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GCA CORPORATION
GCA TECHNOLOGY DIVISION
Bedford, Massachusetts

A STUDY OF PLANETARY METEOROLOGY

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SUMMARY

Preliminary estimates are prepared of the weighting functions of the infrared bands of methane, ammonia, and hydrogen for the Jovian atmosphere. Together with the infrared spectral emission observations of Jupiter, these weighting functions are used to obtain a rough estimate of the vertical temperature profile in the Jovian atmosphere.

1. INTRODUCTION

The primary objectives of the current research program are:

(1) To study the meteorology of the planet Jupiter, with particular emphasis on the deduction of the atmospheric temperature profile through analysis of available infrared and microwave observations of the planet's emission spectrum.

(2) To develop and apply inversion techniques for inferring atmospheric temperature and constituent profiles of planetary atmospheres.

During the first quarter, work was started on the first objective listed above. The available observations of the Jovian emission spectrum having the required spectral resolution consist of the infrared observations of Gillett et al. (1969) and the microwave observations of Law and Staelin (1968) and Wrixon et al. (1971). To invert these observations and deduce the temperature structure requires a knowledge of the absorption (transmission) characteristics of the Jovian atmospheric gases that influence the observed spectrum. These are methane and hydrogen — which absorb in the infrared — and ammonia — which absorbs in both the infrared and microwave portions of the spectrum. Much of the first quarter was spent in developing the transmission models for these gases. These transmission models have been used in preliminary calculations to obtain a picture of what the weighting functions look like. (The concept of weighting function is explained in the next section; briefly, the weighting function indicates where in the atmosphere the observed radiation originates.) These weighting functions, together with the observations were used to obtain a preliminary and rough estimation of the Jovian atmospheric temperature profile.

This work is reviewed in detail in the following section. The last section of this report summarizes research plans for the next quarterly period.

2. DISCUSSION

2.1 Weighting Function Concept

A quick insight into the inversion process can be obtained from an understanding of the weighting function concept. Thus, in this section we shall first discuss the weighting function concept in general, then discuss specific transmission models for the Jovian gases of interest, and then apply these transmission models to approximate Jovian atmospheric conditions to obtain the Jovian weighting functions. From the Jovian weighting functions and the emission spectrum observations, we shall obtain a rough estimate of the temperature profile.

The emitted radiation intensity of a planetary atmosphere at wavenumber ν can be written as

$$I(\nu) = B[\nu,\ \theta(z_s)]\ T(\nu,\ z_s) + \int_{z_s}^{\infty} B[\nu,\ \theta(z)]\ \left[\frac{dT(\nu,z)}{dz}\right] dz \qquad (1)$$

where B is the Planck function, θ is temperature, z is altitude, $T(\nu,z)$ is the transmittance from the altitude z to the top of the atmosphere, and the subscript s refers to the planet's surface. In the presence of a cloud cover that acts as a black body, the cloud top altitude replaces the planet's surface in the equation. For strongly absorbing gases, $T(\nu,\ z_s)$ may be zero, and only the second term on the R.H.S. of (1) — the atmospheric contribution — need be considered. If there is no absorption at all by the atmosphere, then the second term on the R.H.S. of (1) equals 0, $T(\nu, z_s) = 1$, and the emitted intensity is equal to the Planck function at $\theta(z_s)$, from which the planet's surface temperature can easily be retrieved. Such regions of the spectrum with little or no absorption are called window regions.

From the second term on the R.H.S. of (1) it can be seen that the emitted intensity as observed at the top of the atmosphere consists of contributions from levels within the atmosphere. These contributions consist of the Planck function at a level weighted by the vertical gradient of the transmittance, $\frac{dT}{dz}$. Thus, $\left[\frac{dT}{dz}\right]$ as a function of z is called the weighting function.

Physical reasoning can be used to demonstrate that the weighting function for a strongly absorbing gas will have a peak somewhere between the surface and the top of the atmosphere. At the top of the atmosphere, the contribution to the outgoing intensity is negligible because the density of the emitting (absorbing) gas is low. At the base of the atmosphere, the contribution to the outgoing intensity is small since the overlying atmosphere absorbs most of the radiation emitted by the near surface layer. Thus, the weighting function must have a maximum somewhere between the surface and top of the atmosphere.

The level at which the weighting function peaks is the center of the atmospheric layer from which most of the emitted intensity originates. We show below that, for a gray absorbing atmosphere with exponentially decreasing density of absorbing gas with altitude, this peak occurs at the level where the transmittance is equal to e^{-1}.

For a gray atmosphere the transmittance from the level z to the top of the atmosphere is given by

$$T = \exp\left(- \alpha \int_{z}^{\infty} \rho_g \, dz\right) \tag{2}$$

where α is the absorption coefficient, and ρ_g is the density of the absorbing gas. If the absorbing gas has a constant mixing ratio

$$w = \rho_g/\rho \tag{3}$$

where ρ is the atmospheric density, Equation (2) can be transformed with the aid of the hydrostatic equation

$$\frac{dp}{dz} = - \rho g \tag{4}$$

to

$$T = \exp\left(-k \int_{0}^{p(z)} dp\right) = \exp\left(-kp(z)\right) \tag{5}$$

where $k = \frac{\alpha w}{g}$.

The weighting function is

$$\frac{dT}{dz} = -ke^{-kp} \frac{dp}{dz} = \rho g k \, e^{-kp} \tag{6}$$

To determine at what level the weighting function peaks, we differentiate (6) with respect to z, and obtain

$$\frac{d}{dz}\left(\frac{dT}{dz}\right) = gk \frac{d}{dz} \left(\rho e^{-kp}\right)$$

$$= gke^{-kp} \left(\frac{d\rho}{dz} + \rho^2 g k\right) \tag{7}$$

The weighting function is a maximum where $\frac{d}{dz}\left(\frac{dT}{dz}\right) = 0$ or where

$$\frac{1}{\rho} \frac{d\rho}{dz} = - \rho g k \tag{8}$$

For an isothermal atmosphere,

$$\frac{1}{\rho} \frac{d\rho}{dz} = \frac{1}{p} \frac{dp}{dz} = - \frac{1}{H} \tag{9}$$

where H is the scale height. Thus, (8) is satisfied when

$$\frac{1}{H} = \rho g k \tag{10}$$

or when $k = 1/p$. When $k = 1/p$, the transmittance $T = e^{-1}$. The opacity or optical depth, τ, is defined as $(-\ell n\ T)$ and is equal to unity at the level of the peak weighting function.

We shall later use the level of peak weighting function to obtain an estimate of the height from which most of the emitted radiation originates, and from the magnitude of the emitted radiation obtain an estimate of the temperature at this height. Since the level of peak weighting function varies with the absorption coefficient, observations at wavelengths with different absorption coefficients will yield information on the temperature at different heights. Such a usage of the weighting function represents a rudimentary form of inversion. More sophisticated inversion techniques are discussed by Conrath (1968), Chahine (1968), Wark and Fleming (1966), and Smith et al. (1970), among others. In future work, we plan to utilize an iterative inversion technique of the type described by Chahine (1968). Thus, the work reported here represents the simplest approach to the problem, that still yields useful results. It also has the virtue of yielding insight into the inversion problem.

The mathematical derivation above applies to a gray atmosphere. To determine weighting functions for a non-gray atmosphere, and to invert the emission spectrum of a non-gray atmosphere requires a knowledge of the real transmission characteristics of the absorbing gases. We discuss below transmission models and associated weighting functions for CH_4, NH_3, and H_2 on Jupiter.

2.2 Transmittance Models and Weighting Functions

2.2.1 7.7μ CH_4 Band

For the infrared absorption band of CH_4 at 7.7μ we follow McClatchey et al. (1970). McClatchey assumes that the transmission of a 20 cm^{-1} interval can be represented as a function of an absorption coefficient k_ν, a path length interval, ΔL_o, and the pressure p, as follows:

$$T_{\Delta\nu}(\nu) = f\left[k_\nu \, \Delta L_o \, p^n\right] \tag{11}$$

From laboratory transmittance data and from line by line transmittance calculations degraded to 20 cm^{-1} resolution, McClatchey derives empirically the function f. For CH_4, he finds n = 0.75.

The empirical transmittance function is represented in McClatchey et al. (1970) by a graph relating transmittance to wavelength for a path length of one kilometer of air at standard temperature and pressure (STP). The transmittance scale also has a scaling factor which permits reading of transmittances for other path lengths. From McClatchey's empirical transmittance function, we have derived an analytical representation of the CH_4 transmittance as follows.

A vertical increment of pressure corrected path length of CH_4 can be written as

$$dL = \left(\frac{n_{CH_4}}{n}\right) \frac{n}{n_{STP}} \left(\frac{p}{p_{STP}}\right)^{.75} dz \tag{12}$$

where n is the atmospheric number density, and n_{CH_4} is the CH_4 number density.

Letting $c = (n_{CH_4}/n)$, and assuming a constant CH_4 mixing ratio with altitude, we obtain for the pressure corrected CH_4 path length of the atmosphere above height z

$$L = \frac{c}{n_{STP}\, p_{STP}^{\,.75}} \int_z^\infty n\, p^{.75}\, dz \tag{13}$$

With the help of the hydrostatic equation in the form

$$dp = -n \mu g \, dz \tag{14}$$

where μ is the mean molecular mass of the atmosphere, we obtain

$$L = \frac{c}{n_{STP} \, p_{STP}^{.75} \, \mu g} \int_0^p p^{.75} \, dp \tag{15}$$

Letting $A = n_{STP} \, p_{STP}^{.75} \, \mu g$, we obtain, upon integration,

$$L = \frac{c}{1.75A} \, p^{1.75} \tag{16}$$

We shall assume that the transmission follows the law

$$T_\nu = \exp\left[-(k_\nu L)^m\right] \tag{17}$$

where k_ν is an absorption coefficient for the interval $\Delta\nu$ centered at ν, and m is to be determined from the empirical transmittance relationship of McClatchey et al. This form is similar to the strong line approximation of the Goody random model, except that in the strong line approximation the pressure dependence is p^1, not $p^{.75}$, and m = 0.5.

By plotting $\ell n(-\ell n \, T)$ versus $\ell n \, X$, where X is kilometers of air at STP, from McClatchey's empirical transmittance scale, and measuring the slope of the resulting plot we obtain the value of m = 0.56. Values of k_ν are determined from the graph presented in McClatchey et al. These values are shown in Table 1.

TABLE 1

ABSORPTION COEFFICIENTS FOR METHANE FOR USE IN EMPIRICAL TRANSMISSION FORMULA $T_\nu = \exp\left[-(k_\nu L)^{.56}\right]$

$\nu(\mathrm{cm}^{-1})$	$\lambda(\mu)$	$k_\nu(\mathrm{cm_{STP}}^{-1})$
1140	8.77	3.1×10^{-4}
1160	8.62	5.4×10^{-3}
1180	8.47	5.4×10^{-3}
1200	8.33	6.2×10^{-3}
1220	8.20	1.7×10^{-2}
1240	8.06	1.1×10^{-1}
1260	7.94	6.2×10^{-1}
1280	7.81	9.4×10^{-1}
1300	7.69	1.4
1320	7.58	2.8×10^{-1}
1340	7.46	3.1×10^{-1}
1360	7.35	2.4×10^{-1}
1380	7.25	1.7×10^{-3}

To obtain the weighting function, we differentiate (17) with respect to height, making use of (12) and the ideal gas law,

$$\frac{dT}{dz} = \frac{.56(k_\nu L)^{-.44}\ c\ p^{1.75}}{n_{STP}\ p_{STP}{}^{.75}\ k\ \theta} \exp\left[-(k_\nu L)^{.56}\right] \tag{18}$$

where k is Boltzmann's constant, and θ is temperature.

Using (18) we have computed the weighting functions for the Jovian atmosphere. The following assumptions are made:

$c = n_{CH_4}/n = 8.3 \times 10^{-4}$ (after Hogan et al., 1969)

$\theta = 168^\circ K$

$\overline{m} = 3.26$

$\theta/\overline{m} = 51.5$

Here θ is the mean temperature of the atmosphere above the Jovian cloud layer. Together, with the mean molecular weight, it determines the scale height, 16.5 km. The cloud-top pressure is assumed to be either 2 atm or 4 atm and the height scale is referenced to the cloud-top.

The computed weighting functions are shown in Figure 1. The large vertical range in the peaks of the weighting functions suggests that temperature information can be obtained for a layer of the atmosphere about 100 km thick above the clouds. However, overlap of absorption due to other bands (for example, NH_3 at $\lambda > 8.3\mu$) may cut down this range in practice. Also, noteworthy are the relatively large widths

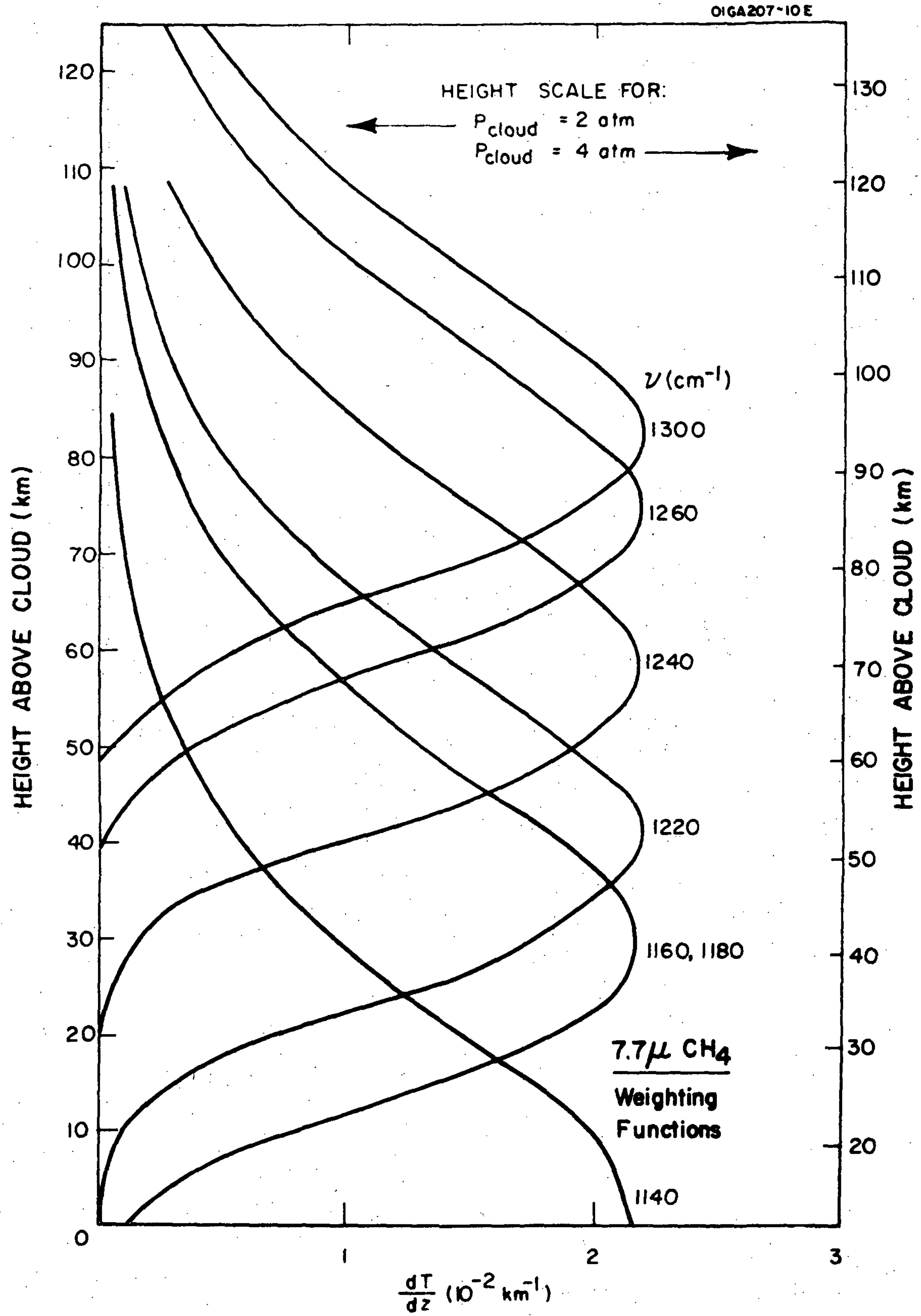

Figure 1. Weighting functions in the Jovian atmosphere for selected wavenumbers of the 7.7μ CH_4 absorption band.

of the weighting functions, of the order of 40 km at $\frac{dT}{dz}$ equal to one-half of peak value. This is due to the relatively large scale height of the Jovian atmosphere, and implies that observations in the 7.7μ CH_4 band will provide relatively low vertical resolution information on the temperature profile.

2.2.2 <u>10.5μ NH_3 Band</u>

For this band, we follow Gille and Lee (1969), who calculated the positions, half-widths and intensities of absorption lines due to ammonia at $\nu < 1400\ cm^{-1}$, and used this information to calculate the parameters of the Goody random absorption model with exponential distribution of intensities. For such a band, the transmittance can be written as

$$T = \exp\left[-2\pi\ yu\ (2\ u + 1)^{-1/2}\right] \tag{19}$$

where

$$u = \frac{a}{8\ P_e\ f(\theta,\theta_r)} \left[\frac{\sum_{i=1}^{N} S_i\ (\theta)}{\sum_{i=1}^{N} \left[S_i(\theta)\ \alpha_{o_i}\ (\theta_r) \right]^{1/2}} \right]^2 \tag{20}$$

$$y = \frac{4\ P_e\ f(\theta,\theta_r)}{\pi\ \Delta\ \nu} \quad \frac{\sum_{i=1}^{N} \left[S_i(\theta)\ \alpha_{o_i}\ (\theta_r) \right]^{1/2}}{\sum_{i=1}^{N} S_i\ (\theta)} \tag{21}$$

where a is the absorber amount (cm STP), P_e is the effective pressure for line broadening (atm), and N is the number of lines in the spectral interval of width $\Delta\nu$. It is assumed that the half width of the ith line at temperature θ is given by

$$\alpha_i(\theta) = \alpha_{o_i}(\theta_r)\, f(\theta,\theta_r)\, P_e \tag{22}$$

where the subscript o refers to a pressure of one atmosphere of NH_3, and $f(\theta,\theta_r)$ is a temperature correction factor.

If we let

$$\sigma \equiv \sum_{i=1}^{N} S_i(\theta) \tag{23}$$

and

$$\beta \equiv \sum_{i=1}^{N} \left(S_i(\theta)\, \alpha_{o_i}\right)^{1/2} \tag{24}$$

we can write (19) as

$$T = \exp\left\{-\frac{\sigma a}{\Delta\nu}\left[\frac{a}{4}\,\frac{1}{P_e\, f(\theta,\theta_r)}\,\frac{\sigma}{\beta}^{2} + 1\right]\right\}^{-1/2} \tag{25}$$

Based upon measurements of NH_3 lines in the microwave region, Gille and Lee assume that

$$P_e = P_{NH_3} + P_{H_2} / 7.9 + P_{H_e} / 11.7 \tag{26}$$

For the temperature dependence, they adopt

$$f(\theta,\theta_r) = \left(\frac{\theta_r}{\theta}\right)^{5/6} \tag{27}$$

for hydrogen broadening.

Based upon their calculations, Gille and Lee tabulate the values of σ and β as a function of wavenumber from 12.50 cm^{-1} to 1387.50 cm^{-1} in 25 cm^{-1} intervals, for temperatures between 100^oK and 225^oK at 25^oK intervals. We reproduce here as Table 2, the portion of their tabulation of application to the present study.

We may use the strong line approximation of Equation (25)

$$T = \exp\left\{-2\ \Delta\nu^{-1}\ \beta\left[a\ P_e\ f(\theta,\ \theta_r)\right]^{1/2}\right\} \tag{28}$$

if

$$R \equiv \frac{a}{4}\ \frac{1}{P_e\ f(\theta,\theta_r)}\left(\frac{\sigma}{\beta}\right)^2 >> 1 \tag{29}$$

For the Jovian atmosphere, we have

$a \cong 700$ cm STP of NH_3

$P_e \cong P_{atm}/10$ (for a predominantly H_2, H_e atmosphere)

$\cong 0.2$ atm near the cloud level

$f(\theta,\theta_r) \cong \frac{300}{150}^{5/6} \cong 2$

at $\nu \cong 1000\ cm^{-1}$, $\sigma = 10^2$, $\beta = 10$

For these values, we obtain

$$R = 4.4 \times 10^4 >> 1$$

TABLE 2

VALUES OF σ AND β FOR THE 10.5μ BAND OF NH_3

($T = 150^{o}K$, $\Delta\nu = 25\ cm^{-1}$) (After Gille and Lee, 1969)

ν (cm^{-1})	μ (microns)	σ ($atm^{-1}\ cm^{-2}$)	β ($atm^{-1/2}\ cm^{-3/2}$)*
837.5	11.9	3.03×10	1.33×10
862.5	11.6	8.68×10	1.81×10
887.5	11.3	7.03×10	1.12×10
912.5	11.0	5.47×10	1.20×10
937.5	10.7	3.25×10^2	5.91×10
962.5	10.4	3.55×10^2	6.62×10
987.5	10.1	9.54×10	9.74
1012.5	9.88	1.37×10^2	1.64×10
1037.5	9.64	2.34×10^2	3.19×10
1062.5	9.41	9.93×10	2.14×10

*There appears to be an error in Gille and Lee's (1969) Table 2b, where the units of β are given as $atm^{-1}\ cm^{-3/2}$.

Hence, we may use the strong line approximation of the Goody random model for the 10.5μ NH_3 band in applications to the Jovian atmosphere. With the substitutions $\Delta\nu = 25$ cm^{-1}, $P_e = r\overline{P}$, where $\overline{P} = \overline{P}_{atm}$, the average pressure along the absorbing path, and $[f(\theta,\theta_r)]^{1/2} \cong 1.4$, Equation (28) becomes

$$T = \exp\left[-.112\ \beta\ (a\ r\ \overline{P})\right]^{1/2} \tag{30}$$

The amount of absorber and effective pressure of a vertical path of NH_3 can be written as

$$a = \frac{\int_z^\infty n_{NH_3}\ dz}{n_{STP}} \tag{31}$$

and

$$\overline{P} = \frac{\int_z^\infty P\ da}{a(z)} \tag{32}$$

Thus,

$$\begin{aligned} a\overline{P} &= \int_z^\infty P\ da \\ &= \frac{1}{n_{STP}} \int_z^\infty P\ n_{NH_3}\ dz \end{aligned} \tag{33}$$

Assuming a scale height H for the distribution of pressure above the cloud-top (c), and a different scale height K for the distribution of NH_3 above the cloud-top (since the NH_3 is probably saturated in at least part of the atmosphere above the major cloud level, and, hence, its number density is an exponential function of the temperature, with a different scale height than that of the atmosphere), we obtain

$$a\overline{P} = \frac{P(c)\ n_{NH_3}(c)}{n_{STP}} \int_z^\infty e^{-z/H}\ e^{-z/K}\ dz \tag{34}$$

If Equation (34) is substituted into the formula for the transmittance, eq. (30), we obtain

$$T = \exp\left\{-A\left[\int_z^\infty \exp(-z/D)\ dz\right]^{1/2}\right. \tag{35}$$

where

$$A = 0.112\ \beta\left[r\ P(c)\ n_{NH_3}(c)/n_{STP}\right]^{1/2} \tag{36}$$

and

$$\frac{1}{D} = \frac{1}{K} + \frac{1}{H} \tag{37}$$

Upon integration of (35), we obtain

$$T(z) = \exp\left[-AD^{1/2}\exp(-z/2D)\right] \tag{38}$$

from which the weighting function is derived as

$$\frac{dT}{dz} = 0.5\ AD^{-1/2}\ e^{-z/2D}\exp\left[-AD^{1/2}\ e^{-z/2D}\right] \tag{39}$$

For our preliminary calculation of the NH_3 weighting functions in the Jovian atmosphere, we have assumed the following values for the Jovian atmospheric parameters:

$n_{NH_3}(c) = 1.1 \times 10^{17}$

$a = 700$ cm STP

From above values, $K = 1.72$ km

$H = 16.5$ km

$\theta/m = 51.5$

$r = 0.1$

$P(c) = 2$ atm and 4 atm

Weighting functions for three wavenumbers in the 10.5μ NH_3 band are plotted in Figure 2. In comparison to the CH_4 weighting functions, the NH_3 weighting functions are much narrower and cover a smaller altitude range. This stems from the fact that due to condensation the NH_3 scale height is much less than the CH_4 scale height.

It is of interest to see what the NH_3 weighting functions would look like if the NH_3 were uniformly mixed with altitude with the same scale height as CH_4. In such a case, equation (33) becomes

$$a\,\overline{P} = \frac{w}{n_{STP}} \int_z^{\infty} P\,n\,dz \tag{40}$$

where w is the NH_3 mixing ratio equal to n_{NH_3}/n, where n is the Jovian atmospheric number density. With the use of the hydrostatic equation, and the relationship

$$dp = 10^6\,dP \tag{41}$$

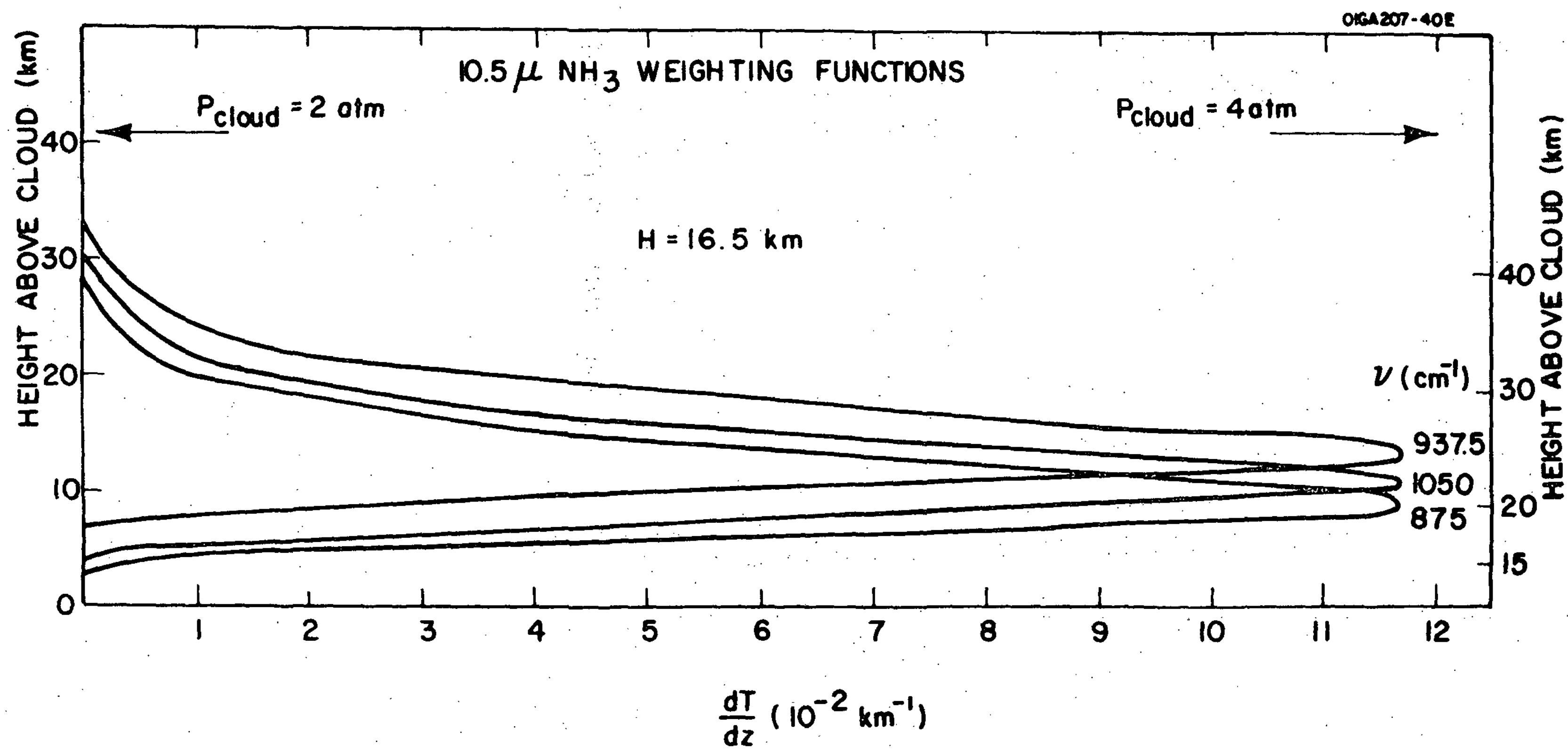

Figure 2. Weighting functions in the Jovian atmosphere for selected wave-numbers of the 10.5μ NH_3 absorption band.

where p is atmospheric pressure in cgs units and P is atmospheric pressure in atmospheres, we obtain

$$a\bar{P} = \frac{10^6 w}{n_{STP}\ \mu g} \int_0^{P(z)} P\ dP \tag{42}$$

Integrating Equation (42), and substituting the result into Equation (30) for T, we obtain

$$T = \exp\left[-0.112 \times 10^3\ \beta\ P\left(\frac{rw}{2n_{STP}\ \mu g}\right)^{1/2}\right] \tag{43}$$

Letting the optical depth

$$\tau = 0.112 \times 10^3\ \beta\ P\left(\frac{rw}{2n_{STP}\ \mu g}\right)^{1/2} \tag{44}$$

differentiating (43) with respect to height, and making use of the equation of state, we obtain the weighting function

$$\frac{dT}{dz} = \frac{0.112 \times 10^3\ \beta\ P}{kT}\left(\frac{rw\ \mu g}{2n_{STP}}\right)^{1/2} e^{-\tau} \tag{45}$$

In evaluating (45), we use a value of w compatible with the same total amount, (a), of NH_3 as used in the previous calculation, that is, 700 cm STP. This value of w may be obtained from

$$a = \frac{1}{n_{STP}} \int_z^{\infty} n_{NH_3}\ dz$$

$$= \frac{w}{n_{STP}} \int_z^{\infty} n\ dz \tag{46}$$

With the use of the hydrostatic equation, (46) becomes

$$a = \frac{w}{n_{STP}\ \mu g} \int_0^p dP = \frac{wp}{n_{STP}\ \mu g} \tag{47}$$

From which,

$$w = \frac{a\ n_{STP}\ \mu g}{p} \tag{48}$$

Taking p equal to 2 atm, we obtain $w = 1.32 \times 10^{-4}$.

The weighting function for the case of constant NH_3 mixing ratio is compared to the weighting function for the case of decreasing NH_3 mixing ratio in Figure 3. For the case of constant NH_3 mixing ratio, the weighting function peaks at a much greater altitude (70 km versus 13 km), is much broader, and has a much lower peak value (2×10^{-2} km^{-1} versus 12×10^{-2} km^{-1}). It is quite obvious that the region of the atmosphere to which 10.5μ emission observations refer depends critically on whether or not NH_3 does indeed condense out and, hence, has a smaller scale height than the major atmospheric constituents. If, as is probably the case, NH_3 does condense and follow a distribution with a small scale height, the narrow weighting functions that result will permit good vertical resolution in the deduction of temperatures from emission observations.

2.2.3 Pressure Induced H_2 Absorption

For the pressure induced absorption of hydrogen, which, at Jovian atmospheric temperatures, peaks between 400 cm^{-1} and 800 cm^{-1},

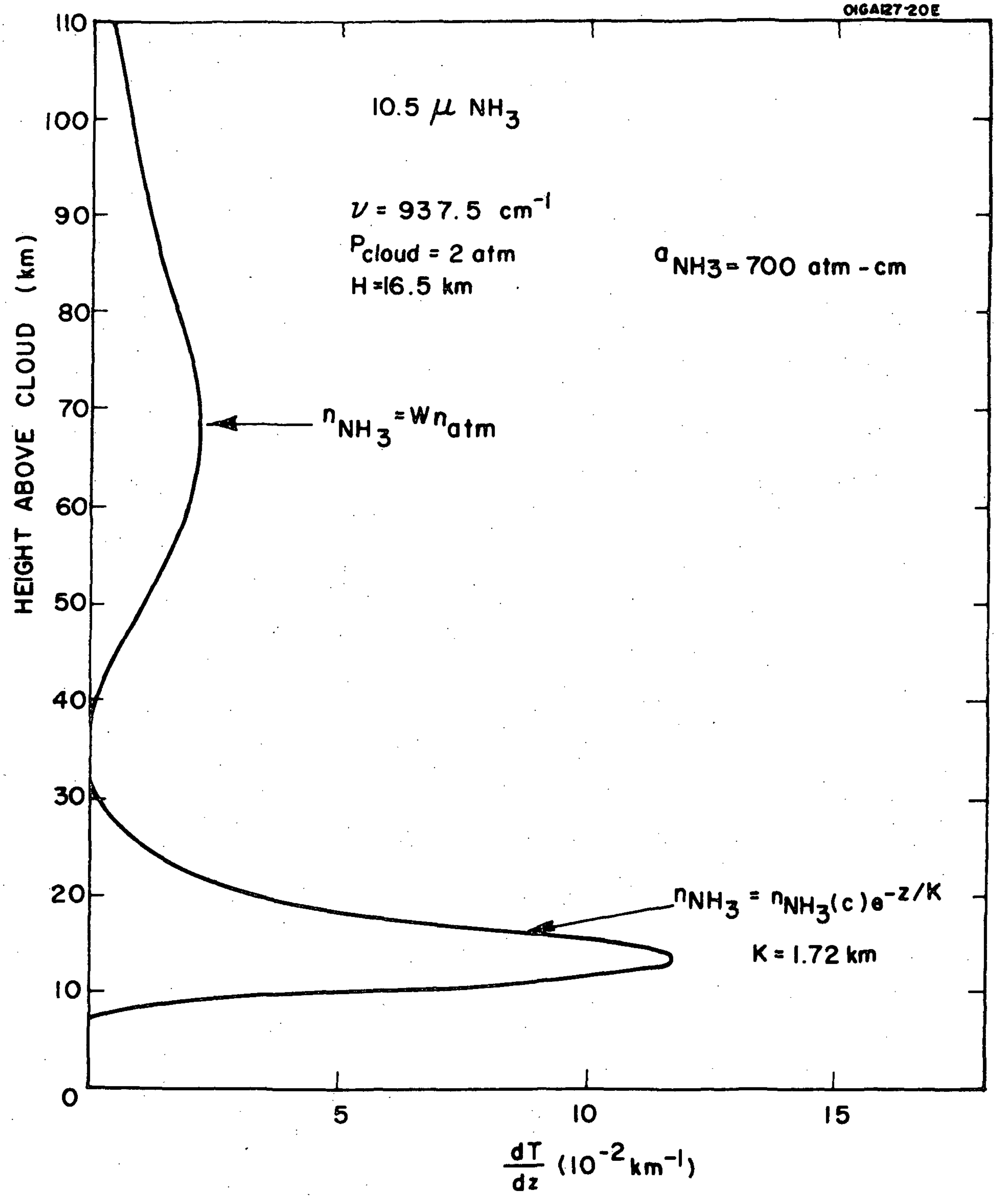

Figure 3. A comparison of weighting functions for NH_3 for two different assumptions on the distribution of NH_3 with altitude.

we follow the work of Trafton (1967). The volume absorption coefficient of H_2 at wavenumber ν in a hydrogen-helium mixture can be written as

$$k_{v,\nu} = \frac{{n_{H_2}}^2}{c} \left(A_\nu + q\ B_\nu\right) \tag{49}$$

where the units of $k_{v,\nu}$ are cm^{-1}, n_{H_2} is the H_2 number density, c is the speed of light, q is the number density ratio of helium to hydrogen, and A_ν and B_ν are the pressure-induced binary absorption coefficients corresponding to absorption in pure H_2 and the enhancement in He-H_2 mixtures, respectively.

The transmittance may be written as

$$T = \exp\left[-\int_z^\infty k_v\ dz\right] \tag{50}$$

where we have omitted the subscript ν for convenience.

Substituting the expression for $k_{v,\nu}$ into Equation (50), we obtain

$$T = \exp\left[-\frac{1}{c}\int_z^\infty (A + q\ B)\ {n_{H_2}}^2\ dz\right] \tag{51}$$

A and B vary womewhat with temperature, and hence, would vary with height. However, for an isothermal atmosphere, they are independent of height and Equation (51) becomes

$$T = \exp\left[- \frac{1(A + q\ B\ )}{c} \int_{z}^{\infty} n_{H_2}{}^2 \right] dz \qquad (52)$$

The number density of H_2 above the clouds can be written as

$$n_{H_2} = n_{H_2}(c) \exp(-z/H) \qquad (53)$$

Integrating Equation (52) with the help of (53), we obtain

$$T = \exp\left[- \frac{(A + q\ B)\ n_{H_2}(c)\ H \exp(-2z/H)}{2c} \right] \qquad (54)$$

From Equation (54), the weighting function can be derived as

$$\frac{dT}{dz} = \frac{(A + q\ B)}{c}\ n_{H_2}{}^2\ (c) \exp(-2z/H)\ e^{-\tau} \qquad (55)$$

where

$$\tau = \frac{(A + q\ B)}{2c}\ n_{H_2}{}^2\ (c)\ H \exp(-2z/H) \qquad (56)$$

We have computed the H_2 weighting functions for the following values of the parameters.

A and B obtained from graphical representation of Trafton (1967) for $\theta = 160^{\circ}$K.

$q = 0.5$

$H = 16.5$ km

$\theta/m = 51.5$

$P_{cloud} = 2$ atm and 4 atm

Shown in Figure 4 are the weighting functions for H_2 for $\nu = 700\ cm^{-1}$ and $\nu = 800\ cm^{-1}$. These wavenumbers are of interest in the analysis of the Jovian infrared spectral observations of Gillett et al. (1969), which cover this region of the H_2 absorption band. The large overlap of the two weighting functions is due to the fact that the absorption coefficient does not vary greatly in this spectral range.

2.2.4 NH_3 Microwave Absorption

For the microwave spectrum of NH_3 there appears to be no simple analytical absorption model from which the weighting functions can be derived. However, information on absorption coefficients and half-widths for individual lines, and pressure broadening, is available in the literature (Townes and Schawlow, 1955; Anderson, 1950; Birnbaum et al., 1953; Legan et al., 1965; and Wrixon et al., 1971). To derive the transmittance at a particular wavenumber from this information requires an integration of the contributions from all the lines that influence the absorption at that wavenumber. This is best done by an electronic computer and we plan to program such computations during the next quarterly period.

2.2.5 Preliminary Estimate of Jovian Temperature Profile

Assuming that the observed brightness temperature at a particular wavenumber represents the temperature at the altitude at which the weighting function for that wavenumber peaks, we can estimate the temperature profile in the Jovian atmosphere from the Jovian infrared

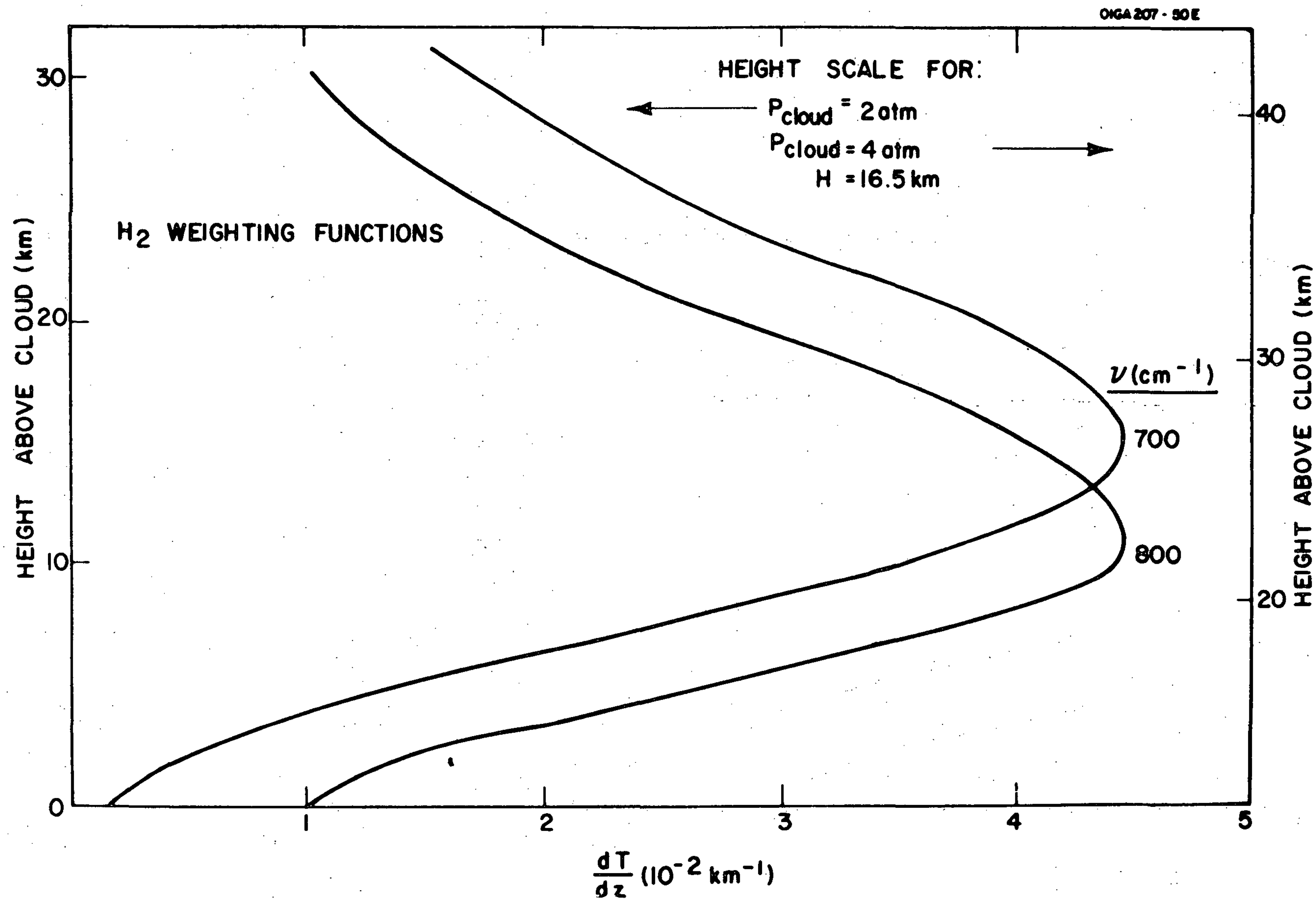

Figure 4. Weighting functions in the Jovian atmosphere for selected wavenumbers of the pressure induced H_2 absorption band.

spectrum (resolution $\Delta\lambda/\lambda = 0.02$, or $\sim 20\ cm^{-1}$ at 8μ) observed by Gillett et al. (1969). We use the weighting functions computed above for CH_4, NH_3, and H_2. Brightness temperatures were read from Hogan et al.'s (1969) plot of Gillett and Low's data. Table 3 lists the observed brightness temperatures and the altitudes to which they refer for selected wavenumbers of the three absorption bands. Two different height scales are listed: one relative to a cloud at $p = 2$ atm and one relative to a cloud at $p = 4$ atm. For both height scales, a scale height of 16.5 km is assumed.

Based upon these temperatures, a preliminary temperature profile for the Jovian atmosphere is plotted in Figure 5. The cloud temperature of $210^{\circ}K$ is based upon the results of Owen (1965; $T_{cloud} = 200 \pm 25^{\circ}K$); Danielson (1966; $T_{cloud} = 210 \pm 15^{\circ}K$; and Gillett et al. (1969; $T_{cloud} \cong 220^{\circ}K$). A cloud top pressure of 4 atm is assumed in this plot. This yields a lapse-rate of in the first 20 km above the cloud $\sim 4^{\circ}C/km$. The adiabatic lapse-rate for a 0.5 H_2 and 0.5 He atmosphere is $3.3^{\circ}C/km$; for a 5/6 H_2 and 1/6 He atmosphere, $2.4^{\circ}C/km$. Had we assumed a cloud pressure of 2 atm, we would have obtained a lapse rate of $\sim 9^{\circ}C/km$ in the first atm, far greater than the probable adiabatic lapse rate on Jupiter. Thus, the inferred temperatures appear to be consistent with a thick cloud at a pressure or 4 atm or more.

The deduced Jovian temperature profile is similar in shape to the mean temperature profile in the earth's atmosphere. That is, a

TABLE 3

OBSERVED JOVIAN BRIGHTNESS TEMPERATURES (AFTER GILLETT et al. 1969) AND THE ALTITUDES TO WHICH THEY REFER FOR SELECTED WAVENUMBERS IN THE CH_4, NH_3, AND H_2 INFRARED ABSORPTION BANDS

Gas	ν (cm^{-1})	T_B (^oK)	z(km) for: P_{cloud} = 4 atm	P_{cloud} = 2 atm
CH_4	1180	140	41	30
	1200	138	41	30
	1225	137	51	40
	1250	140	76	65
	1300	148	91	80
NH_3	875	127	20	9
	1050	125	22	11
H_2	800	126	22	11
	700	118	26	15

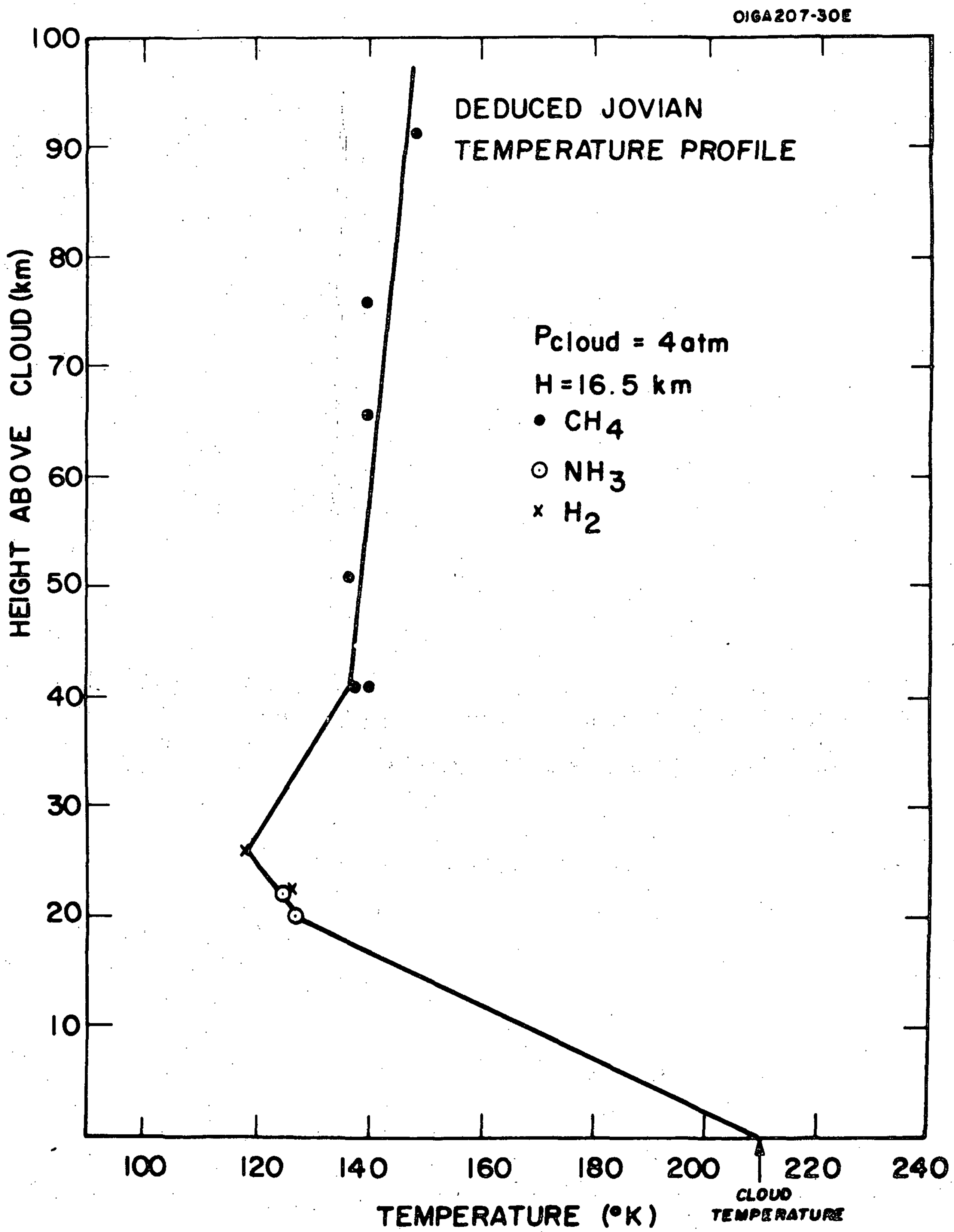

Figure 5. Preliminary estimate of temperature profile in the Jovian atmosphere from analysis of infrared emission observations.

tropospheric layer, in which the temperature decreases with altitude and a stratospheric layer in which the temperature increases with altitude. The presence of such a Jovian stratospheric layer in which the temperature increases with altitude was first suggested by Gillett et al. (1969), on the basis of a qualitative interpretation of their Jovian spectral observations. Theoretical calculations of the Jovian temperature profile with a radiative-convective equilibrium model (Hogan et al., 1969) indicate that the observed stratospheric structure is due to absorption of solar infrared energy by the 3020 cm^{-1} band of methane, a suggestion made by Gillett et al. (1969).

3. FUTURE PLANS

Plans for the next quarterly period include the following:

(1) Programming of the computations of the NH_3 microwave weighting functions for the Jovian atmosphere.

(2) Further analysis of the Jovian temperature profile, utilizing available observations.

(3) Commencement of development of inversion techniques for application to planetary atmospheres emission observations.

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