellites were not used in the M1 solution.

Figure 10. Summary of Satellite Observational Residuals (Initial Satellite Solutions).

TABLE X
Satellite Data Summary (Preliminary Results)

Satellite	Inclination (Degrees)	Semi-Major Axis Kilometers	Eccentricity	Perigee Height km	Number of Arcs 7 to 14 (days arcs)	Number of Observations	RMS[1] of Residuals
1. ANNA 1B (1962)	50	7508	.007	1077	22	4015	5″.2[3]
2. GRS (1963)	50	7237	.062	424	4	338	11.9
3. TRANSIT-4A (1961)	67	7318	.008	885	14	1372	11.1
4. TELSTAR 1 (1962)	44	9672	.241	962	5	1749	11.1
5. GEOS-I (1965)	59	8074	.073	1121	38	23,580[4]	2.7
6. GEOS-II[2](MOTS) (1968)	106	7709	.031	1101	15	9,518[4]	5.3
7. COURIER 1B (1960)	28	7465	.016	965	13	2,966	5.2
8. OVI-2 (1965)	144	8306	.182	416	4	944	9.9
9. OGO-2 (1965)	87	7344	.075	420	7	447	23.9
10. DI-C (1967)	40	7336	.052	579	4	782	8.4
11. BE-B (1964)	80	7362	.012	912	4	476	15.5
12. BE-C (1965)	41	7311	.026	941	12	5111	17.2
13. SECOR-5 (1965)	69	8159	.079	1137	1	127	7.0
14. OSCAR-7 (1966)	89	7417	.023	868	4	1750	5.5
15. ECHO R.B. (1960)	47	7971	.011	1512	2	298	7.4
16. TIROS 9 (Minitrack)[2] (1965)	96	8021	.116	708	2	637	.5 (10^{-3}) direction
17. ALOUETTE 2 (Minitrack)[2] (1965)	80	8097	.151	506	3	650	.6 (10^{-3}) cosines

TOTAL OPTICAL OBSERVATIONS ≃ 54,000 TOTAL RMS ≃ 7.7 seconds of arc
Primarily SAO Baker-Nunn

(1) RMS values are based upon initial satellite arcs where the 1966 SAO-M1 model was used as the starting solution. See Figure 10 and text.

(2) MOTS (Minitrack Optical Tracking System at GSFC) data used on GEOS-II. Minitrack Interferometer data used for satellite resonant analysis with Alouette and TIROS 9.

(3) Units of seconds of arc.

(4) Subset solutions are being examined for heavily weighted satellites. Sum of the squares of the observational residuals are larger on some other satellites, i.e., these satellites do not dominate the solution statistically.

TABLE XI-A

8×8 Geopotential Solution Values in the GSFC First Iteration Result[1]

Geopotential Coefficients			Geopotential Coefficients		
n, m	$\bar{C}_{n,m}$ 10^6	$\bar{S}_{n,m}$ 10^6	n, m	$\bar{C}_{n,m}$ 10^6	$\bar{S}_{n,m}$ 10^6
2,0	-484.177		6,4	-.18	-.50
2,1	.01	-.01	6,5	-.22	-.56
2,2	2.40	-1.36	6,6	-.07	-.19
3,0	.956		7,0	.091	
3,1	1.98	.30	7,1	.24	.14
3,2	.92	-.63	7,2	.33	.16
3,3	.70	1.49	7,3	.37	-.16
4,0	.556		7,4	-.35	-.06
4,1	-.60	-.44	7,5	.19	.04
4,2	.41	.64	7,6	-.35	.13
4,3	.96	-.25	7,7		
4,4	-.23	.31		.10	.11
5,0	.066		8,0	.089	
5,1	.00	-.05	8,1	-.04	.07
5,2	.69	-.29	8,2	.08	.11
5,3	-.33	-.23	8,3	-.07	.03
5,4	-.34	-.15	8,4	-.30	.02
5,5	.46	-.50	8,5	.14	.11
6,0	-.176		8,6	.00	.29
6,1	-.05	.01	8,7	.06	-.06
6,2	.04	-.32	8,8	-.29	.19
6,3	.09	.03			

(1) Higher order satellite resonant coefficients have been determined in the solution but are not presented in the preliminary results.

TABLE XI-B
Geocentric Station Coordinate Solutions For Principal Baker-Nunn Tracking Sites

Station Number(1)	Name	X(2) (meters)	Y	Z	No. of Observations in Solution
9001	ORGAN	-1,535,760.	-5,166,986.	3,400,064.	5,991
9002	OLFAN	5,056,120.	2,716,509.	-2,775,799.	5,773
9003	WOOMER(3)	-3,983,782.	3,743,121.	-3,275,587.	726
9004	SPAIN	5,105,589.	-555,235.	3,769,684.	5,424
9005	TOKYO	-3,946,688.	3,366,276.	3,698,821.	667
9006	NATOL	1,018,211.	5,471,109.	3,109,634.	3,342
9007	QUIPA	1,942,772.	-5,804,077.	-1,796,955.	3,131
9008	SHRAZ	3,376,877.	4,403,979.	3,136,248.	136(3)
9009	CURAC	2,251,833.	-5,816,932.	1,327,166.	20(3)
9010	JUPTR	976,279.	-5,601,396.	2,880,247.	4,175
9011	VILDO	2,280,580.	-4,914,568.	3,355,436.	3,472
9012	MAUIO	5,466,049.	-2,404,282.	2,242,181.	3,619
9023	AUSBAK	-3,977,773.	3,725,101.	-3,303,041.	6,417
					~43,000(4)

(1) See SAO reference 2 for further identification of station location. This enumberation is consistent with that used in the NGSP (National Geodetic Satellite Program) and the NSSDC (National Space Science Data Center) at Goddard where the data is available.

(2) Station coordinates are referenced to a Greenwich Earth Reference System and are consistent with the SAO reference system, except that polar motion adjustment for a mean pole was not applied in our preliminary solution. See Table IX for a polar tilt adjustment.

(3) Additional observations are available but were not properly used because of a problem in station numbering.

(4) The difference between the number here of 43,000 and the total number of observations of 54,000 in Table X for the satellite distribution is the additional set of observations associated with the stations that were utilized, such as the MOTS stations for example, in the geopotential solution.

ACKNOWLEDGMENTS

The authors wish to acknowledge various organizations and personnel for their contributions in the development of the GEOSTAR program system and related data processing activities. In the current data processing for the first geodetic solution and related program development, the following personnel from the Computer Science Corporation are acknowledged for their contributions: (a) For program development and maintenance - Ralph D. Grant and Harold E. Dennis, (b) Data Operation - Wayne A. Taylor, Donald W. McGraw, Calvin V. Engrum, Lionel W. Williams, and Charles S. Kenworthy, and (c) Administration and Theoretical Design - Mark L. Sandson and William E. Strange.

Further acknowledgment for major program contributions to the GEOSTAR system is given to (a) Wolf Research and Development Corporation for the NONAME orbit determination program, (b) Computer Usage Development Corporation for the MERGE and SOLVE programs of the LUNGFISH (Lunar Gravity Field in Spherical Harmonics) system which have been applied to combine and solve the normal equations for geodetic parameters, and (c) International Business Machines Corporation for the initial assembly, documentation, and data operations analysis of the GEOSTAR system.

Acknowledgment is given to the GSFC personnel who were responsible for or contributed to the program development and data analysis performance of the above program systems. In this connection special mention is given to Clarence E. Doll, James G. Marsh, Gail P. Brodsky, Carmelo E. Velez, James P. Murphy, David E. Smith, and Joseph W. Siry of the Trajectory Analysis & Geodynamics Division.

Finally, for the tabular data compiled in the report, special acknowledgment is given to Paul E. Gibbs of Computer and Software, Incorporated. This entailed the development and operations of utility programs to process the solution results from the GEOSTAR system and other geodetic models.

REFERENCES

1. Gaposchkin, E. M. (1966). "Tesseral Harmonic Coefficients and Station Coordinates from the Dynamic Method," Geodetic Parameters for a 1966 Smithsonian Standard Earth, SAO Special Report No. 200, Vol. 2, pp. 105-258.

2. Gaposchkin, E. M., and Lambeck, K. (1970). "1969 Smithsonian Standard Earth (II)," SAO Special Report 315.

3. Pellinen, L. P. (1969). "Estimation and Application of Degree Variances of Gravity Anomalies," Symp. Physical Geodesy, Praka, Preprint.

4. Kaula, W. M. (1967). "Geophysical Implications of Satellite Determinations of the Earth's Gravitational Field." Space Science Reviews, Vol. 7, pp. 769-794.

5. Wagner, C. A. (1970). "Geopotential Coefficient Recovery from Very Long Arcs of Resonant Orbits," Journal of Geophysical Research, Vol. 75-22, pp. 6662-6674.

6. Velez, C. E. and Brodsky, G. P. (1970). "GEOSTAR-II, A Geopotential and Station Position Recovery System," Goddard Space Flight Center Document X-553-70-372. October 1970.

7. Velez, C. E. and Brodsky, G. P. (1969). "GEOSTAR-I, A Geopotential and Station Position Recovery System," Goddard Space Flight Center Document X-553-69-544. November 1969.