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SECOND QUARTERLY REPORT

Contract No. NAS8-26773

ATS-11810

PHENOMENA AFTER METEOROID PENETRATION OF A BUMPER PLATE

**CASE FILE
COPY**

by

F. C. Todd



Submitted to

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

Submitted by

The University of Alabama in Huntsville
Division of Graduate Programs and Research
Research Institute
P. O. Box 1247
Huntsville, Alabama 35807

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FOREWORD

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SECOND QUARTERLY REPORT

May 1 through July 31, 1971

on

PHENOMENA AFTER METEOROID PENETRATION OF A BUMPER PLATE

Phase Report

REACTIVATION OF HARDAGE'S COMPUTER PROGRAM FOR CRATER FORMATION
AND APPLICATION OF PROGRAM TO PLATE PENETRATION

to

Thermal Physics Section
Engineering Branch, Materials Division
Astronautics Laboratory
Marshall Space Flight Center

from

Graduate Programs and Research
The University of Alabama in Huntsville

by

F. C. Todd

GENERAL STATUS OF THE PROJECT

The proposal and the first Quarterly Report indicated that the work on this project would be divided into four phases. These phases are (1) the ablation of particles penetrating the paper insulation, (2) the formation of a cone of debris with a truncated apex at the hole which is formed by the penetration of an initially, spherical rock through a thin plate, (3) the penetration of the above cone, large end first, into the paper insulation with conversion of the paper into a gas, and (4) the effect of the degraded shock at the large end of the cone impacting through the paper onto the back-up plate. Work was reported on phase

one of this project in the first Quarterly Report. This is the second Quarterly Progress Report which consists of three parts. One part is this relatively short introduction and the other parts are two enclosures. These three parts of the report will be discussed below.

This introduction to the enclosures is written to present the status of this project in regard to completion of the project and to show the connection between this short write-up and the work that is included on the enclosures. Since this report dovetails into the work on a project for the Space Sciences Laboratory, a Quarterly Report to SSL is enclosed. In this introduction to the enclosures, the specific details of interest to this project for the Thermal Physics Section are emphasized. The work in the enclosed Quarterly Report to SSL results from an adaptation of a Ph.D. thesis that was completed by Dr. B. A. Hardage in May of 1967. The report to SSL shows that the computer program works very well as it has been reactivated. It is actually working better than the computer program ran for Hardage. The "knit-picking" attitude in the SSL report is to insure that no major error is overlooked. The second enclosure is a copy of the original thesis by Hardage. The foreword in the thesis is included to tell the history of the thesis and to clarify some of the points that would otherwise be rather obscure. For reasons that are given in a foreword in Hardage's thesis, this work was performed on the same series of contracts that are continued to the present time with SSL. This thesis was not forwarded to any NASA representative until this last spring. Hardage's thesis is concerned with the formation of a crater by the hypervelocity impact of a quartz sphere. The penetration of a plate is essentially the same as the first part of the crater formation as is shown by the Quarterly Report to SSL.

The period that is covered by this report includes the summer vacation when this author was absent on vacation for two months. This is largely the reason for the delay in this report. Part of the delay is the fact that progress looked good on the reactivation of the computer program but there was nothing to report. The most difficult problem with this entire project has always been considered to be the reactivation of this computer program. This is now shown to operate better than it ever has in the past. The author is not a programmer, so full credit must be given to Mark Hooker for the actual mechanics of modifying

the program while Dr. Hardage has helped with advice in every way requested. He was not asked to assist in programming. The improvement in Hardage's program comes from a change in the method of differencing. To be technical, Hardage used back differencing which is very good for about 30 cycles. Hooker, on the advice of Hardage and after a demonstration by Dr. R. E. Bruce, made the modifications on the program. Bruce is another ex-student of mine who is a consultant on the SSL project. The practical difference is that Hardage stopped his program about every 50 cycles and adjusted the interfaces. The improvement is that 371 cycles were obtained with no adjustments at all on anything and none were required before continuing the run.

In the remainder of these introductory comments, some discussion is presented on the results as they are related to the project for the Thermal Physics Section as compared to the project for SSL. In particular, there is a recapitulation of the status of the work on the penetration of a particle through paper as indicated by the first Quarterly Report. Then there are a number of comments on the status of the work for the Thermal Physics Section as shown by this second Quarterly Report. In conclusion, there are a few comments on the last two phases of this project.

STATUS OF THE STUDY ON THE ABLATION OF PARTICLES

The first Quarterly Report on the status of the penetration of particles through the paper layers presented two approaches to the problem on the loss of energy and material by the particle. The energy loss per unit length of penetration is very different for these two postulates. The first approach to the problem was programmed for a numerical solution. The boundary condition was the total depth of penetration into the insulation of a particle of a known initial, effective diameter. The second approach with different basic assumptions has a very different rate of degradation along the length of path. In addition, there is another very important observation that may be employed as a boundary condition. The new boundary condition is that the loss of energy is the same per unit length of path at any position along the entire length of path. This new solution is not entirely completed. The next step is to program the solution for a computer and to vary the possible components in the solution and the available variables in order to obtain a consistent solution. An apparently acceptable solution must then be incorporated into the overall solution which may be checked against experiment.

In the preceding paragraph, the new boundary condition is that the loss of energy per unit length of path is the same along the entire length of path. This statement is based on the observation that the diameter of hole through the paper insulation is the same diameter along its entire length. The hole is believed to be formed by penetrating the layers of paper with a combination of rupture and burning. The rupture and the friction from penetration keeps the particle hot and ablating while the burning has the effect of determining the diameter of the damage, i.e., of the hole. The only way for the hole in the paper to remain the same diameter along its length is for the particle to move slower and slower as the particle becomes smaller and smaller.

STATUS OF FORMING A COMPUTER SOLUTION TO REPRESENT THE DEBRIS CONE

The basic study in this second Quarterly Report is involved with the formation of the debris cone. The attached report to the Space Sciences Laboratory shows the progress in obtaining a solution for the penetration of the thin plate. The report to SSL emphasizes the problem of penetration and the corrections that may be added in order to obtain a more accurate representation. Equal, or more emphasis could be placed on obtaining information for the formation of the debris cone. In this section, some discussion is given on the penetration of the thin plate with emphasis on the formation of the debris cone.

In the last section in the SSL report, the short section on "Program for Future Work," presents corrections primarily for the origin of the debris cone. The original program was designed for following the formation of a crater. It was satisfactory for that purpose so very few additional corrections are required. From the graphs of the penetration of the plate, the general features of the problem are clearly delineated. Although unnecessary, a few comments will consolidate the discussion. The graphs show that pieces and blobs of fluid aluminum are being ejected from the thin aluminum plate before the crushed rock starts to flow through the "shock formed" nozzle. At the instant of separation, the direction, velocity and energy content of each separating piece is known from the print-out of the computer program. The summation of these particles is the initial front of the debris cloud. This is the same way that the "effective" increase in momentum was determined during the formation of the crater.

It is now feasible to recognize the order of magnitude of the corrections that are recommended. There are two fundamental corrections which must be considered. These are (1) the proposed change in the equation of state in order to withstand tension and (2) the possible result of considering the viscoelastic properties in some regions of the impact. Consider the quantitative effect of requiring tension to rupture the aluminum. The pressure in the compressed aluminum is as high as 2 megabars. A change in the tension to rupture from 0 to a maximum of 5 kilobars would be expected to have a very small effect on the shape of the hole, or the total energy that is involved in the penetration. It is important for the author to correct every possible error and consider the residue. For the general problem, the solution is already very good and this will be mentioned again in the last paragraph of this section.

The second important correction is for the viscoelastic effect that is included in the program by Rosenblatt. The paper by Whiteside et al, Reference 3 in the Bibliography for the SSL report, indicates that this is not important for high velocities where the pressure is high and the energy content is very large. The viscoelastic forces will have a small effect on the penetrating material. The viscoelastic region will certainly appear but it will be most effective in determining the amount of losses that this region will insert between the edge of the hole and the body of the thin plate.

As an illustration that the principal features of the penetration are reported in the enclosed graphs, consider the graph in Figure 9 of the Quarterly Report to SSL. In the proposal for this project, it was indicated that the wall of the hole through the thin plate would be expected to form a nozzle. The future outline of the nozzle in the wall of the hole is indicated by the rupturing material in Figure 9. At the lower left is already a break in the fluid aluminum so the rupture of two very small sections will form the nozzle that is observed in the final hole.

DIFFERENT CORRECTIONS FOR THE COMPUTER PROGRAM

In the listing of the program for future work in the report to SSL, all six corrections are of benefit to the Thermal Physics Section. Corrections 4 and 6 are exclusively for the Thermal Physics Section with no need for them by SSL, although the latter laboratory

may have a general interest in the conclusions. Some discussions of the reason for the different corrections may indicate their quantitative importance to the solution.

The capability of stopping and restarting the computer program is for the purpose of insuring that the parts from the thin plate break clean and into relatively small pieces. At present, a break occurs when the flow produces tension between adjacent cells. Some of these breaks will not occur when a fixed, small tension must exist between adjacent cells in order for a break to occur. It is doubted, however, that the number of breaks will be significantly affected. The primary forces are so large during this early stage of the impact that it is not anticipated that any substantial difference in the number of breaks will exist. This capability of stopping, revising and restarting the program was developed in the original program so the capability is retained in the present version of the program. Although not yet reported to SSL, the capability has been reinstated, tried and found to operate in an acceptable manner.

Many comments are made on the need for a correction for tension in the equation of state. For the problem from the Thermal Physics Section, the effect of this correction is small. The difference is that slightly more energy is required to break-up the thin plate. This is observable as a lowering of the total energy in the cone of debris, both internal and kinetic energy are lost. Large chunks in the break-up of the plate instead of small chunks would result in this same effect, i.e. would reduce the total energy in the cone of debris. The amount of break-up of the thin plate will depend on the operator who "hand-tailors" the break-up. It is desirable for the computer to divide the energy and the kinetic energy between the separating and the bulk material. While still attached to each other, the laws for the continuum will separate the blobs with the proper division of energy.

The "book-keeping-program" is to assure that all of the pieces of aluminum and of quartz are taken into account in order to obtain a more accurate solution. This phase is not particularly significant for the penetration of a thin plate, but it is of fundamental importance in locating the position, relative to the cone of debris, for pieces that separate from the aluminum and the quartz.

The substitution of the smaller mesh is entirely for accuracy and most of the accuracy is expected to be of second order to the general solution. The top and bottom sides of the thin plate have already been made parallel and perpendicular to the axis. This is of little significance for these problems except the solution will be for a thin plate.

The final correction, in phase 6, is merely a general statement that all obvious corrections are to be added and that the "book-keeping" is important.



F. C. Todd
Principal Investigator

QUARTERLY REPORT

July 1, 1971 to September 30, 1971

on

STUDIES ON DENSE PLASMAS

Contract NAS8-25055

Phase Report

STATUS OF REACTIVATION OF HARDAGE'S COMPUTER PROGRAM FOR HYPERVELOCITY
IMPACT AND ADAPTATION TO PENETRATION OF A THIN PLATE

to

Space Sciences Laboratory

Marshall Space Flight Center

from

Graduate Programs and Research

The University of Alabama in Huntsville

by

M. J. Hooker and F. C. Todd

INTRODUCTION

This project deals with the phenomena that are associated with the hypervelocity impact of microparticles on an aluminum slab. The title of the project, "Study of Dense Plasmas", was applicable when this project was moved to The University of Alabama in Huntsville from Oklahoma State University. For the first year at The University of Alabama in Huntsville, the studies were confined to phases that were concerned with plasmas of the type which are equivalent to the plasmas that are produced by hypervelocity impacts, the energy content results from the increase of entropy which accompanies shock compression of the aluminum and of the material of the particle. The overall objective of the project on hypervelocity impact is to measure the density, the mass and the velocity of the particles that impact on a detector in space at hypervelocity. The expression, hypervelocity impact, is a description for particles that impact on a target with a relative velocity that is greater than the velocity of sound in the target. In addition, an investigation is to be made into possible methods for determining the composition of impacting particles.

The studies of the plasma were initiated in order to employ optical measurements for checking an analytical, computer type solution for the formation of the crater and another analytical computer solution for the expansion of a sphere of plasma. Computations show that a "cold" shell forms about the outside of an "exploding" plasma and this prediction has been confirmed by experiment. This type of work occupied the first two years of this contract at The University of Alabama in Huntsville. Additional comments are not in order in this introduction for this quarterly report will discuss the analytical, computer type solution for the formation of a crater.

At the start of the current fiscal year, June 1, the scope of the contract was enlarged to include two new phases. (1) Studies on a 3-dimensional, quadrupole, ion trap, mass spectrometer. (2) The analytical solution, as far as is possible, of the penetration of a thin plate by a sphere. The latter phase is the so-called, "bumper plate" problem. A complete analytical or computer solution cannot be obtained for this problem. When the incident sphere is crushed and starts to tear the plate apart, the problem is no longer a problem in a continuum. Since the available equations are for a continuum with variable density and pressure, the equations become invalid when holes, slits, and other complete ruptures occur in the media.

In the following report some examples are presented of the computer solution for the formation of a crater. The program has been reactivated from a solution of that problem by a former student, Dr. B. A. Hardage, of the author. Some changes were introduced which have resulted in a better computer solution than was obtained from the original work by Hardage. The change was minor, it was a change from back differencing to central differencing. As a final step, prior to the preparation of this report, the equations for the formation of the crater were applied to the penetration of a plate. The solution cannot be correct, but an analysis of the results will assist in deciding on the additional corrections that are required. In this report, four stages of the penetration of a plate are presented in graphical form in order to permit the solution to be analyzed and to select some of the additional features that are to be added to the program in order to obtain a good computer solution to which "hand-tailored" modifications may be added. The final computer solution must be "hand-tailored" through the region in which the break-up of the continuum occurs. Details of the proposed method for "hand-tailoring" the solution are presented in the body of this report.

COMPUTER PROGRAM FOR THE FORMATION OF A CRATER BY A HYPERVELOCITY IMPACT

An added phase of this contract was to be an attempt to obtain a computer solution for the penetration of a thin aluminum plate by a sphere of rock. The thickness of the aluminum plate is to be roughly one-quarter of the diameter of the sphere. The sphere of rock is really a sphere of quartz. An equation of state for quartz was one of the first that was obtained after equations of state were published for very many of the elements in the periodic table. In the particular problem that is discussed in this report, the quartz is porous instead of being solid. It has a density of 2.00 grams per cm³ in comparison to solid quartz with a density of 2.56 grams per cm³. This amount of porosity is very significant. Several people, starting with the Russians, have shown that the energy to crush the "cold", porous rock is converted into heat energy in the inviscid fluid which the quartz becomes after shock compression. This added energy increases the temperature by a very significant amount. This is more than when a solid sphere of rock is crushed by a shock to the same final pressure on the Hugoniot curve. Details of this subject, with references to the literature, appear in Hardage's thesis which was Interim Report Number 4. This report replaced Monthly Letter Report, Number 23.

There are several unique features of Hardage's solution. A detailed recounting of these features would be a rewrite of Hardage's thesis and this is certainly not desirable. There are, however, a few features which should be discussed in order to more easily interpret the results that are presented later in this report. The computer problem is solved by difference equations that involve a little more than the usual difference between the values of the variables in the cells of a mesh. This subject is discussed in the following subsection. Then there is some discussion of other features, such as possible velocities, and momentum distribution when drops separate from the main body of the material.

Modified Finite Difference Technique

The usual method for finite differences is expanded by a method that was initially started in an article that was written at Los Alamos and which contained some errors.¹ Some three to four years later, a second article on almost the same subject was written

by another author² at Los Alamos. There were no errors in the second article. Mention is made of these two articles since my ex-student, B. A. Sodek, started working with this method before the second article appeared. Sodek had corrected most of the errors but had missed one error, at least, in the first Los Alamos paper before the second Los Alamos report was released. The results from an application of this modification were not reported in either Los Alamos paper. Sodek completed his thesis before Hardage started his work so all of this background information was available to Hardage. As this modification of the simple difference method was applied by Hardage, all of the ways in which the material in a cell may be displaced by air, by another material, or by air and another material was considered. By employing this technique, Hardage was able to give the area of rock, of aluminum, and of rock, aluminum and air in each cell. This procedure located the interfaces between each of these components in his problem with considerable accuracy. He was also able to determine the position of the plastic material with the same accuracy after a few assumptions were made. This calculation required that Hardage distinguish the six ways in which an interface may move across a cell, and to identify every cell with a type number that identified the type of material displacement. Instead of employing the simplest method of differencing, Hardage calculated the radial and the tangential velocity for each cell and calculated the motion of the interface at each intersection with the wall of the cell. For each cycle, he calculated the Courant condition. He employed only 0.1 of the time interval that is given by the Courant condition. The Courant condition, in this type of problem, gives the approximate time interval for the moving interface to travel a distance of one cell dimension.

Basic Velocities in the Solution of the Problem

There are three distinct types of velocities in the problem and these velocities are often mixed in involved ways. There is one velocity that corresponds to the propagation of a longitudinal, elastic shock through solids. This velocity is a constant that

depends only on the elastic properties of the material. For solid aluminum at standard conditions, this velocity is recorded to be 16,740 feet per second. In metric units, this value is approximately 6.59 kilometers per second.

A shear wave through an elastic material has a velocity of propagation which is different from the velocity of a longitudinal wave. In the formation of a crater by a hypervelocity impact, shear waves in the solid aluminum are produced on the sides of the crater near the upper edge of the crater. The solution of the problem shows that the shear results from the flow of fluid from under the impacting particle on its way to being ejected from the crater by blowing off the lip of the crater. The resulting shear waves are properly called Rayleigh waves. These waves propagate with a velocity of a longitudinal shock wave. The Rayleigh waves have another characteristic, they decrease exponentially in energy with the depth into the solid. Confirmation of the existence of Rayleigh waves will be discussed later in this section.

The third type of velocities is the most pertinent to hypervelocity impact and to crater formation. This refers to the velocity of longitudinal shock propagation in inviscid fluids, where shear waves cannot exist. This velocity is defined by the laws of conservation of mass, momentum and energy when they are applied to the jump conditions which exist at the shock front. An equation of state is necessary and is normally obtained from the Mie equation of state. The shock front propagates at a velocity which is greater than the propagation of a longitudinal sound wave in the elastic material into which it is propagating and slower than the longitudinal sound wave velocity in the compressed material which is behind the shock front. The velocity of sound propagation in the inviscid fluid behind the shock front is given by an equation of the same form as it would be in any inviscid, compressible fluid; i.e. it is proportional to the square root of the pressure divided by the density. Since the compressed material is a fluid without viscosity, only longitudinal waves may propagate through this medium.

A search of the literature by Hardage did not disclose information on the pressure, or on the internal energy at which the transformation from inviscid to very viscous flow

occurred. It was found, by the use of the equation of state, that shock compression up the Hugoniot to about 33 kilobars would, after an adiabatic expansion to one atmosphere, leave an internal energy content in the aluminum of the heat of fusion. Since the expansion should be isentropic instead of adiabatic, and since the inviscid fluid must pass through a plastic range, a higher pressure would be required. Since the velocity in the elastic material is equal to the fluid velocity at 37.5 kilobars, this value was selected as the pressure at which the fluid material would be assumed to cease to be inviscid and would become plastic. This assumption is independent of the internal energy in the aluminum at the time of the transition. This was the simplest, arbitrary assumption which could be made at the time of the first solution to the problem. Sharp transitions from inviscid to viscous flow are not anticipated and would be unusual in physics. A more gradual transition over a longer range of pressures is anticipated. The statement in the last sentence is strongly supported by a recent paper in the October issue of a reliable journal.³ Pressure is probably not the only variable. Internal energy should probably also be considered in addition to the pressure.

The presence of Rayleigh waves is tentatively confirmed by two pieces of experimental evidence which are somewhat similar. Early in this project, a crater was obtained in an aluminum target which had been produced by Scully of North American Aviation by impacting a glass sphere on aluminum at a velocity of 28,800 feet per second, which is about 8.78 kilometers per second. The aluminum target was cut and polished through the center of the crater. It was etched to show the extent of the region with obvious grain distortion, which also the region of plastic flow. Plastic flow was found on the bottom and well up the sides of the hemispherical crater and was the same thickness over the entire area of its appearance. The region of distorted grains faded out toward the upper edge of the crater and did not extend up the crater to the original surface of the target. The solutions of Hardage's computer problem shows that this region was primarily in shear. Interference between the spherical, longitudinal wave and the shear wave could explain this disappearance of the plastic region near the surface of the target.

There is another bit of similar evidence that was in a report by a research team at Brown University. Experimentalists made a thin section of a piezo-elastic material which was transparent but was supported on both sides by a transparent material. A physical impact was made on the thin section with a cylinder so interference bands could be photographed. Concentric interference bands were observed which correspond to the calculated pressure decrease. These cylindrical bands fade out at approximately the same position that the evidence of plastic material faded on the etched aluminum in the first experiment. The references for the description of the etched aluminum and of the formal reference to the report from Brown University are listed in a report to Goddard Space Flight Center.

Mixed Velocities in the Plastic Region

Around and in a particle that is entering a target at hypervelocity, the pressure is very high and may often be of the order of several megabars. Behind a shock of this magnitude, the aluminum is assumed to be an inviscid fluid. As a roughly spherical shock expands, the pressure of the shock front decreases in amplitude until the material is only compressed sufficiently to produce a plastic state in which flows are small and the flow is very viscous. As the amplitude of the shock decreases still more, the magnitude of the shock becomes sufficiently small so the material behind the shock remains an elastic solid. A shock from a hypervelocity impact must start in an inviscid fluid. When this same shock is observed in the solid, elastic material, it is known that the shock has passed through a plastic zone. It is necessary, therefore, to consider the factors which influence the shape of the shock in the plastic zone. This is the region of greatest uncertainty in Hardage's computer solution. A great deal of information exists concerning stresses that exceed the elastic limit by a small amount. In these cases, material becomes plastic in order to yield. Almost no information exists on the subject of the material going, under pressure, from the inviscid fluid to the plastic and on to the elastic state.

In the plastic region, two longitudinal waves move simultaneously through the medium. This results in separating the shock into two parts and results in considerable

difficulty in defining the mechanism of this separation of the shock. There is practically no information available in the literature on the viscosity of fluid metals at pressures between several kilobars. The velocity of sound that is calculated from the equation of state is taken as the velocity of sound in inviscid, fluid aluminum. At a pressure of $37\frac{1}{2}$ kilobars, the calculated velocity of sound in the inviscid, fluid aluminum is found to be the same as the measured velocity of sound in elastic, solid aluminum. This pressure was selected as the arbitrary, sudden transition from fluid to plastic material. The preceding transition pressure can only be true provided the specific equation of state in Hardage's thesis is used. As already stated, the selection of $37\frac{1}{2}$ kilobars is quite arbitrary and is not defensible, but it does give a result which may be compared with experiment.

The assumption is more extensive than merely selecting the value of $37\frac{1}{2}$ kilobars as the arbitrary transition pressure. The computer is instructed to search each cycle in the inviscid-fluid zone for the first pressure wave in which the peak value of the pressure does not exceed $37\frac{1}{2}$ kilobars. When this cycle is found, the computer declares that the inviscid fluid is suddenly converted to a very viscous plastic. This sudden transformation is contrary to our knowledge of other physical phenomena. This assumption will certainly require modification in the final solution.

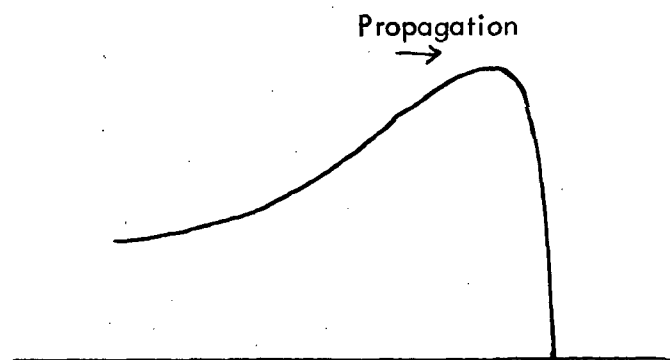
The plastic zone is assumed to support an elastic shock, which propagates as a longitudinal shock wave at the speed of sound in the solid, elastic material. The maximum pressure of the elastic wave was taken as the pressure at which elastic failure occurs in solid aluminum. In addition, another longitudinal shock wave propagates through the plastic zone at the propagation velocity of sound in a compressed, inviscid fluid. As already stated, this fluid shock in the plastic zone has a velocity of propagation that varies as the square root of the ratio of the pressure divided by the density. Recalling that the velocity of propagation in the elastic medium and in the compressed, inviscid fluid are the same at $37\frac{1}{2}$ kilobars, the velocity of propagation of sound at the fluid velocity in the plastic zone must be less than the velocity of propagation in the elastic part of the shock. As a consequence, an elastic shock of limited amplitude reaches the limit of the plastic zone before the fluid shock. This divides the shock into two parts, and this division is very well established by experiment for atmospheric pressures.

The details of the separate shocks are not so well established when the plastic zone is at pressures in the kilobar range. When the pressure in the plastic zone is relatively large compared to atmospheric pressure, the viscosity may be changed and another balance of the velocities may exist.

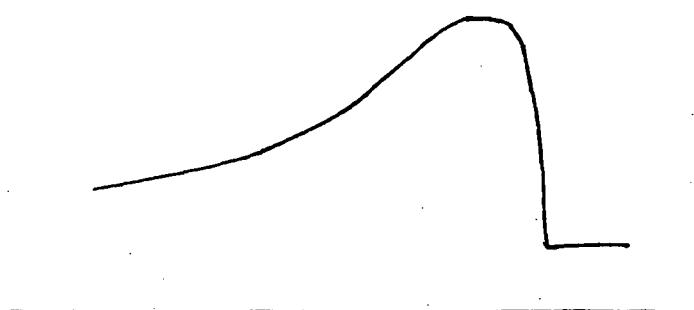
There is some evidence concerning the propagation through a plastic zone in the face of a plate that is exposed to atmospheric pressure. Visual observation is possible in this experiment. This information is presented in the sketches in Figure 1. The upper sketch, Curve A, shows the form of the single shock in the inviscid fluid just before the peak pressure drops to 37-1/2 kilobars. The second sketch, Curve B, shows the pulse of pressure as it is expected to appear as it leaves the plastic region and enters the elastic region according to the assumptions that are presented above and are basic to Hardage's computer program. The curve at the bottom of the page, Curve C, shows the nature of the curve that was actually observed to reach the elastic region when the fluid, plastic and elastic media were all exposed to air at one atmosphere. This shock is of the type that would be expected with variable viscosity in the plastic region and probably with the viscosity variable in the fluid region. To repeat, Curve B is the type of shock which Hardage's computer program predicts in the solid material of the target.

SPECIAL INSTRUCTIONS IN THE COMPUTER PROGRAM

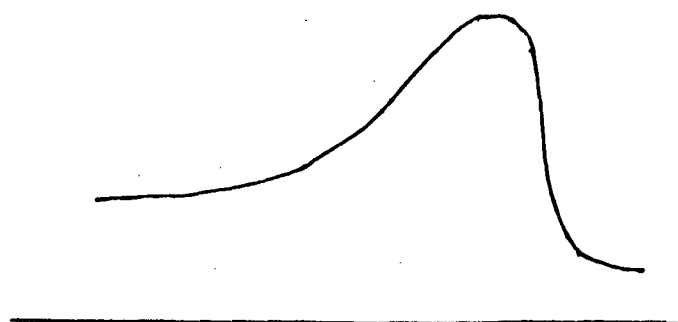
There are several instructions in the computer program which are required to obtain an acceptable computer solution. Some of these instructions are essential for a controlled solution. Experience has shown that central differencing appears to have the computer solution proceed without "blowing-up" unless frequently corrected. Another instruction relates to the minimum pressure in the cells which the computer will recognize. A minimum pressure is one of the easier ways to prevent a spurious, small pressure from "racing" ahead of the true pressure. Without this limit on the minimum, a spurious, low pressure front will advance at about one mesh per cycle. This contrasts with the advance of about 0.1 of a mesh per cycle for the shock front which is set from the Courant condition. In curves in this Quarterly Report, the minimum pressure is



A. Shock in Fluid



B. Shock Leaving Plastic Region, Theoretical



C. Shock Leaving Plastic Region, Observed for Shock at Surface

Figure 1. Transmission of Shock from Fluid through Plastic to Elastic Region

50 kilobars. Our latest runs have employed 10 kilobars with the same general shape of the impact. There are several instructions of this type which do not greatly influence the solution.

There are two sets of instructions which do affect the solution and these must be described in detail since they have this significant influence. One assumption is that the velocity of propagation of sound cannot fall below a minimum value; although the program, without any restriction, permits this to occur. The second condition is applicable when drops or blobs, of fluid separate from the main target. The equations which are applicable only to a continuum then have failed. It is, however, still possible to apply Newton's third law to the target and the separating blob.

Necessity that Minimum Velocity Be Greater than Zero

The preceding paragraphs have discussed the velocities that are observed and are written into the program. There is an additional condition in the program that is very important to the results. The velocities downward are longitudinal, sound propagation in the compressed fluids and have a relatively high velocity, i.e. the velocity is proportional to the square root of the ratio of pressure to density. The high velocity is a consequence of the high pressure. At the top surface of the aluminum target, the pressure will approach atmospheric, so the ratio of the pressure to the density approaches $1/1000$ kilobars. In order to avoid the velocity approaching zero at the surface where the pressure is one atmosphere, there is a command to the computer that the velocity of waves in the aluminum target can never be less than the shear velocity; i.e. the velocity cannot be less than $2/3$ of the constant elastic velocity of a longitudinal wave in elastic material.

In an earlier computer program by Sodek, the preceding instruction was omitted. In his solution, the crushed sphere appears to enter the target through a smaller hole and to expand into a large plate shaped object deeper in the target. The edges of the plate are turned up toward the undisturbed surface. As the solution proceeds, the edge of the plate reaches the surface as the lip of a bowl shaped object and "blows-off" a substantial hunk of the target surface.

General, analytical considerations appear to support the type of solution that appears in this report. In these curves, the expansion of the crushed sphere of rock appears to turn up a lip of the top of the target through which the sphere has entered. The author has seen several examples of targets which have had hyper-velocity impacts here in Huntsville. There is plenty of evidence that the top is blown off. The most startling support of the included curves comes from a picture that was taken on the moon. A meteoroid had penetrated a plate of rock and had lifted out a lip around the entering face. This slide was shown Mr. G. B. Heller, Director of SSL, in a review of report on the trip of Apollo 12.

Instructions to Computer to Separate Blobs and to Form Interfaces

As the crater is formed, drops, or blobs, of rock and/or aluminum are ejected from the target. There are two instructions to the computer which are believed to influence the results. The first requirement on a blob of material that is directed away from the target is that a shock front must have given the blob of material sufficient internal energy to have it contain the energy of fusion at atmospheric pressure. If it contains this much internal energy, the blob of material may separate. Otherwise, the blob will not be released. No energy is assumed to be required to neck and separate the material from the target. The computer program should include energy to form the enlarged surface area but this surface is so difficult to calculate that no correction is added.

After separation of the blob of material from the target, a correction must be added. The blob leaves with a mass, M , and a velocity, V , which gives the momentum, MV . By Newton's third law, an equal and opposite momentum must be transmitted to the target at the position of separation. The vertical component of the momentum must be added to the initial momentum of the sphere to give the effective momentum of the impacting sphere. Since the material is ejected during the formation of the crater, this momentum is part of the overall momentum that is given to the target by the shock wave that is transmitted to the target by the plastic material that surrounds most of the crater.

In the preceding paragraph, the increase in the effective momentum into the target was discussed. This increase in momentum is accompanied by a loss of energy.

Every blob that separates carries the difference between the internal energy content at the time of separation and the initial internal energy. In equation form, this may be written in the following form

$$\text{Internal Energy Lost} = E_{\text{Int}} \text{ at Separation} - E_{\text{Int}} \text{ Initial}$$

In addition, the separating blob has the kinetic energy of separation. This may be expressed by the following relation.

$$E_{\text{Total Lost}} = \text{Internal Energy Lost} + \frac{1}{2}MV^2$$

where M is the mass of the blob and V is its velocity. The energy is balanced in the above manner in order to subtract the above energy from the total energy input by the sphere is the summation of $\frac{1}{2}MV^2$ over the volume of the sphere.

An additional correction is required for the shock separation of a fluid. The shock in the sphere that enters the thick slab of aluminum causes the fluid rock to separate, partially. This is shown by the empty space that has divided the fluid rock at the top of the crushed sphere. This comment refers to the condition that is shown in the rock in Figure 6. Energy is also required for separation of the fluid aluminum that is shown to occur in the last two figures in this report. These figures illustrate the penetration of the crushed sphere of rock through the thin plate of aluminum. The present program does not contain an equation of state that will require energy to oppose this separation. The present equation of state goes to zero pressure and there is a statement that negative pressures (tension) cannot exist. This is expressed by a statement that negative energies must be set equal to zero. When the total energy becomes zero, separation is assumed to occur. The added correction may be of the following form:

- (1) The energy for shock separation of the fluid must be inserted into the problem.
- (2) This energy is inserted by an extension of the equation of state to negative pressures and it requires that negative energy be permitted.

PREDICTIONS OF COMPUTER PROGRAM FOR CRATER FORMATION

The preceding portion of this report presents several of the conditions that are imposed on the computer program for the formation of a crater by the hypervelocity impact of a sphere. Since the details of the crater formation are well established by the work of Bjork and others, the only reason for presenting this small sample of the solution is to permit the reader to judge the merit of this program by comparison of this solution with the generally accepted solutions of others. After the introduction of central differencing, there was no correction to the computer program during the 371 cycles that are reported here. In Hardage's original solution, there would have been some 6 or 7 stops in the program to adjust the solution. Central differencing was introduced into the problem on the recommendation of Dr. R. E. Bruce and with the strong approval of Dr. B. A. Hardage.

The problem that is illustrated is the start of the formation of a crater by the hypervelocity impact of a porous sphere of quartz onto a solid aluminum slab. The porous quartz sphere has an average density of 2.00 grams per cm³. On impacting, the sphere is crushed and adds the energy to crush the sphere, which is

$$\int_{V_1}^{V_2} \rho dV$$

to the kinetic energy of the quartz sphere. The integral is over the change in volume from the large, porous volume, V_1 , to the smaller volume V_2 , of the crushed sphere. The density of the cold, crushed quartz is 2.56 grams per cm³. This type of correction was initially reported by Russians but there are some modifications on the Russian work that Hardage has included. The sphere of quartz impacts on a thick slab of aluminum with a density of 2.785 grams per cm³.

Since symmetry exists, the graphs show the impact of a "pie-shaped" segment of quartz on a thick, "pie-shaped" slab of aluminum. The "piece of pie" of quartz has a thickness of zero at the right margin in the following graphs and a thickness of $d\phi R_{\sin \theta}$ at the center of each mesh. In the preceding relation, R is the radial dis-

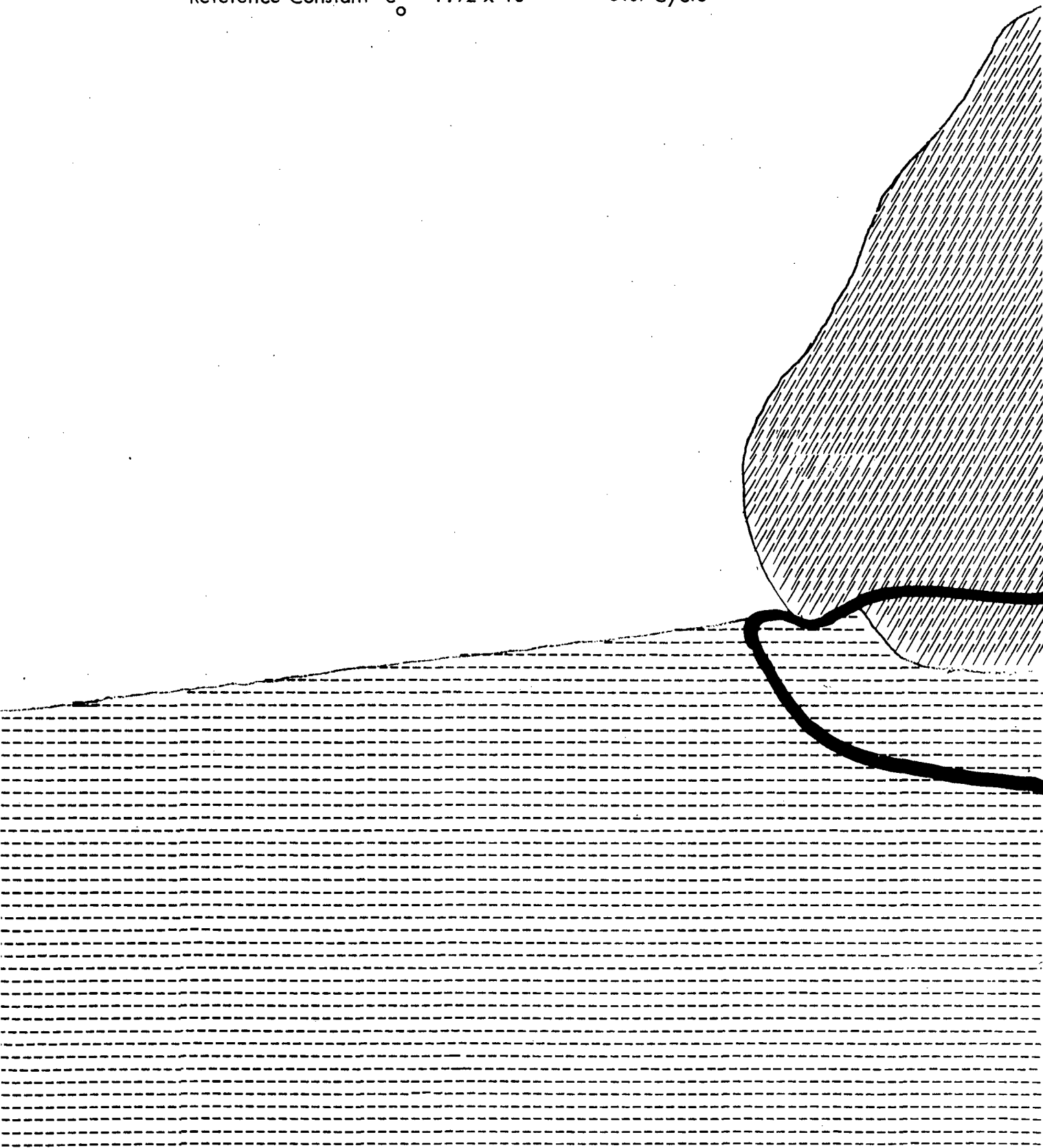
tance of a particular mesh from the origin to the center of the mesh. The angle between the vertical axis and the radial line to the center of the mesh is θ . The angle ϕ is measured about the vertical axis on an arc that is perpendicular to the plane of the paper. The computer program requires that a large amount of information be obtained and recorded. The information includes the area of rock and of aluminum in each cell. From these two values, the area of air in the cell may be calculated. The average density and pressure are recorded. The radial distance to each cell is recorded with the radial and transverse velocities. In addition, the internal and total energy are recorded.

Graph of Impact after 1.74×10^{-10} seconds

The sphere is placed to just touch the aluminum slab on the vertical axis. At time, $t = 0$, the quartz sphere moves into the slab at an initial velocity of 7.2 kilometers per second. After 31 cycles, the shock from the sphere has moved a little more than 3 meshes into the aluminum slab. This is illustrated in the graph, Figure 2. The movement of 3 meshes is a consequence of calculating the Courant condition each cycle and using a time interval between cycles that permits the shock front to move roughly 0.1 meshes for each cycle. In the graph in Figure 2, there are a number of features of interest. These will be discussed in a little detail.

The shape of the partially crushed hemisphere is shown at the top of the graph. The heavy lines enclose the region in which the pressure exceeds 50 kilobars. The elastic limit in aluminum is about 5.4 kilobars so there is much crushing outside of the heavy lines. This explains the bulge on the lower part of the hemisphere of quartz. In future computations, the lowest recorded pressure will be dropped to 10 kilobars in order to show more detail on the edges and in the lightly crushed areas. Trial runs have been obtained at this low pressure and they are satisfactory. The line that separates one kind of cross-hatching from another is calculated by the computer and it is the interface between rock and aluminum. The computer indicates the cells through which this line passes since it must contain only crushed rock and crushed

Figure 2. Graph of Crater Forming Impact at 3.18×10^{-10} Seconds
Initial Velocity 7.2 km/sec
Radius of sphere 7.8×10^{-4} cm. Porous Quartz 2.00 gm/cm^3
Reference Constant $c_o = 1.92 \times 10^4$ 31st Cycle



aluminum; i.e. fluid rock and fluid aluminum. The computer program also gives specific designations to cells in the mesh which contain (1) rock and air, (2) aluminum and air, and (3) rock, aluminum and air. The interface between rock and aluminum will generally end in one, or more cells that contain aluminum, rock and air.

Graph of Impact after 6.38×10^{-10} Seconds

After the impact has been in progress about twice as long as for the preceding graph, the lower end of the hemisphere has been considerably more deformed. The interface between the sphere and the aluminum slab shows some of the consequences of the assumptions that have been described. The penetration is illustrated by the graph in Figure 3. The crushed, fluid rock appears to have "bulged" over the aluminum slab that it is entering. The pull and push of this "bulge" on the aluminum target appears to have put one cell in sufficient tension to empty the cell of all material. It is to be recalled that the equation of state, for this computer solution, cannot withstand tension. It will be interesting to observe this particular defect in the solution after the equation of state is corrected to withstand some tension.

Three arrows are placed on different cells. The arrow length is proportional to the relative velocities of these cells and their direction is the direction of the velocity. The arrow on the cell that is directly above the interface between the rock and aluminum shows that this cell is moving primarily in the downward direction but slightly toward the axis. In contrast, the two cells on the edge of the impact are moving primarily outward at this time in the impact, but with less velocity. The decreased velocity is attributed to a smaller ratio of the pressure over the density. It is to be recalled that the velocity is proportional to the square-root of this ratio. Another example to the same effect is the start of a lip that is being turned up on the surface of the aluminum plate.

Graph of Impact after 1.57×10^{-9}

As the solution for the hypervelocity impact continues, there is an additional feature of interest that is shown in the graph that presents another step in the solution. This graph is presented in Figure 4, and it corresponds to the solution after 151 cycles. As the impact continues, the initial sphere rock is compressed toward becoming a relatively

Figure 3. Graph of Crater Forming Impact at 6.38×10^{-10} Seconds
Initial Velocity 7.2 km/sec
Radius of sphere 7.8×10^{-4} cm. Porous Quartz 2.00 gm/cm^3
Reference Constant $c_0 = 1.92 \times 10^4$ 61st Cycle

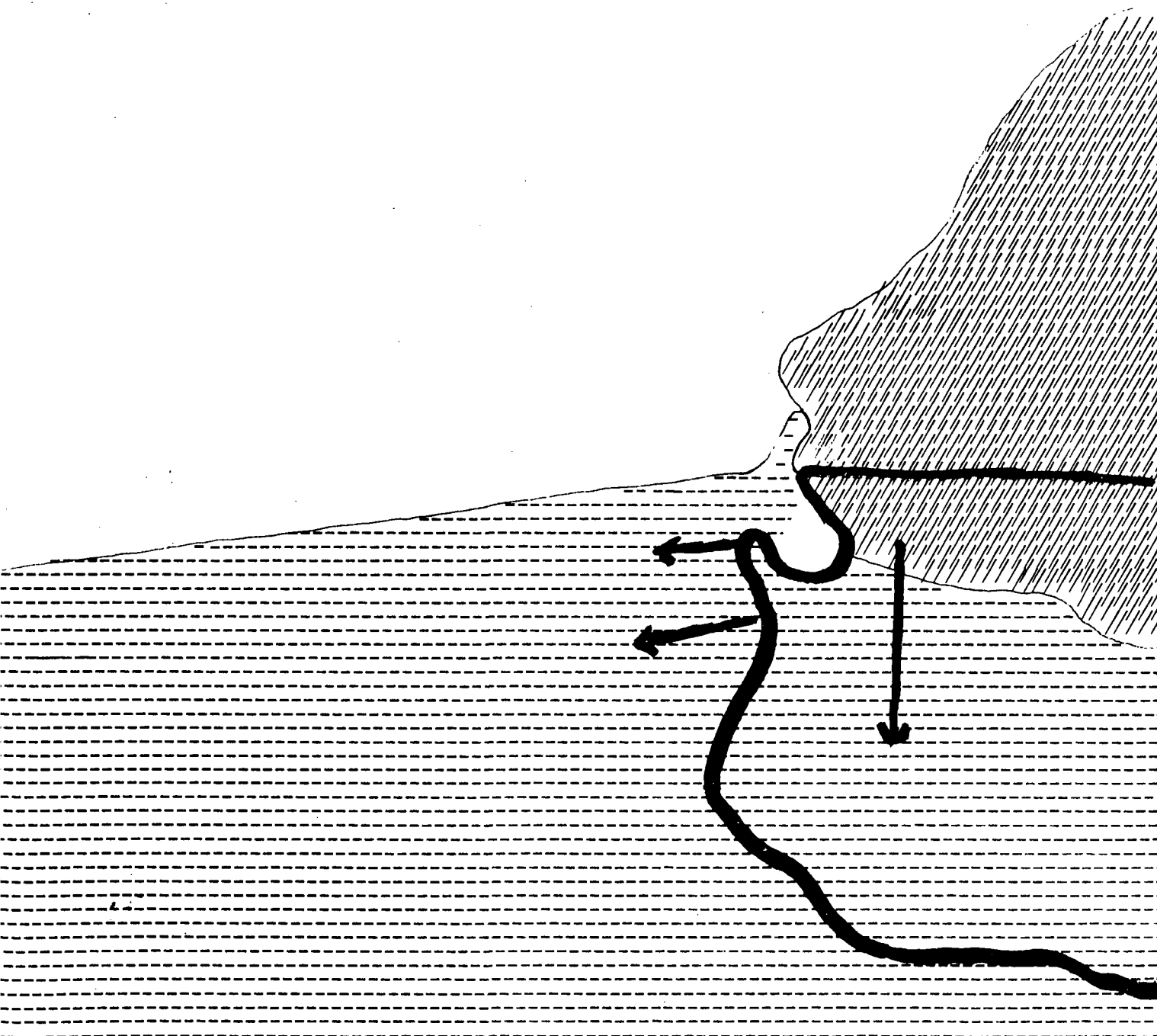
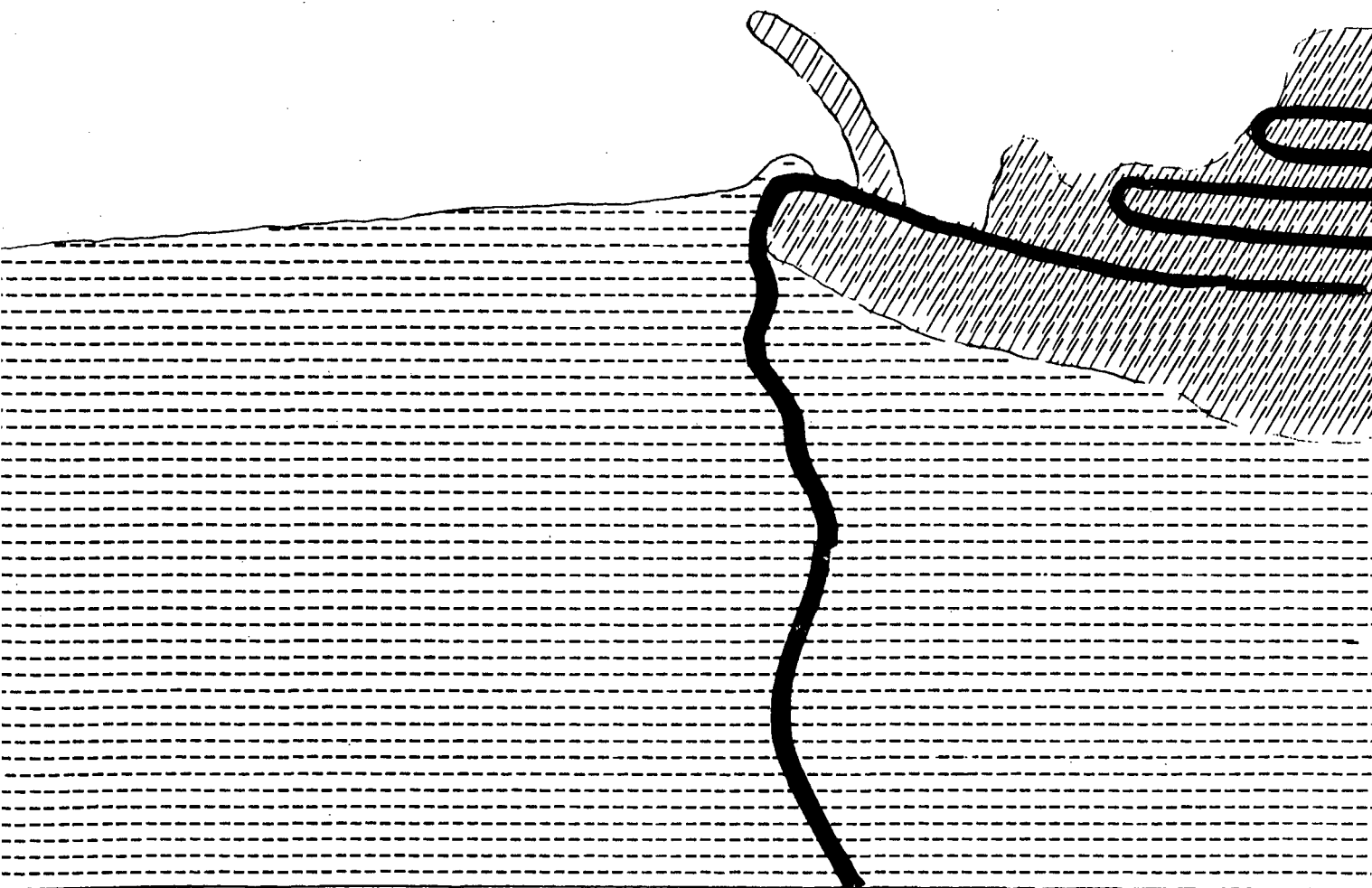


Figure 4. Graph of Crater Forming Impact at 1.57×10^{-9} Seconds
Initial Velocity 7.2 km/sec
Radius of sphere 7.8×10^{-4} cm. Porous Quartz 2.00 gm/cm^3
Reference Constant $c_0 = 1.92 \times 10^4$ 151st Cycle



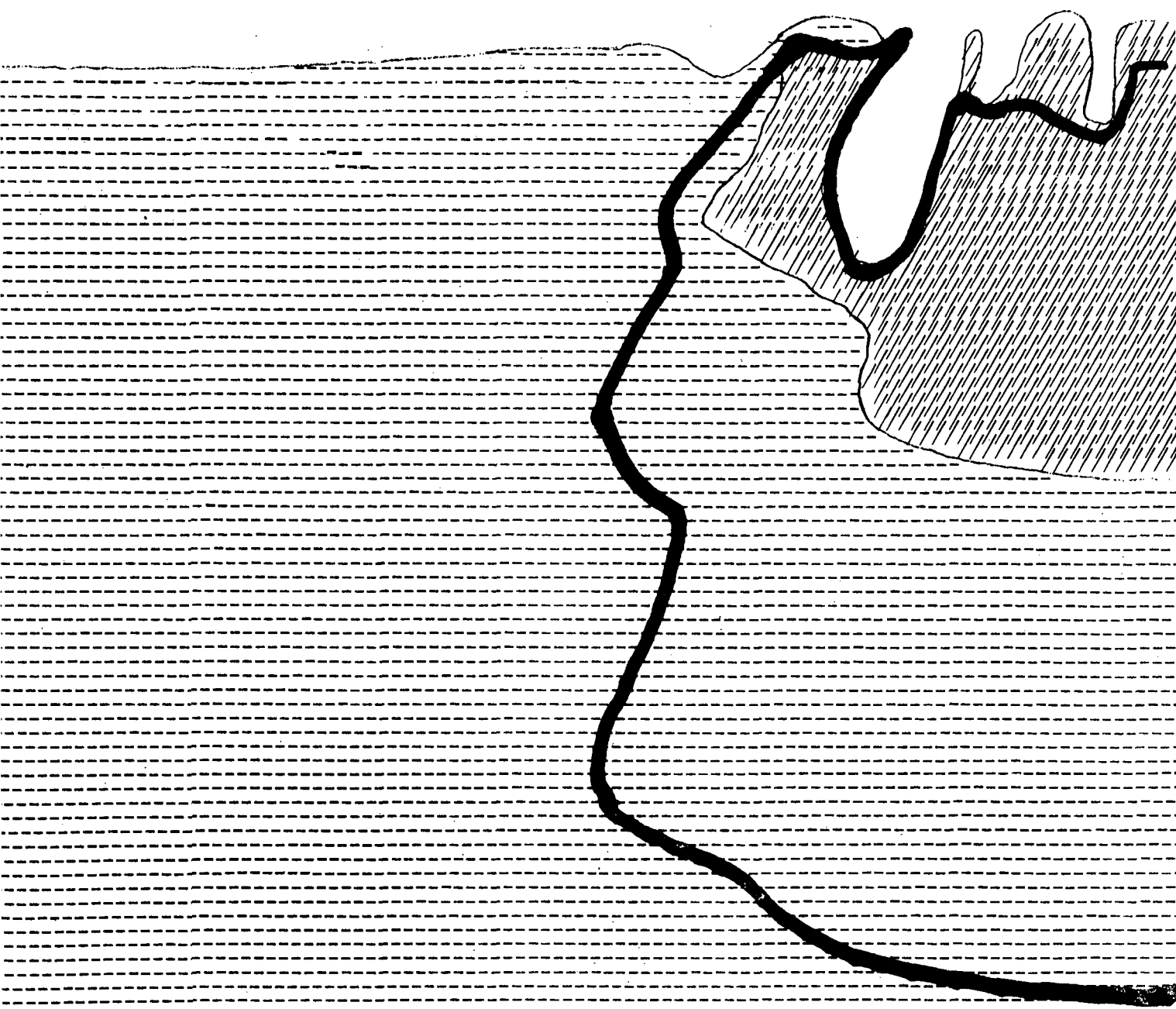
flat plate about the axis. The top of this plate, which was the initial top of the sphere, has oscillations in pressure and density. This is shown by the closed lines that enclose regions of high pressure; i.e. pressures which are in excess of 20 kilobars. They are really equipotential lines, but the lines are at very different pressures, so that nomenclature is not employed.

The reflection of the shock wave from the top of the sphere has caused oscillations that give the low and high pressures. At the initial position of contact, a high pressure region was formed which divided and one shock propagated into the target while the other shock propagated into the rock. The shock into the rock is reflected from the rock-air interface and initiates the pressure, and associated density pulsations that are observed. Since the initial pulse was of large amplitude, the initial pulse was sufficient (over 37-1/2 kilobars) to convert the rock to a fluid. The energy content of the material is certainly sufficient to allow separation in the low pressure region between the pressure pulses provided this region was in tension. Earlier in this report, it was mentioned that separation would occur (1) if a cell had sufficient internal energy to be fluid and (2) if material in the cell was in tension. It is obvious that the low pressure regions are still in compression, although the pressure is less than 20 kilobars. The pressure in the two cells in the upper pressure zone are at about 100 kilobars and in the second pressure zone are about 570 kilobars. The pressure in the main body of rock is about 515 kilobars next to the low pressure zone. The pressures in the two low pressure zones between these three high pressure areas are not known except that they are less than 20 kilobars. A new solution has been obtained with the lowest pressure reduced to 10 kilobars. These new solutions will give more detail in the low pressure regions.

Graph of Impact after 4.29×10^{-9} Seconds

After the computer program has completed 371 cycles with no corrections, the results at the 371st cycle are shown in the graph in Figure 5. The elapsed time since the porous sphere started penetration into the thick slab of aluminum is 2.33×10^{-9} seconds. The rapid oscillations in the shape have decreased but the shape of the penetrating rock is not smooth and regular shaped. The fluid rock appears to be penetrating

Figure 5. Graph of Crater Forming Impact at 4.29×10^{-9} Seconds
Initial Velocity 7.2 km/sec.
Radius of Sphere 7.8×10^{-4} cm. Porous Quartz 2.00 gm/cm^3
Reference Constant $c_o = 1.92 \times 10^4$ 371st Cycle



into the aluminum target in an orderly fashion but with an odd shape. The most unique feature of the impact is the almost complete separation of the fluid rock into two parts. Although the parts are not entirely separated, there is an outer ring and an inner, plate-shaped core of the rock. A detailed study of the velocities shows that the fluid rock and the fluid aluminum are in violent, small-scale oscillations, which are not indicated by the graph. There are general compression and expansion waves, but around the edges of the rock there is considerable turbulence of the fluid rock and the fluid aluminum.

The separation of the fluid rock almost certainly occurred only a short time before the cycle that is illustrated in Figure 5. The basis for this statement is that the cells next to the open space are completely filled with material instead of being only partially filled as are the cells around the remainder of the fluid material in air and next to each other. The velocities and directions of the velocities of the cells next to the break are not regular, but should best be described as extremely turbulent.

The separation of the fluid rock into two parts is very interesting and was not anticipated. In the solutions that were obtained by Hardage, the separation did not appear. This is probably a consequence of Hardage's frequent adjustments of the solution. It may not occur in this problem after an equation of state is introduced which will permit some tension in the fluid. The great difficulty with the introduction of this is to determine the amount of tension which may be introduced.

PREDICTIONS OF THE PRESENT COMPUTER PROGRAM FOR PENETRATION OF A BUMPER PLATE

The preceding curves show the formation of a crater. In Hardage's thesis, the final crater size corresponds roughly with the prediction by Bjork, so no large error is inherent in the computer program when it is employed to calculate the formation of a crater. With no corrections, this same set of equations was applied to the penetration of a sphere of quartz through a bumper plate. The example for consideration is a bumper plate with a thickness that is about $1/4$ of the diameter of the penetrating

sphere. The ratio of plate thickness to sphere diameter is one over four. This ratio defines the range over which the variables may change. There should be no rupture, or separation, of the material that has been made plastic by the shock. The penetration occurs in less than $1/16$ of the time that was given the most consideration by Rosenblatt. The authors obtained a copy of the Rosenblatt report at the time that this report was being prepared. The treatment of viscosity and strain rate in the Rosenblatt report is much preferred to the method of Hardage. This progress report must be completed so this report is not rewritten to include the new information.

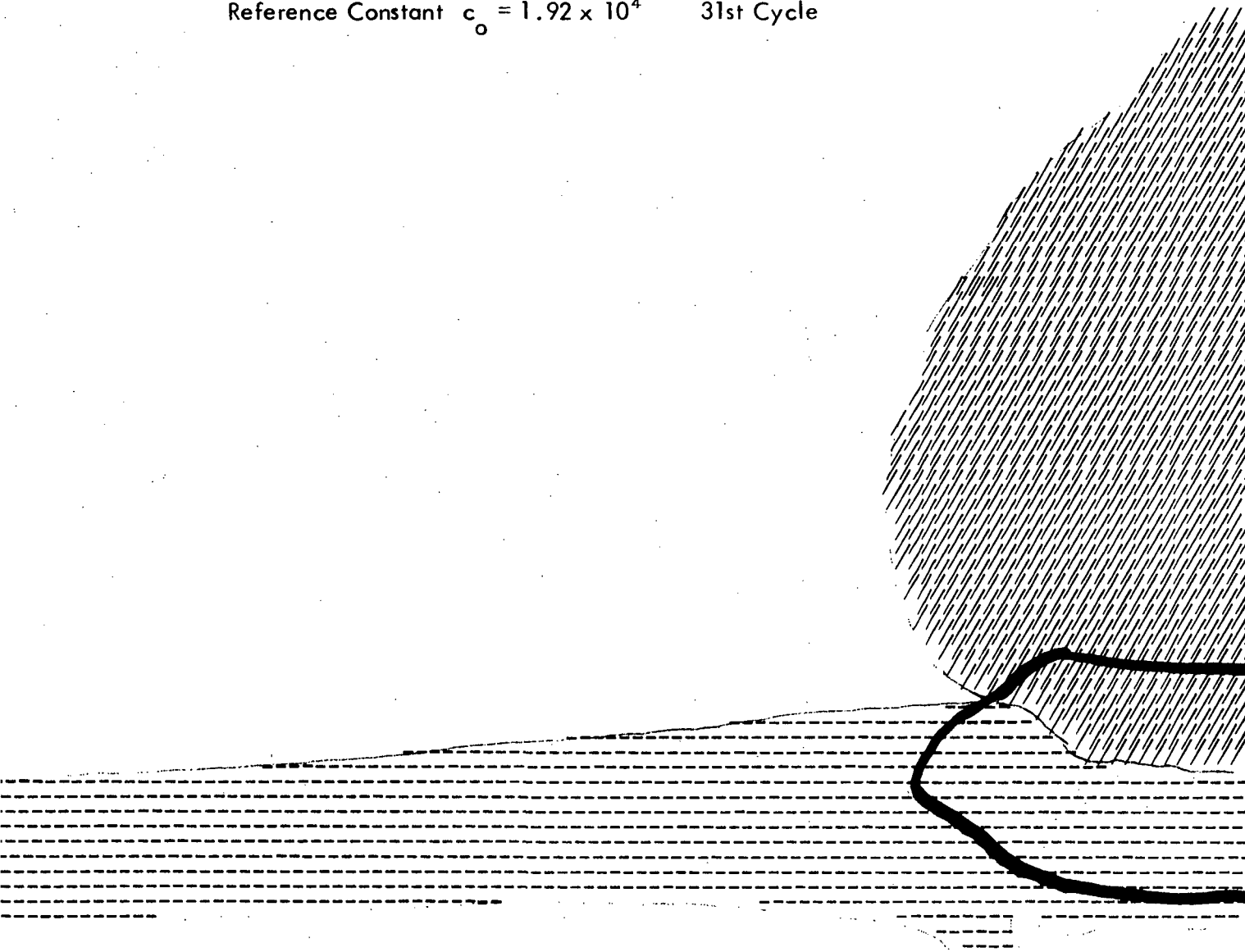
With the exception of hypervelocities which are only a little above the velocity of sound in the target, there should be almost negligible rupture, or separation, of the material of the target while the material is plastic. These comments are based on a cited reference.³ Plastic zones are formed in the target but they form in the region outside of the walls of the hole and not in the material that is ripped apart by the penetrating material from the crushed sphere. The plastic zone is produced in the target as the radial shock decays with propagation away from the position of origin of the shock; i.e. at the wall where the crushed, fluid rock penetrates the plate. For this reason, the plastic region affects the energy coupled to the body of the metal diaphragm more than to the penetration of the rock through the thin plate. The plastic zone would not form in the thin plates with small spheres until after the penetration of the rock is completed.

With neglect of the plastic zone, the primary deficiency in the basic solution appears to be the lack of terms in the equation of state that will permit a limited amount of tension before failure. This was discussed in detail with respect to the preceding graphs for crater formation. The subject will only be mentioned in the discussion of the following graphs for penetration of a plate by a rock of quartz.

Graph of Plate Impact after 3.22×10^{-10}

The first graph for impact on a thin plate corresponds, approximately in time, to the first graph for the formation of a crater. The graph of impact on the plate is shown in Figure 6 for 3.22×10^{-10} seconds after the start of penetration. The preceding graph is to be compared with the one for crater formation in Figure 2 which

Figure 6. Graph of Plate Penetrating Impact at 3.22×10^{-10} Seconds
Initial Velocity 7.2 km/sec.
Radius of Sphere 7.8×10^{-4} cm. Porous Quartz 2.00 gm/cm^3
Reference Constant $c_o = 1.92 \times 10^4$ 31st Cycle



was for 3.18×10^{-10} seconds after the start of the impact. There is very little difference between the graphs for the position of the heavy lines which show the shock fronts at which the pressure exceeds 20 kilobars in both the rock and the target. The heavy line has not reached the back face of the plate, but the heavy line encloses regions at pressures in excess of 20 kilobars. The shock front with pressures of less than 20 kilobars has reached the back face of the plate and produces irregularities. This is possible for the yield strength of 2024 T is about 5.4 kilobars. The irregularities are even more than is shown by the irregular edge. Material is being emptied out of cells that are not on the extreme back surface of the plate. An example is the irregularity that may be designated as the "into-the-metal kink". The internal cell in this kink is partially empty while the cell below it is completely filled.

In all of the graphs for an impact on a thin plate, the bottom face of the thin plate is flat but the top of the plate is cone shaped. This is not important for this trial run of the computer program, but it has already been corrected for future computer calculations on the penetration of a sphere of rock through a thin plate.

There are, at least, two effects which require correction for a more accurate solution to this problem. One correction is to modify the equation of state so the material of the target will withstand some tension. In this report, there have been many comments on this deficiency. There is another difficulty which should be corrected before the final solution is made. By the application of the artificial damping method of von Neumann and Richtmyer,⁴ the shock face is made to slope over two, three, or more cells in a direction that is roughly perpendicular to the shock front. For more detail, more cells are desired. The most logical manner to increase the number of cells would be to divide each present cell into four cells. The division will give cells of $1/2$ the width and $1/2$ of the radial dimension of the present cells. This will roughly quadruple the computer time that will be required for a solution so it is one of the last modifications of the program that will be made.

Graph of Plate Impact after 6.36×10^{-10} Seconds

The second graph of the plate impact corresponds approximately, in time, to the second impact for the formation of a crater. The impact for formation of a crater is presented in Figure 3 after the penetration has proceeded for 6.38×10^{-10} seconds and the impact on the plate is shown in Figure 7 after the penetration has proceeded for 6.36×10^{-10} seconds. The sphere of rock has been crushed to about the same extent in both cases. This means that the shock front has propagated into the rock to an almost identical distance in both cases. In contrast, the shock has not propagated as far into the plate as it did in the thick slab of aluminum. This indicates that the reflected shock from the bottom surface of the plate has slowed the shock advance into the plate.

The bottom of the aluminum plate shows much more extensive damage than was apparent in Figure 6. An empty cell of aluminum is formed next to the axis near the initial bottom of the plate. The empty cell has two partially filled cells next to it, and a full cell of aluminum is displaced downward in the direction of shock propagation. This cell is displaced from its original position at the bottom of the plate. This break in the continuity of the plate requires the deduction of the energy for this break from the energy content just before the break.

Graph of Plate Impact after 1.04×10^{-9} Seconds

The third graph of the impact of a sphere on a plate is illustrated by the graph in Figure 8. This graph represents the condition of the sphere and target at 1.04×10^{-9} seconds after the penetration started. There is no graph in this report of the crater forming impact to compare with this graph of plate penetration. Although it is not included, a comparable graph does exist for the formation of a crater. The extent of crushing of the sphere is approximately the same in the comparable impact; however, the damage to the plate has increased very much over the damage in Figure 7. The increased roughness of the lower edge of the plate and the breaks inside the plate show the effect of the shock reflection from the bottom of the plate. It is particularly interesting that a fluctuation in the pressure and density occurs in the lower part of the fluid rock next to the axis. This is indicated by the closed pressure loop that is

Figure 7. Graph of Plate Penetrating Impact at 6.36×10^{-10} Seconds
Initial Velocity 7.2 km/sec.
Radius of Sphere 7.8×10^{-4} cm. Porous Quartz 2.00 gm/cm^3
Reference Constant $c_0 = 1.92 \times 10^4$ 61st Cycle

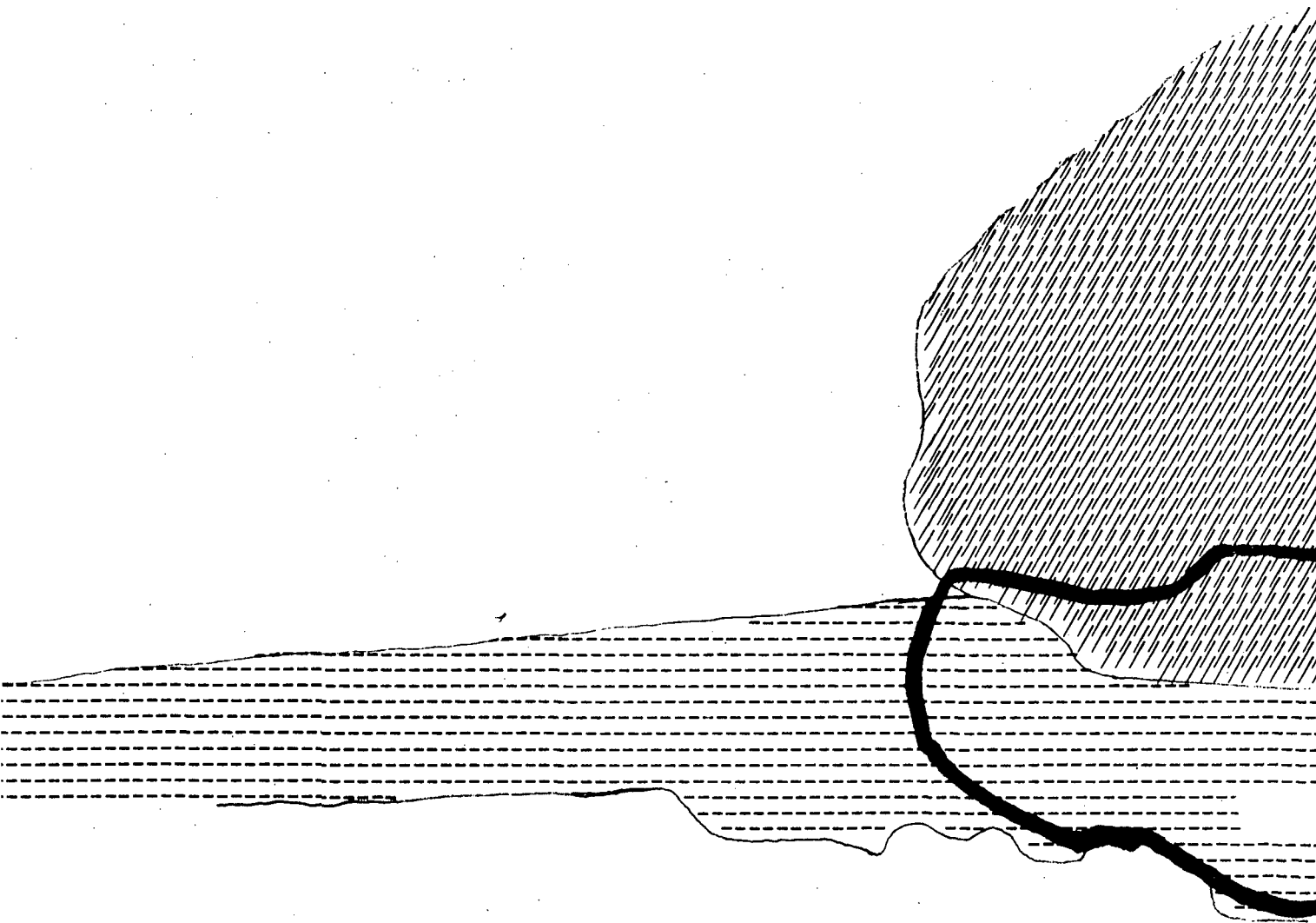
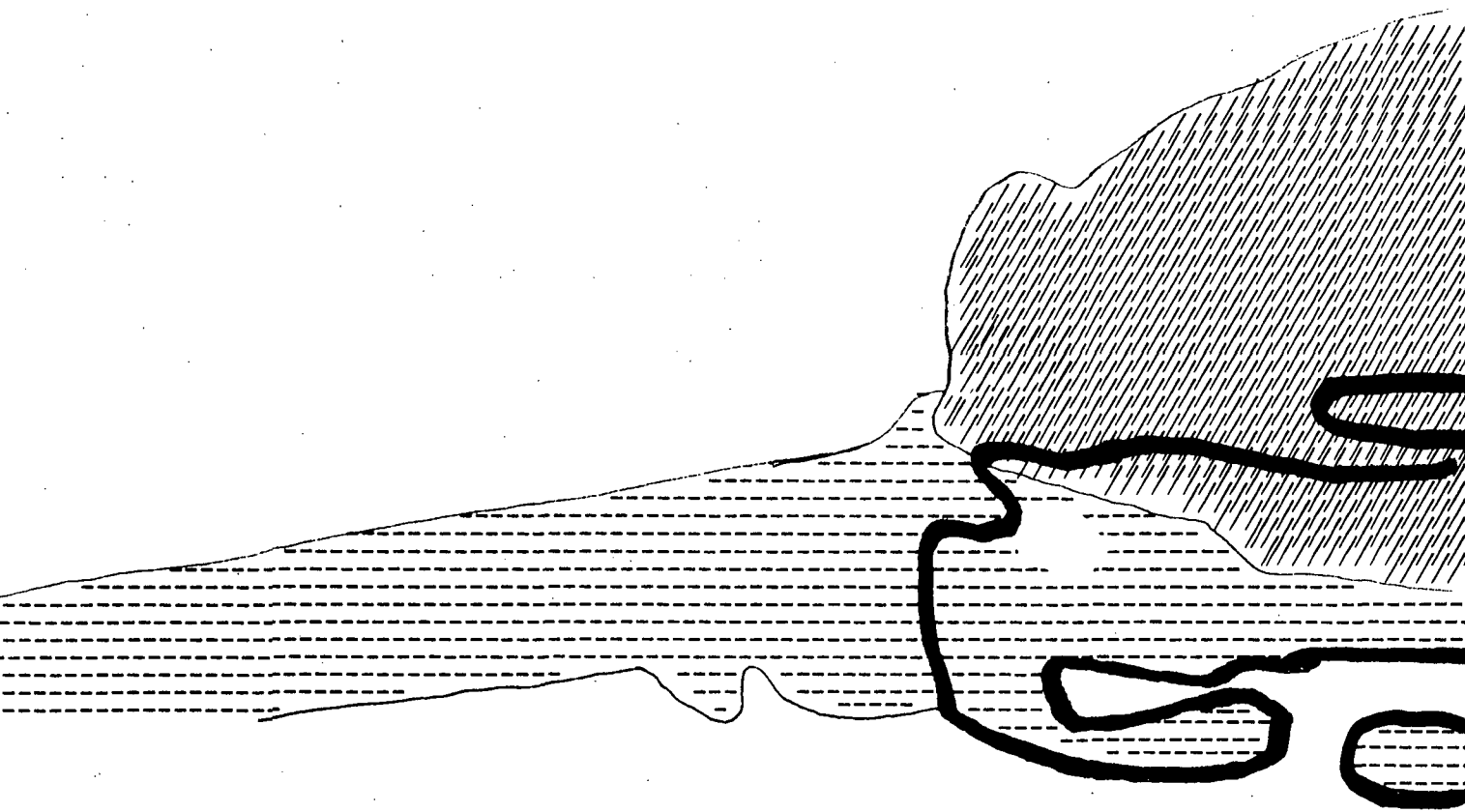


Figure 8. Graph of Plate Penetrating Impact at 1.04×10^{-9} Seconds
Initial Velocity 7.2 km/sec.
Radius of Sphere 7.8×10^{-4} Porous Quartz 2.00 gm/cm^3
Reference Constant $c_o = 1.92 \times 10^4$ 101st Cycle



above and separated from the entirely connected shock front into the rock and aluminum. This pressure fluctuation is to be compared with the one in Figure 4 for the crater forming impact at the much later time of 1.57×10^{-9} seconds after the penetration starts.

In comparison with the plate impact in Figure 7, the pressure has extended a little to the lower left and affected more of the bottom surface of the plate. The entire bottom surface has become more irregular and many more tears and ruptures are observed in the body of the plate.

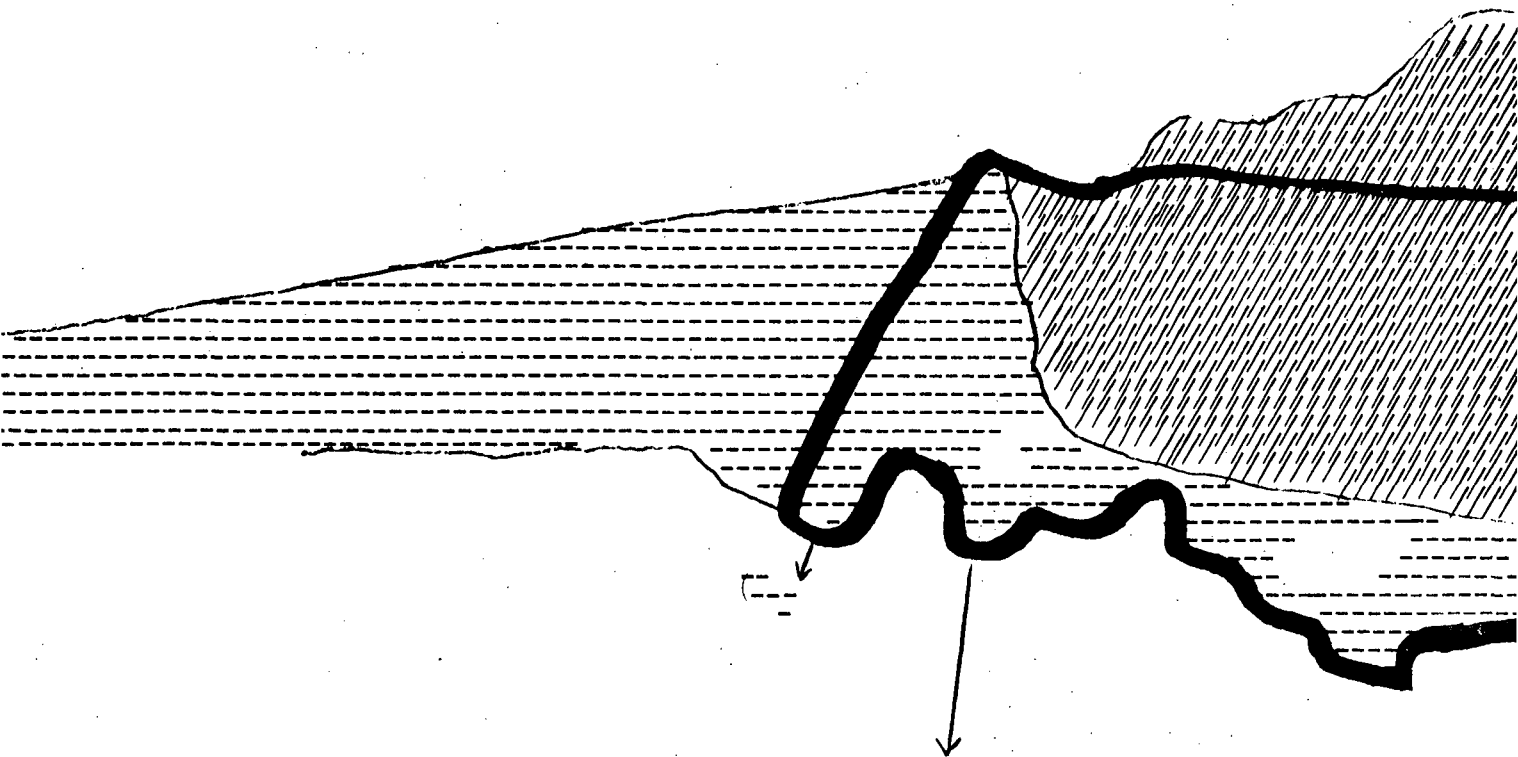
The new phenomena in this impact are the increased number of tears and ruptures in the body of the aluminum plate. Open spaces are being formed in various positions and a piece of the fluid aluminum is completely separated from the bottom of the thin plate. The details of this spalling is not of practical significance in this graph on account of the deficiency in the equation of state and the failure of all equations which apply only to a continuum. The energy for shock rupture of the fluid is not yet deducted from the problem.

Graph of Plate Impact after 1.55×10^{-9} Seconds

The fourth and last graph of the impact of a sphere on a thin plate is shown by the graph in Figure 9. This graph corresponds to conditions at 1.55×10^{-9} seconds after the impact started. In the plate impact, more of the sphere has entered the plate than has entered the semi-infinite slab.

Another significant observation for the plate impact is that the fluid rock has almost penetrated the plate and much of the aluminum plate has disappeared. The shock reflection from the disintegrating plate has reduced the pressure. There is a corresponding increase in the volume of the rock compared to the extremely compressed, relatively smaller volume of rock in the impact on the semi-infinite slab in Figure 4. Another observation is that there is not as much splash from the surface as is found in the crater impact. Too many photographs have been taken to believe that no splash occurs from the plate, it merely indicates more splash for the thick slab than for the thin plate. This is to be expected.

Figure 9. Graph of Plate Penetrating Impact at 1.54×10^{-9} Seconds
Initial Velocity 7.2 km/sec.
Radius of Sphere 7.8×10^{-4} cm. Porous Quartz 2.00 gm/cm^3
Reference Constant $c_0 = 1.92 \times 10^4$ 151st Cycle



The amount of aluminum that has escaped from the graph is so large that a comment is required. With the present computer program, the blobs from the bottom of the plate continue on off the bottom of the graph. There is an exception, after the computer completes calculating a row of cells along a tangential row and finds no change, it will stop calculating. It is programmed not to continue to follow the material at greater radial distances. There is another effect. If the blobs escape with internal pressure, there will be an expansion of the material as if it were a gas. As it expands and the density falls below an arbitrary limit, the blob is eventually dropped from consideration by the computer.

With the present computer program, a "housekeeping" computation must be made to keep track of the mass that is lost from the plate. The energy and the momentum must be measured for each particle at the instant of separation from the main plate.

PROGRAM FOR FUTURE WORK

The preceding considerations of the predictions of the computer program indicate several areas for modifications. Before starting on extensive modifications on the program, the paper by Rosenblatt will be seriously studied in order to ascertain the extent to which the strain rate considerations should be included in the final program.

The corrections to the computer program are of the following types. These items are not listed in their order of importance, or their order of being applied to the program.

(1) The computer program has a capability of being stopped and the results printed out up to the time of stopping. The results may be corrected for several cycles before the last cycle that is printed. The program may then be started at the corrected cycle. This may be necessary in order to "hand-tailor the solution" if that is found to be necessary.

(2) The equation of state is to be corrected so that appreciable tension is applied before rupture occurs. The exact amount of tension for rupture has not yet been determined.

(3) A "housekeeping" program is to be added in order to keep a record of the amount of mass of aluminum and rock that is lost from the vicinity of the penetration.

(4) The mesh is to be modified to show more detail in the reflection of the shock and in the break-up of the plate. It is recommended that each cell be replaced by four cells. Since this will roughly quadruple the amount of computations, this should be enough to ascertain the possible need for a more detailed graph.

(5) As stated in the report, the top and the bottom surfaces of the thin plate have already been made parallel to each other and perpendicular to the vertical axis.

(6) Follow through the penetration in detail in order to calculate the formation of the cone shaped jet of vapor and particulate matter which penetrates the insulation.



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Principal Investigator

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FOREWORD

Basis for Use of Attached Thesis as a Letter Report

The thesis for which this is the Foreword is being submitted as Letter Report No. 23, for the period from May 1 to May 31, 1971, inclusive. This substitution is made after discussion and with the oral permission of the Contracting Officer's Representative. His justification for accepting this old report is based on the requirements of the present contract. The renewal of the contract on June 1, 1971, was obtained on a promise to extend the study to a consideration of the penetration of a bumper plate by a sphere. The solution, for the new, basic analytical problem, is the same as the first part of the solution that is presented in this thesis. There will be future reports that will modify and redirect the solution that is presented in this report. It is desirable to present this report in order to give the basic work on which these modifications are to be based.

The attached thesis is for work which was submitted for a degree in May of 1967. It was never submitted as a report to NASA for reasons which were related to its subject matter and to the origin of my support at the time that this thesis was being completed. Support for my contracts was being shifted from Goddard Space Flight Center to Marshall Space Flight Center. There was much confusion and a small contract on hypervelocity impact was promised from Goddard and this report was being held to forward to Goddard. At that time, Marshall Space had no interest in this particular solution. It was interested in a solution by Bjork which will be mentioned later in this Foreword. Although money was assigned by NASA Headquarters to Goddard Space for support of this study, a number of unusual circumstances resulted in the funds being intercepted so the small contract with Goddard Space did not materialize. As a consequence, this report was never forwarded to any NASA agency.

General Method Selected for Solving Hypervelocity Impact Problems

The attached thesis by B. A. Hardage considers the complete problem of the hypervelocity impact of a sphere of rock on a semi-infinite block of aluminum. The

first objective of the solutions is to show the shape and size of the crater that is produced in the aluminum block. The size is a function of the "density", size and initial velocity of the sphere. The "density" is not the usual concept of the density of a solid. Instead, the same material, quartz, is used in each problem and the "density" is really the average density as the porosity is changed. Two "densities" are used. One value is 2.0 grams per cm.³ and the other is 0.5 grams per cm.³. The density of quartz is 2.65 grams per cm.³ so neither of the selected "densities" is a true solid. One is very porous quartz and the other is nearly solid quartz. The correction for porosity is made in a manner that was originated with this thesis and is derived with a change from the work by the Russians.

The solution for the shape and size of the crater in the aluminum block is comparable with the value that is reported by Bjork in a report which has the date of September 1966. The method for presenting the problem to the computer is entirely different between the two authors. There is so much difference that the next paragraph discusses the fundamental difference.

Bjork employed the point-in-cell method of programming his problem for the computer. This technique considers a point of matter in a mesh of the problem and finds the change in density, and energy as time progresses and the point moves from cell to cell. This method avoids the violent oscillations in the computer solution that are often obtained when the entire mass is considered. Hardage programmed his computer problem with the standard hydrodynamic equations for the material and damped out the oscillations with an artificial, viscous term that was recommended by John von Neumann and R. D. Richtmyer. This pseudo viscosity term gradually disappears, or becomes negligible, and a good solution is obtained after the program has progressed for several time intervals. Nothing would be gained, at this time, by a more general discussion of the relative merits of the two methods. It is sufficient to indicate that both give approximately the same solution, as they must, since both are basically correct.

The method of solution by Hardage has certain properties that are inherent in the pseudo viscosity solution. One property that must be considered when the solution is

presented is that the pseudo viscosity term gives a solution with a broad shock front. The shock front in the solution is usually two to five cells wide, or meshes in the differencing scheme. An investigation has shown that the actual shock front is very steep and is at the position of greatest slope in the solution that is obtained in this thesis. This is a serious disadvantage of this solution. It may be minimized by using very small cells, but this requires more time on the computer.

There is another difference between the Bjork and the Hardage solutions that is important. Hardage can easily introduce the equation of state for plastic flow so he includes plastic flow in his solution. Bjork does not consider plastic flow for some reason, which might be the terms of his contract. Hardage's solution extends through the plastic range and into the elastic range. There are some problems; for example, the pressure in the plastic range should be a tensor and Hardage obtains a solution with consideration of this requirement; but he employs an assumption that an approximate solution may be obtained with a scalar pressure.

Properties of the Plastic Region

Almost no information was found in the literature on the properties of plastic aluminum at pressures which are well above atmospheric pressure of 1 bar. The usual assumption in problems such as Hardage's is that the high pressure of the initial shock produces an inviscid fluid. As a radial shock front of large amplitude advances into the solid, the pressure on the metal is raised and the compressed material becomes a fluid with no viscosity. As the change in pressure at the shock front becomes less, such as by radial expansion of the shock front, the pressure step in the shock front will become so small that the materials are not compressed to an inviscid liquid, but to a very viscous plastic. In this thesis, Hardage considered several factors and selected 37.5 kilobars as the critical pressure, P_p , for aluminum. An aluminum slab which is shock compressed to pressures above this pressure is assumed to become an inviscid liquid. When compressed to just this pressure of 37.5 kilobars, an aluminum slab becomes a viscous plastic. One reason for selecting this value is that solid aluminum becomes a liquid after it is compressed to this pressure and is then released to return to a pressure of one atmosphere.

After selection of the preceding pressure, P_p , the shock front traveling through the plastic region must supply internal energy to the material and dissipative energy to overcome the viscous forces. The approximate thickness of the plastic zone for a spherical shock front is known by an experiment on this program. This thickness, plus the known characteristics of aluminum at the plastic-elastic interface determines the ratio of dissipative to stored energy. The maximum pressure for the elastic region is known, so it is required to reduce the pressure, P_p , between the fluid and plastic region to the critical pressure, P_p , where the later pressure is the elastic limit of the shock material.

As a first approximation and for all of the work in Hardage's thesis, the assumption was employed that 25% of the energy was lost in friction and 75% was stored in the internal energy. From the results in this thesis, this value for the losses is obviously much, much too small. The great thickness of the plastic zone with respect to the observed width is the evidence for this conclusion. One solution was obtained, after the thesis was submitted for a degree, with values of 75% energy lost in friction and 25% of the energy stored in internal energy. The plastic zone was much thinner than was reported in the thesis; but the plastic zone was still slightly thicker than was observed by experiment.

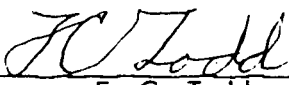
The preceding result would indicate that another solution exists. If the assumed pressure, P_p , is lowered, the ratio of friction loss to energy stored will become smaller. This consequence is expected for there are two unknown variables, P_p and the loss ratio, while there is only one known quantity, the thickness of the plastic zone. Another known quantity must be measured in order to clarify this situation. The arbitrary definition of P_p is hard to justify. The selected value is employed as an expedient to permit the solution without many trial and error methods of reaching the optimum value. The value of P_p was selected with reference to two facts. (1) After aluminum is shock compressed to this pressure and the pressure is released, the aluminum would retain sufficient internal energy to be a liquid. (2) At this pressure, the velocity of propagation of the shock in the fluid aluminum is equal to the velocity of propagation of an elastic shock through the aluminum metal. There is not too much data in this region and it is questionable if a better estimate may be obtained with the data that is available.

Outline of the Work to be Based on this Thesis

The computer program in this thesis is in the process of being reactivated and made operational with a few changes. The primary objective of this program is to obtain a semi-analytical solution for the penetration of a sphere through a bumper plate that is approximately 25% of the thickness of the sphere.

While the program is operational, a new estimate will be made of the thickness of the plastic zone. In addition, another solution will be made for the size of crater that is produced.

The computer program has been partially reactivated. The details of this work and the further necessary work on the computer program will be discussed in a Monthly Letter Report.


F. C. Todd
Principal Investigator

HYPERVELOCITY IMPACT WITH FLOW AND SHOCK
PENETRATION THROUGH FLUID, PLASTIC,
AND ELASTIC ZONES

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Scope of Study: The hypervelocity impact of a porous sphere of silica on a thick slab of aluminum involves several phenomena. On the basis of studies on hypervelocity impact at Oklahoma State University, a complete analytical model for the flow and shock propagation during the impact was formulated and converted to a computer program. The program follows the hypervelocity impact for the duration of the phenomena. The solution shows variations with time of the size and shape of the fluid, plastic and elastic regions of the target; the energy and momentum of the material ejected from the crater; the behavior of the shock wave during the formation of the crater; the final size of the crater; and the shape of the shock wave in the elastic material which surrounds the impact zone. The impacting body is a porous, silicate sphere, and the target is a semi-infinite aluminum slab. Several material interfaces are created during the impact. The program monitors the motion of all of these interfaces as they move through an Eulerian, finite-difference mesh. Solutions are obtained for porous spheres with a pore volume of $1/5$ and of $4/5$. Solutions are obtained for sphere velocities of 6.25, 7.5, 20, and 72 kilometers per second. Two diameters of spheres were employed.

Findings and Conclusions: The final depth of the crater differs from former analytical studies at Oklahoma State University but agrees with the latest nationally accepted analytical studies and semi-empirical predictions. The spray that is ejected from the impact adds up to 12 times the initial momentum of the sphere as the velocity increases and carries away up to $1/4$ of the initial energy of the sphere. The plastic zone in these calculations is too thick, but may be reduced by assuming a larger value for the viscosity in plastic flow. There is some uncertainty in the fluid-plastic transition, but there is no information, either analytical or experimental, in the literature for comparison. Strong pressure oscillations occur in the shocked zone.

ADVISOR'S APPROVAL _____

HYPERVELOCITY IMPACT WITH FLOW AND SHOCK
PENETRATION THROUGH FLUID, PLASTIC,
AND ELASTIC ZONES

Thesis Approved:

Thesis Adviser

Dean of the Graduate College

PREFACE

This work presents a complete numerical solution of the hypervelocity impact of a microparticle on an aluminum surface. The solution follows the flow from the instant of contact and continues until inviscid fluid flow is not a valid assumption. The shock in the inviscid region couples to the plastic region, and that in turn couples to the elastic region. The monitoring of the motion of the physical boundaries of the target and impacting projectile is an important part of the complete solution.

Dr. F. C. Todd acted as my advisor and guide throughout the study. I owe this man more than I can ever repay; the completion of the problem was a result of his wisdom and patience.

The problem involved many hours of computer time. I am grateful to W. M. Alexander and Otto Berg for arranging computer time at Goddard Space Flight Center. Mr. W. F. Cahill of the Theoretical Division at Goddard, and his secretary, Mrs. White, were very helpful to me during my stays there. The long nights with the computers were made enjoyable because of the courtesies extended by Mr. Elmer Terry and his staff of operators: Jim Ridgeley, Sterling Gilmore, Jim Green, and G. Griffith.

Computer programs always involve much "debugging" time. I wish to thank K. O. Baker, Ken Slavin and their staff of operators for arranging many short "debugging" runs on the computer at Continental Oil Company, Ponca City, Oklahoma. Mrs. Marylynn Luther of Conoco was of

much assistance in the removal of these "bugs". Mr. Sam Wax and Mr. McEaddy were also of much help in finding certain "bugs" while at Goddard.

Credit should be extended to Dr. B. A. Sodek, who preceded me in this work. He solved the early time behavior of the impact when the inviscid fluid approximation is valid, and much of his program was incorporated into the present version.

I am grateful to Mr. S. E. Elliott of Phillips Petroleum Company for allowing me to complete the thesis while working for Phillips and for his encouragement during the trying times of converting the computer printout into meaningful physical answers. Don Goff of Phillips prepared several of the figures in the text.

The work was supported by NASA contract NASr-7 which is administered through the Research Foundation of Oklahoma State University.

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CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

Several interesting phenomena are observed when a small particle moving at hypersonic velocity strikes a metal surface (8). Very soon after impact a light flash is emitted, and after this flashing the impact creates a crater several times larger than the projectile. The time at which the light is emitted and the size of the crater depend upon the size of the impacting particle. The crater usually has a raised, curled lip around its periphery. During the crater formation, high speed photography reveals that a fine, high velocity spray is ejected from the circumference of the crater.

Micrometeoroids are small, rapidly moving particles in outer space which have a mass of less than 10^{-4} gram and velocities that range from 30,000 to 240,000 feet per second. The smallest stable volume of a crystal is approximately thirty molecules, so the mass of these microparticles could be as small as 10^{-20} or 10^{-21} gram. Those particles with a mass that is less than 10^{-11} or 10^{-12} gram are pushed beyond the earth's orbit by the excess of radiation pressure from the sun over the gravitational force.

Donn has reported that meteor residue should have a generally spherical shape and should be composed of iron-nickel alloys or silicate minerals, and the latter are mainly enstatite or olivine (12). Donn also states that penetration measurements made by Explorer XVI for

particles in the 10^{-9} gram range are one to two orders of magnitude below "microphone measurements" for these same particles. "Microphone measurements" refer to the detector for micrometeoroids that is mounted on many space vehicles. This discrepancy would arise from impacts of two different kinds of particles: fluffy and compact. All terrestrial samples of space particles, called micrometeorites, are compact, but evidently, porous particles are much more numerous in space.

The NASA project which has supported this thesis began as an analytical study of micrometeoroid impact on a plane aluminum surface. Dr. F. C. Todd, project supervisor, has prepared a model of hypervelocity impact which includes: 1) the formation of a plasma from the impacting particle and the aluminum target, and 2) the creation of strong shock waves that move radially from the point of impact out into the aluminum target and back into the impacting micrometeoroid.

The transition of the hot plasma back to its normal state results in the light flash because some energy is lost by means of visible and UV radiation. The shock waves initiate a fluid flow that ejects some target material to create the crater.

Lake reported on a theoretical solution to the shock propagation problem (18). Sodek make a theoretical study of the properties of the impact of a spherical micrometeoroid on a semi-infinite aluminum target (32). Bruce devised a theoretical determination of the essential properties of an exploding aluminum plasma (7). Lake and Sodek confirm the assumptions of the proposed impact model. In particular, their work predicts that pressures of several megabars are created by the impact. At such pressures, the target and projectile are converted

into a hot, dense plasma; and any material movement can initially be described as nonviscous, hydrodynamic flow. Experimental work is in progress to verify these theoretical findings.

Statement of the Problem

This thesis is concerned with extending the theoretical studies of shock wave propagation and crater formation that result from hyper-velocity impact. Both Lake and Sodek assumed inviscid hydrodynamic flow. Sodek solved the problem of a normal density aluminum sphere impacting on an aluminum surface. This thesis examines the impact of a porous, spherical rock upon an aluminum surface, and the solution is extended beyond the time when inviscid hydrodynamic flow is a valid assumption.

At time $T=0.0$, a porous stone micrometeoroid is assumed to have just made contact with the aluminum target. The target is at rest, and the micrometeoroid is moving toward the target at a velocity V . The appropriate dynamical equations are solved, numerically, to give:

1. The size of the crater created by the impact.
2. An estimate of the mass, momentum, and energy of the material ejected from the crater.
3. The behavior of the shock-wave propagation in the fluid, plastic, and elastic zones of the aluminum target.
4. The deformation of the micrometeoroid.

Presentation of the Problem

The theory of shock waves and the equations needed to describe the

impact process are discussed in Chapter II. Appropriate dynamical equations are developed for use with the fluid, plastic, and elastic states of aluminum.

Equations of state are described for these three states of aluminum and also for the porous, stone micrometeoroid. The viscosity function which is required to describe plastic flow is included in the discussion.

The conversion of the necessary equations to finite difference form is discussed in Chapter III. The finite difference techniques, that allow the physical boundaries of the problem to be accurately monitored, are discussed in detail.

The solutions are presented in Chapter IV in the form of crater and pressure profiles. Five groups of solutions are given; each group for a specific value of micrometeoroid density and velocity. The five impact cases are:

impact velocity = 6.25 km/sec	density = 2.0 gm/cc
impact velocity = 7.5 km/sec	density = 2.0 gm/cc
impact velocity = 20 km/sec	density = 2.0 gm/cc
impact velocity = 20 km/sec	density = 0.5 gm/cc
impact velocity = 72 km/sec	density = 0.5 gm/cc

One solution is obtained that compares the impact of a micrometeoroid of radius R with a micrometeoroid of radius $1.5 R$.

CHAPTER II

DYNAMICAL EQUATIONS

Conservation Equations

The equations expressing conservation of mass, momentum, and energy are the fundamental dynamical equations used in this study. These equations can be expressed in two different forms depending upon the type of coordinate frame that is employed for reference. The Eulerian coordinate frame describes the values of the flow variables at fixed points in space. Such a coordinate frame is fixed, and the material moves through the stationary space points of the coordinate system. The Lagrangian coordinate system describes dynamical variables in terms of the motion of individual elements of the material. This type of reference frame is not fixed in space but moves with the material. In two and three dimensional problems, the Eulerian representation is preferred from both a mathematical and a physical point of view.

Derivations of the conservation equations can be found in many books; e. g., Bird, et al., (5) or Handbook of Physics (10). In their vector form, the equations in Eulerian coordinates are:

conservation of mass, or the conservation equation

$$(1) \quad \frac{D\rho}{Dt} + \rho \vec{\nabla} \cdot \vec{V} = 0$$

conservation of momentum, or the equation of motion

$$(2) \quad \rho \frac{D\vec{V}}{Dt} + \vec{\nabla} p - \vec{\nabla} \cdot \vec{S}_{ij} = 0$$

conservation of energy, or the energy equation

$$(3) \quad \rho \frac{DE}{Dt} + \vec{\nabla} \cdot (\rho \vec{V}) - \vec{\nabla} \cdot (S_{ij} V_i) = 0$$

This form for the energy equation was derived by Steward (36) by the assumption of adiabatic energy flow. With this equation to describe the behavior of a shocked material, the decompression and any subsequent compression of the material remains on a single adiabat. This equation does not affect the increase in entropy by the initial shock front.

The velocity vector, $\vec{V}$, of the material under consideration has the components (v_i, v_j, v_k) , ρ is the density, p is the pressure, and E is the total energy. These four variables, V, ρ, p , and E , are dependent variables, so a fourth equation is needed in order to obtain a solution. The fourth equation is the equation of state, and it is discussed later in this chapter.

Several other terms should be explained. The operator, $\frac{D}{Dt}$, is defined by the following relation.

$$(4) \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla}$$

The quantity, S_{ij} , is the stress tensor. For inviscid fluid flow, $S_{ij}=0$. For plastic, or viscous flow, it is given by

$$(5) \quad S_{ij} = \left(\eta_B - \frac{2}{3} \eta \right) d_{ij} \delta_{ij} + 2\eta d_{ij}$$

Where η_B is the bulk viscosity, η is the shear viscosity, and δ_{ij} is the Kronecker delta. It is generally assumed that $\eta_B=0$ (11). The d_{ij} are the strain rate components

$$(6) \quad d_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

If the material is elastic and not strained above its elastic limit, then the following relation is employed for S_{ij}

$$(7) \quad S_{ij} = \lambda e_{ij} \delta_{ij} + 2\mu e_{ij}$$

The constant, μ , is the modulus of rigidity, and λ is Lamé's lambda. The values of the tensor, e_{ij} , are the strain components

$$(8) \quad e_{ij} = \frac{1}{2} \left(\frac{\partial r_i}{\partial x_j} + \frac{\partial r_j}{\partial x_i} \right)$$

The r_i and r_j are the components of the displacement vector of the material resulting from plastic deformation, only. A uniform body displacement from the body moving through space at constant velocity does not affect the magnitudes of r_i and r_j .

The Rankine-Hugoniot Equations

The shock front appears in the dynamical equations as the positions at which the velocity, the density, and the other dependent variables are discontinuous. It is necessary to have conditions that relate the state of the material on one side of the shock front to the state on the other side. These conditions are called the Rankine-Hugoniot jump conditions.

Bradley (6) gives a good derivation of these jump equations. In essence, the equations simply state the conservation of mass, momentum, and energy. Let p_0 , ρ_0 , and E_0 be the values of the pressure, the density and the specific internal energy ahead of the shock front, and

p_1 , ρ_1 , and E_1 be the values behind the front. Assume that the material ahead of the front is at rest, that the front moves at a velocity V , and that the material behind the front moves with a velocity u_1 .

The Rankine-Hugoniot equations are then

conservation of mass flow

$$(9) \quad \rho_0 V = \rho_1 (V - u_1)$$

conservation of momentum

$$(10) \quad p_1 - p_0 = \rho_0 V u_1$$

conservation of energy flow

$$(11) \quad p_1 u_1 = \frac{1}{2} \rho_0 V u_1^2 + \rho_0 V (E_1 - E_0)$$

A fourth boundary condition must be included in the Rankine-Hugoniot conditions, and this condition is that the entropy must increase across a shock front. An entropy increase occurs across a shock front because more energy is expended in shock compression than is expended in an adiabatic compression. Any equation of state to describe shock compression must account for this increase in entropy.

These above, Rankine-Hugoniot equations relate the variables on the two sides of a shock front, and the Eulerian dynamical equations relate the variables at points away from the shock front. A shock wave propagation problem may be solved with the Rankine-Hugoniot equations as boundary conditions at the shock front. This technique is called shock fitting, and it is a cumbersome way to obtain a solution. If the shock front is changed from a true discontinuity to a narrow region in which all variables have large but continuous gradients, then it is not necessary to use the shock fitting technique. The Rankine-Hugoniot

equations still hold, but their explicit use as boundary conditions is not needed. This technique was suggested by J. von Neumann and R. D. Richtmyer (37).

In the plastic and elastic regions of aluminum, the smearing out of the shock front is accomplished by including the viscous effects of the stress tensor in the equations of motion and the energy equation.

In the early stages of the impact process, when inviscid fluid flow may be assumed, this smearing of the shock is accomplished by introducing a psuedo viscosity effect. This effect is an artificial dissipation that is added to the pressure. A form for the artificial dissipation was suggested by Landshoff (20), and it has the form

$$(12) \quad q = -C_1^2 \vec{\nabla} \cdot \vec{V} (C_2 + |\vec{\nabla} \cdot \vec{V}|)$$

C_1 and C_2 are adjustable constants, and V is the material velocity vector. The psuedo viscosity effect is obtained by replacing p in the fluid flow equations by the sum ($p + q$). The value of q is always chosen as positive for compression; i.e., when $\vec{\nabla} \cdot \vec{V} < 0$. It is made equal to zero for expansion; i.e., when $\vec{\nabla} \cdot \vec{V} > 0$.

Equations for Fluid Region

When a micrometeoroid strikes an aluminum surface at hypervelocity, pressures of several million atmospheres are created in both the aluminium and the micrometeoroid (18), (32). At such pressures, both materials may be treated analytically, as inviscid fluids. When inviscid fluid flow is assumed, the dynamical equations simplify to

$$(13) \quad \frac{\partial \rho}{\partial t} = -\nabla(\rho \vec{V})$$

$$(14) \quad \rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right) = - \nabla p$$

$$(15) \quad \rho \left(\frac{\partial \vec{E}}{\partial t} + (\vec{v} \cdot \nabla) \vec{E} \right) = - \nabla \cdot (\rho \vec{v})$$

These equations are in Cartesian coordinates. In agreement with the preceding members of this group at O. S. U., the solution for the considerably expanded problem in this thesis was solved in spherical coordinates. In spherical coordinates, the conservation of mass equation assumes the following form.

$$(16) \quad \frac{\partial \rho}{\partial t} = - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho w \sin \theta)$$

The equation for the conservation of momentum reduces to two equations which are for radial and polar flow. The azimuthal, or longitudinal flow is assumed to be zero; i.e., in the velocity relation, $V(u, w, v)$, V_ϕ is zero so the relation for the velocity becomes $V(u, w, 0)$. The other two components of the velocity are u_r and w_θ . In the discussion of the conversion of the equations to the polar form with this restriction on the velocity, the following vector identity is useful

$$(17) \quad (\vec{v} \cdot \nabla) \vec{v} = \frac{1}{2} \nabla V^2 - \vec{v} \times \nabla \times \vec{v}$$

When v_ϕ is set equal to zero

$$(18) \quad \frac{1}{2} \nabla V^2 = \frac{1}{2} \vec{r}_0 \frac{\partial}{\partial r} (u^2 + w^2) + \frac{1}{2} \vec{\theta}_0 \frac{1}{r} \frac{\partial}{\partial \theta} (u^2 + w^2)$$

and

$$(19) \quad \vec{v} \times \vec{\nabla} \times \vec{v} = \vec{r}_0 \left[\omega \frac{\partial \omega}{\partial r} + \frac{\omega^2}{r} - \frac{\omega}{r} \frac{\partial u}{\partial \theta} \right] - \vec{\theta}_0 \left[u \frac{\partial \omega}{\partial r} + \frac{u\omega}{r} - \frac{u}{r} \frac{\partial u}{\partial \theta} \right]$$

Substitute the identity in Equation 17 into Equation 14 and use Equations 18 and 19 to evaluate $(\vec{v} \cdot \vec{\nabla}) \vec{v}$. The radial component of the resulting equation for flow in the radial direction, r_0 , becomes

$$(20) \quad \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial r} + \rho \frac{u}{r} \frac{\partial u}{\partial \theta} - \rho \frac{\omega^2}{r} = - \frac{\partial p}{\partial r}$$

Multiply the relation for the conservation of mass, Equation 16, by u and add the resulting relation to Equation 20. After these manipulations, the radial component for the equation of motion may be written

$$(21) \quad \frac{\partial(\rho u)}{\partial t} = - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u^2) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho u \sin \theta) + \rho \frac{\omega^2}{r} - \frac{\partial p}{\partial r}$$

Return to Equations 17, 18, and 19 and the polar, or θ , component of the equation of motion may be written

$$(22) \quad \rho \frac{\partial \omega}{\partial t} + \rho \frac{u}{r} \frac{\partial \omega}{\partial \theta} + \rho u \frac{\partial \omega}{\partial r} + \frac{\rho u \omega}{r} = - \frac{1}{r} \frac{\partial p}{\partial \theta}$$

Multiply the conservation equation by ω , add the result to Equation 22 and the angular equation of motion becomes

$$(23) \quad \frac{\partial(\rho \omega)}{\partial t} = - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u \omega) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho \omega^2 \sin \theta) - \frac{\rho u \omega}{r} - \frac{1}{r} \frac{\partial p}{\partial \theta}$$

The energy equation may be written

$$(24) \quad \rho \frac{\partial E}{\partial t} + \rho u \frac{\partial E}{\partial r} + \frac{\rho w}{r} \frac{\partial E}{\partial \theta} = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho w \sin \theta)$$

Multiply the conservation Equation 16 by E, add to 24, and the energy equation becomes

$$(25) \quad \frac{\partial (\rho E)}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u E) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho w E \sin \theta) - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho w \sin \theta)$$

Equations 16, 21, 23, and 25 are the forms of the dynamical equations that are used to describe the inviscid fluid flow which occurs early in the impact process. The equations are now in the "conservative" form which converts easily into accurate finite difference equations. The pseudo viscosity term, q , can be added to the pressure p in these equations in order to smear out the shock front and remove the jump discontinuity across the front.

The equation of state used for the fluid flow region is discussed in a later section of this chapter.

Equations for Plastic Region

The inviscid fluid model is not valid after the shock wave pressure has decreased to a magnitude of one to two hundred kilobars. This transition occurs after the shock wave has propagated a distance of two to several micrometeoroid radii into the aluminum target. The exact

distance depends on the initial velocity and mass. The appropriate model for this region is the plastic flow model.

The stress tensor in the equation of motion and in the energy equation must be evaluated in order to accurately describe plastic flow. The divergence of the stress tensor is given by the following relation (3).

$$(26) \quad \text{div } S_{ij} = \frac{1}{3} \eta \vec{\nabla} (\vec{\nabla} \cdot \vec{V}) + \eta \nabla^2 (\vec{V}) - \frac{2}{3} (\vec{\nabla} \cdot \vec{V}) \vec{\nabla} \eta \\ + 2 (\vec{\nabla} \eta \cdot \vec{\nabla}) \vec{V} + \vec{\nabla} \eta \times (\vec{\nabla} \times \vec{V}).$$

where $\vec{V}$ is the material velocity, and η is the viscosity.

The radial, or r , component of this divergence must be added to the radial equation of motion in Equation 21, and the tangential, or θ , component must be added to Equation 23. In Appendix A, it is shown that the equations of motion are

$$(27) \quad \frac{\partial(\rho u)}{\partial t} = - \frac{\partial(\rho^2 \rho u^2)}{\rho^2 \partial r} - \frac{\partial(\rho \omega u \sin \theta)}{\rho \sin \theta \partial \theta} + \rho \frac{\omega^2}{r} \\ - \frac{\partial \phi}{\partial r} + \frac{1}{3} \eta \frac{\partial}{\partial r} [\vec{\nabla} \cdot \vec{V}] + \eta \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial u}{\partial r}) \right. \\ \left. + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\frac{\partial u}{\partial \theta} \sin \theta \right) - \frac{2u}{r^2} - 2 \frac{\partial(\omega \sin \theta)}{r^2 \sin \theta \partial \theta} \right] \\ - \frac{2}{3} \frac{\partial \eta}{\partial r} [\vec{\nabla} \cdot \vec{V}] + 2 \frac{\partial \eta}{\partial r} \frac{\partial u}{\partial r} + \frac{2}{r^2} \frac{\partial \eta}{\partial \theta} \frac{\partial u}{\partial \theta} \\ + \frac{1}{r^2} \frac{\partial \eta}{\partial \theta} \left[\frac{\partial}{\partial r} (r \omega) - \frac{\partial u}{\partial \theta} \right]$$

and

$$\begin{aligned}
 \frac{\partial(\rho w)}{\partial t} &= -\frac{(\rho w^2 \sin \theta)}{r \sin \theta \partial \theta} - \frac{\partial(r^2 \rho u w)}{r^2 \partial r} - \frac{\rho u w}{r} \\
 &\quad - \frac{1}{r} \frac{\partial p}{\partial \theta} + \frac{1}{3} \eta \frac{1}{r} \frac{\partial}{\partial \theta} [\vec{\nabla} \cdot \vec{V}] \\
 &\quad + \eta \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial w}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\frac{\partial w}{\partial \theta} \sin \theta \right) \right. \\
 &\quad \left. - \frac{w}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial u}{\partial \theta} \right] - \frac{2}{3} \frac{\partial \eta}{r \partial \theta} [\vec{\nabla} \cdot \vec{V}] \\
 &\quad + 2 \frac{\partial \eta}{\partial r} \frac{\partial w}{\partial r} + \frac{2}{r^2} \frac{\partial \eta}{\partial \theta} \frac{\partial w}{\partial \theta} \\
 &\quad - \frac{\partial \eta}{r \partial r} \left[\frac{\partial}{\partial r} (r w) - \frac{\partial u}{\partial \theta} \right]
 \end{aligned}
 \tag{28}$$

The inclusion of the strain rate tensor in the energy equation adds many more terms to Equation 25. In Appendix A, it is shown that

the energy equation for the plastic zone is

$$\begin{aligned}
 \frac{\partial(\rho E)}{\partial t} = & - \frac{\partial(r^2 \rho u E)}{r^2 \partial r} - \frac{\partial(\rho w E \sin \theta)}{r \sin \theta \partial \theta} - \frac{\partial(r^2 \rho u)}{r^2 \partial r} \\
 & - \frac{\partial(\rho w \sin \theta)}{r \sin \theta \partial \theta} + [\vec{\nabla} \cdot \vec{V}]^2 \\
 & + \frac{1}{2} \left(\frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial r} - \frac{w}{r} \right) [\vec{\nabla} \cdot \vec{V}] \\
 & + u \left\{ \frac{1}{3} \eta \frac{\partial}{\partial r} [\vec{\nabla} \cdot \vec{V}] + \eta \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial u}{\partial r}) \right. \right. \\
 (29) \quad & + \left. \frac{\partial \left(\frac{\partial u}{\partial \theta} \sin \theta \right)}{r^2 \sin \theta \partial \theta} - \frac{2u}{r^2} - \frac{2 \frac{\partial}{\partial \theta} (w \sin \theta)}{r^2 \sin \theta} \right] \\
 & - \frac{2}{3} \frac{\partial \eta}{\partial r} [\vec{\nabla} \cdot \vec{V}] + 2 \frac{\partial \eta}{\partial r} \frac{\partial u}{\partial r} + \frac{2}{r^2} \frac{\partial \eta}{\partial \theta} \frac{\partial u}{\partial \theta} \\
 & + \left. \frac{1}{r^2} \frac{\partial \eta}{\partial \theta} \left[\frac{\partial}{\partial r} (r w) - \frac{\partial u}{\partial \theta} \right] \right\} + w \left\{ \frac{\eta}{3r} \frac{\partial}{\partial \theta} [\vec{\nabla} \cdot \vec{V}] \right. \\
 & + \eta \left[\frac{\frac{\partial}{\partial r} (r^2 \frac{\partial w}{\partial r})}{r^2} + \frac{\frac{\partial}{\partial \theta} \left(\frac{\partial w}{\partial \theta} \sin \theta \right)}{r^2 \sin \theta} - \frac{w}{r^2 \sin^2 \theta} \right. \\
 & + \left. \left. \frac{2}{r^2} \frac{\partial u}{\partial \theta} \right] - \frac{2}{3r} \frac{\partial \eta}{\partial \theta} [\vec{\nabla} \cdot \vec{V}] + 2 \frac{\partial \eta}{\partial r} \frac{\partial w}{\partial r} \right. \\
 & + \left. \left. 2 \frac{\partial \eta}{\partial \theta} \frac{\partial w}{\partial \theta} - \frac{1}{r} \frac{\partial \eta}{\partial r} \left[\frac{\partial}{\partial r} (r w) - \frac{\partial u}{\partial \theta} \right] \right\}
 \end{aligned}$$

Equations for the Elastic Region

In the plastic region, there is viscous flow of the material which results in a permanent distortion of the grain structure of the metal. As the peak pressure in the shock front continues to decrease with propagation away from the position of impact of a small body on a very large target, the pressure is not sufficiently large to produce a permanent distortion of the grain structure of the target. The material of the target in this undistorted region remains in the elastic state. The equations for the propagation of the shock through this region are the familiar equations for an elastic region.

The stress tensor was given in Equation 7; it is

$$S_{ij} = \lambda e_{ij} \delta_{ij} + 2\mu e_{ij}$$

where μ is the modulus of rigidity, and λ is Lamé's lambda. The components of the strain tensor, e_{ij} , are defined in Equation 8. The divergence of the stress tensor must be added to the equation of motion, and the rotation for the divergence is (26)

$$(30) \quad \nabla \cdot S_{ij} = (\lambda + \mu) \vec{\nabla}(\vec{\nabla} \cdot \vec{r}) + \mu \nabla^2(\vec{r})$$

The radial component of this vector must be added to the radial component of motion, Equation 21, and the tangential component, θ , must be added to Equation 23. In Appendix B, it is shown that the radial equation of motion is

$$\begin{aligned}
\frac{d(pu)}{dt} &= -\frac{1}{r^2} \frac{d}{dt}(r^2 p u^2) - \frac{\frac{d}{d\theta}(p u \sin \theta)}{r \sin \theta} + \frac{p u^2}{r} - \frac{dp}{dr} \\
&\quad + (\lambda + \mu) \frac{d}{dr} \left[\frac{1}{r^2} \frac{d}{dr}(r^2 y_r) + \frac{\frac{d}{d\theta}(y_\theta \sin \theta)}{r \sin \theta} \right] \\
(31) \quad &+ \mu \left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dy_r}{dr} \right) + \frac{\frac{d}{d\theta} \left(\frac{dy_\theta}{d\theta} \sin \theta \right)}{r^2 \sin \theta} \right. \\
&\quad \left. - \frac{2 y_r}{r^2} - \frac{\frac{d}{d\theta}(y_\theta \sin \theta)}{r^2 \sin \theta} \right]
\end{aligned}$$

The angular, or tangential equation of motion is

$$\begin{aligned}
\frac{d(p\omega)}{dt} &= -\frac{1}{r^2} \frac{d}{dt}(r^2 p \omega) - \frac{\frac{d}{d\theta}(p \omega^2 \sin \theta)}{r \sin \theta} - \frac{p \omega u}{r} - \frac{1}{r} \frac{dp}{d\theta} \\
&\quad + (\lambda + \mu) \frac{d}{r d\theta} \left[\frac{1}{r^2} \frac{d}{dr}(r^2 y_r) + \frac{\frac{d}{d\theta}(y_\theta \sin \theta)}{r \sin \theta} \right] \\
(32) \quad &+ \mu \left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dy_\theta}{dr} \right) + \frac{\frac{d}{d\theta} \left(\frac{dy_r}{d\theta} \sin \theta \right)}{r^2 \sin \theta} \right. \\
&\quad \left. - \frac{y_\theta}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{dy_r}{d\theta} \right]
\end{aligned}$$

The energy equation is also derived in Appendix B.

$$\begin{aligned}
 \frac{\partial(\rho E)}{\partial t} = & -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u E) - \frac{\frac{\partial}{\partial \theta} (\rho u E \sin \theta)}{r \sin \theta} - \frac{1}{r^2} \frac{\partial (r^2 \rho u)}{\partial r} \\
 & - \frac{\frac{\partial}{\partial \theta} (\rho u \sin \theta)}{r \sin \theta} + \kappa \left\{ (\lambda + \mu) \frac{\partial}{\partial r} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 y_r) \right. \right. \\
 & \left. \left. + \frac{\frac{\partial}{\partial \theta} (y_\theta \sin \theta)}{r \sin \theta} \right] + \mu \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial y_r}{\partial r}) \right. \right. \\
 & \left. \left. + \frac{\frac{\partial}{\partial \theta} (\frac{\partial y_\theta}{\partial \theta} \sin \theta)}{r^2 \sin \theta} - \frac{2 y_r}{r} - \frac{2 \frac{\partial}{\partial \theta} (y_\theta \sin \theta)}{r^2 \sin \theta} \right] \right\} \\
 & + \omega \left\{ (\lambda + \mu) \frac{\partial}{\partial \theta} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 y_r) + \frac{\frac{\partial}{\partial \theta} (y_\theta \sin \theta)}{r \sin \theta} \right] \right. \\
 & \left. + \mu \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial y_\theta}{\partial r}) + \frac{\frac{\partial}{\partial \theta} (\frac{\partial y_\theta}{\partial \theta} \sin \theta)}{r^2 \sin \theta} - \frac{y_\theta}{r^2 \sin^2 \theta} \right. \right. \\
 & \left. \left. + \frac{2}{r^2} \frac{\partial y_r}{\partial \theta} \right] \right\} + [\vec{\nabla} \cdot \vec{V}] \left[\frac{\partial y_r}{\partial r} + \frac{1}{r} \frac{\partial y_\theta}{\partial \theta} + \frac{2 y_r}{r} \right. \\
 & \left. + \frac{y_\theta}{r} \cot \theta + \frac{1}{2} \left(\frac{\partial y_\theta}{\partial r} - \frac{y_\theta}{r} + \frac{1}{r} \frac{\partial y_r}{\partial \theta} \right) \right]
 \end{aligned}
 \tag{33}$$

Equations of State

This thesis is directed to obtain a solution for the impact of a sphere of porous rock onto a solid, thick slab of aluminum. To obtain this solution, four equations and a condition must be employed for each condition of matter. Three of the four equations are for the conservation of mass, momentum and energy. In the sections that immediately precede this section, these equations were obtained in spherical coordinates for each of the three media; inviscid fluid, plastic and elastic materials. The fourth equation for the solution is an equation of state for each medium. The condition, mentioned above, is that the entropy content of the material must be increased by the passage of the shock front.

Four equations of state are required to obtain the desired solution. These equations of state are:

1. an equation for fluid aluminum
2. an equation for plastic aluminum
3. an equation for elastic aluminum
4. an equation for porous stone

Equations of state are presented for each of these four cases in the following discussion.

Equation of State for the Fluid Region of Aluminum

An equation of state from the work of Tillotson is used for the inviscid fluid range (35). The equation has the form

$$(34) \quad P = \left(a + \frac{\tau}{\frac{E}{E_0 \eta^2} + 1} \right) \rho E + A \mu + B \mu^2$$

P is the pressure in megabars; E is the specific internal energy; ρ is the specific density; η is the ratio ρ/ρ_0 , where ρ_0 is the normal density, and $\mu = (\eta - 1)$. The letters E_0 , A, B, a and b represent constants. Their values for aluminum are:

$$a = 0.5$$

$$b = 1.63$$

$$A = 0.752 \text{ mb}$$

$$B = 0.650 \text{ mb}$$

$$E_0 = 0.05 \text{ mb cm}^3/\text{gm.}$$

Equation of State for the Elastic Region of Aluminum

An equation of state was suggested by Lundergan for elastic aluminum (23). It has the form

$$(35) \quad P = 1044 \left(1 - \frac{\rho_0}{\rho}\right)$$

Equation of State for Plastic Region of Aluminum

For the plastic range of aluminum, Lundergan (23) suggests the equation of state:

$$(36) \quad P = 744 \left(1 - \frac{\rho_0}{\rho}\right) + 1.6$$

Equation 36 does not agree well with Tillotson's equation in the fluid-plastic transition zone (see Figure 1). Since the equation is an empirical fit for experimental data below 31 kb, it should not be expected to be accurate in the vicinity of the fluid-plastic transition.

Lundergan suggests that for pressures greater than 31 kb, the plastic equation have the form

$$(37) \quad P = \left(\lambda + \frac{2}{3}\mu\right) \left[1 + b\left(1 - \frac{P_0}{P}\right)\right] \left(1 - \frac{P_0}{P}\right)$$

In this thesis, the equation of state for the plastic zone was chosen to have the form

$$(38) \quad P = C_1 + C_2\left(1 - \frac{P_0}{P}\right) + C_3\left(1 - \frac{P_0}{P}\right)^2$$

Lundergan (23) uses the equation of state proposed by Walsh, et al.

(39) which is

$$P = 765\left(1 - \frac{P_0}{P}\right) + 1659\left(1 - \frac{P_0}{P}\right)^2 + 428\left(1 - \frac{P_0}{P}\right)^3$$

to predict that the high pressure at which the dual shock wave, plastic region begins. This equation predicts a pressure of 113 kb. When Tillotson's equation of state is used, the dual shock wave initiates at approximately 138 kb.

A numerical curve fitting technique was used on the IBM 7040 computer at Oklahoma State University to fit Equation 38 with Tillotson's equation at 138 kb. The constants in equation 38 were found to have the values

$$C_1 = 1.6 \text{ kb}$$

$$C_2 = 845.5 \text{ kb}$$

$$C_3 = 2145.7 \text{ kb.}$$

The resulting form of Equation 38 is plotted in Figure 1.

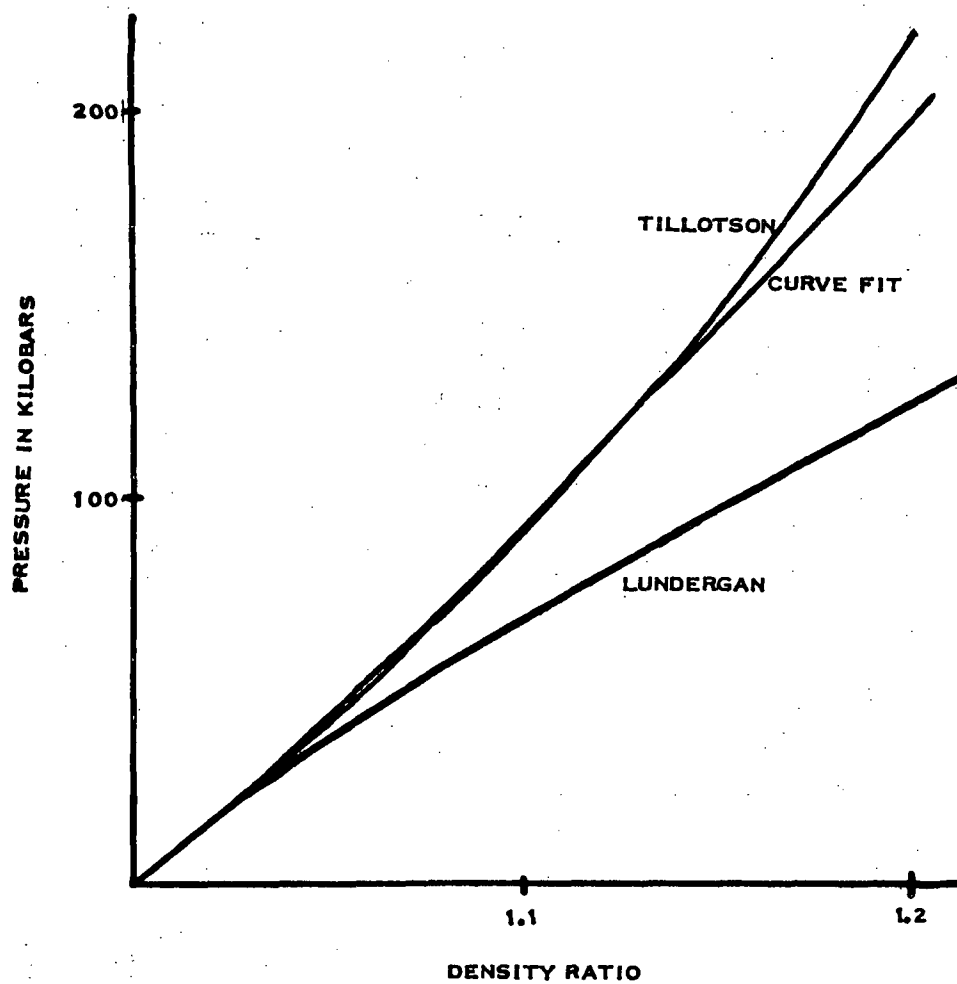


FIGURE 1 - PLASTIC EQUATIONS OF STATE

Lundergan states that the constant b in equation 37 should be approximately 2.5. The ratio of the constants in Equation 38 is

$$\frac{c_3}{c_2} = 2.54$$

From these considerations, Equation 38

1. fits Tillotson's equation at 138 kb
2. fits the elastic yield point at 6.4 kb.
3. gives the approximate values of the constants: $(\lambda + \frac{2}{3}\mu)$ and b .

It should be noted that Bell also proposes that the equation of state for the plastic zone of aluminum should be parabolic (4).

Equation of State for Porous Stone

A somewhat complicated equation of state is employed to describe the pressures created in the impacting, porous micrometeoroid. Since the internal pressures in the micrometeoroid will be quite high and fluid flow behavior may be assumed, it appears that Tillotson's form of the equation of state would be appropriate. However, the following table, compiled by Wagner, et al., shows that the Hugoniot points of a porous body predicted by the Tillotson equation are not sufficiently close to the experimental points to warrant its use without modification (38).

Several Soviet papers have proposed an equation of state for porous materials which fits experimental data quite well (1), (2), (16), (17). McClosky (25) has modified and extended the Russian work, and Wagner, et al., (38), have corrected McClosky's work.

TABLE I
COMPARISON OF RUSSIAN EXPERIMENTAL DATA WITH
TILLOTSON'S EQUATION OF STATE

Porosity	Compression	Pressure (Mb)	
		Experimental	Tillotson
Aluminum			
1.00	2.185	4.93	4.813
1.43	1.498	1.391	1.404
2.08	1.176	1.003	0.790
2.98	1.015	0.702	0.559
Copper			
1.00	1.960	9.55	10.321
1.57	1.395	2.626	3.457
1.57	1.595	7.01	9.309
2.00	1.219	2.204	2.975
2.00	1.402	5.95	8.231
3.01	1.045	1.582	2.595
4.00	0.927	1.260	2.228
4.00	1.018	3.54	5.676
Nickel			
1.00	1.946	9.56	8.811
1.43	1.364	2.908	2.678
1.75	1.261	2.469	2.372
1.75	1.295	6.87	4.517
3.00	0.941	1.639	1.251
3.00	0.949	4.67	2.845

The proposed equation of state assumes that the pressure, P , and the internal energy, E , can be written as the sum of three components.

$$(39) \quad P(\eta, T) = P_c(\eta) + P_n(\eta, T) + P_e(\eta, T)$$

$$(40) \quad E(\eta, T) = E_c(\eta) + E_n(\eta, T) + E_e(\eta, T)$$

In these equations, T is the temperature, η is the compression ratio ρ/ρ_0 , where ρ is the density, and ρ_0 is the density at normal conditions. The subscripts have the following meanings: c indicates the cold compression; i.e., the effect of atomic lattice interactions at 0°K ; n indicates the contribution due to thermal vibrations of lattice ions; and e represents the contribution of the thermal excitation of electrons.

At 0°K , only the electrons contribute to the pressure. The ions contribute only through their kinetic energy, and at 0°K their kinetic energy may be assumed to be zero. The contribution of the electrons arises from their resistance to compressive forces, since compression raises the quantum states.

There is no unique way of representing repulsive force potentials. An early estimate was that the repulsive potential had the following form

$$(41) \quad E = -A \left[\frac{R_0}{r} - \frac{1}{n} \left(\frac{R_0}{r} \right)^n \right]$$

where E is the interaction energy of the crystal, and r_0 is the interatomic distance when pressure and temperature are zero. Different authors use different values for n , but generally

$$7.84 \leq n \leq 9.1$$

The Russians found that repulsive forces could not be fitted by a power series because n must be changed as r changes. No single value of n will give the complete solution for a wide range of pressures. A more versatile equation is

$$(42) \quad E = -A \left[\frac{r_0}{r} - \frac{P}{r_0} \exp \left\{ \frac{1}{P} (r_0 - r) \right\} \right]$$

This form gives good agreement with experimental data when the constants are properly adjusted. The pressure is given by

$$(43) \quad P = - \frac{\partial E}{\partial V}$$

Since $V = r^3$, then the "cold" pressure component is

$$(44) \quad P_c(\eta) = A_1 \left\{ \eta^{2/3} \exp \left[b_1 (1 - \eta^{-1/3}) \right] - \eta^{4/3} \right\}$$

The first term in Equation 44 represents the force from the repulsive potential of ions, and the second term represents the attractive potential of the ions. McClosky (25) describes the employment of certain physical constants to evaluate the constants A_1 and b_1 .

The internal energy from this "cold" compression is represented by the following relation

$$(45) \quad E_c = - \int P_c dV$$

Since $V = 1/\rho$ and $\eta = \frac{\rho}{\rho_0}$, change Equation 45 to an integration over η

$$(46) \quad E_c = \frac{1}{\rho_0} \int_1^\eta \frac{P_c}{\eta^2} d\eta$$

or

$$(47) \quad E_c(\eta) = \frac{3A}{\rho_0} \left[\frac{1}{t_1} \exp\left[t_1(1-\eta^{-\frac{1}{3}})\right] - \eta^{\frac{1}{3}} - \frac{1}{t_1} + 1 \right]$$

Equations 44 and 47 are valid only for $\eta \leq 2.5$. For higher compressions, the equations must agree with Thomas-Fermi statistics. McClosky makes the transition from Equation 44 to Thomas-Fermi statistics by writing

$$(48) \quad P_c(\eta) = P_c(\eta_b) + \left. \frac{dP_c}{d\eta} \right|_{\eta=\eta_b} (\eta - \eta_b) + A_2 (\eta - \eta_b)^2$$

The first term on the right is Equation 44, which is evaluated at $\eta = \eta_b$.

The constant A_2 may be evaluated by referring to Thomas-Fermi data to find a pressure $P_c(\eta_{TFC})$ at a high value for the compression ratio, η_{TFC} . The relation for A_2 becomes:

$$(49) \quad A_2 = \frac{1}{(\eta_{TFC} - \eta_b)^2} \left[P_c(\eta_{TFC}) - P_c(\eta_b) \right] - \left. \frac{1}{(\eta_{TFC} - \eta_b)} \frac{dP_c}{d\eta} \right|_{\eta=\eta_b}$$

For the nuclear component, the Debye theory must be valid at low temperatures. This requires that these relations hold:

$$(50) \quad E_n(\eta, T) = 9RT \left(\frac{T}{\Theta} \right)^3 \int_0^{\Theta/T} \frac{x^3 dx}{e^x - 1}$$

and

$$(51) \quad P_n(\eta, T) = \gamma p E_n$$

where γ is the Gruneisen ratio.

At high temperatures, the thermal motion of the nuclei is properly described by the equation for a perfect gas. To describe the transition from the solid phase to the gas phase, the Soviets have proposed the following:

$$(52) \quad E_n(\eta, T) = \frac{3}{2} \frac{2+z}{1+z} RT$$

$$(53) \quad P_n(\eta, T) = \eta p_0 \frac{3\gamma+z}{1+z} RT$$

where

$$(54) \quad z = \frac{\rho_0 R T A}{K \left(\frac{\partial \rho}{\partial \eta} \right)}$$

The atomic weight is A, and K is an empirical constant. These equations give the proper limiting forms because when T is small

$$E_N \sim 3RT$$

Debye Theory

$$P_N \sim \eta \rho_0 \gamma E_N$$

and when T is large

$$E_N \sim \frac{3}{2} RT$$

Perfect Gas.

$$P_N \sim \eta \rho_0 RT$$

The electronic thermal contribution at low temperature is given by Gilvarry (14)

$$(55) \quad E_e(\eta, T) = \frac{1}{2} \beta(\eta) T^2$$

$$(56) \quad P_e(\eta, T) = \frac{1}{2} \eta \rho_0 g(\eta) \beta(\eta) T^2$$

where β is the coefficient of electronic specific heat, and g is the electronic Grueisen's ratio; i.e., $g = P_e / (E_e / V)$

Latter's work shows that at high temperatures, the pressure and internal energy approach asymptotic values which are closely approximated by

$$(57) \quad E_e = \frac{3}{2} n T$$

$$(58) \quad P_e = \eta \rho_0 n T$$

where

$$(59) \quad n = \frac{0.85 X^{0.59}}{1 + 0.85 X^{0.59}} z R$$

and

$$(60) \quad X = \frac{T}{1.16 \times 10^4 Z^{4/3}}$$

The atomic number is Z (21). To interpolate between the low and the high temperature range, the Russians propose

$$(61) \quad E_e(\eta, T) = \frac{9n^2}{4\beta} \log \left[\cosh \left(\frac{2\beta T}{3n} \right) \right]$$

$$(62) \quad P_e(\eta, T) = \eta \rho_0 \frac{n^2}{g\beta} \log \left[\cosh \left(\frac{g\beta T}{n} \right) \right]$$

These equations give the proper limiting forms. For low temperatures, Equations 61 and 62 reduce to

$$E_e = \frac{1}{2} \beta T^2$$

$$P_e = \frac{1}{2} \eta \rho_0 g \beta T^2$$

and for high temperatures, they become

$$E_e = \frac{3}{2} n T$$

$$P_e = \eta \rho_0 n T$$

A valid criticism of Equations 61 and 62 has been presented by Wagner, et al., (38). Any thermodynamic equation must satisfy the equation of "thermodynamic consistency"

$$(63) \quad \left(\frac{\partial E}{\partial V} \right)_T = T^2 \frac{\partial}{\partial T} \left(\frac{P}{T} \right)$$

which is perhaps better known as the Maxwell equation

$$(64) \left(\frac{\partial S}{\partial V} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V$$

Equations 61 and 62 do not obey this relation. In these equations, E is internal energy, S is entropy, T is temperature, P is pressure, and V is volume. In retrospect, it should be mentioned that the interpolation equations for the nuclear contribution, equations 52 and 53, and the equations for the cold component, Equations 44 and 45, do obey the law of "thermodynamic consistence."

Since the electronic contribution is large at high pressures, this inaccuracy may cause significant errors. The alternative suggested by Wagner, et al., is to numerically interpolate Latter's high temperature data in tabular form.

Equations 44 to 62 were incorporated into a computer code, and tables of $P(\eta, T)$ and $E(\eta, T)$ were calculated. From these tables, a third pressure table was constructed with P having the coordinates ρ and E . This table was read in as input data to the impact problem. Selected pressure isotherms for a porous silicate rock are shown in Figures 2 and 3.

The dynamical equations are employed to calculate the density, ρ , and the internal energy, E . These two variables determine the coordinates of the pressure elements in the table, and the appropriate value of pressure is found by numerical interpolation.

Viscosity Models

The viscosity of a material is a function of the thermodynamic

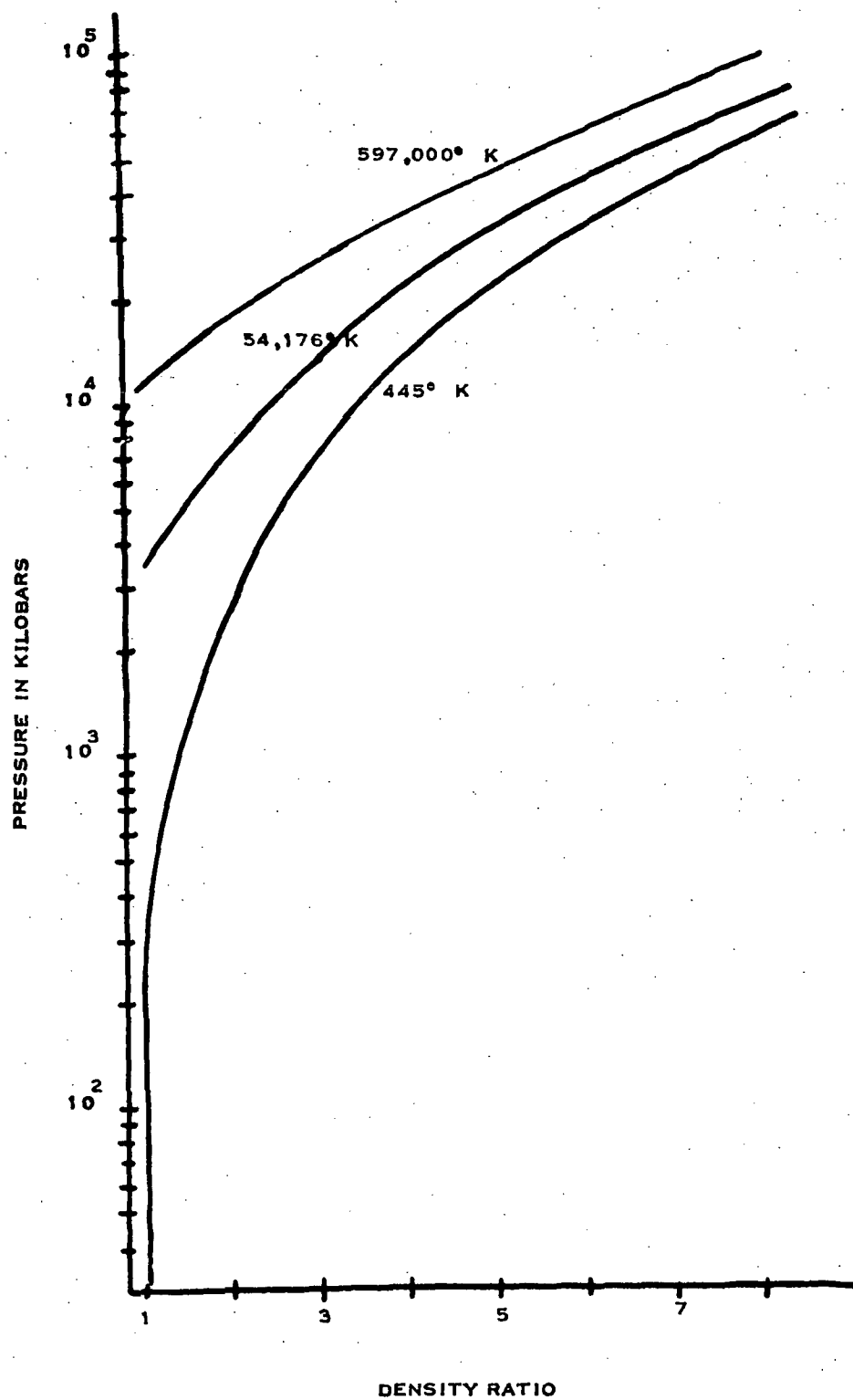


FIGURE 2 - PRESSURE ISOTHERMS FOR ROCK OF POROSITY 1

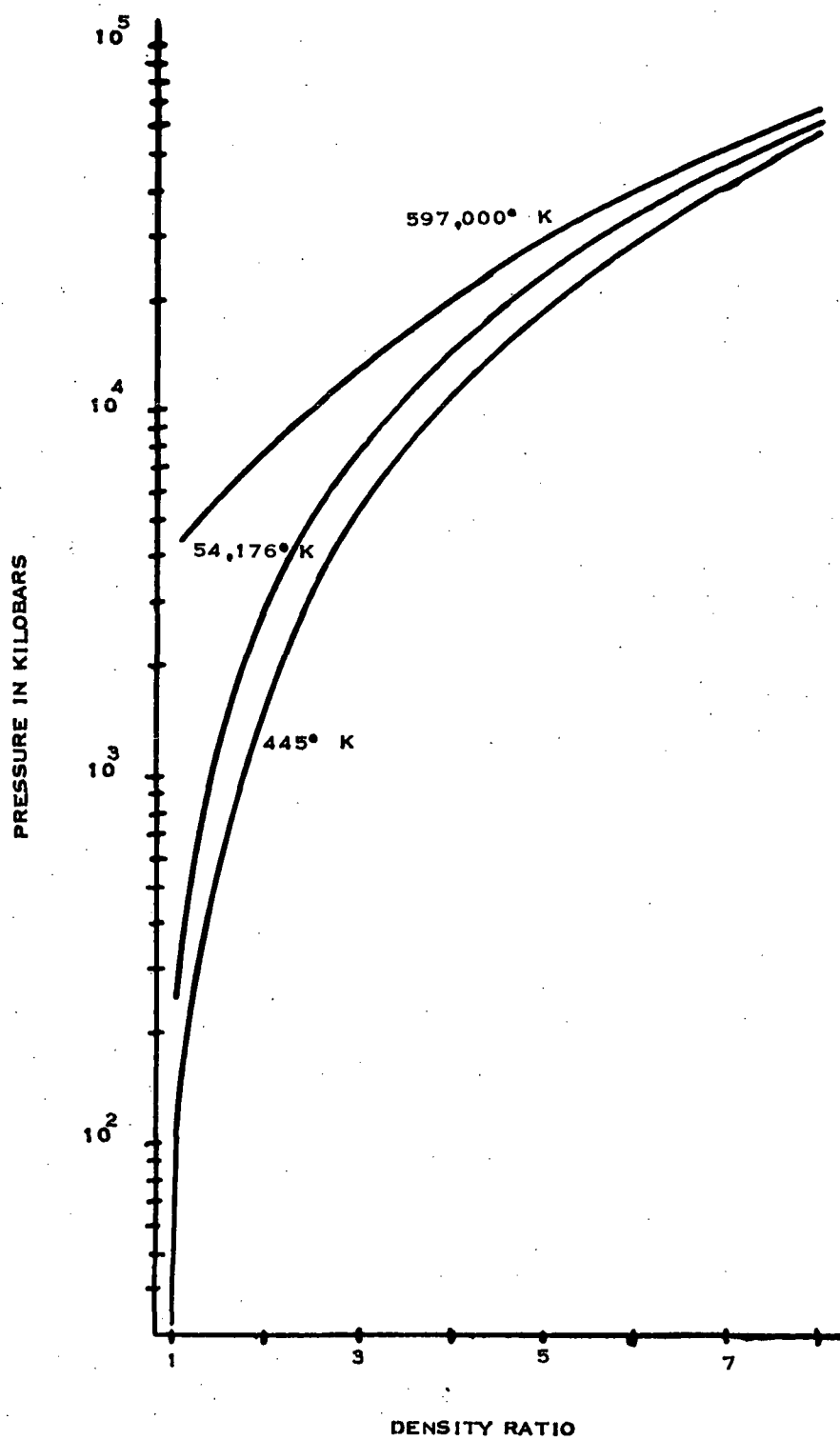


FIGURE 3 - PRESSURE ISOTHERMS FOR ROCK OF POROSITY 4

state of the material. For a homogeneous medium, the viscosity is a function of two thermodynamic properties. The usual choice for these variables are:

$$\eta = \eta(\rho, T) \quad \text{OR} \quad \eta = \eta(p, T)$$

Some general facts about the viscosity are:

1. The viscosity of a solid increases if the temperature increases and also increases if the pressure increases.
2. The viscosity of a liquid increases if the pressure increases but decreases if the temperature increases.
3. The viscosity of a gas increases if the temperature increases and also if the pressure increases.

Non-Newtonian flow can be explained in terms of a single viscosity coefficient if this coefficient is considered to be a function of the local rate of strain. For a liquid,

$$\eta = F(S)$$

where S is the strain rate tensor, and F is a positive, even function (13). In general notation

$$(65) \quad S = \frac{1}{2}(S_{11}^2 + S_{22}^2 + S_{33}^2) + S_{12}^2 + S_{13}^2 + S_{23}^2$$

where

$$(66) \quad S_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

For a plastic solid, the viscosity is a decreasing function of the strain rate tensor. Eirich (13) proposes the following model for the viscosity of a plastic solid

$$(67) \quad \eta = \eta_0 + \frac{S_0}{|\dot{\epsilon}|}$$

This model is used by Riney (31) to describe the plastic flow which occurs during a hypervelocity impact, and it is the model that is employed in this thesis. In the literature, plastic flow described by this model is called Bingham plastic flow. The terms in Equation 67 have the following meanings:

η_0 is the simple viscosity effect

S_0 is the yield value in shear

$|\dot{\epsilon}|$ is the value of the strain rate tensor as defined in Equation 65.

S_0 and η_0 are not constants but are functions of pressure and temperature. For a plastic solid

$$\frac{\partial \eta_0}{\partial p} > 0 \quad \text{and} \quad \frac{\partial S_0}{\partial p} > 0$$

but

$$\frac{\partial \eta_0}{\partial T} < 0 \quad \text{and} \quad \frac{\partial S_0}{\partial T} < 0$$

The equations that describe the pressure and temperature dependence of η_0 and S_0 in this thesis are

$$\eta_0 = a_1 + a_2 p + a_3/E$$

$$S_0 = b_1 + b_2 p + b_3/E$$

E is the internal energy, which is a measure of the temperature. As a consequence, the conditions $\frac{\partial \eta_0}{\partial T} < 0$ and $\frac{\partial S_0}{\partial T} < 0$ are satisfied. The

constants $a_1, a_2, a_3, b_1, b_2,$ and b_3 were picked by trial and error so that nominal values for η_0 and ζ_0 result at standard temperature and pressure.

If a pressure, or an energy gradient exist, then there will be a viscosity gradient since

$$\vec{\nabla} \eta = \frac{\partial \eta}{\partial p} \vec{\nabla} p + \frac{\partial \eta}{\partial E} \vec{\nabla} E$$

CHAPTER III

FINITE DIFFERENCE METHODS

Finite Difference Mesh

For the solution of the impact problem, the differential equations are converted to finite difference equations. In seeking the solution, the finite difference equations were applied to each cell, in turn, in a regularly arranged network, or mesh, of cells. Before discussing the conversion of the dynamic flow equations to finite difference equations, it is desirable to describe the arrangement of the mesh of cells. The selection of the coordinate system for the mesh of cells is made after the problem is fully defined. The problem is stated in the next paragraph.

At time $t = 0$, it is assumed that the spherical micrometeoroid has just touched the aluminum target. The target is assumed to be semi-infinite and to be at rest. The micrometeoroid has a velocity V normal to the target, and the point of impact determines the axis of symmetry. The physical situation is depicted in Figure 4. The line OZ represents the axis of symmetry for the fluid flow and shock wave behavior. The point P is the point of contact between the sphere and the aluminum.

The basic problem in obtaining a numerical solution is the conversion of the physical situation into a mathematical model that allows finite difference techniques to be employed to represent the physical

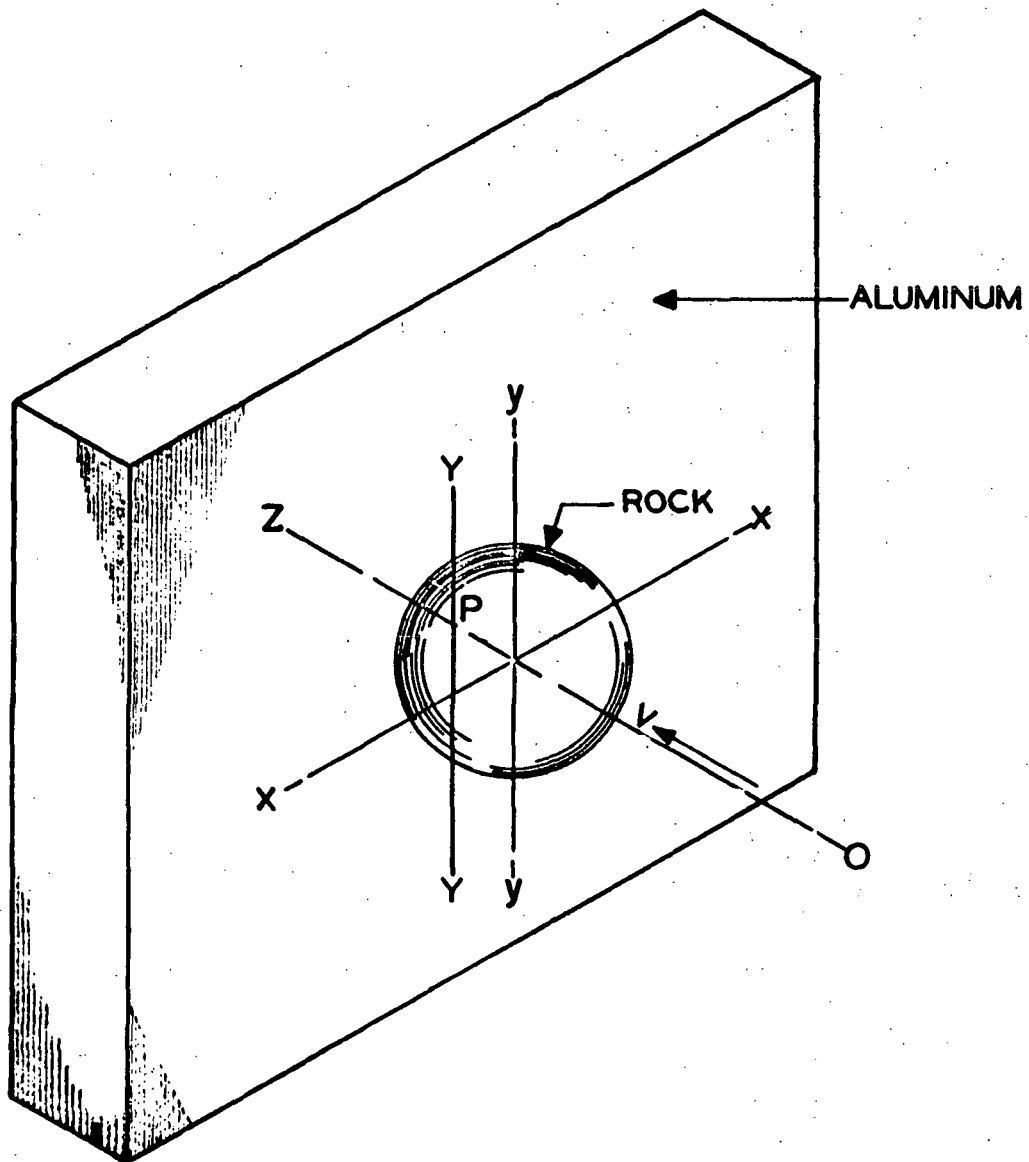


FIGURE 4 - IMPACT OF STONE SPHERE ON ALUMINUM TARGET

system. The problem is made to appear to be reduced to a two dimensional problem by considering only the plane determined by the lines OZ and YPY. Because of symmetry, the solution plane is further restricted by using only the half plane lying above the line OZ.

In order to apply finite difference techniques to the dynamical equations that describe the impact, the solution space must be covered by a grid of finite difference cells. This grid is constructed by placing the origin at point O and drawing lines of constant angle and lines of constant radius. The resulting picture of the impact is shown in Figure 5. The center of each cell in the mesh is given two coordinates (L,M). L represents the radial coordinate, and M the angular coordinate. The directions of increasing L and M are shown. Up to this position in this section, it has not been clearly emphasized that the cells in the mesh are of different thicknesses. The thickness may be explained by reference to the mesh in Figure 5. A typical radial line from the origin makes an angle, θ , with the line of symmetry, OZ. At any distance, r, from the origin along this typical radial line, the thickness of the cell is

$$r \sin \theta d\theta$$

After finding the solution in the single layer of cells that are shown in the figure, the complete solution is found by rotating the wedge shaped solution about the axis of symmetry, OZ. Since the thickness of the wedge is zero at the OZ-axis, there is no flow across the axis.

Equations of Dynamics in Finite Difference Form

The completed computer program for a long and complicated problem

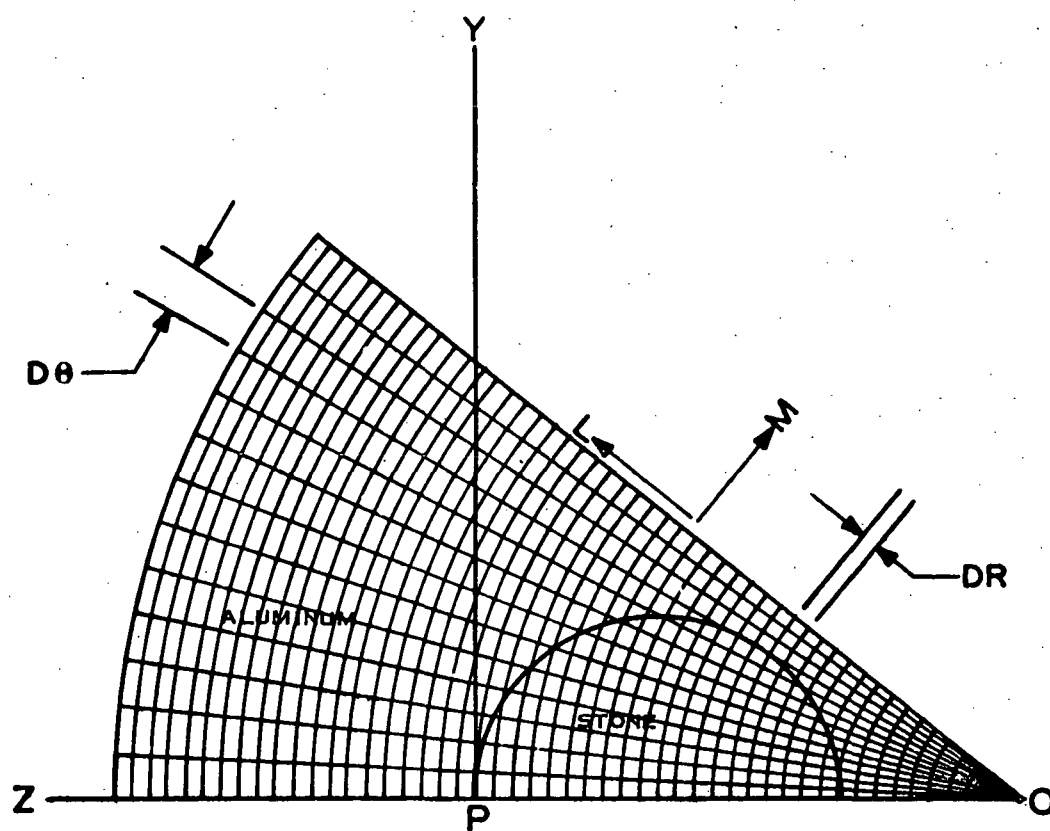


FIGURE 5 - FINITE DIFFERENCE MESH

shows many characteristics which are typical of the individual who wrote the program. There are, nevertheless, many basic techniques which are common to all good computer programs. The mixture of the basic technique and the individual characteristics for formulating the difference equations for a computer program are, in part, presented in this section as well as the conversion of the differential equations for dynamics into difference equations.

Consider a function, f , which depends on a space dimension, x , and the time, t . Express this function in the usual functional notation.

$$(1) \quad f = f(x, t)$$

The partial derivative of this function with respect to x may be approximated in finite difference form by either one of three expressions.

$$(2) \quad \frac{\partial f}{\partial x} = \frac{f(x, t) - f(x - \Delta x, t)}{\Delta x}$$

or

$$(3) \quad \frac{\partial f}{\partial x} = \frac{f(x + \frac{1}{2}\Delta x, t) - f(x - \frac{1}{2}\Delta x, t)}{\Delta x}$$

or

$$(4) \quad \frac{\partial f}{\partial x} = \frac{f(x + \Delta x, t) - f(x, t)}{\Delta x}$$

These representations, 2, 3, and 4, are called backward difference, central difference, and forward difference; respectively. The smaller the value of Δx , the better will be the approximation. Similarly, the partial derivative of f with respect to time may be approximated by either of the following expressions.

$$(5) \quad \frac{\partial f}{\partial t} = \frac{f(x, t) - f(x, t - \Delta t)}{\Delta t}$$

or

$$(6) \quad \frac{\partial f}{\partial t} = \frac{f(x, t + \frac{1}{2}\Delta t) - f(x, t - \frac{1}{2}\Delta t)}{\Delta t}$$

or

$$(7) \quad \frac{\partial f}{\partial t} = \frac{f(x, t + \Delta t) - f(x, t)}{\Delta t}$$

Again, Equation 5 is a backward difference, Equation 6 is a central difference, and Equation 7 is a forward difference. The accuracy of the approximation varies inversely with the magnitude of Δt .

In this thesis, all space derivatives are approximated by central differences, and all time derivatives are approximated by forward differences.

A typical cell in the two dimensional mesh for describing the problem is illustrated in Figure 6. The center of the cell has coordinates (R, θ) which may be calculated from the indices (L, M) . Side 1 is determined by the ray, $\theta - \frac{1}{2}\Delta\theta$, and side 3 is represented by the ray, $\theta + \frac{1}{2}\Delta\theta$. Side 2 is determined by a circle of radius, $R + \frac{1}{2}\Delta R$; and side 4 is determined by a circle of radius, $R - \frac{1}{2}\Delta R$.

A shorthand notation will simplify further discussion in regard to the conversion of partial differential equations to finite difference equations. Let N represent the time, t ; and $(N+1)$ represent the time, $(t + \Delta t)$. Let L represent the radial distance, R ; $L - \frac{1}{2}$ the distance, $R - \frac{1}{2}\Delta R$; and $L + \frac{1}{2}$ the distance, $R + \frac{1}{2}\Delta R$. Also let M represent the angle, θ ; $M - \frac{1}{2}$ the angle, $\theta - \frac{1}{2}\Delta\theta$; and $M + \frac{1}{2}$ the angle, $\theta + \frac{1}{2}\Delta\theta$.

Consider the function,

$$f = f(r, \theta, t)$$

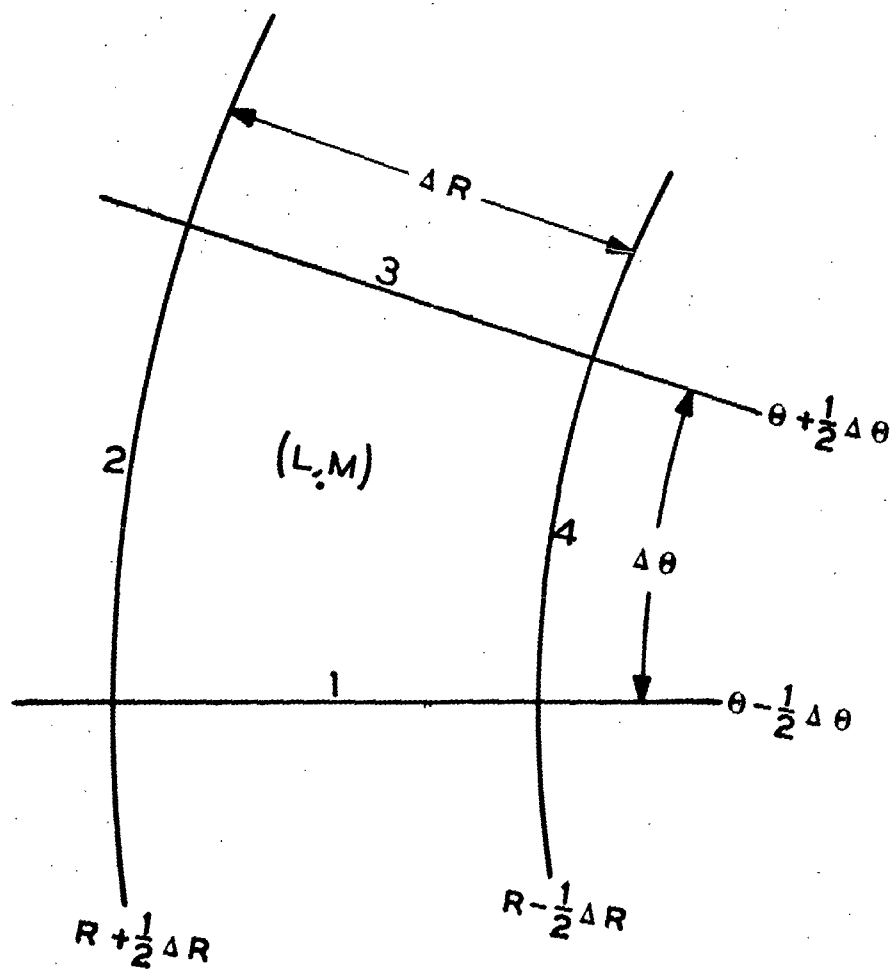


FIGURE 6 - FINITE DIFFERENCE CELL

then the following shorthand notation will be employed

$$f_N^{L,M} \equiv f(r, \theta, z) \quad (8)$$

$$f_{N+1}^{L,M} \equiv f(r, \theta, z + \Delta z) \quad (9)$$

$$f_N^{L+1/2, M} \equiv f(r + \frac{1}{2}\Delta r, \theta, z) \quad (10)$$

$$f_N^{L-1/2, M} \equiv f(r - \frac{1}{2}\Delta r, \theta, z) \quad (11)$$

$$f_N^{L, M+1/2} \equiv f(r, \theta + \frac{1}{2}\Delta \theta, z) \quad (12)$$

$$f_N^{L, M-1/2} \equiv f(r, \theta - \frac{1}{2}\Delta \theta, z) \quad (13)$$

The partial derivatives of the function, f , at the center of cell

(L, M) with respect to the space variables, r and θ , are

$$\frac{\partial f}{\partial r} = \frac{f_N^{L+1/2, M} - f_N^{L-1/2, M}}{\Delta r} \quad (14)$$

and

$$\frac{\partial f}{\partial \theta} = \frac{f_N^{L, M+1/2} - f_N^{L, M-1/2}}{\Delta \theta} \quad (15)$$

and the time derivative is

$$\frac{\partial f}{\partial t} = \frac{f_{N+1}^{L, M} - f_N^{L, M}}{\Delta t} \quad (16)$$

The conservation of mass

$$(17) \quad \frac{\partial \rho}{\partial t} = -\frac{1}{r} \frac{\partial (r^2 \rho u)}{\partial r} - \frac{\partial}{\partial \theta} (\rho u \sin \theta) \quad \frac{r \sin \theta}{r}$$

may be represented in finite difference form

$$\frac{D_{L,M}^{N+1} - D_{L,M}^N}{\Delta t} = - \frac{(R^2 D_U)_{L+1/2,M}^N - (R^2 D_U)_{L-1/2,M}^N}{R^2 \Delta R} - \frac{R^2 \Delta R}{(D W \sin \theta)_{L,M+1/2}^N - (D W \sin \theta)_{L,M-1/2}^N} R \sin \theta \Delta \theta$$

$$(18) \quad - \frac{R \sin \theta \Delta \theta}{(D W \sin \theta)_{L,M+1/2}^N - (D W \sin \theta)_{L,M-1/2}^N}$$

The other conservation equations for inviscid fluid flow which are:

$$(19) \quad \frac{\partial p_u}{\partial t} = -\frac{1}{r} \frac{\partial (r^2 p_u)}{\partial r} - \frac{\partial}{\partial \theta} (p_u r \sin \theta) + p_u^2 - \frac{\partial p}{\partial r}$$

$$(20) \quad \frac{\partial p_w}{\partial t} = -\frac{1}{r} \frac{\partial (r^2 p_w)}{\partial r} - \frac{\partial}{\partial \theta} (p_w^2 \sin \theta) - p_w u - \frac{r}{r} \frac{\partial p}{\partial \theta}$$

$$(21) \quad \frac{\partial p_E}{\partial t} = -\frac{1}{r} \frac{\partial (r^2 p_E)}{\partial r} - \frac{\partial}{\partial \theta} (p_E \sin \theta) - \frac{r \sin \theta}{r} \frac{\partial}{\partial \theta} (p_w \sin \theta) - \frac{r \sin \theta}{r}$$

may be transformed into the next three finite difference equations.

$$\begin{aligned}
 (22) \quad \frac{(DU)_{L,M}^{N+1} - (DU)_{L,M}^N}{\Delta t} = & - \frac{(R^2 DU^2)_{L+1/2,M}^N - (R^2 DU^2)_{L-1/2,M}^N}{R^2 \Delta R} \\
 & - \frac{(DW \sin \theta)_{L,M+1/2}^N - (DW \sin \theta)_{L,M-1/2}^N}{R \sin \theta \Delta \theta} + \frac{(DW^2)_{L,M}^{N+1/2}}{R} \\
 & - \frac{P_{L+1/2,M}^N - P_{L-1/2,M}^N}{\Delta R}
 \end{aligned}$$

$$\begin{aligned}
 (23) \quad \frac{(DW)_{L,M}^{N+1} - (DW)_{L,M}^N}{\Delta t} = & - \frac{(R^2 DW)_{L+1/2,M}^N - (R^2 DW)_{L-1/2,M}^N}{R^2 \Delta R} \\
 & - \frac{(DW^2 \sin \theta)_{L,M+1/2}^N - (DW^2 \sin \theta)_{L,M-1/2}^N}{R \sin \theta \Delta \theta} - \frac{(DWU)_{L,M}^{N+1/2}}{R} \\
 & - \frac{P_{L,M+1/2}^N - P_{L,M-1/2}^N}{R \Delta \theta}
 \end{aligned}$$

$$\begin{aligned}
 (24) \quad \frac{(DE)_{L,M}^{N+1} - (DE)_{L,M}^N}{\Delta t} = & - \frac{(R^2 DUE)_{L+1/2,M}^N - (R^2 DUE)_{L-1/2,M}^N}{R^2 \Delta R} \\
 & - \frac{(DWE \sin \theta)_{L,M+1/2}^N - (DWE \sin \theta)_{L,M-1/2}^N}{R \sin \theta \Delta \theta} \\
 & - \frac{(R^2 PU)_{L+1/2,M}^N - (R^2 PU)_{L-1/2,M}^N}{R^2 \Delta R} \\
 & - \frac{(PW \sin \theta)_{L,M+1/2}^N - (PW \sin \theta)_{L,M-1/2}^N}{R \sin \theta \Delta \theta}
 \end{aligned}$$

Equations 18, 22, 23, and 24 are made dimensionless by selecting

$$D = \rho/\rho_0$$

$$P = \bar{p}/\bar{p}_0$$

$$R = r/a$$

$$U = u/c_0$$

$$W = w/c_0$$

$$E = e/e_0$$

$$T = t/t_0$$

$$c_0^2 = \bar{p}_0/\rho_0$$

$$e_0 = c_0^2$$

$$t_0 = a/c_0$$

where

ρ = actual mass in grams per cubic centimeters

$\bar{p}$ = actual pressure in kilobars

r = actual distance in centimeters

u = actual radial flow velocity in centimeters/second

w = actual angular flow velocity in centimeters/second

e = actual total energy in joules per gram

t = actual time in seconds

ρ_0 = normal density of target

$\bar{p}_0$ = 1 kilobar

a = radius of micrometeoroid in centimeters

The computer solution which yields D , U , W , and E as functions of time and space is obtained in the following manner:

1. Specify $D_{L,M}^N$, $U_{L,M}^N$, $W_{L,M}^N$, $E_{L,M}^N$, and $P_{L,M}^N$; and the values of D , U , W , and P at the center of each cell of the mesh at time N .

2. Solve equations 18, 22, 23, and 24 for $D_{L,M}^{N+1}$, $U_{L,M}^{N+1}$, $W_{L,M}^{N+1}$, and $E_{L,M}^{N+1}$; the values of D , U , W , and E at the center of each cell in the mesh at time $(N + 1)$.
3. Solve the equations of state for $P_{L,M}^{N+1}$.
4. Print out the values of $D_{L,M}^{N+1}$, $U_{L,M}^{N+1}$, $W_{L,M}^{N+1}$, $E_{L,M}^{N+1}$, and $P_{L,M}^{N+1}$.
5. Replace each value $D_{L,M}^N$ with $D_{L,M}^{N+1}$, $U_{L,M}^N$ with $U_{L,M}^{N+1}$, $W_{L,M}^N$ with $W_{L,M}^{N+1}$, $E_{L,M}^N$ with $E_{L,M}^{N+1}$, and $P_{L,M}^N$ with $P_{L,M}^{N+1}$.
6. Increase time by an amount Δt .
7. Repeat steps 2 through 5 until the desired time, t_F .

The digital process is illustrated in Figure 7.

Let X represent any of the variables D , U , W , E , or P . In order to solve equations 18, 22, 23, and 24 for $D_{L,M}^{N+1}$, $U_{L,M}^{N+1}$, $W_{L,M}^{N+1}$, and $E_{L,M}^{N+1}$, it is necessary to be able to evaluate terms of the form

$$X_{L + \frac{1}{2}, M}^N$$

$$X_{L - \frac{1}{2}, M}^N$$

$$X_{L, M + \frac{1}{2}}^N$$

$$X_{L, M - \frac{1}{2}}^N$$

which are the values of the variables D , U , W , E , and P at the cell sides 2, 4, 3, and 1 respectively. Three types of averaging are used to evaluate these cell side terms from the known values of the variables at the centers of the cells. The averaging schemes are discussed in references 22 and 32. Three averaging schemes are listed below:

Type I

$$X_{L+\frac{1}{2}, M}^N = \frac{1}{2} (X(L, M) + X(L+1, M))$$

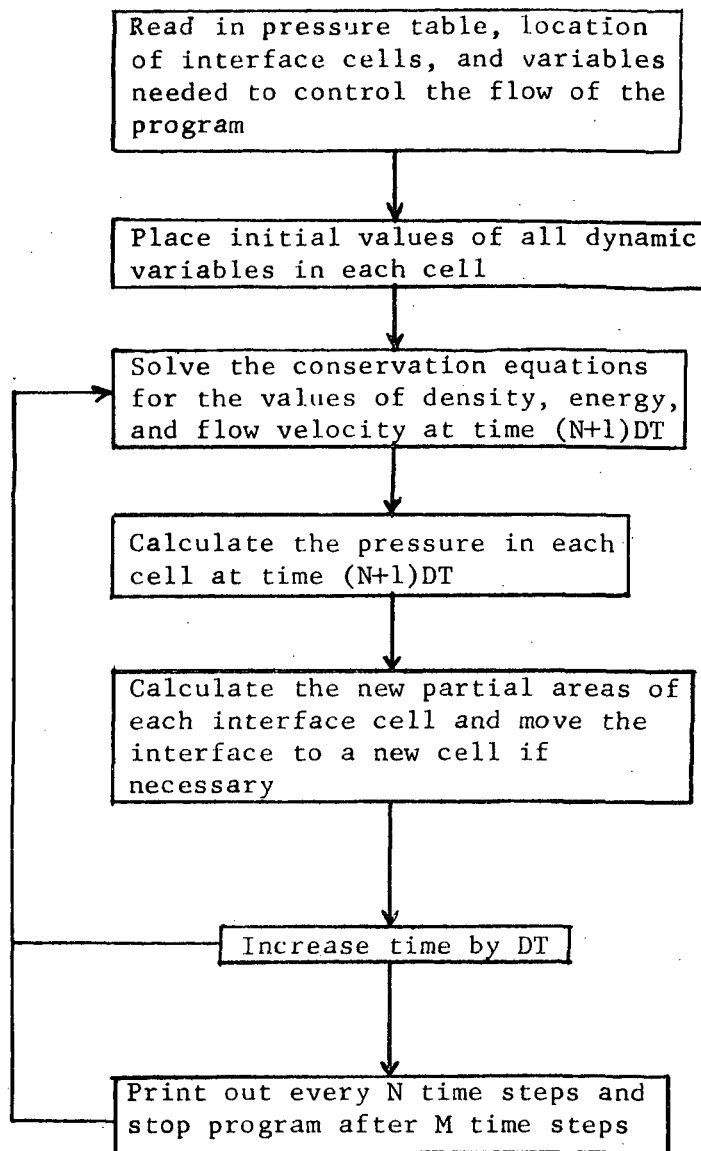


Figure 7. Flow Chart of Computer Program

$$X_{L,M+\frac{1}{2}}^N = \frac{1}{2} (X(L,M) + X(L,M+1))$$

Type II

$$\begin{aligned} X_{L+\frac{1}{2},M}^N &= X(L,M) & V > 0 \\ &= 0 & V = 0 \\ &= X(L+1,M) & V < 0 \end{aligned}$$

$$\begin{aligned} X_{L,M+\frac{1}{2}}^N &= X(L,M) & V > 0 \\ &= 0 & V = 0 \\ &= X(L,M+1) & V < 0 \end{aligned}$$

Type III

$$\begin{aligned} X_{L+\frac{1}{2},M}^N &= \frac{3}{4} X(L,M) + \frac{3}{8} X(L+1,M) - \frac{1}{8} X(L-1,M) & V > 0 \\ &= 0 & V = 0 \\ &= \frac{3}{4} X(L+1,M) + \frac{3}{8} X(L,M) - \frac{1}{8} X(L+2,M) & V < 0 \end{aligned}$$

$$\begin{aligned} X_{L,M+\frac{1}{2}}^N &= \frac{3}{4} X(L,M) + \frac{3}{8} X(L,M+1) - \frac{1}{8} X(L,M-1) & V > 0 \\ &= 0 & V = 0 \\ &= \frac{3}{4} X(L,M+1) + \frac{3}{8} X(L,M) - \frac{1}{8} X(L,M+2) & V < 0 \end{aligned}$$

A test velocity, V , is defined as

$$V = U(L,M) + U(L+1,M)$$

for $X_{L+\frac{1}{2},M}^N$; and as

$$V = W(L,M) + W(L,M+1)$$

for $X_{L,M+\frac{1}{2}}^N$. The equations for $X_{L-\frac{1}{2},M}^N$ and the test velocity V are obtained by subtracting 1 from the L index. The equations for $X_{L,M-\frac{1}{2}}^N$ and its test velocity are obtained by subtracting 1 from the M index.

Special averaging techniques must be used to evaluate these cell side terms at the boundaries of the finite difference mesh since the number of neighboring cells is reduced there. The mesh coordinates are confined to

$$1 \leq L \leq LN$$

$$1 \leq M \leq MN$$

As an example, if $L=1$, then there is no $X(L-1,M)$ so Types I or III cannot be used to evaluate $X_{L-\frac{1}{2},M}^N$. Similarly, if $M=1$, there is no $X(L,M-1)$; if $L=LN$, there is no $X(L+1,M)$; and if $M=MN$, there is no $X(L,M+1)$. Different but similar problems exist. The following system is employed to evaluate the cell side variables at the mesh boundaries:

$$L=1 \quad X_{L-\frac{1}{2},M}^N = X(1,M)$$

$$X_{L+\frac{1}{2},M}^N = \text{TYPE I}$$

$$L=LN \quad X_{L-\frac{1}{2},M}^N = \text{TYPE I}$$

$$X_{L+\frac{1}{2},M}^N = X(LN,M)$$

$$M=MN \quad X_{L,M-\frac{1}{2}}^N = \text{TYPE I}$$

$$X_{L,M+\frac{1}{2}}^N = X(L,MN)$$

$$M=1 \quad X_{L,M-\frac{1}{2}}^N = X(L,1)$$

$$X_{L,M+\frac{1}{2}}^N = \text{TYPE I}$$

The following exception was made for the boundary, $M=1$, which is the Z axis. This axis is an axis of symmetry so the variables on one side of the axis are mirror images of the variables on the opposite side,

and there is no flow of material across the Z axis. As an example, for $M=1$, $X_{L,M} - \frac{1}{2} = 0$ if X is W, the angular flow velocity.

The finite difference forms of equations 18, 22, 23, and 24 that are incorporated into the computer program consist of the following.

$$(25) \quad CD(L,M) = D(L,M) + DT * ((R4 * R4 * D4 * U4 - R2 * R2 * D2 * U2) / (R * R * DR) + (D1 * W1 * XT1 - D3 * W3 * XT2) / (R * XT * D\theta))$$

$$(26) \quad CU(L,M) = (D(L,M) * U(L,M) + DT * ((R4 * R4 * D4 * U4 * U14 - R2 * R2 * D2 * U2 * U12) / (R * R * DR) + (D1 * W1 * XT1 * U11 - D3 * W3 * XT2 * U13) / (R * XT * D\theta) + XND * XNW * XNW / R + (P4 - P2) / DR)) / CD(L,M)$$

$$(27) \quad CW(L,M) = (D(L,M) * W(L,M) + DT * ((R4 * R4 * D4 * U4 * W14 - R2 * R2 * D2 * U2 * W12) / (R * R * DR) + (D1 * W1 * XT1 * W11 - D3 * W3 * XT2 * W13) / (R * XT * D\theta) - XND * XNW * XNW / R - (P3 - P1) / (R * D\theta))) / CD(L,M)$$

$$(28) \quad CE(L,M) = (D(L,M) * E(L,M) + DT * ((R4 * R4 * D4 * U4 * E14 - R2 * R2 * D2 * U2 * E12) / (R * R * DR) + (D1 * W1 * XT1 * E11 - D3 * W3 * XT2 * E13) / (R * XT * D\theta) + (R4 * R4 * P4 * U14 - R2 * R2 * P2 * U12) / (R * R * DR) + (P1 * W1 * XT1 - P3 * W3 * XT2) / (R * XT * D\theta))) / CD(L,M)$$

The unsubscripted variables in the equations are the cell side values. The definition of each variable and the type of averaging used to evaluate it are listed below:

$D1 = D_{L,M-1/2}^N$	TYPE III
$D2 = D_{L+1/2,M}^N$	TYPE III
$D3 = D_{L,M+1/2}^N$	TYPE III
$D4 = D_{L-1/2,M}^N$	TYPE III
$U2 = U_{L+1/2,M}^N$	TYPE III
$U4 = U_{L-1/2,M}^N$	TYPE III
$W1 = W_{L,M-1/2}^N$	TYPE III
$W3 = W_{L,M+1/2}^N$	TYPE III
$P1 = P_{L,M-1/2}^N$	TYPE I
$P2 = P_{L+1/2,M}^N$	TYPE I
$P3 = P_{L,M+1/2}^N$	TYPE I
$P4 = P_{L-1/2,M}^N$	TYPE I
$U11 = U_{L,M-1/2}^N$	TYPE II
$U12 = U_{L+1/2,M}^N$	TYPE II
$U13 = U_{L,M+1/2}^N$	TYPE II
$U14 = U_{L-1/2,M}^N$	TYPE II
$W11 = W_{L,M-1/2}^N$	TYPE II
$W12 = W_{L+1/2,M}^N$	TYPE II
$W13 = W_{L,M+1/2}^N$	TYPE II
$W14 = W_{L-1/2,M}^N$	TYPE II
$E11 = E_{L,M-1/2}^N$	TYPE II
$E12 = E_{L+1/2,M}^N$	TYPE II
$E13 = E_{L,M+1/2}^N$	TYPE II
$E14 = E_{L-1/2,M}^N$	TYPE II

Some variables are not averaged. These variables are listed below.

$$XT1 = \sin \theta_{L,M-1/2}^N$$

$$XT = \sin \theta_{L,M}^N$$

$$XT2 = \sin \theta_{L,M+1/2}^N$$

$$R4 = R_{L-1/2,M}^N$$

$$R2 = R_{L+1/2,M}^N$$

$$R = R_{L,M}^N$$

In addition, some variables are linear time averages, rather than space averages. The time averaged quantities are

$$XND = D_{L,M}^{N+1/2}$$

$$XNU = U_{L,M}^{N+1/2}$$

$$XNW = W_{L,M}^{N+1/2}$$

These same differencing schemes are employed to convert the elastic and the plastic terms into the finite difference form. The converted elastic and plastic equations will not be given because they are quite lengthy and cumbersome to write out in detail.

Tag System

A system of cell tags was devised in order to follow the motion of the physical boundaries through the two dimensional mesh. The sign

and the magnitude of a cell tag determine the material in the cell, and the computer operations which may occur in the cell. The tags and their meanings are listed below.

Tag (L,M)	Meaning
-3.0	Cell (L,M) is completely filled with fluid rock and fluid aluminum.
-2.0	Cell (L,M) is completely filled with fluid rock.
-1.0	Cell (L,M) is partially filled with fluid rock.
0.0	Cell (L,M) is completely empty.
+1.0	Cell (L,M) is partially filled with fluid aluminum.
+2.0	Cell (L,M) is completely filled with fluid aluminum
+3.0	Cell (L,M) is partially filled with fluid rock and fluid aluminum.
+4.0	Cell (L,M) is filled with plastic aluminum
+5.0	Cell (L,M) is filled with elastic aluminum

The boundary of the rock micrometeoroid is determined by the coordinates of the cells whose tags are -3.0, -1.0, and +3.0. It must be possible to connect these coordinates with a smooth, unbroken line, or a bookkeeping error has occurred within the computer program. Every cell inside this boundary must have a tag of -2.0. The aluminum surface is determined by the coordinates of those cells whose tags are -3.0, +1.0, or +3.0.

Suppose that the boundary of the rock is to be determined. Let T represent any of the values -3.0, -1.0, or 3.0; any one of which

would imply that a rock boundary is in a particular cell. There are then six possible ways that the T's may be arranged in the two dimensional mesh so that the rock would have a continuous boundary. Each one of these six arrangements represents two possible ways that the rock could be located in the cell. The possibilities are shown in Figure 8; the shaded region represents the rock in the cell.

The twelve possible rock boundary configurations are identical to the twelve possibilities that Rich employs to describe fluid flow in a two dimensional mesh (28). A system of bookkeeping was devised which allows only these six possible arrangements of the tags -3.0, -1.0, and +3.0. The same system was used for the tags -3.0, +1.0, and +3.0 for the aluminum surface.

The rock material is moving in such a manner that a cell partially filled with rock will either completely fill, or empty. The tag of this cell will then change from -1.0 to some other tag value. The new tag value is completely determined by examining the values of $ARO(L,M)$, the area of the cell (L,M) that is occupied by rock; and $AAL(L,M)$, the area of cell (L,M) that is occupied by aluminum. These values are compared with the total area of the cell (L,M). Let X represent the area of cell (L,M). The following table shows the manner in which changes in ARO, or AAL may change the tag of -1.0. The methods employed to calculate the changes in ARO and AAL will be discussed in the next section.

Other allowed tag changes that involve physical boundaries are:

-3.0 may change to -2.0, +2.0 or +3.0

-2.0 may change to -1.0, or -3.0

0.0 may change to -1.0, +1.0 or +3.0

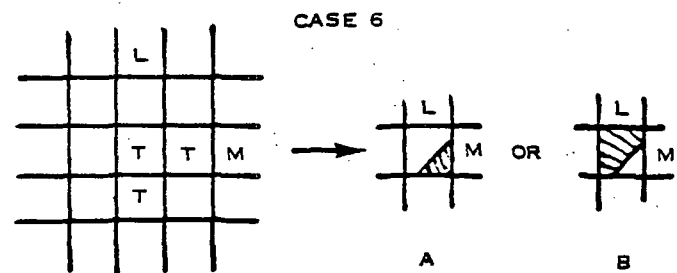
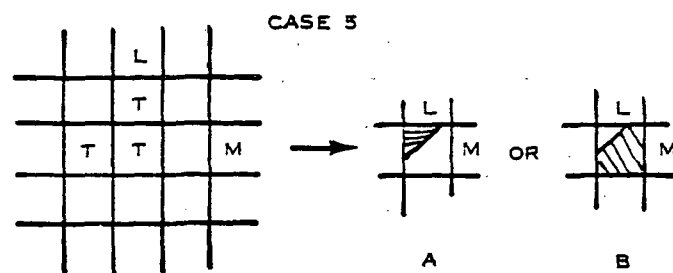
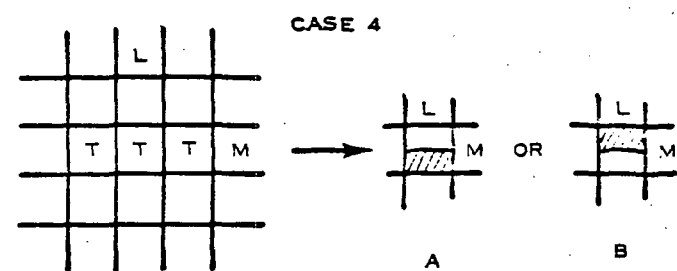
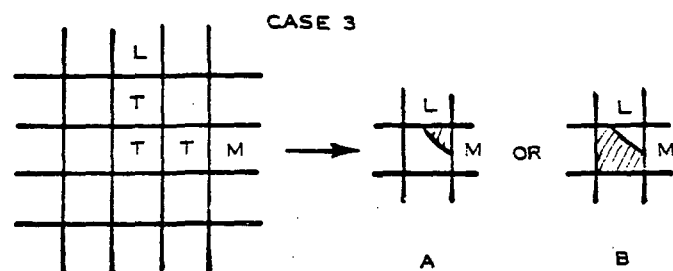
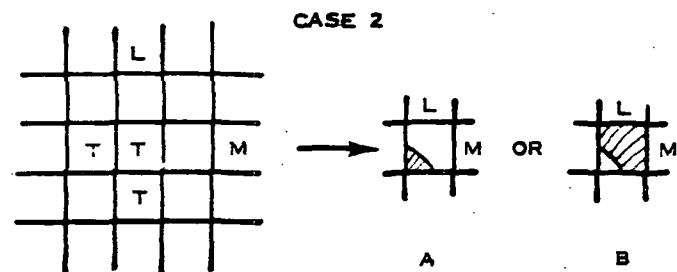
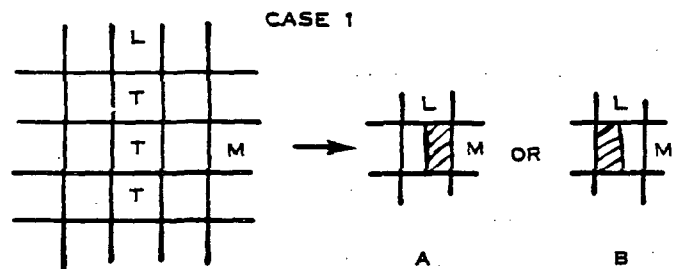


FIGURE 8 - SIX TYPES OF BOUNDARY CELLS

TABLE II

POSSIBLE TAG CHANGES FOR A CELL PARTIALLY FILLED WITH ROCK

Time = t	Time = t + Δt	New Tag Value For Cell (L,M)
AAL(L,M) = 0.0	AAL(L,M) = 0.0	0.0
0.0 < ARO(L,M) < X	ARO(L,M) = 0.0	
AAL(L,M) = 0.0	AAL(L,M) = 0.0	-2.0
0.0 < ARO(L,M) < X	ARO(L,M) $\geq$ X	
AAL(L,M) = 0.0	AAL(L,M) > 0.0	+3.0
0.0 < ARO(L,M) < X	ARO(L,M) > 0.0	
	ARO(L,M) + AAL(L,M) < X	
AAL(L,M) = 0.0	AAL(L,M) > 0.0	-3.0
0.0 < ARO(L,M) < X	ARO(L,M) > 0.0	
	ARO(L,M) + AAL(L,M) $\geq$ X	

+1.0 may change to +2.0, +3.0, -3.0 or 0.0

+2.0 may change to +1.0, or -3.0

+3.0 may change to -3.0, -1.0, 1.0 or 0.0

A change in the tag value of the cell (L,M) will always induce a tag change in neighboring cells. For example, suppose that the tag of cell (L,M) is -1.0, and the cell is represented by Case 1 of Figure 8. If TAG(L,M) changes to 0.0, then TAG(L-1,M) changes to -1.0 if the configuration is Case 1a, or the tag of cell (L+1,M) changes to -1.0 if the configuration is Case 1b. The induced tag changes for Cases 2, 3, 5, and 6 are more complicated. The induced changes when the tag of cell (L,M) is -3.0, or +3.0, is more difficult to determine for all of the Cases. The induced tag change is always governed by the fact that the final tag arrangement in cell (L,M) and its eight immediate neighbors must reduce to one of the six allowed possibilities.

Almost all numerical operations are controlled by the tag value. For example, when the pressure in cell (L,M) is calculated, there are different equations of state that could be used for the calculation. The tag of cell (L,M) determines which equation of state is used. If TAG(L,M)=+2.0, Tillotson's equation for aluminum should be used. If TAG(L,M)=-2.0, the tabular equation of state for rock should be used. If TAG(L,M)=+4.0, the plastic equation of state is used. If TAG(L,M)=0.0, the cell contains no material so the program skips the cell and does no numerical calculations.

This tag system is the foundation upon which the entire computer program is built.

Methods to Change Partial Areas of Boundary Cells

In this thesis, a boundary cell is defined as any cell which contains an interface. A cell contains a partial area if the area of rock and/or aluminum in the cell is less than the area of the cell. The interface may be either rock-aluminum, rock-vacuum, or aluminum-vacuum. The fundamental difficulty in solving a fluid flow problem is in following the motion of such interfaces. An interface is made to move through a cell by altering the area of rock and/or aluminum in the cell. The problem of following physical boundaries is basically the determination of sufficiently accurate methods to be employed to approximate changes in the partial areas of boundary cells.

Rich suggests a method to calculate the partial area of a cell that is good for rectangular mesh cells (28). A fundamental assumption in Rich's method is that a physical boundary such as ABC in Figure 9 may be replaced by the straight line AC. This assumption allows the partial areas of cells to be calculated as the areas of triangles or parallelograms. From these areas, the intercepts of the boundary with the cells sides, such as points a and b, may be determined. The interface motion may be followed by monitoring the motion of the intercept points.

Rich's method would work quite well for any coordinate system which would have rectangular mesh cells such as a Cartesian x-y system or the plane of a cylindrical system which contains the z axis. The method would not work well for the spherical coordinate system that is employed in this study. The partial areas of the mesh cells, such as the one in Figure 10, cannot be accurately represented as the areas of triangles or parallelograms except for large radii of curvature.

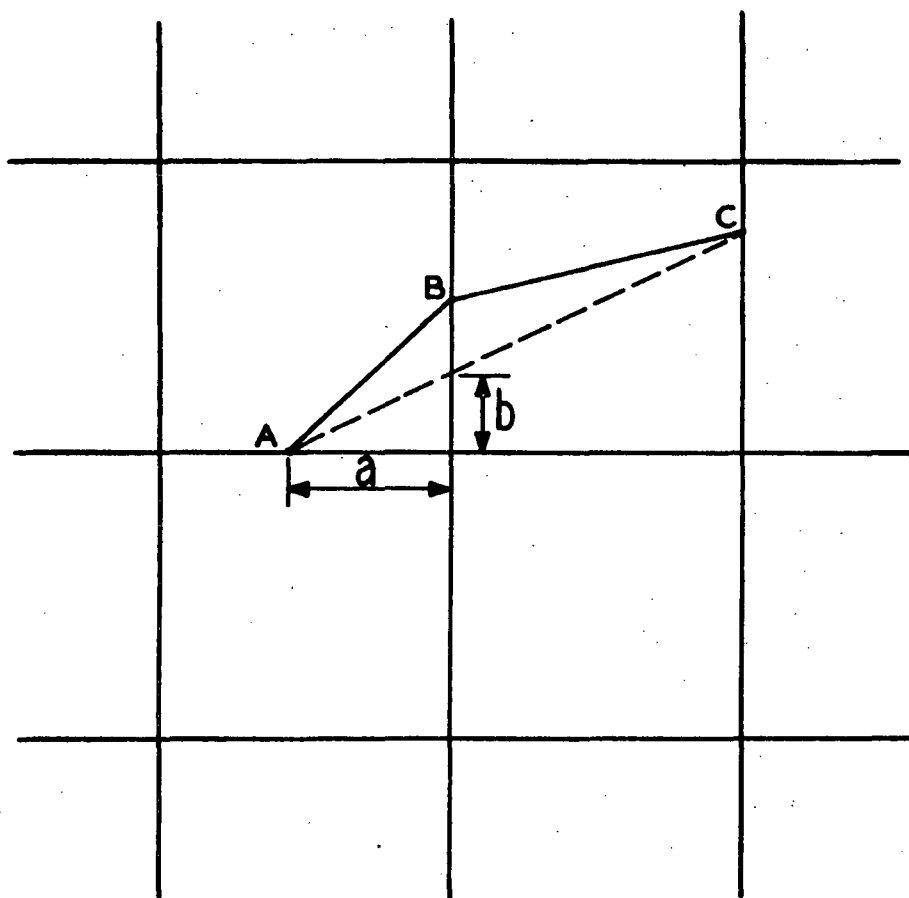


FIGURE 9 – APPROXIMATION OF PARTIAL CELL AREA
IN RECTANGULAR MESH

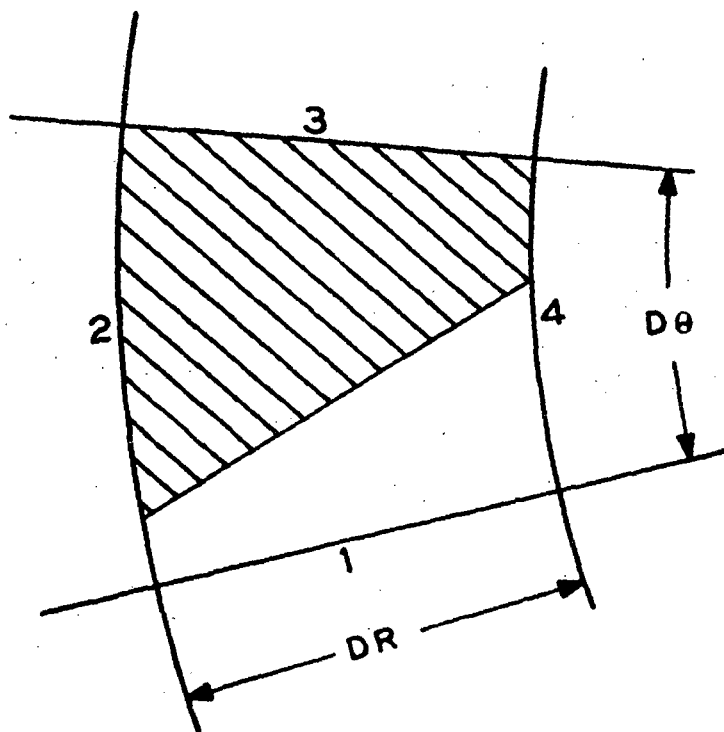


FIGURE 10 - PARTIAL AREA IN A SPHERICAL COORDINATE SYSTEM

Let A^N represent the partial area of a boundary cell at time $N\Delta t$ and A^{N+L} the area at time $(N+1)\Delta t$. Let X be the total area of the cell, $D\theta$ the angle between cell sides 1 and 3, and DR the radial distance between sides 2 and 4. These cell sides are shown in Figure 10. Let R_2 and U_2 represent the radial distance and radial flow velocity at cell side 2, and R_4 and U_4 represent the radial distance and radial flow velocity at cell side 4. Let W_1 and W_3 be the angular flow velocities at sides 1 and 3 respectively.

The change in cell area is found by calculating the difference in the flow in and the flow out of the cell. The flow in or out of side 2 during time DT is

$$\pm U_2 * R_2 * DTH_2 * DT$$

where $(R_2)(DTH_2)$ is the length of side 2 that is open to cell $(L+1,M)$.

The flow in or out of side 4 during time DT is

$$\pm U_4 * R_4 * DTH_4 * DT$$

where $(R_4)(DTH_4)$ is the length of side 4 open to cell $(L-1,M)$. These quantities are illustrated in Figure 11. The flow in or out of side 1 during time DT is

$$\pm W_1 * XL_1 * DT$$

where XL_1 is the length of side 1 open to cell $(L,M-1)$, and the flow in or out of side 3 during time DT is

$$\pm W_3 * XL_3 * DT$$

where XL_3 is the length of side 3 open to cell $(L,M+L)$. See Figure 12.

The change in cell area is given by

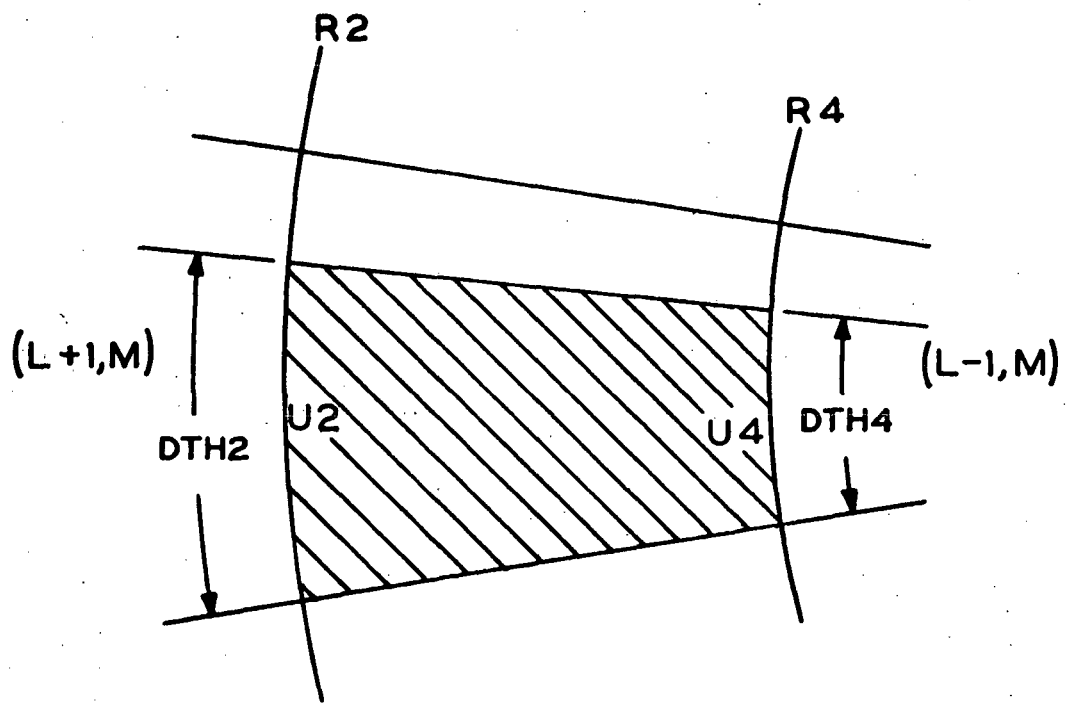


FIGURE 11 - RADIAL FLOW IN AND OUT OF CELL

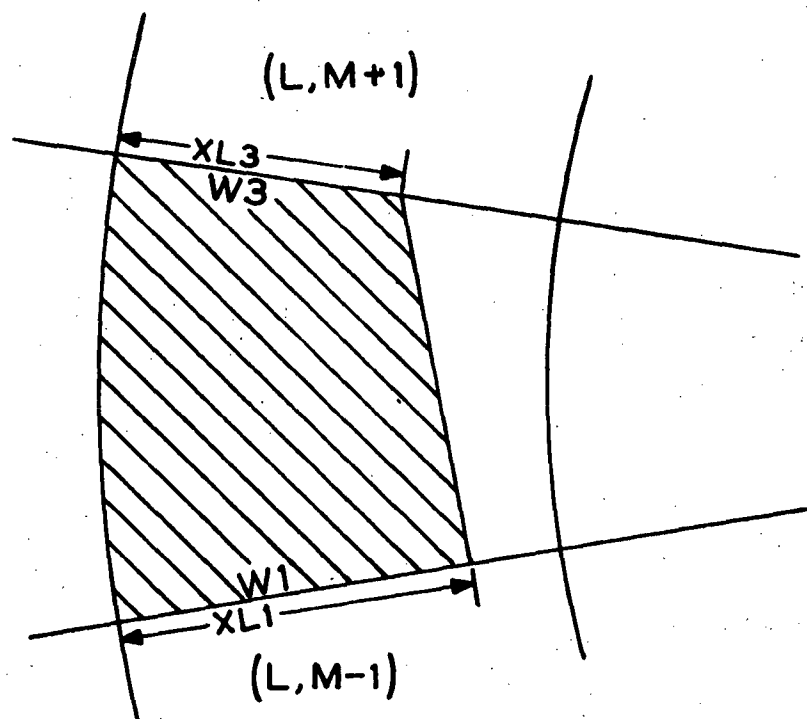


FIGURE 12 - ANGULAR FLOW IN AND OUT OF CELL

$$A^{n+1} = A^n + \left[\pm U_2 * R_2 * DTH_2 \quad \pm U_4 * R_4 * DTH_4 \right. \\ \left. \pm W_1 * XL_1 \quad \pm W_3 * XL_3 \right] * DT$$

The algebraic sign of each term is determined by the directions of U_2 , U_4 , W_1 , and W_3 as well as the particular case of the possible six cell tag arrangements that is being considered. The calculations of DTH_2 , DTH_4 , XL_1 , and XL_3 for each of the six cases are summarized in Figure 13.

These interpolation equations are simple and in many cases probably do not accurately describe changes in area. It is difficult to defend these equations from a purely mathematical argument. For this reason, the equations were subjected to the following test to determine if they would perform satisfactorily. A semicircle was placed on the Z axis of Figure 5 (Chapter III) so that its center was in cell 20. Each cell in the semicircle was given U and W components of velocity so that the resultant velocity was parallel to the Z axis. A computer program was then formulated to make the semicircle move along the Z axis until its center was in cell 30. The boundary of this semicircle was moved through the mesh by using the interpolation equations to increase, or decrease, the partial areas of the boundary cells. After moving 10 cells, the final semicircle was slightly distorted, but it represented the original semicircle accurately enough for this author to have complete confidence in the simple linear interpolation that is employed to evaluate changes in the partial areas of interface cells.

The interface motion depends very strongly upon the size of the time increment DT . If DT is large enough so material traverses a cell in only two or three time steps, the interface becomes misshapened and

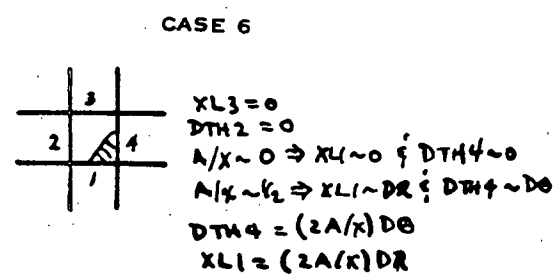
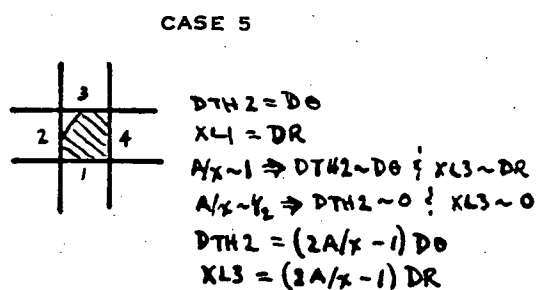
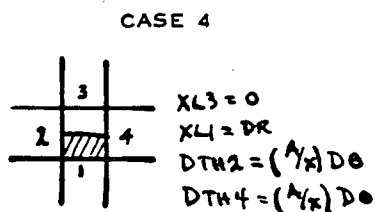
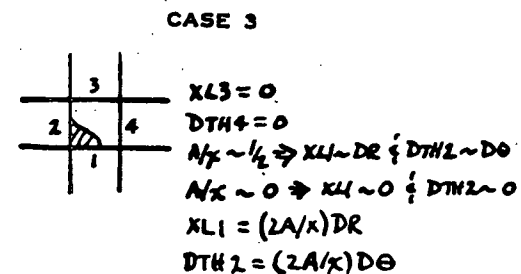
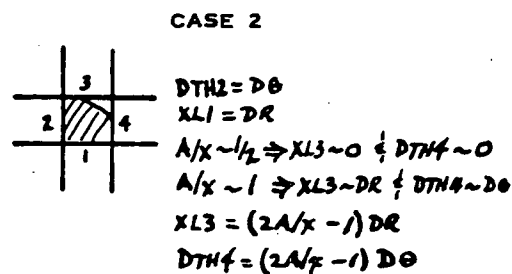
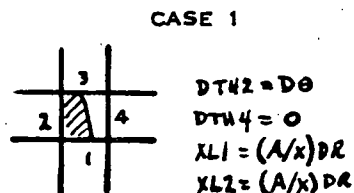


FIGURE 13 - SIX CASES OF PARTIAL AREA

broken. To maintain a smooth continuous interface, and thus, to maintain a continuous impacting sphere, the time increment is chosen so that at least ten time steps are required for material to cross a mesh cell. In the computer program, DT is calculated from the equation

$$DT = \frac{DR}{C * UMAX}$$

where C is given a value between 10 and 15, and UMAX is the maximum value of the flow velocity in any cell in the mesh at time step N. Time steps of this size also result in smaller oscillations in the density, energy, and pressure profiles. The evaluation of the time step will be discussed further in the next chapter.

If the interface cell under consideration is completely filled by both aluminum and rock, the new value of the area occupied by rock is found from the interpolation equations, and the new value of the area occupied by aluminum is found from

$$AAL_{L,M}^{N+1} = X - ARO_{L,M}^{N+1}$$

where AAL is the aluminum area, ARO is the rock area, and X is the total cell area.

To conserve computer memory, the computer program that calculates the partial areas of the cells occupied by rock along the rock-vacuum interface and by aluminum along the aluminum-vacuum interface at time $t = 0$ was formulated and executed separately from the main impact program. The output of this area calculation program was then used as input data for the impact program.

At time $t = 0.0$, the rock-vacuum interface is the boundary of a semi-circle with its center at $(r, \theta, \phi) = (t, 0, 0)$ on the z axis, and it

has the radius, a . See Figures 5 and 14. The target surface is represented by a plane normal to the z axis and intersecting the axis at $z = (a+b)$. The equation of the circle in the yz plane is

$$(29) \quad r^2 = a^2 + 2br \cos \theta - b^2$$

and the equation of the straight line DE that represents the aluminum surface is

$$(30) \quad r = (a+b) / \cos \theta$$

The area occupied by rock at boundary cells such as at point A is evaluated as shown in Figure 15. The function $f(r, \theta)$ is the equation of the circle from Equation 29. For points such as D and C, the limits of r are changed to an integration from R_1 to $f(r, \theta)$. The area that is occupied by aluminum at a point such as E is calculated by the same expression except $f(r, \theta)$ is the equation of the straight line from Equation 30.

The complexity of the integration increases when $f(r, \theta)$ intersects either sides 2 or 4 of the cell. In such cases, the angle θ_s in Figure 16 serves as one limit of integration. This angle can be easily found by setting $f(r, \theta) = c$ where c is set equal to either R_1 or R_2 .

Expansion of Mesh

Limitations of computer memory permitted only 1200 cells in the finite difference mesh. The last radial cell on the z -axis lies three sphere radii below the original position of the surface of the target (Figure 17). The mesh scale in Figure 17 is greatly exaggerated. When the shock front has propagated a distance equal to $3R$, the mesh is filled, and no further time steps of the solution may be calculated

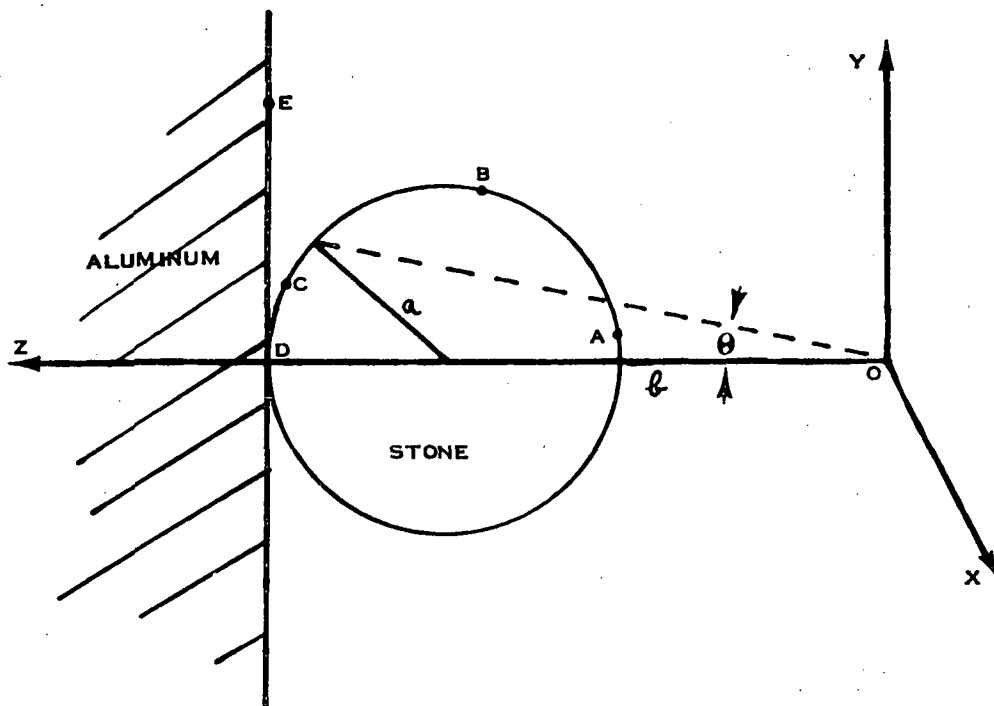


FIGURE 14 - PHYSICAL BOUNDARIES AT TIME $T = 0$

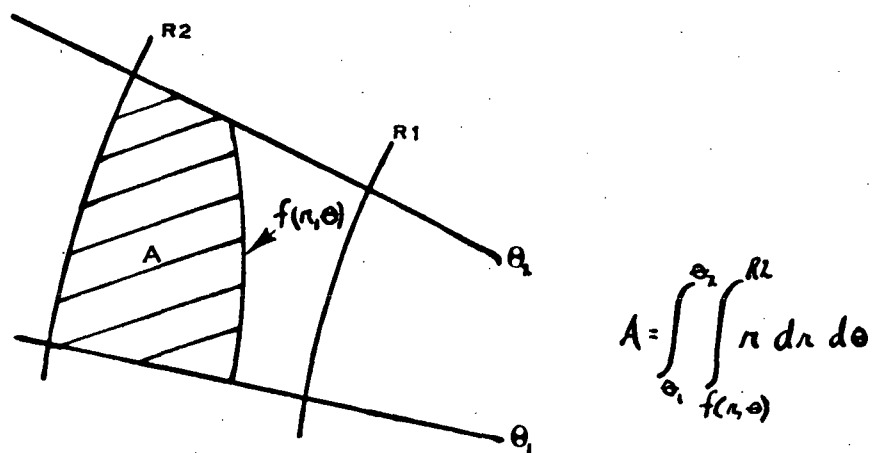


FIGURE 15 – CALCULATION OF PARTIAL AREA

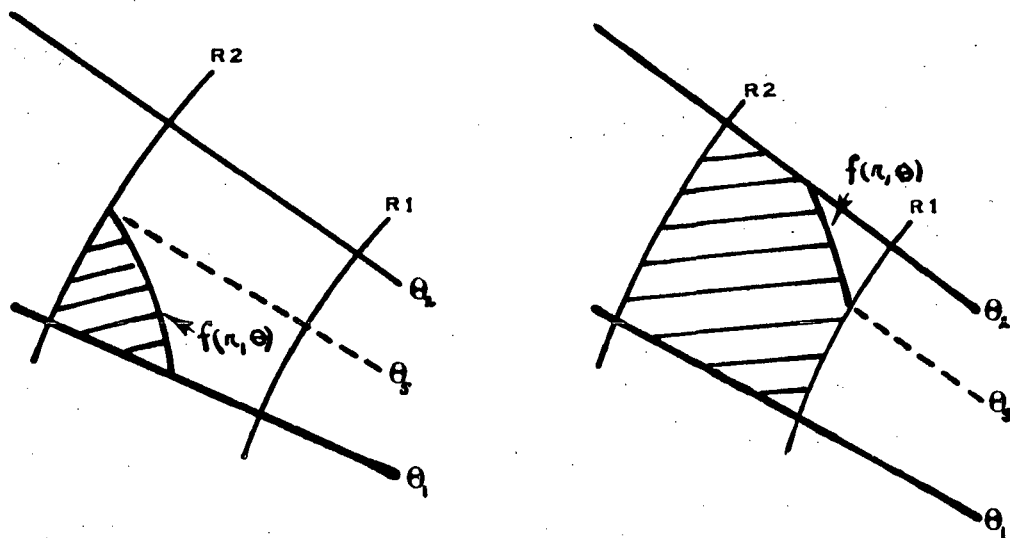


FIGURE 16 - CALCULATION OF PARTIAL AREA

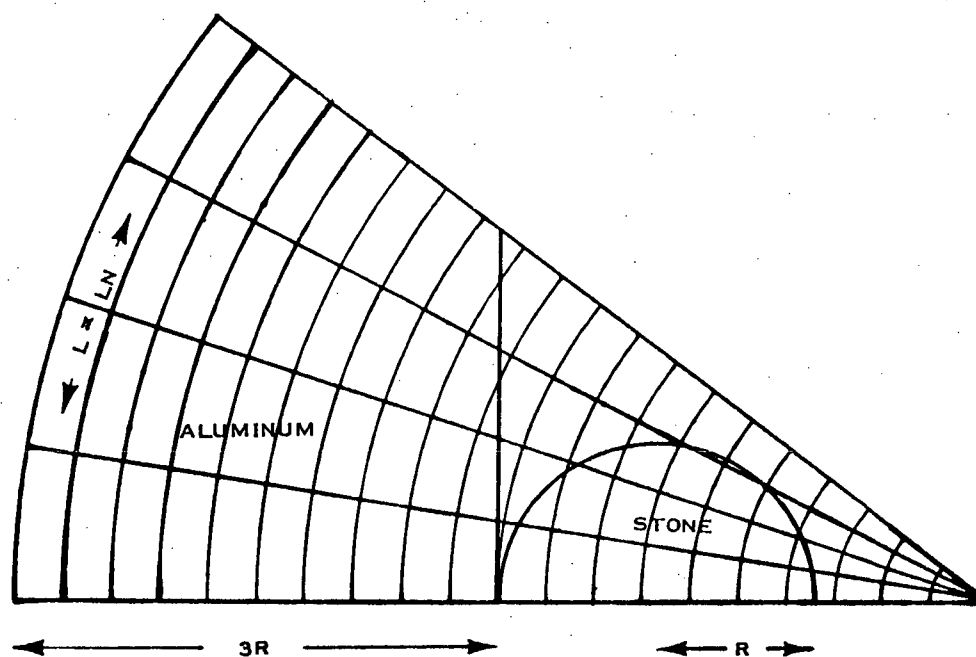


FIGURE 17 - DEPTH TO WHICH FINITE DIFFERENCE MESH
PENETRATES TARGET AT TIME $T = 0$

until undisturbed cells are placed in front of the shocked region.

An undisturbed region is created ahead of the shock front by a condensation of the mesh. This condensation process averages the flow variables in cells (L,M) and $(L-1,M)$, places this average in cell $(L-1,M)$, and doubles the radial dimension of each cell. This process reduces the number of radial cells needed to describe the shocked region from LN to $LN/2$ and moves the shock front from $L = LN$ to $L = LN/2$. A cell with $L = LN/2$ is still $3R$ below the original target surface position because the radial dimension of each cell has doubled. The radial cells from $(LN/2+1,M)$ to (LN,M) , now lies ahead of the shock front. They may be described as undisturbed cells, and the shocked region can expand into these cells. This condensation process is repeated until the shock parameters and flow variables have decreased to desired values.

This mesh condensation may be reversed if space resolution is required for any reason. Instead of combining two cells into one cell and doubling the cell size, a single cell could be divided into N cells. This mesh resolution is performed when the dual shock wave region is created. The plastic region is quite thin, and its physical width could lie totally within a single cell if several mesh condensations have occurred. A single cell could not describe the details of this region; therefore, the cell widths are decreased so that several cells are needed to traverse the width of the region. This resolution technique is illustrated in Figure 18. The widths of the cells preceding the zone (those cells for $L < L_F$) are not changed.

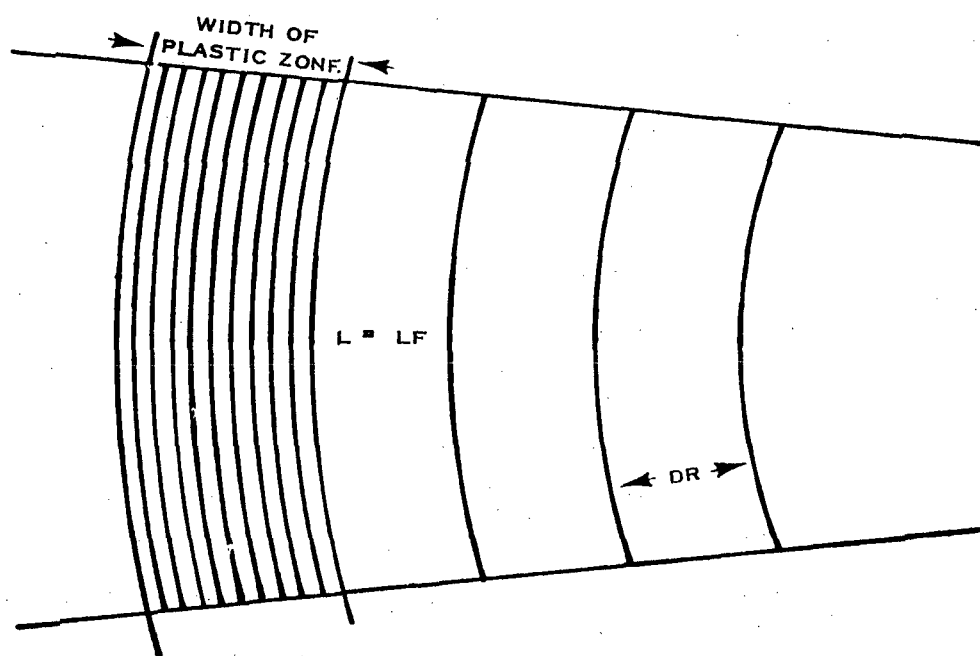


FIGURE 18 – DECREASING CELL WIDTHS TO OBTAIN RESOLUTION
IN PLASTIC ZONE

CHAPTER IV

DISCUSSION OF SOLUTIONS

Uniqueness and Convergence

The solution of any initial value problem should be subjected to the following question:

1. Is the solution unique?

A finite difference solution of an initial value problem should be subjected to the additional investigation:

2. Is the finite difference solution an accurate approximation to the true solution?

Closely associated with the first question is the concept of a "well-set" initial value problem. A one dimensional initial value problem is said to be well-set in a domain D when there is one and only one solution $y = f(x, c)$ in D of the given differential equation for each given (x, c) in D , and when this solution varies continuously with c . To show that the problem is well-set; therefore, requires proving theorems of existence (there is a solution); uniqueness (there is only one solution); and continuity (the solution depends continuously on the initial value). Solutions which do not have these properties are useless physically, because no physical measurement is exact. The concept of a well-set initial value problem gives a precise mathematical interpretation to the physical concept of determinism. It is generally believed that the inviscid, compressible-fluid equations give rise to a

system of partial differential equations which define a well-set initial value problem. The effect on these properties that result from the addition of elastic, or plastic, stress tensor terms to the inviscid fluid equation is difficult to determine in a rigorous mathematical sense.

The second question, the convergence of the finite difference solution to the true solution, may be established by the following methods:

1. Compare the approximate solution to the true solution.
2. Compare the solution obtained using different values of the mesh parameters ΔR , $\Delta \theta$, and ΔT .
3. Compare the solution with the conditions demanded by the Theorem of Lax.

The first method is the most powerful of the three if a true solution exists for comparison. In the case of hypervelocity impact problems, there is no true solution against which to compare the finite difference solution because no closed mathematical solution has been obtained. The "true" solution, which must be employed, is the solution that results via experimental studies of the problem. Extensive, experimental studies have been performed at low velocities (< 10 km/sec), but it has been difficult to explore the upper range of micrometeoroid velocities (> 36 km/sec). The impact parameter measured in experimental studies is almost always the crater size. This author knows of no experimental solution which has obtained the details of the impact process; i.e., the peak pressure of the shock wave, the quantity of momenta and energy carried away by the target ejecta, the direction and magnitude of the material flow, etc. As a consequence, a comparison of

the finite difference solution to the "true" solution is necessarily confined to a comparison of the size and shape of the crater which is predicted by the computer solution to the size and shape which is established by experimental studies. These comparisons will be emphasized later in this chapter.

The second method insures that the solution is not just a set of numbers dependent upon the parameters of the finite difference mesh. If a mesh of ΔR_1 , $\Delta \theta_1$, and a mesh of ΔR_2 , $\Delta \theta_2$, result in substantially different numerical solutions then no unique solution exists. If the solutions do agree, the solution may or may not be unique.

The third test uses the Equivalence Theorem of Lax which states (29):

Given a properly posed initial value problem and a finite difference approximation to it that satisfies the consistency conditions, stability is the necessary and sufficient condition for convergence.

The consistency condition requires:

1. The numerical solution depends continuously on the initial data, and
2. The difference equations convert into the partial differential equations as ΔR , $\Delta \theta$, and ΔT approach zero.

Both of these requirements are satisfied by the finite difference solution. The fact that the second requirement is satisfied is obvious. The first requirement is also satisfied since each iteration of the solution proceeds from updated initial data.

A numerical solution is stable if there are no unbounded numerical oscillations in the space and time profiles. A stable solution may be

oscillatory as long as the oscillations are small, or grow smaller with time.

Richtmyer has shown that stability can be insured by a proper choice of the time step ΔT (29). If $\vec{\Delta X}$ is the vector between two space points of interest in the finite difference mesh, then a relation that must be satisfied in order to have stable solutions in the interval $\vec{\Delta X}$ is the Courant condition

$$\Delta T \leq \frac{\Delta X}{C + |\vec{V}|}$$

where C is the speed of sound in the medium, $\vec{V}$ is the material velocity, and ΔT is the time step. Tests conducted at Oklahoma State University have shown that stability is insured if the time step is calculated from the condition

$$\Delta T = \frac{\Delta R}{k(|V_{\max}| + C)}$$

where k is a constant with a value between 10 and 15, $V_{\max}$ is the largest value of the material velocity which was found in the finite difference mesh, and ΔR is the smallest dimension of any cell.

The solutions obtained in this thesis have been subjected to these three tests, and the solutions do imply uniqueness and convergence to the true solution.

Presentation of Solutions

In the study of crater formation and shock propagation for this thesis, all computer solutions are for impacts onto a very thick slab of solid aluminum (semi-infinite solid). Three parameters are varied for the incident sphere (micrometeoroid); these parameters are:

1. Velocity of the incident sphere
2. Density of the incident sphere
3. Diameter of the incident sphere.

The particular combination of numerical values for these parameters in each of the solutions are listed in Table III. In the table, V is the velocity of the impacting sphere, D is the mean density of the sphere, and M is the mass of the sphere. A comment is necessary in regard to the numerical values for the density. The density of the solid aluminium target is 2.78 grams per cubic centimeter. The impacting spheres are of quartz which has a solid density of 2.5 grams per cubic centimeter. The values for the density of the spheres in the table are all for porous quartz spheres with a pore volume of $1/5$ for a density of 2.0 and a pore volume of $4/5$ for a density of 0.5. The treatment of the equation of state for the porous materials follows the American improvements on the initial Russian suggestions, as was discussed earlier in this thesis.

TABLE III

VALUES OF THE VELOCITY, DENSITY, AND MASS OF THE IMPACTING SPHERE

	V (km/sec)	D (gm/cc)	M (gram)
Case 1	6.25	2	10^{-9}
Case 2	7.5	2	10^{-9}
Case 3	2.0	2	10^{-9}
Case 4	20	0.5	10^{-9}
Case 5	72	0.5	10^{-9}
Case 6	20	0.5	3.4×10^{-9}

The computer output data is reduced to curves which are divided into three groups for presentation:

1. crater solutions
2. pressure profiles
3. calculation of the momentum and energy of the ejecta

Crater Solutions

Effect of Velocity and Density

The sizes and shapes of the craters that are formed by the impacts are presented in the first group of curves since the crater is doubtlessly the most prominent feature of the impact. The crater diagrams show the direction and magnitude of the velocity of flow; the interface between the rock and the aluminum; the amount of rock in the crater; as well as the volume and the position of the fluid, plastic, and elastic regions of the target. The size of the impacting sphere is shown to scale in some figures in order to illustrate the relative size of the sphere and crater. The cross-hatched region in each figure represents the volume that is filled with rock. The position of the free surface of the aluminum target is indicated by the letter S. The material velocity is represented by arrows interspersed through the shocked region. The length of the arrow shows the velocity of flow to the scale that is shown on each figure. An arrowhead with no tail means that the material velocity at that point is much smaller than the velocity scale that is placed on the figure. The labeled dots along the z-axis are spaced at intervals of one, or two radii of the impacting sphere, as is designated on each figure. The shock front is drawn as a dashed line.

The crater solutions are arranged in the order of increasing impact velocity. The solutions for low impact velocities are of particular interest since they are in the range which permit comparison with experimental studies. The formation of the craters for an impact velocity of 6.25 kilometers per second are presented in Figures 19 to 21, and those for an impact velocity of 7.5 kilometers per second are presented in Figures 22 to 25. These craters are discussed as a group since they are just above the transition zone from high velocity to hypervelocity impact. True hypervelocity impact is usually not achieved until the impact velocity is more than twice the speed of sound in the impacting target. For aluminum, this would require a velocity that is greater than 10 kilometers per second.

The mass of micrometeoroids is defined to be less than 10^{-4} grams. The mass of the impacting spheres in these two solutions and in most subsequent solutions was selected to be 10^{-9} grams. The radius of a sphere of this mass is of the order of 10^{-4} centimeters. The magnitude of this diameter is very significant in order to appreciate the small time increments between each stage of the crater formation. The times are of the order of nanoseconds. The lowest velocity sphere (6.25 km/sec) will require less than two nanoseconds to travel a distance equal to its diameter when in free flight. The very small diameter explains the very short time for the early stages of the cratering. Most solutions in the literature are for larger projectiles which have masses of several grams, and the cratering times are of the order of microseconds.

These computer solutions for the formation of craters may be compared with experimental studies. One research group has studied the impact of aluminum projectiles on an aluminum target where the impact

velocity was 7.32 kilometers per second (34). The depths of the craters formed in these impacts averaged $2\frac{1}{2}$ times larger than the radius of the impacting projectile. This crater size lies between the crater sizes calculated for 6.25 and 7.5 kilometers per second impacts, and this suggests that the numerical solutions are good. Kinslow has also studied hypervelocity impact on aluminum (15). At an impact velocity of 25,000 feet per second, the crater depth was slightly less than three projectile radii. This result agrees well with the computer calculations. In both of these studies, the aluminum projectiles had a density 25% greater than the density of the rock sphere used in this study.

The shock front propagates at approximately the same velocity as the velocity of the sphere until the sphere has penetrated a distance about equal to one sphere radius. At this point, the shock front separates from the rock-aluminum interface and steadily moves away from the slower traveling rock material. This behavior is illustrated in Figures 19 and 22. When the separation between the shock wave and the rock-aluminum interface is about R (R is the radius of the incident sphere), there is a distinct movement of material toward the periphery of the crater. Figures 20 and 23 illustrate this movement. This movement is the initiation of the ejecta and the resulting crater lip. Practically all of the material behind the shock front is in the fluid state.

The shock wave travels slower along the free surface where the pressure is zero, and this results in the shocked region assuming the shape of a pear. This shape occurred for all of the impact solutions that were obtained. The shaded regions in the figures represent the

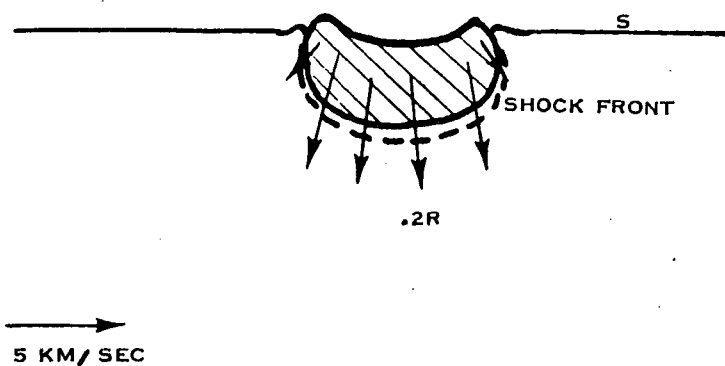


Figure 19. Crater at 7.9×10^{-10} second for sphere
of density 2.0 gm/cc impacting at
 6.25 km/sec .

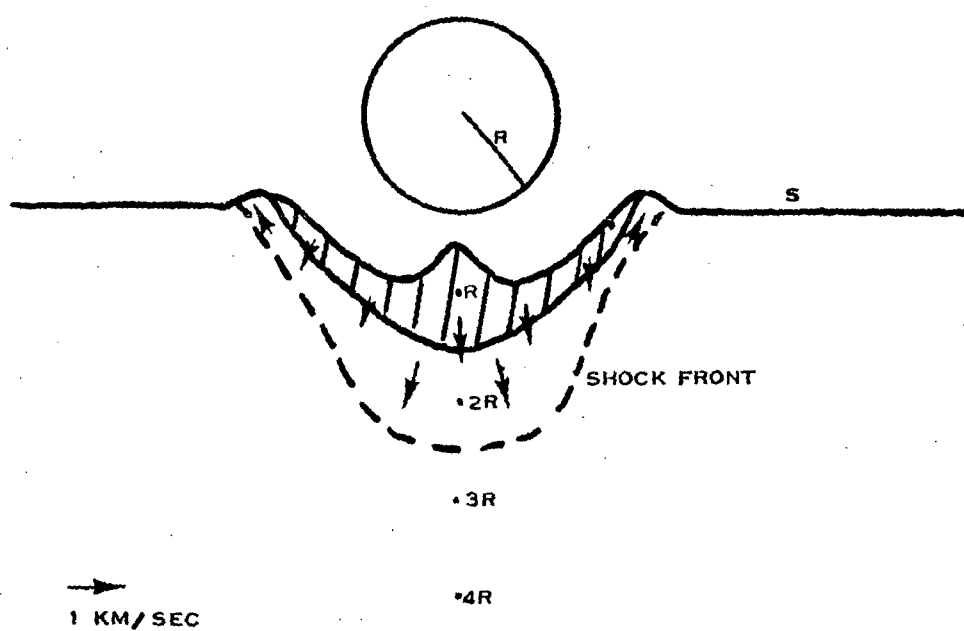


Figure 20. Crater at 1.6×10^{-9} second for sphere of density 2.0 gm/cc impacting at 6.25 km/sec .

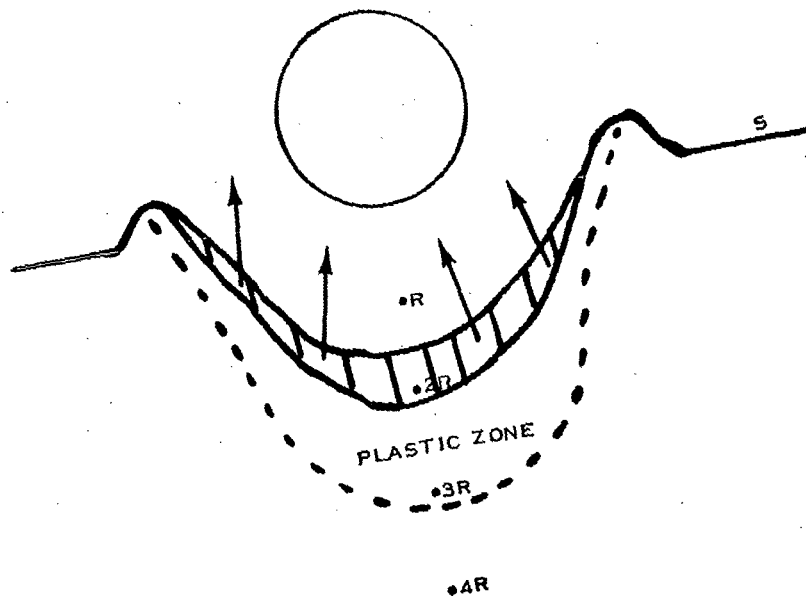
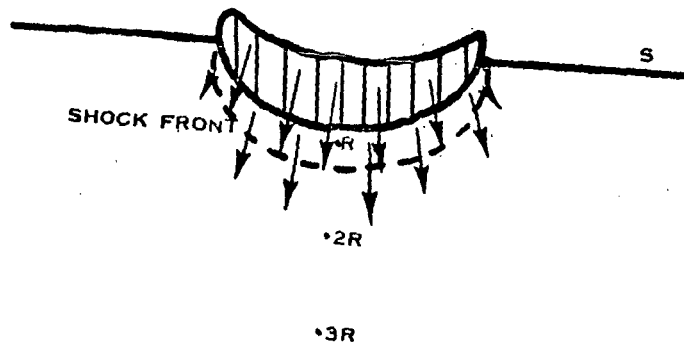


Figure 21. Crater at 2.5×10^{-9} second for sphere of density 2.0 gm/cc impacting at 6.25 km/sec .



→
3.5 KM/SEC

Figure 22. Crater at 6.5×10^{-10} second for sphere of density 2.0 gm/cc impacting at 7.5 km/sec .

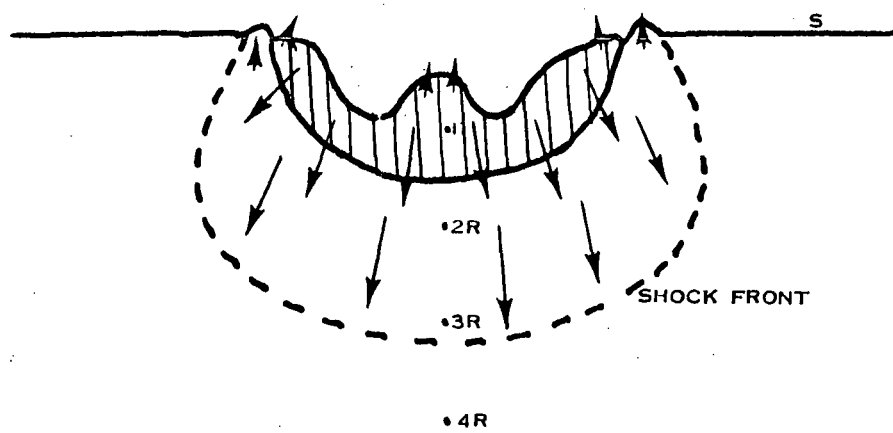


Figure 23. Crater at 1.9×10^{-9} second for sphere of density 2.0 gm/cc impacting at 7.5 km/sec .

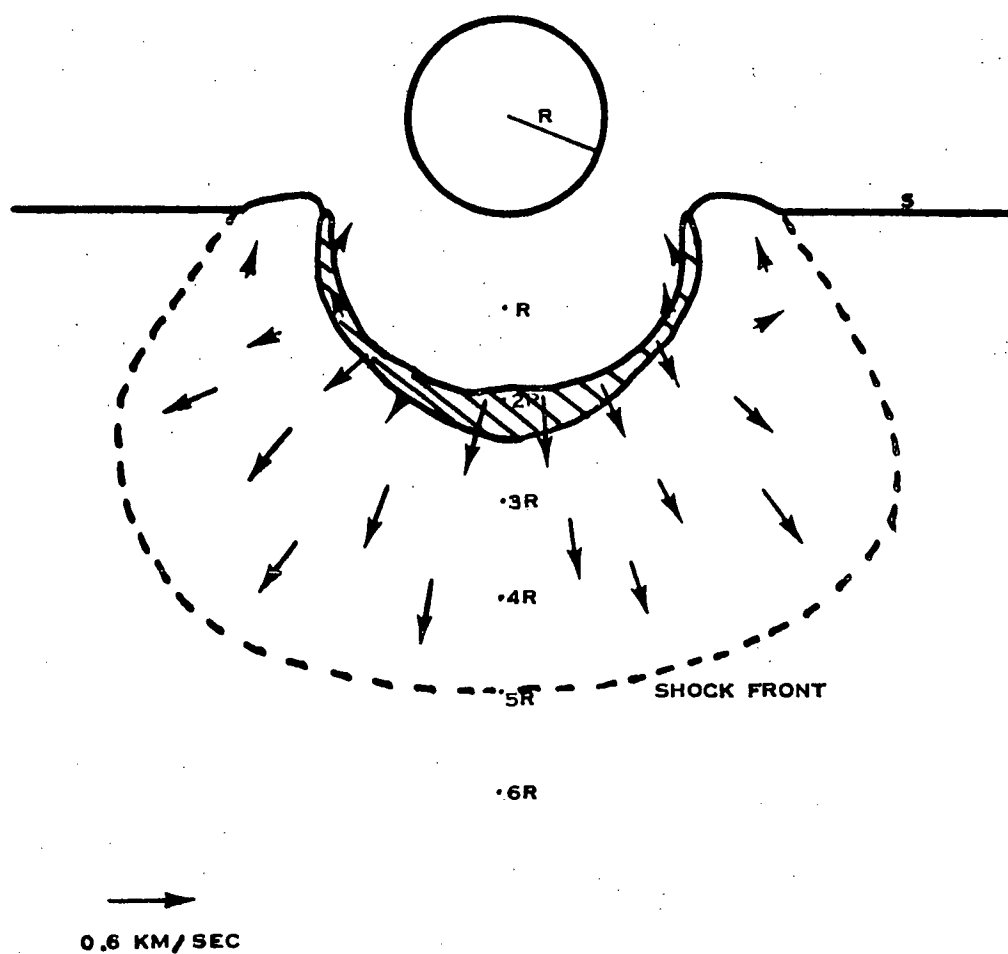
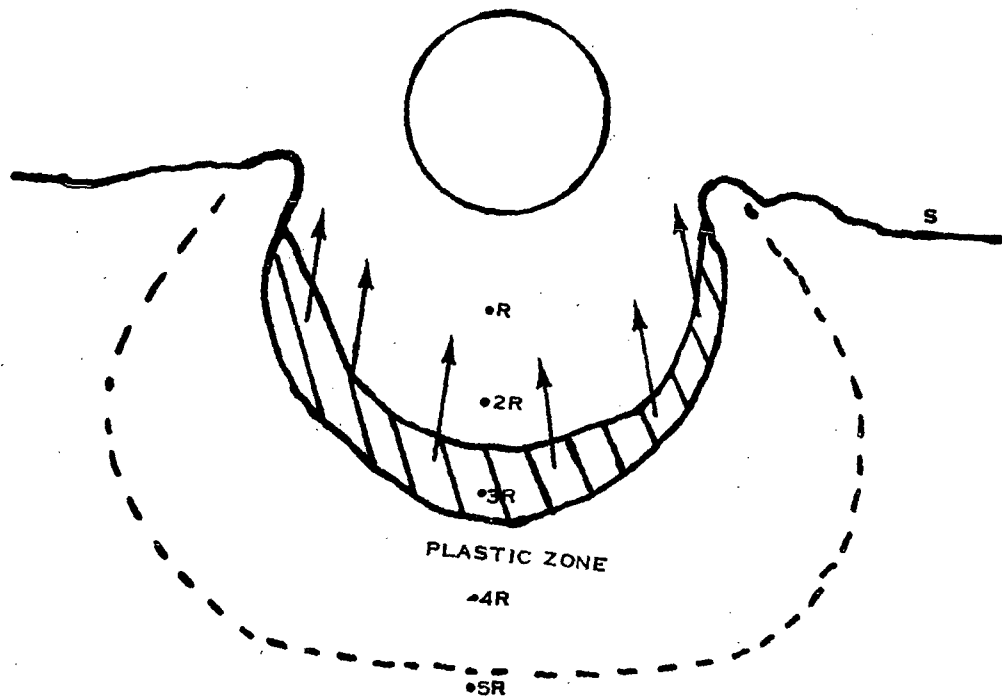


Figure 24. Crater at 3.0×10^{-9} second for sphere of density 2.0 gm/cc impacting at 7.5 km/sec .



→
0,2 KM/SEC

Figure 25. Crater at 4.7×10^{-9} second for sphere of density 2.0 gm/cc impacting at 7.5 km/sec .

volume occupied by rock. After the sphere penetrates a short distance into the target, the rock material becomes fluid, and the majority of it converts to a plasma.

The crater profiles in the last figure of the solution for each set of parameters shows a region which is labeled the "plastic zone". The boundary of this region that is nearer the target surface was determined with the following instructions to the computer. The computer was programmed to monitor the peak pressure of the propagating shock wave in the inviscid fluid until the peak pressure decreased to 137 kb. At that time, the energy in each cell was investigated and compared to the energy which is required to melt aluminum at zero pressure. The fluid-plastic boundary was placed on the line which separated the molten from the solid aluminum, as determined by the preceding instructions. This position is always in the region which has been over-run by the shock with a pressure in excess of 137 kb. The boundary between the plastic and the elastic regions was the position at which the pressure of the shock front decreases below 5.9 kb.

There is considerable uncertainty in the indicated procedure but this is a first approximation. A solution with this approximation serves to indicate the nature of the difficulties that are involved and to compare the results with experiment. The energy content to melt aluminum is probably somewhat dependent on the pressure (6). Available references are not very clear. Measurements on gallium show a 27.4 °C decrease in the melting point with a 12 kb increase in the pressure (40). In the form of the periodic table that shows the long periods, the elements from top to bottom in the same group are B, Al, Ga, In and Tl.

Between the two boundaries that define the plastic region, the shock wave has a dual wave structure which consists of an elastic precursor wave that travels at the speed of sound in the elastic region and this wave is followed by a slower plastic wave. The viscosity terms were included in the equations of motion in this region. The total stress in this region and in the elastic region is defined as the quantity, τ_{ij} , which is given by

$$\tau_{ij} = -P\delta_{ij} + S_{ij}$$

where P is the hydrodynamic pressure which is calculated from an equation of state, and S_{ij} is the stress from Equation 5 in Chapter II.

The difference between these solutions and those in the literature is in the location and the size of the plastic region. Very few hyper-velocity impact solutions have been published which attempt to reveal the elastic-plastic conditions at long solution times. Previous solutions by Wagner, et al., and by Kinslow ignore the existence of any plastic material and place the elastic region immediately below the crater (38, 15). Riney has published a visco-plastic solution, but the solution time is too short to reveal the extent of the plastic region (30). His solution was run only 10 time cycles, and the impacting body entered only half way into the target. The elastic-plastic solution of these low velocity impacts may be compared with experiment. A metallographic study of a crater which was supplied by Souilly has been prepared. A picture of the crater was published by Sodek (33). The plastic region appears to be considerably thinner than is obtained from the solutions in Figures 21 and 25. More experimental work should be performed to accurately define the location and extent of the plastic

deformation region.

The comments and descriptions of these low velocity impacts also apply to the remaining impact solutions. The crater solutions for these solutions are shown as Figures 26 to 38. In each impact solution, the material behind the shock front, shortly after impact, is in a fluid state except for the last figure in each solution. These figures are 29, 33, and 38. A good illustration of the violent ejection of the fluid material from the crater lip is shown in Figures 27 and 28. A particularly large amount of ejecta is shown in Figure 37. The dashed line in this latter figure marks the boundary of the ejecta and the position at which the free surface of the crater will be after the material is ejected. The conditions to release the ejecta are presented in a following section on ejecta.

The last figure in each sequence shows the elastic and plastic regions. The final crater surface will lie at the shallower boundary of the plastic region because the melted material above this region will be forced out of the target.

The effects of the velocity and the density of the impacting body on the crater parameters is summarized in Table IV. The symbol R in Table IV indicates the radius of the impacting sphere.

Effect of Sphere Diameter on the Crater

In the previous section, the effects are summarized for the velocity and the density of the impacting body. One effect has not been examined, and that effect is the diameter of the impacting sphere. The concept of late stage equivalence, which is the foundation of several proposed scaling laws, is based on the assumption that the influence of

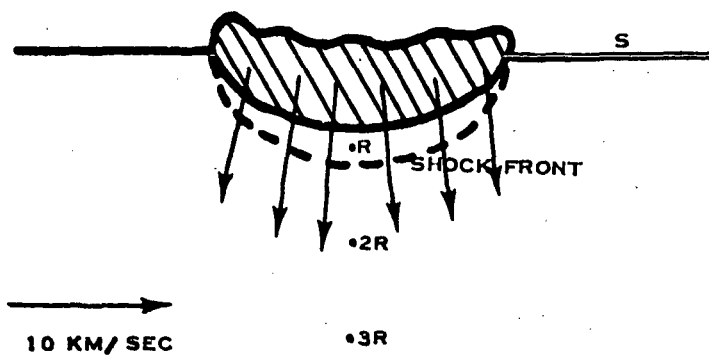


Figure 26. Crater at 2.6×10^{-10} second for sphere of density 2.0 gm/cc impacting at 20 km/sec .

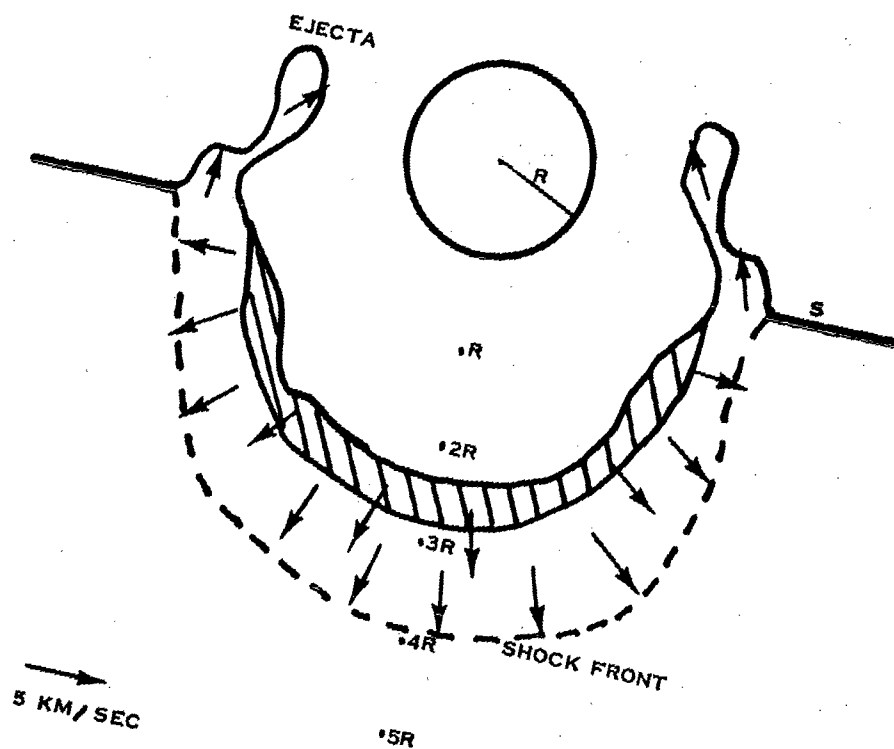


Figure 27. Crater at 1.3×10^{-9} second for sphere of density 2.0 gm/cc impacting at 20 km/sec .

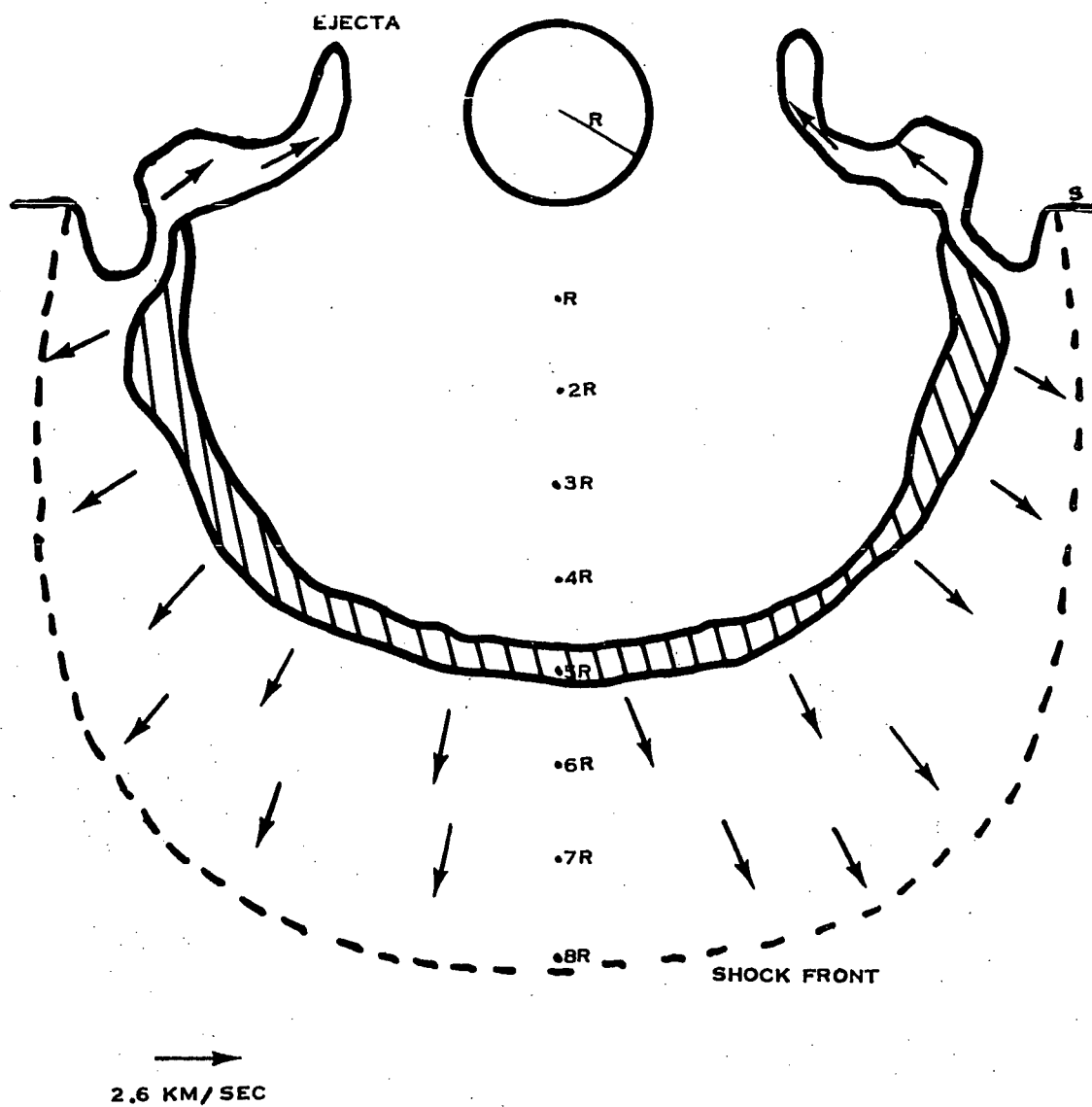
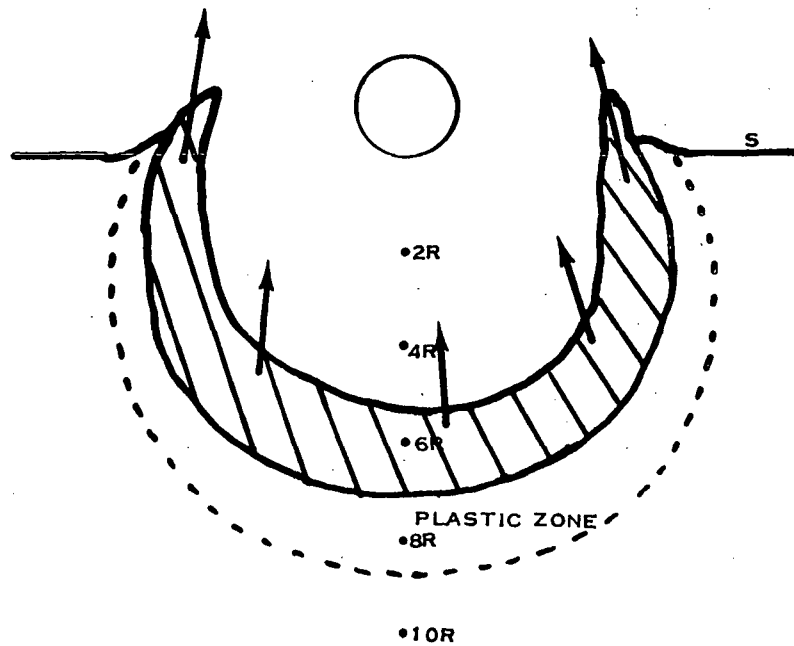
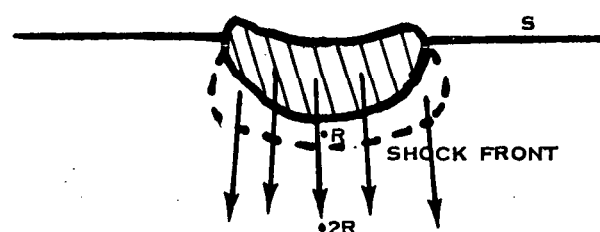


Figure 28. Crater at 3.6×10^{-9} second for sphere of density 2.0 gm/cc impacting at 20 km/sec .



→
0.1 KM/SEC

Figure 29. Crater at 6.5×10^{-9} second for sphere of density 2.0 gm/cc impacting at 20 km/sec .



→
10 KM/ SEC

Figure 30. Crater at 4.0×10^{-10} second for sphere
of density 0.5 gm/cc impacting at
20 km/sec.

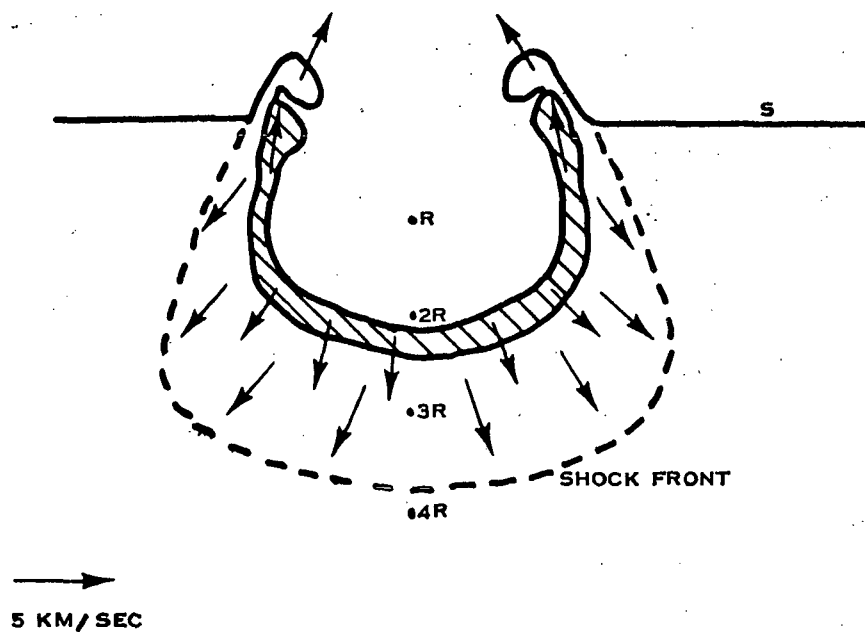


Figure 31. Crater at 1.1×10^{-9} second for sphere of density 0.5 gm/cc impacting at 20 km/sec.

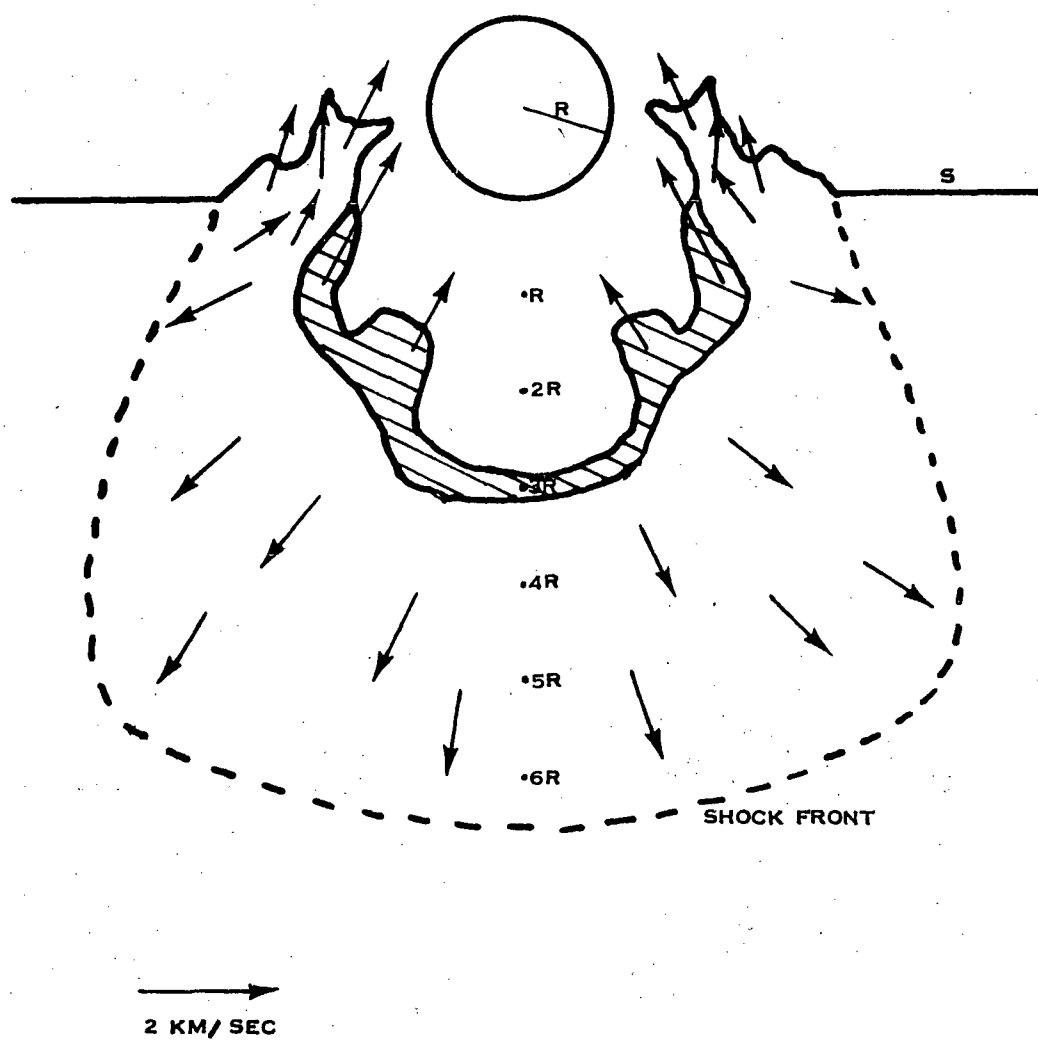


Figure 32. Crater at 4.9×10^{-9} second for sphere of density 0.5 gm/cc impacting at 20 km/sec .

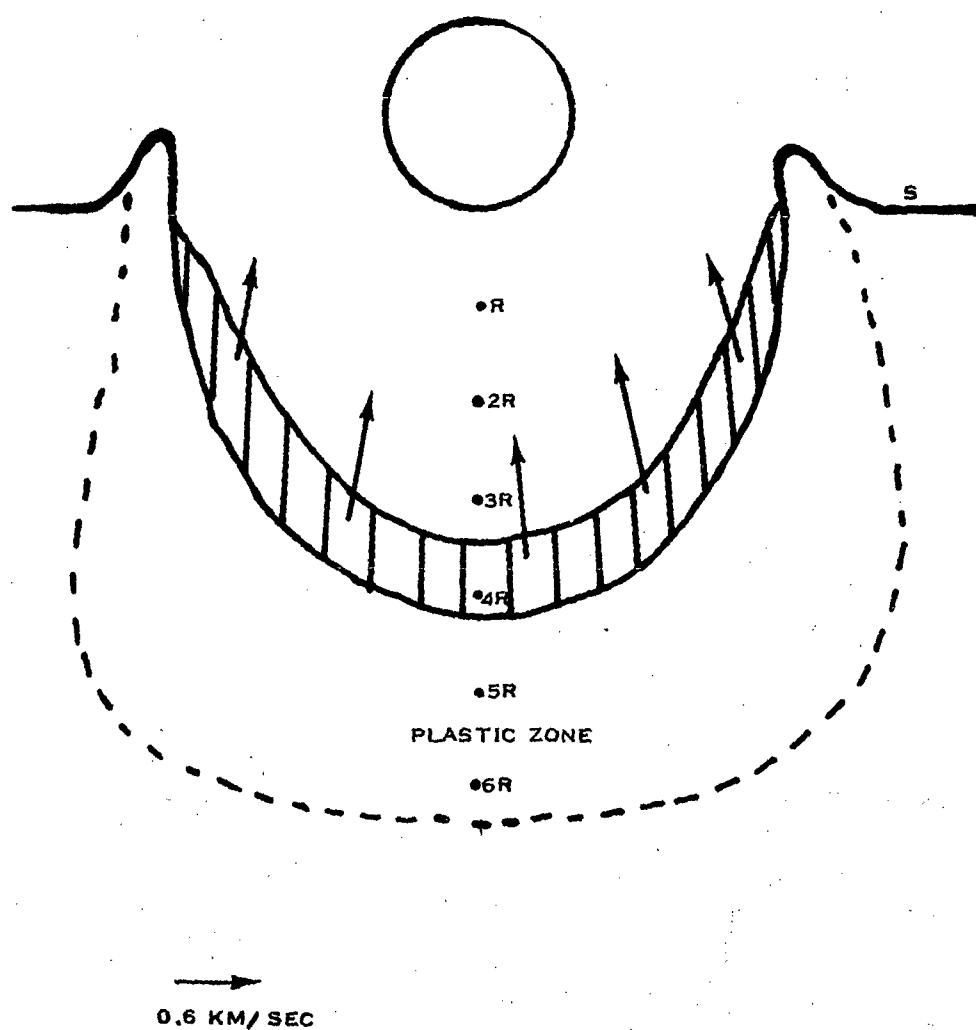


Figure 33. Crater at 6.3×10^{-9} second for sphere of density 0.5 gm/cc impacting at 20 km/sec .

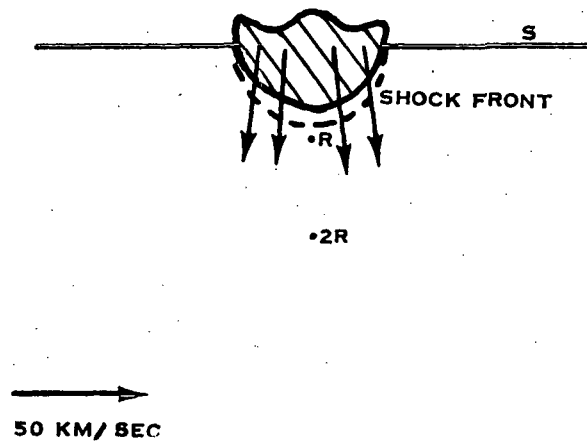


Figure 34. Crater at 1.2×10^{-10} second for sphere
of density 0.5 gm/cc impacting at
 72 km/sec .

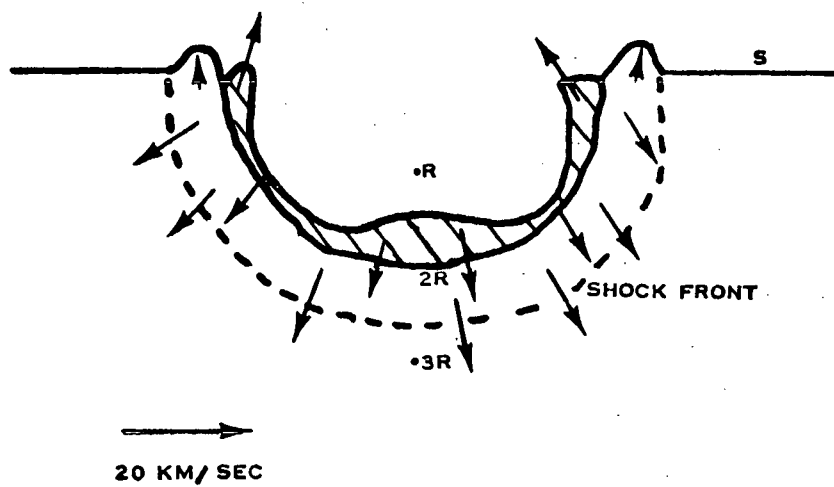


Figure 35. Crater at 9.3×10^{-10} second for sphere of density 0.5 gm/cc impacting at 72 km/sec .

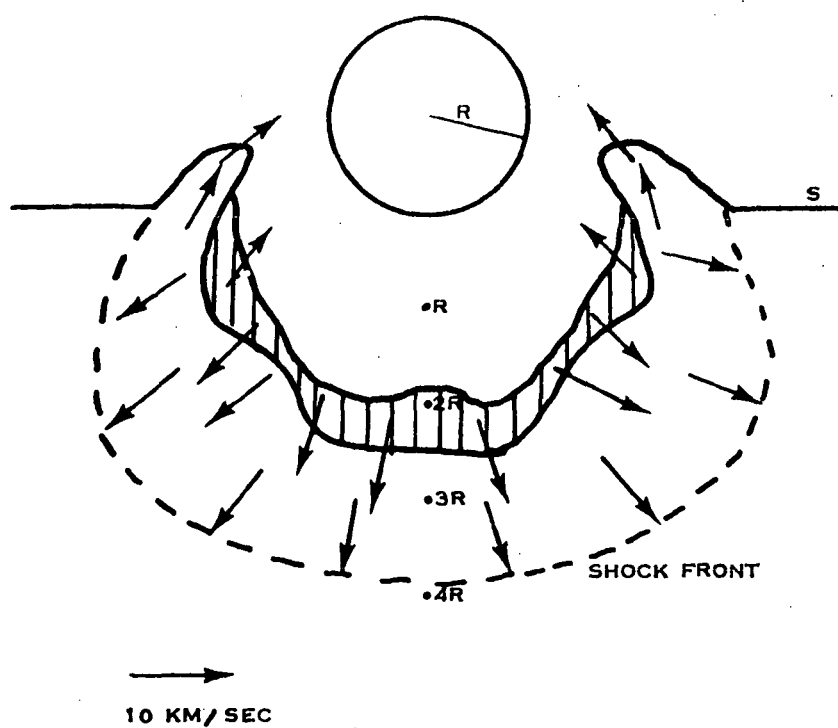


Figure 36. Crater at 1.5×10^{-9} second for sphere of density 0.5 gm/cc impacting at 72 km/sec .

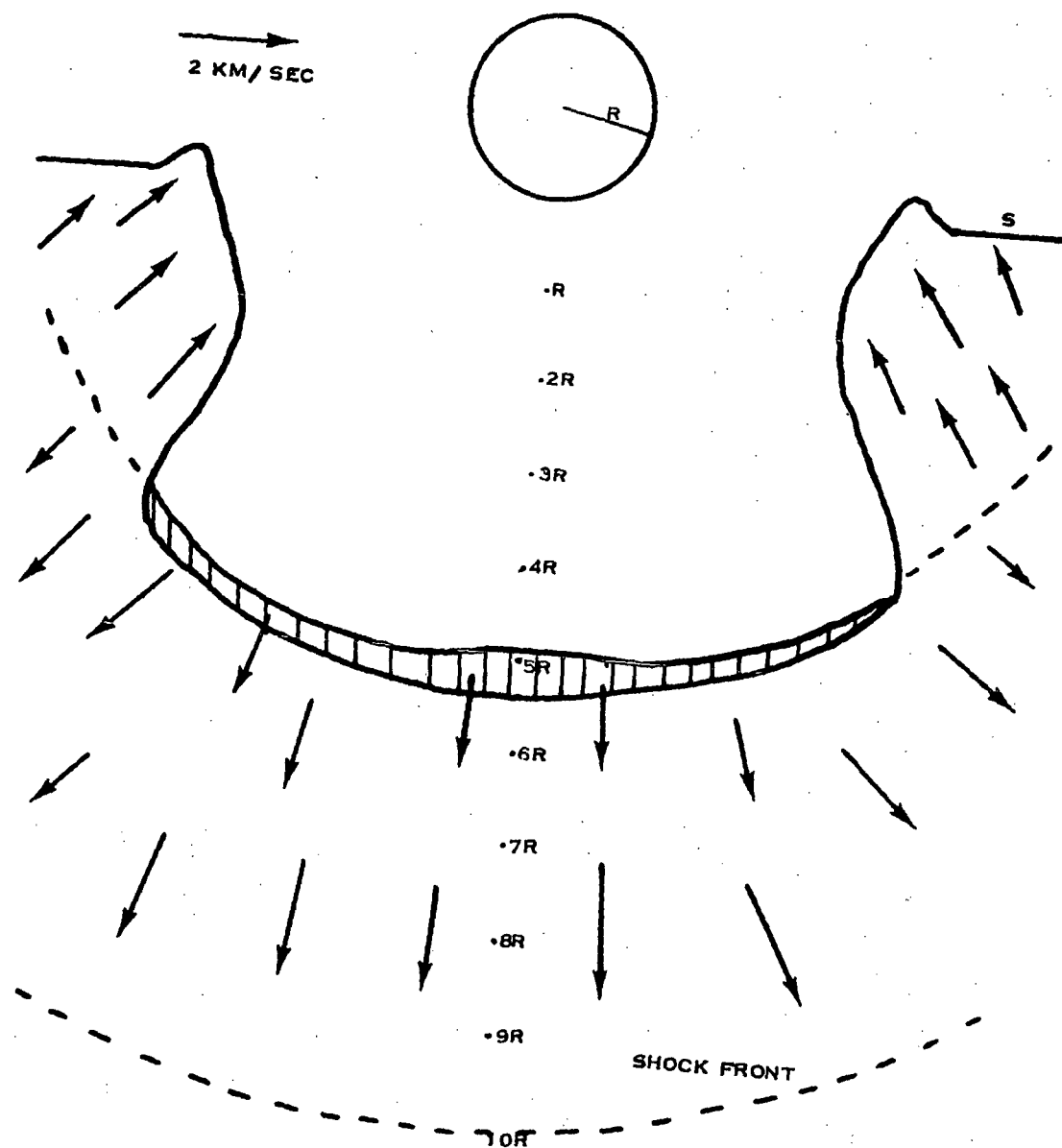


Figure 37. Crater at 5.9×10^{-9} second for sphere of density 0.5 gm/cc impacting at 72 km/sec .

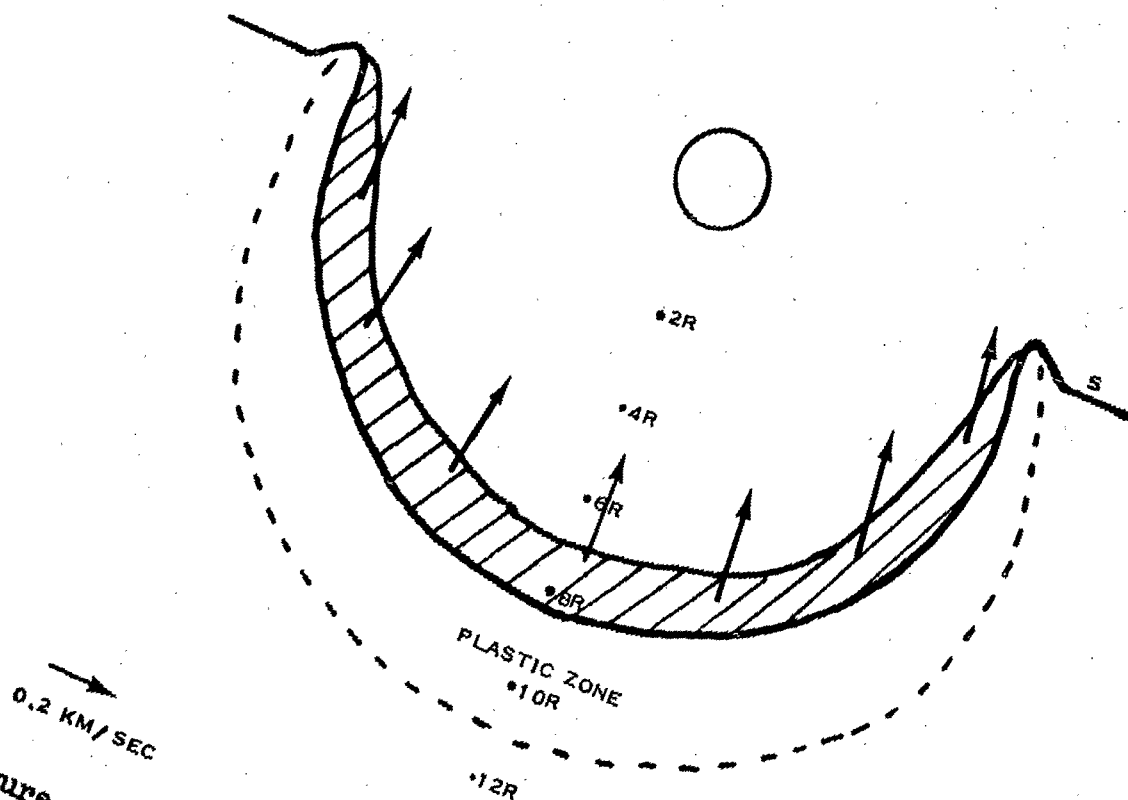


Figure 38. Crater at 1.2×10^{-8} second for sphere of density 0.5 gm/cc impacting at 72 km/sec.

material viscosity and strength effects on crater dimensions scales with size. The verification of this assumption is virtually impossible by experiment. This statement follows since late stage equivalence is asserted to be valid only at impact velocities above $2C$, where C is the velocity of sound in the target. Experimental data is difficult to obtain for most metals at such velocities.

TABLE IV
CRATER PARAMETERS RESULTING FROM IMPACTS

Velocity	Density	Depth of Crater	Width of Plastic Zone
6.25 km/sec	2.0 gm/sec	2.1 R	R
7.5 km/sec	2.0 gm/sec	3.3 R	1.5 R
20.0 km/sec	2.0 gm/sec	7.1 R	1.8 R
20.0 km/sec	0.5 gm/sec	4.3 R	2.1 R
72.0 km/sec	0.5 gm/sec	8.3 R	2.4 R

Several research groups have proposed simple scaling laws. Riney and Heyda state that the flow fields are identical for bodies of different size provided the kinetic energy of the impacting body is held constant, and that the density of the impacting body has no effect on the late stages of the impact process if the mass of the body does not change (31). Their scaling law proposes that the crater depth is proportional to the $2/3$ power of the impact velocity. Bjork's recent paper refutes the idea that a simple, density scaling law of the form $K(\rho/\rho_T)^B$ can suitably account for the dependence of penetration on density (38). The work of other groups could be cited, but their scaling laws would only be another form of the equation $k_1(V/C)^{2.1}$.

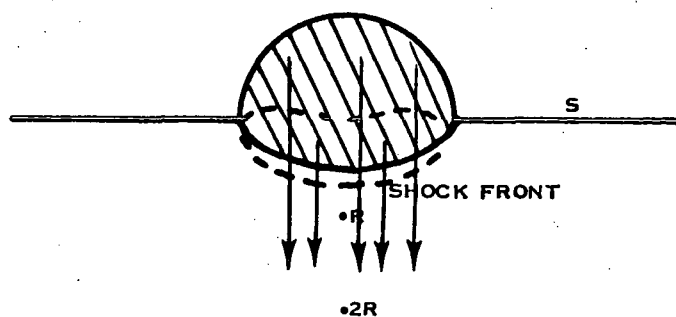
or $K_i(\rho_p/\rho_t)^{\beta_i}$. These scaling laws are based on the concept that the mechanical coupling between the projectile and the target arises only through the agency of a shock. The scaling assumptions do not account for the increase in input momentum from the effect of the ejected material.

The fundamental fact, on which the investigators in this area agree, is that there is some type of power dependence between crater penetration and impact velocity. If this assumption is true, then the crater created by a sphere of radius $1.5R$ should be 1.5 times deeper than a crater created by a sphere of radius R . To test this assumption, a solution was sought for a sphere of density 0.5 gram/centimeter and a radius 11.7×10^{-4} centimeter which impacted at 20 kilometers per second. The crater solutions are shown in Figures 39 to 42. These solutions should be compared with Figures 30 to 33, which are the solutions for a sphere that is identical except for a radius of 7.8×10^{-4} centimeter.

The first figure in the series was chosen to illustrate the shock wave in the rock sphere. This wave is present in the preceding impacts but was not illustrated. It never moves much farther away from the original aluminum surface than the position in Figure 39.

The development of the crater does not differ from any which has been reported. The last stage of the crater shows an interesting picture of the ejecta. The previous solutions show a thin layer of rock vapor lining the crater. In Figure 42, this rock lining has been broken, and a considerable quantity of aluminum vapor is ejecting through this rupture in the rock lining.

The comparison of the parameters of this crater with the crater



→
10 KM/ SEC

Figure 39. Crater at 4.5×10^{-10} second for sphere of density 0.5 gm/cc impacting at 20 km/sec .

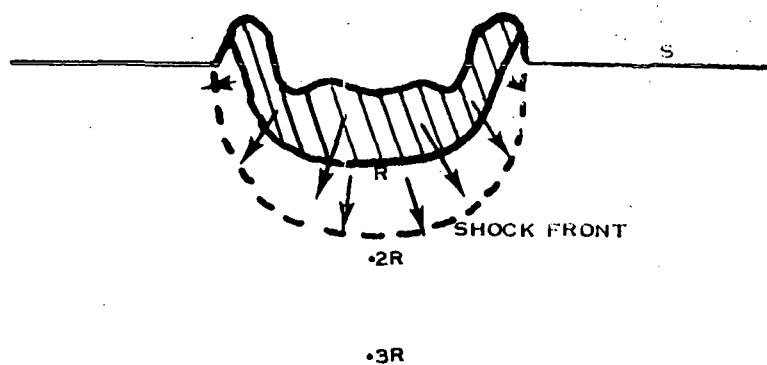


Figure 40. Crater at 2.5×10^{-9} second for sphere of density 0.5 gm/cc impacting at 20 km/sec.

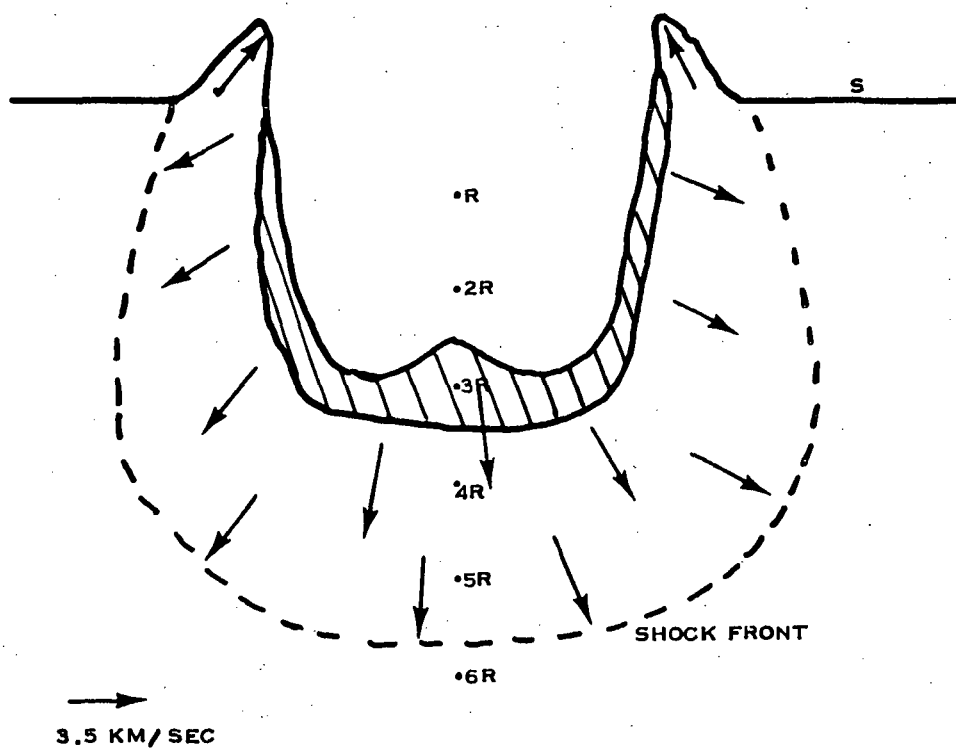


Figure 41. Crater at 1.0×10^{-8} second for sphere of density 0.5 gm/cc impacting at 20 km/sec.

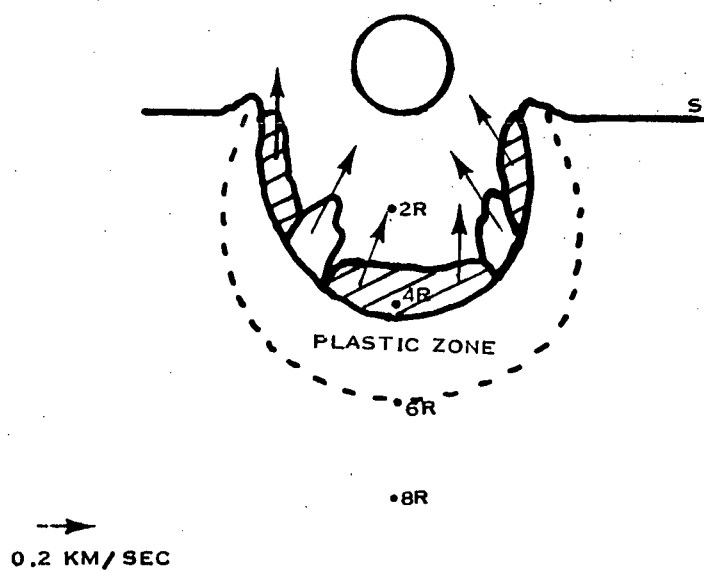


Figure 42. Crater at 1.9×10^{-8} second for sphere of density 0.5 gm/cc impacting at 20 km/sec .

from the impact of the smaller sphere is summarized in Table V. All of the crater parameters agree quite well; therefore, the crater size scales linearly or practically linearly with projectile size in this instance. More solutions are required to prove that the dependence is linear when the diameter is varied further.

TABLE V

CRATER PARAMETERS CREATED BY SPHERES OF RADIUS R AND RADIUS $3/2 R$

Radius of Sphere	Depth of Crater	Width of Plastic Zone
R	$4.3 R$	$2.1 R$
$3/2 R$	$4.2 R$	$1.8 R$

Pressure Profiles

Certain features of the impact process are best illustrated by examining the distribution of pressure behind the shock front. The pressure profiles emphasize the violent churning and mixing of the highly compressed material, and they also show the existence of a dual wave structure in the plastic zone. Neither of these phenomena are revealed in detail in the crater diagrams. The pressure profiles created by the impacts are shown in Figures 43 to 66. The letter, S, in the figures indicates the original position of the aluminum surface; the letter R represents a distance equal to one sphere radius.

The profiles are plotted for a fixed value of θ which is $7\frac{1}{2}^\circ$ away from the Z axis. The churning and mixing of the shocked material is indicated in these figures by the change in amplitude of the pressure profiles. Specific examples are shown in Figures 46, 48, 53, 61, 65,

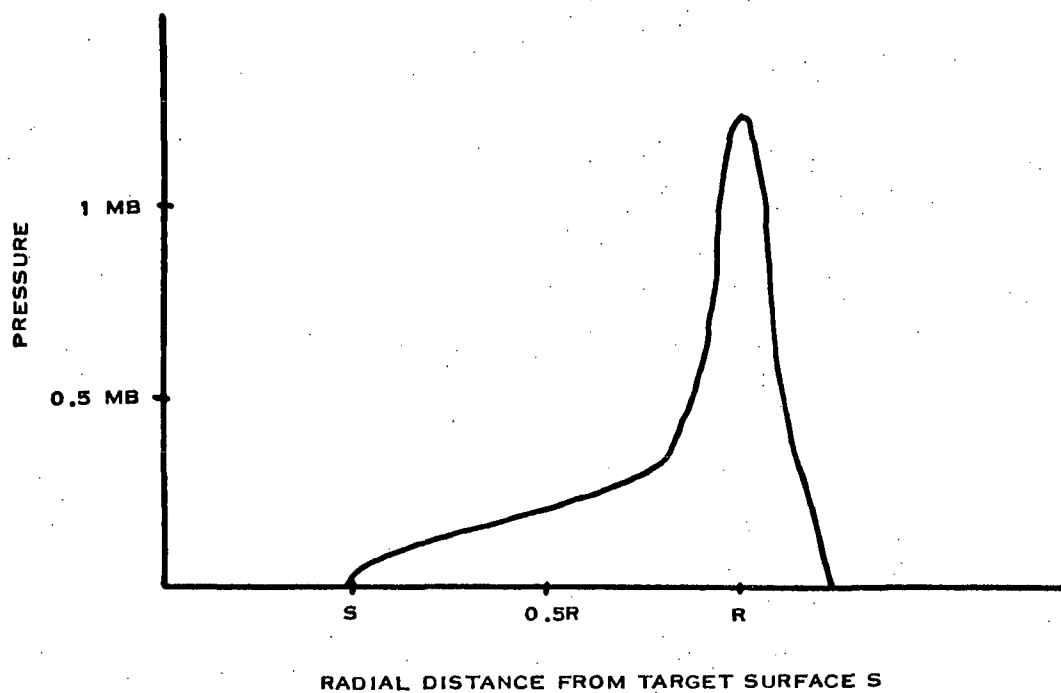


Figure 43. Pressure profile at 7.9×10^{-10} second
for sphere of density 2.0 gm/cc
impacting at 6.25 km/sec .

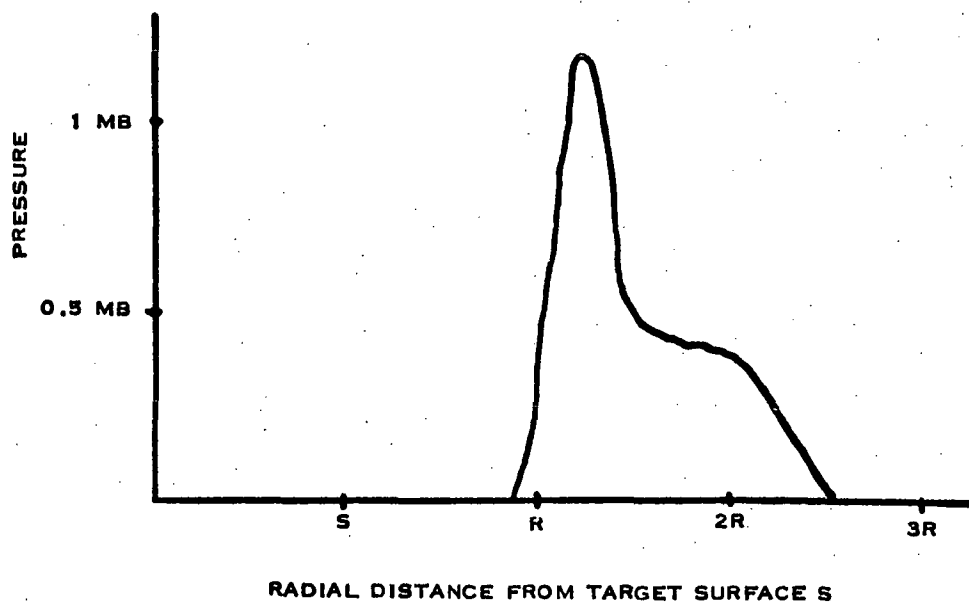


Figure 14. Pressure profile at 1.6×10^{-9} second for sphere of density 2.0 gm/cc impacting at 6.25 km/sec .

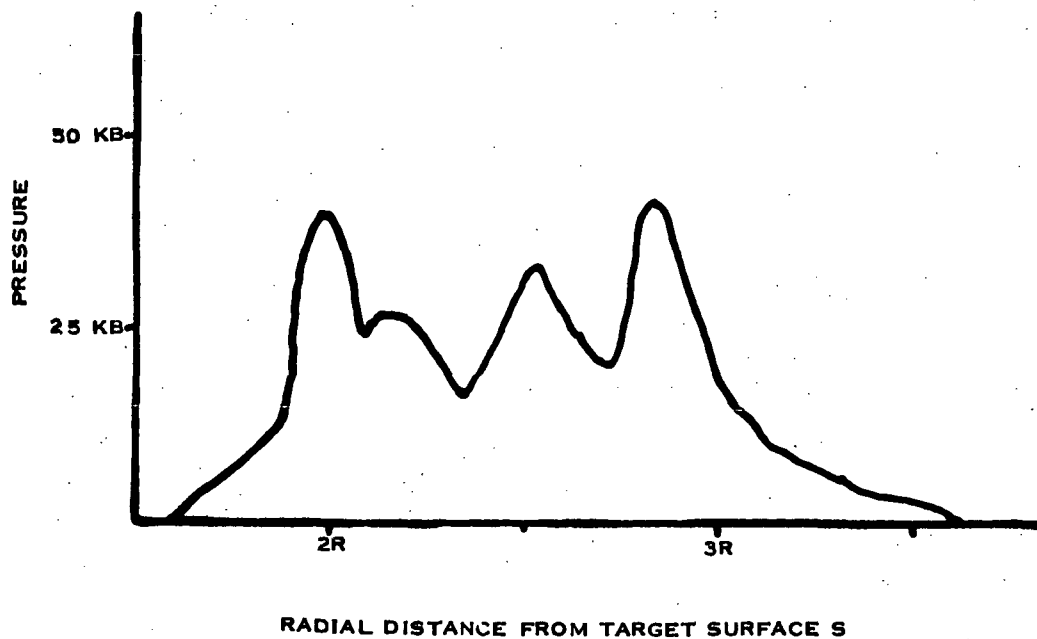


Figure 45. Pressure profile at 2.5×10^{-9} second
for sphere of density 2.0 gm/cc
impacting at 6.25 km/sec.

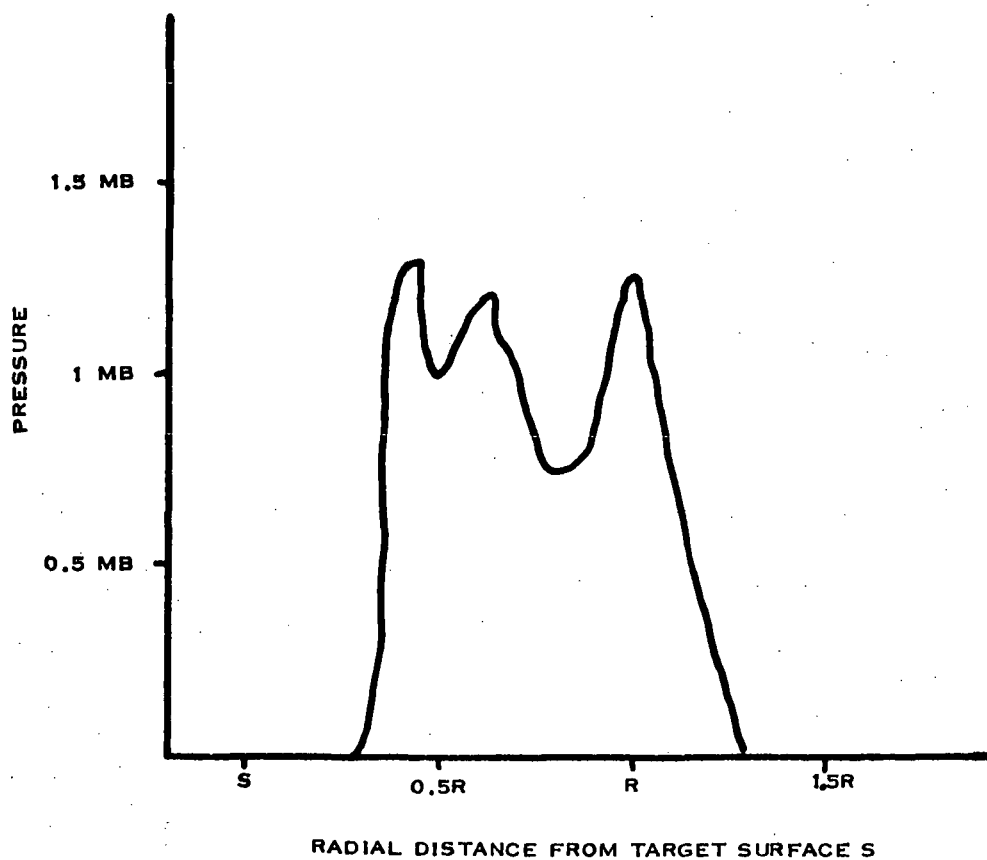


Figure 46. Pressure profile at 6.5×10^{-10} second
for sphere of density 2.0 gm/cc
impacting at 7.5 km/sec.

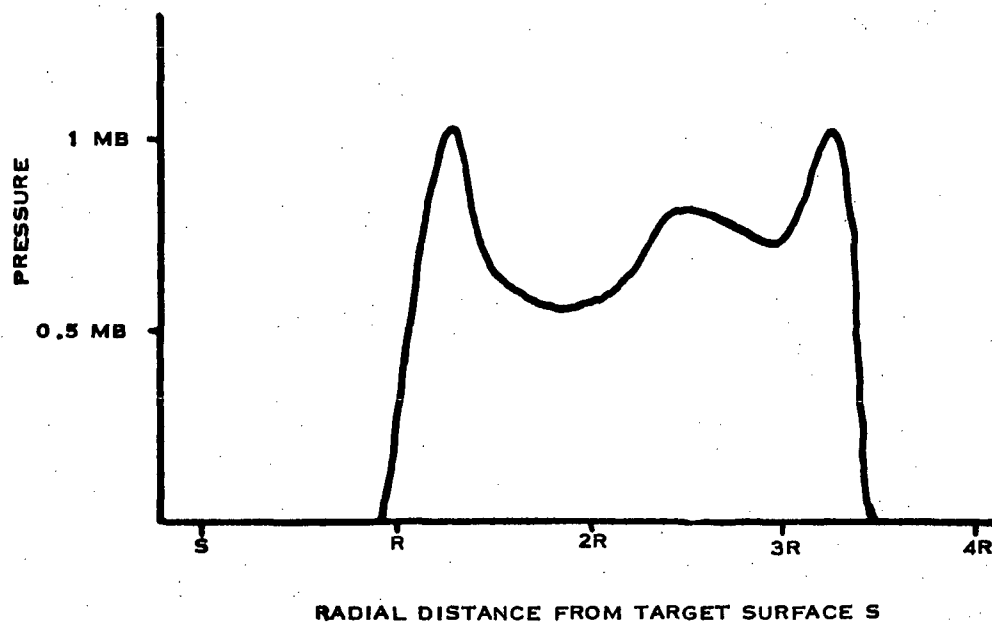


Figure 47. Pressure profile at 1.9×10^{-9} second
for sphere of density 2.0 gm/cc
impacting at 7.5 km/sec .

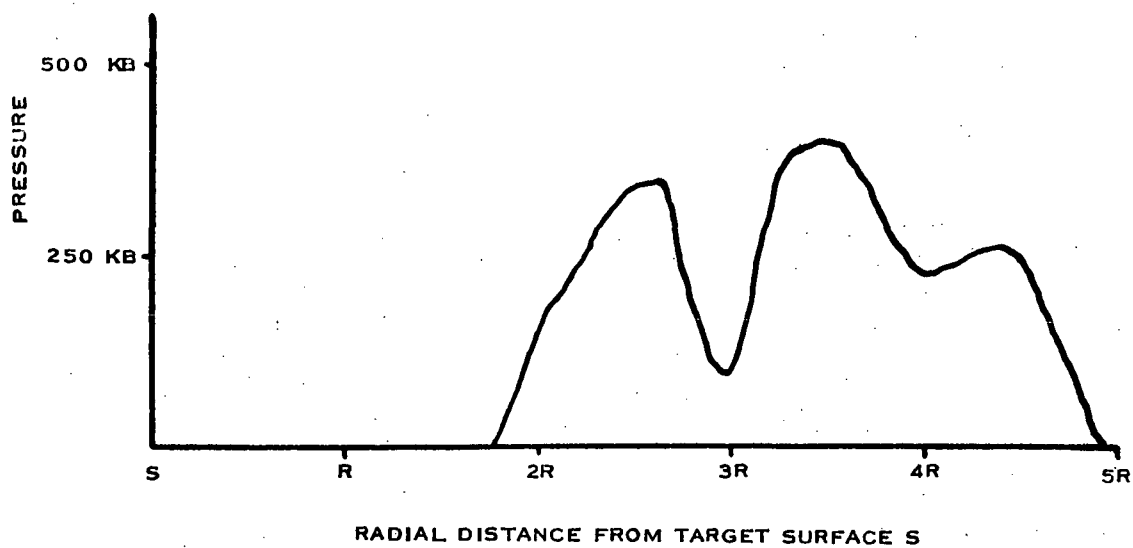


Figure 48. Pressure profile at 3.0×10^{-9} second
for sphere of density 2.0 gm/cc
impacting at 7.5 km/sec .

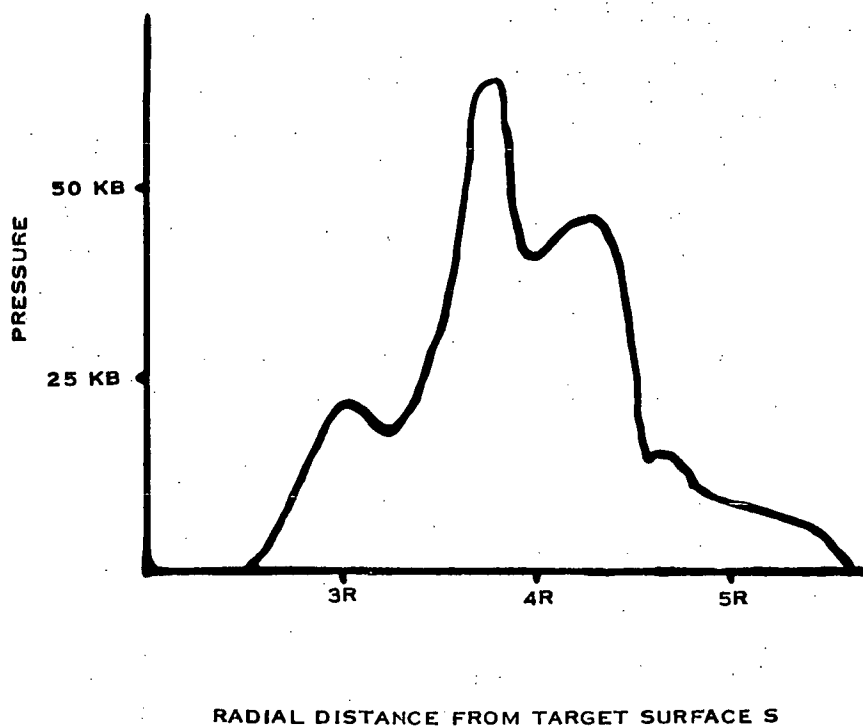


Figure 49. Pressure profile at 4.7×10^{-9} second
for sphere of density 2.0 gm/cc
impacting at 7.5 km/sec .

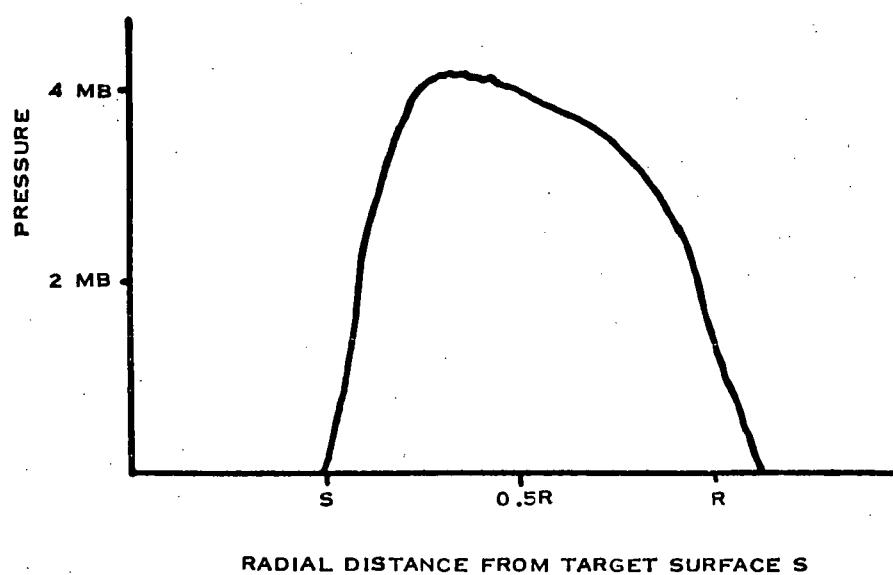


Figure 50. Pressure profile at 2.6×10^{-10} second
for sphere of density 2.0 gm/cc
impacting at 20 km/sec .

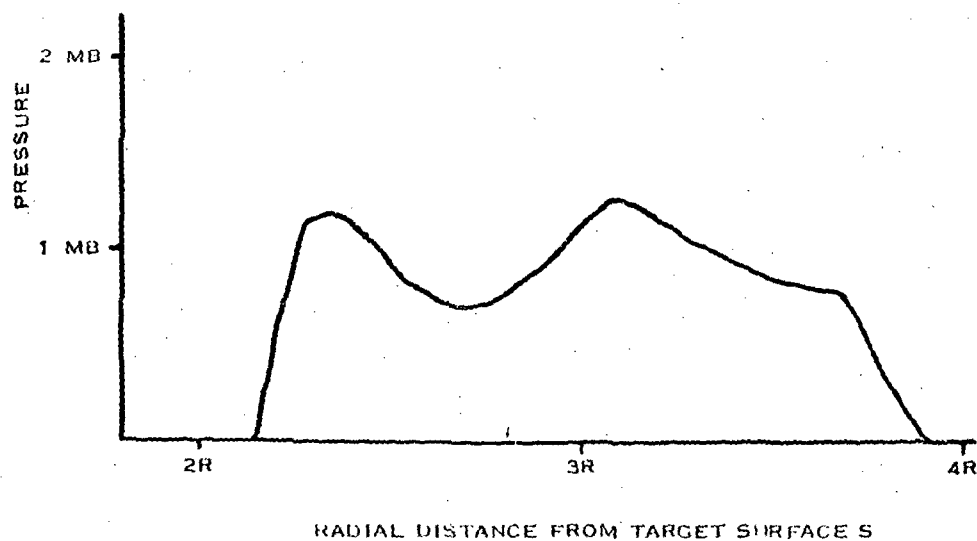


Figure 51. Pressure profile at 1.3×10^{-9} second
for sphere of density 2.0 gm/cc
impacting at 20 km/sec .

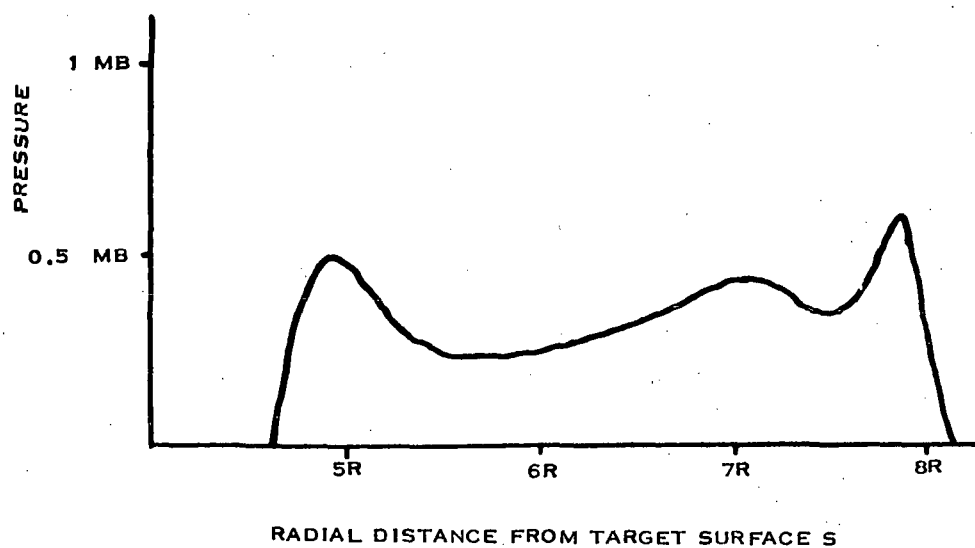


Figure 52. Pressure profile at 3.6×10^{-9} second
for sphere of density 2.0 gm/cc
impacting at 20 km/sec .

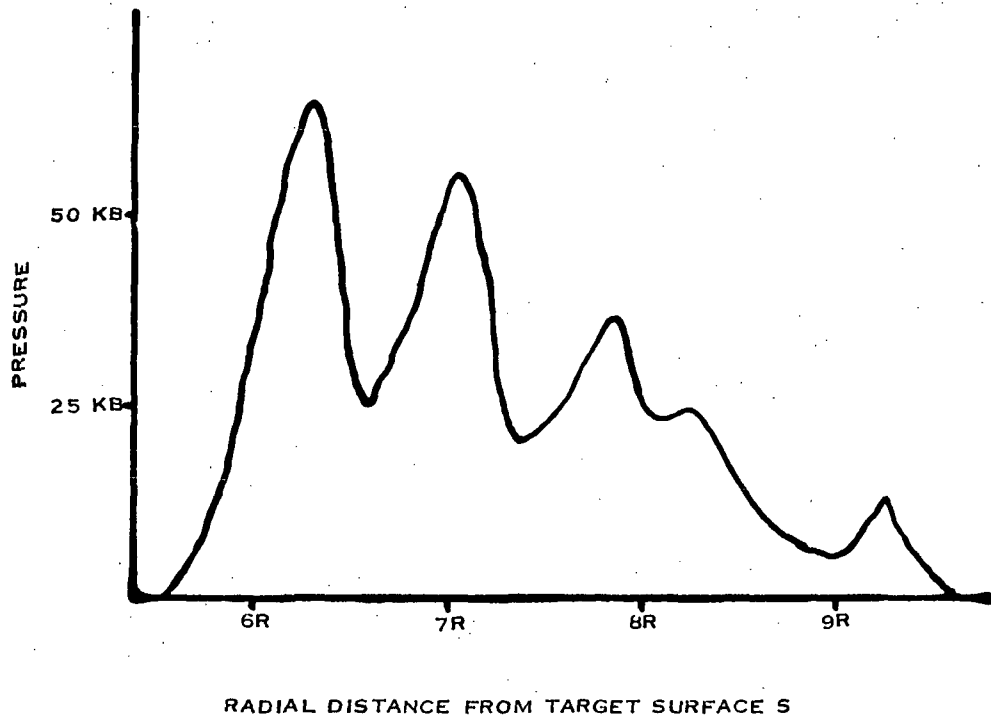


Figure 53. Pressure profile at 6.5×10^{-9} second
for sphere of density 2.0 gm/cc
impacting at 20 km/sec.

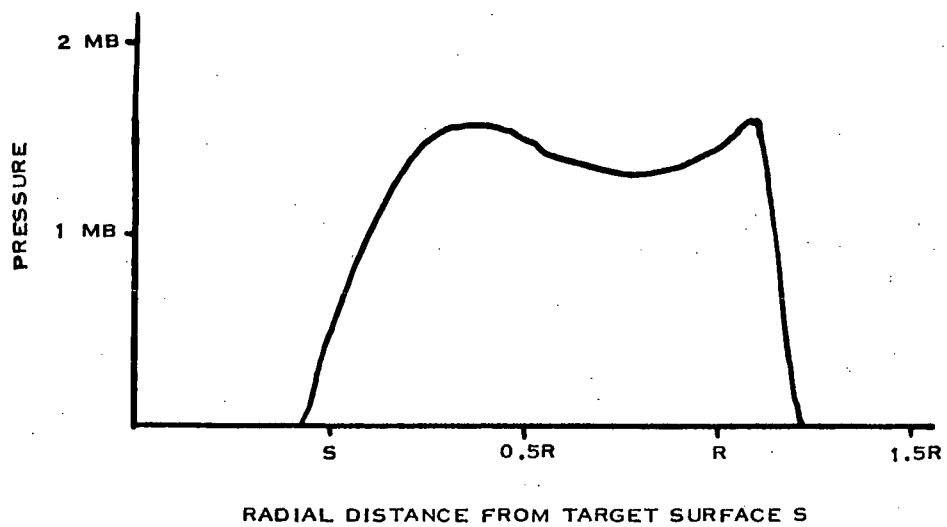


Figure 54. Pressure profile at 4.0×10^{-10} second
for sphere of density 0.5 gm/cc
impacting at 20 km/sec.

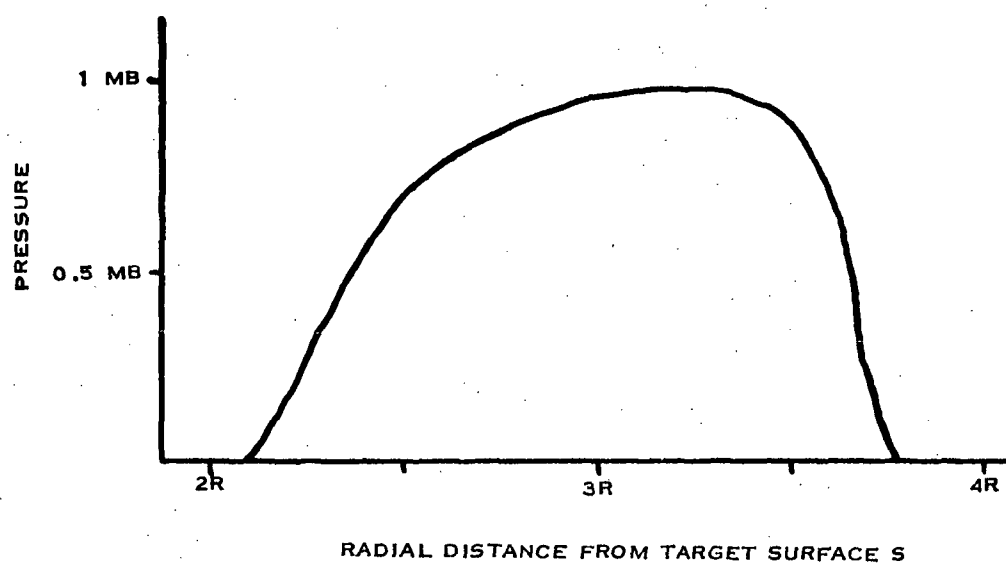


Figure 55. Pressure profile at 1.1×10^{-9} second
for sphere of density 0.5 gm/cc
impacting at 20 km/sec.

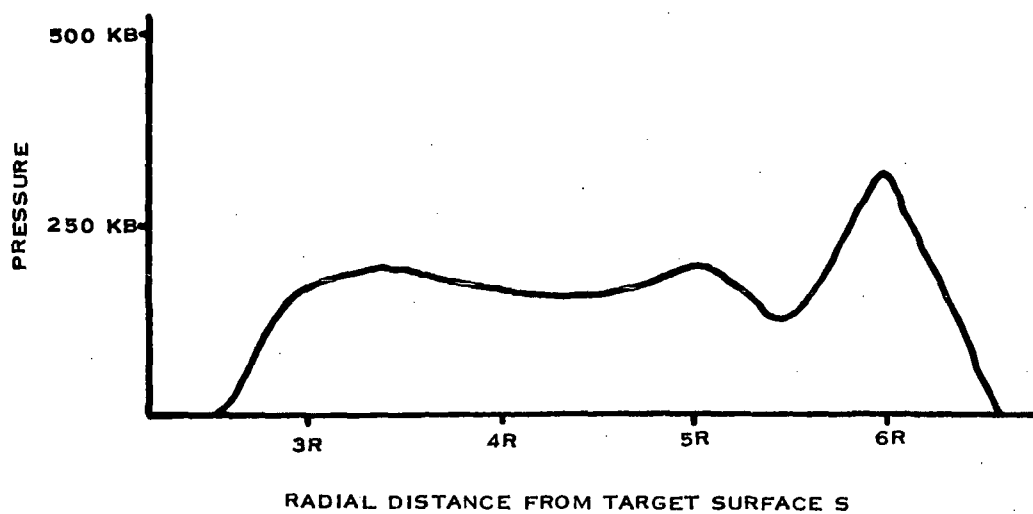


Figure 56. Pressure profile at 4.9×10^{-9} second for sphere of density 0.5 gm/cc impacting at 20 km/sec .

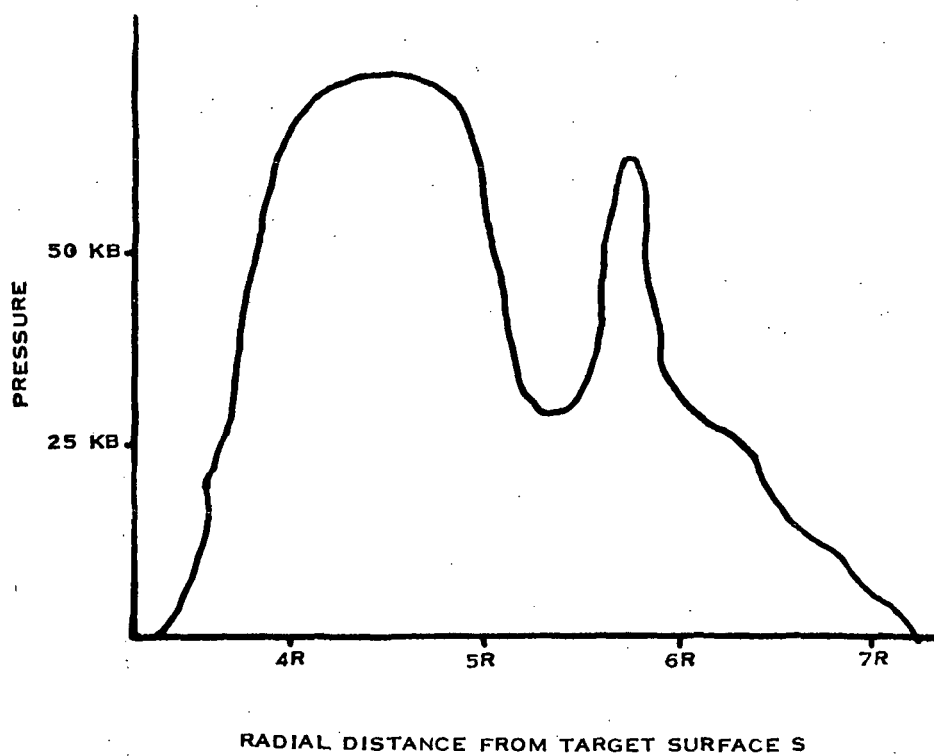


Figure 57. Pressure profile at 6.3×10^{-9} second
for sphere of density 0.5 gm/cc
impacting at 20 km/sec.

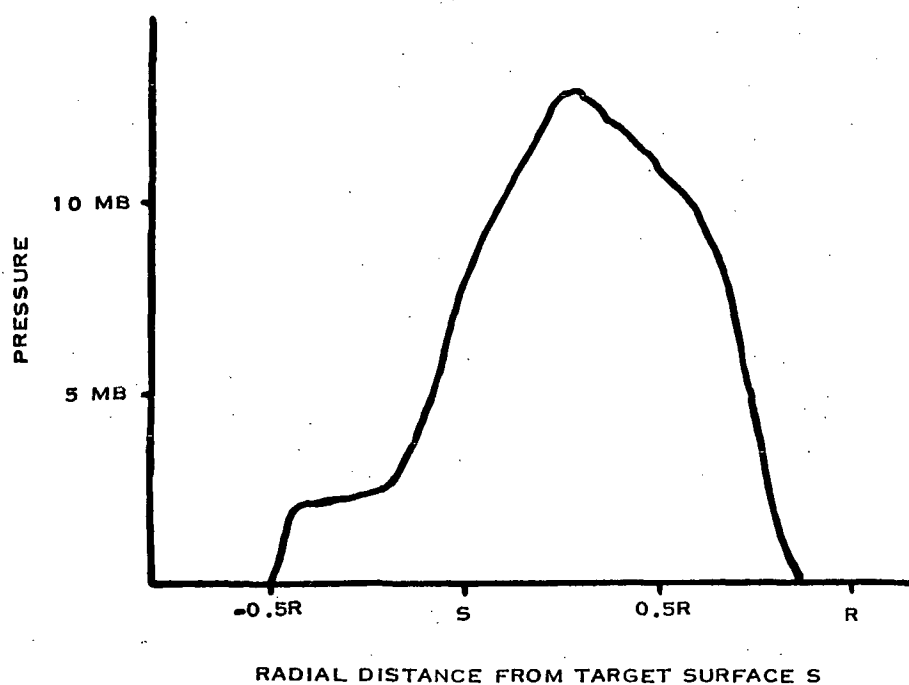


Figure 58. Pressure profile at 1.2×10^{-10} second
for sphere of density 0.5 gm/cc
impacting at 72 km/sec .

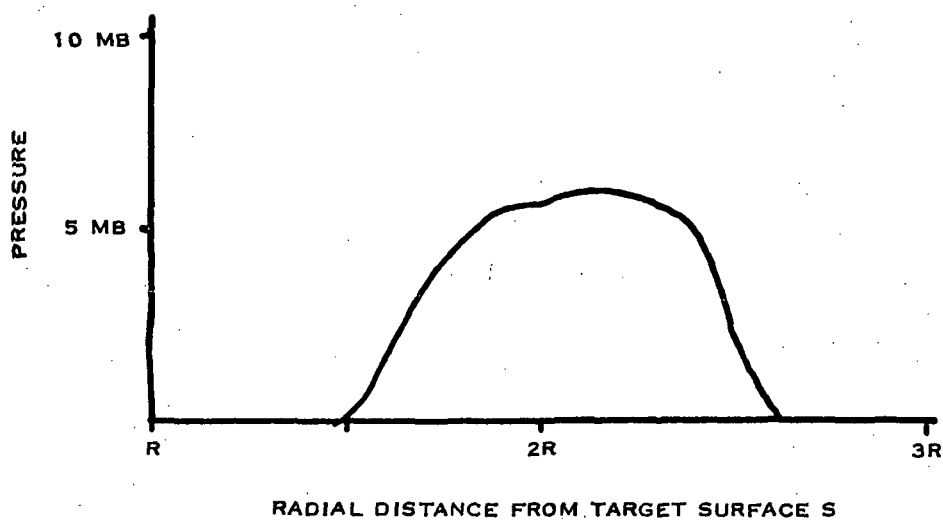


Figure 59. Pressure profile at 9.3×10^{-10} second
for sphere of density 0.5 gm/cc
impacting at 72 km/sec.

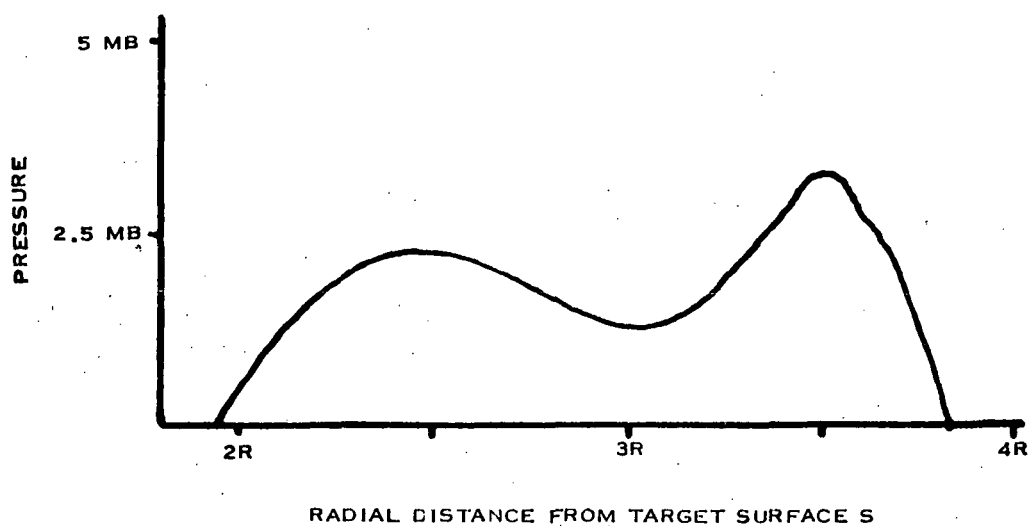


Figure 60. Pressure profile at 1.5×10^{-9} second
for sphere of density 0.5 gm/cc
impacting at 72 km/sec.

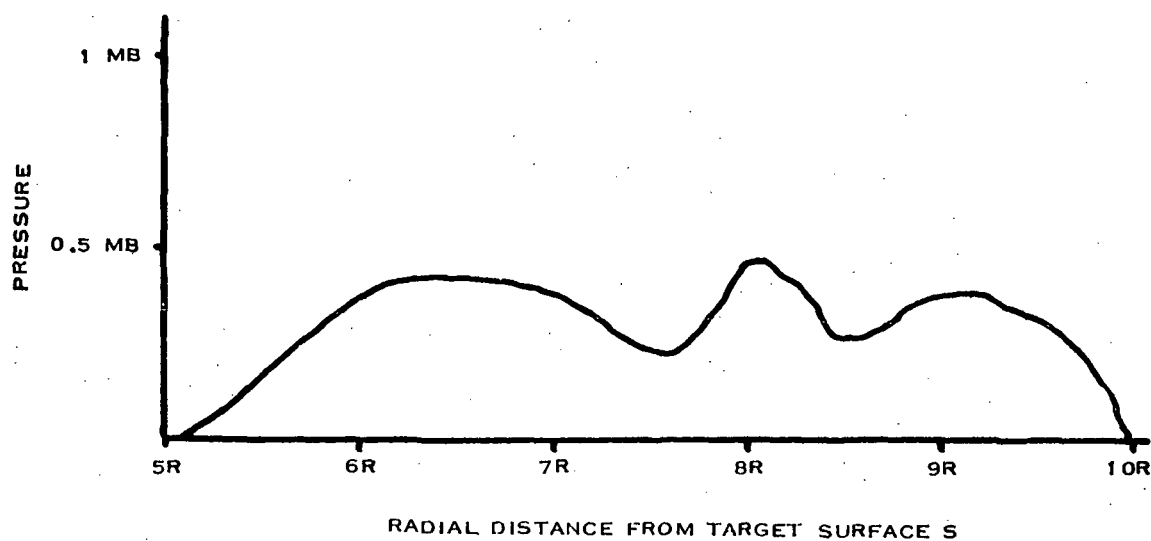


Figure 61. Pressure profile at 5.9×10^{-9} second
for sphere of density 0.5 gm/cc
impacting at 72 km/sec.

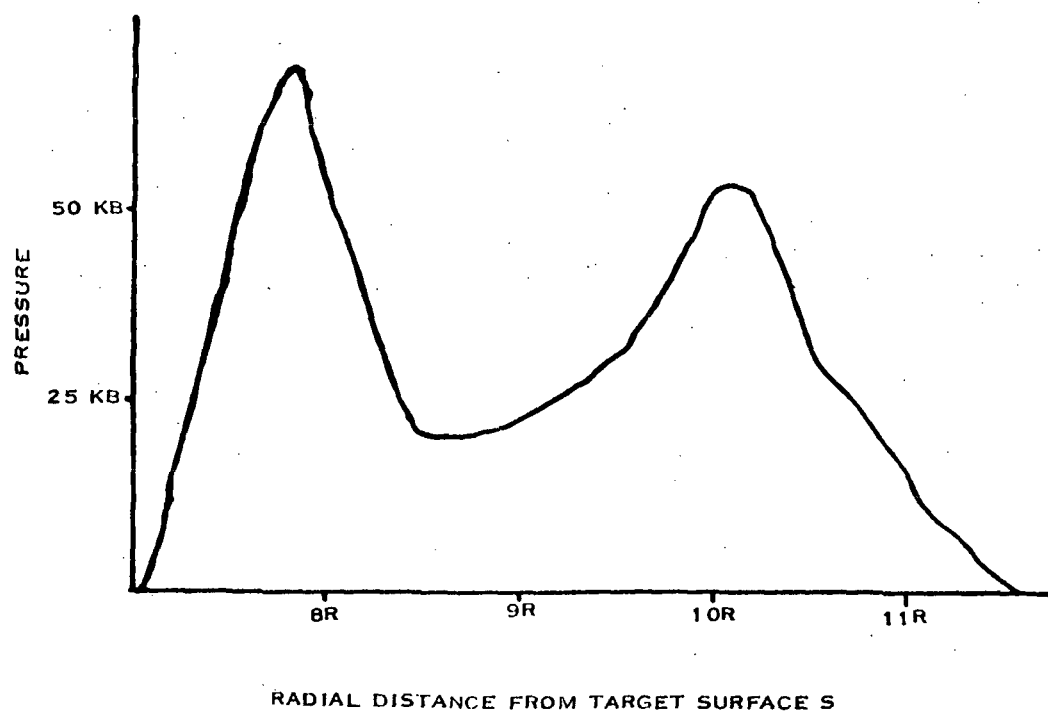


Figure 62. Pressure profile at 1.2×10^{-8} second
for sphere of density 0.5 gm/cc
impacting at 72 km/sec.

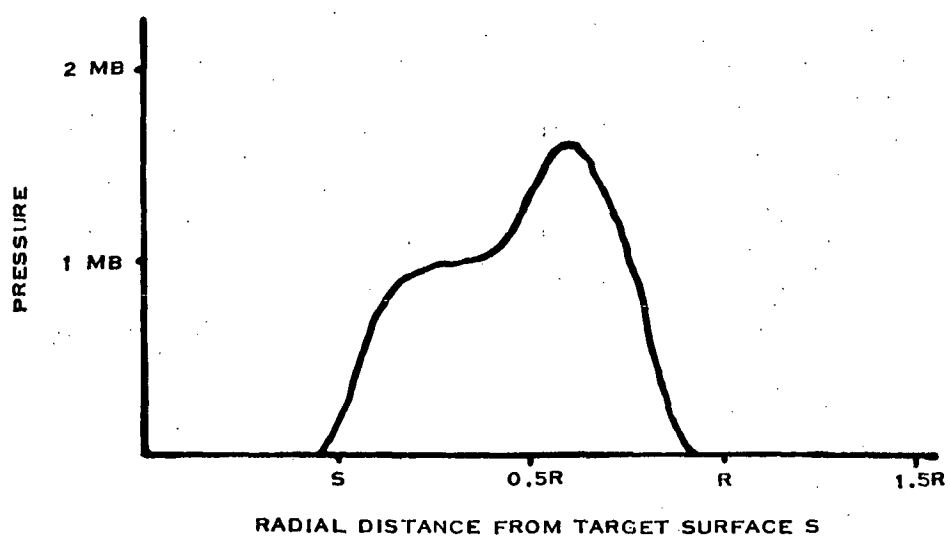


Figure 63. Pressure profile at 4.5×10^{-10} second
for sphere of density 0.5 gm/cc
impacting at 20 km/sec.

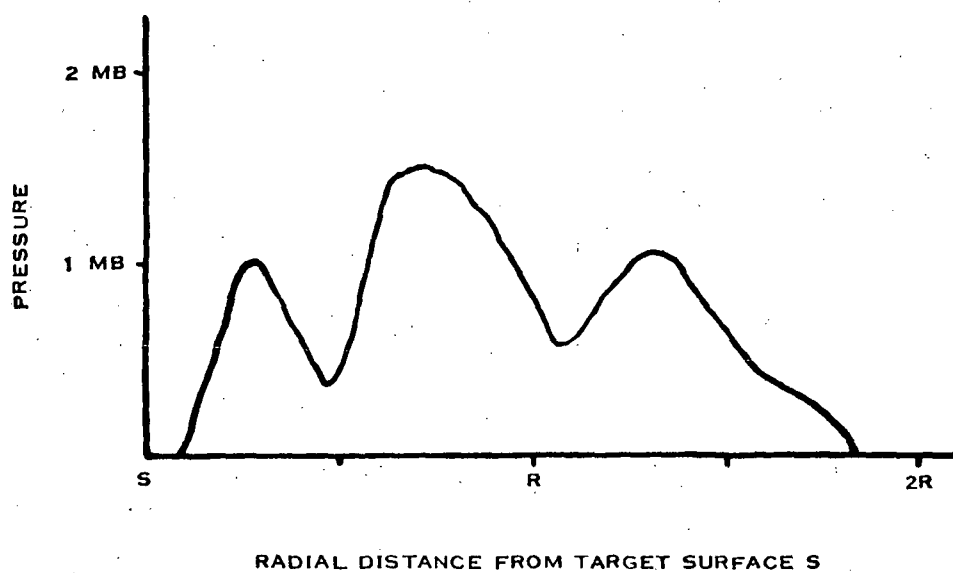


Figure 64. Pressure profile at 2.5×10^{-9} second
for sphere of density 0.5 gm/cc
impacting at 20 km/sec .

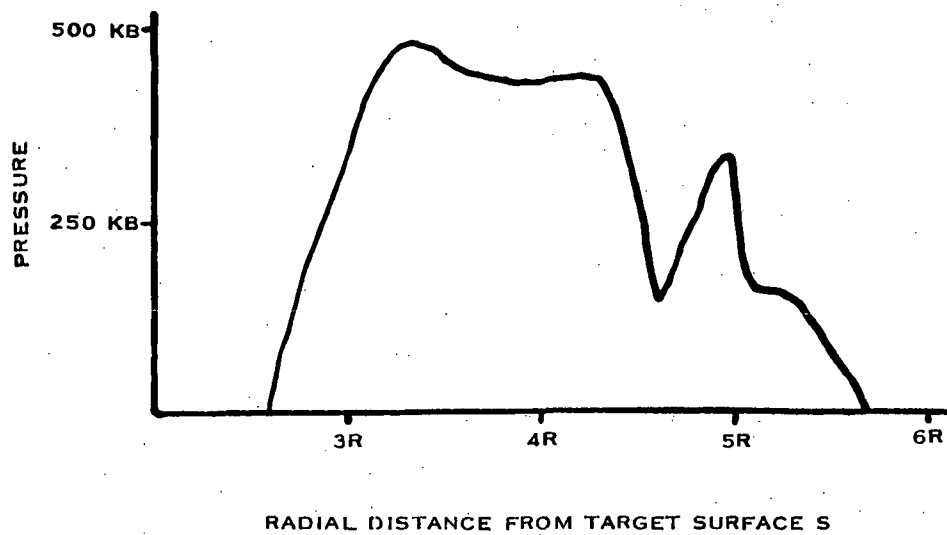


Figure 65. Pressure profile at 1.0×10^{-8} second
for sphere of density 0.5 gm/cc
impacting at 20 km/sec.

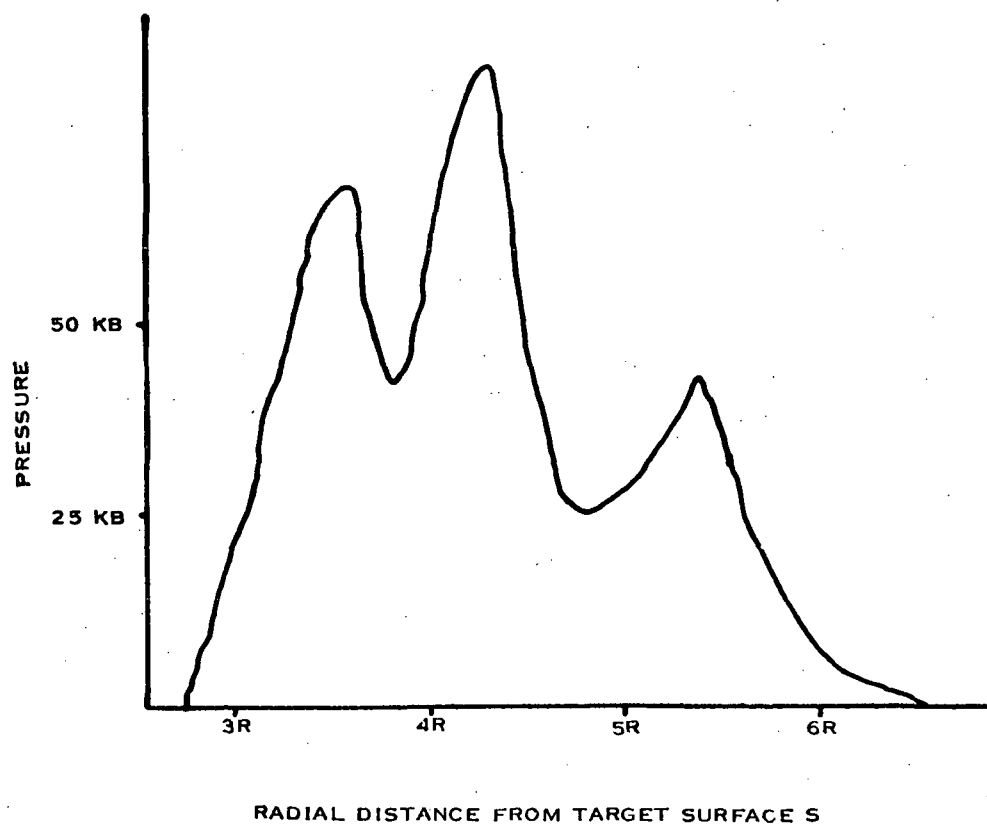


Figure 66. Pressure profile at 1.9×10^{-8} second
for sphere of density 0.5 gm/cc
impacting at 20 km/sec .

and 66. A certain amount of oscillation results in any numerical solution, but these oscillations are too extreme to believe that they are completely a result of numerical techniques. The presence of the pressure oscillations in the solution, independent of the computer characteristics, was demonstrated by computing a second solution with a different size of cell. The pressure amplitudes in the second solution were roughly the same as in the first solution. Kinslow has made excellent photographs of an impact at 21,000 feet per second, and these photographs show rough edges around the shocked material (15). These non-symmetrical extrusions imply the existence of vortices and swirls in the material which would result from such oscillations. Kinslow states that the spalling surface moves erratically after the shock has reached it. This behavior also indicates that there must be strong oscillations in the pressure wave that is incident on the surface. The data collected from atomic bomb blasts reveals that swirls and eruptive outbreaks occur inside the blast sphere. This, too, implies that there are strong pressure concentrations at certain spots in a highly shocked region. A close examination of the figures will show that the oscillations move to and fro and are not stationary.

It is well known that a shock wave exists as a single wave when the peak pressure is of the order of several megabars. After the shock pressure decreases to 137 kilobars (according to previous assumptions), there is a transition from the fluid state to the plastic state. At this position, the shock ceases to exist as a single wave. The shock develops the dual wave structure that is illustrated in Figure 67.

There is a precursor wave that raises the pressure to the elastic limit of the material, and behind this wave is a stronger wave that

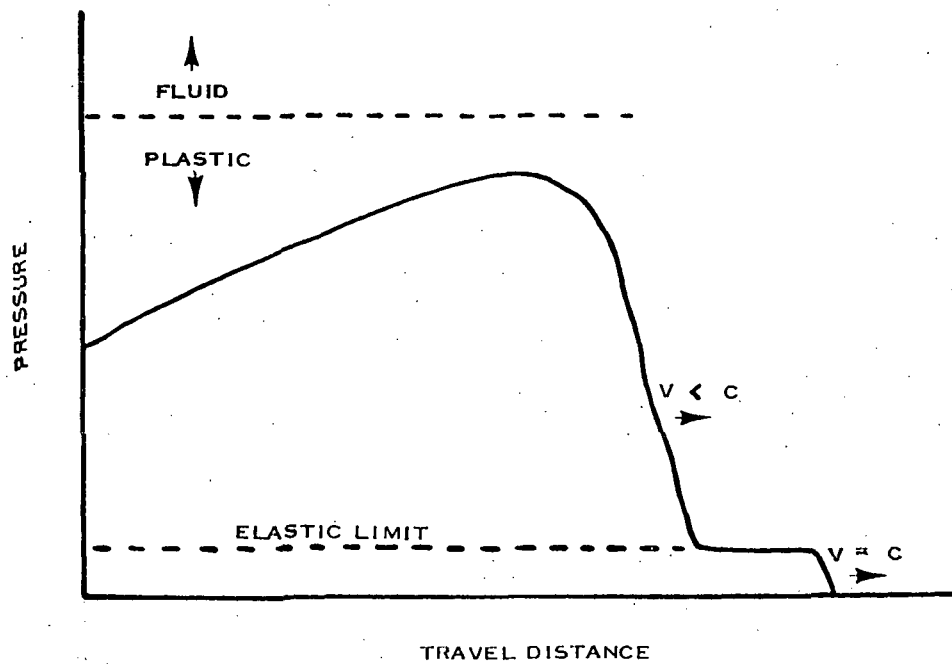


FIGURE 67 – DUAL WAVE STRUCTURE IN PLASTIC REGION

raises the pressure to its peak value. The first wave is called the elastic precursor, and it travels with the velocity of sound in the medium. The trailing wave is called the plastic wave, and its velocity of propagation is less than the speed of sound. As the shock continues onward, the plastic wave is attenuated until its peak decreases to the elastic limit. The shock wave now consists of a single acoustic wave. This elastic-plastic wave is shown in Figures 45, 49, 53, 57, 62, and 66.

Calculation of the Momentum and Energy of the Ejecta

An important part of this solution is an estimation of the momentum and energy carried away by the ejected target material. The shock process in any cell in the finite difference mesh may be described by consideration of the pressure cycle in the cell. A typical cycle is illustrated in Figure 68. As the shock moves through the cell, the pressure rises along the Hugoniot curve from A to B. After the shock front passes the cell, the pressure releases along the adiabat BCD. A question to be answered when the material returns to some point between C and D is: "Is the material fluid or solid?" If it is solid, then it cannot be ejected from the target; if it is fluid, it may be ejected provided the velocity components are such that the fluid would be carried away from the shocked region. The decision of whether the material is fluid or solid is made in the following manner:

1. Check the density, d , in each cell behind the shock front.
2. Calculate the ratio $R=d/d_0$ where d_0 is the undisturbed density.

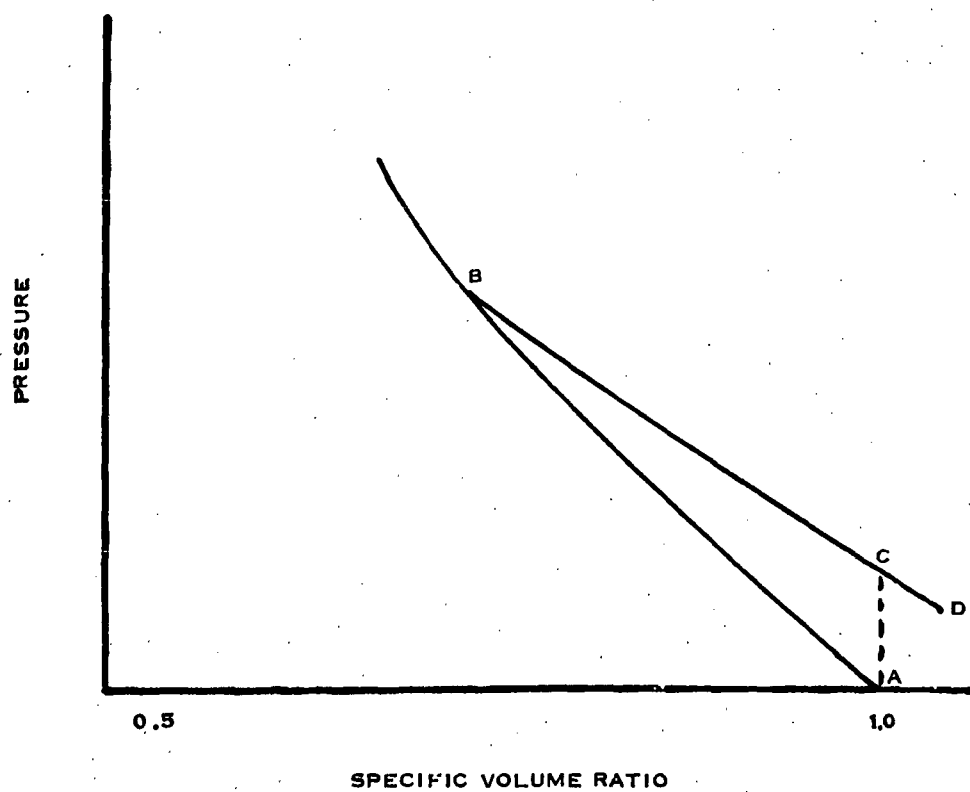


FIGURE 68 - PRESSURE CYCLE IN CELL

3. If $R > 1.0$, skip the ejecta calculation because the material has not released to point C.
4. If $R \leq 1.0$, check the internal energy, EIN, in the cell.
5. If EIN is less than the latent heat of fusion, skip the ejecta calculation because the material is a solid under tension and is not free to be ejected.
6. If EIN is greater than the latent heat of fusion, allow the material to be ejected since it is a fluid.

Whenever the material in a cell is ejected, the tag value of that cell is set equal to zero so that the computer program records that the material has left the cell. The energy and momentum in the cell are stored in the computer, and their values are printed when desired. A running total of these quantities is kept throughout the "run" of the program.

This technique with the computer conserves energy and momentum, but volume is not conserved. It is obvious in the latter stages of cratering that the volume of the rock layer which lines the crater is greater than the volume of the incident sphere. This increase in volume results primarily from the numerical method that was employed to follow the interface motion. The present computer code cannot permit a rock-vacuum interface and a rock-aluminum interface to be in one cell at the same time. With this restriction, it is necessary for the rock layer to always be thicker than one cell width; e.g., the rock-aluminum interface can be in cell (L,M), but the rock-vacuum interface can be no closer than cell (L-1,M). As the crater increases in size, the dimensions of the cell increase (see Expansion of Mesh, Chapter III). Forcing the rock layer to always be thicker than one cell width causes the

rock layer to increase in thickness, since the cells periodically increase in size.

It should be emphasized that in each solution, the rock will be entirely ejected from the crater, even though it is still shown in the crater in the last figure of each solution. The velocity vectors in each of these figures indicate that the fluid rock is ejected away from the "set" plastic aluminum. The amounts of momenta and energy found to be carried away by the ejected material, both rock and aluminum, are summarized in Table VI.

TABLE VI
ENERGY AND MOMENTUM OF THE EJECTA AND
ENERGY NEEDED TO MELT CRATER

V(km/sec)	D(gm/cc)	RP	RE	RM	E/E _{inc}
6.25	2.0	0.18	0.0001	0.352	0.65
7.5	2.0	0.45	0.002	0.660	0.34
20	2.0	8.99	0.176	0.713	0.11
20	0.5	3.12	0.152	0.590	0.26
72	0.5	12.11	0.240	0.545	0.215

In the table, V is the initial velocity of the sphere and D is the density. The numbers listed under RP are the ratios of the normal component of the momentum that is ejected perpendicularly away from the target to the momentum of the incident sphere. It is quite interesting to note that, for high velocities, the ejected momentum is several times larger than the incident momentum. This momentum from ejecta in the -z direction imparts an equal momentum to the target in the +z

direction. Any experiment which measures the impulse that is imparted to the target should measure an impulse that is larger than the incident momentum. This fact must be kept in mind when interpreting space probe experiments that measure micrometeoroid velocities.

The numbers listed under RE are the ratios of the ejected energy to the energy of the incident sphere. The numbers under RM are the fractions of the energy of the incident sphere that is required to melt a quantity of aluminum which is equal to the volume of the crater. The energy to create the plastic region and to provide the acoustic energy is

$$E = (1 - RE - RM) E_{inc}$$

where E_{inc} is the incident energy of the sphere. It is evident that only a small percentage of the energy appears as plastic deformation and acoustic energy at the higher impact velocities.

CHAPTER V

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

Conclusions

This thesis has presented a numerical solution to extend the theory of shock wave propagation and crater formation that results from hypervelocity impacts. The solution examines the impact of porous, spherical, stone spheres upon a solid aluminum slab. The solution is extended beyond the time when inviscid fluid flow is a valid assumption. Appropriate dynamical equations are developed that account for the viscosity effects of the plastic and elastic zones. Considerable effort is exerted to obtain a valid equation of state for compressed porous rock. An equation of state that was proposed by Tillotson is employed for the fluid range of aluminum, and a curve fitted equation of state is employed for the plastic region of aluminum. The dynamic equations are converted to finite difference equations in an Eulerian coordinate system.

A tag system is developed that permits the physical boundaries of the problem to be accurately tracked through the finite difference mesh. Certain approximations are introduced in order to calculate the partial areas of the cells that are occupied by the physical boundary.

The uniqueness and convergence of the numerical solutions are investigated. The theorem of Lax and the Courant condition are of paramount importance in these investigations.

The numerical solutions are presented in the form of crater diagrams and pressure profiles. Each of these presentations of the numerical results illustrates the effects of the velocity and of the density on the solution. The solutions reveal the presence of strong oscillations within the shocked region and a dual shock wave structure in the plastic region of the problem.

One solution compares the crater created by a sphere of radius $(3/2)R$ with a crater created by a sphere of radius R . The crater depths scale linearly with the diameter of the impacting sphere.

Probably the most important part of the solution is the calculation of the momentum increase and energy decrease by the ejecta. A knowledge of the ejected energy and momentum permits an accurate interpretation of the data from ballistic type micrometeoroid detectors. The ejected momentum is larger than the momentum of the incident sphere, except at low impact velocities. The ratio of the ejecta to sphere momentum increases as the impact velocity increases. The energy of the ejecta is negligible at the lower hypervelocities. It becomes a significant fraction of the incident energy at the higher hypervelocities. The ratio of energy lost in the ejecta to the incident energy also increases as the impact velocity increases.

Suggestions for Future Work

The size of the plastic region obtained in the numerical calculations appears to be too large. The extent of this zone is controlled by the viscosity model that is used in the conservation equations. The viscosity effects of aluminum at pressures above 100 kilobars requires further investigation in order to obtain a more accurate

viscosity model. The pressure at which aluminum converts from a fluid to a plastic material also needs to be established. The calculated thickness of the plastic zone could be in error because the viscosity effects were not included at higher pressures.

More impact solutions are required in order to reach any definitive conclusions about the scaling laws that some have proposed to predict crater depths. The one comparison that was made implies a linear scaling with projectile size, but it cannot be assumed that this linear behavior is valid over the entire range of velocities and densities.

The calculations of the ejected momentum and energy are extremely interesting. Only six points in the entire domain of allowed sphere velocities and densities were evaluated. Similar solutions are desired for several velocity and density values. With these values, perhaps some type of functional dependence between ejected momentum and energy and impact velocity and density may be established.

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APPENDIX A

EVALUATION OF VISCOUS AND STRAIN RATE TERMS TO USE IN DYNAMICAL EQUATIONS DESCRIBING PLASTIC FLOW

For inviscid fluid flow the radial equation of motion is

$$(A1) \quad \frac{d}{dt}(\rho u) = -\frac{1}{r^2} \frac{d}{dr}(r^2 \rho u^2) - \frac{\frac{d}{d\theta}(\rho w u \sin \theta)}{r \sin \theta} + \frac{\rho w^2}{r} - \frac{dp}{dr}$$

and the angular equation of motion is

$$(A2) \quad \frac{d}{dt}(\rho w) = -\frac{1}{r^2} \frac{d}{dr}(r^2 \rho u w) - \frac{\frac{d}{d\theta}(\rho w^2 \sin \theta)}{r \sin \theta} - \frac{\rho w u}{r} - \frac{1}{r} \frac{dp}{d\theta}$$

In plastic flow, viscous terms must be added to these equations of motion. These terms are the r and θ components of the divergence of the stress tensor which is

$$(A3) \quad \begin{aligned} \text{div } S_{ij} = & \frac{1}{3} \eta \vec{\nabla} (\vec{\nabla} \cdot \vec{v}) + \eta \nabla^2 (\vec{v}) - \frac{2}{3} (\vec{\nabla} \cdot \vec{v}) \vec{\nabla} \eta \\ & + 2 (\vec{\nabla} \eta \cdot \vec{\nabla}) \vec{v} + \vec{\nabla} \eta \times (\vec{\nabla} \times \vec{v}) \end{aligned}$$

In spherical coordinates

$$(A4) \quad \vec{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{d}{dr}(r^2 u) + \frac{\frac{d}{d\theta}(w \sin \theta)}{r \sin \theta}$$

so the first term of A3 is

$$(A5) \quad \frac{1}{3} \eta \vec{\nabla} (\vec{\nabla} \cdot \vec{V}) = \frac{1}{3} \eta \left\{ \vec{a}_n \frac{\partial}{\partial r} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) + \frac{\frac{\partial}{\partial \theta} (w \sin \theta)}{r \sin \theta} \right] \right. \\ \left. + \vec{a}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) + \frac{\frac{\partial}{\partial \theta} (w \sin \theta)}{r \sin \theta} \right] \right\}$$

The second term is given by Morse and Feshbach as (26)

$$(A6) \quad \eta \nabla^2 \vec{V} = \eta \vec{a}_n \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial u}{\partial r}) + \frac{\frac{\partial}{\partial \theta} (\frac{\partial u}{\partial \theta} \sin \theta)}{r^2 \sin \theta} \right. \\ \left. - \frac{2u}{r^2} - \frac{2}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (w \sin \theta) \right] \\ + \eta \vec{a}_\theta \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial w}{\partial r}) + \frac{\frac{\partial}{\partial \theta} (\frac{\partial w}{\partial \theta} \sin \theta)}{r^2 \sin \theta} \right. \\ \left. - \frac{w}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial u}{\partial \theta} \right]$$

The third term of A3 is

$$(A7) \quad -\frac{2}{3} (\vec{\nabla} \cdot \vec{V}) \vec{\nabla} \eta = -\frac{2}{3} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) + \frac{\frac{\partial}{\partial \theta} (w \sin \theta)}{r \sin \theta} \right] \\ * \left[\vec{a}_n \frac{\partial \eta}{\partial r} + \vec{a}_\theta \frac{\partial \eta}{r \partial \theta} \right]$$

and the fourth term is

$$\begin{aligned}
 2(\vec{\nabla}\eta \cdot \vec{\nabla})\vec{V} &= 2\vec{a}_r \left[\frac{\partial\eta}{\partial r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial\eta}{\partial\theta} \frac{\partial u}{\partial\theta} \right] \\
 (A8) \quad &+ 2\vec{a}_\theta \left[\frac{\partial\eta}{\partial r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial\eta}{\partial\theta} \frac{\partial w}{\partial\theta} \right]
 \end{aligned}$$

To obtain the last term of A3 let

$$\vec{F} = \vec{\nabla} \times \vec{V} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \vec{a}_r & \vec{a}_\theta & r \sin\theta \vec{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & 0 \\ u & r\omega & 0 \end{vmatrix}$$

then

$$\vec{\nabla}\eta \times (\vec{\nabla} \times \vec{V}) = \begin{vmatrix} \vec{a}_r & \vec{a}_\theta & \vec{a}_\phi \\ \frac{\partial\eta}{\partial r} & \frac{1}{r} \frac{\partial\eta}{\partial\theta} & 0 \\ 0 & 0 & F_\phi \end{vmatrix}$$

or

$$\vec{\nabla}\eta \times (\vec{\nabla} \times \vec{V}) = \vec{a}_r \frac{1}{r} \frac{\partial\eta}{\partial\theta} (F_\phi) - \vec{a}_\theta \frac{\partial\eta}{\partial r} (F_\phi)$$

or

$$\vec{\nabla} \eta \times (\vec{\nabla} \times \vec{v}) = \vec{a}_r \left\{ \frac{1}{r^2} \frac{\partial \eta}{\partial \theta} \left[\frac{\partial}{\partial r} (rw) - \frac{\partial u}{\partial \theta} \right] \right\}$$

(A9)

$$+ \vec{a}_\theta \frac{\partial \eta}{r \partial r} \left[\frac{\partial u}{\partial \theta} - \frac{\partial}{\partial r} (rw) \right]$$

The radial equation of motion describing plastic flow is the sum of A1 and the 4 components of A5, A6, A7, A8, and A9 and is

$$\frac{\partial(\rho u)}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u^2) - \frac{\frac{\partial}{\partial \theta} (\rho w u \sin \theta)}{r \sin \theta} + \frac{\rho w^2}{r} - \frac{\partial p}{\partial r}$$

$$+ \frac{1}{3} \eta \frac{\partial}{\partial r} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) + \frac{\frac{\partial}{\partial \theta} (w \sin \theta)}{r \sin \theta} \right]$$

$$+ \eta \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial u}{\partial r}) + \frac{\frac{\partial}{\partial \theta} (\frac{\partial u}{\partial \theta} \sin \theta)}{r^2 \sin \theta} - \frac{2u}{r^2} \right.$$

(A10)

$$\left. - \frac{2 \frac{\partial}{\partial \theta} (w \sin \theta)}{r^2 \sin \theta} \right] - \frac{2}{3} \frac{\partial \eta}{\partial r} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) \right.$$

$$\left. + \frac{\frac{\partial}{\partial \theta} (w \sin \theta)}{r \sin \theta} \right] + 2 \left[\frac{\partial \eta}{\partial r} \frac{\partial u}{\partial r} + \frac{\partial \eta}{r^2 \partial \theta} \frac{\partial u}{\partial \theta} \right]$$

$$+ \frac{1}{r^2} \frac{\partial \eta}{\partial \theta} \left[\frac{\partial}{\partial r} (rw) - \frac{\partial u}{\partial \theta} \right]$$

The θ component of the equation of motion is the sum of A2 and the θ components of A5, A6, A7, A8, and A9 and is

$$\begin{aligned}
 \frac{\partial(\rho w)}{\partial t} = & -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u w) - \frac{\frac{\partial}{\partial \theta} (\rho w^2 \sin \theta)}{r \sin \theta} - \frac{\rho w u}{r} - \frac{1}{r} \frac{\partial p}{\partial \theta} \\
 & + \frac{1}{3} \eta \frac{1}{r} \frac{\partial}{\partial \theta} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) + \frac{\frac{\partial}{\partial \theta} (w \sin \theta)}{r \sin \theta} \right] + \eta \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial w}{\partial r}) \right. \\
 & + \left. \frac{\frac{\partial}{\partial \theta} (\frac{\partial w}{\partial \theta} \sin \theta)}{r^2 \sin \theta} - \frac{w}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial u}{\partial \theta} \right] \\
 & - \frac{2}{3} \frac{\partial \eta}{r \partial \theta} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) + \frac{\frac{\partial}{\partial \theta} (w \sin \theta)}{r \sin \theta} \right] \\
 & + 2 \left[\frac{\partial \eta}{\partial r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial \eta}{\partial \theta} \frac{\partial w}{\partial \theta} \right] - \frac{1}{r} \frac{\partial \eta}{\partial r} \left[\frac{\partial}{\partial r} (r w) - \frac{\partial u}{\partial \theta} \right]
 \end{aligned}
 \tag{A11}$$

For inviscid fluid flow, the energy equation is

$$\begin{aligned}
 \frac{\partial(\rho E)}{\partial t} = & -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u E) - \frac{\frac{\partial}{\partial \theta} (\rho w E \sin \theta)}{r \sin \theta} \\
 & - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) - \frac{\frac{\partial}{\partial \theta} (\rho w \sin \theta)}{r \sin \theta}
 \end{aligned}
 \tag{A12}$$

The viscous effects of plastic flow are included in the energy equation by adding $\nabla \cdot (S_{ij} V_i)$ to A12. $S_{ij} V_i$ is the vector of rank one resulting from the product of the dyad S with the flow velocity V .

$$\nabla \cdot (S_{ij} V_i) = S_{ij} \nabla \cdot V_i + V_i \nabla \cdot S_{ij}
 \tag{A13}$$

The term $\nabla \cdot S_{ij}$ has already been evaluated in equations A3 to A9. The last term of A13 is then

$$\begin{aligned}
 (A14) \quad V_i \nabla \cdot S_{ij} = & u \left\{ \frac{1}{3} \eta \frac{\partial}{\partial r} [\text{div } \vec{V}] + \eta \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial u}{\partial r}) \right. \right. \\
 & + \frac{\frac{\partial}{\partial \theta} (\frac{\partial u}{\partial \theta} \sin \theta)}{r^2 \sin \theta} - \frac{2u}{r^2} - \frac{2 \frac{\partial}{\partial \theta} (w \sin \theta)}{r^2 \sin \theta} \\
 & - \frac{2}{3} \frac{\partial \eta}{\partial r} \text{div } \vec{V} + 2 \left[\frac{\partial \eta}{\partial r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial \eta}{\partial \theta} \frac{\partial u}{\partial \theta} \right] \\
 & \left. + \frac{1}{r^2} \frac{\partial \eta}{\partial \theta} \left[\frac{\partial}{\partial r} (rw) - \frac{\partial u}{\partial \theta} \right] \right\} \\
 & + w \left\{ \frac{1}{3} \eta r \frac{\partial}{\partial \theta} [\text{div } \vec{V}] + \eta \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial w}{\partial r}) \right. \right. \\
 & + \frac{\frac{\partial}{\partial \theta} (\frac{\partial w}{\partial \theta} \sin \theta)}{r^2 \sin \theta} - \frac{w}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial u}{\partial \theta} \left. \right] \\
 & - \frac{2}{3} \frac{\partial \eta}{r \partial \theta} [\text{div } \vec{V}] + 2 \left[\frac{\partial \eta}{\partial r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial \eta}{\partial \theta} \frac{\partial w}{\partial \theta} \right] \\
 & \left. - \frac{1}{r} \frac{\partial \eta}{\partial r} \left[\frac{\partial}{\partial r} (rw) - \frac{\partial u}{\partial \theta} \right] \right\}
 \end{aligned}$$

In the two dimensions, r and θ , used in this problem, the components of S_{ij} are

$$\begin{aligned}
 S_{rr} &= \frac{\partial u}{\partial r} \\
 S_{\theta\theta} &= \frac{1}{r} \frac{\partial w}{\partial \theta} + \frac{u}{r} \\
 S_{r\theta} &= \frac{1}{2} \left(\frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial r} - \frac{w}{r} \right) \\
 S_{\theta r} &= \frac{u}{r} + \frac{w \cot \theta}{r}
 \end{aligned}
 \tag{A15}$$

A useful identity is

$$\operatorname{div} \vec{V} = \frac{1}{r} \frac{\partial}{\partial r} (r^2 u) + \frac{\frac{\partial}{\partial \theta} (w \sin \theta)}{r \sin \theta} = \frac{\partial u}{\partial r} + \frac{2u}{r} + \frac{\partial w}{r \partial \theta} + \frac{w \cot \theta}{r}$$

or

$$\operatorname{div} \vec{V} = S_{rr} + S_{\theta\theta} + S_{\theta r}
 \tag{A16}$$

Thus the first term of A13 is

$$S_{ij} \vec{\nabla}_i \vec{V} = (S_{rr} + S_{\theta\theta} + S_{\theta r} + S_{r\theta}) \operatorname{div} \vec{V}$$

or

$$S_{ij} \vec{\nabla}_i \vec{V} = \left[\operatorname{div} \vec{V} + \frac{1}{2} \left(\frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial r} - \frac{w}{r} \right) \right] \operatorname{div} \vec{V}
 \tag{A17}$$

The energy equation for the plastic zone is obtained by adding

A14 and A17 to the right side of A12.

The conservation of mass equation for the plastic zone is identical to the form used in the inviscid fluid model.

APPENDIX B

EVALUATION OF THE ELASTIC STRESS TENSOR TERMS TO INCLUDE IN THE
DYNAMICAL EQUATIONS DESCRIBING THE ELASTIC ZONE

In the elastic zone, the elastic stress tensor must be included in the equation of motion and the energy equation. The conservation of mass equation is not changed from the form used in the inviscid fluid model.

The elastic stress tensor is

$$(B1) \quad S_{ij} = \lambda e_{ij} \delta_{ij} + 2\mu e_{ij}$$

where the e_{ij} are the components of the strain tensor

$$(B2) \quad e_{ij} = \frac{1}{2} \left(\frac{\partial y_i}{\partial x_j} + \frac{\partial y_j}{\partial x_i} \right)$$

The y_i and y_j are the components of the displacement vector of the material resulting from elastic deformation only. Let u be the flow velocity of the material in the radial direction, and w be the flow velocity in the θ direction. The radial elastic displacement during a short time dt is then

$$(B3) \quad y_r = u dt$$

and the elastic displacement in the θ direction during dt is

$$(B4) \quad y_\theta = w dt$$

It should be emphasized that u and w are not the components of the elastic wave velocity but are the components of the material flow velocity behind the elastic wave front.

The elastic constants λ and μ are given by

$$(B5) \quad \lambda = \frac{E\sigma}{(1-2\sigma)(1+\sigma)}$$

and

$$(B6) \quad \mu = \frac{E}{2(1+\sigma)}$$

where E is Young's modulus, and σ is Poisson's ratio.

The term which must be added to the equation of motion is (27)

$$(B7) \quad \nabla \cdot S_{ij} = (\lambda + \mu) \vec{\nabla} (\vec{\nabla} \cdot \vec{r}) + \mu \nabla^2 \vec{r}$$

$\vec{r}$ is the displacement vector, and in the two dimensions r and θ

$$\vec{r} = (y_r, y_\theta, 0)$$

Therefore

$$\vec{\nabla} \cdot \vec{r} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 y_r) + \frac{\frac{\partial}{\partial \theta} (y_\theta \sin \theta)}{r \sin \theta}$$

and the first term of B7 is

$$(B8) \quad (\lambda + \mu) \vec{\nabla} (\vec{\nabla} \cdot \vec{r}) = \vec{a}_r (\lambda + \mu) \frac{\partial}{\partial r} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 y_r) + \frac{\frac{\partial}{\partial \theta} (y_\theta \sin \theta)}{r \sin \theta} \right] + \vec{a}_\theta (\lambda + \mu) \frac{\partial}{\partial \theta} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 y_r) + \frac{\frac{\partial}{\partial \theta} (y_\theta \sin \theta)}{r \sin \theta} \right]$$

The second term in B7 can be found from a vector identity (26)

$$\mu \nabla^2(\vec{r}) = \vec{a}_r \mu \left[\nabla^2 y_r - \frac{2}{r^2} y_r - \frac{2 \frac{\partial}{\partial \theta} (y_\theta \sin \theta)}{r^2 \sin \theta} \right] \\ + \vec{a}_\theta \mu \left[\nabla^2 y_\theta - \frac{y_\theta}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial y_r}{\partial \theta} \right]$$

or

$$\mu \nabla^2(\vec{r}) = \vec{a}_r \mu \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial y_r}{\partial r} \right) + \frac{\frac{\partial}{\partial \theta} \left(\frac{\partial y_\theta}{\partial \theta} \sin \theta \right)}{r^2 \sin \theta} \right. \\ (B9) \quad \left. - \frac{2 y_r}{r^2} - \frac{2 \frac{\partial}{\partial \theta} (y_\theta \sin \theta)}{r^2 \sin \theta} \right] + \vec{a}_\theta \mu \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial y_r}{\partial r} \right) \right. \\ \left. + \frac{\frac{\partial}{\partial \theta} \left(\frac{\partial y_\theta}{\partial \theta} \sin \theta \right)}{r^2 \sin \theta} - \frac{y_\theta}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial y_r}{\partial \theta} \right]$$

The radial motion equation describing elastic flow is obtained by adding the r components of B8 and B9 to the inviscid fluid radial motion equation; the result is

$$\frac{\partial(\rho u)}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u^2) - \frac{\frac{\partial}{\partial \theta} (\rho w \sin \theta)}{r \sin \theta} + \frac{\rho w^2}{r} - \frac{\partial p}{\partial r} \\ (B10) \quad + (\lambda + \mu) \frac{\partial}{\partial r} \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 y_r) + \frac{\frac{\partial}{\partial \theta} (y_\theta \sin \theta)}{r \sin \theta} \right] + \mu \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial y_r}{\partial r} \right) \right. \\ \left. + \frac{\frac{\partial}{\partial \theta} \left(\frac{\partial y_\theta}{\partial \theta} \sin \theta \right)}{r^2 \sin \theta} - \frac{2 y_r}{r^2} - \frac{2 \frac{\partial}{\partial \theta} (y_\theta \sin \theta)}{r^2 \sin \theta} \right]$$

Adding the θ component of B8 and B9 to the θ motion equation describing inviscid fluid flow gives the elastic motion equation

$$\begin{aligned}
 \frac{\partial(\rho w)}{\partial t} = & -\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \rho u w) - \frac{\frac{\partial}{\partial \theta}(\rho w^2 \sin \theta)}{r \sin \theta} - \frac{\rho w v}{r} - \frac{1}{r} \frac{\partial p}{\partial \theta} \\
 & + (\lambda + \mu) \frac{\partial}{\partial \theta} \left[\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 y_r) + \frac{\frac{\partial}{\partial \theta}(y_\theta \sin \theta)}{r \sin \theta} \right] + \mu \left[\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \frac{\partial y_\theta}{\partial r}) \right. \\
 & \left. + \frac{\frac{\partial}{\partial \theta}(\frac{\partial y_\theta}{\partial \theta} \sin \theta)}{r^2 \sin \theta} - \frac{y_\theta}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial y_r}{\partial \theta} \right]
 \end{aligned}
 \tag{B11}$$

The elastic energy equation must include the term $\text{div}(S_{ij} V_i)$ which is

$$\text{(B12) } \text{div}(S_{ij} V_i) = S_{ij} \nabla_i V_i + V_i \nabla_i S_{ij}$$

The S_{ij} components are (19)

$$S_{rr} = \frac{\partial y_r}{\partial r}$$

$$S_{\theta\theta} = \frac{1}{r} \frac{\partial y_\theta}{\partial \theta} + \frac{y_r}{r}$$

$$S_{\phi\phi} = \frac{y_\theta}{r} \cot \theta + \frac{y_r}{r}$$

$$S_{r\theta} = \frac{1}{2} \left(\frac{\partial y_\theta}{\partial r} - \frac{y_\theta}{r} + \frac{1}{r} \frac{\partial y_r}{\partial \theta} \right)$$

The first term of B12 is then

$$(B13) \quad \left[\frac{\partial y_n}{\partial n} + \frac{1}{n} \frac{\partial y_\theta}{\partial \theta} + 2 \frac{y_n}{n} + \frac{1}{2} \left(\frac{\partial y_\theta}{\partial n} - \frac{y_\theta}{n} + \frac{1}{n} \frac{\partial y_n}{\partial n} \right) + \frac{y_\theta}{n} \cot \theta \right] \text{div } \vec{V}$$

The second term of B12 is the dot product of the flow vector

$$\vec{V} = (u, w, 0)$$

with the vector $\nabla \cdot \vec{S}_i$ evaluated in B7 to B9. Adding this dot product and the terms in B13 to the energy equation for inviscid fluid flow gives the energy equation for the elastic zone.

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