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A SUBORBITAL ABORT GUIDANCE SCHEME FOR ORBITER RETURNS TO THE LAUNCH SITE

Flight Analysis Branch

MISSION PLANNING AND ANALYSIS DIVISION

MANNED SPACECRAFT CENTER
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By W. R. Lacy

1.0 SUMMARY

A suborbital abort guidance scheme is presented for returning the orbiter vehicle to the launch site in the event of a booster malfunction. The guidance scheme described, referred to as FBGUID, represents a simple closed-form technique to the flyback abort problem. In addition to returning to the launch site, objectives of the formulation are to preclude excessive structural and heating loads. A demonstration of the feasibility of this abort guidance concept is included for a 150-second abort time.

2.0 INTRODUCTION

The flyback abort technique involves performing a combined lifting and thrusting maneuver to achieve a 180° reorientation of the thrust vector and a return to the launch site. FBGUID provides a simple closed-form guidance scheme for achieving this. The formulation for the bank angle commands was based on the return to launch site guidance concept presented in reference 1; however, several modifications were made to improve the possibility of not violating heating and g-loading constraints. First, a pitch-up maneuver was initiated at apogee and the angle of attack was modulated during descent to soften the initial plunge of the orbiter into the atmosphere; and second, the limiting value for bank commands and thrust level was adjusted to attempt to satisfy normal load constraints during the turn phase. An overall view of how the flyback scheme might fit into the total suborbital abort guidance plan is illustrated in figure 1.

The FBGUID program is a three-degree-of-freedom point mass digital computer program. The basic computational structure of FBGUID, the Modular Atmosphere Simulation Program (MASP) (ref. 2), is modular in design, thus providing flexibility to easily incorporate guidance schemes. A fourth-order Runge-Kutta integration technique is used to solve the equations of motion. The computation of position and velocity derivatives is performed in an earth-relative coordinate system. The provision for forces acting on the vehicle, other than gravity, are aerodynamic forces through tabular input data and thrust forces computed on the basis of mass flow rate and specific impulse.

The normal force constraint used in the study was 954 000 pounds, approximately 3.0g times the empty orbiter weight. Thrust performance guidelines under which the FBGUID technique was designed include the following.

a. The engine throttle capability varies from 50 to 100 percent.

b. Main engine thrusting is continuous through propellant depletion.

c. Thrust initiation starts within 10 seconds after abort staging.

d. Total propellant depletion occurs prior to landing.

The North American high cross-range delta wing orbiter configuration and engine characteristics used in this flyback study are presented in figure 2. The results presented for an abort at 150 seconds were generated using FBGUID, a modified version of an existing program, MASP, developed for the Flight Analysis Branch (ref. 2). The program was run on the Comshare time sharing computer, SDS-940.

3.0 DISCUSSION

FBGUID is composed of three computational phases. The first phase, the preturn trajectory conditioning phase, covers the time period from abort initiation up to initiation of the first bank angle command. This phase terminates shortly after the orbiter plunge into the atmosphere has been recessed and the altitude rate starts increasing. The second phase, the 180° turning maneuver phase, is nominally completed prior to propellant depletion. The turning angle logic is designed to compensate for range requirements following completion of the turn. The third phase, the post-turn thrust and glide phase, involves burning off any propellant remaining after the desired target azimuth has been achieved and using maximum lift-to-drag (L/D) angle of attack control for the terminal glide phase.

A view of the turning angle geometry and the three abort phases is presented in figure 3.

3.1 Preturn Trajectory Conditioning Phase

The first phase of FBGUID is a trajectory conditioning phase. Since the orbiter main engine ignition occurs within 10 seconds after staging, the thrust level up to the time of apogee is an important factor. It must be adequate for getting the fully loaded orbiter out of the maximum dynamic pressure regions on early aborts and limited such that on exo-atmospheric aborts the resulting down-range position does not exceed the orbiter return capability. Fifty percent thrust of one main engine was used for aborts after 125 seconds to conserve propellant for the turn maneuver. Zero degree angle of attack α was commanded for this portion of flight.

At apogee, α is increased at a rate of 2 deg/sec for the pullout maneuver. During the pullout maneuver α is modulated at 1 deg/sec to compensate for the increased normal force. Also, once the orbiter has encountered a 0.5g load the thrust level is increased to a value between 80 percent and 100 percent of one engine. Once the altitude rate becomes positive, α is commanded to the desired attitude for initiation of the turn maneuver and the thrust level is adjusted. Alpha was limited to a maximum value of 45° during this portion of the trajectory to prevent too drastic an altitude drop during the early segment of the turn maneuver.

3.2 180° Turning Maneuver Phase

The bank angle control logic is designed to modulate the direction of lift and thrust vectors by rolling the orbiter vehicle about the relative velocity vector V_E. The bank control law attempts to balance the effects of drag and thrust acceleration and result in a constant altitude turn. A constant thrust level is maintained throughout the turn. Normal force limiting logic was also included in the bank angle computation scheme.

The bank angle computations are based on the control law

$$\text{BANK} = \text{BANKR} + \text{C1} * (\text{VI} - \text{VREF}) + \text{C2}\ \dot{R}$$

where the calculation of the reference velocity (VREF) is based on a constant radius, constant centrifugal force turn. VREF is computed by

$$\text{VREF} = (\text{TGO} - \text{TMAR}) * \text{F1}/\text{THETA}$$

TMAR = time margin constant for post-turn ranging requirement, sec

F1 = constant centrifugal acceleration, ft/sec^2

THETA = total turn angle required to orient the relative velocity vector toward the desired target azimuth, rad

The value of TMAR varies as a function of abort time since for early aborts less range to target requirements exist after completing the turn and more for later aborts. The study indicated that large values of TMAR ($\geq$50 sec) yield large bank angle ϕ_B commands during the early turn phase, thus increasing the normal forces. This is especially true at later abort times.

In the ϕ_B calculation, C1 is a gain constant to compensate for differences between the current and desired inertial velocity; C2 $\dot{R}$ represents a damping term for altitude changes. The assumption of a constant centrifugal acceleration is an oversimplification of facts as explained in reference 1, but does provide a workable solution. Since the bank angle commands are based on a finite radius turn, the calculation of THETA θ must include an additional correction angle ALPT A_T incurred during the turn. The turn angle geometry for θ was presented earlier in figure 3. The derivation of θ and A_T is included in the appendix.

The ϕ_B commands are limited in accordance with the normal force constraint, FNMAX = 954 000 pounds, by the following.

a. ϕ_B limit = 80° if:

actual NFORCE $\leqq$ (954 000 - 0.3 * current orbiter weight)

b. ϕ_B limit = 55° if:

actual NFORCE $\geqq$ (954 000 - 0.3 * current orbiter weight)

FNMAX is severely tested for late abort times during the early turn phase because of the orbiter weight and the thrust level required to stabilize altitude. The initial angle of attack is maintained constant through the time when maximum normal force is achieved. Results indicate this will occur within the first 125 seconds of the turn for aborts up through the time of nominal staging (214 sec). Once the point of maximum normal force has been passed, α is commanded to 30° for the remainder of the turn. A certain amount of altitude loss occurs during the early turn phase because of the increased angle of attack, but this is required to alleviate the normal forces. The orbiter altitude stabilizes after the maximum normal force region is passed.

The turn phase is terminated once the relative velocity vector has reached a direction colinear with the desired launch site target azimuth.

3.3 Post-Turn Thrust and Glide Phase

As previously stated, the amount of burn time available following completion of the turn is a function of the time-margin constant TMAR. It follows for late aborts that the burn time requirement is proportional to the down-range distance. The final computational phase is designed for powered or unpowered flight along the target azimuth. Maximum lift trajectories were flown during the glide phase. A successful orbiter return trajectory was defined at 60 000 feet as follows: the range to target $\leq$ 80 miles; the Mach number $\cong$ 1.0; and the flight-path angle $\cong$ 0.0°. During the glide phase, any residual difference between the desired and actual heading azimuth was trimmed out using bank commands limited to $\pm 8°$.

4.0 INITIALIZATION DATA

The orbiter state vectors at various abort initiation times were generated by the Flight Analysis Branch GEMASS program. These data for initializing the 150-second flyback abort case are presented in table I. Guidance constants incorporated into FBGUID for computing bank angle commands are included in table I. All the flyback cases investigated in this study were aborted from a 55° inclination launch and targeted for the Kennedy Space Center (KSC), Florida launch site.

The aerodynamic data coefficients are presented in table II. These data reflect trimmed lift and drag values for a 6650-ft^2 area (ref. 3).

5.0 RESULTS FOR A 150-SECOND ABORT

Table III presents the flyback sequence of events for returning to KSC following an abort at 150 seconds. This set of events represents the standard sequence used in FBGUID to solve the flyback problem during this study phase.

During the preturn trajectory conditioning phase, an 85 percent thrust level of one main engine was used for the pullout maneuver. This thrust level, coupled with α control, was adequate to decrease altitude rate without excessive buildup of g-loads and down-range distance. An attempt was made to minimize propellant consumption prior to the turning phase. The resulting trajectory prior to turn initiation indicated a maximum normal force of 970 962 pounds, approximately 1.8 percent in excess of the NFORCE constraint. The total g-load and weight at this point (310.0 sec) were 1.614g and 686 948 pounds, respectively.

A constant 70 percent thrust level was used during the turning maneuver phase. This provided adequate thrust to maintain altitude above 100 000 feet and to compensate for the slightly increased α (38°) flown during the initial portion of the turn. A range of α values, 45° to 23°, was examined for the early turn phase up to the time of maximum normal force encounter. The results varied from rapid decreases in altitude to excessive buildup in normal force and down-range distance.

Attempts to hold the normal force below the NFORCE constraint were not successful during the critical early turn phase. The maximum normal force encountered was 1 168 111 pounds at 436 seconds; orbiter weight and total g-load indicated at this time were 559 304 pounds and 2.305g, respectively. Bank commands were limited to 55° during this time period in an attempt to hold normal forces down and at the same time provide enough turn command to rotate the velocity vector. Down-range distance builds up rapidly during this time when the Mach number is approximately 5.0. The desired target azimuth changed from 219° initially to 253° at turn completion as a result of the finite radius turn.

The final thrust and glide phase was terminated at an altitude of 11 931 feet and a range to target of 4.94 n. mi. Normal forces were well within the limit during this final phase.

Figure 4 presents trajectory results for the 150-second flyback sequence. Figure 5 represents the aerodynamic loading and heating results. The total g-loads shown in figure 5(a) satisfies a 3.0g total load constraint; however, as shown in figures 5(b), 5(c), and 5(d), the normal loads and forces violated normal constraints. The shaded portions of these curves represent the regions where the 954 000-pound constraint was exceeded.

Figures 6 and 7 represent time histories of guidance parameters computed during the turn angle phase. A comparison of figures 6(a) and 6(b) illustrates the fairly constant decrease in turn angle parameters as the turn is completed following the initial iteration period. The computed bank angle commands and the unlimited guidance commands are presented in figures 6(e) and 7. A comparison of the two figures indicates the normal force limiting logic was in effect from 406 seconds to 490 seconds. This covers a time period starting 30 seconds prior to the maximum normal force trajectory point.

6.0 CONCLUSIONS

This initial guidance study phase of the flyback abort mode has been directed toward developing a closed-form solution to the flyback problem while satisfying normal vehicle design constraints established for the space shuttle by Phase B contractors. Results presented for the 150-second abort case indicate the feasibility of this approach; however, certain planned improvements in trim and throttle control may better adapt the scheme to the overall flyback abort problem. Studies have indicated improvements to the scheme may be made through the bank angle control law by adjusting the gain constants TMAR, C1, and C2. Future efforts will investigate the requirement for variable gains as a function of time of abort.

Studies performed to date using FBGUID indicate reasonable success at early abort times but no acceptable results have been achieved for aborts near nominal staging time (214 sec). In the later abort cases, the down-range distance back to the launch site following completion of the turn has exceeded the return capability of the orbiter main engines. Future studies will include the use of any other thrusting capability available during the final phase, such as translational attitude control thrust.

TABLE I.- INITIALIZATION DATA FOR 150-SECOND FLYBACK ABORT CASE

Initial state vector* - earth relative coordinates	
Time	150.0 sec
CLAT	28.94054 deg
CLONG	-80.18531 deg
ALT	149 479.3 ft
V_E	5 332.8 fps
GAME	18.98769 deg
AZCUR	38.51452 deg
WGHT	823 813. lb
WFUEL	505 943. lb
AREA	6 650. ft^2
Input constants for bank control logic*	
DT**	2.0 sec
BANKL	55.0 deg
BANKL1	80.0 deg
BANKR	45.0 deg
C1	.0195
C2	.075
FL	55.0 fps^2
FNMAX	954 000. lb
TMAR	5.0 sec
VREF	5 000. fps

*Refer to the discussion, section 3.2, for an explanation of these parameters.

**A 2-second integration step size was used for all FBGUID phases.

TABLE II. - HIGH CROSS-RANGE ORBITER AERODYNAMICS

(a) Trimmed lift coefficient aero data

α, deg \ Mach	Trimmed lift coefficient*											
	0.25	0.60	0.90	1.2	1.5	2.0	3.0	4.0	5.0	7.5	10.0	20.0
0	-.0250	-.0340	-.0460	-.0030	+.0004	-.0030	-.0280	-.0297	-.0403	-.0295	-.0259	-.0269
5	-.1600	+.1440	+.1430	+.1394	+.1486	+.1200	+.0850	+.0616	+.0460	+.0277	+.0270	+.0220
7.5	+.2450	+.2320	+.2280	+.2169	+.2198	+.1780	+.1430	+.1150	+.0965	+.0630	+.0615	+.0485
10.0	+.3360	+.3240	+.3210	+.2905	+.2912	+.2365	+.2040	+.1720	+.1450	+.0998	+.0968	+.0809
12.5	+.4260	+.4100	+.4140	+.3679	+.3672	+.2990	+.2660	+.2254	+.2003	+.1430	+.1411	+.1202
15.0	+.5170	+.4980	+.5100	+.4536	+.4421	+.3640	+.3280	+.2850	+.2550	+.1936	+.1895	+.1586
20.0	+.6900	+.6560	+.6850	+.6422	+.6012	+.4950	+.4490	+.3939	+.3600	+.2990	+.2870	+.2600
25.0	+.8000	+.7630	+.8150	+.8080	+.7565	+.6250	+.5600	+.5106	+.4700	+.4120	+.3930	+.3760
30.0	+.9030	+.8190	+.9100	+.9378	+.8910	+.7510	+.6660	+.6210	+.5690	+.5260	+.5120	+.4970
35.0	+1.0100	+.9080	+.9820	+1.0262	+1.0090	+.8590	+.7620	+.7150	+.6750	+.6340	+.6250	+.6160
40.0	+1.0100	+.9030	+.9750	+1.0610	+1.0770	+.9370	+.8460	+.7920	+.7690	+.7360	+.7290	+.7230
45.0				+1.0520	+1.1000	+1.0120	+.9100	+.8500	+.8420	+.8170	+.8120	+.8020
50.0				+1.0330	+1.0780	+1.0520	+.9430	+.8910	+.8820	+.8610	+.8620	+.8520
55.0						+1.0620	+.9550	+.8970	+.8830	+.8690	+.8720	+.8610
60.0						+1.0430	+.9290	+.8780	+.8550	+.8400	+.8420	+.8360

*Includes flared rudder where applicable.

TABLE II. - HIGH CROSS-RANGE ORBITER AERODYNAMICS - Concluded

(b) Trimmed drag coefficient aero data

	Trimmed drag coefficient*											
α, deg \ Mach	0.25	0.60	0.90	1.2	1.5	2.0	3.0	4.0	5.0	7.5	10.0	20.0
0	+.0208	+.0210	+.0300	+.0667	+.0683	+.0667	+.0560	+.0489	+.0457	+.0349	+.0325	+.0309
5	+.0242	+.0265	+.0360	+.0814	+.0909	+.0825	+.0640	+.0536	+.0473	+.0366	+.0355	+.0333
7.5	+.0325	+.0304	+.0350	+.0918	+.1062	+.0902	+.0752	+.0630	+.0549	+.0422	+.0409	+.0380
10.0	+.0453	+.0444	+.0480	+.1093	+.1198	+.1048	+.0913	+.0793	+.0662	+.0507	+.0490	+.0454
12.5	+.0610	+.0618	+.0640	+.1309	+.1438	+.1240	+.1131	+.0984	+.0835	+.0642	+.0624	+.0559
15.0	+.0825	+.0830	+.0865	+.1579	+.1702	+.1488	+.1415	+.1254	+.1068	+.0833	+.0801	+.0705
20.0	+.1620	+.1430	+.1650	+.2276	+.2396	+.2155	+.2105	+.1921	+.1665	+.1369	+.1307	+.1195
25.0	+.2980	+.3010	+.3490	+.4008	+.3578	+.3109	+.2982	+.2792	+.2514	+.2192	+.2082	+.1999
30.0	+.5900	+.5285	+.5450	+.5956	+.5380	+.4620	+.4167	+.3892	+.3597	+.3286	+.3194	+.3119
35.0	+.8170	+.6825	+.7050	+.7806	+.7454	+.6271	+.5564	+.5223	+.4948	+.4645	+.4609	+.4533
40.0	+.9505	+.7940	+.8330	+.9530	+.9487	+.8148	+.7292	+.6806	+.6611	+.6341	+.6273	+.6213
45.0				+1.1265	+1.1359	+1.0395	+.9230	+.8653	+.8558	+.8272	+.8286	+.8114
50.0				+1.3259	+1.3261	+1.2867	+1.1395	+1.0790	+1.0665	+1.0355	+1.0350	+1.0220
55.0						+1.5564	+1.3840	+1.3020	+1.2900	+1.2540	+1.2565	+1.2410
60.0						+1.8616	+1.6350	+1.5510	+1.5095	+1.4775	+1.4800	+1.4680

*Includes flared rudder where applicable.

TABLE III.- ORBITER FLYBACK SEQUENCE OF EVENTS FOR 150-SECOND ABORT TIME

G.e.t., sec	Event
150	Abort initiation
155	Orbiter main engine ignition to 50 percent thrust (1 engine)
211	Initiate pitch up to 60 deg at apogee (2 deg/sec)
244	Increase one main engine thrust level to 85 percent when the total load factor reached .5g's
338	Altitude rate became positive and angle of attack for turn phase was commanded to 38 deg (5 deg/sec)
340	Initiate turn phase and reduce thrust level to 70 percent of one engine
688	Desired target azimuth was achieved and postturn thrust and glide phase initiated
698	Depletion of main engine propellant
1652	FBGUID terminated for range to target less than 5 miles

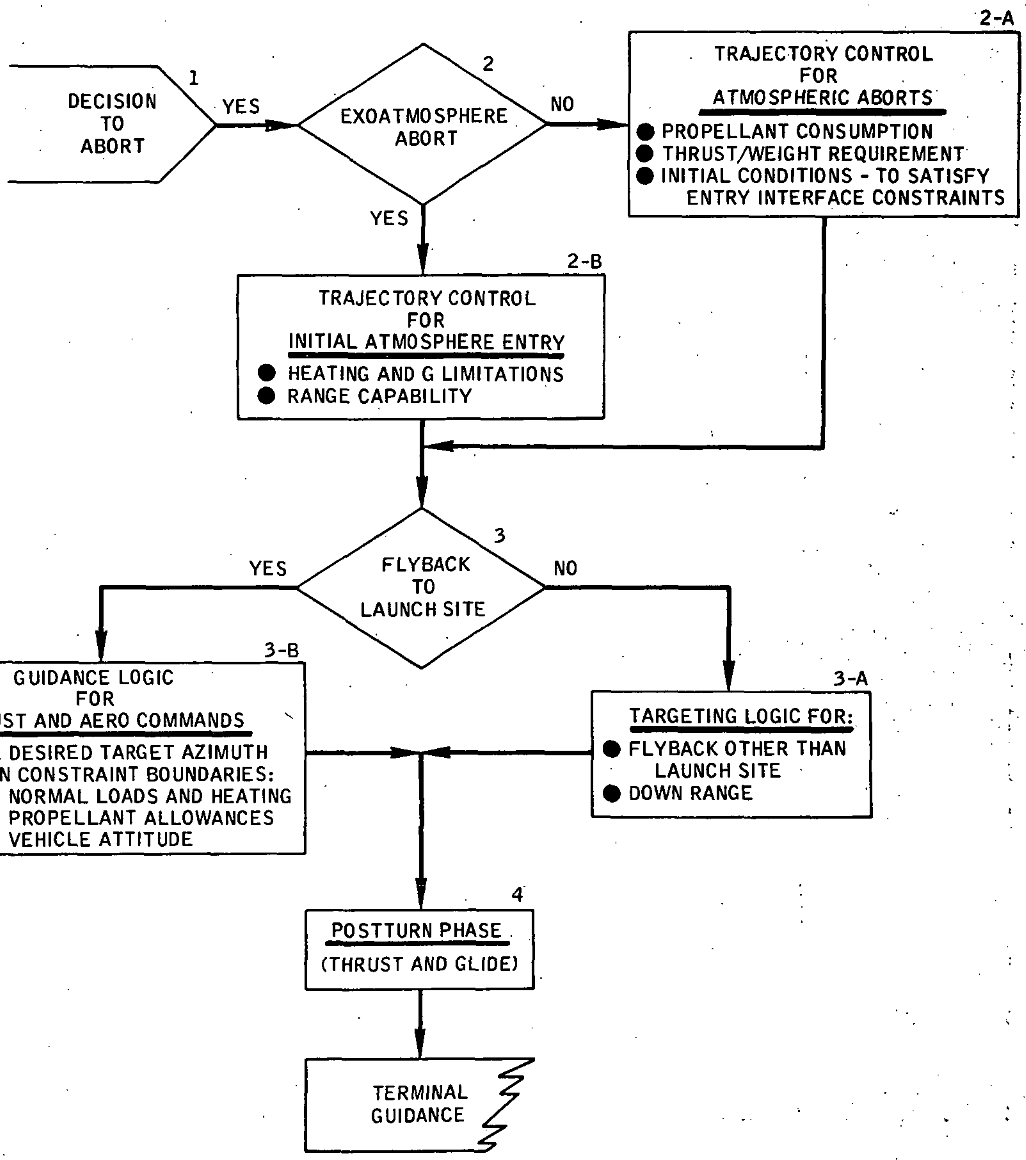
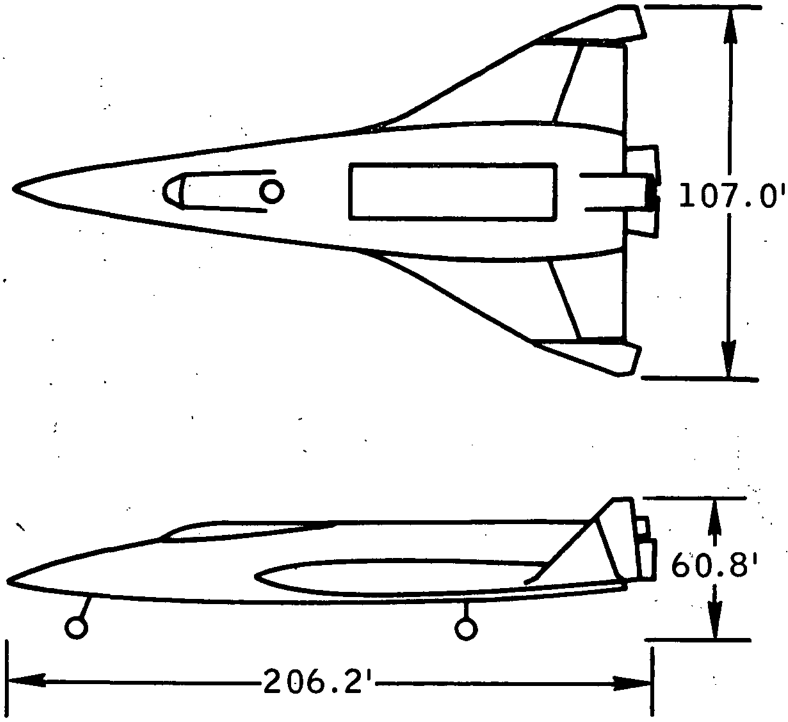


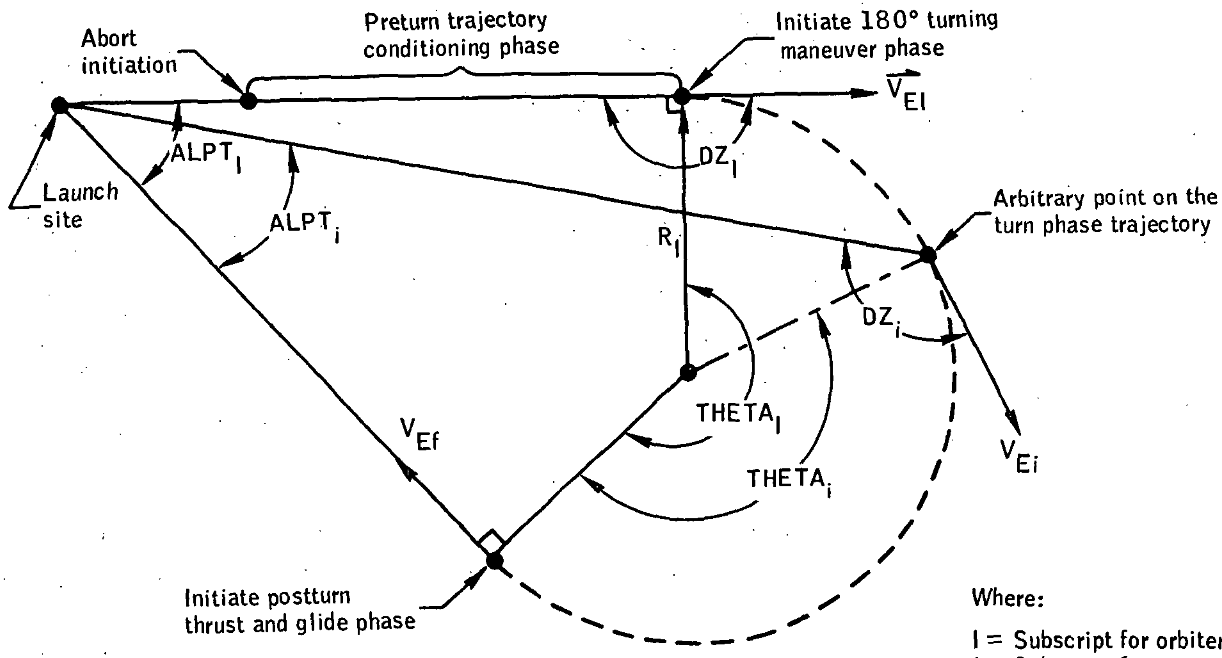
Figure 1.- Interface between the orbiter flyback guidance scheme and the total suborbital abort logic.



Vehicle parameters	Value	
Weight at abort initiation, lb	823 813	
Propellant weight, lb	505 943	
Weight at propellant depletion, lb	317 870	
Reference area, ft^2	6 650	
Number of engines	1	2
*Engine sea-level thrust, lb	488 000	976 000
*Engine vacuum thrust, lb	632 000	1 264 000
*Engine sea-level I_{sp}, sec	350	350
*Engine vacuum I_{sp}, sec	459	459

* Values are for nozzle skirt extended

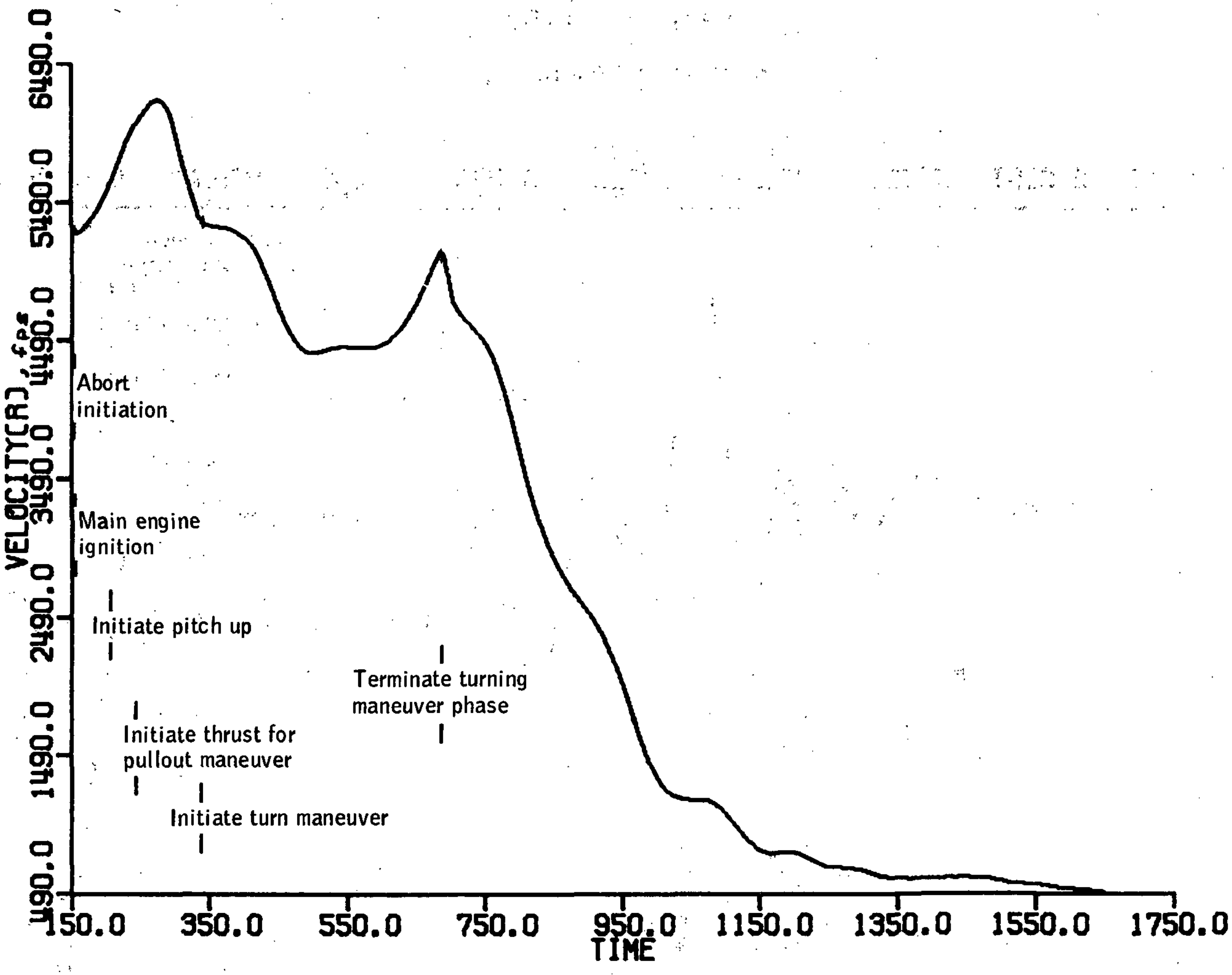
Figure 2.- North American high cross-range orbiter configuration data.



Where:

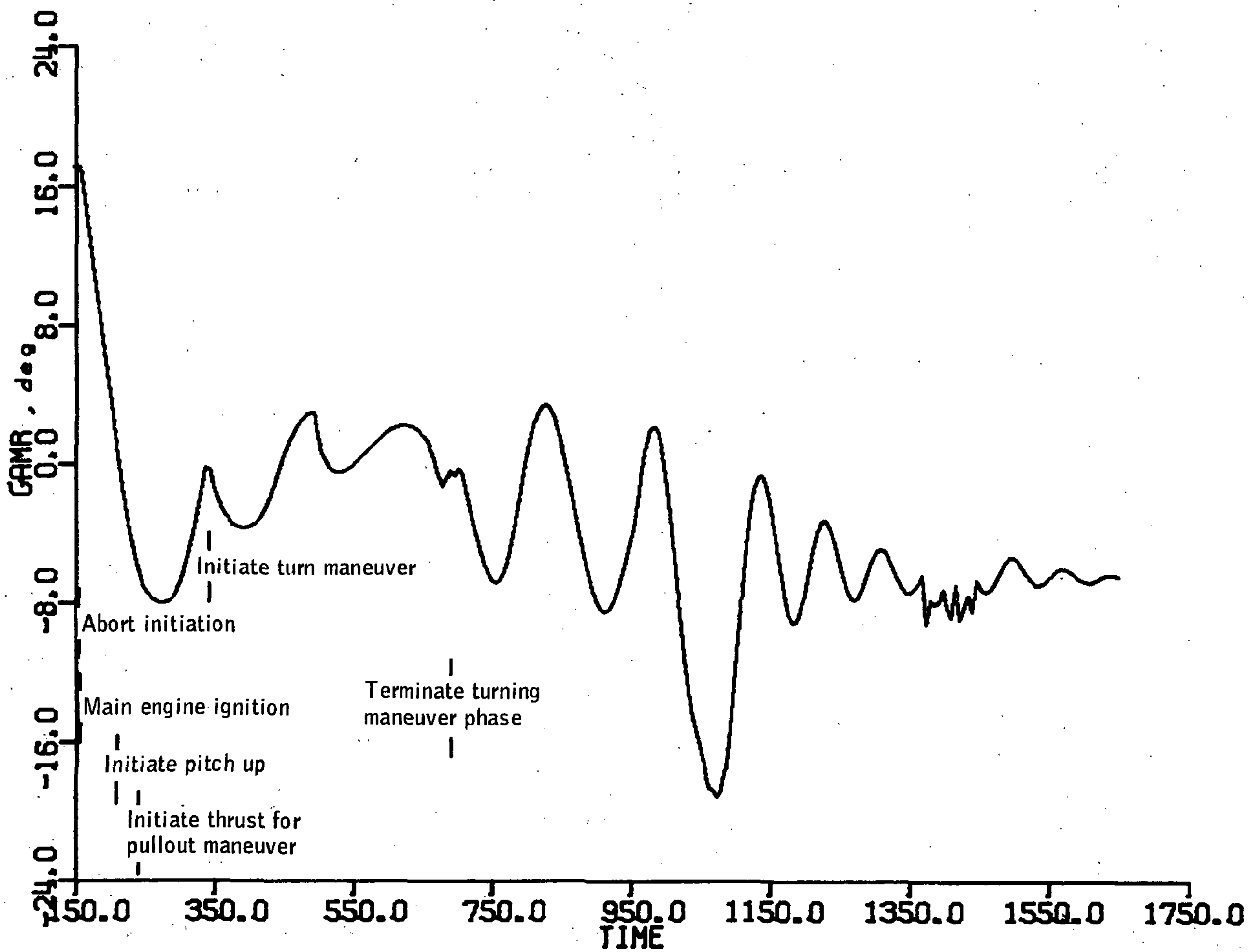
I = Subscript for orbiter at turn initiate
i = Subscript for any point during turn
f = Subscript to designate turn completion
DZ = Angle between the *current relative* velocity vector, V_E, and the *target azimuth* ($DZ_I \cong 180°$)
ALPT = Correction angle accrued during *finite radius turn*
THETA = DZ + ALPT

Figure 3.- Flyback turning angle geometry.



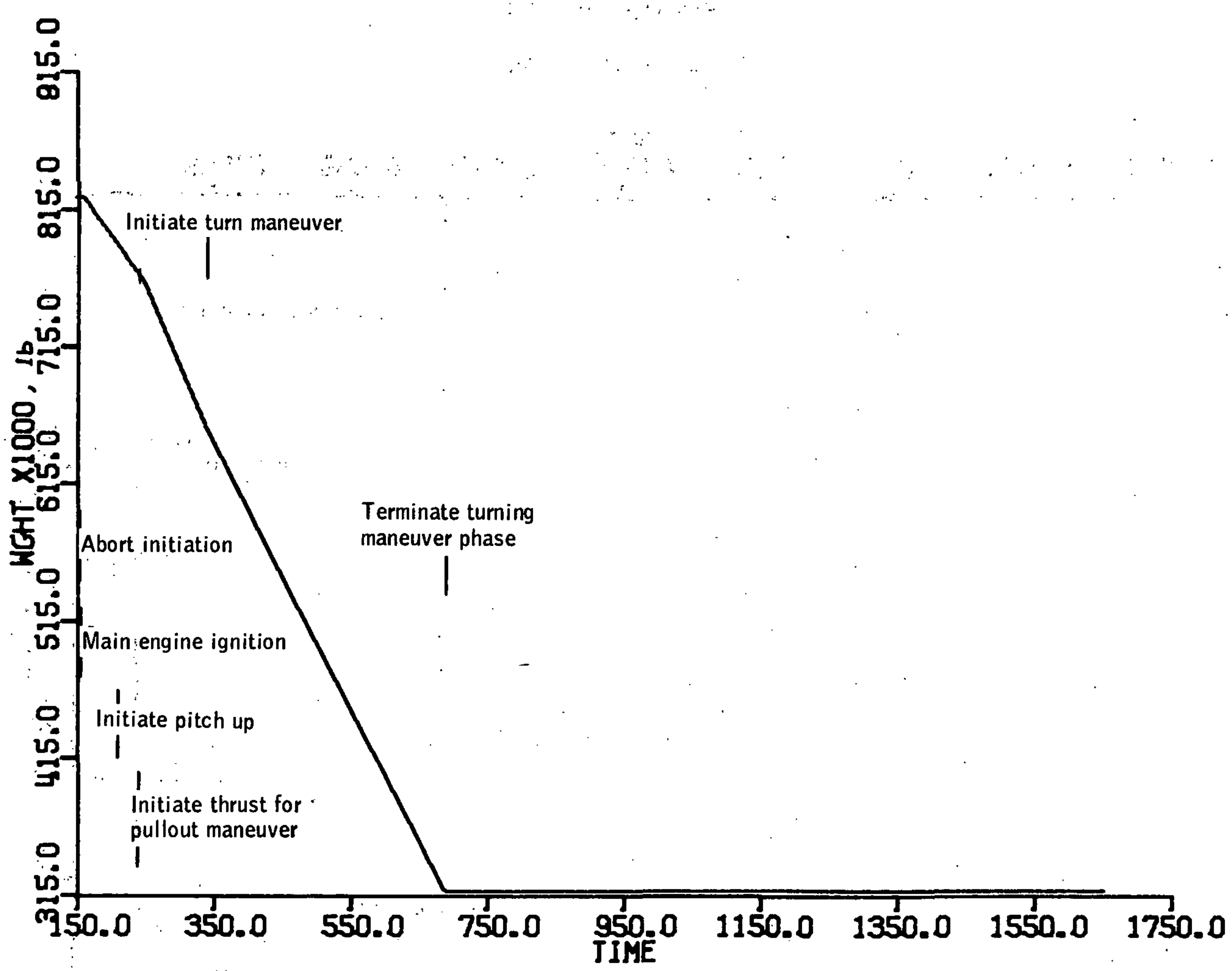
(a) Relative velocity time history.

Figure 4.- Flyback trajectory parameter time histories for abort at 150 seconds.



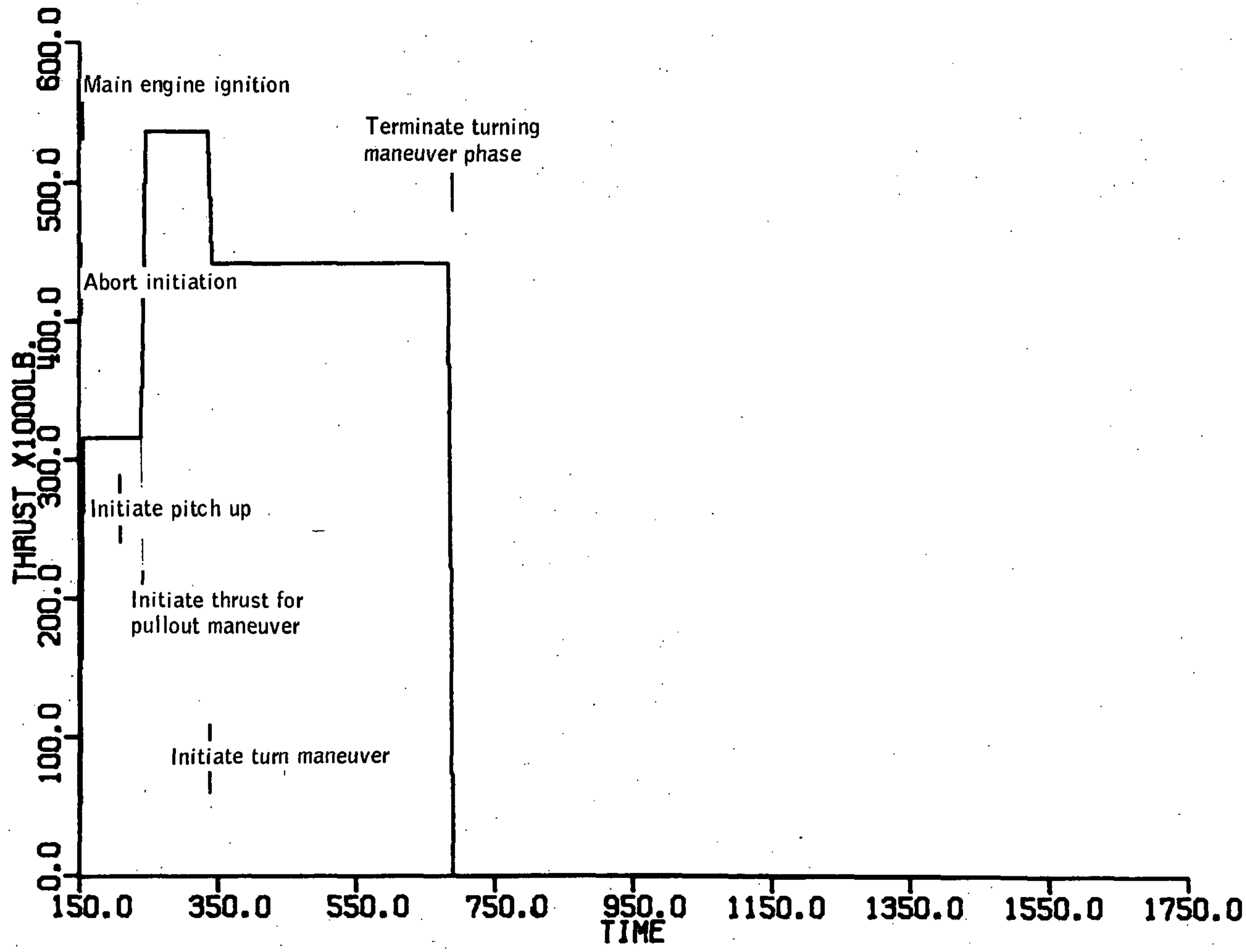
(b) Relative flight-path angle.

Figure 4.- Continued.



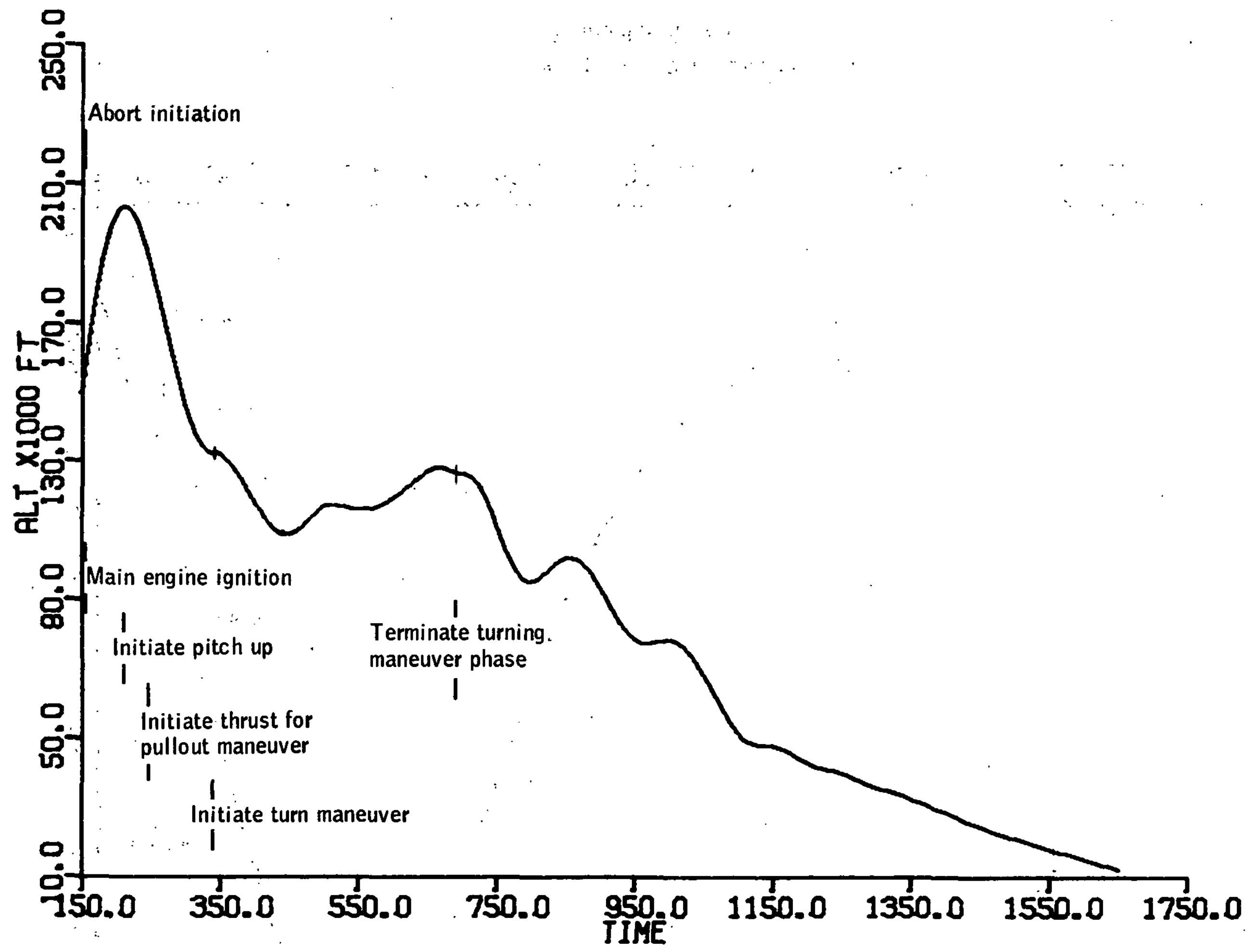
(c) Orbiter weight time history.

Figure 4.- Continued.



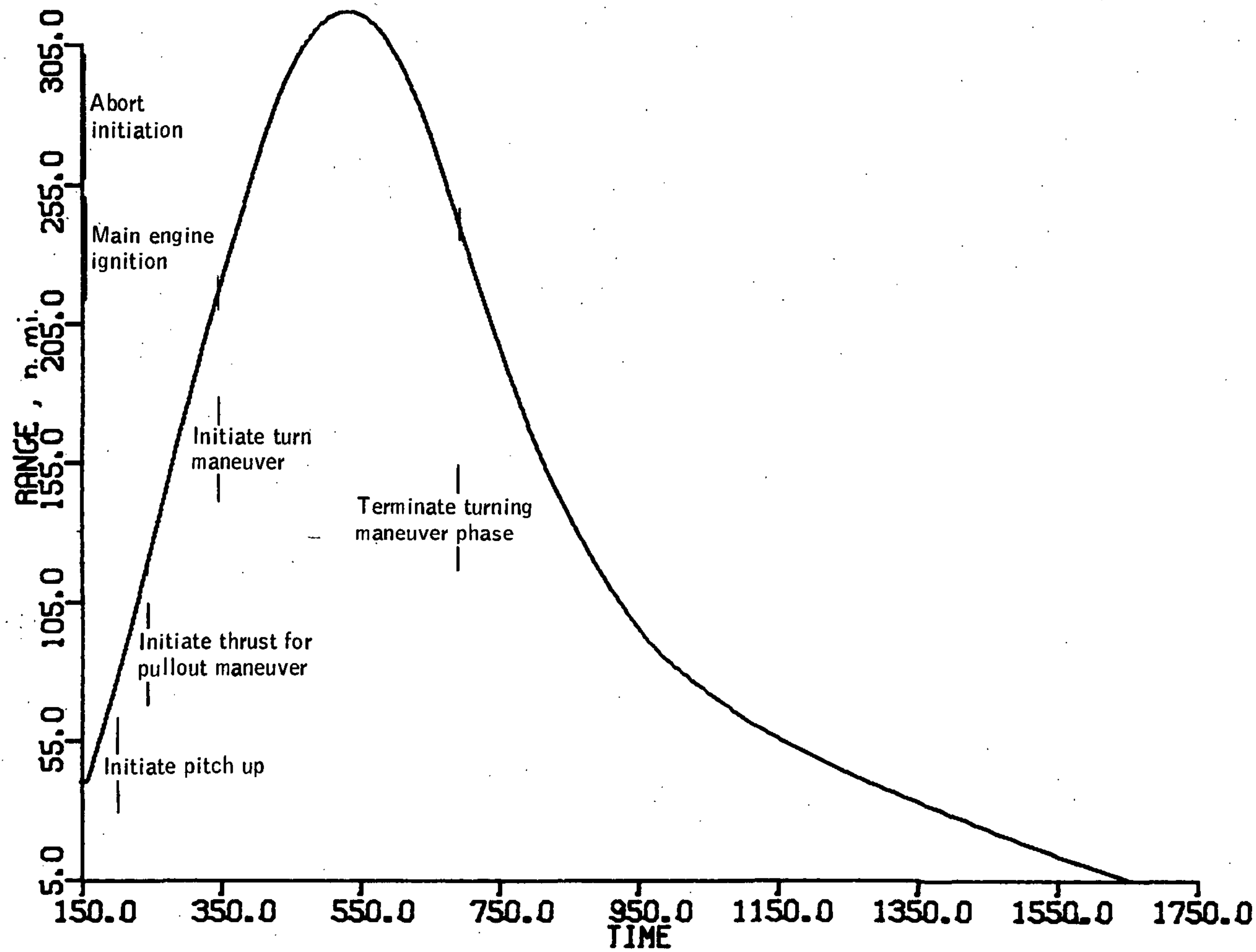
(d) Thrust time history.

Figure 4.- Continued.



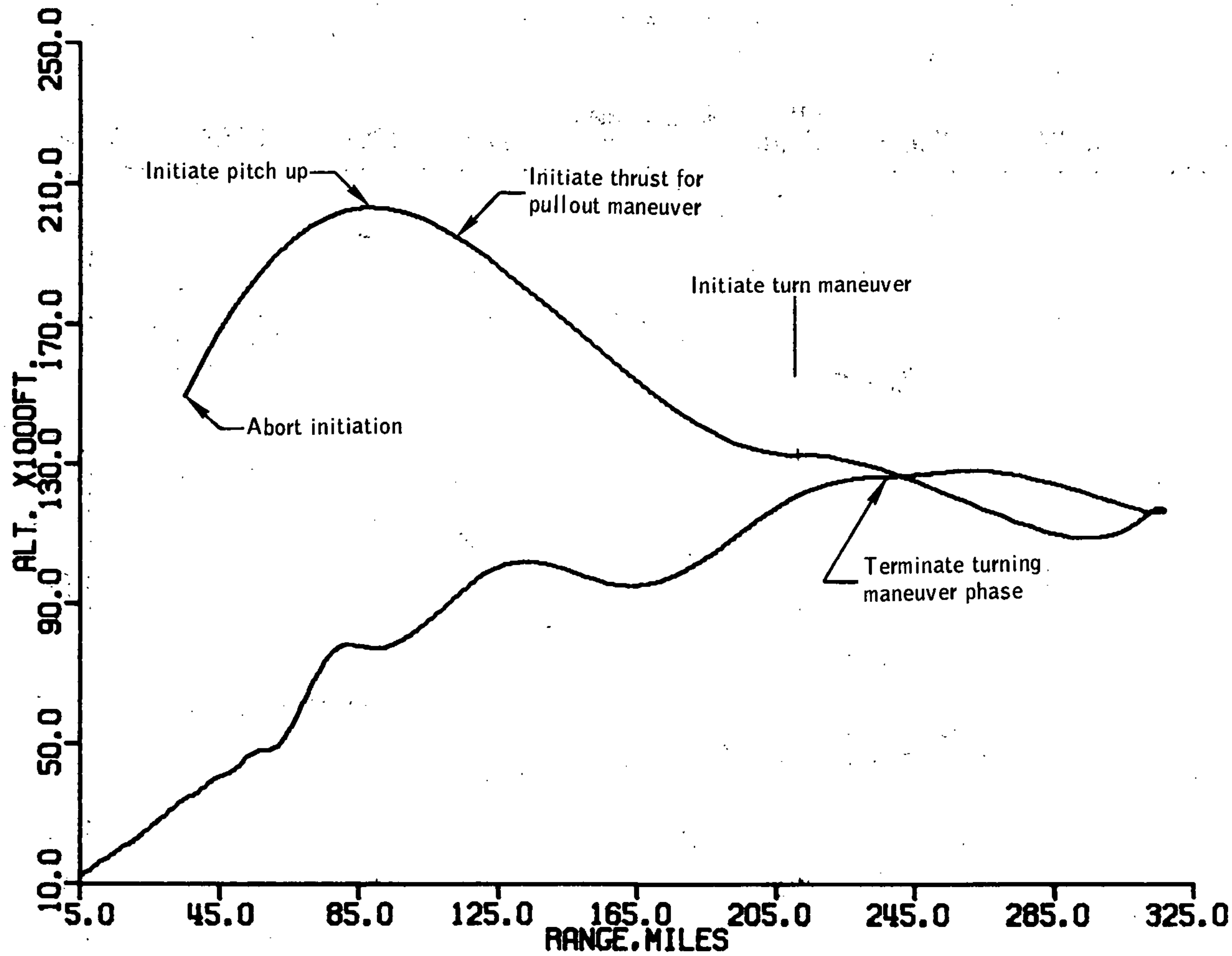
(e) Altitude time history.

Figure 4.- Continued.



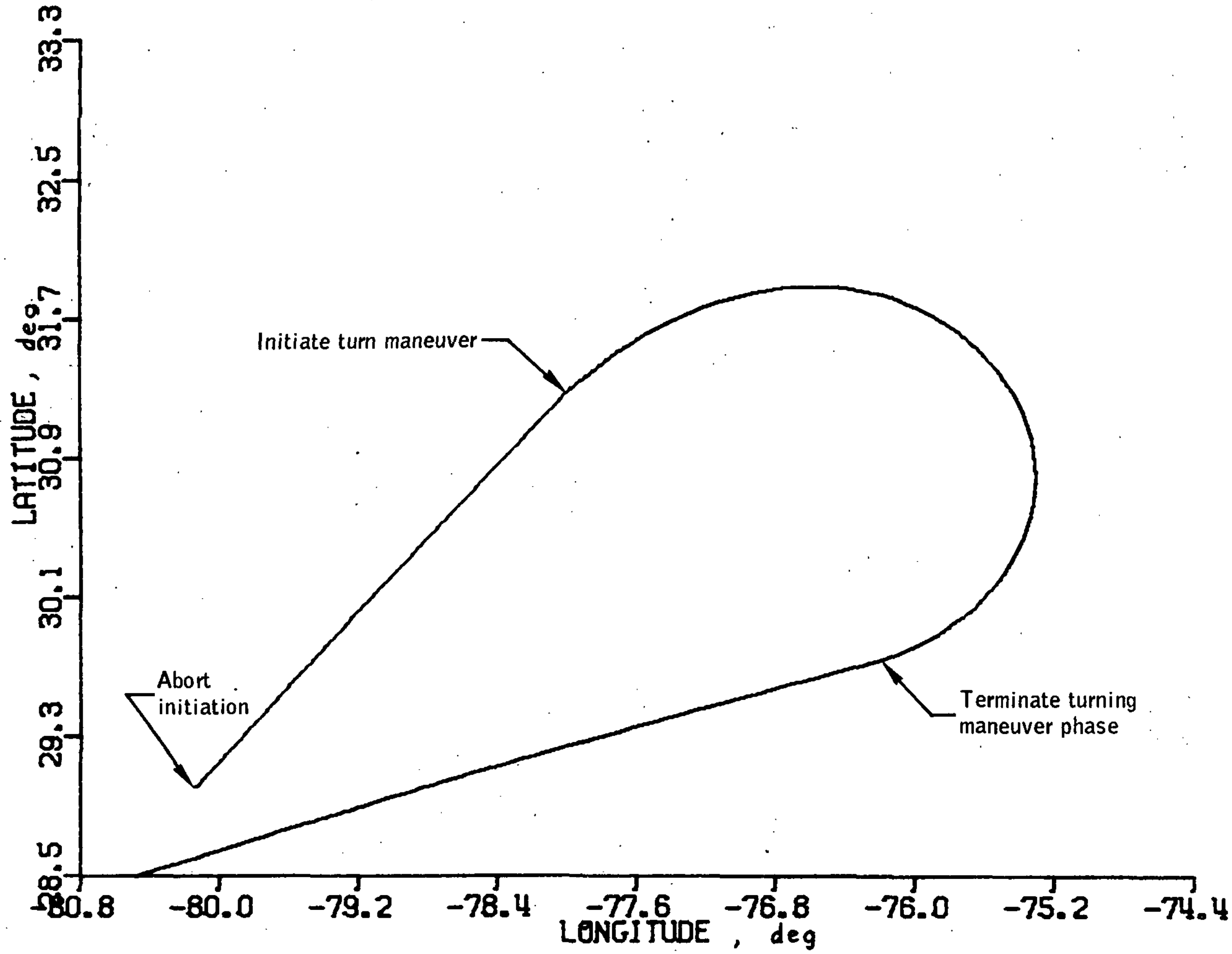
(f) Down-range distance time history.

Figure 4.- Continued.



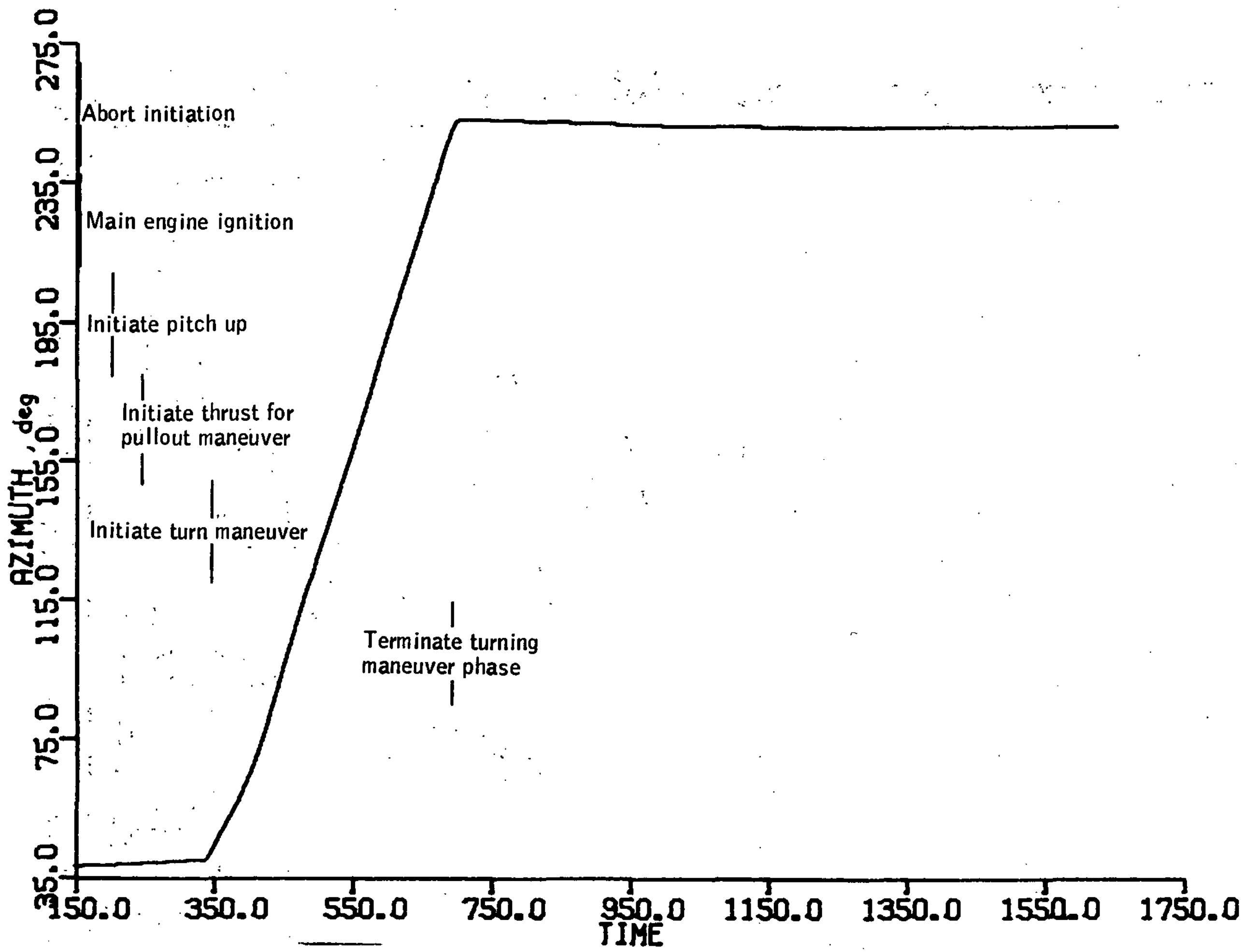
(g) Altitude versus down-range distance.

Figure 4.- Continued.



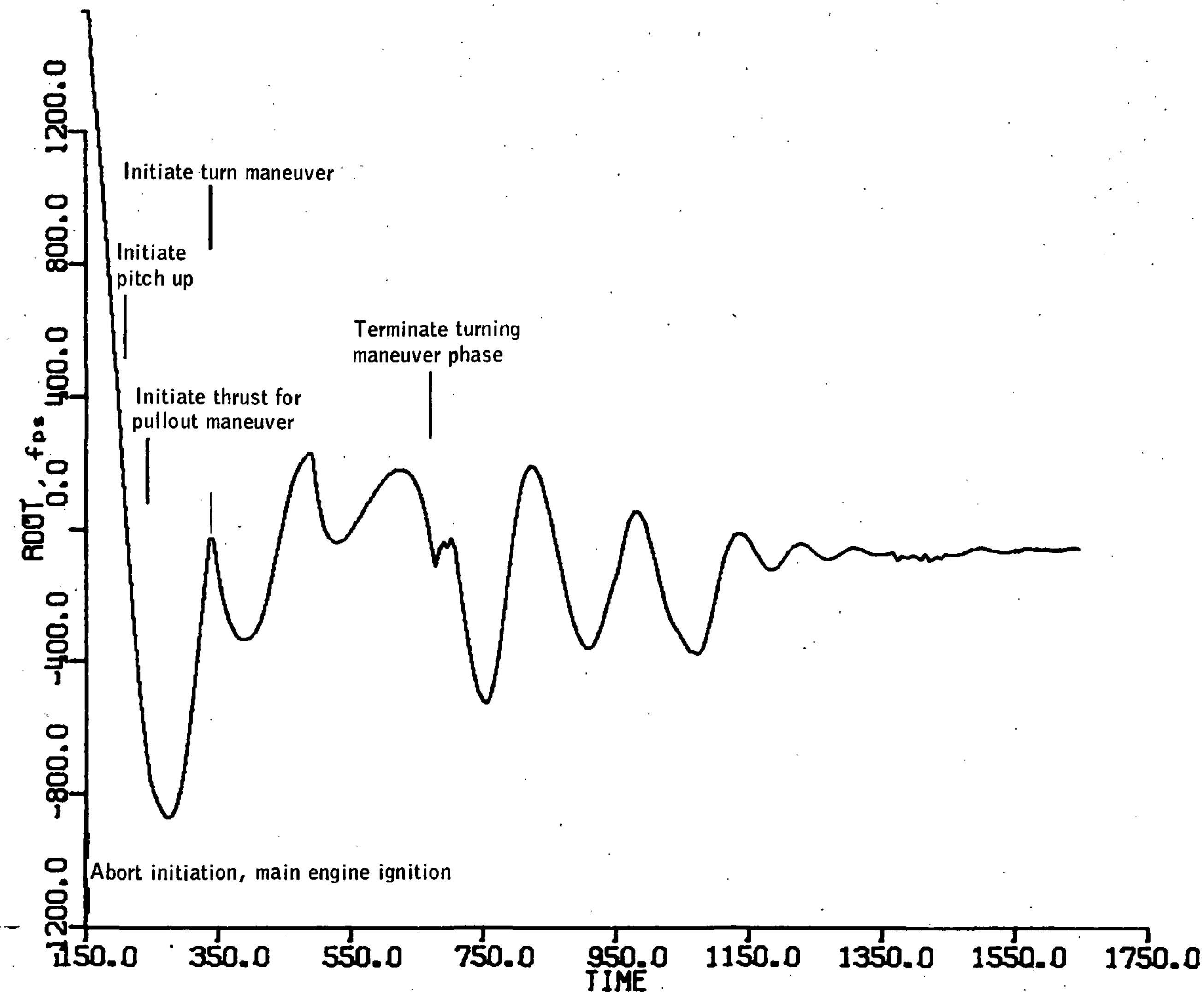
(h) Flyback groundtrack.

Figure 4.- Continued.



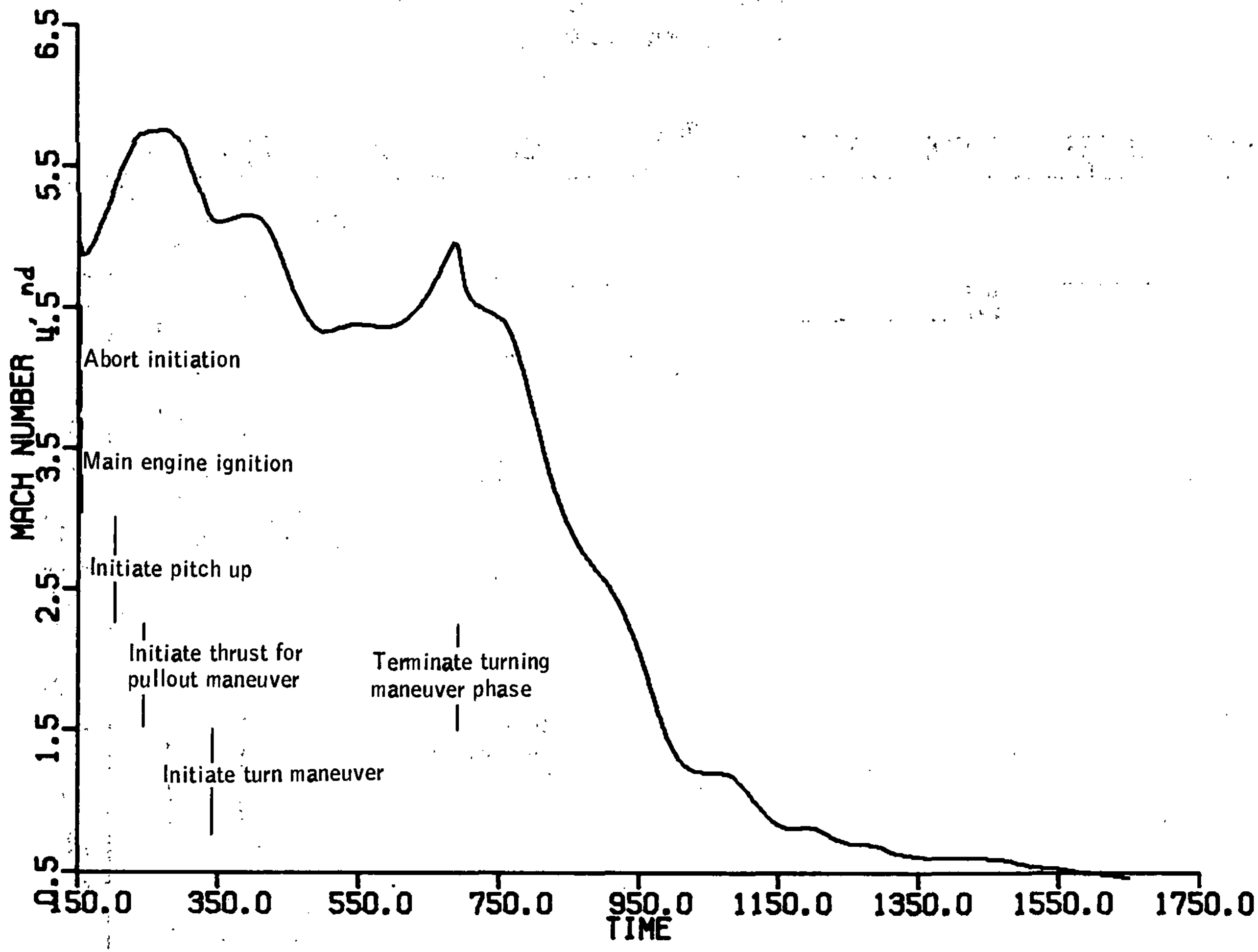
(i) Current flyback azimuth time history.

Figure 4.- Continued.



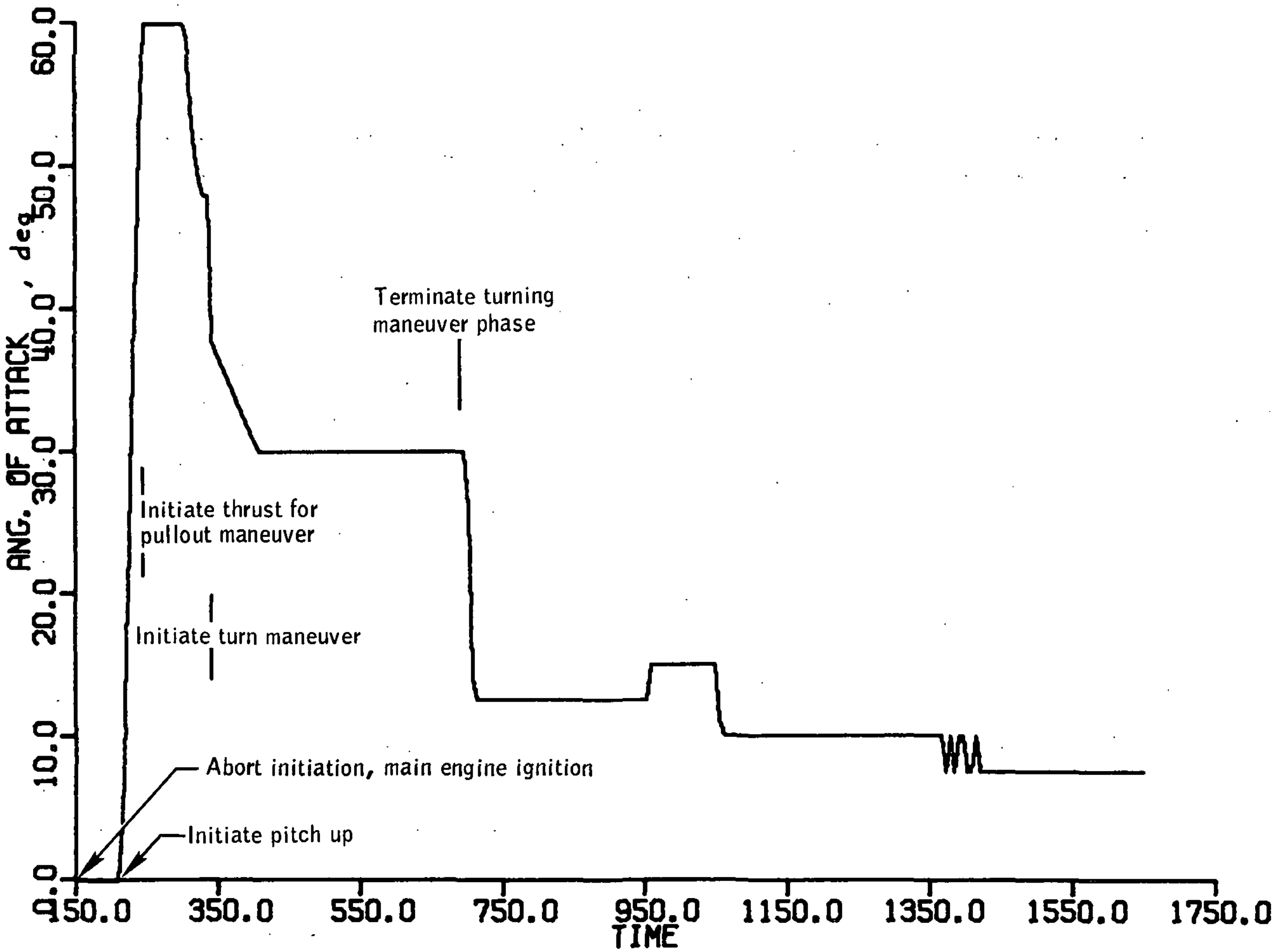
(j) Altitude rate time history.

Figure 4.- Continued.



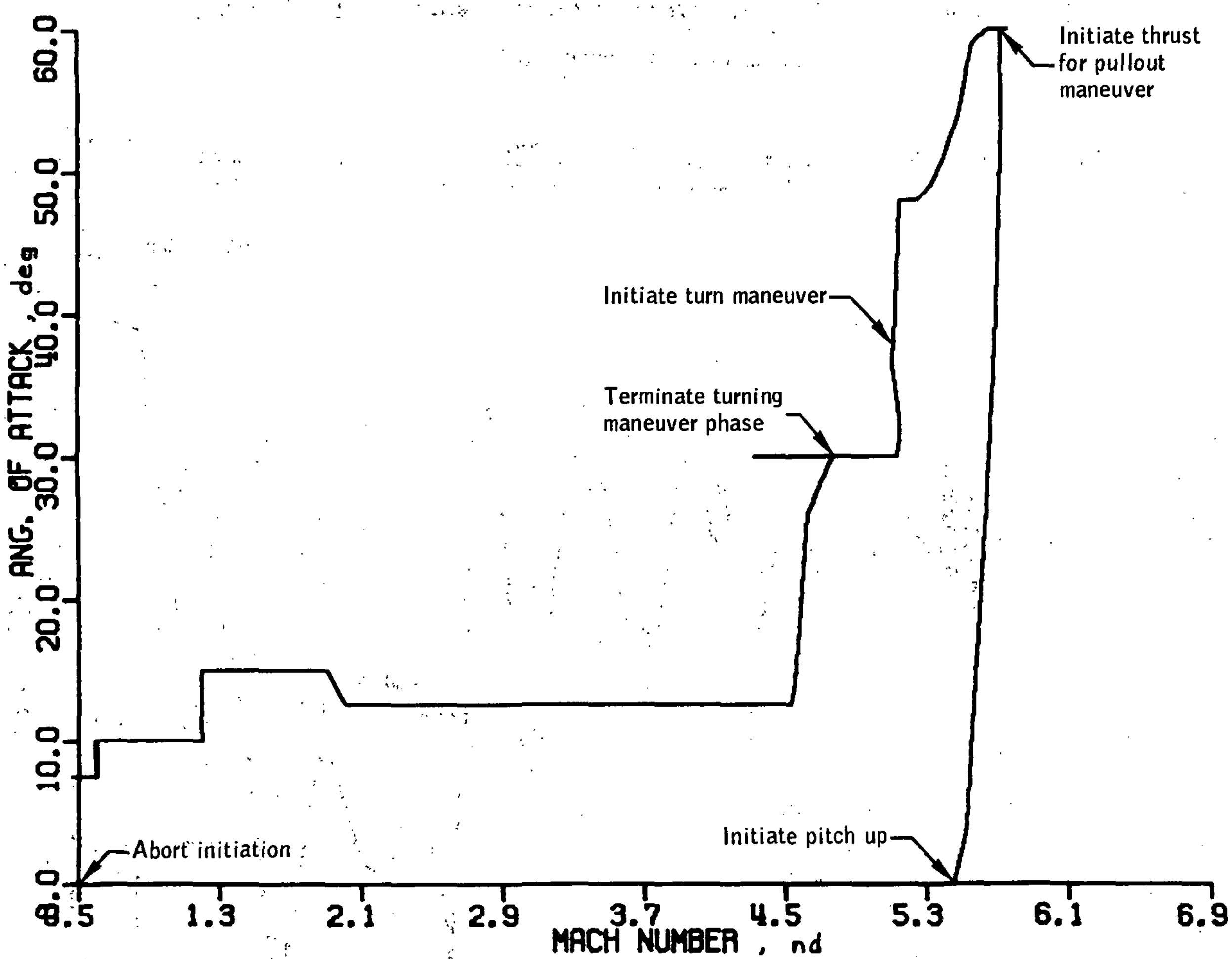
(k) Mach number time history.

Figure 4.- Continued.



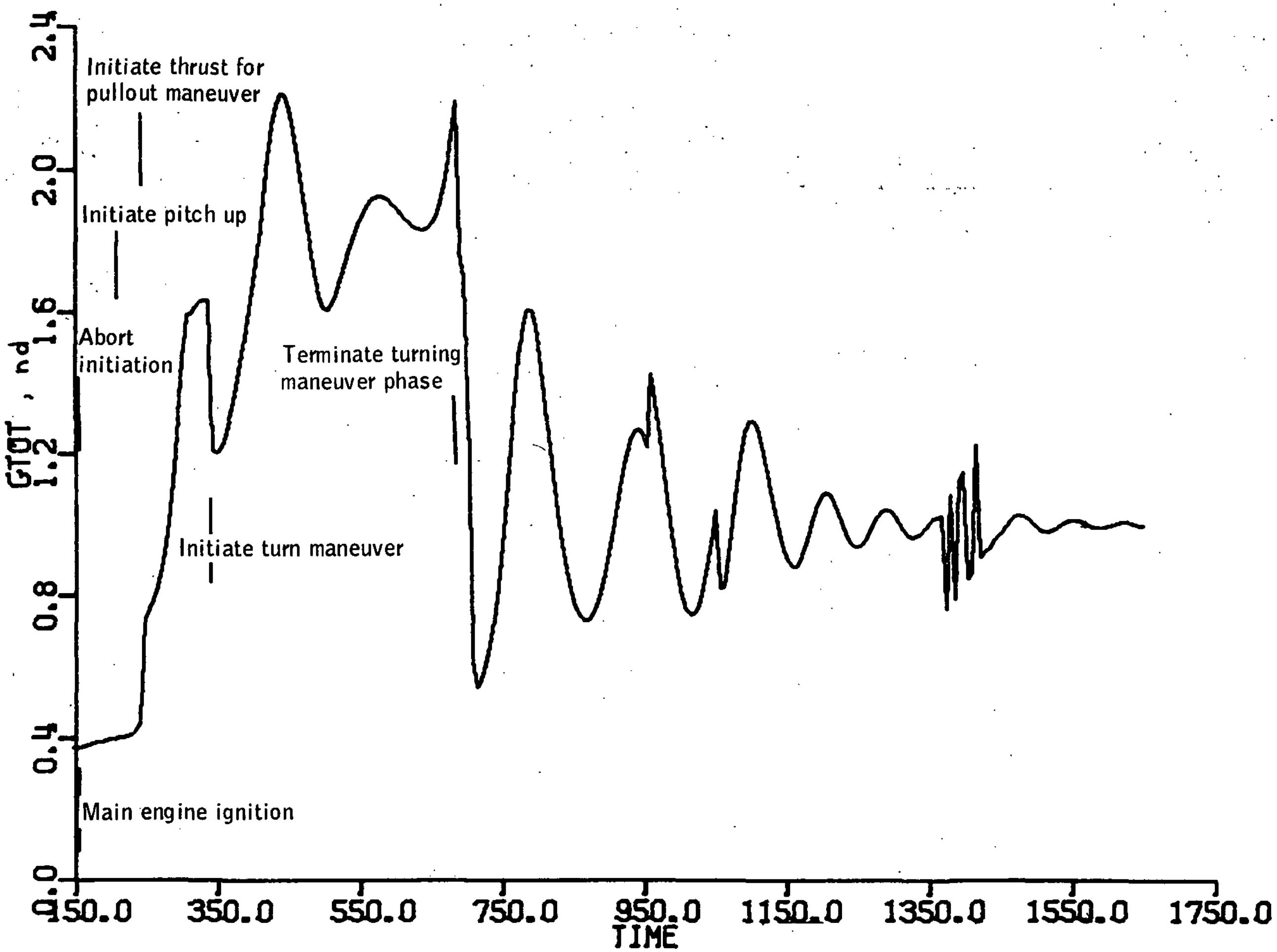
(l) Angle of attack time history.

Figure 4.- Continued.



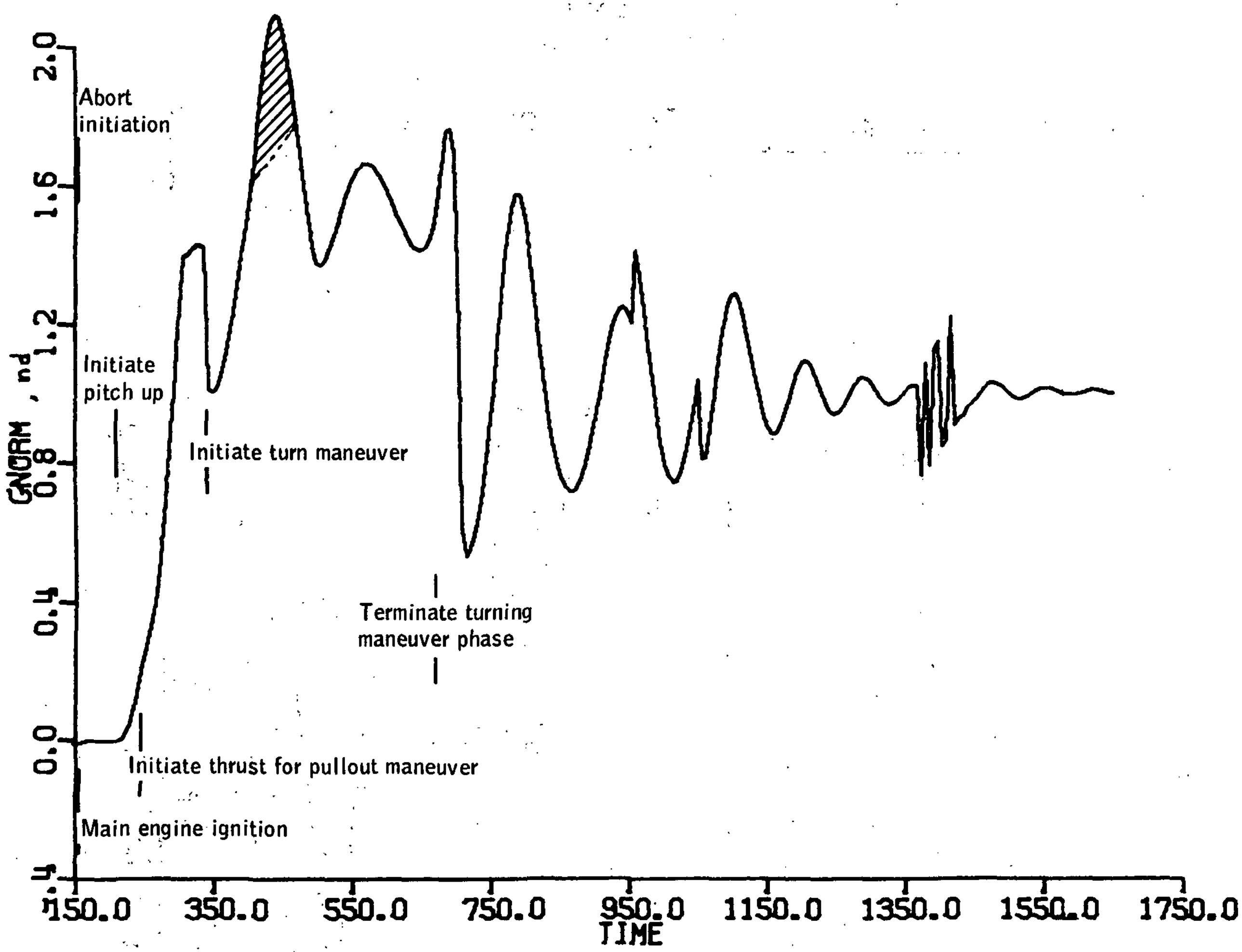
(m) Angle of attack versus Mach number.

Figure 4.- Concluded.



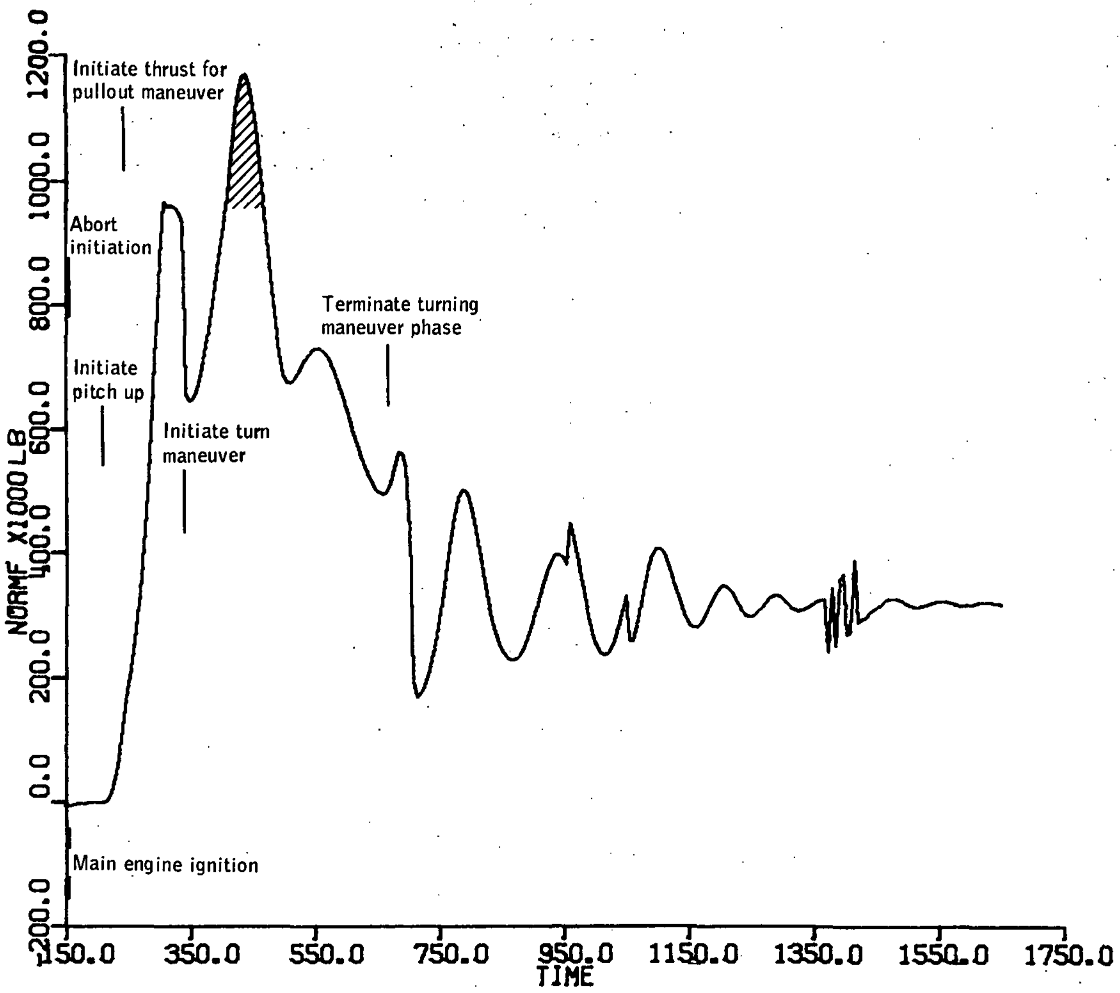
(a) Total vehicle loading time history.

Figure 5.- Time histories of aerodynamic loading parameters for abort at 150 seconds.



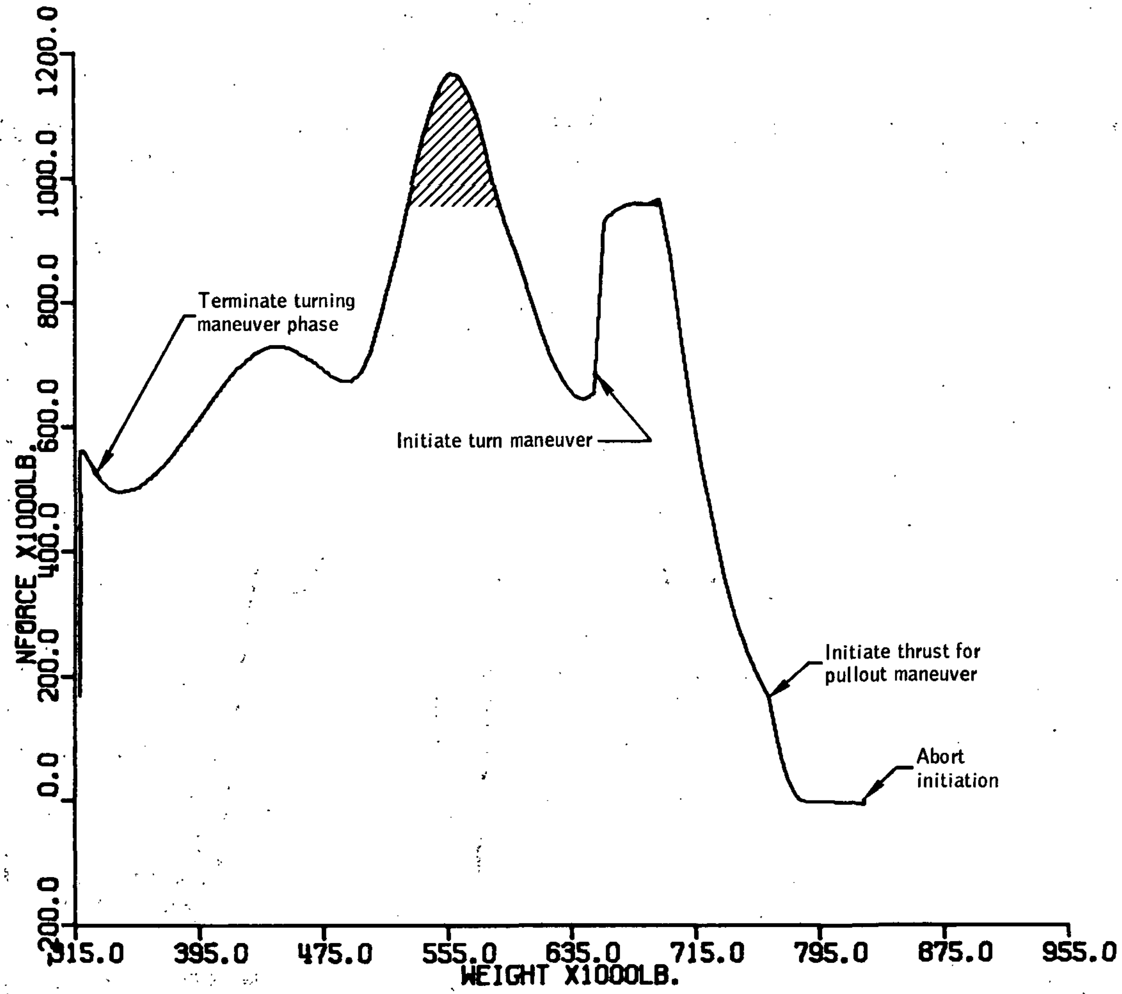
(b) Normal load time history.

Figure 5.- Continued.



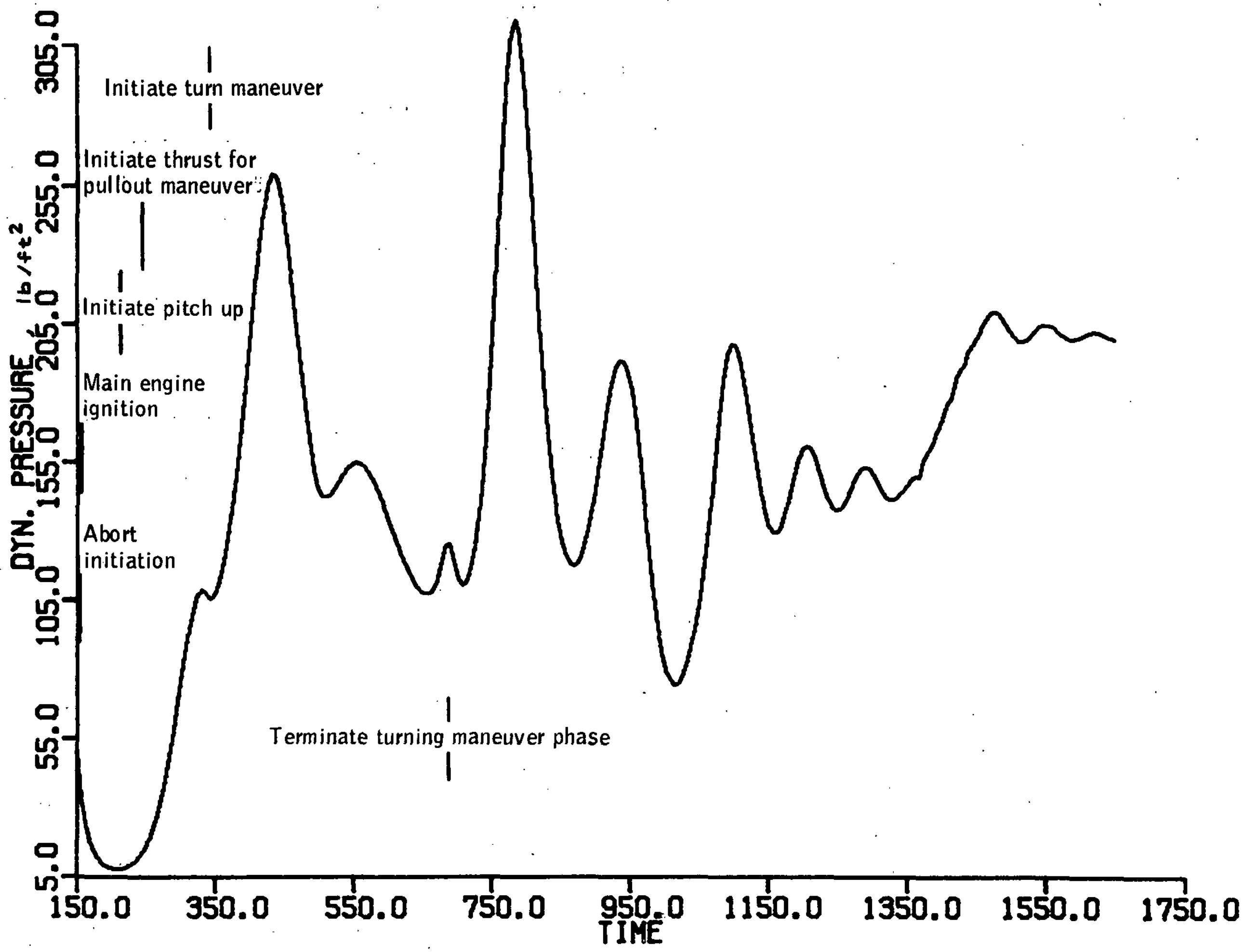
(c) Normal force time history.

Figure 5.- Continued.



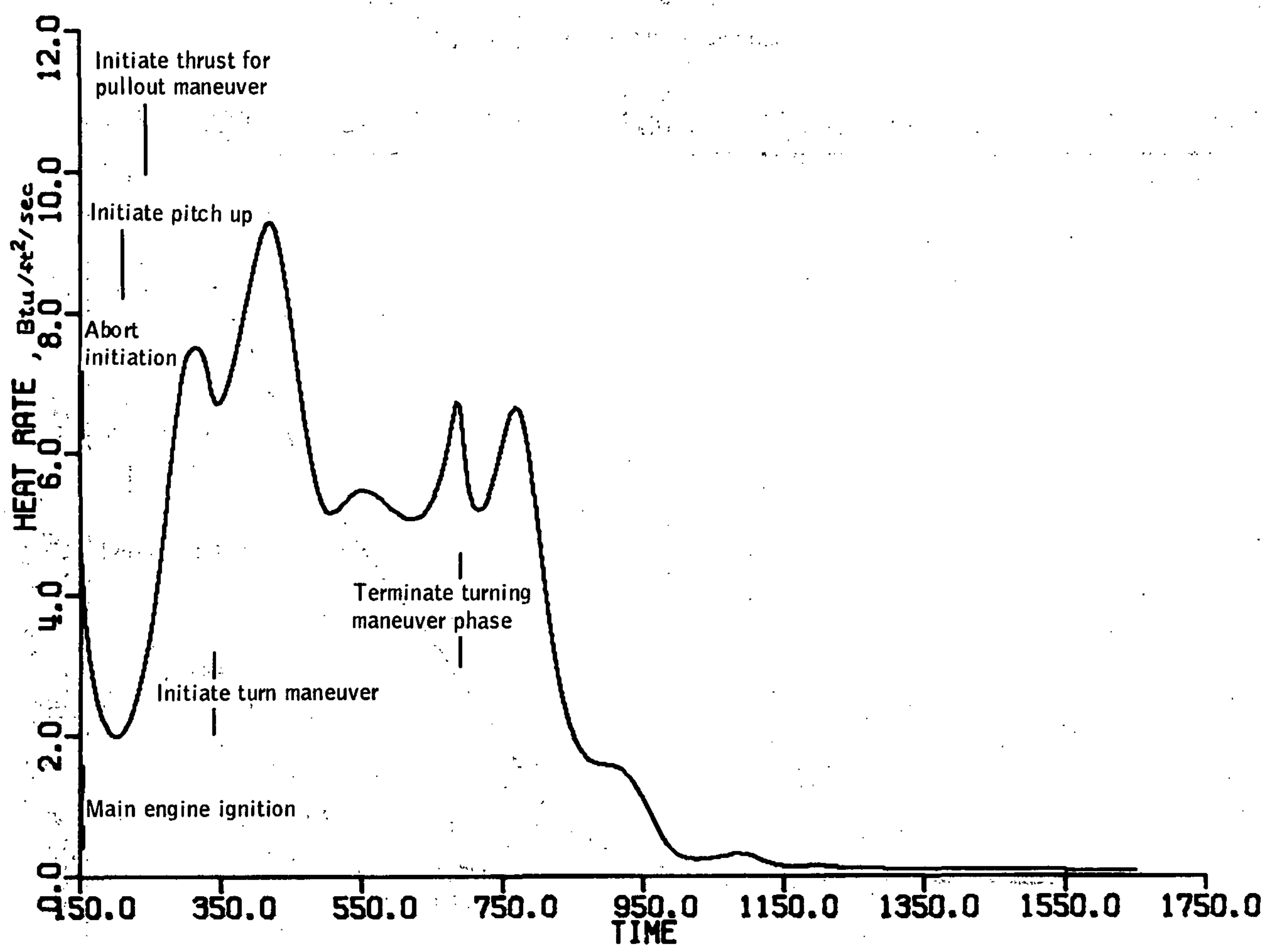
(d) Normal force versus vehicle weight.

Figure 5.- Continued.



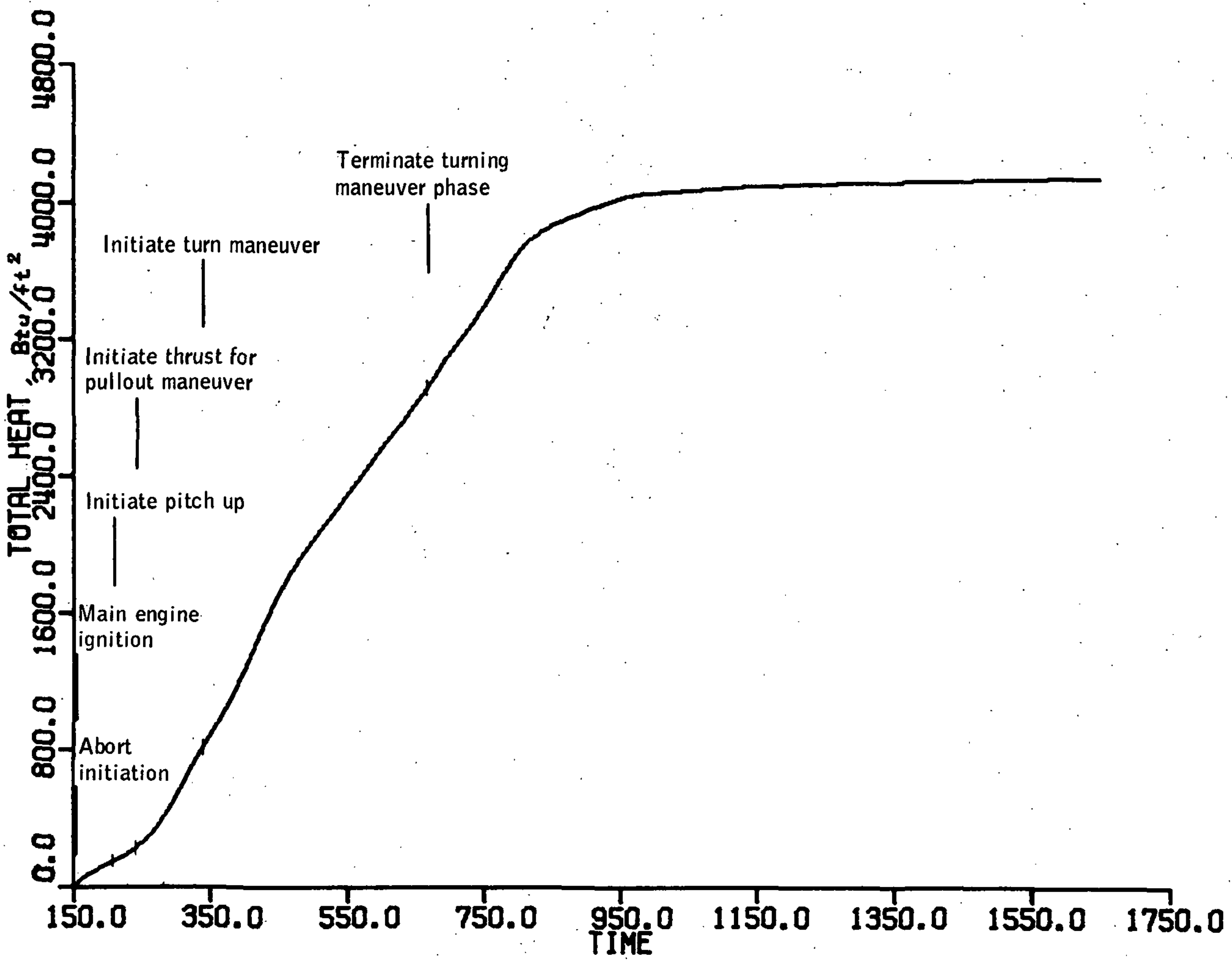
(e) Dynamic pressure time history.

Figure 5.- Continued.



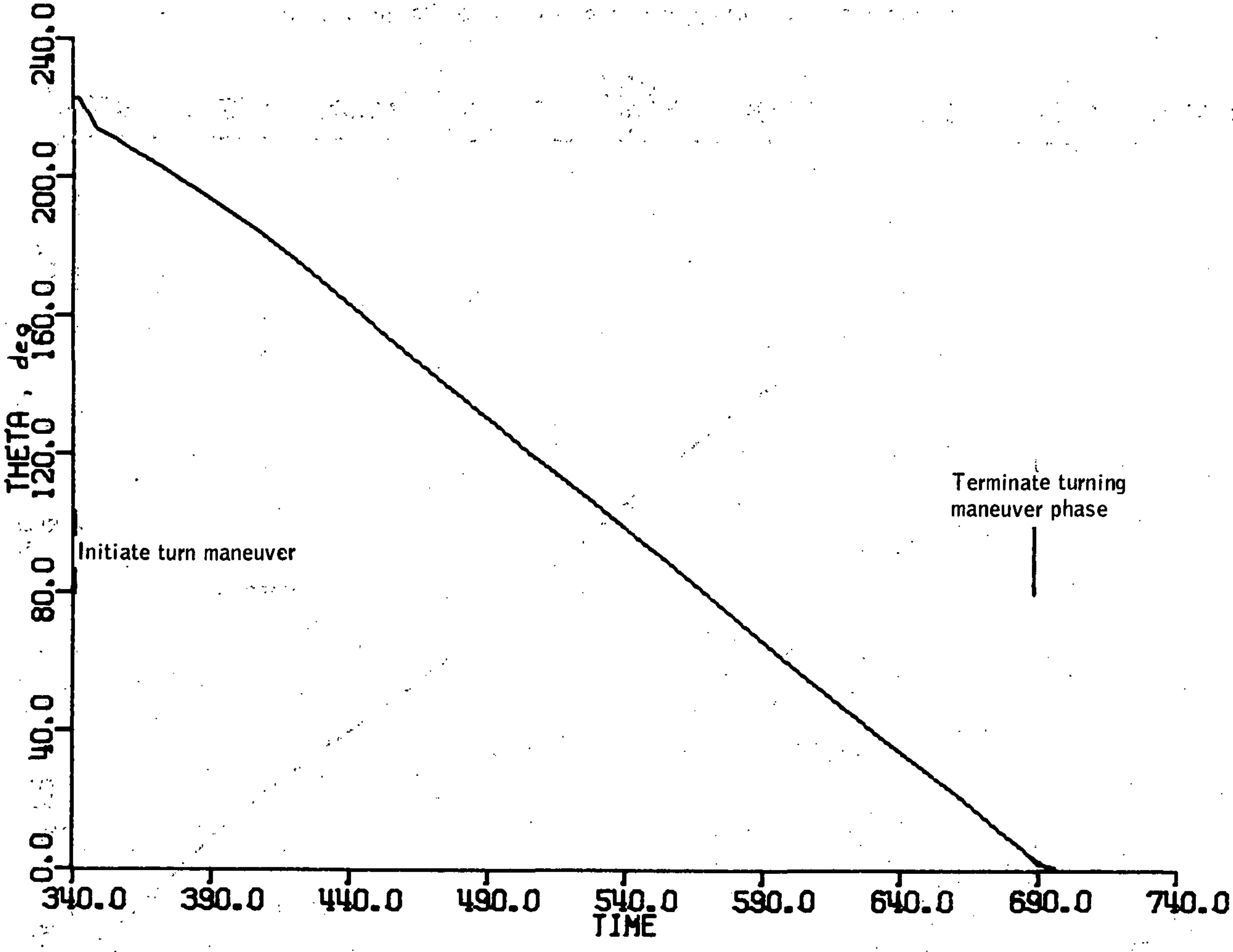
(f) Convective heat rate time history.

Figure 5.- Continued.



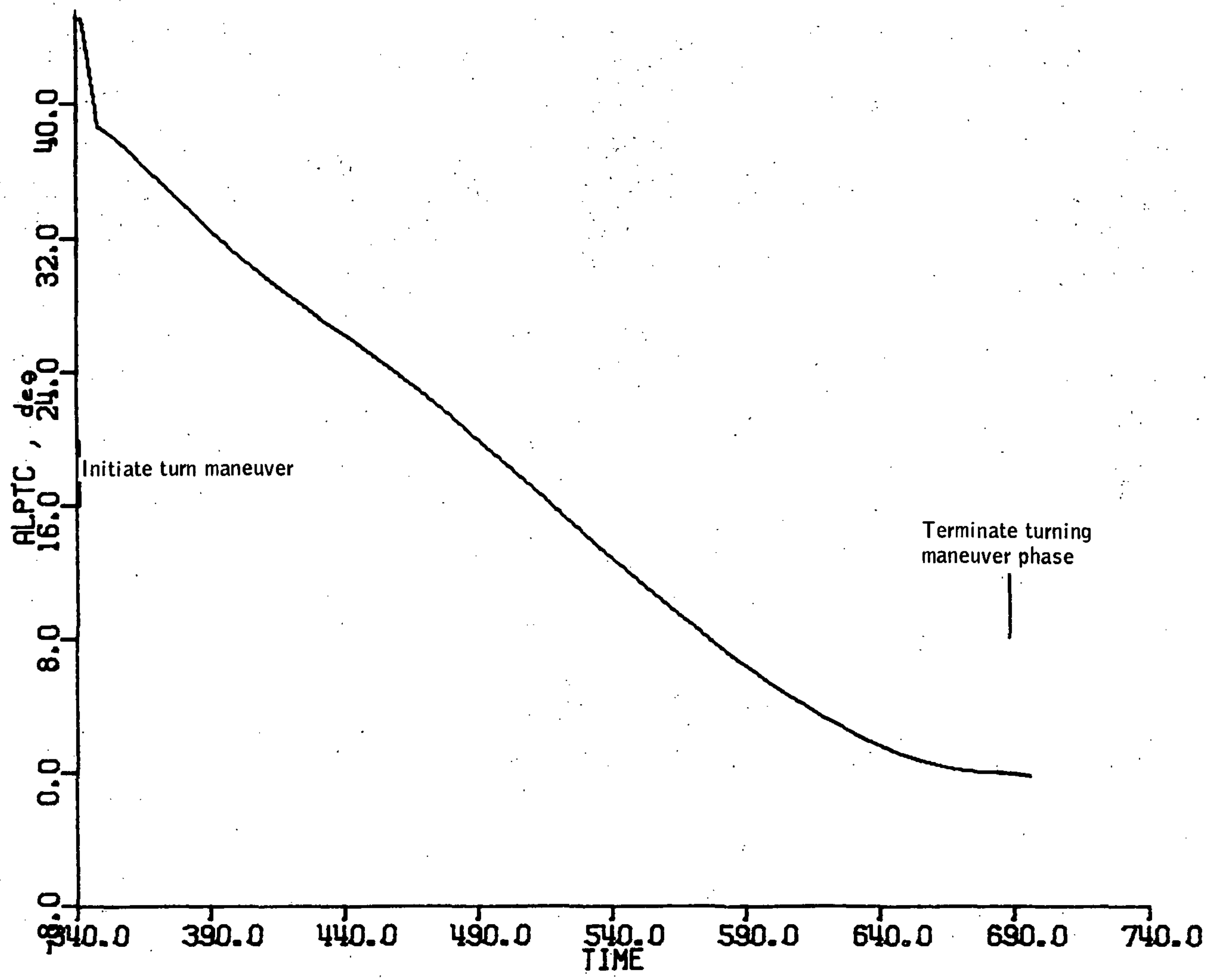
(g) Total convective heat load time history.

Figure 5.- Concluded.



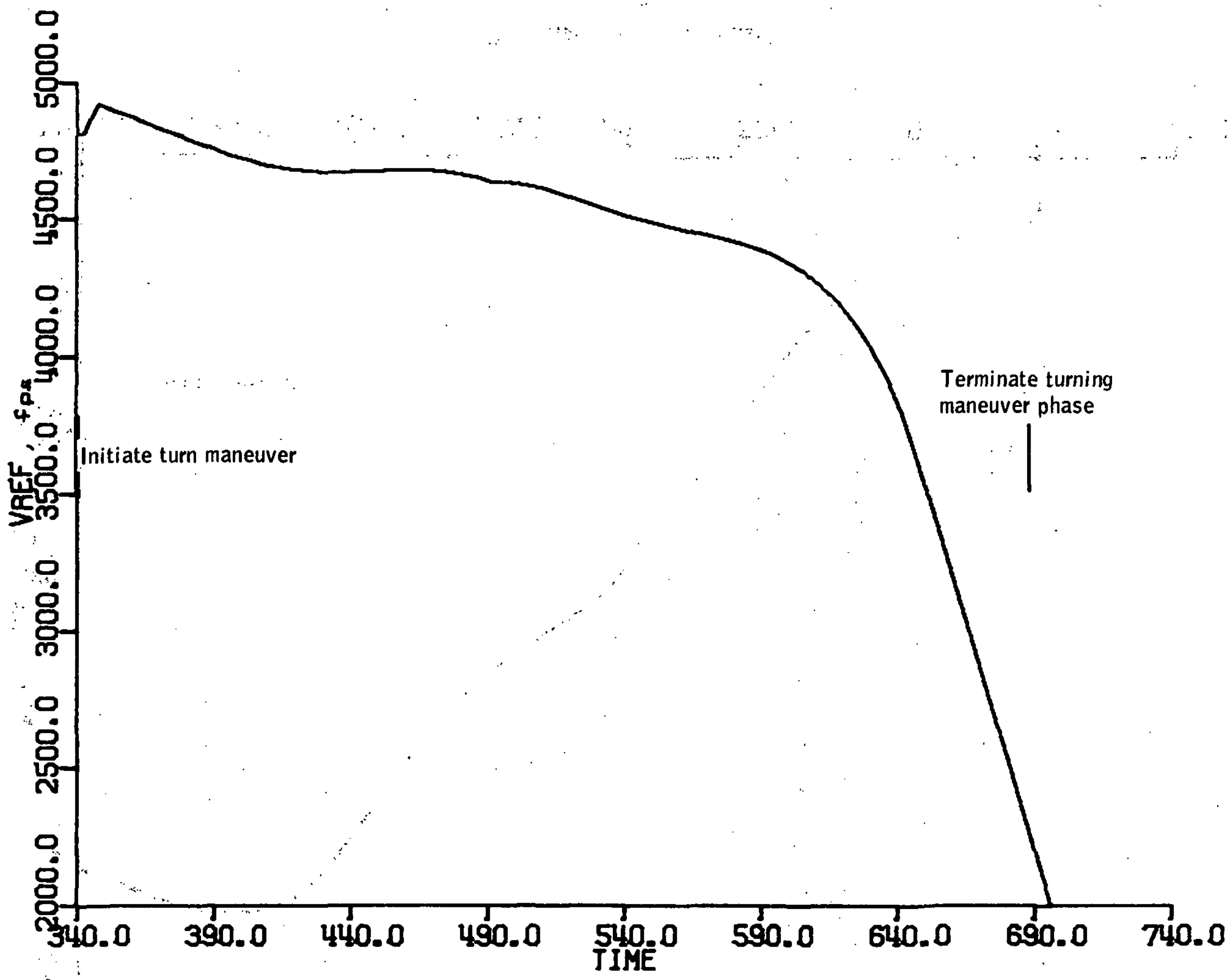
(a) Total turn angle time history.

Figure 6.- Time histories of guidance parameters computed during the turn angle phase.



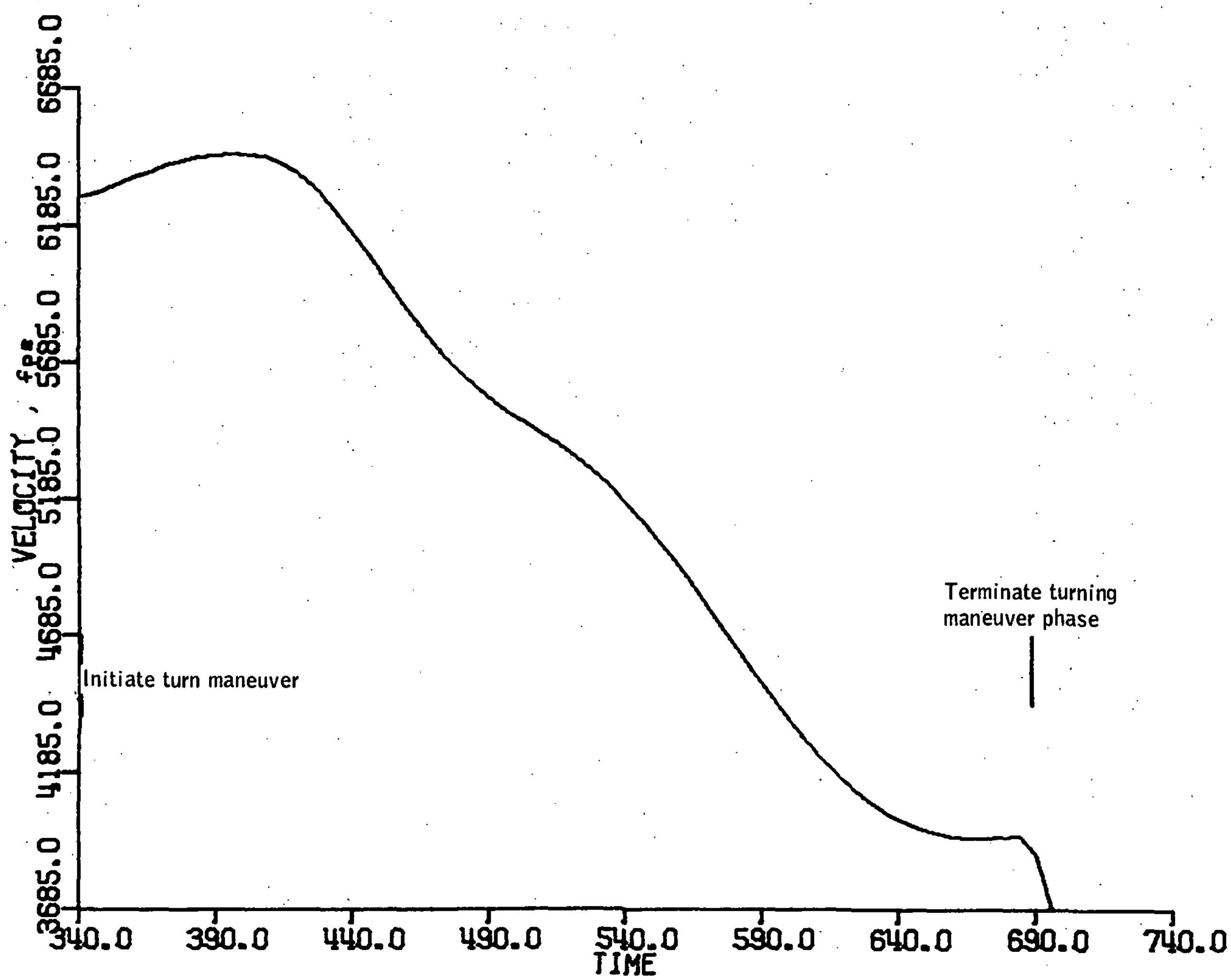
(b) Time history of correction angle used in computing total turn angle.

Figure 6.- Continued.



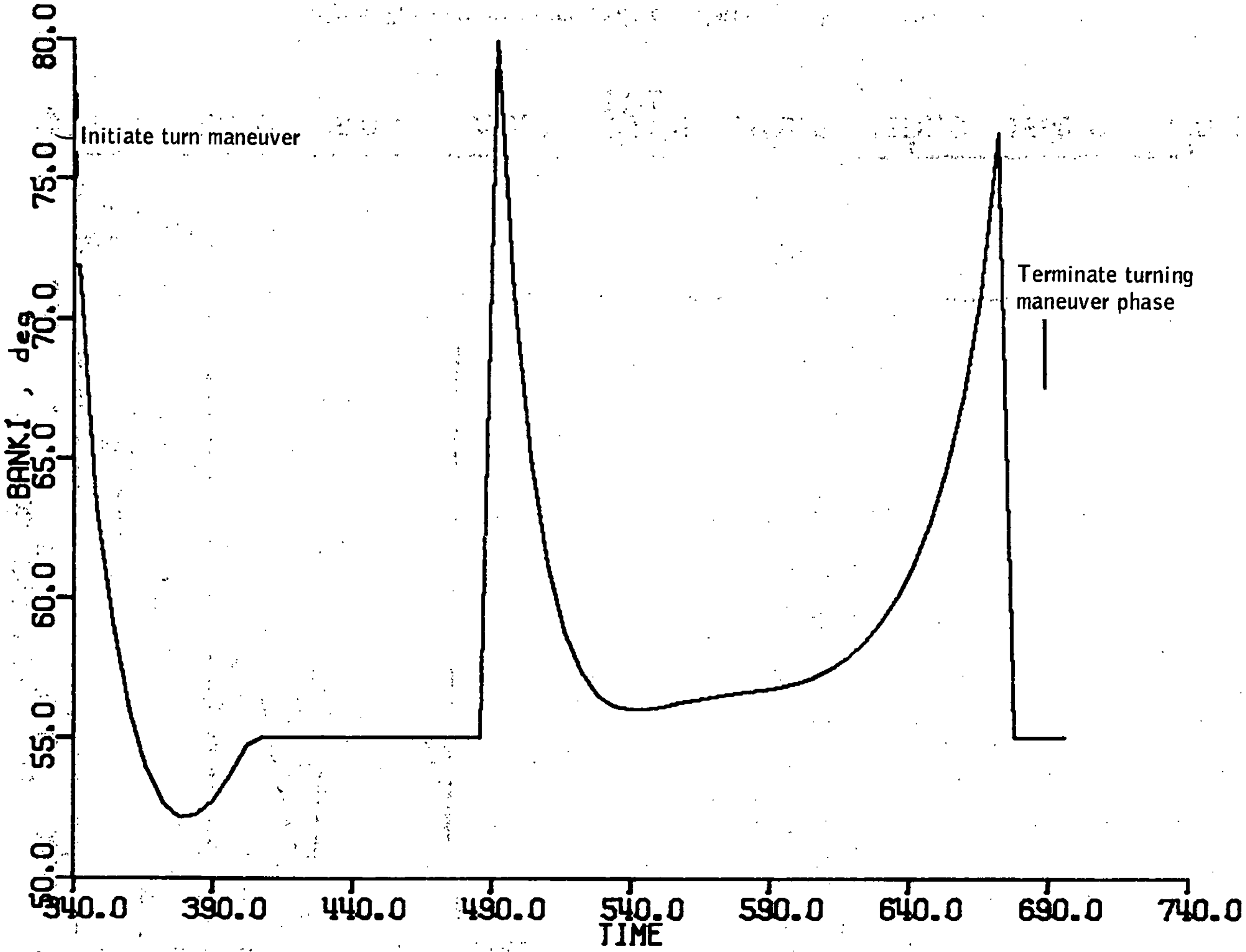
(c) Desired reference velocity time history.

Figure 6.- Continued.



(d) Inertial velocity time history.

Figure 6.- Continued.



(e) Limited bank angle command time history.

Figure 6.- Concluded.

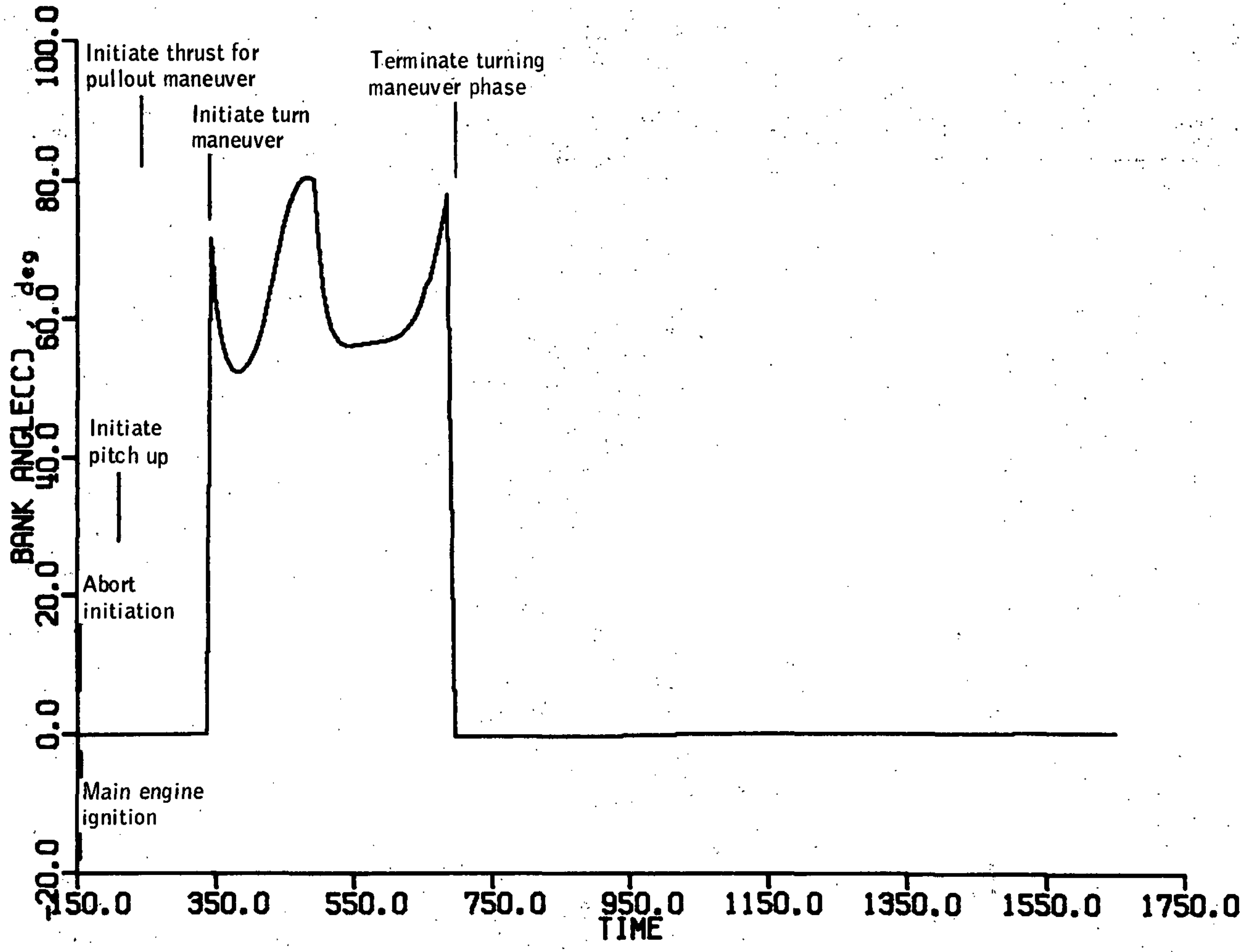


Figure 7.- Unlimited bank angle calculations for 150-second abort.

APPENDIX

DESCRIPTION OF THETA AND ALPT TURNING ANGLE RELATIONSHIPS

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The computation of THETA θ and ALPT A_T incorporated in FBGUID, is based on the assumption that certain geometrical relations hold true for the flyback trajectory path. In reality, the orbiter trajectory deviates from this assumption because of vehicle constraints which perturbate the geometry; however, the deviations do not prevent convergence to an acceptable solution.

Referring to figure 3, the geometrical relations for A_T and θ can be obtained as follows.

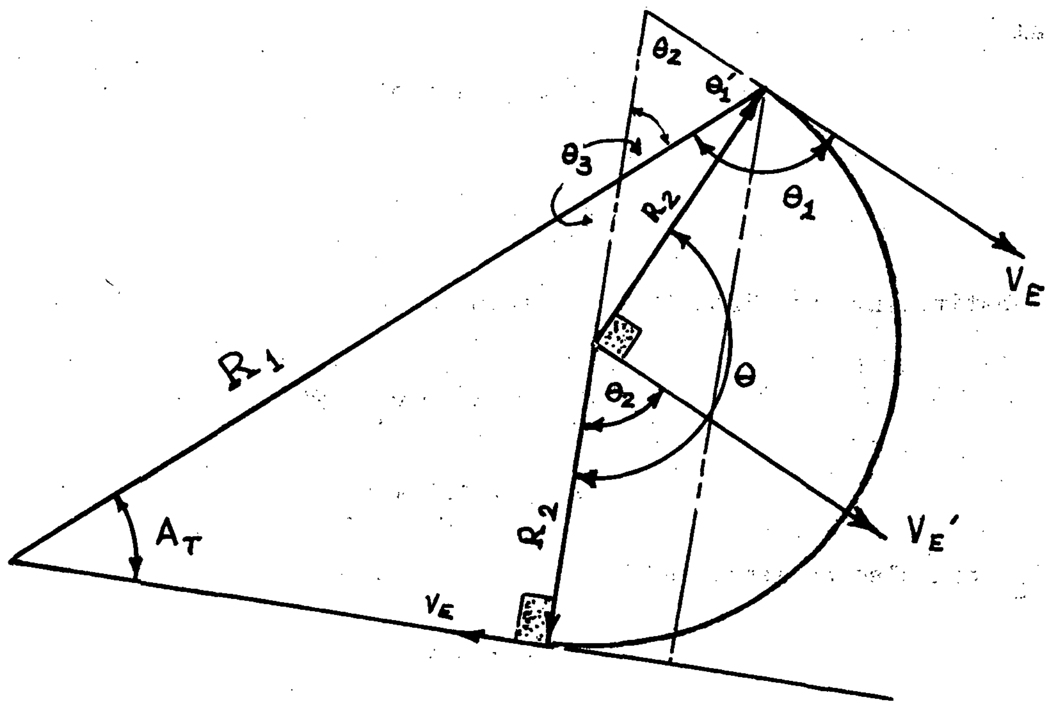

Figure A-1.- THETA and ALPT trigonometric relations.

$$R_1 \sin A_T = R_2 + R_2 \cos \theta' \qquad (1)$$

since

$$\cos \theta' = -\cos \theta$$

$$R_1 \sin A_T = R_2 (1 - \cos \theta) \qquad (2)$$

to equate θ in terms of θ_1 and A_T, consider

$$\theta_1' + \theta_2 + \theta_3 = 180° \qquad (3)$$

also

$$A_T + 90° + \theta_3 = 180° \qquad (4)$$

equating (3) and (4) we obtain

$$\theta_2 = A_T - \theta_1' + 90° \qquad (5)$$

also

$$\theta_1' + \theta_1 = 180°$$

$$\theta_1' = 180° - \theta_1$$

Substituting θ_1' into (5) we obtain

$$\theta_2 = A_T - 180° + \theta_1 + 90°$$

$$\theta_2 = A_T - 90° + \theta_1 \qquad (6)$$

θ_2 can also be expressed by

$$\theta_2 = \theta - 90°$$

therefore,

$$\theta - 90^\circ = A_T - 90^\circ + \theta_1$$

or

$$\theta = A_T + \theta_1 \qquad (7)$$

Substituting θ into (2)

$$R_1 \sin A_T = R_2[1 - \cos (A_T + \theta_1)] \qquad (8)$$

expanding the right side

$$= R_2(1 - \cos A_T \cos \theta_1 + \sin \theta_1 \sin A_T)$$

assuming a small correction angle, A_T is achieved as the turn progresses as compared to θ_1

$$\cos A_T = 1$$

and

$$\sin A_T = A_T$$

then equation (8) becomes

$$R_1 A_T = R_2(1 - \cos \theta_1 + A_T \sin \theta_1) \qquad (9)$$

$$A_T = \frac{R_2}{R_1} (1 - \cos \theta_1) + \frac{R_2}{R_1} A_T \sin \theta_1$$

$$A_T(1 - \frac{R_2}{R_1} \sin \theta_1) = \frac{R_2}{R_1}(1 - \cos \theta_1)$$

$$A_T = \frac{R_2}{R_1} \frac{(1 - \cos \theta_1)}{(1 - \frac{R_2}{R_1} \sin \theta_1)}$$

finally

$$A_T = (1 - \cos \theta_1)/\frac{R_1}{R_2}(1 - \sin \theta_1) \qquad (10)$$

Expression (10) is computed in conjunction with expression (7) in FBGUID to yield turn angle values.

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