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NASA <u>TN D-6570</u> C.1



# SIMULATION OF FLIGHT MANEUVER-LOAD DISTRIBUTIONS BY UTILIZING STATIONARY, NON-GAUSSIAN RANDOM LOAD HISTORIES

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION . WASHINGTON, D. C. . NOVEMBER 1971

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			[]	33345		
1.	Report No. NASA TN D-6570	2. Government Accession No.	3. Recipient's Catalog	No.		
<b>4</b> .	Title and Subtitle SIMULATION OF FLIGHT MA	NEUVER-LOAD DISTRIBUTIONS	5. Report Date November 1971			
	BY UTILIZING STATIONARY HISTORIES	, NON-GAUSSIAN RANDOM LOAD	6. Performing Organiz	ation Code		
7.	Author(s)		8. Performing Organiza	ation Report No.		
	Herbert A. Leybold		L-7810			
9.	Performing Organization Name and Address		10. Work Unit No.	01		
	NASA Langley Research Cent	er	11 Contract or Grant No			
	Hampton, Va. 23365		The contract of Grant	140.		
			13. Type of Report an	d Period Covered		
12.	Sponsoring Agency Name and Address		Technical Note			
	National Aeronautics and Space	e Administration	14. Sponsoring Agency	Code		
	Washington, D.C. 20546					
15.	15. Supplementary Notes This paper is based in part upon a thesis entitled "Simulation of Service Load Distributions Utilizing Stationary, Non-Gaussian Random Time Histories" submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering, North Carolina State University, Raleigh, North Carolina, 1969.					
16.	Abstract	-				
	Random numbers were generated with the aid of a digital computer and transformed such that the probability density function of a discrete random load history composed of these random numbers had one of the following non-Gaussian distributions: Poisson, binomial, log-normal, Weibull, and exponential. The resulting random load histories were analyzed to determine their peak statistics and were compared with cumulative peak maneuver-load distributions for fighter and transport aircraft in flight.					
17.	Key Words (Suggested by Author(s))	18. Distribution Stateme	nt			
	Random load histories	Unclassified	– Unlimited			
	Maneuver-load spectra					
19.	Security Classif. (of this report)	20. Security Classif, (of this page)	21. No. of Pages	22. Price*		
	Unclassified	Unclassified	25	\$3.00		

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### SIMULATION OF FLIGHT MANEUVER-LOAD DISTRIBUTIONS BY UTILIZING STATIONARY, NON-GAUSSIAN RANDOM LOAD HISTORIES\*

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#### SUMMARY

The basic load history used to conduct random-load fatigue tests is, in general, Gaussian in nature and its peak distributions, both maximum and minimum, are inherently symmetrical about its mean. This symmetry is a characteristic of gust loading. However, for maneuver loads, maximum and minimum peak distributions are asymmetric. Thus the basic Gaussian random load history does not afford an accurate simulation of the aircraft maneuver-load environment. The purpose of the present investigation was to determine the feasibility of utilizing some stationary, non-Gaussian random load histories to simulate this environment.

Five stationary, non-Gaussian random load histories were generated with the aid of a digital computer. The peak distributions of the generated random histories were determined and compared with maneuver-load flight data. The results indicated that non-Gaussian random load histories could be used to simulate slightly asymmetric aircraft maneuver-load spectra (i.e., spectra of transport aircraft), whereas highly asymmetric aircraft maneuver-load spectra (i.e., spectra of fighter aircraft) could not be simulated as well with these generated random histories.

### INTRODUCTION

In recent years, random-load fatigue testing of aircraft structural parts was proposed. A major factor which influenced this approach was the realization that maneuver loads vary randomly with time and that, under the existing state of the art, fatigue life could not be predicted in an accurate, deterministic fashion. Currently, with more sophisticated fatigue testing machines available (machines capable of applying complex load histories which more nearly simulate actual flight experience), random-load fatigue tests have become rather common.

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<sup>\*</sup>This paper is based in part upon a thesis entitled "Simulation of Service Load Distributions Utilizing Stationary, Non-Gaussian Random Time Histories" submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering, North Carolina State University, Raleigh, North Carolina, 1969.

The random load history most widely used for random-load fatigue testing is Gaussian in nature. The Gaussian random load history is so called because its instantaneous loads are distributed normally; this distribution of instantaneous loads can be characterized by the well-known Gaussian probability density function. But the peak loads, which are considered to be the significant fatigue-inducing parameter of a random load history, are not Gaussian in nature. Rice (ref. 1) has developed analytical methods for predicting the peak-load distributions of Gaussian random load histories. Both maximum and minimum peak-load distributions are symmetrical about the mean of the random load history. This symmetry is a characteristic of gust loading. However, the maximum and minimum peak loads from maneuvers are not symmetrical about the mean. For maneuver loads, these distributions are asymmetric, and thus the basic Gaussian random load history does not simulate the maneuver-load flight environment. The purpose of the present investigation is to determine if certain stationary, non-Gaussian random load histories can simulate the aircraft maneuver-load flight environment.

Since no analytical methods are available for determining the maximum and minimum peak distributions of non-Gaussian random load histories, these distributions were determined numerically. A random-number generator was used to generate random numbers having a constant density function and having limits of 0 and 1. These random numbers were then transformed so that the probability density function of a discrete random load history composed of these random numbers had one of the following non-Gaussian distributions: Poisson, binomial, log-normal, Weibull, and exponential. These five forms of the density function were arbitrarily selected because they are widely discussed in the literature and are mathematically describable. The transformed random histories were analyzed to determine their peak statistics and were compared with peak statistics encountered in flight.

### SYMBOLS

E()	expected value
F(x)	cumulative probability density function
f(x)	probability density function
g	acceleration due to gravity, 9.8 m/sec $^2$
n	number of events (positive integer)
р	probability of success

q	probability of failure
RMS	root mean square
Δt	time increment
x	random variable
<b>{y}</b>	set of uniformly distributed random numbers
α,β	parameters of Weibull probability density function
Г()	gamma function
θ	parameter of exponential probability density function
μ	parameter of Poisson probability density function
$\mu_{ m log}$	mean of logarithms
σ	standard deviation
$\sigma_{\log}$	standard deviation of logarithms
σ <b>2</b>	variance

#### GENERATION AND ANALYSIS OF RANDOM LOAD HISTORIES

Uniformly distributed random numbers were generated digitally and then transformed to have a nonuniform distribution. The set of uniformly distributed random numbers was transformed into sets having Poisson, binomial, log-normal, Weibull, and exponential density functions. These functions were chosen because they are common, their characteristics are well known, and they are analytic. The transformed numbers are considered to be the instantaneous values of load at discrete time intervals and therefore form a load history (see fig. 1). The final step was to study the peaks in the five load histories generated. These peaks are not distributed according to the distribution of the set of transformed random numbers; for example, a load history with instantaneous values that are Poisson distributed has maximum and minimum peaks that are not Poisson distributed. Figure 1 shows the differences between what is meant by a peak (maximum or minimum) and the instantaneous value of the load.

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A set of random numbers  $\langle y \rangle$  bounded by the limits 0 and 1 and having a constant probability density function were generated with a standard random-number generator. This set of uniformly distributed numbers was then transformed into five new sets; each new set was distributed according to one of the five previously mentioned non-Gaussian probability density functions. The numbers, when drawn from a particular set, represent the instantaneous values of the load history; obviously, these instantaneous values of the history are distributed the same as the numbers of the set.

The set of uniformly distributed random numbers was transformed by

$$\mathbf{F}(\mathbf{x}) = \int_{-\infty}^{\mathbf{x}} f(\gamma) d\gamma$$
 (1)

where  $f(\gamma)$  is the specified density function. The cumulative probability density function F(x) is bounded by the limits 0 and 1, where x can take on values of  $-\infty$  to  $+\infty$ . The random numbers from the set  $\langle y \rangle$  having a constant probability density distribution and having limits of 0 and 1 were substituted for F(x) in equation (1). Equation (1) could then be evaluated for x (the transformed random number). For this investigation, each random history generated contained 10 000 random numbers.

Two of the probability density functions considered (Poisson and binomial) were discrete functions, and hence the integral in equation (1) was changed to a summation to determine F(x); that is,

$$\mathbf{F}(\mathbf{x}) = \sum_{0}^{\mathbf{x}} \mathbf{f}(\gamma) \tag{2}$$

For the Poisson distribution, the following recurrence formula was used to compute F(x):

$$f(x + 1) = \frac{\mu}{x + 1} f(x) \qquad (x = 0, 1, 2, ..., \infty; \mu > 0) \qquad (3)$$

Similarly, for the binomial distribution, the recurrence formula used was

$$f(x + 1) = \left(\frac{n - x}{x + 1}\right) \frac{p}{q} f(x) \qquad (x = 0, 1, 2, ..., n; 0$$

The density function of the log-normal distribution was

$$f(\mathbf{x}) = \frac{1}{\mathbf{x}\sigma_{\log}\sqrt{2\pi}} \exp\left[\frac{-\left(\log_{e} \mathbf{x} - \mu_{\log}\right)^{2}}{2\sigma_{\log}^{2}}\right] \qquad (\mathbf{x} > 0; \ \sigma_{\log} > 0; \ -\infty < \mu_{\log} < \infty) \qquad (5)$$

where x was evaluated with a numerical approximation of the integral in equation (1).

For the other two density functions, x was determined by directly integrating equation (1). For the Weibull distribution,

$$f(x) = \alpha \beta x^{\alpha - 1} \exp(-\beta x^{\alpha}) \qquad (x > 0; \beta > 0; \alpha > 0) \qquad (6)$$

and

$$\mathbf{F}(\mathbf{x}) = \mathbf{1} - \exp(-\beta \mathbf{x}^{\boldsymbol{\alpha}}) \tag{7}$$

Solving for x in equation (7) yielded

$$\mathbf{x} = \left\{ -\frac{1}{\beta} \log_{\mathbf{e}} \left[ \mathbf{1} - \mathbf{F}(\mathbf{x}) \right] \right\}^{1/\alpha}$$
(8)

For the exponential distribution,

$$f(x) = \theta \exp(-\theta x) \qquad (\theta > 0; x > 0) \qquad (9)$$

and

$$\mathbf{F}(\mathbf{x}) = \mathbf{1} - \exp(-\theta \mathbf{x}) \tag{10}$$

Solving for x in equation (10) gave

$$\mathbf{x} = -\frac{1}{\theta} \log_{\mathbf{e}} \left[ \mathbf{1} - \mathbf{F}(\mathbf{x}) \right]$$
(11)

It should be noted that the exponential distribution is a special case of the Weibull distribution when  $\alpha = 1$ .

The first 100 transformed random numbers for each of the five different load histories generated are plotted in sequence at equal intervals of time in figures 2 to 6. The mean, the root mean square, and the standard deviation are shown for each history.

The mean indicates the average or expected value of the random variable x and is mathematically expressed as

$$E(x) = \int_{-\infty}^{\infty} xf(x)dx$$
(12)

The root mean square is a measure of the amplitude of a time history and is the square root of the mean of the squares of a random variable x. The mean square (that is, the mean of the squares) is mathematically expressed as

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$
(13)

The standard deviation is a measure of the dispersion from the mean and is the square root of the variance of a random variable x. The variance is defined mathematically as

$$\sigma^2 = E\left\{ \left[ x - E(x) \right]^2 \right\}$$
(14)

which reduces to

$$\sigma^{2} = E(x^{2}) - [E(x)]^{2}$$
(15)

All three properties were calculated for each of the five random load histories generated. The results of these calculations are shown in table 1. These same properties for the selected functions were calculated on the basis of the analytic equations summarized in table 2. The resulting theoretical values showed good agreement with the values calculated for the generated histories, as can be seen in table 1.

### COMPARISON OF GENERATED AND THEORETICAL HISTOGRAMS OF INSTANTANEOUS VALUES

The frequency of occurrence of the instantaneous loads was determined (see fig. 7(a)) for each of the five random load histories generated, and these frequencies are plotted as histograms in figures 2 to 6. The distributions of the generated numbers are compared with theoretical histograms. The theoretical histograms were obtained by first taking an incremental area under the particular probability-density-function curve and then multiplying the incremental area by 10 000 so that the frequency of occurrence has the same base as the generated numbers. Figures 2 to 6 indicate that excellent agreement was obtained. In addition, a check on the randomness of the number generator was performed with a chi-square test. In all cases, the chi-square value is inside the accepted region specified by the 95-percent confidence level.

Some of the parameters selected for the binomial, Weibull, and Poisson density functions tended to make them look Gaussian. This result was to be expected because the normal or Gaussian density function is frequently used to approximate these other functions. For example, the normal function approximates the binomial function quite well when the binomial parameters np and n(1 - p) are both greater than 5, as verified in figure 5(b).

### DETERMINATION OF PEAK AND CUMULATIVE PEAK DISTRIBUTIONS FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES

Peak values, both maximum and minimum, were counted in equal intervals as shown in figure 7(b). The frequency of occurrence of these peak values, for each of the five random histories generated, are plotted on the left-hand side of figures 8 to 12. The maximum and minimum peak distributions intersect at the mean of the random history. Moreover, for Gaussian random histories, both maximum and minimum peak distributions are approximately normal in shape and are mirror images of each other. As mentioned previously, some distributions of instantaneous values show a trend toward being normal. Therefore, it could be anticipated that their maximum and minimum peak values are approximately normally distributed and symmetrical about the mean (see, for example, figs. 5(b) and 11(b)).

Cumulative peak distributions were also determined and are plotted on the righthand side of figures 8 to 12. Some of these distributions are subsequently compared with actual maneuver-load data from aircraft in flight.

### COMPARISON OF CUMULATIVE PEAK DISTRIBUTIONS WITH MANEUVER-LOAD DATA

Actual maneuver-load data are generally presented on the basis of either the incremental acceleration (in g units) or the percent limit load. The data are usually plotted on semi-log paper with the cumulative number of exceedances plotted on the log scale and the incremental acceleration or percent limit load on the linear scale. The digitally generated data discussed previously have been converted to these scales for purposes of comparison with actual maneuver-load data. In order to cover a wide range of asymmetry of actual loading spectra, two different types of spectra, one for a fighter aircraft and one for a transport aircraft, were selected for comparison. The fighter spectrum is highly asymmetric whereas the transport spectrum is only slightly asymmetric. For the range of parameters covered in this investigation, all peak distributions generated were compared with these two types of spectra.

Figure 13 presents the operational maneuver-load spectrum for transport-type aircraft as obtained from reference 2. This spectrum is compared with the digitally generated data which have a log-normal probability density function. Good agreement between the digitally simulated and actual maneuver-load data was obtained. The simulated data resulted in more occurrences than did the actual flight data, particularly at the lower levels of incremental acceleration. Figure 14 presents the operational maneuver-load spectrum for fighter-type aircraft as obtained from reference 3. This spectrum is compared with the digitally generated data which have a Weibull probability density function. Fair agreement between the digitally simulated and actual maneuver-load data was obtained, the best agreement being achieved for the positive half of the spectrum.

### CONCLUSIONS

Five stationary, non-Gaussian random histories have been generated with the aid of a digital computer. Distributions of maximum and minimum peaks as well as probability density functions have been obtained for each load history. The following conclusions have been drawn from an analysis of the generated digital data:

(1) When the probability density function of a random history approached a Gaussian distribution, the maximum and minimum peak distributions of that random history were mirror images of each other about its mean and were distributed essentially normally.

(2) When the probability density function of a random history was decidedly non-Gaussian in shape, the maximum and minimum peak distributions of that random history were asymmetric about its mean and were also non-Gaussian in shape.

(3) Of the five random histories generated, the one having a log-normal distribution compared most favorably with the transport-type spectrum whereas the one having a Weibull distribution compared most favorably with the fighter-type spectrum.

Langley Research Center,

National Aeronautics and Space Administration, Hampton, Va., October 22, 1971.

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Density	Parameters	Mean, E(x)		Root mean square, RMS		Standard deviation, $\sigma$	
function		Generated	Theoretical	Generated	Theoretical	Generated	Theoretical
Weibull	$\alpha = 2; \beta = 2$	0.6207	0.6267	0.7004	0.7071	0.3245	0.3276
	$\alpha = 5; \beta = 5$	.6631	.6655	.6801	.6827	.1511	.1524
Exponential	$\theta = 1$	0.9811	1.0	1.3917	1.4142	0.9870	1.0
	$\theta = 2$	.4906	.5	.6959	.7071	.4935	.5
Poisson	$\mu = 10$	9.948	10.0	10.436	10.488	3.130	3.162
	$\mu = 66$	65.867	66.0	66.365	66.498	8.032	8.124
Binomial	n = 10; p = 0.1	0.979	1.0	1.357	1.378	0.941	0.949
	n = 500; p = 0.1	49.892	50.0	50.340	50.448	6.626	6.708
Log-normal	$\mu_{\rm log} = 0;  \sigma_{\rm log}^2 = 0.1$	1.0453	1.0513	1.0983	1.1052	0.3372	0.3409
	$\mu_{\rm log}=0;\sigma_{\rm log}^2=0.05$	1.0214	1.0253	1.0468	1.0513	.2296	.2322

### TABLE 1.- PROPERTIES OF GENERATED AND THEORETICAL DENSITY FUNCTIONS

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Density function	Mean, E(x)	Mean square, $E(x^2)$	Variance, $\sigma^2$
Weibull	$\left(\frac{1}{\beta}\right)^{1/\alpha}\Gamma\left(\frac{\alpha+1}{\alpha}\right)$	$\left(\frac{1}{\beta}\right)^{2/lpha}\Gamma\left(\frac{lpha+2}{lpha}\right)$	$\left(\frac{1}{\beta}\right)^{2/\alpha} \left\{ \Gamma\left(\frac{\alpha+2}{\alpha}\right) - \left[\Gamma\left(\frac{\alpha+1}{\alpha}\right)\right]^2 \right\}$
Exponential	$\frac{1}{\theta}$	$\frac{2}{\theta^2}$	$\frac{1}{\theta^2}$
Poisson	μ	$\mu(\mu + 1)$	μ
Binomial	np	n(n - 1)p <sup>2</sup> + np	np(1 - p)
Log-normal	$\exp\left(\mu_{\log} + \frac{1}{2} \sigma_{\log}^2\right)$	$\exp\left(2\mu_{\log}+2\sigma_{\log}^{2}\right)$	$\exp\left(2\mu_{\log} + \sigma_{\log}^{2}\right) \left[\exp\left(\sigma_{\log}^{2}\right) - 1\right]$

### TABLE 2.- SUMMARY OF PROPERTIES OF DENSITY FUNCTIONS



Time

Figure 1.- Discrete random history showing maximum and minimum peaks.

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(a)  $\alpha = 2; \beta = 2.$ 



Figure 2.- Sequences of first 100 random numbers and histograms of first 10 000 random numbers for load histories having a Weibull probability density function.



Figure 3.- Sequences of first 100 random numbers and histograms of first 10 000 random numbers for load histories having an exponential probability density function.

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(a)  $\mu = 10$ .



(b)  $\mu = 66$ .

Figure 4.- Sequences of first 100 random numbers and histograms of first 10 000 random numbers for load histories having a Poisson probability density function.



(b) n = 500; p = 0.1.

Figure 5.- Sequences of first 100 random numbers and histograms of first 10 000 random numbers for load histories having a binomial probability density function.





(b) 
$$\sigma_{\log}^2 = 0.1; \ \mu_{\log} = 0.1$$

Figure 6.- Sequences of first 100 random numbers and histograms of first 10 000 random numbers for load histories having a log-normal probability density function.



Time

(a) Frequency of occurrence of instantaneous loads.

Figure 7.- Frequency of occurrence in prescribed intervals.



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(b) Frequency of occurrence of maximum and minimum peaks.

Figure 7.- Concluded.



(a)  $\alpha = 2; \beta = 2.$ 



(b)  $\alpha = 5; \beta = 5.$ 

Figure 8.- Peak and cumulative peak distributions for random histories having a Weibull probability density function.

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(a)  $\theta = 1$ .



Figure 9.- Peak and cumulative peak distributions for random histories having an exponential probability density function.



Figure 10.- Peak and cumulative peak distributions for random histories having a Poisson probability density function.







(b) n = 500; p = 0.1.

Figure 11.- Peak and cumulative peak distributions for random histories having a binomial probability density function.



(b) 
$$\sigma_{\log}^2 = 0.1; \ \mu_{\log} = 0.1$$

Figure 12.- Peak and cumulative peak distributions for random histories having a log-normal probability density function.



Incremental acceleration, g units

Figure 13.- Log-normal digitally generated data compared with operational maneuver-load spectrum (for transport aircraft) from reference 2.



Figure 14.- Weibull digitally generated data compared with operational maneuver-load spectrum (for fighter aircraft) from reference 3.

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