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# TECHNICAL MEMORANDUM

## PROBING ATMOSPHERIC WATER VAPOR PROFILES VIA MULTIPLE SCATTERING OF ELECTROMAGNETIC WAVES



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ABSTRACT

A theoretical analysis on the multiple scattering of electromagnetic waves propagating in a finite inhomogeneous medium is presented and applied to the study of wave propagation in a clear atmosphere (fine weather conditions).

It is shown that the analysis offers a method of synthesizing the water vapor density profile in a clear atmosphere by measuring the resultant reflections from the density profile at several different frequencies. It is also shown that the resultant reflection emerges as the consequence of multiple scattering of partial reflections from various parts of the inhomogeneous medium.

The solutions of the multiple scattering approach are shown to be more accurate than those of the WKB approach, which neglects the multiple scattering effects.

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TECHNICAL MEMORANDUM

## I. INTRODUCTION

When an electromagnetic plane wave of centimeters or longer wavelength propagates vertically through a clear atmosphere (fine weather conditions) and produces a reflection, the reflection is always associated with some identifiable and abnormally steep gradients of dielectric constant due to inhomogeneities in the water vapor density in the atmosphere. Anomalies in dielectric constant gradients of the atmosphere, however, are not always accompanied by reflections. These observed features cannot be explained satisfactorily by present available theories that do not take into consideration the effects of small internal reflections or multiple scattering of electromagnetic waves propagating in a finite inhomogeneous medium. However, these observed features can be explained readily by the theoretical analysis, in the next section, that includes multiple scattering effects. The analysis indicates that in general the resultant reflection from these steep dielectric constant gradients is not large enough to cause any significant loss of transmission, but is large enough to be used in probing water vapor concentration profiles in the atmosphere. It is shown that the interference effect of atmospheric water vapor layers with an undulating density profile is responsible for the measurable resultant reflection or total cancellation of partial reflections. For a given profile of water vapor density, the reflection characteristics are found to be sensitive to frequencies used in probing. For some models of idealized water vapor density profiles, each model of profile can be determined or synthesized by one or two reflection measurements at one or two appropriate frequencies. To determine the water vapor density profile of arbitrary shape, reflection measurements at more than two appropriate frequencies will be necessary.

Local anomalous variations of water vapor density in a clear atmosphere due to temperature inversion, wind shear, and clear air turbulence (CAT) are usually detectable as radar echos.[1-4] The determination of these anomalous water vapor density profiles in a clear atmosphere is important in characterizing the nature of transmission of infrared radiation between the surface of the earth and space, since water vapor is one of the atmospheric gases that cause the bulk of infrared resonant molecular absorption.

In the following section we shall present a general formulation applicable to the analysis of multiple scattering of electromagnetic waves propagating in a finite inhomogenous medium which has a slow and smoothly continuous variation in its dielectric properties along the axis of propagation.

## II. FORMULATION OF THE PROBLEM

A medium which is homogeneous and infinite in extent in both the x and y directions but finite in extent and inhomogeneous in the z direction is impinged by a plane electromagnetic wave in the z direction. The inhomogeneity is assumed to be in the dielectric property $\varepsilon$ of the medium only and the profile of the inhomogeneity in the z direction is assumed to be arbitrary except for nonvanishing $\varepsilon$ and continuous profile derivatives. The region of the inhomogeneous medium extends from $z = 0$ to $z = \ell$. The Maxwell equations for the problem, with the time factor $e^{i\omega t}$ understood are:

$$\frac{dE_y}{dz} = i\omega\mu_o H_x \quad , \qquad \frac{dH_x}{dz} = i\omega\varepsilon_o \varepsilon E_y \quad ; \tag{1}$$

$$\frac{dE_x}{dz} = -i\omega\mu_o H_y \ , \qquad \frac{dH_y}{dz} = -i\omega\varepsilon_o \varepsilon E_x \ , \tag{2}$$

where E and H are, respectively, the electric and magnetic field, $\omega$ is the angular frequency of the incident wave, $\mu_o$ and $\varepsilon_o$ are, respectively, the magnetic and electric permittivity of the free space, $\varepsilon$ is the complex relative electric permittivity, which is a function of z, and the subscripts indicate the direction of the field.

Assuming the electric field of the incident plane wave is linearly polarized in the y direction only, we obtain the wave equations as:

$$\frac{d^2E_y}{dz^2} + k_o{}^2\varepsilon E_y = 0 \quad , \tag{3}$$

and

$$\frac{d^2H_x}{dz^2} - \frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\frac{dH_x}{dz} + k_o{}^2\varepsilon H_x = 0 \quad , \tag{4}$$

where $k_o = \omega\sqrt{\varepsilon_o\mu_o}$ is the intrinsic wave number of free space and $k_o\sqrt{\varepsilon}$ is the real phase constant for real $\varepsilon$. Note that Equations (3) and (4) are no longer symmetric in $E_y$ and $H_x$ as in the case of a homogeneous medium, which admits plane wave solutions with $E_y$ and $H_x$ of constant amplitudes and of the same phase constant. In an inhomogeneous medium we cannot assume the existence of a plane wave, since a plane wave is not a solution of Equation (3) or Equation (4). To show that $E_y$ and $H_x$ do not have the same phase constant, we transform Equation (4) into the normal form by the substitution

$$H_x = \sqrt{\varepsilon}\, H_{xt} \quad , \tag{5}$$

and obtain

$$\frac{d^2H_{xt}}{dz^2} + \left[k_o{}^2\varepsilon + \frac{1}{2\varepsilon}\frac{d^2\varepsilon}{dz^2} - \frac{3}{4}\left(\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\right)^2\right]H_{xt} = 0 \tag{6}$$

Since both Equations (3) and (6) are in the same normal form, their solutions will be of the same form also.

This will be the basis on which we shall proceed to attack the problem by solving Equations (3) and (6).

The phase integral of $E_y$ along the direction z, for Eq.(3) with varying $\varepsilon$, is assumed to be of the form

$$\rho = \int_0^z k_o \sqrt{\varepsilon}\, dz \, , \tag{7}$$

and the phase integral of $H_{xt}$, from Eq.(6), is thus of the similar form

$$\int_0^z k_o \sqrt{\varepsilon} \sqrt{1 + \frac{1}{k_o^2 \varepsilon}\left[\frac{1}{2\varepsilon}\frac{d^2\varepsilon}{dz^2} - \frac{3}{4}\left(\frac{d\varepsilon}{\varepsilon dz}\right)^2\right]} \quad dz. \tag{8}$$

It is evident that for a varying $\varepsilon$, the fields $E_y$ and $H_x$ are no longer in phase and the phase difference between them is

$$m = \int_0^z k_o \sqrt{\varepsilon}\, p \, dz = \int_0^\rho p\, d\rho \, , \tag{9}$$

where $k_o \sqrt{\varepsilon}\, p$ is the difference between the phase constants of $E_y$ and $H_x$ and p has the dimensionless magnitude

$$p = \frac{\frac{1}{k_o^2 \varepsilon}\left[\frac{1}{2\varepsilon}\frac{d^2\varepsilon}{dz^2} - \frac{3}{4}\left(\frac{d\varepsilon}{\varepsilon dz}\right)^2\right]}{1 + \sqrt{1 + \frac{1}{k_o^2 \varepsilon}\left[\frac{1}{2\varepsilon}\frac{d^2\varepsilon}{dz^2} - \frac{3}{4}\left(\frac{d\varepsilon}{\varepsilon dz}\right)^2\right]}} \tag{10}$$

Note that p is an exact expression and can be evaluated easily when $\varepsilon$ is a prescribed function of z. On the other hand, if

$\varepsilon$ is a slowly varying function of z such that $\frac{d\varepsilon}{dz}$ and $\frac{d^2\varepsilon}{dz^2}$ are all very small, p can take the following simple form:

$$p \doteq \frac{1}{4k_o^2\varepsilon}\left[\frac{1}{\varepsilon}\frac{d^2\varepsilon}{dz^2} - \frac{3}{2}\left(\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\right)^2\right] \qquad (11)$$

The criterion to use Eq. (11) is that $|p|$ itself is much smaller than unity. It is seen from Eq. (9) that for the same length of an inhomogeneous medium the phase difference between $E_y$ and $H_x$ for high frequencies is smaller than that for low frequencies, or for the same phase difference high-frequency waves can propagate through a longer distance than low-frequency waves. In fact, for cases where $|p| << 1$ holds along the path, the phase difference m is inversely proportional to the frequency, as shown in Eq. (25) later. In passing, we mention that for a complex $\varepsilon$, Eqs. (7) through (11) are also complex quantities.

The qualitative concept of the preceding paragraphs is intended to depict a clear picture of the nature of electromagnetic wave propagation in an inhomogeneous medium. The actual solution of the problem, however, can be obtained only be solving the wave equations (3) and (6). It is well known that Eq. (3), with arbitrary $\varepsilon(z)$, has no exact solution in terms of a finite number of elementary functions. The conventional method of solving Eq. (3) approximately is either the WKB method[5-7] or the perturbation method.[8] These approximate methods do not and cannot consider the effect of internal reflections or multiple scatterings of waves. In the light of the concept of the preceding paragraphs, an attempt is being made in this paper to obtain solutions more accurate than previously possible by taking internal reflections into account. The solutions will appear in a form of two coupled first order differential equations between the forward wave and backward wave.

In the formulation that follows we shall use the phase difference term m of Eq.(9) as a "vehicle" by which we try to reach the unexplored territory of internal reflections or multiple scattering of waves in inhomogeneous media. The solution of Eq.(3) can be cast in a general form as the sum of a forward wave $E_{yf}$ and a backward wave $E_{yb}$

$$E_y = E_{yf} + E_{yb} = A(z)\, e^{i\int_0^z k_o\sqrt{\varepsilon}\, dz} + B(z) e^{-i\int_0^z k_o\sqrt{\varepsilon}\, dz}, \tag{12}$$

and the solution of Eq.(4), with the aid of Eqs.(5) and (6), has the similar form

$$H_x = H_{xf} + H_{xb} = \sqrt{\frac{\varepsilon_o}{\mu_o}}\sqrt{\varepsilon}\left[C(z) e^{i\int_0^z k_o\sqrt{\varepsilon}(1+p)dz} + D(z) e^{-i\int_0^z k_o\sqrt{\varepsilon}(1+p)dz}\right], \tag{13}$$

where A(z), B(z), C(z), and D(z) are functions to be determined and the multiplying constant $\sqrt{\frac{\varepsilon_o}{\mu_o}}$ in Eq. (13) is used to obtain the correct dimensions for $H_x$.

Substitution of Eqs. (7) and (9) into Eqs. (12) and (13) results in concise forms:

$$E_y = A(z)e^{i\rho} + B(z)e^{-i\rho}, \tag{14}$$

$$H_x = \sqrt{\frac{\varepsilon_o\varepsilon}{\mu_o}}\left[C(z)\, e^{i(\rho+m)} + D(z)e^{-i(\rho+m)}\right]. \tag{15}$$

For homogeneous media m vanishes and we have C=A and D=-B. For inhomogeneous media, it is plausible to assume that C and D can be expressed, respectively, in terms of A and B, e.g.,

$$C(z) = A(z)u(z) \qquad \text{and} \quad D(z) = -B(z)v(z); \quad (16)$$

or

$$C(z) = A(z)[1 + \xi(z)], \text{ and } D(z) = -B(z)[1 + \eta(z)] . \quad (17)$$

For slowly varying $\varepsilon$, it is expected that the ratio of A/C or B/D is close to unity, and thus u and v will be close to unity and $\xi$ and $\eta$ much less than unity. In anticipation of the nonlinear differential equations to be solved later, it appears advisable to use the forms shown in Eq. (17), so that omission of nonlinear terms such as $\xi^2$ or $\eta^2$ can be justified.

We now proceed to (i) express the exact first order coupled differential equations between A(z) and B(z) in terms of the new functions $\xi$ (z) and $\eta$ (z) [Eqs. (18) through (24)], (ii) simplify these differential equations by expanding exponential terms and neglecting higher order terms [Eqs. (25) through (35)], (iii) obtain the approximate solutions of the functions $\xi(z)$ and $\eta(z)$ under proper boundary conditions [Eqs. (36) through (48)], and (iv) solve the simplified equations by an iterative scheme [Eqs. (49) through (57)]. Readers not interested in these details may turn to the final solutions, Eqs. (56) and (57), on page 19.

Substitution of Eq. (17) into Eq. (15) yields

$$H_x = \sqrt{\frac{\varepsilon_o \varepsilon}{\mu_o}} \left[ A(1+\xi) e^{i(\rho+m)} - B(1+\eta) e^{-i(\rho+m)} \right]. \tag{18}$$

Differentiating Eq.(14) and making use of the first equation of Eq.(1), we obtain

$$H_x = \left( \sqrt{\frac{\varepsilon_o \varepsilon}{\mu_o}} A + \frac{1}{i\omega\mu_o} \frac{dA}{dz} \right) e^{i\rho} - \left( \sqrt{\frac{\varepsilon_o \varepsilon}{\mu_o}} B - \frac{1}{i\omega\mu_o} \frac{dB}{dz} \right) e^{-i\rho} . \tag{19}$$

Differentiating Eq.(18) and making use of the second equation of Eq.(1), we obtain

$$E_y = \left\{ (1+p)(1+\xi)A + \frac{1}{ik_o\varepsilon} \frac{d}{dz} [\sqrt{\varepsilon}(1+\xi)A] \right\} e^{i(\rho+m)}$$
$$+ \left\{ (1+p)(1+\eta)B - \frac{1}{ik_o\varepsilon} \frac{d}{dz} [\sqrt{\varepsilon}(1+\eta)B] \right\} e^{-i(\rho+m)} \tag{20}$$

Equating Eq. (18) to Eq. (19) and Eq. (14) to Eq. (20) yield, respectively,

$$\frac{dA}{dz} + \frac{dB}{dz}\, e^{-i2\rho} = ik_o\sqrt{\varepsilon}\left\{\left[(1+\xi)e^{im}-1\right]A + \left[1-(1+\eta)e^{-im}\right]e^{-i2\rho}B\right\} \qquad (21)$$

$$\frac{dA}{dz} - \frac{(1+\eta)}{(1+\xi)}\frac{dB}{dz}\, e^{-i2(\rho+m)} = \left\{ik_o\sqrt{\varepsilon}\left[\frac{e^{-im}}{(1+\xi)} - (1+p)\right] - \left[\frac{1}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{1}{(1+\xi)}\frac{d\xi}{dz}\right]\right\}A + \left\{ik_o\sqrt{\varepsilon}\left[e^{im} - (1+p)(1+\eta)\right] + \left[\frac{(1+\eta)}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{d\eta}{dz}\right]\right\}\frac{e^{-i2m}}{(1+\xi)}Be^{-i2\rho}. \qquad (22)$$

Combining Eqs. (21) and (22), we obtain the sought-after coupled first order differential equations:

$$\frac{dA}{dz} = \frac{1}{(1+\xi)+(1+\eta)e^{-i2m}}\left\{ik_o\sqrt{\varepsilon}\left[e^{-im} - (1+p)(1+\xi) - (1+\eta)\left[1-(1+\xi)e^{im}\right]\cdot e^{-i2m}\right]A - \left[\frac{(1+\xi)}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{d\xi}{dz}\right]A + \left[\frac{(1+\eta)}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{d\eta}{dz}\right]e^{-i2m}Be^{-i2\rho} + ik_o\sqrt{\varepsilon}\left[(1+\eta)\left[1-(1+\eta)e^{-im}\right] - (1+p)(1+\eta) + e^{im}\right]e^{-i2m}Be^{-i2\rho}\right\}, \qquad (23)$$

$$\frac{dB}{dz} = \frac{1}{(1+\xi)+(1+\eta)e^{-i2m}}\Bigg\{ - ik_o\sqrt{\varepsilon}\Bigg[e^{im} - (1+p)(1+\eta) - (1+\xi).$$

$$[1-(1+\eta)e^{-im}]e^{i2m}\Bigg]e^{-i2m}\,B - \left[\frac{(1+\eta)}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{d\eta}{dz}\right]e^{-i2m}\,B$$

$$+\left[\frac{(1+\xi)}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{d\xi}{dz}\right]Ae^{i2\rho} - ik_o\sqrt{\varepsilon}\Bigg[(1+\xi)\,[1-(1+\xi)e^{im}]$$

$$-(1+p)(1+\xi) + e^{-im}\Bigg]A\,e^{i2\rho}\Bigg\}. \qquad (24)$$

Eqs.(23) and (24) are the formal exact coupled differential equations which take into consideration the effect of internal reflections or multiple scattering. These equations can be simplified somewhat if the exponential terms involving m are expanded in a series form for small m. This implies that the total phase difference between $E_y$ and $H_x$ must be much less than one radian at any distance z. From Eq.(9) and (11), we have the phase difference

$$m = \int_0^z \frac{1}{2k_o\sqrt{\varepsilon}}\left[\frac{1}{2\varepsilon}\frac{d^2\varepsilon}{dz^2} - \frac{3}{4}\left(\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\right)^2\right]dz = \frac{1}{4k_o\varepsilon\sqrt{\varepsilon}}\frac{d\varepsilon}{dz}\,, \qquad (25)$$

for cases where the first derivative of $\varepsilon$ vanishes at z=0.

It is thus seen that $m^2$ is of the same order of magnitude as p in Eq.(11) for slowly varying $\varepsilon$ and $m^2$ terms shall be retained at least for the moment in the expansion of $e^{\pm im}$

and $e^{\pm i2m}$ in Eqs.(23) and (24). Neglecting terms of $m^3, m^4$, etc., we obtain under the constraint $m^2 \ll 1$

$$\frac{dA}{dz} = \frac{1}{2}\left(1+im - \frac{\xi+\eta}{2}\right)\left\{ik_o \sqrt{\varepsilon}[m^2+\xi\eta-p-im(\xi-\eta)]A - \left(\frac{1+\xi}{2\varepsilon}\frac{d\varepsilon}{dz}\right) A \right.$$
$$+ \left(\frac{1+\eta}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{d\eta}{dz}\right)(1-i2m)Be^{-i2\rho}$$
$$\left. - ik_o\sqrt{\varepsilon}[2\eta+\eta^2-4m^2+p-i2m(1+3\eta)]Be^{-i2\rho}\right\}, \tag{26}$$

$$\frac{dB}{dz} = \frac{1}{2}\left(1+im - \frac{\xi+\eta}{2}\right)\left\{- ik_o\sqrt{\varepsilon}[m^2+\xi\eta-p-im(\xi-\eta)]B\right.$$
$$- \left(\frac{1+\eta}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{d\eta}{dz}\right)(1-i2m)B + \left(\frac{1+\xi}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{d\xi}{dz}\right)Ae^{i2\rho}$$
$$\left. + ik_o\sqrt{\varepsilon}[2\xi+\xi^2+p+i2m(1+\xi)]Ae^{i2\rho}\right\}. \tag{27}$$

In Eqs.(26) and (27) $\xi$ and $\eta$ are still unknown functions to be determined. In an attempt to simplify Eqs.(26) and (27) further, we shall assume for the moment and justify later that $\xi$, $\eta$, $\frac{d\xi}{dz}$ and $\frac{d\eta}{dz}$ are of the same order of magnitude as m. We thus obtain

$$\frac{dA}{dz} = \frac{1}{2}\left\{ik_o\sqrt{\varepsilon}[m^2+\xi\eta-p-im(\xi-\eta)] - \left(\frac{1}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{d\xi}{dz}\right)\left(1+im - \frac{\xi+\eta}{2}\right) - \frac{\xi}{2\varepsilon}\frac{d\varepsilon}{dz}\right\}A$$
$$+ \frac{1}{2}\left\{\left(\frac{1}{2\varepsilon}\frac{d\varepsilon}{dz} + \frac{d\eta}{dz}\right)\left(1-im - \frac{\xi+\eta}{2}\right) + \frac{\eta}{2\varepsilon}\frac{d\varepsilon}{dz}\right.$$
$$\left. - ik_o\sqrt{\varepsilon}[2\eta-2m^2-\xi\eta+p-im(2-\xi+\eta)]\right\}Be^{-i2\rho}, \tag{28}$$

$$\frac{dB}{dz} = \frac{1}{2}\left\{-ik_o\sqrt{\varepsilon}[m^2+\xi\eta-p-im(\xi-\eta)]-\left(\frac{1}{2\varepsilon}\frac{d\varepsilon}{dz}+\frac{d\eta}{dz}\right)\left(1-im-\frac{\xi+\eta}{2}\right)-\frac{\eta}{2\varepsilon}\frac{d\varepsilon}{dz}\right\}B$$

$$+\frac{1}{2}\left\{\left(\frac{1}{2\varepsilon}\frac{d\varepsilon}{dz}+\frac{d\xi}{dz}\right)\left(1+im-\frac{\xi+\eta}{2}\right)+\frac{\xi}{2\varepsilon}\frac{d\varepsilon}{dz}\right.$$

$$\left.+ik_o\sqrt{\varepsilon}[2\xi-2m^2-\xi\eta+p+im(2+3\xi-\eta)]\right\}Ae^{i2\rho}\ . \tag{29}$$

Noting the fact that, with the aid of Eq. (25),

$$k_o\sqrt{\varepsilon}m = k_o\sqrt{\varepsilon}\,\frac{1}{4k_o\varepsilon\sqrt{\varepsilon}}\frac{d\varepsilon}{dz} = \frac{1}{4\varepsilon}\frac{d\varepsilon}{dz}\ , \tag{29a}$$

we have

$$\frac{dA}{dz} = \frac{1}{2}\left[ik_o\sqrt{\varepsilon}(\xi\eta+m^2-p)-\left(\frac{1}{2\varepsilon}\frac{d\varepsilon}{dz}+\frac{d\xi}{dz}\right)(1+im)+\frac{\xi+\eta}{2}\frac{d\xi}{dz}\right]A \tag{30}$$

$$+\frac{1}{2}\left[\frac{d\eta}{dz}\left(1-im-\frac{\xi+\eta}{2}\right)-\frac{im}{2\varepsilon}\frac{d\varepsilon}{dz}-ik_o\sqrt{\varepsilon}(2\eta-2m^2-\xi\eta+p)\right]Be^{-i2\rho},$$

$$\frac{dB}{dz} = \frac{1}{2}\left[-ik_o\sqrt{\varepsilon}(\xi\eta+m^2-p)-\left(\frac{1}{2\varepsilon}\frac{d\varepsilon}{dz}+\frac{d\eta}{dz}\right)(1-im)+\frac{\xi+\eta}{2}\frac{d\eta}{dz}\right]B$$

$$+\frac{1}{2}\left[\frac{d\xi}{dz}\left(1+im-\frac{\xi+\eta}{2}\right)+\frac{im}{2\varepsilon}\frac{d\varepsilon}{dz}-\frac{\xi}{2\varepsilon}\frac{d\varepsilon}{dz}\right.$$

$$\left.+ik_o\sqrt{\varepsilon}\left(2\xi-2m^2-\xi\eta+p\right)\right]Ae^{i2\rho}\ . \tag{31}$$

Further simplification and rearrangement result in

$$\frac{dA}{dz} = \frac{1}{2}\left\{i\left[k_o\sqrt{\varepsilon}(\xi\eta-m^2-p)-m\frac{d\xi}{dz}\right]-\left[\frac{1}{2\varepsilon}\frac{d\varepsilon}{dz}+\left(1-\frac{\xi+\eta}{2}\right)\frac{d\xi}{dz}\right]\right\}A$$

$$+\frac{1}{2}\left\{\left(1-\frac{\xi+\eta}{2}\right)\frac{d\eta}{dz}-i\left[k_o\sqrt{\varepsilon}(2\eta-\xi\eta+p)+m\frac{d\eta}{dz}\right]\right\}Be^{-i2\rho}\ , \tag{32}$$

$$\frac{dB}{dz} = \frac{1}{2}\left\{-i\left[k_o\sqrt{\varepsilon}(\xi\eta - m^2 - p) - m\frac{d\eta}{dz}\right] - \left[\frac{1}{2\varepsilon}\frac{d\varepsilon}{dz} + \left(1 - \frac{\xi+\eta}{2}\right)\frac{d\eta}{dz}\right]\right\}B$$

$$+ \frac{1}{2}\left\{\left[\left(1 - \frac{\xi+\eta}{2}\right)\frac{d\xi}{dz} - \frac{\xi}{2\varepsilon}\frac{d\varepsilon}{dz}\right] + i\left[k_o\sqrt{\varepsilon}(2\xi - \xi\eta + p) + m\frac{d\xi}{dz}\right]\right\}Ae^{i2\rho}. \tag{33}$$

If terms of the order of $m^2$ and $p$ are neglected now, the following simple equations are obtained:

$$\frac{dA}{dz} = -\left(\frac{1}{4\varepsilon}\frac{d\varepsilon}{dz} + \frac{1}{2}\frac{d\xi}{dz}\right)A + \left(\frac{1}{2}\frac{d\eta}{dz} - ik_o\sqrt{\varepsilon}\eta\right)e^{-i2\rho}B\ , \tag{34}$$

$$\frac{dB}{dz} = -\left(\frac{1}{4\varepsilon}\frac{d\varepsilon}{dz} + \frac{1}{2}\frac{d\eta}{dz}\right)B + \left(\frac{1}{2}\frac{d\xi}{dz} + ik_o\sqrt{\varepsilon}\xi\right)e^{i2\rho}A\ . \tag{35}$$

In order to gain more insight into the problem, we shall let both $\xi$ and $\eta$ vanish, for the moment, in Eqs. (32) through (35) and obtain, respectively,

$$\frac{dA}{dz} = \left[-i\frac{1}{2}k_o\sqrt{\varepsilon}(m^2+p) - \frac{1}{4\varepsilon}\frac{d\varepsilon}{dz}\right]A - i\frac{1}{2}k_o\sqrt{\varepsilon}\,pe^{-i2\rho}B\ , \tag{32a}$$

$$\frac{dB}{dz} = \left[i\frac{1}{2}k_o\sqrt{\varepsilon}(m^2+p) - \frac{1}{4\varepsilon}\frac{d\varepsilon}{dz}\right]B + i\frac{1}{2}k_o\sqrt{\varepsilon}\,pe^{i2\rho}A\ ; \tag{33a}$$

$$\frac{dA}{dz} = -\frac{1}{4\varepsilon}\frac{d\varepsilon}{dz}A\ , \quad \text{and} \quad A = 1/\varepsilon^{\frac{1}{4}}\ ; \tag{34a}$$

$$\frac{dB}{dz} = -\frac{1}{4\varepsilon}\frac{d\varepsilon}{dz}B\ , \quad \text{and} \quad B = 1/\varepsilon^{\frac{1}{4}}\ . \tag{35a}$$

We note that Eqs.(34a) and (35a) are the WKB solutions [5-7] for the amplitudes of both the forward and backward waves. In this case, the forward and backward waves are not coupled and thus cannot exhibit the effects of internal reflections. The terms containing $m^2$ and p, shown in Eqs. (32a) and (33a), have been neglected.

Eqs.(32) and (33) are the almost exact coupled first order differential equations in A and B, which must now be solved in terms of the yet unknown functions $\xi(z)$ and $\eta(z)$. So our next task is to find approximate expressions for both $\xi$ and $\eta$ as a function of z. To obtain approximate $\xi$ and $\eta$, we shall make use of Eqs.(18) and (19) by equating the forward wave component of Eq.(18) to that of Eq.(19) and the backward wave component of Eq.(18) to that of Eq.(19). The relations thus obtained are:

$$\sqrt{\frac{\varepsilon_o \varepsilon}{\mu_o}}\, A(1+\xi)e^{im} = \sqrt{\frac{\varepsilon_o \varepsilon}{\mu_o}}\, A + \frac{1}{i\omega\mu_o}\frac{dA}{dz}\,, \tag{36}$$

$$\sqrt{\frac{\varepsilon_o \varepsilon}{\mu_o}}\, B(1+\eta)e^{-im} = \sqrt{\frac{\varepsilon_o \varepsilon}{\mu_o}}\, B - \frac{1}{i\omega\mu_o}\frac{dB}{dz}\,; \tag{37}$$

and

$$A = e^{-i\int_0^z k_o\sqrt{\varepsilon}\,[1-(1+\xi)e^{im}]dz}\,, \tag{38}$$

$$B = e^{i\int_0^z k_o\sqrt{\varepsilon}\,[1-(1+\eta)e^{-im}]dz}\,. \tag{39}$$

Using Eqs.(38) and (39) in Eq.(12), we have

$$E_{yf} = e^{i\int_0^z k_o\sqrt{\varepsilon}\,(1+\xi)e^{im}dz}\,, \tag{40}$$

and

$$E_{yb} = e^{-i\int_0^z k_o\sqrt{\varepsilon}(1+\eta)e^{-im}dz} \tag{41}$$

Both $E_{yf}$ and $E_{yb}$ should individually satisfy Eq.(3), and we have for $E_{yf}$

$$\frac{d^2E_{yf}}{dz^2} = \left\{ \left(ik_o\sqrt{\varepsilon}\right)^2 (1+\xi)^2 \; e^{i2m} + ik_o\left[\frac{1+\xi}{2\sqrt{\varepsilon}}\frac{d\varepsilon}{dz} + \sqrt{\varepsilon}\,\frac{d\xi}{dz} + ipk_o\varepsilon(1+\xi)\right] e^{im}\right\} E_{yf} \tag{42}$$

Neglecting terms of the order of $\xi^2$ and $m^2$, we have from Eq.(42)

$$\frac{d\xi}{dz} \doteq - i2k_o\sqrt{\varepsilon}\;\xi - ik_o\sqrt{\varepsilon}\;p \;, \tag{43}$$

and

$$\xi = e^{-i2\rho}\left[K_\xi - i\int_0^z k_o\sqrt{\varepsilon}\;pe^{i2\rho}dz\right] \;,$$

where $K_\xi$ is the constant to be determined by the boundary condition that the incident fields $E_{yf}$ and $H_{xf}$ must be in phase at z=0. $\xi$ thus must be zero at z=0, i.e., $K_\xi$=0 and

$$\xi = -ie^{-i2\rho}\int_0^z k_o\sqrt{\varepsilon}\;pe^{i2\rho}dz \;. \tag{44}$$

Similarly, we have for $E_{yb}$

$$\frac{d^2E_{yb}}{dz^2} = \left\{\left(-ik_o\sqrt{\varepsilon}\right)^2(1+\eta)^2 e^{-i2m} - ik_o\left[\frac{1+\eta}{2\sqrt{\varepsilon}}\frac{d\varepsilon}{dz} + \sqrt{\varepsilon}\frac{d\eta}{dz}\right.\right.$$

$$\left.\left. - ipk_o\varepsilon(1+\eta)\right] e^{-im}\right\}E_{yb} , \qquad (45)$$

$$\frac{d\eta}{dz} \doteq i2k_o\sqrt{\varepsilon}\,\eta + ik_o\sqrt{\varepsilon}\,p , \qquad (46)$$

and

$$\eta = e^{i2\rho}\left[K_\eta + i\int_0^z k_o\sqrt{\varepsilon}\,p\,e^{-i2\rho}dz\right] ,$$

where $K_\eta$ is the constant to be determined by the boundary condition that the phase difference between the reflected fields $E_{yb}$ and $H_{xb}$ at $z=\ell$ must be identical to that of the forward fields $E_{yf}$ and $H_{xf}$ at $z=\ell$. The phase reversal of $H_{xb}$ has been taken into consideration by the minus sign in Eq. (17) already. Only at $z=\ell$ are the local phases and magnitudes of the reflected waves not influenced by reflected waves at other locations in the profile. This implies that $\eta(\ell) = \xi(\ell)$ and thus

$$K_\eta = \xi(\ell)e^{-i2\rho(\ell)} - i\int_0^\ell k_o\sqrt{\varepsilon}\,pe^{-i2\rho}dp, \qquad (47)$$

and

$$\eta = i\,e^{i2\rho}\left[\int_\ell^z k_o\sqrt{\varepsilon}\,pe^{-i2\rho}dz - i\xi(\ell)e^{-i2\rho(\ell)}\right] . \qquad (48)$$

Eqs. (43), (44), (46) and (48) verify the fact that $\xi$, $\eta$, $\frac{d\xi}{dz}$, and $\frac{d\eta}{dz}$ are indeed of the order of m as assumed earlier.

With the aid of Eqs.(44) and (48), Eqs.(32) through (35) can be expressed in terms of known functions $\xi$ and $\eta$. For the purpose of simple illustration we shall use Eqs.(34) and (35) to obtain the final solutions for A and B. If we use Eqs.(32) and (33), the solutions obtained should be more accurate than those obtained from Eqs.(34) and (35). Taking advantage of the relations shown in Eqs.(43) and (46), we have from Eqs.(34) and (35)

$$\frac{dA}{dz} = -\left(\frac{1}{4\varepsilon}\frac{d\varepsilon}{dz} + \frac{1}{2}\frac{d\xi}{dz}\right)A + ik_o\sqrt{\varepsilon}p\ e^{-i2\rho}B\ , \tag{49}$$

$$\frac{dB}{dz} = -\left(\frac{1}{4\varepsilon}\frac{d\varepsilon}{dz} + \frac{1}{2}\frac{d\eta}{dz}\right)B - ik_o\sqrt{\varepsilon}p\ e^{i2\rho}A\ ; \tag{50}$$

and

$$A = \varepsilon^{-\frac{1}{4}}\ e^{-\frac{\xi}{2}}\left[C_1 + i\int_0^z k_o\sqrt{\varepsilon}\ p\ \varepsilon^{\frac{1}{4}}\ e^{\frac{\xi}{2}}\ e^{-i2\rho}B\ dz\right]\ , \tag{51}$$

$$B = \varepsilon^{-\frac{1}{4}}\ e^{-\frac{\eta}{2}}\left[C_2 - i\int_0^z k_o\sqrt{\varepsilon}\ p\ \varepsilon^{\frac{1}{4}}\ e^{\frac{\eta}{2}}\ e^{i2\rho}A\ dz\right]\ , \tag{52}$$

where $C_1$ and $C_2$ are constants to be determined by the boundary conditions for A and B. The first order iteration solutions of Eqs.(51) and (52) yield

$$A = \varepsilon^{-\frac{1}{4}}\ e^{-\frac{\xi}{2}}\left[C_1 + iC_2\int_0^z k_o\sqrt{\varepsilon}\ p\ e^{-i2\rho}dz\right]\ , \tag{53}$$

and

$$B = \varepsilon^{-\frac{1}{4}}\ e^{-\frac{\eta}{2}}\left[C_2 - iC_1\int_0^z k_o\sqrt{\varepsilon}\ p\ e^{i2\rho}dz\right]\ . \tag{54}$$

At $z=0^+$, the normalized boundary condition requires that A=1 (amplitude) and thus $C_1 = [\varepsilon(0)]^{\frac{1}{4}}$. At $z=\ell^-$, we have in general $B=B(\ell)$ and thus

$$C_2 = [\varepsilon(\ell)]^{\frac{1}{4}} B(\ell) + i[\varepsilon(0)]^{\frac{1}{4}} \int_0^{\ell} k_o\sqrt{\varepsilon}\, pe^{i2\rho}dz \ , \quad (55)$$

where $B(\ell)$ vanishes if the first derivative of $\varepsilon$ vanishes at $z=\ell^-$. Accordingly, we have the final solutions, for the case $B(\ell)=0$,

$$A(z) = \left[\frac{\varepsilon(0)}{\varepsilon(z)}\right]^{\frac{1}{4}} e^{-\frac{\xi}{2}}\left[1 - \int_0^{\ell} k_o\sqrt{\varepsilon}\, pe^{i2\rho}dz \cdot \int_0^{z} k_o\sqrt{\varepsilon}\, pe^{-i2\rho}dz\right] , \quad (56)$$

and

$$B(z) = -i\left[\frac{\varepsilon(0)}{\varepsilon(z)}\right]^{\frac{1}{4}} e^{-\frac{\eta}{2}} \int_{\ell}^{z} k_o\sqrt{\varepsilon}\, pe^{i2\rho}dz \ . \quad (57)$$

With the aid of Eqs.(44),(47), and (48), we can write A and B explicitly in terms of $\xi$ and $\eta$:

$$A(z) = \left[\frac{\varepsilon(0)}{\varepsilon}\right]^{\frac{1}{4}} e^{-\frac{\xi}{2}}\left[1 - \xi(\ell)e^{i2\rho(\ell)}(\eta e^{-i2\rho}-K_\eta)\right] , \quad (58)$$

and

$$B(z) = \left[\frac{\varepsilon(0)}{\varepsilon}\right]^{\frac{1}{4}} e^{-\frac{\eta}{2}}\left[\xi e^{i2\rho}-\xi(\ell)e^{i2\rho(\ell)}\right] . \quad (59)$$

With A and B as known functions of z, the electric and magnetic fields can be obtained from Eqs.(14) and (18).

We can calculate the power incident and reflected at z=0 and the power transmitted at $z=\ell$ by using the Poynting theorem for complex vectors. From Eqs.(14) and (18), we have the average power from the cross product (defined by X)

$$P_{av}\hat{z} = \frac{1}{2}\,\mathrm{Re}(E_y\hat{y}XH_x^*\hat{x}) = \frac{1}{2}\,\mathrm{Re}(Ae^{i\rho}+Be^{-i\rho})\sqrt{\frac{\varepsilon_o\varepsilon^*}{\mu_o}}\Big[A^*(1+\xi^*)e^{-i(\rho^*+m^*)}$$

$$-B^*(1+\eta^*)e^{i(\rho^*+m^*)}\Big]\hat{z}\ , \qquad (60)$$

where Re denotes the real part, ^ the unit vector, and * the complex conjugate. At z=0, $\xi(0)=0$, $A(0)=1$, $\rho(0)=0$, and $m(0)=0$,

$$P_{av}(0) = \frac{1}{2}\,\mathrm{Re}\sqrt{\frac{\varepsilon_o\varepsilon(0)^*}{\mu_o}}[1+B(0)]\{1-B(0)^*[1+\eta(0)^*]\} \qquad (61)$$

$$= \frac{1}{2}\,\mathrm{Re}\sqrt{\frac{\varepsilon_o\varepsilon(0)^*}{\mu_o}}\Big\{1-B(0)B(0)^*[1+\eta(0)^*]+B(0)-B(0)^*[1+\eta(0)^*]\Big\}.$$

Thus the incident power is $\frac{1}{2}\,\mathrm{Re}\sqrt{\frac{\varepsilon_o\varepsilon(0)^*}{\mu_o}}$ and the reflection coefficient at z=0 is

$$R = -\mathrm{Re}\{B(0)B(0)^*[1+\eta(0)^*] - B(0) + B(0)^*[1+\eta(0)^*]\} \qquad (62)$$

If the medium is homogeneous, $\eta(0)$ vanishes, B(0) is real, and the reflection coefficient R reduces to the standard form $B(0)^2$. At $z=\ell$, $B(\ell)$ must vanish for a smooth transition, and we have the transmission coefficient

$$T = \mathrm{Re}\left\{\sqrt{\frac{\varepsilon(\ell)^*}{\varepsilon(0)^*}}\, A(\ell)A(\ell)^*[1+\xi(\ell)^*]e^{i[\rho(\ell)-\rho(\ell)^*-m(\ell)^*]}\right\} . \qquad (63)$$

For homogeneous lossless media, T reduces to the standard form $A^2\sqrt{\frac{\varepsilon(\ell)}{\varepsilon(0)}}$ . We point out that Eqs.(62) and (63) are valid only under the constraints $|p| << 1$, $|m^2| << 1$, and $B(\ell)=0$.

It is evident that both the transmission coefficient T and the reflection coefficient R can be easily obtained once $\xi$ and $\eta$ are explicitly evaluated. Since the dimensionless parameter $\rho$ [phase integral of Eq.(7)] in Eqs.(44) and (48) is an integral function of z for a prescribed $\varepsilon(z)$, double integrations must be performed to evaluate these equations. To avoid the double integration, we can transform Eqs.(44) and (48) explicitly in term of $\rho$ alone and use $\rho$ as the independent variable:

$$\xi(\rho) = -ie^{-i2\rho}\int_0^{\rho} p(\rho)e^{i2\rho}d\rho \ , \qquad (64)$$

$$\eta(\rho) = ie^{i2\rho}\int_{\rho_\ell}^{\rho} p(\rho)e^{-i2\rho}d\rho + \xi(\rho_\ell)e^{i2(\rho-\rho_\ell)} \ , \qquad (65)$$

where, from Eq.(11),

$$p(z) = \frac{1}{4k_o^2\varepsilon}\left[\frac{1}{\varepsilon}\frac{d^2\varepsilon}{dz^2} - \frac{3}{2}\left(\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\right)^2\right]$$

$$= \frac{1}{4}\left[\frac{1}{\varepsilon}\frac{d^2\varepsilon}{d\rho^2} - \left(\frac{1}{\varepsilon}\frac{d\varepsilon}{d\rho}\right)^2\right] = p(\rho) \ , \qquad (66)$$

and

$$\rho_\ell = \rho(\ell) = \int_0^\ell k_o \sqrt{\varepsilon}\, dz \ . \tag{67}$$

Eqs.(56) and (57) then become, respectively,

$$A(\rho) = \left[\frac{\varepsilon(0)}{\varepsilon(\rho)}\right]^{\frac{1}{4}} e^{-\frac{\xi(\rho)}{2}} \left[1 - \int_0^{\rho_\ell} p(\rho) e^{i2\rho}\, d\rho \cdot \int_0^{\rho} p(\rho) e^{-i2\rho}\, d\rho\right], \tag{68}$$

and

$$B(\rho) = i \left[\frac{\varepsilon(0)}{\varepsilon(\rho)}\right]^{\frac{1}{4}} e^{-\frac{\eta(\rho)}{2}} \int_\rho^{\rho_\ell} p(\rho) e^{i2\rho}\, d\rho \ . \tag{69}$$

For a prescribed $\varepsilon(\rho)$, Eqs.(64)-(69) can be evaluated without double integration and the relation between z and $\rho$ can be expressed through the equation

$$k_o z = \int_0^\rho \frac{d\rho}{\sqrt{\varepsilon(\rho)}} \tag{70}$$

## III. SOLUTION ACCURACY

The formal solution of the wave equation, Eq.(3), is Eq.(14), which can be explicitly expressed in terms of z with the aid of Eqs.(56) and (57). We now ask how accurate this solution is. Since the exact solution for Eq.(3) with arbitrary $\varepsilon(z)$ is not known in analytic form it is impossible to state an error bound of Eq.(14). On the other hand, we can compare the relative accuracies of the solution in the form of Eq.(14) and that obtained by the WKB method. The WKB solution of

Eq.(3) is $\varepsilon^{-\frac{1}{4}} e^{\pm i\rho}$. If we substitute $\varepsilon^{-\frac{1}{4}} e^{\pm i\rho}$ into Eq.(3), we obtain

$$\frac{d^2E_y}{dz^2} + k_o^2 \varepsilon E_y = \left[5\left(\frac{1}{4\varepsilon}\frac{d\varepsilon}{dz}\right)^2 - \frac{1}{4\varepsilon}\frac{d^2\varepsilon}{dz^2}\right]E_y \; . \tag{71}$$

The residue error terms on the right side of Eq. (71) indicate that the WKB solution does not satisfy Eq. (3) exactly. The conventional criterion for using the WKB approximation is

$$\frac{1}{4k_o^2\varepsilon}\left|\frac{1}{\varepsilon}\frac{d^2\varepsilon}{dz^2} - \frac{5}{4}\left(\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\right)^2\right| << 1 \; . \tag{72}$$

It is interesting to note the almost exact identity between the conditions of Eqs.(11) and (72).

If we substitute $Ae^{i\rho}$ (with A expressed by Eq.(56)) or $Be^{-i\rho}$ (with B expressed by Eq.(57)) into Eq.(3) we obtain, neglecting terms of $\frac{d^3\varepsilon}{dz^3}$ and $\left(\frac{d\varepsilon}{dz}\right)^3$ , respectively,

$$\frac{d^2E_y}{dz^2} + k_o^2 \; \varepsilon E_y = - \left[\frac{1}{16}\left(\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\right)^2 + \xi^2 \; k_o^2\varepsilon\right]E_y \qquad \text{or}$$

$$- \left[\frac{1}{16}\left(\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\right)^2 + \eta^2 \; k_o^2\varepsilon\right]E_y \; . \tag{73}$$

With the aid of Eqs.(44) and (48), it can be shown that both $\xi^2k_o^2\varepsilon$ and $\eta^2k_o^2\varepsilon$ vary between 0 and $-\left(\frac{1}{4\varepsilon}\frac{d\varepsilon}{dz}\right)^2$ . Accordingly, Eq.(73) can be written conservatively with a maximum possible residue error of $-\left(\frac{1}{4\varepsilon}\frac{d\varepsilon}{dz}\right)^2$ as

$$\frac{d^2E_y}{dz^2} + k_o^2\varepsilon\, E_y = -\left(\frac{1}{4\varepsilon}\frac{d\varepsilon}{dz}\right)^2 E_y \tag{74}$$

Comparisons indicate that the residue error of Eq.(74) is at least five times smaller than that of Eq.(71) for a slowly varying $\varepsilon$ where $\frac{1}{\varepsilon}\frac{d^2\varepsilon}{dz^2} << \left(\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\right)^2$. It is known that if residue error terms vanish in a differential equation the solution is exact. We infer that solutions with small residue error are more accurate than those with large residue error, but we cannot state the bound of accuracy. This heuristic argument provides evidence that our present solutions, which take the effect of internal reflections or multiple scattering into consideration, are more accurate than the WKB solutions which neglect this effect.

## IV. SAMPLE PROFILES

If we prescribe a Gaussian type of nonvanishing, even distribution profile for $\varepsilon(\rho)$, we have

$$\varepsilon(\rho) = a\, e^{-b\left(\frac{\rho}{\rho_\ell} - \frac{1}{2}\right)^2}, \tag{75}$$

where a and b are constants to be determined by the prescribed values at $\varepsilon(\rho=\frac{1}{2}\rho_\ell)$ and $\varepsilon(\rho=0 \text{ or } \rho_\ell)$. It turns out that for Eq.(75) the evaluation of $p(\rho)$ from Eqs.(66) yields the constant

$$p(\rho) = -b/(2\rho_\ell^2) \quad , \tag{76}$$

where the values of b and $\rho_\ell$ must be such that $|p|$ is much smaller than unity. $\xi$ and $\eta$ from Eqs.(64) and (65) become, respectively,

$$\xi(\rho) = \frac{b}{4\rho_\ell^2}(1-e^{-i2\rho}) \ , \tag{77}$$

and

$$\eta(\rho) = \frac{b}{4\rho_\ell^2}\left[1-e^{i2(\rho-2\rho_\ell)}\right] . \tag{78}$$

From Eqs.(68) and (69), we have, respectively,

$$A(\rho_\ell) = \left\{\exp\left[-\frac{b}{8\rho_\ell^2}\left(1-e^{-i2\rho_\ell}\right)\right]\right\}\left[1-i\frac{b}{4\rho_\ell^2}\left(e^{i2\rho_\ell}-1\right)\left(-i\frac{b}{4\rho_\ell^2}\right)\left(e^{-i2\rho_\ell}-1\right)\right]$$

$$= \left\{\exp\left[-\frac{b}{8\rho_\ell^2}\left(1-e^{-i2\rho_\ell}\right)\right]\right\}\left[1-\left(\frac{\sqrt{2}b}{4\rho_\ell^2}\right)^2\left(1-\cos 2\rho_\ell\right)\right] , \tag{79}$$

and

$$B(0) = \left\{\exp\left[-\frac{b}{8\rho_\ell^2}\left(1-e^{-i4\rho_\ell}\right)\right]\right\}\left(\frac{b}{4\rho_\ell^2}\right)\left(1-e^{i2\rho_\ell}\right) . \tag{80}$$

The transmission and reflection coefficients for lossless cases are, from Eqs. (63) and (62)

$$T = \mathrm{Re}\left\{\exp\left[-\frac{b}{4\rho_\ell^2}(1-\cos 2\rho_\ell)\right]\right\}\left[1-\left(\frac{\sqrt{2}b}{4\rho_\ell^2}\right)^2(1-\cos 2\rho_\ell)\right]^2$$

$$\left[1+\frac{b}{4\rho_\ell^2}\left(1-e^{i2\rho_\ell}\right)\right]e^{ib/2\rho_\ell} , \tag{81}$$

$$R = -\mathrm{Re}\left\{\exp\left[-\frac{b}{4\rho_\ell^2}(1-\cos 4\rho_\ell)\right]\right\}\left(\frac{\sqrt{2}b}{4\rho_\ell^2}\right)^2(1-\cos 2\rho_\ell)\left[1+\frac{b}{4\rho_\ell^2}\left(1-e^{i2\rho_\ell}\right)\right]$$

$$-\left\{\exp\left[-\frac{b}{8\rho_\ell^2}\left(1-e^{-i4\rho_\ell}\right)\right]\right\}\left(\frac{b}{4\rho_\ell^2}\right)\left(1-e^{i2\rho_\ell}\right)$$

$$+\left\{\exp\left[-\frac{b}{8\rho_\ell^2}\left(1-e^{i4\rho_\ell}\right)\right]\right\}\left(\frac{b}{4\rho_\ell^2}\right)\left(1-e^{-i2\rho_\ell}\right)\left[1+\frac{b}{4\rho_\ell^2}\left(1-e^{i2\rho_\ell}\right)\right] . \quad (82)$$

Next, we prescribe for $\varepsilon(\rho)$ another nonvanishing odd distribution function of the form

$$\varepsilon(\rho) = 1 + a \sin(n\pi\rho/\rho_\ell) , \quad (83)$$

where a is an arbitrary constant smaller than unity and n is a nonzero integer. We have from Eq. (66)

$$p(\rho) = -\left(\frac{n\pi}{2\rho_\ell}\right)^2 \frac{a[a+\sin(n\pi\rho/\rho_\ell)]}{[1+a \sin(n\pi\rho/\rho_\ell)]^2} , \quad (84)$$

where n, a, and $\rho_\ell$ must be so chosen that $|p(\rho)| << 1$.

To obtain $\xi$ and $\eta$ for $p(\rho)$ of Eq. (84), numerical integrations have to be performed. Since $\xi$ and $\eta$ are directly proportional to p, it is important to note from Eq. (84) that p is proportional to $n^2$, indicating that in general the reflected power increases as the number of "ripples" increases in the dielectric profile of the finite inhomogeneous medium.

For a delta function type of symmetrical distribution model

$$\varepsilon(\rho) = \frac{ab}{\left(\frac{\rho}{\rho_\ell} - \frac{1}{2}\right)^2 + b^2}\,, \tag{85}$$

we have

$$p(\rho) = \frac{\left(\frac{\rho}{\rho_\ell} - \frac{1}{2}\right)^2 - b^2}{2\left[\left(\frac{\rho}{\rho_\ell} - \frac{1}{2}\right)^2 + b^2\right]^2 \rho_\ell^2}\,. \tag{86}$$

For an antisymmetrical distribution function of the form

$$\varepsilon(\rho) = 1 + a\left(\frac{\rho}{\rho_\ell} - \frac{1}{2}\right) e^{-\left(\frac{\rho}{\rho_\ell} - \frac{1}{2}\right)^2} \tag{87}$$

we have

$$p(\rho) = \frac{-ae^{-\left(\frac{\rho}{\rho_\ell}-\frac{1}{2}\right)^2}\left\{2\left(\frac{\rho}{\rho_\ell}-\frac{1}{2}\right)\left[3-2\left(\frac{\rho}{\rho_\ell}-\frac{1}{2}\right)^2\right] + ae^{-\left(\frac{\rho}{\rho_\ell}-\frac{1}{2}\right)^2}\left[1+4\left(\frac{\rho}{\rho_\ell}-\frac{1}{2}\right)^2\right]\right\}}{\left[1+a\left(\frac{\rho}{\rho_\ell} - \frac{1}{2}\right)e^{-\left(\frac{\rho}{\rho_\ell} - \frac{1}{2}\right)^2}\right]^2 \rho_\ell^2}\,. \tag{88}$$

For an amplitude modulated sinusoidal distribution function of the form

$$\varepsilon(\rho) = 1 + a[1 + b \sin(\beta\pi\rho/\rho_\ell)]\sin(n\pi\rho/\rho_\ell) \tag{89}$$

we have

$$p = -a\left(\frac{\pi}{2\rho_\ell}\right)^2 \Bigg\{ a\left(1+b \sin\beta\pi\frac{\rho}{\rho_\ell}\right)\left[n^2\left(1+b \sin\beta\pi\frac{\rho}{\rho_\ell}\right)+\beta^2 b\sin^2 n\pi\frac{\rho}{\rho_\ell}\sin\beta\pi\frac{\rho}{\rho_\ell}\right]$$

$$+a\left(\beta b\sin n\pi\frac{\rho}{\rho_\ell}\cos\beta\pi\frac{\rho}{\rho_\ell}\right)^2 + \left[n^2+b(n^2+\beta^2)\sin\beta\pi\frac{\rho}{\rho_\ell}\right]\sin n\pi\frac{\rho}{\rho_\ell}$$

$$-2n\,\beta b\cos n\pi\frac{\rho}{\rho_\ell}\cos\beta\pi\frac{\rho}{\rho_\ell}\Bigg\} \Bigg/ \left[1 + a\left(1+b\sin\beta\pi\frac{\rho}{\rho_\ell}\right)\sin n\pi\frac{\rho}{\rho_\ell}\right]^2 \tag{90}$$

A generalization of Eq.(83) by raising the sinusoidal variation to $J^{th}$ power yields the following undulating profile:

$$\varepsilon(\rho) = 1 + a \sin^J (n\pi\rho/\rho_\ell) \tag{91}$$

where J = 1, 2, 3, 4, etc.

From Eq.(66), we obtain

$$p(\rho) = aJ\left(\frac{n\pi}{2\rho_\ell}\right)^2 \{\varepsilon(\rho)[(J-1)\cdot\sin^{(J-2)}(n\pi\rho/\rho_\ell)\cdot\cos^2(n\pi\rho/\rho_\ell) - \sin^J(n\pi\rho/\rho_\ell)] - aJ[\cos(n\pi\rho/\rho_\ell)\cdot\sin^{(J-1)}(n\pi\rho/\rho_\ell)]^2\}/[\varepsilon(\rho)]^2 \tag{92}$$

V. Computational Results

We shall now use the above models of profile distribution for $\varepsilon(\rho)$ to compute the plane wave transmission and reflection in the presence of dielectric constant gradients in a clear atmosphere.

For practical purposes the refractive index $\underline{n}$ of a parcel of atmosphere at absolute temperature T and pressure P (in millibars) is given by

$$\underline{n} = \sqrt{\varepsilon} = 1 + \frac{c}{T}\left(P + \frac{de}{T}\right)\cdot 10^{-6} , \tag{93}$$

where c = 77.6°K/millibar, d=4810°K, and e is the partial pressure of the water vapor in millibars. It is customary to speak of the refractivity N in the form

$$N = (\underline{n}-1)\cdot 10^6 . \tag{94}$$

The meteorological conditions necessary for the production of patches of atmosphere with anomalously large refractive index gradients have yet to be firmly established. A steep lapse-rate in water vapor content is probably essential since the correlation between patches of large scattering cross section and temperature inversion is quite evident. Simultaneous radar and refractometer soundings have shown[1-3] that the lower atmosphere often contains, in clear air, parcels or strata which exhibit steep refractive index gradients. Similar stratification of the lower atmosphere is also observed in recent soundings by lidars (laser radars)[9-10] and acoustic probing.[11] Radar observations[2] have shown that layer structures of the atmosphere are seen almost all of the time, regardless of season, at various elevations up to 20,000 feet. Lane[12] reports that refractive index changes of 5N units over a vertical interval of about 10 cm were observed. We shall use a refractive index change of 50N units (i.e., dielectric constant change of about 100N units) over a vertical distance of one meter as a realistic maximum allowable limit in computation (maximum $\varepsilon=1.0001$).

We shall use the Gaussian profile of Eq. (75), the delta function profile of Eq. (85), and the undulating profile of Eq. (91), since these profiles all represent actual profiles found in a clear atmosphere. The boundary conditions prescribed for these profiles are the same, i.e.,

$$\text{at } \rho = 0 \text{ and } \rho_\ell , \qquad \varepsilon(0) = \varepsilon(\rho_\ell) = 1 ,$$

$$\text{at } \rho = \tfrac{1}{2}\rho_\ell , \qquad \varepsilon\left(\tfrac{1}{2}\,\rho_\ell\right) = 1.0001 .$$

Accordingly, we have for

Gaussian profile: $\varepsilon_1(\rho)=ae^{-b(\rho/\rho_\ell-0.5)^2}$ , (75)

where $a=1.0001$, $b=4\ \ell og_e a=0.00039998$.

Delta profile: $\varepsilon_2(\rho)=ab/2[b^2+(\rho/\rho_\ell-0.5)^2]^2\rho_\ell^2$ , (85)

where $b=0.5/\sqrt{1.0001-1}=50$, $a=1.0001b=50.005$.

undulating profile: $\varepsilon_3(\rho)=1+a\ \sin^J(n\pi\rho/\rho_\ell)$ , (91)

where $a=0.0001$, $n=1, 2, 3$, etc.

$J=1, 2, 3$, etc.

We note that the derivatives at $\rho=0^-$ and $\rho_\ell^+$ are not continuous for the above equations except for cases where $J>1$ in Eq.(91). It turns out that the Gaussian and delta function profiles are nearly identical, as shown in the profile curves of Figure 1, and the reflection characteristics shown in Figure 1a are accordingly also nearly identical. Figure 1a also shows the reflection characteristics for undulating profiles with n=1 and various J, and Figure 2 shows those for cases with J=1 and various n. From Figure 2 we note that the absolute maximum occurs at $\rho_\ell=0$ and a relative maximum appears to occur in the vicinity of $\rho_\ell=n\pi/2$ except for cases with n=1 and 2. Figure 3 shows for n=20 and 21 that the relative maxima indeed occur at $\rho_\ell=20\pi/2=10\pi$ and $\rho_\ell=21\pi/2=10.5\pi$ respectively. We therefore conclude that reflection maxima do not always decrease with increasing $\rho_\ell$, where $\rho_\ell = \int_0^\ell k_o\sqrt{\varepsilon}dz$ as shown in Eq.(67), and the location of a relative maximum depends strongly on the profile involved. For very large $\rho_\ell$, reflection maxima, however, do decrease with increasing $\rho_\ell$. The small discontinuity in derivatives at the ends of the profile introduces only negligible error, although it does not meet the boundary conditions imposed by the analysis.

Since Eq.(91) can represent various shapes of undulating profile by assigning different J, we show the reflection characteristics for various n in Figure 4 for J=2, in Figure 5 for J=3, and in Figure 6 for J=4. Figure 4 indicates that for J=2 and various n the absolute maximum is the relative maximum and occurs at $\rho_\ell = n\pi$. Both Figure 5 for J=3 and Figure 6 for J=4 show that there is always a relative maximum at $\rho_\ell = J \cdot n \cdot \pi/2$ and the absolute maximum always occurs at $\rho_\ell = Jn\pi/2 - n\pi = \frac{n\pi}{2}(J-2)$.

Summarizing these results, we can generalize that there is always a relative maximum at $\rho_\ell = Jn\pi/2$ for any J and the absolute maximum occurs at $\rho_\ell = \frac{n}{2}(J-2)$ for $J>2$. For J=2 the absolute maximum merges with the relative maximum and for J=1 the absolute maximum is at $\rho_\ell = 0$. Minima occur at intervals of $\pi$, except where there is a relative maximum.

From Figures 1a through 6 we can conclude that reflection maxima occur when partial internal reflections from all locations of the inhomogenous medium are added in phase, and no resultant reflection occurs when partial internal reflections result in complete cancellation. To further prove the validity of the above statement and the correctness of this theoretical analysis on multiple scattering, we shall extract some findings from Figures 1a through 6 and arrange them in Table I below for various J and n:

Table I. Magnitudes and Locations for the Absolute Maxima

| J | n=1 | n=2 | n=3 | n=4 | n=6 | n=20 |
|---|---|---|---|---|---|---|
| 1 | | -67(0) | | -61(0) | -57(0) | -47(0) |
| 2 | -79(π) | -73(2π) | -70(3π) | -67(4π) | | -53(20π) |
| 3 | | -75(π) | -72(1.5π) | -69(2π) | | -55(10π) |
| 4 | -79(π) | -73(2π) | -70(3π) | -67(4π) | | -53(20π) |

The number within the parentheses is the location of the absolute maximum and the number before the parentheses is the reflection in db. Table I reveals that in general when n is doubled the reflection increases by 6db; when n is an order of magnitude larger the reflection increases by 20db. These numerical findings are clearly consistent with the reasoning that when two (or ten) partial reflections of almost identical magnitudes are added in phase the resultant reflected power increases by 6db (or 20db).

Figures 1a through 6 are obtained by plotting the computed results of power reflection coefficient R of Eq.(62). The power transmission coefficient T is computed from Eq.(63). In computing these coefficients numerical integration is used in evaluating A(z) of Eq.(68) and B(z) of Eq.(69), since $p(\rho)$ is in general not analytically integrable. The power transmission coefficient T is not plotted since it is very close to unity. The following Table II shows the numerical values of a sample calculation:

Table II. Sample Calculations for the case a=.0001, n=20, J=2

| | | | |
|---|---|---|---|
| RHOL=68.00 | RT=.99999982 | RR=.00000014 | LR=-68.62 |
| RHOL=66.00 | RT=.99999999 | RR=.00000000 | LR=-94.82 |
| RHOL=64.00 | RT=.99999630 | RR=.00000300 | LR=-55.22 |
| RHOL=62.00 | RT=.99999548 | RR=.00000394 | LR=-54.04 |
| RHOL=60.00 | RT=.99999992 | RR=.00000006 | LR=-72.23 |
| RHOL=58.00 | RT=.99999970 | RR=.00000023 | LR=-66.47 |
| RHOL=56.00 | RT=.99999995 | RR=.00000003 | LR=-74.93 |
| | KZ=67.99633503 | RS=.99999996 | |
| | KZ=65.99656296 | RS=.99999999 | |
| | KZ=63.99673271 | RS=.99999930 | |
| | KZ=61.99684858 | RS=.99999943 | |
| | KZ=59.99697590 | RS=.99999998 | |
| | KZ=57.99709606 | RS=.99999993 | |
| | KZ=55.99721003 | RS=.99999998 | |

RT = Real part of the transmission coefficient.

RR = Real part of the reflection coefficient.

LR = 10 $\log_{10}$(RR) in db

KZ is evaluated from Eq.(70)

RS is the sum of RT and RR and is extremely close to unity. This also indicates the validity of the theoretical analysis.

VI. APPLICATION TO PROBING OF ATMOSPHERIC WATER VAPOR PROFILE

To illustrate the nature of the problem, we shall start with a hypothetical case: given two profiles of the forms $\varepsilon_A=1+0.0001 \sin^2(4\pi\rho/\rho_\ell)$ and $\varepsilon_B=1+0.0001 \sin^4(4\pi\rho/\rho_\ell)$, how can we distinguish the profiles from each other by simple reflection measurements? Inspection of Figures 4 and 6 indicates that for n=4 at $\rho_\ell=4\pi$ the reflections for both J=2 and J=4 should be the same, whereas at $\rho_\ell=8\pi$ the reflection for J=2 is zero and that for J=4 is about -73db. To translate $\rho_\ell$ back to physical parameters we use the definition of $\rho_\ell$ from Eq.(67), i.e.,

$$\rho_\ell = \int_0^\ell k_o\sqrt{\varepsilon(z)}dz = \int_0^\ell \frac{2\pi}{\lambda_o} \sqrt{\varepsilon(z)}dz$$

where $\lambda_o$ is the free space wavelength.

For $\varepsilon(z)$ extremely close to unity along the profile, $\rho_\ell = \frac{2\pi}{\lambda_o}\ell$. Accordingly, $\rho_\ell$ is directly proportional to the length $\ell$ of the profile and inversely proportional to the wavelength. For $\ell=10$ meters, $\lambda_o=2.5$ meters for $\rho_\ell=8\pi$ and $\lambda_o=5$ meters for $\rho_\ell=4\pi$. In other words, reflection measurements at $\lambda_o=5$ meters should yield identical values for both profiles whereas at $\lambda_o=2.5$ meters no reflection should be observed for the profile with J=2.

In actual cases, the situation is reversed since the profile is the unknown to be found. In such cases we have to measure the actual reflections at a number of wavelengths and then find the correct profile to fit these measurement points. The process of curve fitting could be started by assuming an initial guess profile and then reducing the difference between the measured points and computed points by an iterative procedure such as the method of least squares. The questions of stability and rapidity of convergence and uniqueness of solutions will of course arise in this kind of synthesis problem but will not be discussed in this paper. Similar questions are also raised in determining atmospheric parameters for inverse problems in radiative transfer[13].

## VII. SUMMARY AND DISCUSSIONS

A theoretical analysis of the multiple scattering of electromagnetic wave propagation in a finite inhomogeneous medium has been presented.

The present analysis not only offers a way to synthesize the water vapor density profile in a clear atmosphere by reflection measurements at different frequencies, but also offers a satisfactory explanation of the facts that in a clear atmosphere(i) a radar echo is always associated with some identifiable steep gradients of refractive index, and (ii) steep gradients of refractive index, however, are not always accompanied by radar echos. Because of the multiple scattering effect of waves, radar echoes depend strongly on the profile of refractive index and the frequency used. For a given profile the echo at one frequency may be the maximum, and at another frequency there may be no reflection at all.

It is implicitly assumed in the analysis that the clear atmosphere has a nonuniform but stationary distribution of molecules at various height levels. This means that not only is the time-average distribution of the density of the molecules uniform local-wise, but also at any instant the density of the molecules also does not exhibit statistical fluctuations about the local average. Another approach[1-4][12][14-16] to this scattering problem is based on random fluctuations of refractive index produced in some way by turbulence in the atmosphere and relates radar echoes from the clear atmosphere to incoherent back-scattering from local patches with large irregular fluctuations in the refractive index. Since it has been recognized that no single model or mechanism is likely to furnish a complete explanation, the present approach represents an attempt to provide additional insight into the problem.

We have assumed in the models of water vapor profile that the refractive index is real because the attenuation of electromagnetic radiation of wavelength longer than a few centimeters by water vapor in the atmosphere is negligibly small. For electromagnetic waves passing through thin layers of light clouds, we might also use a real refraction index for water vapor at low microwave frequencies. For light clouds with sparsely distributed small water droplets, the Mie scattering effect of the water droplets might be neglected and only the effect of water vapor need be considered.

For thick and heavy clouds, attenuation will be appreciable and at the same time frequency dependent. The analysis in this paper could be applied directly to a thick and heavy cloud or even rain if an equivalent complex dielectric constant could be obtained for the heavy cloud or rain. It is clear that in these cases the complex dielectric constant itself is frequency dependent as well as height dependent.

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C.C.H. Tang

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Attachment
References
Figures 1-6

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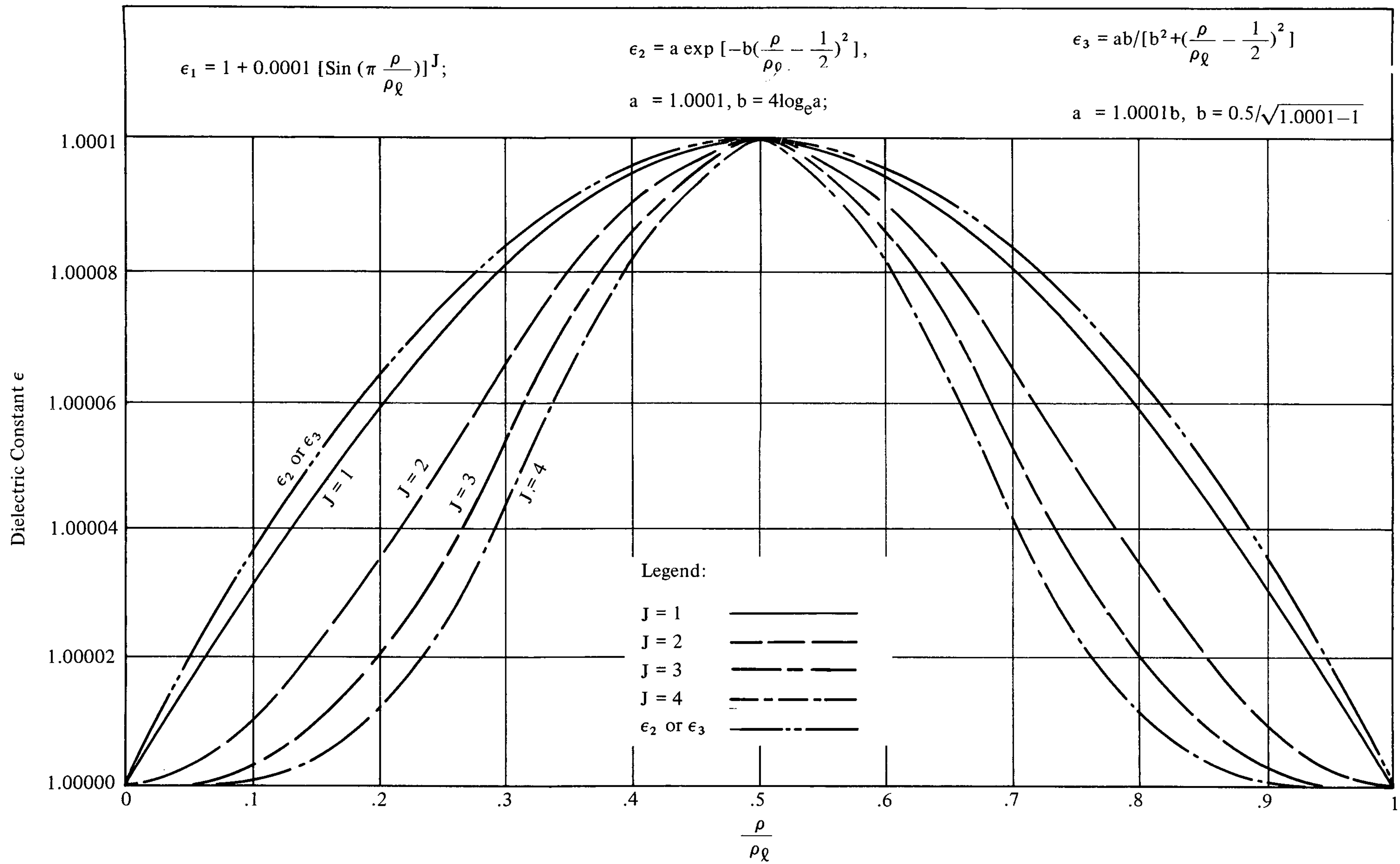


FIGURE 1 - ATMOSPHERIC DIELECTRIC CONSTANT PROFILE

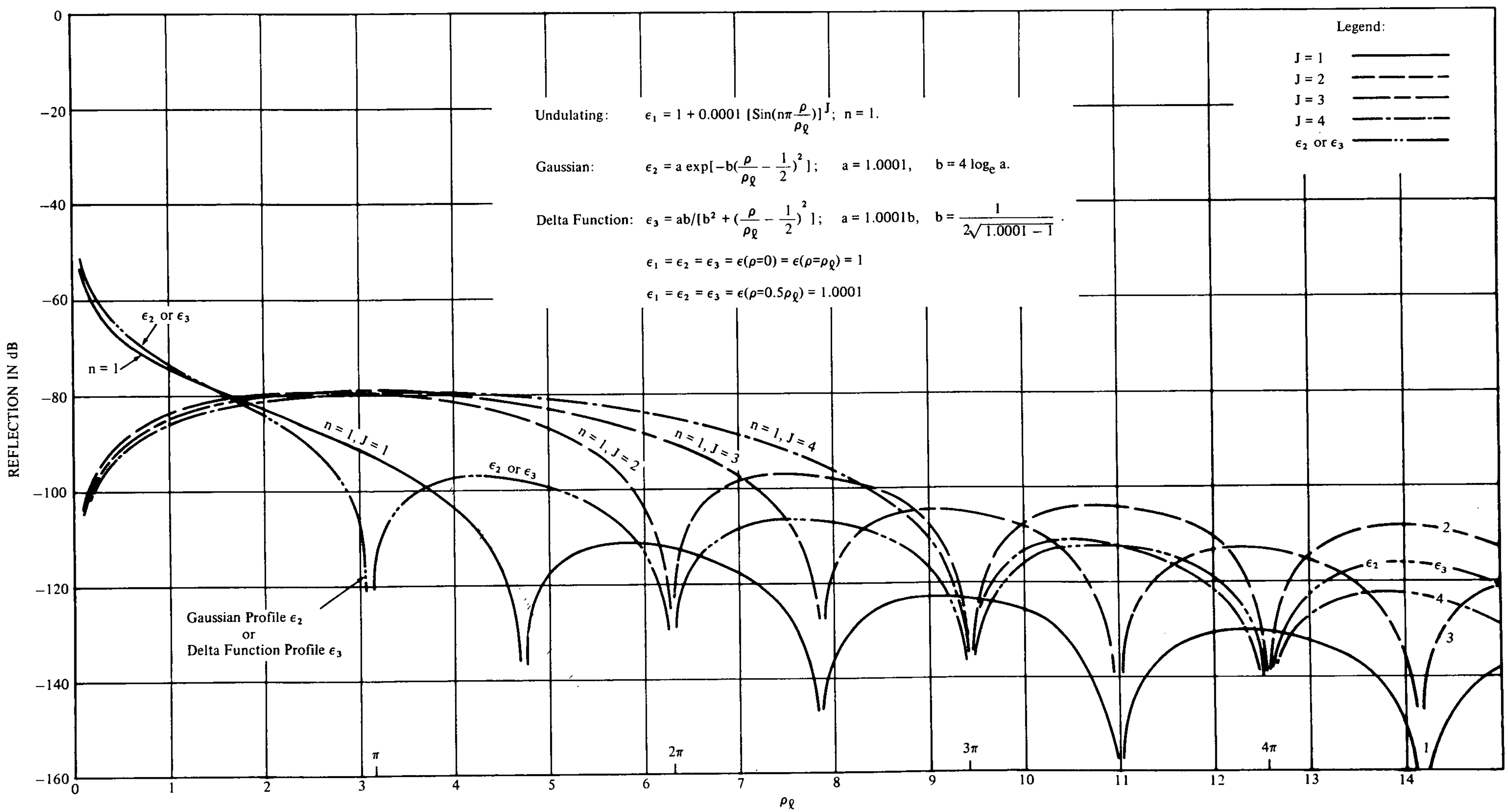


FIGURE 1a - REFLECTION CHARACTERISTICS FROM FINITE INHOMOGENEOUS MEDIA

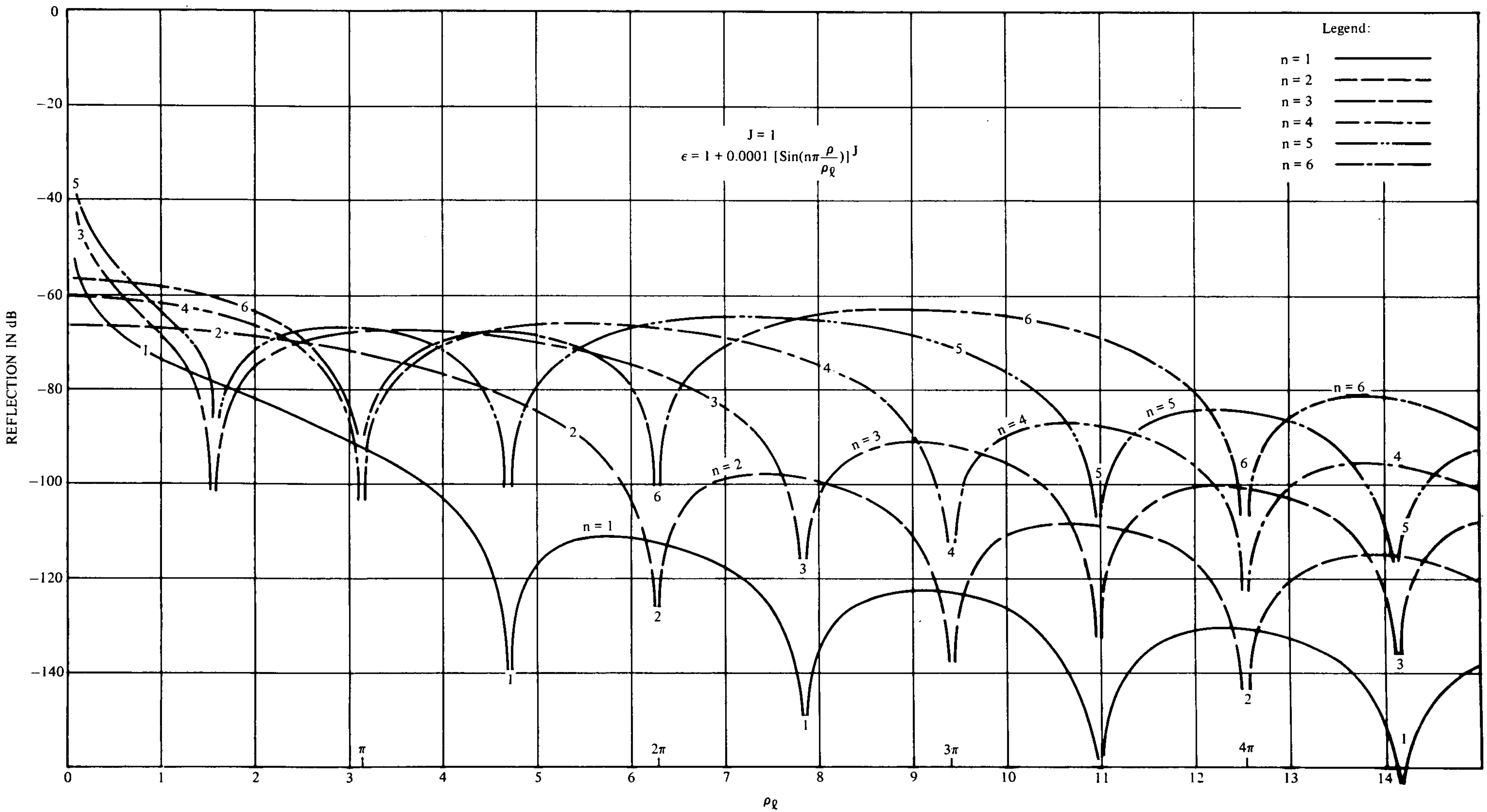


FIGURE 2 - REFLECTION CHARACTERISTICS FROM FINITE INHOMOGENEOUS MEDIA (J=1)

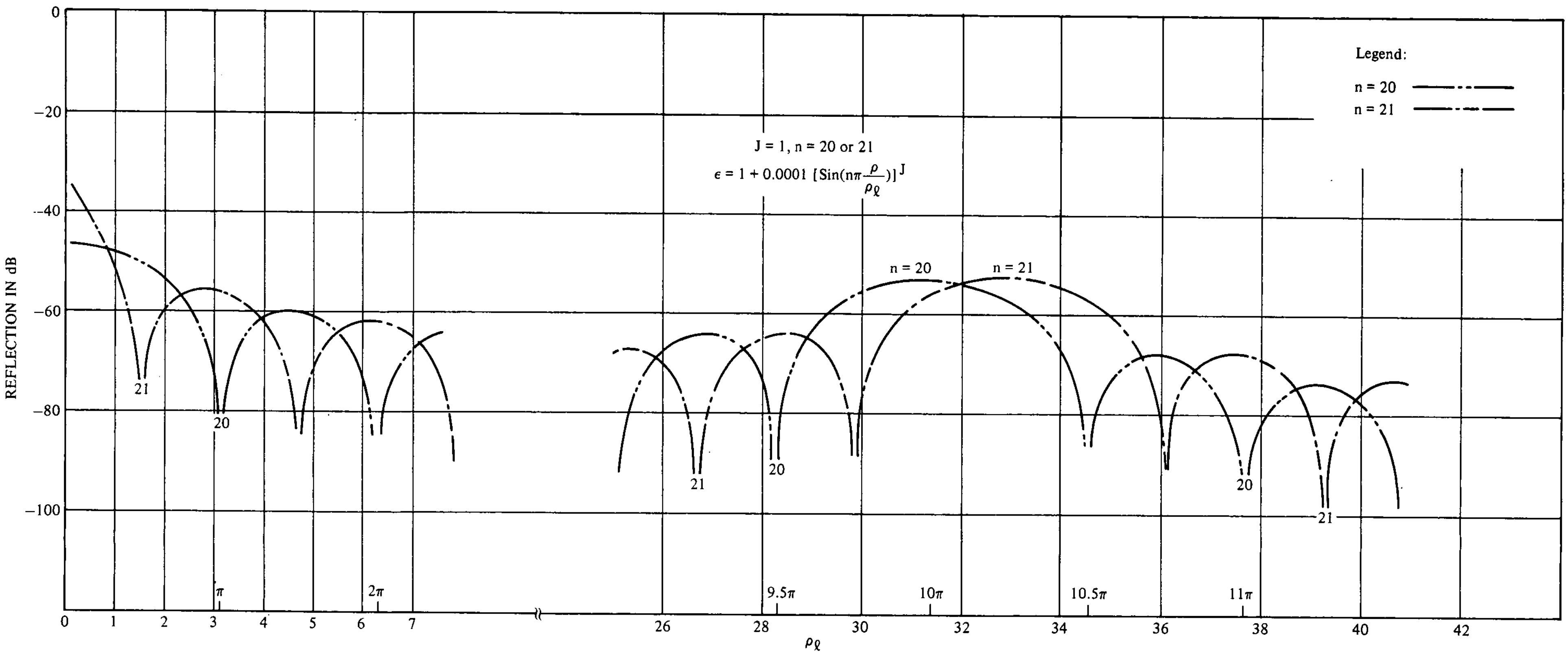


FIGURE 3 - REFLECTION CHARACTERISTICS FROM FINITE INHOMOGENEOUS MEDIA (J=1)

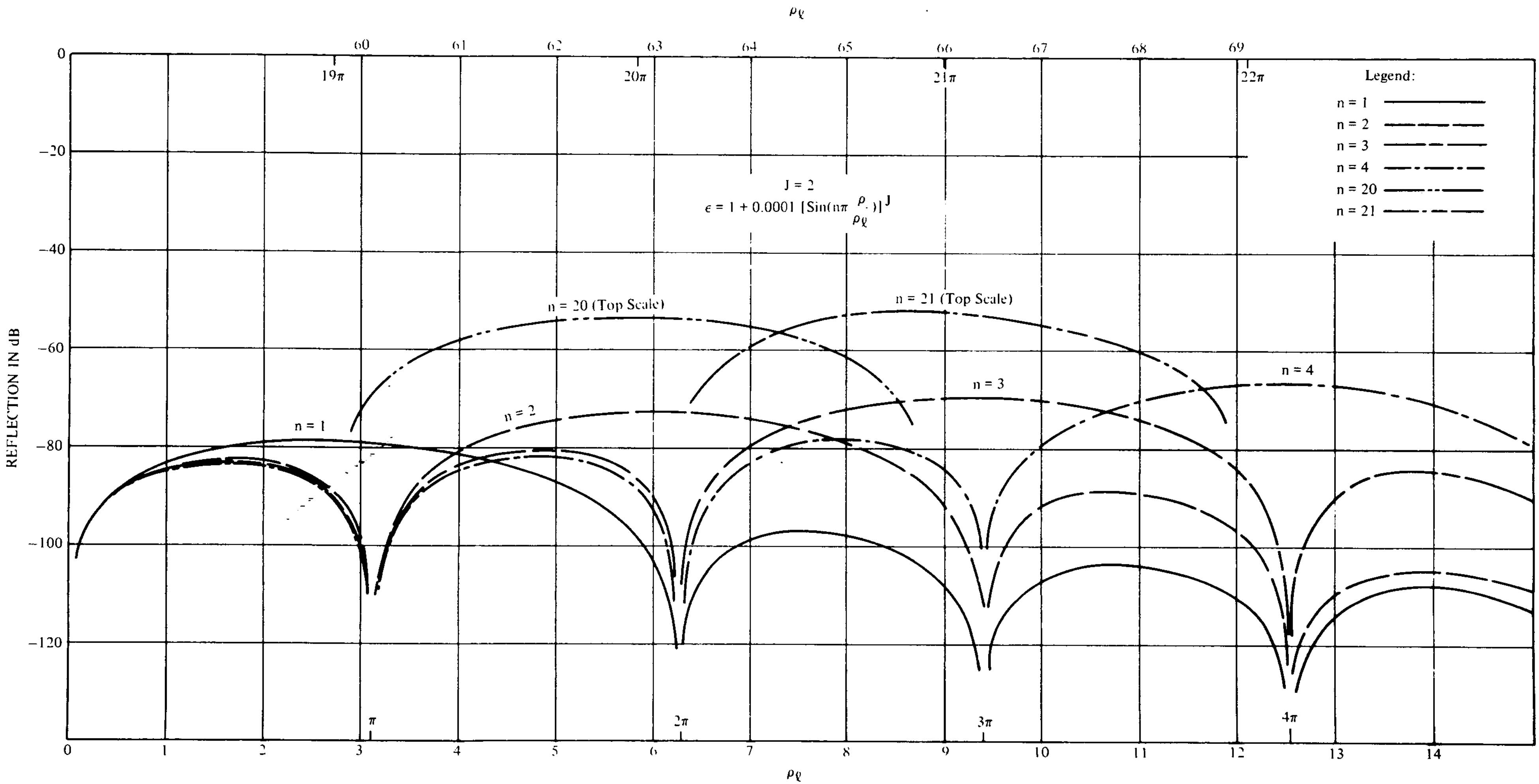


FIGURE 4 - REFLECTION CHARACTERISTICS FROM FINITE INHOMOGENEOUS MEDIA (J=2)

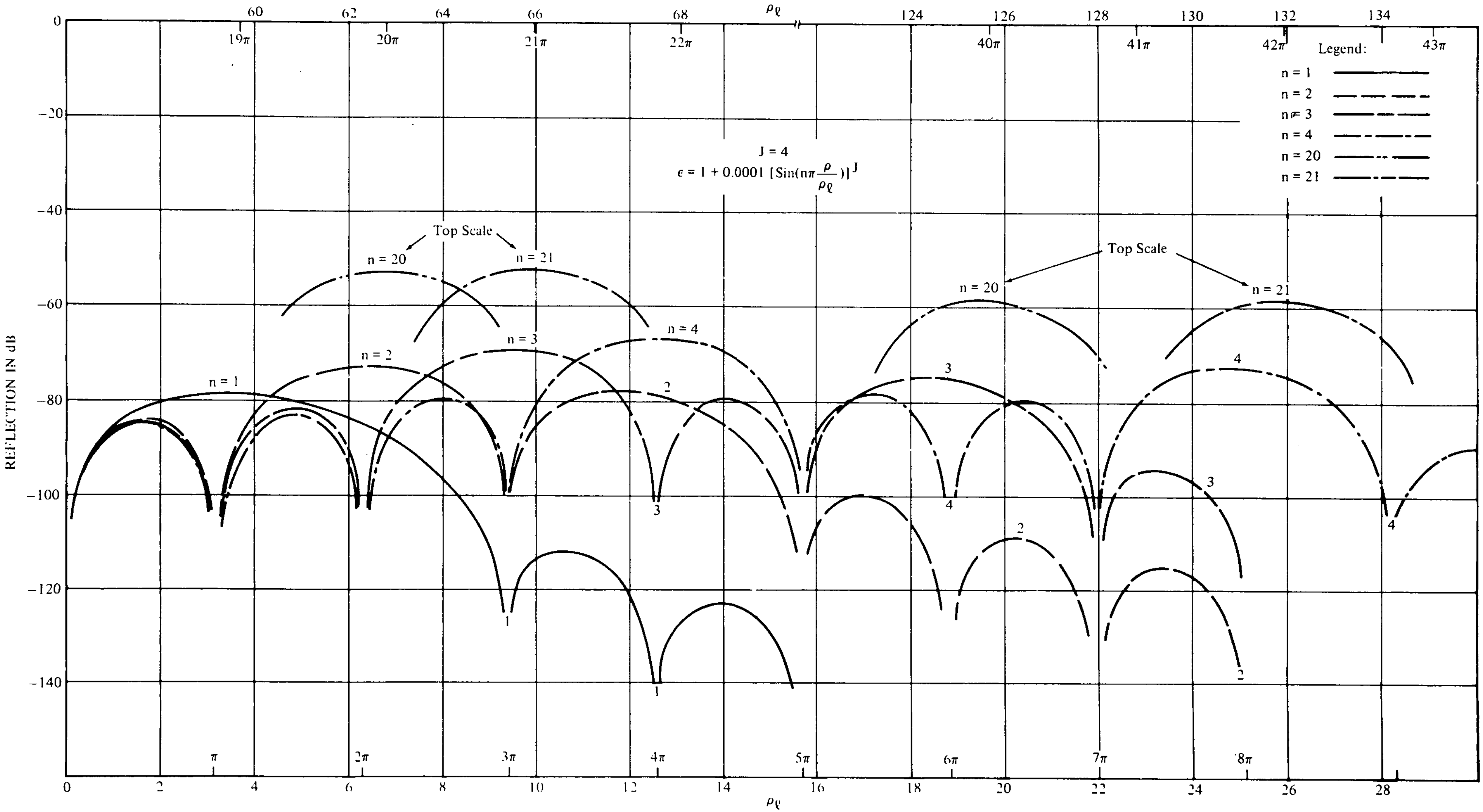


FIGURE 6 - REFLECTION CHARACTERISTICS FROM FINITE INHOMOGENEOUS MEDIA (J=4)

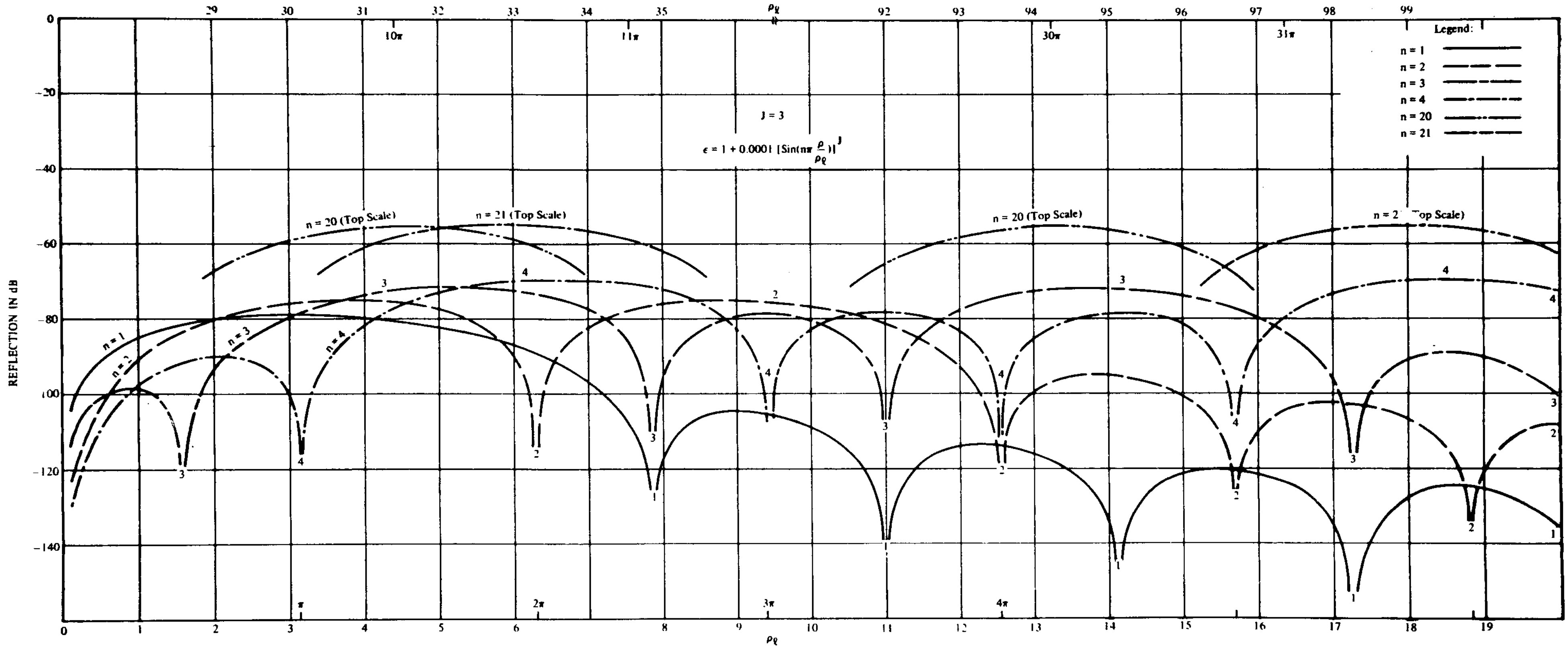


FIGURE 5 - REFLECTION CHARACTERISTICS FROM FINITE INHOMOGENEOUS MEDIA (J=3)