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CONCERNING THE ORIGIN OF NOVAE AND U GEMINORUM STARS

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ABSTRACT

The dynamical stability of the contact component in a semi-detached binary system is investigated by a linear time dependent analysis. The boundary condition imposed on the photosphere by the Roche lobe is assumed to be one of constant pressure over the whole surface of the lobe, which is treated as being spherically symmetric. The eigenvalues of the linear equation of motion are evaluated for sequences of stellar envelope models under the constraint of this boundary condition. An extensive region in the Hertzprung-Russell diagram is found to be dynamically unstable. This region coincides almost precisely with that predicted by a previous quasi-static study. Timescales for the growth of the instability are also evaluated. Color observations of several novae and U Geminorum stars are discussed in an attempt to provide semi-empirical tests of the proposed theoretical model of the outbursts. In particular the track in the H-R diagram of the U Geminorum star, EM Cygni, is shown to be consistent with an outburst which is purely a temperature change at constant radius, as is demanded by the theory.

I. INTRODUCTION

During the past decade strong evidence has accumulated to suggest that all novae and nova-like variables are binary systems of extremely short perid. The model generally proposed to account for the spectroscopic and photoelectric variations is one in which the cooler red component is in contact with its bounding zero velocity surface, or Roche lobe, whilst the blue dwarf component is well detached. This semi-detached binary model was first suggested by Crawford and Kraft (1956) to account for the observations of AE Aqr and has been well substantiated by later investigations of other systems by Kraft (1963), Krzeminski (1965), Mumford (1967) and others.

Determining which of the two stars is the seat of the explosion in these systems has proved exceptionally difficult. However in the case of U Germinorum Krzeminski's (1965) photoelectric observations at different phases of eclipse indicate that the colour and luminosity changes occur in the cool companion filling its Roche lobe, and not in the blue component. These observations have been confirmed by both Paczynski (1965) and Mumford (1967). Mumford further suggests that following each luminosity increase the blue component is surrounded by a ring of material which is most extensive following an outburst and becomes more compact later. Such a conclusion would appear to indicate that during each outburst the red component overflows its lobe, spilling material into the Roche lobe of the companion.

In the light of these detailed observations it becomes essential to examine the dynamical stability of semi-detached binary systems from a theoretical point of view. Any dynamical instability caused by mass loss from the surface of the contact component must produce rapid and, if of any significant scale, dramatic effects within the system. Since the photosphere must suffer drastic changes in structure during the course of such an instability, with continual replenishment of material expanding from lower layers as the surface region is stripped off, large changes in the surface temperature and consequently of the luminosity must be expected within a dynamical timescale.

Such a model is very different in nature from the many suggested theoretical schemes previously proposed to account for the nova phenomena. These all associate the outburst with the blue component and implicitly ignore the observed semi-detached nature of these systems. Historically they are developments of theoretical models, such as the shock-wave theory of nova investigated by Hazlehurst (1962) for example, originating before the binary structure had been identified. However in the light of this identification a fundamental reexamination is obviously demanded of the intrinsic stability of such semi-detached binary systems.

Most numerical studies of semi-detached systems, by Kippenhahn et al. (1967), Plavec (1968), and others, have been exclusively restricted to studies of thermal instabilities by the very nature of their method of analysis. Bath (1969), henceforth referred to as Paper 1, studied the possibility of dynamical mass loss from the surface of the contact component. As a consequence of the radius of the cool star initially increasing as material is removed from the surface, explosive behaviour was predicted for initial models in certain regions of the log L/log Te diagram. With the simplifications implicit in the method of analysis used, the properties of the outburst by the contact component were found to be similar to those observed in novae and U Germinorum stars.

However this preliminary analysis was simplified in its assumptions, particularly that of quasi-static equilibrium during the mass loss process. Thus the acceleration terms were ignored, and the radius changes resulting from adiabatic readjustment to hydrostatic equilibrium following mass loss from stellar model envelopes were compared with the radius of the restricting Roche lobe. Furthermore the assumed constancy of the ratio of specific heats in any mass layer during the perturbation process is a strictly invalid approximation in the nonlinear regime for which the perturbation equations were derived. These criticisms of the analysis of Paper 1, taken together with the broad parallels which were shown there between the theory and the observable features of novae and nova-like variables demand further, if more circumspect, study of the stability problem.

II. BOUNDARY CONSTRAINTS ON THE CONTACT STAR

The acceleration term may be easily introduced if the analysis is restricted to a linear approach. Such a formulation allows the contact star to be tested for inherent instabilities to infinitesimal perturbations, but of course gives no information about the properties of the insuing instability apart from initial growth times and velocity distributions. In order to treat the nonstatic case a reinvestigation is required of the physical conditions imposed on the contact star by the nature of the potential fields in the rotating system. As is well known, the form of the combined centrifugal and gravitational fields is such that the contact star in a semi-detached system is confined within the limiting zero velocity surface. Expansion beyond this results in mass loss through the inner Lagrangian point, the point of potential maximum between the two stars. Thus the surface of the star is, in a naive sense, confined within a fixed surface defined by the spatial position of the inner Lagrangian point. The position of this will change as the mass ratio of the component alters, but if the mass loss is small, this will be negligible.

The question to be answered is what are the conditions that this

Lagrangian point exerts on the contact star? If the full problem could be treated with asymmetric models fitted self-consistently within the correct potential fields, the initial pressure structure would be such that the pressure gradient fell to zero at the Lagrangian point. In the following analysis the potential field is idealized to that of a single non-rotating spherically symmetric star. The detailed effects of asymmetry and the peculiar potential fields acting on the star are ignored.

The boundary condition on the outer surface which has been applied to simulate the properties of these stars in which the outer layers are no longer gravitationally bound when expanded over the Roche lobe, is a condition of constant pressure at constant radius over the whole outer photospheric surface of the star. Thus it is assumed that overflowing material is immediately swept away into the lobe of the companion star, the back pressure across the Lagrangian point being unaffected by this infall of material. The effect of the peculiar potential configuration at the zero velocity surface is therefore simulated by the heuristic mechanism of a continuous vacuum pump, which holds this surface at constant pressure. Under this constraint of a constant pressure acting at a fixed Eulerian boundary a linear analysis of the stability of the contact star may be performed in which the acceleration terms are included.

Such a condition of constant surface pressure will only be valid if the overflowing material is swept away rapidly from the surface into the lobe of the companion star. If this is not the case then the instability may well be inhibited or at least occur on a timescale determined by the rate of this mass flow. Order of magnitude estimates of this timescale suggest that it is comparable to the dynamical timescale of the contact star. Kriz (1970) obtains a stream velocity in the vicinity of the Lagrangian point of about 10 km/sec for an initially static star, implying a timescale in that region of something like a day. Though the analysis is not strictly applicable to the dynamic case being investigated here, it does give a lower bound to the mass loss rate, which in a dynamical outburst would be expected to be even greater.

III. LINEAR ADIABATIC RADIAL MOTION OF THE CONTACT STAR

The linearized equation of adiabatic radial motion has been derived, for example by Rosseland (1949). The well known equation (1) results

$$\rho \frac{d^2v}{dt^2} = \frac{\partial}{\partial r} (\Gamma_1 P \operatorname{div} \underline{v}) + \frac{4\rho g v}{r} \tag{1}$$

where the symbols have their usual meaning. Since time enters only through v, solutions of the form

$$v = v(r)e^{i\sigma t} \tag{2}$$

are possible. The constant σ may be either real or complex. If real then the solution is oscillatory as normally obtained for example in a star with a free boundary, or constant pressure boundary to the left of the Hayashi track. However if σ is complex, then an exponentially growing disturbance is obtained. It is this form of solution which determines definitively the instability of any model in the circumstances previously described. Substitution of (2) into (1) results in the standard wave equation for radial motion under gravity.

$$\frac{\partial}{\partial r} (\Gamma_1 P \operatorname{div} \underline{v}) + (\sigma^2 + \frac{4g}{r}) \rho v = 0 \tag{3}$$

In the present analysis the velocity rather than the displacement, δr, has been taken as the dependent variable since the latter is not such a meaningful concept in the Eulerian sense in which equation (3) will be applied. Equation (3) presents an eigenvalue problem to be solved with a boundary condition of constant pressure, constant that is at a fixed radius given by the initial radius of the photosphere in the model. Thus the outer boundary condition to be applied is that at this surface,

$$\frac{\partial P}{\partial r} = 0 \qquad (4)$$

In order to apply this condition to the solution of equation (3) it must be transformed to a constraint upon the dependence of v, the velocity amplitude, on the radius, r. For radial motions the equation of continuity is

$$\frac{d\rho}{dr} = -\rho \operatorname{div} \underline{v} \qquad (5)$$

and for purely adiabatic changes

$$\frac{dP}{dr} = \frac{\Gamma_1 P}{\rho} \frac{d\rho}{dr} \qquad (6)$$

Substitution of (6) into (5), together with the boundary condition (4) leads to

$$v \frac{\partial P}{\partial r} = - \Gamma_1 P \operatorname{div} \underline{v} \qquad (7)$$

Substituting for the pressure gradient from the equation of motion of a single star and introducing the homology invariant, $V_0 = d \log P/d \log r$, gives to first order in v,

$$\frac{\partial v}{\partial r} = \frac{v}{r} \left(\frac{V_0}{\Gamma_1} - 2\right) \qquad (8)$$

Thus the peculiar boundary condition imposes a steep velocity gradient on the contact star at the surface. For since V_0 tends to values of the order $\sim 10^4$ at the photosphere, (and to infinity of course for a surface at zero pressure), the velocity at the boundary grows in proportion to the radius to a very high power.

Thus if in fact the solution of equation (3) results in values of σ^2 which are negative, then a growing instability will result with an extremely steep velocity gradient at the surface. Only those layers actually close to the surface will suffer, and contribute to, the explosion. With this in mind, the inner boundary condition applied to envelope models with an inner boundary at $r \sim 0.1$ R has been taken as a rigid surface. Thus the velocity is assumed to fall to zero in the deep interior.

IV. ORIGIN OF THE INSTABILITY

To investigate the influence of the Eulerian constant pressure boundary condition on the stability of a star from an analytical point of view, equation (3) is best transformed to a standard Sturm-Louiville form. This is achieved by defining a new variable, ξ, the relative amplitude of the velocity variation v/r. Equation (3) then becomes

$$\frac{d}{dr}\left(r^4\Gamma_1 P \frac{d\xi}{dr}\right) + \left\{r^3 \frac{d}{dr}\left[\left(3\Gamma_1 - 4\right)P\right]\right\} \xi + \sigma^2 \rho r^4 \xi = 0 \tag{9}$$

with corresponding conditions at the inner boundary

$$\xi = 0 \tag{10}$$

and the outer boundary

$$\left(\frac{V_0}{\Gamma_1} - 3\right) \xi = \frac{d\xi}{dr} \tag{11}$$

This outer boundary condition compares with the more normal Lagrangian constant pressure condition which has the form,

$$-3\xi = \frac{d\xi}{dr} \tag{12}$$

Now the origin of dynamical instabilities in single stars is associated with the coefficient multiplying the amplitude in the second term of equation (9) [see, for example, Ledoux and Walraven (1958)]. If this is positive definite then negative eigenvalues for certain modes may occur. However a finite number of negative eigenvalues may also arise if the coefficient multiplying the amplitude in the outer boundary condition is positive. This has been demonstrated for the generalized Sturm-Louiville problem by Courant and Hilbert (1953). Consideration of the boundary conditions, equations (11) and (12), shows that the coefficients have opposite sign in the Lagrangian and Eulerian constant pressure boundary condition cases, and, in particular, in the latter case this coefficient is a large ($\sim 10^4$) positive number. Thus, in those models for which the eigenvalues are positive when studied as normal isolated single stars, negative eigenvalues causing growing dynamical disturbances may occur when the pressure at some fixed radius is held constant. Thus it is seen that the form of the outer boundary condition directly leads to the possibility of dynamical disturbances being generated in stars which would normally be stable.

V. METHOD OF SOLUTION FOR DETAILED MODELS

The eigenvalues and eigenfunctions of equation (3) have been found for sequences of detailed envelope models of various masses. The models were constructed using the program of Moss (1968) with minor modifications to include radiation pressure. The composition was taken as X = 0.695, Y = 0.280, and Z = 0.025, as in the models of Paper 1. The envelope models were fitted to Iben's (1967) evolutionary tracks except in the case of the 30 M$_\odot$ sequence for which the track of Stothers (1966) was used to supply values of luminosity and effective temperature.

The method of solution was a convergence procedure based upon the fitting point method. Equation (3) was integrated both inward and outward from the boundaries to the fitting point. Defining α as

$$\alpha = \left.\frac{\frac{dv}{dr}}{v_1}\right|_1 - \left.\frac{\frac{dv}{dr}}{v_2}\right|_2$$

where 1 and 2 are solutions at the fitting point resulting from inward and outward integrations, then the roots for which α is zero define the solutions, and corresponding eigenvalues for any model. These roots were found in the computational procedure by the Newton method, starting with an initial guessed value of σ^2.

If such a procedure led to the first harmonic being obtained then the fundamental was found by successively decreasing the eigenvalues of the first harmonic. If necessary this was reduced to a negative value until the root finding procedure converged on the fundamental. This method allows, within the accuracy of the numerical method, all modes with real roots to be found, and treats both boundary conditions explicitly.

VI. RESULTS

Solutions were obtained for all models in Table 1, shown in the log L/log Te diagram of Figure 1. The 1 and 5 M$_\odot$ models correspond to those discussed in Paper 1 on the basis of the quasi-static method. In the case of the fundamental mode both unstable and stable solutions were obtained, with eigenvalues tabulated in Table 1.

The fundamental question relating to this analysis is how does the extent of the unstable region compare with that predicted by the quasi-static analysis in the H-R diagram? The region for which σ^2 is negative is shown in the shaded area of Figure 1. As expected the unstable region occurs only at lower surface temperatures in models with extensive ionization regions, but the boundary is well to the left of the Hayashi track, in the region of dynamical stability for single stars. The unstable region found by the quasi-static analysis lies leftwards of the dashed line. This boundary agrees closely with that of the shaded region predicted by the time dependent study described here. In fact only one model, number 4 of 5 $M_\odot$, has been found to show different behavior according to the method of analysis used, and this is close to the border of the dividing region. From this close correspondence between the results of this analysis and that of Paper 1 it would appear that the approximations inherent in the latter are valid, at least to determine the stability of models initially. The inclusion of the acceleration terms does not radically alter the conditions of stability for the models. The closeness of agreement is almost surprising considering the fundamental differences between the two methods.

The pattern of the eigenfunction for several unstable models of 1 $M_\odot$ is shown in Figure 2. The steep velocity gradients at the surface imposed by the outer boundary condition are evident from this figure. They also demonstrate that for the unstable models the depth at which the velocities become negligible increases as the surface temperature of models decreases. In other words the instability becomes stronger and more deep seated in the star with decreasing surface temperature. In the case of the stable models, shown in Figure 3 for some of the 15 $M_\odot$ envelopes, the same feature is evident, but these of course correspond to pulsational oscillations which may not necessarily grow.

In the case of the higher modes, no value of σ^2 negative was obtained. It seems that in normal circumstances the higher harmonics are stable. The pattern of the solutions for the fundamental, and 1st and 2nd harmonics of the 1 $M_\odot$ model 7 are shown in Figure 4.

Determination of the eigenvalues of unstable models allows a growth time for the disturbance to be derived. The velocity increases by a factor two in a time τ given by

$$\tau = \frac{\log_e 2}{i\sigma} \tag{13}$$

The values of this growth time are given in Table 1. A trend of increasing τ with decreasing surface temperature for models of a given mass is evident, as was also found in the crude estimates of Paper 1. In the case of those models that were class I unstable in the quasi-static study, that is dynamically unstable until the removal of material above the HeI ionization zone, the timescale is seen to be extremely short, of the order of minutes and tens of minutes. Those models found to be unstable down to the HeII zone have longer timescales, between hours and several days.

It is concluded therefore that on the basis of the simplifications necessary to treat spherically symmetric models, stars within the unstable region of the H-R diagram of Figure 1 will be dynamically unstable as contact members of semi-detached binary systems. If these contact components do indeed have their photospheres at the Roche surface, and the companion is well within the neighboring lobe, then a growing disturbance in which the surface of the contact star is progressively stripped off would be expected to occur on a dynamical timescale. Only a full time-dependent non-linear analysis will show whether the further conclusions of Paper 1 are correct, in particular that the instability continues until the ionization regions are exposed at the surface. Such an analysis would also allow a full investigation of the mode whereby stability is restored. If non-adiabatic it would also be possible to study the important effects of thermal readjustment and recombination during and following the initial dynamical outburst. A forthcoming paper will treat the non-linear time dependent adiabatic case, and it is hoped to treat the non-adiabatic problem in future work.

VII. SEMI-EMPIRICAL TESTS

The proposed model of novae and explosive variable stars allows direct semi-empirical tests to be performed using the observations of individual systems. The basis of these tests arises from the differences in the nature of the explosion as described by the proposed binary model, and the previously accepted single star, shock-wave or accretion theory of novae. In the latter model the luminosity change is envisaged to arise primarily from a radius expansion of the photosphere, this photosphere remaining at constant temperature or even cooling during the explosion. The pre-nova, considered as the blue star alone, is regarded as having expanded spherically until roughly comparable to a supergiant star at low temperature. The atoms that have already passed outward through the photosphere surround the nova with an expanding envelope giving rise to wide, bright emission lines.

In the theory presented in Paper 1 and corroborated by the linear analysis presented here, the pre-nova, identified as the cool, distended red giant component of a semi-detached binary system, suffers dynamical instability when it first starts losing mass through the inner Lagrangian point. This results in a rapid welling of deep hot material to the surface as the outer layers are stripped off into the lobe of the companion star. The luminosity change occurs as a result of temperature changes in the exposed layers, the photosphere being restricted, at least in the initial stages to within the Roche lobe. As a result the surface changes in nature from that of a cool giant to a hot, extremely over-luminous supergiant photosphere, whose radius is essentially constant at all times.

Emission lines will be produced in material surrounding the blue star, and in that which has sufficient energy from the initial instability to avoid accretion by the dwarf component. If the exploded material co-rotates initially and is concentrated in regions of lowest potential then the greatest mass loss will occur through the outer Lagrangian points of the binary system. This suggests that the observed shell structure of the emission lines may be rather that of expanding spirals. This would agree with the conclusion of Payne-Gaposhkin (1957) that the saddle shaped profiles of the emission lines can only be accounted for satisfactorily according to a model in which the emitting atoms are concentrated in a ring that is both rotating and expanding. The line formation process in such a system has been discussed by Sobelov (1947) in connection with Be stars, but such an expanding, rotating ring system must necessarily arise in novae if the mechanism being proposed is correct. Baade [see Payne-Gaposhkin (1957)] has in fact reconstructed the structure of the emitting region of V603 Aql as a series of axially symmetric rings, expanding away from the central nova. It would be of interest to discover whether the plane of this ring system coincided with the orbital plane of a central binary system.

However the essential feature of the proposed model is the constancy of the radius of the emitting photosphere. The difference between the behavior predicted by the classical theory and this model is obviously open to observational tests, if the variation in photospheric radius can be derived. This may be obtained if the changes in the continuum are known. However the interpretation of color observations of novae and nova-like variables is notoriously difficult because of the influence of overlying emission lines.

In the case of the nova-like variable EM Cygni observed by Mumford and Krzeminski (1969) the color changes during the outburst remain close to the black body line in the two color diagrams. If the radiation is assumed to be black body then bolometric corrections may be applied to obtain the bolometric magnitude changes, and the color index transformed to an effective temperature. This has been done for the observed points in Figure 5 using the values of Harris (1963). The temperature is seen to vary from 9,600°K near maximum to 5,750°K near minimum, with the apparent magnitude declining from 12.25 to 13.73. This is the sort of variation predicted for class I instabilities in Paper 1. No account has been taken of interstellar absorption which would tend to increase both the effective temperature and the magnitude range.

If the radiation is indeed black body then the bolometric magnitude varies as,

$$m_{bol} = - 10 \log_{10} Te - 5 \log_{10} R + K$$

If the explosion does proceed at essentially constant radius, then the variation of bolometric magnitude with the logarithm of the effective temperature should be linear with a gradient of - 10. In Figure 5 the continuous line is of such a slope. Comparison between the observed points and the theoretically expected change is remarkable considering the lack of sophistication in the analysis.

Application of this analysis to other explosive stars appears to indicate tremendous differences of behavior. However in most cases where all three UBV colors have been observed simultaneously those systems which do not agree with the predicted behavior also diverge by large amounts from the black body line in the two color diagram. To take some examples, Mumford's (1966a, 1966b, 1968) observations of recent light variations in Nova GK Persei agree well with an outburst at constant radius, as do the observations of Zuckermann (1961) and Grant and Abt (1959) of SS Cygni, though only in this latter case with large scatter and not at all near maximum. Paczynski's (1965) observations of U Geminorum, combined with those of Krzeminski (1965), though indicating an increase in temperature initially, also diverge rapidly from a line of slope -10 in the region of maximum light.

However the difficulty of analyzing the observations with standard photoelectric filters is dramatically illustrated by reference to the observations of Nova DQ Her by Chincarini, van Genderen, Shen Liang-Zhao, and Kreiner, Kurpinska and Winiarski (1966), all of which are discussed in the latter paper. Vast differences in B-V values during the decline are found by different observers. Only those of the last group, whose V filter has a large spectral sensitivity in the yellow, agree well with the interpretation that during decline the photosphere remains at constant radius.

The above simplistic semi-empirical analysis would be expected to show some disagreement between the theory and observations, even if the UBV values were not influenced by emission lines. For no account has been taken of the contribution to the luminosity by the blue component, which must certainly play a role in determining the energy distribution of the underlying continuum. Accretion by this component would be expected to give rise to X-ray emission according to the suggestion of Shklovsky (1967) and the later work of Zel'dovitch and Shakura (1969) and Cameron and Mok (1967). However it is interesting to note that the optical variation of Cyg X-2 observed by Kristian et al. (1967) agrees approximately with the interpretation of a luminosity change at constant radius. The slope of the linear relation in the magnitude/color diagram observed by Mook (1970) in Sco X-1 does not show such satisfactory agreement, though it would indicate some temperature increase accompanying the luminosity rise.

The application of the model to the so-called symbiotic variables, with explosive outbursts similar to but more irregular than U Geminorum stars is also tempting. Their suggested binary nature, with one cool red giant component, and the other a blue component, obviously fits the requirements of the theory, but it remains to be shown whether they are indeed semi-detached.

A major problem remains in the observed stability of Algol systems, the classic semi-detached systems. If the contact components do indeed have their photospheres at the Roche lobe, then according to this study they would be expected to lie in the stable region of Figure 1. Although this is indeed the case for the best observed systems known to be semi-detached, it is not true generally. This is particularly the case for those systems examined by Kopal (1959) whose luminosity and temperatures were derived on the assumption that they were semi-detached. Whether this latter assumption is incorrect, or whether the stability of these systems is related to the properties of the companion B or A main sequence star is presently unknown. Certainly heating of the contact component by the companion through the reflection effect must influence the structure of the contact star. The periods of Algol systems are generally longer than those of U Geminorum stars and there is evidence [Kopal and Shapley (1955)] that the secondary of Algol itself may be slightly smaller than its corresponding Roche limit. If this were true of all these stable systems with cool subgiant secondaries the problem would necessarily be resolved. The stability of Algol systems to dynamical mass loss might on the other hand be due to changes of structure that are indicated by the thermal timescale calculations of Kippenhahn et al. (1967) and others. Such stars, having suffered considerable mass exchange to the companion, or perhaps mass loss from the system, cannot be simulated by the envelope models fitted to Iben's tracks in this paper.

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Table 1

Eigen values

Model	$M^*/M_\odot$	$L^*/L_\odot$		Te	σ^2	τ sec
1	1	7.94	10^{-1}	5,754	-4.48×10^{-4}	3.27×10
2	1	1.58		6,025	-2.22×10^{-4}	4.65×10
3	1	2.82		5,370	-2.44×10^{-5}	1.40×10^{2}
4	1	2.63		4,898	-8.12×10^{-6}	2.43×10^{2}
5	1	1.00	10	4,265	-1.41×10^{-7}	1.83×10^{3}
6	1	5.37	10	3,981	-3.94×10^{-9}	1.10×10^{4}
7	1	3.98	10^{2}	3,548	-4.53×10^{-11}	1.03×10^{5}
1	5	6.31	10^{2}	18,621	1.71×10^{-6}	
2	5	1.00	10^{3}	15,488	2.66×10^{-7}	
3	5	1.26	10^{3}	14,791	1.40×10^{-7}	
4	5	1.05	10^{3}	10,000	-1.34×10^{-8}	6.00×10^{3}
5	5	7.94	10^{2}	6,310	-9.45×10^{-8}	2.25×10^{3}
6	5	6.92	10^{2}	4,467	-3.14×10^{-9}	1.24×10^{4}
7	5	1.26	10^{3}	3,981	-2.62×10^{-10}	4.28×10^{4}
8	5	1.99	10^{3}	7,943	-3.75×10^{-8}	3.58×10^{3}
9	5	1.78	10^{3}	4,786	-1.54×10^{-9}	1.77×10^{4}
1	9	4.47	10^{3}	25,119	9.71×10^{-7}	
2	9	7.94	10^{3}	20,893	3.54×10^{-8}	
3	9	1.00	10^{4}	18,197	1.22×10^{-7}	
4	9	9.33	10^{3}	9,332	4.82×10^{-10}	
5	9	5.75	10^{3}	4,169	-9.23×10^{-11}	7.21×10^{4}
6	9	1.00	10^{4}	3,802	-1.20×10^{-11}	2.00×10^{5}
7	9	1.15	10^{4}	6,309	-1.72×10^{-9}	1.67×10^{4}
8	9	1.66	10^{4}	14,125	3.23×10^{-9}	
9	9	1.66	10^{4}	9,333	2.46×10^{-10}	

Table 1 (continued)

Model	$M^*/M_\odot$	$L^*/L_\odot$		Te	σ^2	τ sec
1	15	2.09	10^4	32,359	7.38×10^{-7}	
2	15	4.47	10^4	26,302	1.47×10^{-8}	
3	15	5.37	10^4	22,385	5.49×10^{-8}	
4	15	5.75	10^4	17,378	2.73×10^{-9}	
5	15	6.31	10^4	15,488	1.16×10^{-9}	
6	15	7.08	10^4	12,589	2.65×10^{-10}	
7	15	7.94	10^4	9,120	3.07×10^{-11}	
8	15	7.41	10^4	6,130	-1.22×10^{-10}	6.27×10^4
9	15	7.07	10^4	5,012	-2.46×10^{-11}	1.40×10^5
10	15	7.94	10^4	3,981	-1.31×10^{-12}	6.05×10^5
1	30	1.34	10^5	43,651	5.63×10^{-7}	
2	30	3.39	10^5	25,119	3.20×10^{-9}	
3	30	3.71	10^5	12,590	3.65×10^{-11}	
4	30	3.76	10^5	24,271	2.23×10^{-9}	
5	30	3.89	10^5	12,590	3.65×10^{-11}	
6	30	3.97	10^5	8,913	4.28×10^{-12}	
7	30	3.98	10^5	7,499	4.22×10^{-13}	
8	30	3.98	10^5	6,310	-1.79×10^{-11}	1.64×10^5
9	30	3.98	10^5	5,012	-3.74×10^{-12}	3.59×10^5
10	30	3.98	10^5	3,981	-2.57×10^{-13}	1.37×10^6

FIGURE CAPTIONS

Figure 1. Region of dynamical instability predicted by the linear analysis is shown shaded in the H-R diagram above. The instability region predicted by the quasi-static method lies to the right of the dashed line. The dash-dotted line indicates the region of demarcation between class I and class II instabilities from this latter method. Numbers refer to masses in solar units of envelope models fitted at the points shown to Iben's or Stothers' evolutionary tracks. Individual models are referred to by number in the text, starting with model 1 on the main sequence and increasing consecutively along the track.

Figure 2. Eigenfunctions for the fundamental mode of some unstable 1 $M_\odot$ models near the surface.

Figure 3. Eigenfunctions for the fundamental mode of some 15 $M_\odot$ models. All these are dynamically stable.

Figure 4. Eigenfunctions for the fundamental and 1st and 2nd harmonics in model 7 of 1 $M_\odot$. Only the fundamental has a negative value of σ^2.

Figure 5. Points show the observed variation of temperature and magnitude of the nova-like variable EM Cygni as deduced from UBV observations. The continuous line indicates the expected slope for changes occurring at constant radius.

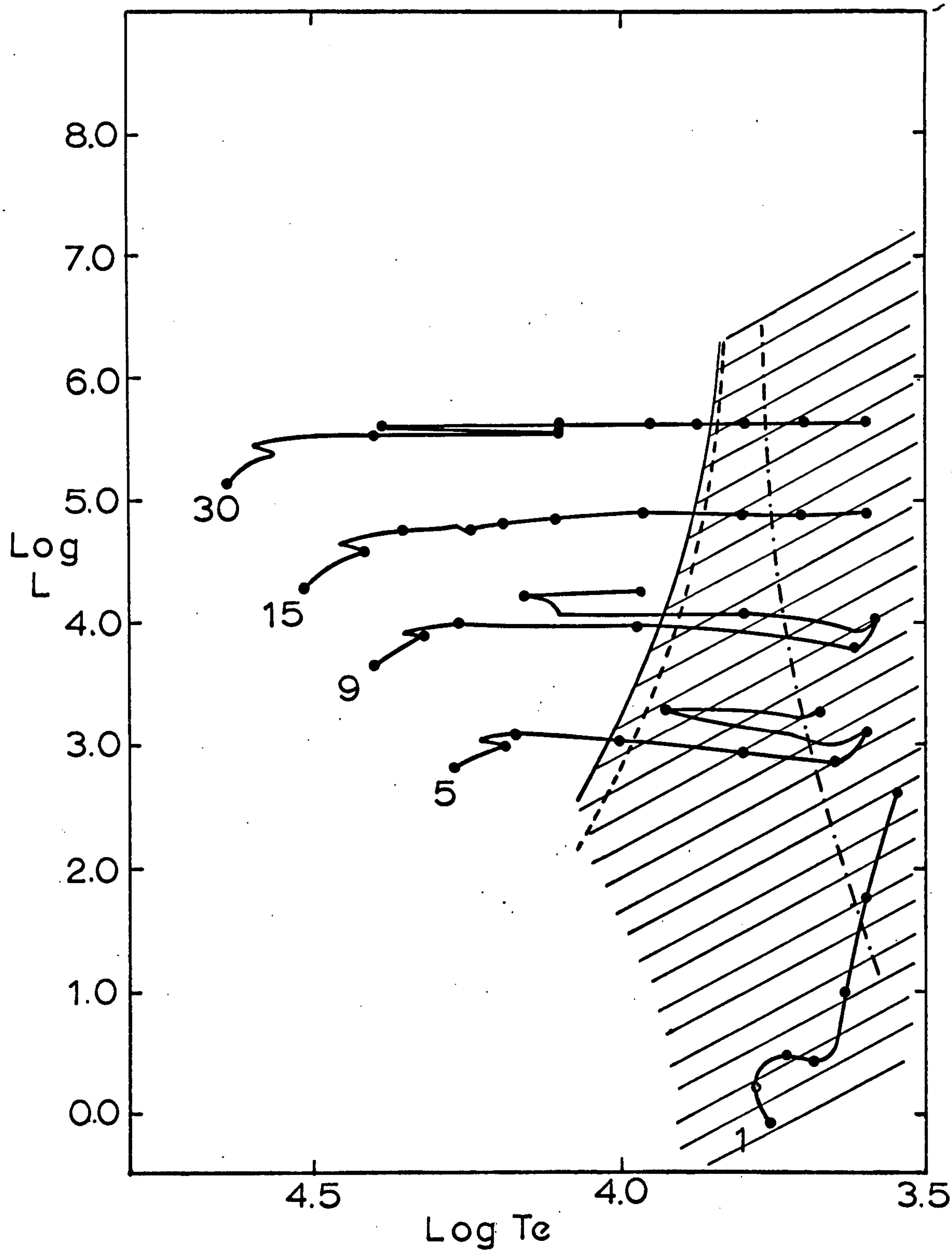


Figure 1

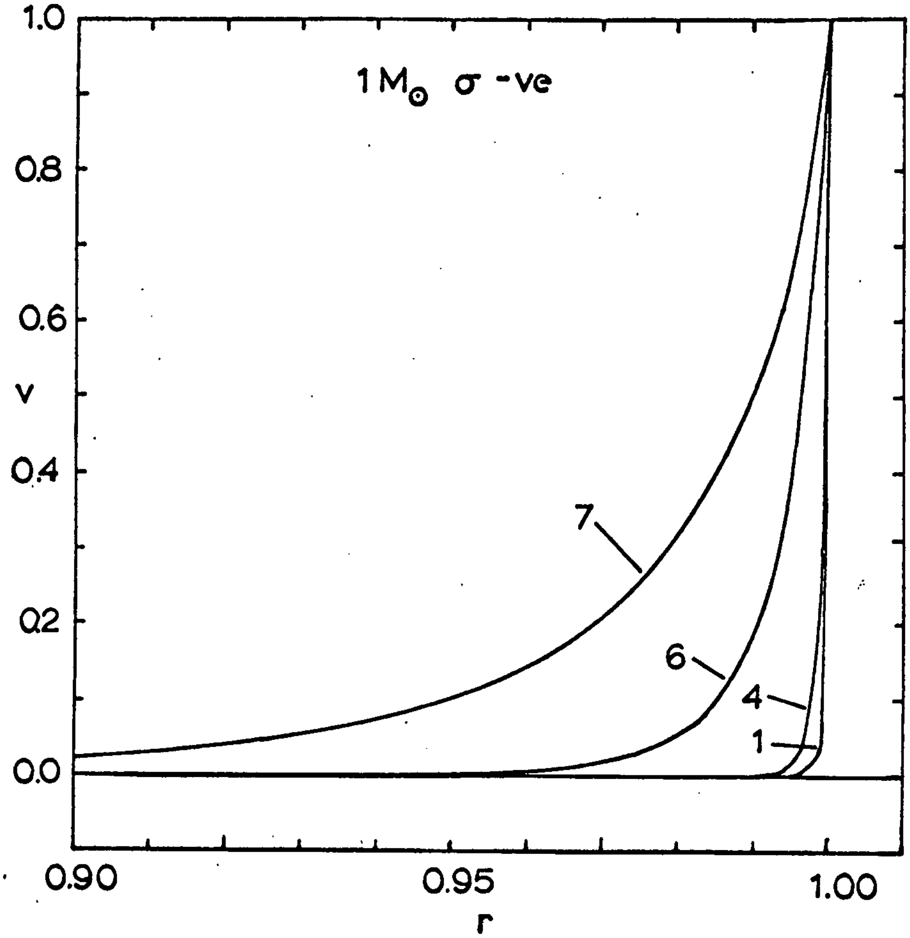


Figure. 2

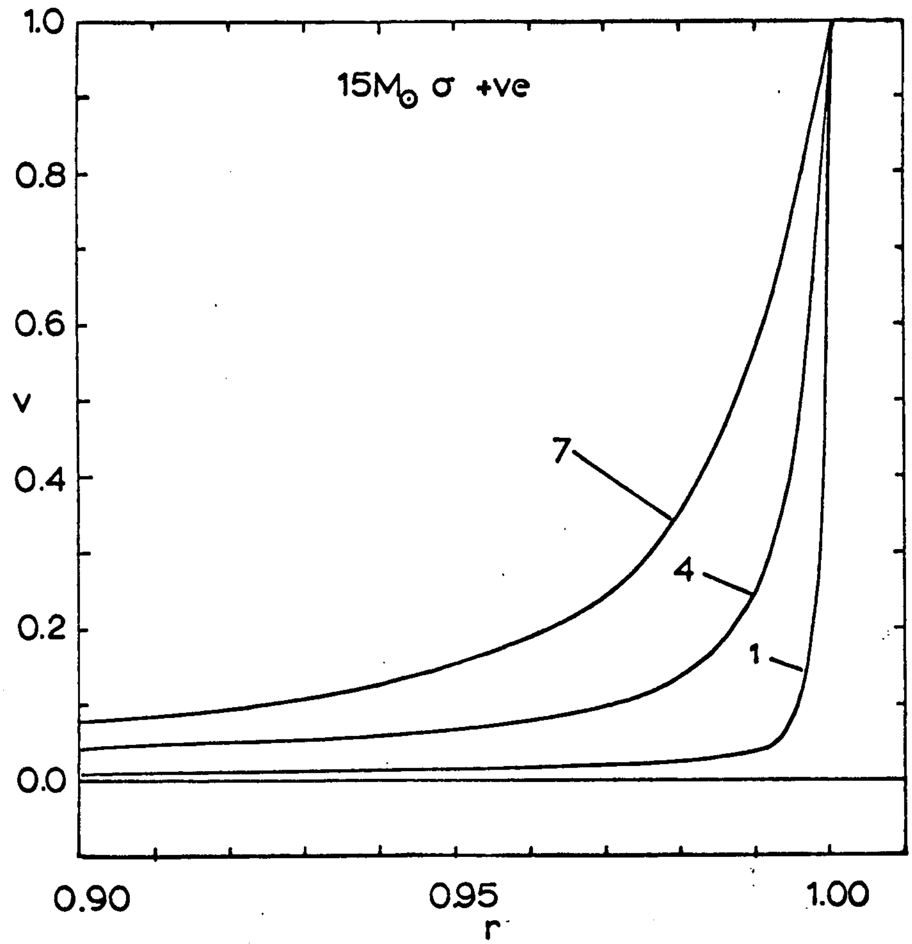
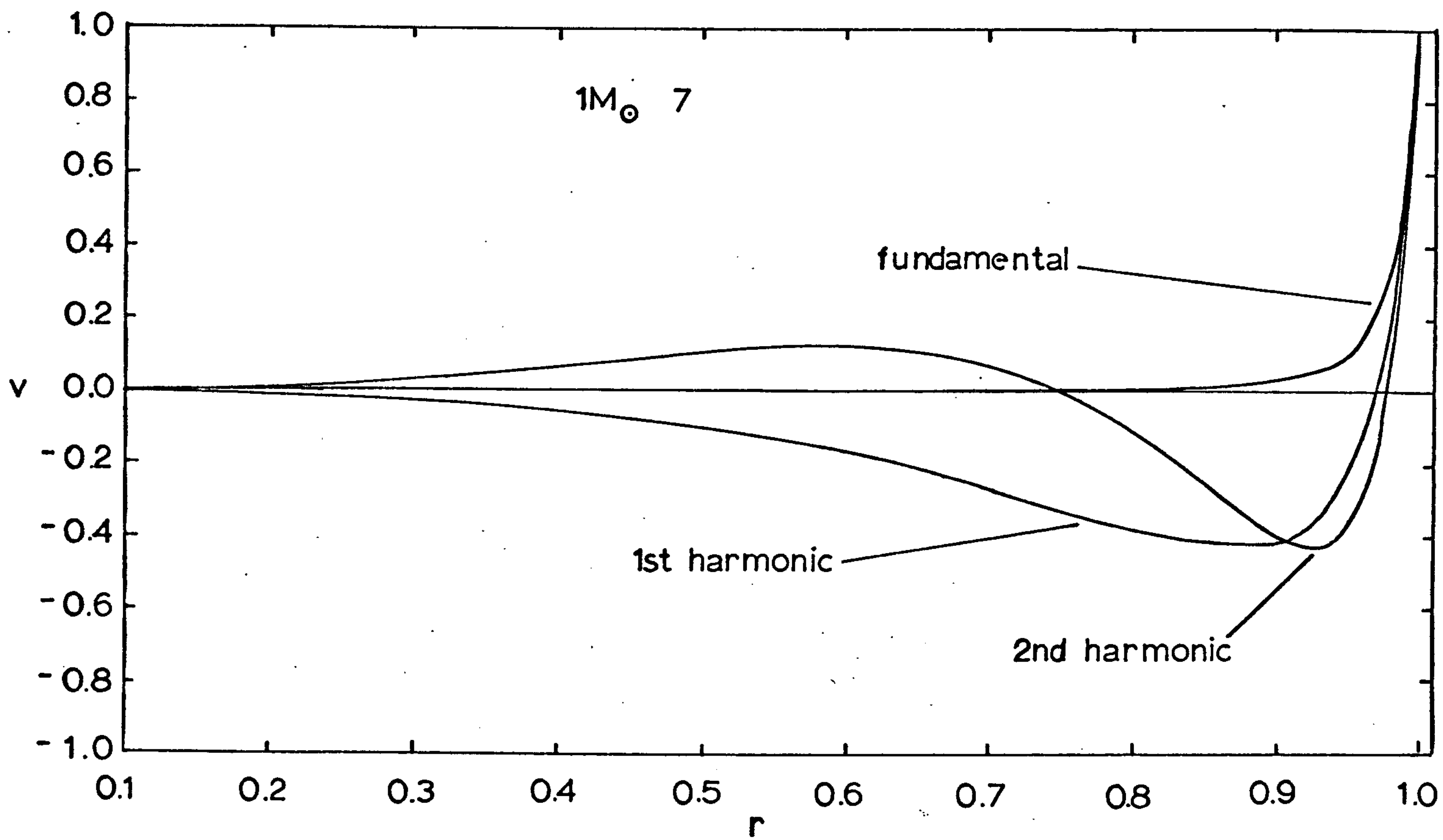


Figure 3



1M⊙ 7
fundamental
1st harmonic
2nd harmonic
v
r
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Figure 4

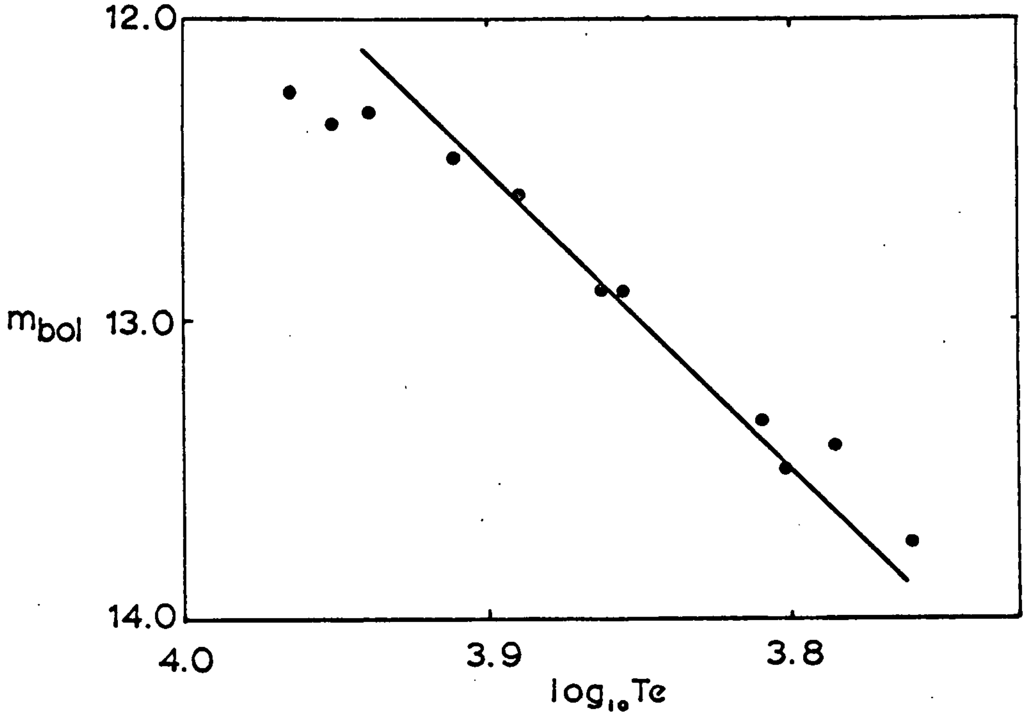


Figure 5