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A PROPOSED CRITERION FOR AIRCRAFT

FLIGHT IN TURBULENCE

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SUMMARY

The objectives of this study were to: (1) review the problem of flight safety in turbulence, (2) suggest a new criterion for turbulence flight if existing ones seemed inadequate, and (3) assess the computational problems associated with any new criterion. Primary emphasis was on catastrophic occurrences in subsonic cruise, and it was assumed that the aircraft was under automatic control. The numerous and interesting questions relating to human pilot responses, false cues, etc., were excluded. It was, however, desired to develop a means of taking account of the action of the automatic control system, insofar as it contributes to the safety question.

Milder turbulence incidents, resulting in passenger discomfort or even injury were not considered. Rather, it was desired to consider the much rarer, but more serious cases in which the aircraft is destroyed, either directly through excessive turbulence-induced loads, or through combined turbulence and control action.

It was found that existing criteria are deficient in several respects. The traditional 1-cosine gust approach does not model the turbulence well, and relates only to structural problems. The traditional power-spectrum approach uses a better model of the turbulence and it can include linear control effects. It cannot, however, account for nonlinearities. Furthermore, there are some serious questions of statistical significance in calibrating the power spectrum model against actual flight experience. Perhaps the most serious deficiency in past methods is the lack of emphasis on control problems. It seems probable that, in recent turbulence-related aircraft losses, control was an important, and perhaps the critical factor.

After reviewing past work in the field, it was concluded that a new criterion is needed. It should be the probability of survival of an encounter with a patch of turbulence of given statistical character. The nature of this patch is based on flight data taken in thunderstorm and mountain-wave turbulence. Aircraft loss could come from any of three mechanisms: (1) exceedance of the ultimate load at some point in the structure, (2) excessive speed, or (3) excessive altitude loss. The action of the control system should be included in determining the probability of survival.

To compute this probability, it is concluded that a Monte Carlo simulation would be required. The size of the problem is such that it seems necessary to use an analog or hybrid computer, at least in the initial stages. It may be found that certain simplifications can be made which would permit use of a digital computer to determine survival probabilities.

INTRODUCTION

The problems of flight in turbulence have occupied aircraft designers almost from the beginning of aviation. Several generations of criteria have been developed and modified, and this process is continuing. The problem has not been completely solved, as aircraft losses from turbulence continue.

To give some idea of the present status, it is interesting to review some safety data. Eastburn (ref. 1) summarizes a ten-year period ending in 1968, in which there were about 70 hull losses of jet aircraft, world-wide, from all causes. Of these, three seem to have been definitely related to turbulence encounters in up-and-away flight: two were thunderstorms and one a mountain-wave situation. There have been, in addition, a number of turbulence-related incidents in which hull losses were averted by rather narrow margins.

Thus, while turbulence is by no means the leading unsolved problem in flight safety, it nonetheless warrants continuing attention. Our understanding of turbulence phenomena is rather imperfect, as this gives rise to substantial uncertainties when aircraft are considered, which are radically different from those for which operating experience has been gained.

The objective of the study reported here was to review the history and present status of the problem, identify aspects that needed improvement, and suggest means for better specification of safety in turbulence. Also, the feasibility of such new specification criteria were to be investigated.

There were several additional restrictions which were perhaps partly arbitrary, but nonetheless reasonable. First, it was decided to focus primarily on catastrophic accidents, rather than the more common turbulence incidents or the even more common routine gust experience. The latter is quite important for fatigue considerations, but is not included here except insofar as it relates to severe extremes.

Also, it was decided to exclude turbulence-related landing and takeoff problems. Not that these are insignificant; indeed, accidents are substantially more common in these regimes than in climb, cruise, or descent. However, it appeared that the low-altitude problems could reasonably be severed from the high-altitude, as the type of turbulence and the nature of the threat were substantially different.

Perhaps a more debatable limitation was exclusion of the problems relating to the human pilot. However, again, it was felt that this problem was severable. The types of investigation required would be quite different, and the nature of the threat would be changed. There are a number of interesting problems to be investigated relative to purely automatic control, and these can be studied more effectively without including the human aspect as well.

One additional item of information may be helpful in laying bare the prejudices of the authors. This study is an outgrowth of the earlier one by Porter, Loomis, and Robinson (ref. 2). In that study, a more or less specific turbulence criterion was stated at the outset, and it was evaluated for a simplified model of the airframe (linear, rigid-body longitudinal only). It made use of a stationary random turbulence model, and frequency-domain analysis methods. A variety of threats to the aircraft was considered, not only the usual structural failure. Thrust and control considerations were included.

The results of that first study seemed quite reasonable, but before pursuing it further, it was decided to take a broader look at the problem, and in particular to examine more carefully what other investigators had done and were doing in this area. To this end, a comprehensive literature survey was carried out. Also, a number of individuals involved in aircraft design problems were interviewed. Based on this information, a new approach was generated. It is outlined in the following pages.

It is perhaps not surprising that this approach turns out to be a rather straightforward generalization of the earlier one. The earlier study was based on two major premises: (1) the threat was essentially probabilistic (so analysis methods would have to be probabilistic also) and (2) a variety of threats should be treated in a common framework (control as well as structural problems). As a result of the current study, we have become convinced of an additional premise: nonlinearities are an essential part of the problem. If these three premises are accepted, most of our proposed approach follows in straightforward fashion.

On many subjects related to turbulence, reasonable men can and do differ. The data are fragmentary and inconclusive. As will be seen later, even those data which do exist have not been carefully interpreted in most cases. In this setting, there is considerable scope for unrefutable opinions. In the following sections, we cite certain data, and draw conclusions as to the best way to proceed. The reader may well differ with our conclusions, or even with our judgement of what constitutes data. We hope, however, that at least we have set the stage for a useful discussion of this subject.

HISTORICAL REVIEW OF U.S. CIVIL GUST CRITERIA

Until the jet-upset incidents of the early 1960's, concern with atmospheric turbulence was largely confined to the problem of ensuring adequate structural design criteria. There were exceptions, notably the efforts of NACA to explore gust alleviation systems for ride quality improvement; but the main thrust of research in turbulence flying has traditionally been towards achieving a satisfactory expectation of remaining within the design loads of the aircraft during its lifetime.

In this section, a brief review is given of the development of these gust design criteria, with emphasis on the design of civil aircraft.

Discrete Gust Concepts

A valuable summary of earlier work in gust research is contained in Reference 3, from which much of the following discussion has been extracted.

Technically, the performance of gust structural design eventually includes the tasks of:

(1) Describing the gust environment within which the aircraft is to operate and

(2) Computing the structural loads resulting from the reaction of the aircraft to these gust.

Even with the highly developed technology and computational capability of today, the performance of neither of these tasks is completely reliable. Yet, armed only with the relatively primitive tools of the early 1930's, the aeronautical industry was forced to generate an immediate practical answer to the question of appropriate gust design loads; the development of efficient civil air transport aircraft demanded a rational approach based upon available data and techniques.

In response to this challenge, an ingenious rationale was conceived which has persisted, through several modifications, to the present time. This is the concept of an "effective" gust velocity for transferring data on measured loads into predictions of loads on a new aircraft.

The Sharp-Edged Gust Formula.— A formula was developed (ref. 4) which included only the most gross effects of airplane characteristics and airspeed as the aircraft traversed a sharp-edged gust. The assumptions included:

(1) An instantaneous change in wind direction and speed, normal to the lateral axis and uniform across the span of the airplane;

(2) No aircraft response to alleviate the load--the airplane, in effect, being driven along a fixed track through the gust; and

(3) No unsteady aerodynamic effects--the lift coefficient being considered a unique function of angle of attack, independent of time.

Under these assumptions, the load factor is given by,

$$\Delta n = \frac{\rho_o U_e V_e C_{L_\alpha}}{2(W/S)} \tag{1}$$

where

Δn = normal load factor in "g" units

ρ_o = sea level air density, slugs/ft^3

U_e = "effective" gust velocity, ft/sec

V_e = equivalent airspeed, ft/sec

C_{L_α} = slope of wing lift curve, per radian

W = aircraft weight

S = wing area, ft^2.

In essence, the early technique was to measure the accelerations experienced by one type of aircraft and, through the sharp-edged gust formula, compute the "effective" gust velocities. Then, applying these same gust velocities to another aircraft, through the same formula, the gust loads on the new aircraft were predicted.

Fundamentally, this rationale has persisted to this day, although the formula has gone through several revisions; the central objective being to transfer loads data from a reference airplane to a new airplane without deriving the true nature of the turbulence structure. Implicit in this scheme is the assumption that the new airplane is "similar" to the aircraft from which the measured loads were obtained. For, if the two aircraft respond to the turbulence in the same way, the errors in computing the "effective" gust velocities are cancelled by the errors in predicting the loads on the new aircraft.

Obviously, the sharp-edged gust formula does not account for the alleviating effects of either the motion of the aircraft or the lag in lift build-up due to unsteady aerodynamic phenomenon. When this formula was used for design load calculations of gliders and other aircraft of widely varying wing loadings and other characteristics, it was found to be unsatisfactory.

Extended Gust Equations.— Motivated by the shortcomings of the simple sharp-edged gust formula, Rhode (ref. 5) developed a more comprehensive formulation of the aircraft response, using the transient aerodynamics work of Küssner (ref. 6). Using this new formulation, the intent was to compute true gust velocities from flight records based upon the following assumptions:

(1) The gust gradient is linear up to a maximum steady gust velocity,

(2) The aircraft is free to heave but not to pitch,

(3) The two-dimensional unsteady-lift functions of Küssner and Wagner are applicable with only a correction for finite-wing lift curve slope, and

(4) The acceleration peak coincides with the first attainment of the maximum gust velocity, except for the very short gradients.

An approximate closed form solution was derived for the ratio of actual load factor increment to that of the sharp-edged gust formula. This ratio was found to be a function of the gust-gradient distance, measured in wing chord lengths, and an airplane mass parameter given by,

$$\mu_g = \frac{2(W/S)}{\rho g \bar{c} C_{L_\alpha}} \quad . \tag{2}$$

This technique was largely confined to research investigations of gust structure, rather than for design load calculations. The need to use flight recordings from which gust-gradient distances could be measured [using assumption (4) above] precluded the use of the usual NACA V-G recorder data which were beginning to be obtained at that time. Nevertheless, important relationships between apparent gust-gradient distance and aircraft responses were obtained from flight research investigations and the extended gust equation.

Data on discrete gust structure (true intensity, gradient distance, and spacing) were obtained from records of load factor increment and airspeed as functions of time for five aircraft of widely varying characteristics. About 48 hours of research turbulence flying data were available at the time of writing of Reference (3).

Although every acceleration peak of these data could be evaluated to obtain the "effective" sharp-edged gust velocity from Equation (1), only those acceleration peaks preceded by a smooth segment could be used to estimate gust-gradient distance and, therefore, the true gust velocity. An analysis of the measured gradient distances showed no significant correlation of true gust velocity with actual gradient distance, measured in feet. On the other hand, a relatively good statistical relationship between true gust velocity and average gradient distance appeared if the gradient were measured in terms of mean chord lengths. In other words, for those gusts which cause well-defined peaks of acceleration response, the gust-gradient distance (measured in wing mean chords) appeared to be independent of the airplane type within the classes of aircraft observed. Furthermore, these data were obtained under a wide variety of weather conditions and altitudes and neither of these variables appeared to have a significant effect. The most significant conclusion was that the average gradient distance, measured in wing chords, is essentially constant for large gusts, but decreases rapidly for smaller gust velocities.

As a result of his analysis of this early research, Donely (ref. 3) concluded that the most probable gust-gradient distance for large gusts was about 10 chord lengths, and that "...a wedge-shaped gust with the gust velocity uniform across the span and either triangular or sinusoidal in shape with a base of 20 chords is believed to be the proper type for most load calculations". This recommendation is quite similar to the current discrete gust model which is discussed later.

<u>The Gust Alleviation Factor</u>.— As mentioned previously, the application of the extended gust equations demanded a more detailed flight record and data reduction technique than possible with the standard NACA V-G recorder. Yet, extensive records from commercial airline flights were available beginning in 1937. To utilize these data, a simple means of accounting for the main alleviation effects was established.

The concept of the gust alleviation factor (K) was developed by NACA and was applied in the Federal airworthiness requirements as early as 1941 (ref. 7). The gust alleviation factor was defined as the relative response of two airplanes encountering the same gust with a linear gradient distance defined in wing chords. For this purpose, the Boeing Model B-247 was chosen as the reference airplane, and a gradient distance of 10 chord lengths was selected.

The use of the gust alleviation factor implied that all airplanes were similar to the Boeing 247, a twin-engine airliner of the 1930's which weighed less than 14,000 lb and cruised at about 200 mph. The only dissimilarity provided for was in wing loading, which was used as a parameter for evaluating the gust alleviation factor. The referenced airplane had a wing loading of 16 lb per square ft, consequently, the gust alleviation factor was unity for this value. For other wing loadings, the gust alleviation factor was given by,

$$K = 1/2(W/S)^{1/4} \text{ for } W/S < 16 \text{ lb/ft}^2 \tag{3}$$

or

$$K = 1.33 - \frac{2.67}{(W/S)^{3/4}} \text{ for } W/S > 16 \text{ lb/ft}^2 \, . \tag{4}$$

In essence, the sharp-edged gust formula [Equation (1)] was used, with the "effective" sharp-edged gust velocity being equal to the product of the alleviation factor and the design gust velocities prescribed in the Federal regulation.

The Revised Discrete Gust Formula.— In computing the alleviation factor, K, the assumption was made that wing loading is proportional to the mass parameter [Equation (2)] and that the effect of pitching on gust load increment is the same for all airplanes. The inadequacy of these assumptions was acknowledged in Reference (3), wherein the suggestion was made that different gust alleviation factors should be applied according to the class of aircraft being considered.

Because of this deficiency, the alleviation factor was modified by the various governmental agencies in establishing their design requirements. This lack of uniformity created some confusion when the design gust velocities used by various agencies were compared. To remedy this situation, the ANC-1 Panel on Flight Loading Conditions agreed to adopt a new standard gust alleviation factor and NACA agreed to develop the new parameter and use it in a revised gust load formula for the reduction of relevant gust data. The development of the revised formula and its initial application are described in Reference (8). This work culminated in the discrete gust design load method currently specified in the Federal Air Regulations for all civil aircraft certificated in the United States.

The revised "gust factor" differs from the previous "alleviation factor" in two important respects:

(1) The gust factor is prescribed as a function of mass ratio [Equation (2)] rather than wing loading and

(2) The discrete gust is assumed to have a "1-cosine" shape with a period of 25 chord lengths, instead of a linear ramp with a length of 10 chords as before.

The NACA approach, in deriving the new gust factor, was to solve the equations of motion numerically for a wide range of mass parameters under the assumptions that:

(1) The forward speed remains constant,

(2) The airplane is free to heave, but not pitch,

(3) The lift movements of the fuselage and horizontal tail are insignificant compared to the wing lift increment, and

(4) The transient lift functions for infinite aspect ratio from Reference (9) are applicable.

The last assumption not only removed aspect ratio as an explicit parameter in defining the gust factor, but seems to be justifiable since it had been observed in earlier work (ref. 3) that the transient aerodynamic responses observed in experiments with finite wings with fuselages tended to agree more closely with the theoretical predictions for infinite aspect ratio.

The gust shape designated by the ANC-1 Panel and used in the derivation was:

$$\frac{\mu(s)}{U} = 1/2(1 - \cos \frac{\pi s}{H}) \ , \ (0 \leq s \leq 2H) \tag{5}$$

and

$$\frac{\mu(s)}{U} = 0 \ , \ (s > 2H) \tag{6}$$

where

μ = the instantaneous vertical gust velocity,

s = the distance traveled in chord lengths, and

H = 12.5 chords.

The revised gust factor is labeled K_g and is defined as the ratio of maximum acceleration experienced by the aircraft, flying through the prescribed gust, to the acceleration predicted by the simple sharp-edged gust formula with the same maximum gust velocity. Consequently, the acceleration increment is given by the formula:

$$\Delta n = \frac{\rho_o C_{L_\alpha} V_e U_{de}}{2(W/S)} K_g \ . \tag{7}$$

The subscript "e" is used to indicate that both the airspeed and the gust velocity are equivalent airspeeds. That is, if σ denotes the atmospheric density ratio,

$$V_e = \sqrt{\sigma} \times V_{true} \tag{8}$$

$$U_{de} = \sqrt{\sigma} \times U_{true} \ . \tag{9}$$

With the gust velocity, the subscript "d" is added to emphasize the point that when the gust velocity is evaluated from measured accelerations, the velocity obtained is "derived". For application to design, of course, values of U_{de} are specified directly.

Since K_g was computed by a numerical process, no closed form analytical expression can be written. A close approximation is widely used, however, and is:

$$K_g = \frac{0.88\ \mu_g}{5.3 + \mu_g} \quad . \tag{10}$$

Horizontal Tail Discrete Gust Loads.— Reference (3) concluded that the accurate estimation of horizontal tail loads was not possible at the time of writing (1950), but suggested that the load be computed as the increase in lift due to the gust alone, multiplied by the downwash factor $1 - \frac{d\epsilon}{d\alpha}$, where ϵ is the average downward on the horizontal tail.

This suggestion is still implemented in the Airworthiness Standards (ref. 10). In effect, the current regulation assumes an upward translation of the aircraft, caused by the wing lift response to the prescribed gust velocity; the angle of attack at the horizontal tail being considered as an instantaneous static angle of attack multiplied by the downwash factor.

Vertical Tail Discrete Gust Loads.— From the structural standpoint, the lateral gust is important primarily because of the aerodynamic loads on the vertical tail. Compared to the vertical gusts, the lateral disturbances appear to have received scant explicit attention. There are reasons for this apparent lack of emphasis; measurements of atmospheric turbulence have shown an isotropic characteristic, except very near the ground. Consequently, the derived gust velocity data would seem to be equally applicable to wing loads and vertical tail loads.

Donely (ref. 3), in 1950, suggested that the loads on the vertical tail be estimated by neglecting the alleviation effects of airplane motion and unsteady-lift phenomena. This suggestion was supported by some experimental data on two aircraft, the XB-15 and the O-2H.

More recently, the airworthiness standards (ref. 10) have incorporated a somewhat less conservative method for computing the vertical tail gust load. Although no specific reference could be found which develops the vertical tail gust load formula of Reference (10), a comparison of the lateral formula to the normal load formula indicates that the following assumptions are incorporated:

(1) The aircraft is free to yaw but not to translate laterally or to roll,

(2) The alleviation effects of two-dimensional unsteady lift build-up on the vertical tail and yawing velocity are included, and

(3) The alleviation effect of yaw angle is neglected.

Gust Criteria for Continuous Turbulence

The application of random process theory to studies of aircraft responses in turbulence was made possible by the improved computational tools which became available in the 1950's. In particular, the power spectral methods for computing the stochastic responses of aircraft to continuous random gusts have become highly developed and have been widely employed in special studies of structural loads in turbulence and for fatigue analyses. Reference (11) summarizes the techniques and reviews pertinent experimental data up to 1964.

A long recognized and inherent weakness of the discrete gust criteria has been the need to specify the state of the aircraft when it first encounters the gust. In fact, the gust structural design criteria just discussed have clearly assumed an initial condition of undisturbed equilibrium flight. Furthermore, the discrete gust concept demands the selection of a particular gust shape, whereas both intuition and experience admit true gust profiles of infinite variety. A structural gust criterion based upon a consideration of turbulence as a continuous random process seems desirable not only for these reasons, but because the existing discrete gust criteria presume a similarity of aircraft responses which has become increasingly difficult to accept.

Aircraft gust load criteria based upon power spectral methods have been explored in research sponsored by the Federal Aviation Agency. The results of this work are reported in References (12) and (13). A brief summary of the work follows.

<u>FAA-Sponsored Research</u>.— The problem of establishing absolute values of structural strength is greatly complicated by the fact that no absolutely reliable stochastic model of atmospheric turbulence is known. To circumvent this difficulty, the best available power spectral model of the atmosphere was applied in an analysis of the structural loads for three existing transport aircraft which have demonstrated an acceptable level of safety. The basic rationale for recommending the new gust criteria is stated in Reference (12): "...in modifying existing criteria or devising new criteria, a practical objective is the achievement of a level of safety with respect to gust loads just equal to that of earlier satisfactory airplanes. ...the new criteria must be of a severity such that when these criteria are applied to the older, satisfactory airplanes, these airplanes are found to be just adequate. A criterion of any greater severity would indicate these airplanes to be inadequate, in contradition to their satisfactory service records. A criterion of any lower severity would have permitted less strength; with the reduced strength the safety record might not have been satisfactory".

It is interesting to note that this philosophy is not unlike that embodied in the discrete gust criteria; both methods are based upon past experience and an assumption that the prescribed technique is capable of transferring past experience into realistic structural stresses in a new aircraft. Perhaps the strongest argument for the power spectral approach is that, for all of its admitted shortcomings, its validity is probably far less sensitive to large differences in aircraft dynamic characteristics than its discrete counterpart.

Three existing aircraft were selected for analysis: the Lockheed Model 749 (Constellation), the Lockheed Model 188 (Electra), and the Boeing Model 720B.

These aircraft satisfied the desired conditions that they have exhibited satisfactory structural integrity in turbulence over long service lives, and that they are gust critical at normal operating speeds. The latter condition was desirable so that strength provided for other reasons would not be interpreted as needed for safety in turbulence.

The stick-fixed longitudinal responses of these aircraft were computed for vertical gust excitation. Both pitch and plunge were permitted and appropriate structural elastic modes were included. For the lateral analyses, structural dynamics were neglected for the Electra and Constallation and the rigid-body responses were computed on the basis of excitation from side-gusts only. For the 720B, airframe flexibility was retained in the lateral response computations. The atmospheric turbulence was described by the Von Karman power spectral model for both vertical and lateral disturbances, with a scale length of 2500 feet.

Two distinct types of structural criteria were developed in Reference (12), although it was suggested that some combination of the two might be useful in practice.

The "design envelope criterion" is most like the current discrete gust criterion in that the specific operational usage of the airplane is not considered. Instead, a design envelope of speed, altitude, gross weight, fuel weight, and c.g. position is specified and a point-by-point analysis made throughout the envelope. This criterion specifies the power-spectral density function of the gust disturbances and design values of true rms gust velocity. The true gust intensity is given for V_B, V_C, and V_D as functions of altitude, where these velocities are the speed for maximum gust intensity, the design cruise speed, and the design dive speed as defined in Reference (10). In applying this criterion, the design load at any selected point is found by multiplying the computed ratio of rms load to rms gust intensity by the design gust value. The design gust intensities are somewhat fictitiously large, since they would result in an rms load equal to the limit load at the structural point in question.

The "mission analysis criterion" accounts for the intended mission profiles to be flown by the aircraft and dictates an acceptable frequency of exceedance of limit structural load. As a result of the analysis of the reference aircraft, the recommended value for frequency of exceedance of limit structural load is given as 2×10^{-5} exceedances per hour.

The recommended exceedance rate is close to the highest frequency of exceedance of vertical loads for the three reference aircraft. On the other hand, the highest lateral load frequency of exceedance was an order of magnitude greater (for the Constellation); but some uncertainty existed in the validity of the lateral analyses and these results were, in effect, disregarded, with the design recommendation of 2×10^{-5} exceedances per hour being recommended for both longitudinal and lateral loads.

Existing Gust Design Standards for Civil Aircraft

The discrete gust, with a "1-cosine" shape and period of 25 mean chords, is still the backbone of the airworthiness standards for civil transport aircraft in the United States.

In the absence of a rational investigation of the response to the prescribed gust, the gust factor (K_g) may be computed on the basis of airplane mass ratio, Equations (2) and (10), and applied to the standard formula, Equation (7), to obtain the design normal load factor for wing structural design. The horizontal tail loading can be computed similarly with the alleviating effect of wing downwash included. The loadings on the wing and horizontal tail caused by unsymmetrical vertical gusts are obtained by assuming that 100 percent of the symmetrical design load acts on one side of the airplane and 80 percent on the other.

The vertical tail load due to lateral gusts is based upon the same gust shape and intensities as the normal load. In the absence of a rational investigation, the gust loading is computed from a formula similar to that used for normal load [see 25.351 (b) of Reference (10)]. As mentioned earlier, this formula is apparently based upon an aircraft free only in yaw, and with the alleviating effects of unsteady aerodynamics and yawing rate included.

No specific consideration of the possible synergistic effects of angled discrete gusts is required, although some structural coupling of lateral and normal loads is provided for in 25.427 (b)(2) for horizontal tails with dihedral or for aircraft on which the horizontal tail is mounted on the vertical fin.

Following the study of power spectral methods discussed above, the FAA, in December, 1966, proposed a revision to the airworthiness standards for transport category aircraft. The modified version of Section 25.305 would have applied the procedures developed in References (12) and (13), although the specified maximum frequency of exceedance of limit load, for use in the mission analysis criterion, would have been decreased to 1×10^{-5} exceedances per hour. This proposed amendment was subsequently withdrawn.

At this time, the suggested power spectral methods have not been implemented in the airworthiness standards. Instead, Amendment 25-23, effective May 8, 8, 1970, in its only reference to continuous turbulence requires that "the dynamic response of the airplane to vertical and lateral continuous turbulence must be taken into account" [25.305 (d)].

It has been suggested by several authors, in Reference (14) for example, that the power spectral technique has not been sufficiently developed to provide a high degree of confidence; at least, not in the absolute sense that is required of a structural design criterion. This opinion was apparently shared by several commentators, as described in the Preamble to Amendment 25-23. These comments were received by FAA in response to a notice of proposed rule-making published in August, 1968. Nevertheless, in adopting Amendment 25-23, the FAA position was given as, "The addition of a continuous turbulence analysis to the already required static discrete gust analysis is a necessary step forward in flight structural safety. Although this type of analysis is still developing, the technique has been and is presently being applied to the design of transport airplanes. As more knowledge becomes available, the analysis techniques can be refined, but in the meantime, the maximum degree of safety available with the state-of-the-art should be designed into airplanes".

In effect, a continuous turbulence analysis is required for all transport category aircraft; but neither the turbulence model, the exact methodology, nor the numerical criterion is explicitly stated at this time. This form of a continuous turbulence requirement is also found in the tentative standards for powered lift transport aircraft (ref. 15).

DEFICIENCIES IN EXISTING METHODS

If the methods used in the past are satisfactory, there is no particular reason to search for new ones. The purpose of this section is to indicate that existing methods do not provide a credible and realistic picture of the actual gust threat. It appears that these methods have been successful because they provided a means of approximately translating experience with older aircraft into new designs.

It may be argued that this "comparison" process can be continued in the future, and indeed in some form it appears to be inescapable. However, this method is always suspect when radically different types of vehicles are considered. In these cases it is more desirable to have a true picture of the threat which the aircraft will face, and a means of evaluating the unconventional aircraft against this threat. Accordingly, in the following paragraphs, we concentrate on the relationship between existing design procedures and the actual gust-related potential for catastrophic events. We conclude that usual procedures are not closely related to the true gust threat, and that their success must be due entirely to the comparison aspect.

Use of Deterministic Disturbances

As indicated in the previous section, various deterministic (nonrandom) gust disturbances were the principal tool for design against gusts. The principal US requirement today involves a 1-cosine gust of specified length and height, and the British standard involves a ramp gust. It is immediately obvious from any examination of flight records that such gusts seldom, if ever, actually occur in nature. That is not to say that such idealizations are not useful for design purposes. In fact, they have proved extremely useful, but as a representation of the actual gusts, they have little standing.

The Power Spectrum Method

Apparently, it was misgivings of the type mentioned above that provided the impetus for the power spectrum approach. This type of gust representation was first proposed in the middle 1950's, and developments of it have continued to the present time. The random gust structure embodied in this method is certainly a more credible representation of the real world than is the 1-cosine gust. The general approach is to model the gust experience as consisting of flight through a number of patches of turbulence of different lengths and intensities. The amount of time spent in turbulence by an aircraft is a random variable, as is the magnitude of that turbulence. These random variables can be described only statistically. Within each patch the gusts are represented by a stationary Gaussian random process, in one, two, or three dimensions, depending on circumstances.

The power spectrum of the Gaussian process is determined from flight measurements, and the probability of occurrence of various levels of turbulence is estimated from VGH recorder data.

With this model established for the atmosphere, the assumption is made that aerodynamics and structural responses of the aircraft are linear. It then follows that the stress at any point in the aircraft is a stationary Gaussian random process (in any one patch of turbulence). The power spectrum of this random process may be computed from the spectrum of the turbulence and the transfer operator relating stress to turbulence inputs. Once the spectrum of the stress random process is known, the probability of exceeding any specivied stress level can be computed. It is also possible to determine the expected number of exceedances per flight hour. From this, it is possible to determine (in principle) the average amount of flying time before the ultimate load is exceeded. In this way, it is possible to compute the likelihood of a catastrophic structural failure. For much more comprehensive accounts of the method, see References (12), (13), and (15).

In the following paragraphs, various aspects of this procedure will be examined, with special emphasis on those which seem to be most open to question. It should be remembered, however, that all of this procedure is concerned purely with structural problems, though linear representations of control actions can be included.

The Gaussian Representation.— Essential to the use of Rice's formula for the expected number of exceedances per unit time (ref. 16) is the assumption that the stress is a Gaussian random process. This assumption is based on two others: (1) that the turbulence is a Gaussian random process and (2) turbulence has a linear influence on stress. The first assumption is the subject of this subsection. The second will be treated in the next.

The question of whether atmospheric turbulence can be treated as Gaussian has been considered by a number of investigators, beginning with Clementson (ref. 17) who was apparently the first to suggest the power spectral approach. To the present time there is no statistically satisfactory answer to this question. It is obvious that representation of an airplane's gust experience by a single stationary random process is unwarranted. Most aircraft spend much of their time in flight with no perceptible turbulence. On the other hand, the notion of patches of more or less uniform stationary Gaussian turbulence is more tenable. In several instances, the first distribution of the gust history in a suitably short portion of flight has been measured and found to be approximately Gaussian, but even here proper hypothesis testing has not been carried out. See, for examples, the studies by Press and Mazelsky (ref. 18), Press (ref. 19), and Crane and Chilton (ref. 20).

However, other investigators have come to contrary conclusions. For example, Burnham (ref. 21) and King (ref. 22) find that the Gaussian hypothesis is not well verified, even for short samples. There are theoretical (ref. 23) and experimental (ref. 24 and 25) reasons for believing that uniform isotropic turbulence, such as that produced behind a wire grid in a wind tunnel, is Gaussian.

With regard to natural turbulence in the free atmosphere, it is not yet possible to be certain. The departures from Gaussian-ness usually seem to take the form of a larger number of large disturbances than would be expected with a Gaussian process. A firm answer, however, is not yet available.

The Linearity Assumption.— Another foundation for the traditional power spectrum approach is the use of a linear dependence of structural stress and control action on turbulence. When applied to the study of catastrophic occurrences, this means that this relationship is linear up to ultimate stress of the structure, and that the control system is linear for disturbances large enough to produce this stress. There are really three parts to this linearity assumption: (1) linearity of the aerodynamic forces on the structure, (2) linearity of the response of the structure itself, and (3) linearity of the control action.

For catastrophic occurrences, the linearity of aerodynamic forces is highly questionable. For instance, one of the turbulence upset accidents studied by Theisen and Haas (ref. 26) involved both positive and negative stall during the maneuver which ultimately led to loss of the aircraft.

Similarly, linearity of the control system does not seem to be a reasonable approximation in these very violent maneuvers. Indeed, control nonlinearities are suspected as an important contributing factor in some jet upsets. Thus, linearizing the control would give results which were perhaps qualitatively different from those to be expected under the same circumstances in flight.

Linearity of the structure itself, on the other hand, seems to be a more useful approximation. Indications are that the stress-strain relation is linear until the vicinity of the yield stress is reached. If, however, there are significant nonlinearities in aerodynamics and control, the usual procedures, which are based on linearity, should not be used.

Distribution of Turbulence Intensities.— Under the "Gaussian patch" hypothesis", it is assumed that all turbulence has the same spectrum, but there are several discrete magnitudes of turbulence power which can be encountered.

The variances of the various magnitudes are designated σ_i^2. Usually two or three levels are used, but a greater number is possible. If the aircraft spends a fraction P_i of its flight time flying in turbulence of variance σ_i^2, then it is easily shown that the expected number of exceedances of incremental load factor a_n (per second of flight) is of the form

$$\overline{M}(a_n) = C \sum_{i=1}^{k} P_i \exp\left\{ -a_n^2/2\overline{A}^2\sigma_i^2 \right\} \tag{11}$$

where k is the number of discrete levels of turbulence being considered. $\overline{A}$ is the ratio between the rms incremental load factor response and the rms value of the turbulence. Therefore, $\overline{A}^2\sigma_i^2 = \beta_i^2$ is the variance of the load factor excursions when the aircraft is flying in turbulence of variance σ_i^2. $\overline{A}$ is determined by the aircraft dynamics and the spectrum of the turbulence. If CP_i and σ_i are known, the above equation can be used to compute the expected frequency of exceeding any given level of a_n. If a certain level of a_n can be established which corresponds to a catastrophic occurrence, then this equation can be used to evaluate the likelihood of catastrophe. By this means, the Gaussian patch model can be related to the subject of interest here.

There are two questions to be considered: (1) is the Gaussian patch model realistic and (2) if so, how can the parameters in the above equation be determined. Both questions can be approached by seeing how well the above equation can be made to fit some set of experimental data. The data usually used are taken from the VGH recorders routinely installed on many aircraft. These instruments record the normal acceleration at the center of gravity, together with the velocity and altitude.

Data of this type have been analyzed for many thousands of hours of flight, by NACA, NASA, USAF, the RAE, and others. A number of authors have used these data to test a Gaussian patch model, and to select the parameters for that model. In the following paragraphs, a brief outline is given of the process by which this is done.

Use of VGH Data to Test the Hypothesis.— The simplest operation on the VGH recorder traces is measurement of the magnitudes of the peaks, in the incremental load factor and the sorting of those magnitudes into class intervals. From this, the number of peaks exceeding a given level can be computed per second of flight, or per mile of flight as desired. Plots of the data obtained from this sort of operation are given in Figure 1. The solid curve is for Operation 1 of Press, Meadows, and Hadlock (ref. 27), while the dashed curve is the cruise data for Aircraft 2 of Hunter and Fetner (ref. 28). These curves are picked somewhat at random from a very large number of similar ones, available in the literature.

Operation 1 is a propeller-driven transport, operating at about 5000 feet on northern transcontinental routes (U.S.). Aircraft 2 is a four-engined jet transport cruising at about 32,000 feet in the eastern U.S. These two cases perhaps represent the extremes of the types of data available for commercial transport aircraft.

The vertical bars through the data points at the lower ends of the curves indicate 60 percent confidence limits. That is to say that, given the data which have been obtained, there is a 60 percent probability that the actual point lies between the ends of the bar. These bounds are based on sample size alone. There has been no consideration of measurement errors, biasing of the sample, or other sources of uncertainty. The method of computing these confidence bounds is given in Appendix A.

Press, Meadows, and Hadlock (ref. 27) fitted the upper data of Figure 1 with an expression of the form of Equation (11) with the following parameters:

$$k = 3, \quad CP_1 = 3.18 \times 10^{-5}, \quad CP_2 = 2.36 \times 10^{-3}, \quad CP_3 = 4.94 \times 10^{-2},$$

$$\beta_1 = 0.430 \qquad \beta_2 = 0.247 \qquad \beta_3 = 0.147.$$

This is plotted as the upper curve of Figure 1. It agrees fairly well with the data, so it can be said that the Gaussian patch hypothesis can explain the data obtained in flight.

If a shape is adopted for the power spectrum of the turbulence, it is possible to work backwards from the parameters of this fitted curve to get the three turbulence magnitudes and the fractions of time spent in each, as shown by Press, Meadows, and Hadlock (ref. 29). Thus, the hypothesis can explain the data, and the data can be used to determine the parameters in the model.

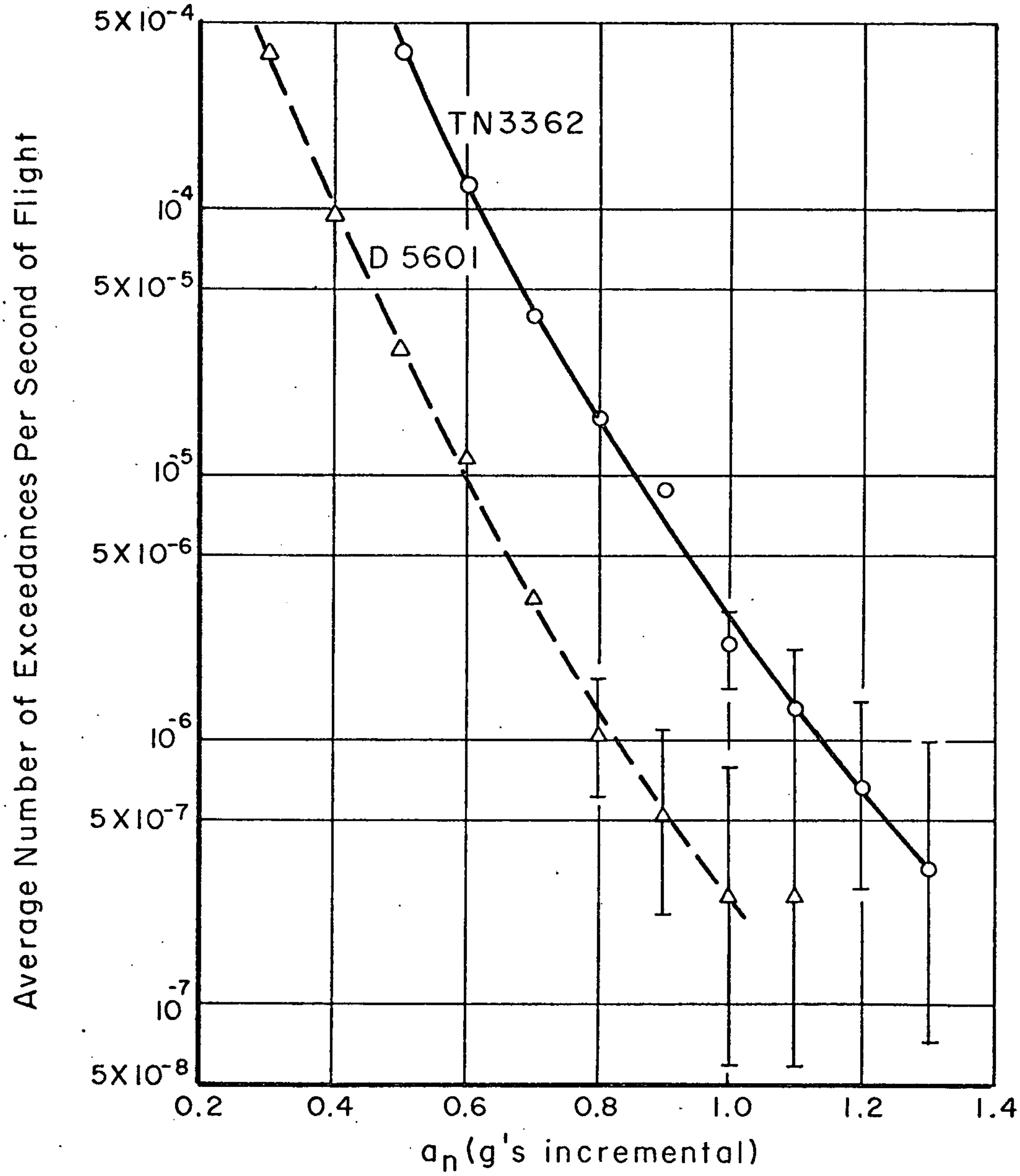

FIGURE I. NORMAL LOAD FACTOR EXCEEDANCES

Application to Catastrophes. — The next objective is to see how this relates to our problem. There are a number of turbulence-related ways that an aircraft can be destroyed, but for the moment, we consider only the traditional one: excessive normal load factor. The first question is one of determining just what normal load factor magnitude is required for aircraft failure.

As will be seen later, high accuracy is not necessary here, so the development will be rather elementary. As noted earlier, the FAR requires that aircraft be capable of withstanding a 25-chord 1-cosine gust of magnitude 50 ft/sec. For both the aircraft used in Figure 1, in their respective cruise configurations, it requires a gust magnitude of about 30 ft/sec to produce 1g incremental load factor. It follows that the design limit load would correspond to about +1.67g incremental, or +2.67 total. The ultimate should be about 50 percent higher, or 4g. Flight experience suggests that aircraft are frequently designed to be actually stronger than this required minimum, so we take the range from 4g to 5g as being representative of a catastrophic magnitude. This would be 3 to 4g's incremental. This range is far off-scale in Figure 1. There are no data in the range of interest to our problem.

It is, of course, highly desirable to bring practical data to bear if at all possible. To this end, we suggest the following approach. It is quite crude, but may nonetheless be illustrative.

In 1968, Burnham (ref. 21 and 30) published a list of 20 gust-related accidents occurring world-wide during the period 1950-1968. This includes both jet and piston-engine experience. During that 18-year period, we estimate that the world's transport fleets logged about 5.4×10^7 flight hours or 1.94×10^{11} flight seconds. If it is assumed that all the accidents on Burnham's list were in fact due to excessive normal load factor, the expected number of exceedances per second would be $20/1.94 \times 10^{11}$ or about 10^{-10} per second. Going to the other extreme, if only two of the accidents were due to excesive normal load factor, the figure would be less by a factor 10.

In a plot of the type of Figure 1, then, it appears that the region of interest would lie between 3g and 4g horizontally, and between 10^{-10} and 10^{-11} on the logarithmic vertical scale. Figure 2 shows the relationship between this region and the region in which the VGH data lie. It is quite obvious that there is a great gap between the two regions, so that any extrapolations based on the VGH data are highly suspect.

Two overall flight safety objectives are shown on the same figure: the suggested value given by Edwards (ref. 31) and the design objective for the C-5A (ref. 32). In both cases, the objective refers to all gust-related exceedances of ultimate load, not only in the normal load factor direction. The objective for normal loads should accordingly lie below the lines plotted.

At any rate, it appears that the design objectives being proposed are at least roughly consistent with the operating experience obtained in commercial transports. This is hardly surprising, since that operating experience no doubt played a role in setting the objective.

It might be suggested that the solution to this gap would be to take more VGH data. This appears to be very unlikely. The situation shown in Figure 2

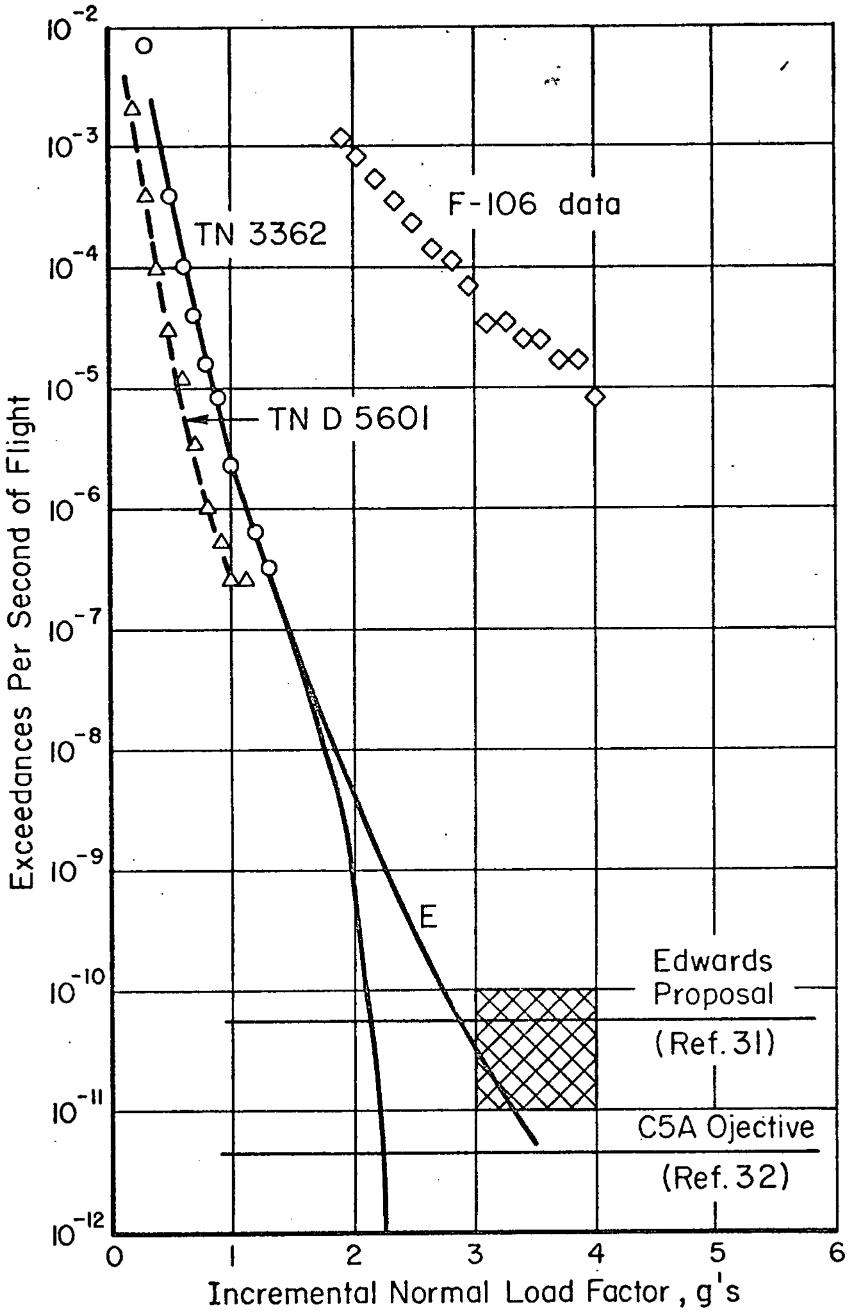

FIGURE 2. LOAD FACTOR EXCEEDANCE COMPARISONS

suggests that the size of sample required is not greatly different from the amount of flying hours logged by the world's transports over two decades. Possibly a sample smaller by as much as ten times would be adequate, but even this would be an immense program, involving perhaps five million flight hours of data.

Peckham (ref. 33) reports that about 300,000 hours of flight data are available, taking all the NATO countries' data together. It seems quite unlikely that a new program of the required size would be undertaken. Aside from the recording and data reduction problems, there is a considerable risk that the data of greatest interest (large load factors) will be truncated through loss of the aircraft, or malfunction of the recorder. There is some possibility that the greatest turbulence-related risk is not normal load factor exceedance at all, but rather lateral gusts, and loss of control. The VGH program would not be expecially helpful in these areas, even if it were carried out. It seems assured, then, that VGH data of the scope needed will not be available for closing the gap in the data of Figure 2.

While operational VGH data do not reach to the region of interest, there are certain special types of data which do. For example, the research program undertaken in the Sangre de Cristo mountains of South Central Colorado in 1964 showed results in this region. The upper data in Figure 2 were taken from this source (ref. 34). These accelerations were observed on an F-106A aircraft so they are not strictly comparable with the other data in the figure, which come from transports. Note, however, that if this aircraft had an ultimate load of 3g, there would be a failure approximately once each three hours of flight. This is, of course, a very severe environment. The aircraft was flown with the express purpose of probing the abnormally severe turbulence found in the lee of the the mountain range. On some particular flights, the experience was substantially worse than that indicated here (which is the total over all flights).

It is perhaps of greater interest to examine the gust velocity itself, rather than the normal load factor. The F 106 used in this study was instrumented to record true gust velocity in three directions, so gust exceedances can be plotted also. The over-all gust experience in the program (positive and negative gusts lumped together) is shown in Figure 3. The circled points are the actual flight data taken from Reference (34). The curve is an approximate fit which is given by

$$N(U) = 2.83 \times 10^{-3}\exp\left\{-U^2/2(31.4)^2\right\} + 6.03 \times 10^{-2}\exp\left\{-U^2/2(19.9)^2\right\} + 0.576\exp\left\{-U^2/2(10.25)^2\right\} \quad (12)$$

which corresponds to a Gaussian patch model with three kinds of patches having rms magnitudes of 31.4, 19.9, and 10.25 ft/sec, respectively. Of principal concern in subsequent discussions is the largest of these: 31.4 ft/sec.

The rms gust velocities for each individual data run are tabulated in Reference (34). There is no single run with a value as large as 30, but there is one at 26 ft/sec, and a number of others above 20 ft/sec. Apparently the individual runs were not in a homogeneous gust field. The degree to which the gusts were Gaussian and/or stationary was not investigated in that study. Indeed, a careful statistical analysis of the excellent body of data taken in that

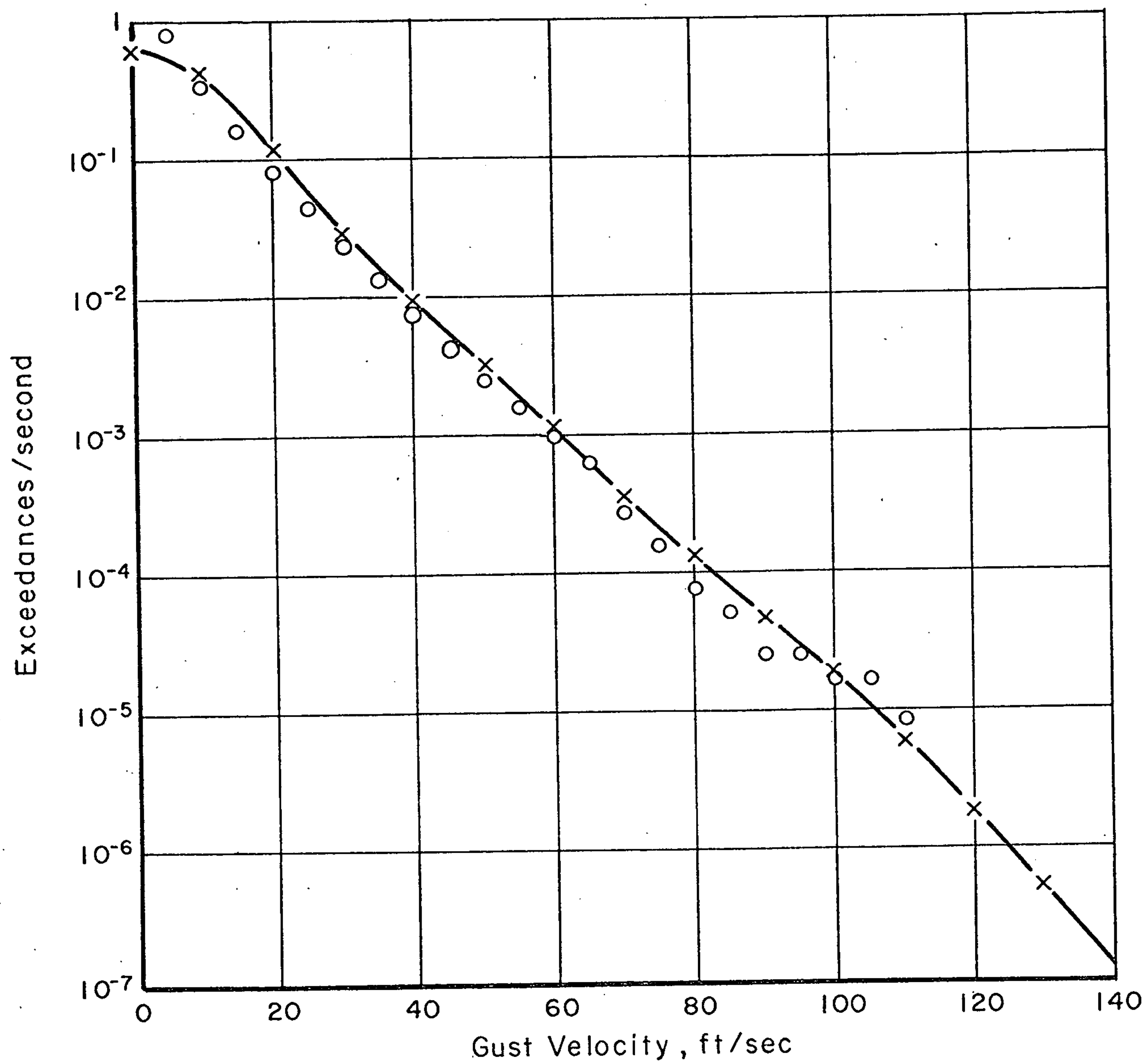

FIGURE 3. GUST EXCEEDANCES, F106 FLIGHT PROGRAM

program has apparently not been made. Such an analysis would be most valuable, since this flight program shows the highest levels of turbulence ever properly recorded in the atmosphere. The same methods have been used in thunderstorms, and the levels of turbulence there sometimes approach, but do not exceed, the magnitudes obtained in the Sangre de Cristos.

Returning now to discussion of Figure 2, the solid curve is the fitted curve of Figure 1. It has a downward curvature for large a_n and hence falls well below any straight-line extrapolation of the VGH data.

At this point there is an almost irresistable temptation to make some such straight-line extrapolation, and to see what the resulting curve might imply. This is the origin of the curve labelled E in Figure 2. The significant part of this curve is the portion near the point at which it crosses the 3g line. If an attempt is made to fit the curve with a Gaussian exceedance curve in this region, it turns out that the rms load factor would be about 0.86. This may be computed strictly from the slope of curve E in that vicinity. Using the value of $\bar{A}$ = 0.01755, taken from Reference (29), this implies an rms gust velocity of about 50 ft/sec.

This would suggest that once each 10 million flying hours, an aircraft will encounter a gust field as large as 50 ft/sec rms. If the most severe recorded gusts (from the F-106 program) can be fitted by a Gaussian patch model with a maximum patch magnitude of about 30 ft/sec rms, the existence of 50 ft/sec rms patches does not seem probable, but cannot be ruled out.

Our own opinion, and it can be only an opinion at present, is that the curve E of Figure 2 is too pessimistic. The true curve probably lies below it. This means that most of the gust-related catastrophes experienced in flight operations are not simple questions of exceeding the ultimate normal load factor. They are probably more complex failures, probably having important contributions from improper control. The "jet upset" accidents of the past decade are now widely thought to be almost exclusively control problems. Indeed, it has recently put forward that they may even not be particularly turbulence related.

It seems possible, then, that present practices may be conservative with respect to structural strength under normal load factor stress, based on actual operating experience. The fact remains, however, that aircraft are still lost to turbulence-related causes. So long as this is true, and so long as the gust threat is not well understood, the gust design problem has not been solved.

A NEW CRITERION FOR FLIGHT IN TURBULENCE

With the background of the foregoing sections, it is now necessary to address the central problem of the study: what criterion should be used for aircraft flight in turbulence. Before giving the result, we outline the principal considerations which led to the choice which was finally made.

Characteristics the Criterion Should Have

A major part of the list of desirable features for a new criterion is contained in the preceeding discussion of deficiencies of existing criteria. First, it is evident that if a high degree of realism is required, the new criterion must be a stochastic one. It should not be restricted to linear systems, or to time-invariant systems. It should not be inherently restricted to Gaussian turbulence models, because it is not yet clear that this is valid. Indeed, the entire turbulence model should be as adaptable as possible, since new data may require changes. It should permit consideration of both control and structural problems; in fact, it may be that control is the more critical. Aircraft flexibility effects must be included, as they affect both control and structural problems.

In addition to these, there are some considerations which should apply to almost any criterion such as (1) it should reflect the true objective of the design process, (2) it should be practical (i.e., implementable at reasonable cost), (3) it should be convincing and consistent with past experience in the field, and (4) it should be selective (sufficiently sensitive to design parameter changes, that those parameters can be specified with reasonable precision).

While it is hardly to be expected that all these requirements can be met simultaneously, they formed the background for the selection of the path to be pursued.

Selection of a New Criterion

Of course, the primary element in the criterion, is the statement of the objective to be attained. In general terms, an aircraft/control system will be considered satisfactory if it is adequately safe in turbulent air, without being excessively conservative, to the point that other aircraft objectives are compromised. These are qualitative ideas at this point, but the following discussion indicates a path to a more quantitative statement.

First, what is "adequately safe"? It is most attractive to set up a direct, absolute over-all safety specification such as "not more than one catastrophic gust-related accident per 10 million flight hours (or per hundred million, or whatever). Some agreement could probably be obtained on a value of this type, but the problem is in determining whether a given design meets the specification or not.

The fundamental difficulty is that the statistics of the atmosphere are not known with sufficient accuracy to permit a direct prediction of flight experience, even if a way could be found to factor in the effect of avoidance procedures. We doubt that the present data base would permit a prediction to within four orders of magnitude, and even to do this would require solution of some formidable computational and theoretical problems.

If an absolute safety prediction cannot be made, it is necessary to fall back on a comparison method. That is to say, a new airplane would be considered "adequately safe" if it is as safe in turbulence as a Boeing 727 or whatever existing airplane can be accepted as having demonstrated adequate safety. This, in effect, is what has always been done in gust criteria. All criteria have been essentially comparison methods which make it possible to relate past experience to the design of a new airplane. However, it is hoped that the comparison method proposed here will be more realistic, and hence more accurate than has been the case before.

The type of representation required for the airplane has already been suggested. Nonlinearities, structural flexibility, and control effects should be included. It is less clear what the model of turbulence should be. What is the nature of the atmospheric threat? This is reviewed in Appendix B, but briefly, the relevant problems are storm-related turbulence, clear-air turbulence, and mountain-wave turbulence. The first and last are of primary concern for catastrophes. CAT occasionally causes troublesome incidents, and some injuries, but its threat to the fundamental integrity of the aircraft appears minimal.

Both thunderstorm turbulence and mountain-wave turbulence are rather sharply localized in space. Typical dimensions are of the order of a few miles. Accordingly, the notion of steady-state flight in severe turbulence is of doubtful realism. Encounters with turbulence tend to be short, and the degree to which a statistical equilibrium is established is open to question. The principal gust-related threat, then, seems to be associated with entering a restricted region in which severe turbulence exists.

The actual mechanisms for catastrophic occurrences seem to be three in number: (1) excessive speed (inducing structural failure via flutter), (2) exceeding ultimate load at some point in the structure (whether by virture of air loads or a combination of air loads and control actions), and (3) excessive altitude loss. Transgression of each of these limits must be considered in determining the safety of flight in turbulence.

While it might be thought desirable to assess the probability of survival under a variety of levels of turbulence, we feel it is more useful to pick a single, severe turbulence magnitude for use as a testing environment. The reasons for this conclusion are the following.

The situation is somewhat different with respect to the two major sources of trouble: structure and control. Considering the structural problem first, certainly the larger gusts are the only ones which are able to generate loads exceeding the ultimate. Roughly speaking, there are two ways in which the large gusts could be encountered: (1) as very unusual events in regions of low or moderate turbulence or (2) as moderately common events in regions of severe turbulence. Since regions of severe turbulence are rather rare, however, it might well be asked in which of the two settings will most of the large gusts experienced by an aircraft be found?

The present body of data does not permit a definitive answer to this question, but some indications can be obtained in various ways. For one thing, it can be observed that, when VGH data are fitted with the "Gaussian patch" hypothesis, it is found that almost all the large gust magnitudes come from the most severe patches. For example, in the fit given for the data of Figure 3, there were three types of patches used, having rms values of 31.4, 19.9, and 10.25 ft/sec, respectively. To evaluate the contributions of these three types of patches to the exceedances of the level of 100 ft/sec, it is necessary only to compute the values of the separate terms of Equation (12). The 31.4 fps patches contribute 1.77×10^{-5} exceedances per second; the 19.9 fps patches give 1.9×10^{-9} exceedances per second; while the 10.25 fps patches give only 1.485×10^{-21} exceedances per second. Thus, virtually all the gusts 100 fps or larger would come from encounters with the most severe turbulence patches, despite the fact that they are some twenty times less frequent than the next larger type of patch.

Another, less quantitative observation leading to the same conclusion, is the common opinion among pilots that thunderstorms are the only real turbulence problem in cruise flight. This opinion is based on flight experience by many pilots over many years, though it is hardly proper statistical data. It is, however, another indication that severe turbulence is the threat, not moderate or light turbulence.

Turning now to control problems, the situation is more clear. If the control system operated improperly in low or moderate turbulence, it would be modified before being placed in service. Any autopilot installed in a commercial transport can be assumed (if it has not malfunctioned) to present no threat to flight safety for the milder disturbances. Control action will usually be linear, as will the aerodynamics. If the controller is stable for small disturbances, it must be stable throughout the linear range. Trouble can be encountered only in the nonlinear region, i.e., under large disturbances.

If this reasoning is valid, it still does not answer the question of just how large the turbulence level should be. It does not seem possible to answer this question at this time, but it is suggested that the region between 20 fps rms and 30 ft/sec rms is probably the most fruitful. The larger figure is consistent with the most severe recorded cases, while the smaller is more typical of severe thunderstorms in general.

Statement of the Criterion

Based on these considerations, the following is proposed as a specific setting for evaluating aircraft safety in turbulence. Consider an aircraft trimmed for 1g level flight, and flying in smooth air. It suddenly enters a region in which there is a uniform isotropic Gaussian turbulence of some specific magnitude and spectrum. This turbulence environment extends along the flight path for some fixed distance L, after which the air abruptly becomes smooth again. For each such turbulence encounter, it will happen that either (1) the aircraft exceeded one or more of the allowable limits in speed, structural load, or altitude change, or (2) it did not. Note that the actual exceedance of a limit could come after the turbulence ceases. In the "jet upset" type of maneuver, this might well be the case.

The probability of survival for this type of turbulence encounter is proposed as the turbulence safety criterion for further investigation.

While specific numbers cannot be given at this time, it is felt that L should be of the order of three to five miles, as indicative of the extent in space of severe turbulence patches. It has already been mentioned that rms values of 20 to 30 ft/sec were indicated. The turbulence spectrum can be selected, based on measured spectra in thunderstorms and mountain-wave turbulence. Of course, it would be unrealistic to "switch on" the turbulence instantaneously at the start of the encounter and "switch off" the turbulence at the end. In both cases, a gradual change, covering perhaps 50 chord lengths, should be employed. Final selection of numerical parameters for the turbulence could be based on generating an encounter sufficiently severe that various existing aircraft, which are widely accepted as satisfactory in turbulence penetration, have a survival probability on the order of 50 percent.

Evaluating new aircraft in this context will once again give a comparison of the new aircraft with older types, for which operational experience has been gained. In that respect, it is no different from the traditional 1-cosine gust or the conventional power spectrum approach. It does differ, however, in using a threat much closer to the one encountered in the real flight situation.

In fact, this proposed gust structure can be viewed as a generalization of both these older methods. It is a stochastic replacement for the traditional step gust, and it is an improvement over the usual random gust. The new criterion is furthermore quite consistent with the current version of the Federal Aviation Regulations, Part 25, Section 25.305(d) which states, "The dynamic response of the airplane to vertical and lateral continuous turbulence must be taken into account". For cruise flight, the proposed criterion offers a rational means of approaching the FAR requirements for turbulence.

Of course, various types of aircraft control representations could be played against this sort of turbulence encounter. In particular, rigid-body or linear flexible models could be used. This would permit correlation of results obtained with the new criterion with those obtained by conventional means. It would also be possible then, to separately evaluate the effect of nonlinearities on the safety of flight in turbulence.

An idea somewhat related to this was put forward by Taylor (ref. 35). He also suggested that a sort of "standard turbulence" be used to replace the cosine or ramp gust used in the past. However, he suggested that this standard turbulence be selected based on the body of VGH data. As he correctly states, this body of data is adequate for fatigue considerations, but not for ultimate loads. We feel that this lack will prove permanent, so we are suggesting a standard turbulence based on recorded extremes.

The principal remaining question relates to the practicality of this new criterion. Can it be evaluated at reasonable expense? Most of the remainder of this report is devoted to a discussion of various aspects of this question. A complete answer cannot be given, short of an actual trial. We offer, however, some indications of what would be involved in such a trial.

ANALYSIS ALTERNATIVES

The dynamics of an aircraft in flight may be represented by a finite set of nonlinear ordinary differential equations. Since the proposed disturbance is random, we are in the field of nonlinear stochastic systems. While this is a comparatively difficult field of analysis, there are, nonetheless, several techniques which have been developed for solving some of the problems of this class.

The specific criterion proposed, however, narrows the field further. The response of the aircraft control system combination will be a multidimensional random process. In this model, a catastrophe is represented by the system point crossing a certain hypersurface in the state space (e.g., moving across the hyperplane representing the maximum allowable speed). In random process theory, this is known as a level crossing problem. In particular, this is a first crossing problem, as only the first boundary violation is of interest. Problems of this type have received considerable attention, starting with the work of Schroedinger and Smoluchowski in 1915. In 1951, Siegert (ref. 36) gave a rather general approach to determination of the probability density function of the first-passage time, for the one-dimensional case. However, a review of more recent work, such as that of Slepian (ref. 37), Strakhov and Kurz (ref. 38), and Rainal (ref. 39), shows that exact solutions in physically meaningful one-dimensional problems are quite rare. There are, however, various approximations and bounds available. For multidimensional problems, apparently nothing has been done.

Application of level-crossing ideas in the fields of structural vibration and flight safety are not new, though they have not been widely applied. The earlier Battelle study (ref. 2) made use of a two-dimensional random process to analyze a combination of threats. Fuller (ref. 40) applied a similar approach to to a combination of structural loads. Houbolt (ref. 41) considered a similar problem, though without the Gaussian assumption. Bolotin (ref. 42) discusses one-dimensional problems of a more complex type, involving accumulated fatigue damage from a number of large loads.

This work, however, made use of the average, or expected, number of load exceedances, not the first passage. These two ideas are closely related. In fact, they become virtually identical in the limit as exceedances become very rare. However, in the application proposed here, they will not necessarily be rare, and the first passage time seems to be more closely related to the objective of the study.

One has to deal, then, with the first crossing of a hypersurface by a multidimensional random process. This random process is the state vector of a nonlinear noise-forced system of ordinary differential equations. At present, there is only one method which offers any prospect of dealing with such problems: Monte Carlo simulation.

The plan would be to simulate the flight of a flexible aircraft (with nonlinear aerodynamics and controls) through the type of turbulence encounter described above. During each such simulated flight, a determination can be made of whether any of the boundaries were exceeded. This simulation is then repeated as often as required to obtain the desired confidence bounds of the estimate of survival probability.

This is a formidable computational task. It is not obvious at present that it can be done economically. However, there is no simpler way to meet the objective. Most of the rest of this report is devoted to an examination of various aspects of the computational problem posed here. While it seems that such a computation has never been carried out, there is reason to believe that it can be done with reasonable economy, if the proper approach is taken.

SIMULATION REQUIREMENTS

Before discussing some of the more critical problems in detail, it is advisable to consider the general makeup of the simulation, and what it should be designed to accomplish. We first consider the major elements which should be present.

Aircraft Representation

The principal requirement is a set of differential equations representing a flexible aircraft, with nonlinear aerodynamics, unrestricted rigid-body motion and no assumption of uncoupling between lateral and longitudinal motion.

The major computational penalty is associated with the representation of flexibility, so this needs to be carefully considered. First is the question of how the flexibility representation should be organized. It is suggested that the normal-mode scheme be used rather than the panel-point, structural influence coefficient method. A wide variety of aircraft can be handled in the normal-mode method with minimal modification. Also, the number of degrees of freedom is much smaller with normal modes. This is a principal determinant of computation time.

The next question is that of how many normal modes should be included. To get accurate indications of structural loads, it is not unusual to include twenty to thirty modes longitudinal, and similar numbers for lateral motion. For control design problems, it is more common to use perhaps five to eight modes in each. Since the preliminary estimate is that the principal problems are in the control area, it is suggested that the smaller number be used, at least in the preliminary investigation of computational feasibility.

Control Representation

A set of differential equations representing the control system will be required. These equations should model all significant nonlinearities, particularly position, rate and acceleration limits on controls, nonlinearities and cross-couplings in sensors (such as gyros). Small-motion nonlinearities, such as dead bands and hysteresis are of less significance here.

Turbulence Representation

The simulation must generate three stationary uncorrelated Gaussian random processes, each with the proper spectrum, and modulate them at the two ends of the run. There are two major options in a digital simulation, and one in an analog. In the digital case, one could 1) generate a sequence of uncorrelated Gaussian variables, run them through a digital filter to get the proper spectrum or 2) approximate the random process by a sum of a finite number of sinusoidal components, each component having random amplitude and phase. The first option is a more accurate representation of the ideal, but it may be more expensive in computer time. It is, however, suggested for initial implementation.

Exceedance Determination

Throughout each run, a check must be kept on excursions beyond the boundaries stated. If none are recorded on a particular run, the aircraft survived. If one is recorded, the run can be terminated at that point. In either case, of course, a record is made of the outcome, as a contribution to the overall probability estimation.

It will not be possible to terminate the run when the turbulence goes to zero. The aircraft could be "alive" at that point, but in a divergent oscillatory motion, which could lead to a later exceedance, even in still air. It should be possible, however, to set up a check on whether the aircraft is converging to its trim position or not, once the turbulence ends. If it ever begins to converge, the run can be terminated at that point.

The Number of Runs

While it is not possible to estimate the running time required in the Monte Carlo simulation, it is possible to get an idea of the number of runs which might be required to develop the necessary confidence in the results. The individual runs will have "success" or "failure" outcomes. Either the aircraft will survive the encounter or it will not. The objective is to estimate the probability of survival. The estimate would be the number of successes divided by the number of runs. This estimate will likely be in error, and the probable error is less, the larger the number of runs.

This is a standard estimation problem, and standard techniques are available for determining confidence bounds. See, e.g., Burington and May (ref. 43), Chapter XIV. If the confidence requirement is set too stringently, there will be a considerable penalty in the number of runs required. Accordingly, it is advisable not to over-specify the confidence desired.

There are two types of uses to which the results of the Monte Carlo simulation could be put. One is to estimate the overall safety experience of the aircraft. This requires knowledge of how often severe turbulence will be encountered. Since this is not known to within even the factor ten, this use places essentially no requirement on the confidence bounds for an individual result.

The other use for the results would be in comparing one aircraft with another, comparing two different control gain settings in the same airplane, comparing the results at two different penetration speeds, etc. This is the application which controls the confidence bounds required. Suppose, for example that a set of n Monte Carlo runs gives an estimate P_1 for the probability of survival of a certain aircraft. Similarly, a set on n runs gives an estimate P_2 for the probability of survival of a second airplane. The question to be answered is: are P_1 and P_2 significantly different in the statistical sense? For n sufficiently large, the answer will be affirmative. For n sufficiently small, it will be negative. The problem is to select n to give the desired confidence in the separation. The value of n will depend on both the confidence level desired, and the difference between P_1 and P_2. The most important influence is the latter. Just how small a difference in probability do we want to detect?

The most favorable case is one in which both P_1 and P_2 are in the vicinity of 0.5. Accordingly, it is suggested that the parameters of the turbulence encounter be adjusted so that the survival probability for a known "good" aircraft is of this order.

The level of discrimination in probability which would be required depends somewhat on the type of study being carried out. It hardly seems, however, that a level much in excess of 0.1 would be of much use for any purpose. There are several ways of converting this into requirements on n, but the following should serve for a preliminary estimate. Suppose that $P_1 = P_2 = 0.5$. How large should n be, in order that the two estimates of P do not differ by more than 0.1 with 95 percent confidence. Application of standard techniques yields the result that n should be at least 200.

If one takes five minutes of computer time as a target for the over all probability estimation, a single run would have to be completed in about 1.5 seconds. Each such run would represent 20 seconds or more of flight time. Thus the simulation would have to run more than ten times faster than real time. It is not at all difficult to envision cases in which four times as many runs are needed. This makes the computational problem even more challenging.

AIRCRAFT REPRESENTATION

At present there exists a fairly substantial background of methods for dealing with motions and stresses in large flexible aircraft. The survey by Ashley (ref. 44) gives some idea of the scope of these methods, and the book by Bisplinghoff and Ashley (ref. 45) is the standard reference in the field. It deals more with the underlying theoretical ideas than with the practical details of building a suitable model. There is a series of reports by Boeing (ref. 46) (ref. 47) (ref. 48) (ref. 49) which treats this latter aspect, and discusses the problems of obtaining the desired degree of accuracy. The representations used by Dempster and Roger (ref. 50), Theisen and Haas (ref. 51), and Wykes, et al (ref. 52) are indicative of the level of detail which can be included.

Some familiarity is assumed on the part of the reader with the body of knowledge contained in the above-cited references. It is not possible here to discuss all the options in construction of a model for the airplane. However, some mention will be made of the reasons for the major choices. The objective here is to present a representative set of equations in order to a) indicate in more detail the type of simulation being suggested and b) lay a basis for some estimation of the difficulty of carrying out the simulation. The general organization, and much of the notation has been drawn from Reference (45) but a number of modifications have been made.

The setting for the equations is the following. There are two orthogonal coordinate systems which will be useful. One is an inertial system, which can be taken as fixed in the earth. The other is moving with the aircraft. The moving coordinate system has its origin at the center of mass of the aircraft. The center of mass is always simply and uniquely defined, whether the structure is strained or not. Orientation of the moving system is a little more involved. Consider some particular instant, at which the structure is in some arbitrary attitude, and with some arbitrary structural deflection. If all the strain energy could be removed from the structure instantaneously, and without changing the angular momentum, then the body would have its unstrained shape (sometimes called the jig shape) and have a certain unique attitude. This attitude is used to define attitude of the moving coordinate system. In particular, a set of principal axes imbedded in the (hypothetical) jig shape is taken as the moving coordinate system. In real motions, the vehicle will seldom, if ever, assume its jig shape; but that shape can nonetheless be used to define uniquely a moving coordinate system associated with a flexible structure. This coordinate system is not fixed with respect to any particular point in the structure. In fact, even the center of mass moves about within the vehicle because of structural deformation.

The center of mass velocity, when expressed in the moving coordinate system has components U, V, and W. The angular velocity of the moving system with respect to the inertial system has components P, Q, and R in the moving system. The orthogonal matrix A transforms vectors from the inertial system into the moving system, and hence A is an expression of the orientation of the moving system. The position of the center of mass, with respect to the inertial system is designated x_I, y_I, z_I. The normal mode displacements are designated ξ_i, and the normal mode shapes by $\overline{Q}(\overline{r})$, where $\overline{r}$ ranges over the jig shape. A bar above a quantity indicates that the quantity is a vector. A particle at some point $\bar{r}$ in the jig shape moves by an amount $\delta\bar{r}$ in the deformation, where

$$\delta\bar{r} = \sum_{i=1}^{k} \bar{Q}_i(\bar{r})\ \xi_i$$

and k is the number of normal modes being considered.

The state of the aircraft is then defined by the position of the center of mass, the velocity of the center of mass, the attitude and angular velocity of the moving coordinate system, and the deflection magnitudes for the normal modes. The differential equations for these quantities are as follows:

$$\dot{U} = VR - WQ + \frac{F_x(\bar{J},\bar{h})}{m}$$

$$\dot{V} = WP - UR + \frac{F_y(\bar{J},\bar{h})}{m}$$

$$\dot{W} = UQ - VP + \frac{F_z(\bar{J},\bar{h})}{m} \qquad (13)$$

$$\dot{x}_I = a_{11}U + a_{21}V + a_{31}W$$

$$\dot{y}_I = a_{12}U + a_{22}V + a_{32}W$$

$$\dot{z}_I = a_{13}U + a_{23}V + a_{33}W \qquad (14)$$

$$I_{xx}\dot{P} = (I_{yy} - I_{zz})\ QR + L_x(\bar{J},\bar{h})$$

$$I_{yy}\dot{Q} = (I_{xx} - I_{zz})\ PR + L_y(\bar{J},\bar{h})$$

$$I_{zz}\dot{R} = (I_{xx} - I_{yy})\ PQ + L_z(\bar{J},\bar{h}) \qquad (15)$$

$$\dot{e}_1 = \frac{1}{2}\ [-e_4P - e_3Q - e_2R] + \lambda e_1(1 - e_1^{\ 2} - e_2^{\ 2} - e_3^{\ 2} - e_4^{\ 2})$$

$$\dot{e}_2 = \frac{1}{2}\ [-e_3P + e_4Q + e_1R] + \lambda e_2(1 - e_1^{\ 2} - e_2^{\ 2} - e_3^{\ 2} - e_4^{\ 2})$$

$$\dot{e}_3 = \frac{1}{2}\ [e_2P + e_1Q - e_4R] + \lambda e_3(1 - e_1^{\ 2} - e_2^{\ 2} - e_3^{\ 2} - e_4^{\ 2})$$

$$\dot{e}_4 = \frac{1}{2}\ [e_1P - e_2Q + e_3R] + \lambda e_4(1 - e_1^{\ 2} - e_2^{\ 2} - e_3^{\ 2} - e_4^{\ 2}) \qquad (16)$$

$$M_i\ddot{\xi}_i + M_i\omega_i^2\xi_i = \Gamma_i(\bar{J},\bar{h}) \qquad i = 1, 2, \ldots k \tag{17}$$

where

$$\bar{F}(\bar{J},\bar{h}) = \iint_s \bar{f}\, ds + \iiint_v \bar{g}\, dv$$

$$\bar{L}(\bar{J},\bar{h}) = \iint_s \bar{r} \times \bar{f}\, ds + \iiint_v \bar{r} \times \bar{g}\, dv$$

$$\bar{\Gamma}_i(\bar{J},\bar{h}) = \iint_s \bar{f} \cdot \bar{\varphi}_i\, ds + \iiint_v \bar{g} \cdot \bar{\varphi}_i\, dv \tag{18}$$

and the components of A are determined by the e's:

$$a_{11} = e_1^2 - e_2^2 - e_3^2 + e_4^2; \quad a_{12} = 2(e_1e_2 + e_3e_4);$$

$$a_{13} = 2(e_2e_4 - e_1e_3)$$

$$a_{21} = 2(e_3e_4 - e_1e_2); \quad a_{22} = e_1^2 - e_2^2 + e_3^2 - e_4^2;$$

$$a_{23} = 2(e_2e_3 + e_4e_1)$$

$$a_{31} = 2(e_1e_3 + e_2e_4); \quad a_{32} = 2(e_2e_3 - e_1e_4);$$

$$a_{33} = e_1^2 + e_2^2 - e_3^2 - e_4^2 \quad . \tag{19}$$

The generalized masses M_i are given by

$$M_i = \iiint_v \rho\, \bar{\varphi}_i \cdot \bar{\varphi}_i\, dv \quad .$$

$\bar{J}$ is used to denote a state vector consisting of U, V, W, P, Q, R, x_I, y_I, z_I, e1, e2, e3, e4, ξ_i, $\dot{\xi}_i$. $\bar{h}$ is a three-dimensional random process representing the atmospheric turbulence. The vector $\bar{f}$ is the force per unit area acting on the surface, and $\bar{g}$ is the force per unit volume (gravity) acting throughout the structure.

Equations (13) are the usual rigid-body equations for the velocity vector. Equations (14) are the usual equations for position. The velocity vector in the moving coordinate system is transformed to the inertial frame and integrated. The required direction cosines, a_{ij} are determined from the e's via Equations (19). Equations (15) are the Euler rotational equations, again the same as rigid-body motion. Equations (16) permit determination of the four quaternion components. These are used rather than the more common Euler angle equations because the Euler angles become singular at one orientation, and it is desired to have unrestricted attitude motion for the airplane. The quaternion method is described by Robinson (ref. 53) and has been used in a number of applications. It offers several advantages over the other possible approaches. The second term in each of Equations (16) are for the purpose of assuring that the quaternion components continue to satisfy the orthogonality condition. Equations (17) determine the normal mode amplitudes.

Writing the equations in this way tends to gloss over the fact that determination of the forcing functions $\overline{F}$, $\overline{L}$, and Γ_i is a very involved process. In fact most of the labor in preparing a simulation of this kind goes into this determination. In the usual linear case, these are linear functions, and it is necessary to determine the coefficients. In the present problem, these may well be nonlinear functions, so a more complex relationship is required. The nonlinear aspect is the only part specifically related to the criterion evaluation, however. The linear part would have to be done in any case, in order to apply traditional methods. We do not consider these questions further here, as our major concern is to get an idea of the simulation problem involved. The complexity of the forcing functions will not make a major contribution to the difficulty of the simulation problem.

There are 13 first-order equations associated with the rigid-body motion. The usual figure would be 12, in view of the six degrees of freedom, but there is the additional equation for the redundant quaternion component. There would be 2k first-order equations for the flexible modes, so that the total order of the system of equations would be 13+2k. If k = 16, which is a reasonable number, one would have to deal with a nonlinear dynamical system of the 45th order. Of course, the residual flexibility technique should be applied to account in part for the normal modes which are not included.

This might be reduced somewhat. The longitudinal and lateral position components x_I and y_I might not be needed. Also, the number of normal modes might be reduced slightly. On the other hand, the control system has not been considered yet, and this will add orders to the system. It may also be necessary to introduce additional variables to represent unsteady aerodynamic effects. It is clear that a very sizeable set of equations will be required.

TURBULENCE REPRESENTATION

In the Monte Carlo simulation, it is essential to have a means of generating sample turbulence histories for use as inputs to the airframe. The generation is comparatively straightforward, once a selection has been made of the desired statistical properties for those histories. The general objective is to state the properties of storm and mountain-wave turbulence with a sufficient degree of completeness as to define uniquely a random process. The principal guide should naturally be flight observations.

It is unfortunate that the picture which emerges from study of the available flight data leaves a very great deal to be desired. A reasonable number of flights have been made, with properly instrumented aircraft, but only a very minor amount of data reduction has been accomplished. Even that which has been done has not made use of standard statistical techniques such as hypothesis testing and confidence bounds.

The types of questions which should be addressed in data reduction are the following: (1) to what degree is the turbulence isotropic, (2) to what degree is it homogeneous, (3) what is the first (and possibly higher) distribution function, (4) what is the autocorrelation, (5) are the three components independent, and (6) if the components are stationary, what are their spectra. In all cases, statistical significance and confidence questions should be answered.

About all that are available are some spectra (without confidence limits or computation of variance of the estimates) and very limited consideration of the first distribution function, without any significance test.

With this degree of uncertainty in the measured results, a broad range of choices can be made with relative freedom from experimental refutation.

Storm Measurements

For reviews of turbulence measurement programs in thunderstorms, see Reference (54) through (59) and the additional references contained therein. Most of these papers contain estimated spectra, though without confidence bounds. In addition, Rhyne and Steiner (ref. 56) and Burnham and Lee (ref. 58) give some limited consideration to the question of whether the turbulence is Gaussian. They come to contrary conclusions, but since neither has done a hypothesis test, the issue is not settled. RMS gust velocities of up to 26 ft/sec are reported, and the variation between separate encounters is substantial. Gray's results (ref. 60) for hurricanes indicate that the turbulence there is comparable to the thunderstorm results.

The spectra as plotted seem to have about the same shape, but the curves are quite ragged, indicating data and/or reduction difficulties. The estimation of power spectra is a subject which has received a great deal of attention from statisticians. The books by Grenander and Roseblatt (ref. 61), Blackman and Tukey (ref. 62), and Jenkins and Watts (ref. 63), and the survey papers by Tukey (ref. 64) and Zaremba (ref. 65) give a picture of the available results. There is not universal agreement on the best procedure, in all details, but there is a good deal of similarity. Also, error estimates have been given for a variety of situations, including all those encountered in the measurement programs mentioned above. It is regrettable that this extensive theory was not used in any effective way in interpreting the flight data.

Another flight measurement program is reported by King, see Reference (22). It consists of flight records taken in normal transport service. The most severe turbulence encounters in 23,000 hours of flight data are analyzed, presumably most of the severe encounters are thunderstorm related. King finds that the distribution of peak magnitudes appears inconsistent with the Gaussian hypothesis, but again there is no proper statistical argument.

Treddenick (ref. 66) attempts to investigate the stationarity of storm turbulence by analyzing the observed data with a sort of weighted moving average. He concludes that it is nonstationary, but again with no confidence analysis.

Mountain Wave Measurements

The best recorded data for turbulence in mountain waves was taken in an Air Force program in the Sangre de Cristo mountains of south-central Colorado (ref. 67), see Reference (34). This program was motivated by an incident in the same vicinity, a few months previously. In that incident, a B-52 lost most of its vertical tail due to a large lateral gust.

The measurement program utilized well-instrumented fighter aircraft to probe the region in which intense turbulence had been found. Three components of turbulence were recorded, and power spectra were computed, though again without confidence considerations. The spectra appear to be consistent with those obtained in thunderstorms, both as to spectrum shape and rms magnitude. The mountain wave magnitudes were perhaps somewhat larger than those for thunderstorms, but not by a great amount. There were several individual runs with rms magnitudes of the order of 30 ft/sec, and one over 37 ft/sec.

The other principal investigation of mountain waves was the Sierra Wave Project (ref. 68). Instrumentation in that project was not as complete as in the subsequent Air Force program, and actual turbulence measurements were not made. However, extreme turbulence was encountered on several occasions, and one of the test aircraft was lost due to this cause. The qualitative reports on turbulence seem to be consistent with the Sangre de Cristo data, but a quantitative comparison is not possible.

Sometimes regions of mountain wave turbulence are indicated by the presence of clouds, sometimes not. Accordingly, mountain waves may present more of a threat inadvertent penetration than do storms.

Selected Turbulence Model

Based on the available data on storm and mountain wave turbulence, together with some rather arbitrary decisions, the following is proposed as a model for the turbulence to be used in the simulation. Fundamentally, the turbulence is a "patch" or region within which the turbulence is homogeneous and isotropic, except near the boundaries. Within the interior, the turbulence is assumed Gaussian, with the Dryden spectrum (ref. 69) and no correlation between the three orthogonal components. The extent of the patch should be about five miles, and the rms value of the turbulence should decrease linearly from the full value in the interior of the patch to zero at the edges, and this decrease should cover a distance of some 50 chord lengths. The reasons for the various choices are discussed in the following paragraphs.

Finite Extent of Turbulence.— The use of a patch of finite size is certainly indicated by flight experience. Almost every recent paper of turbulence of any kind mentions the patchy character of the turbulence distribution. This is certainly the case for storm and mountain wave turbulence. Both occur in sharply defined regions. The principal question is the size of the region to be used. Thunderstorm dimensions vary from 3 to 5 miles for small storms up to 15 or 20 miles for very large ones, with more typical dimensions of the order of 10 miles. From what is known of the structure of storms, it is most unlikely that the turbulence would be homogeneous over distances of the order of 10 miles.

In the Sangre de Cristo work, on the other hand, the turbulence core was found to have a diameter of the order of 2000 feet, though it might extend several miles parallel to the direction of the mountain range. The report of the Sierra wave project suggests somewhat larger dimensions for the roll cloud rotors, up to several miles.

All of this suggests no particular conclusion as to the extent of the turbulence to be used. Dimensions from 2000 feet to 20 miles might be defensible. Indeed, a final selection should await some simulation experience.

One question relates to the speed with which the effects of initial condition are eliminated. If the turbulence were semi-infinite in extent, the probability density function of the system state vector would approach a steady state, and not change further. The question is how long this change would require, starting from the undisturbed and trimmed condition at turbulence entry. The size of the turbulence patch should be roughly the same as the distance required to reach the statistical steady-state unless that distance turns out to be very small, say less than a mile. If that turns out to be the case, it might be preferable to base the criterion on the steady-state situation and derive something like the probability of loss per mile of flight. The computational problem would be almost exactly the same.

At present, it seems that dimensions of the order of 3 to 5 miles might be the most useful range, but this would have to be confirmed in further work.

Homogeneity, Isotropy, and Normality.— The few investigators who have investigated these questions in flight measurements have obtained mixed results, but none have made convincing statistical arguments. Until such arguments are made, or new data are available, there seems no valid reason for departing from the most convenient option, a homogeneous, isotropic turbulence field, with a Gaussian distribution in each dimension.

Dryden Spectrum.– This rather unconventional choice is made for about the same reason. The flight data do not permit a confident choice between the Dryden spectrum and the von Karman (the two principal candidates) and the Dryden spectrum is substantially more convenient to mechanize in either analog or digital simulations. Random processes with Dryden spectra can be produced by passing white noise through lumped-parameter filters. Processes with von Karman spectra can be approximated by this means, but this is an additional complication. When using frequency-domain methods, as in traditional power spectrum operations, either spectrum is about equally convenient. In the time domain, however, the Dryden is more substantially easier to implement.

Representation of the Boundary.– At the edge of the turbulence patch, it would not be reasonable to begin a stationary turbulence instantaneously as with a switch. This would generally give a sharp-edged gust, which is more severe than those encountered in the atmosphere. Accordingly, it is suggested that the increase in turbulence level from zero to the steady-state occupy some 50 chord lengths. This is still a fairly rapid onset, but not so rapid as to give abnormal loads at the start.

Taylor's Turbulence Model

As mentioned previously, Taylor, see Reference (35), has proposed the use of a "standard turbulence" in studying structural problems. In a later paper (ref. 70) he gave a more explicit suggested model. He proposed that each patch be considered homogeneous and isotropic, with the von Karman spectrum. He also proposed that the peak magnitude of the total gust velocity vector have a Rayleigh distribution, rather than a Rayleigh distribution on the peaks of the individual components, as is more common. This gives a somewhat better fit of the observed relation between zero crossings, and the number of large gusts, though again statistical arguments are lacking.

While he doesn't explicitly state how this model would be used, apparently the steady-state response would be evaluated as he does not propose a finite size for the gust encounter.

His conclusions are based on analysis of the severe portions of the 23,000-hour sample reported by King (see Reference 22). While this was normal transport operating experience, it seems fair to conclude that most of these severe portions represented thunderstorm encounters.

For frequency-domain analyses, his model would be a reasonable one. For our purposes, the finite-extent and Dryden spectrum are more convenient, and equally consistent with observed data.

COMPUTATIONAL ALTERNATIVES

The Computational Problem

Based on the discussion of the preceding sections, it is possible to form some idea of the magnitude of the computational task to be performed. A set of perhaps 50 first-order ordinary differential equations must be solved several hundred times, for each criterion evaluation required. These equations have the property that the response will contain widely separated frequencies. The frequencies associated with the higher order elastic modes are much higher than those of the rigid-body motion. Equations with this property are called "stiff" equations.

Internal to the simulation is also the problem of generating random processes of the desired distribution and spectrum to represent the three components of turbulence.

The linearized and stationary version of this problem has been extensively studied. The computations in this case are carried out in the frequency domain and the computational situation is quite different. Time-domain simulations of the complexity required have also been done. For example, see Theisen and Haas, Reference (51). However, such simulations have apparently not been used in a Monte Carlo mode. For this type of use, the running time requirements are far more stringent than in the case where only a few dozen runs with various deterministic inputs are required.

There are two major alternatives for carrying out these computations: analog (or hybrid) and digital computation. Incidentally, both techniques were used by Theisen and Haas, see Reference (51). The prospects for each are discussed in the following paragraphs.

Analog Simulation

The most promising means for meeting the requirements seems to be use of a modern analog, or preferably a hybrid computer. There are a number of advantages. Because of the parallel operation of this type of device, the entire simulation does not have to proceed at a pace determined by the largest eigenvalue. There is, of course, a limit to the bandwidth which a given computer can handle, but this is quite large for modern machines. Generally, the objective would be to speed up the solution time until the highest vibrational mode begins to lose accuracy from the bandwidth limit. It seems quite likely that, even with a number of modes included, it would be possible to run faster than real time. Also, running time on such a computer is generally much less expensive than for a large digital computer.

Noise generation is also simpler on an analog machine, as all the noise channels required may be generated in parallel. However, it would not be possible to repeat a noise run exactly, as could be done on a digital computer quite easily if the noise is based on a pseudorandom sequence.

Run control and data manipulation might present some difficulty on a pure analog machine, but use of a hybrid computer would eliminate this problem. The digital part of such a simulation would, however, be quite simple.

As with any large analog simulation, there will exist a validation problem. This would probably require use of a digital simulation to compare results on a few runs with deterministic inputs. These inputs could include random-appearing sums of sine waves.

Digital Simulation

Experience suggests that digital solution of such large stiff sets of differential equations cannot be done quickly enough for economical use in a Monte Carlo simulation. It would be necessary to have improvements in run time somewhere between one and two orders of magnitude for this to become practical.

The mere size of the set of differential equations is a substantial difficulty, but probably more serious is the extreme stiffness of the equations. The entire simulation is paced by the period of the highest-frequency structural mode included. Of course, substantial efforts have gone into developing numerical methods for handling this type of situation. For reviews of this work, see, e.g., Liniger and Willoughby (ref. 71) or Seinfeld, Hwang, and Lapidus (ref. 72).

The fundamental problem with stiff equations is that conventional numerical integration methods become unstable when the step size is too large. The maximum allowable step size is determined by the highest eigenvalue of the system. Accordingly, the special methods developed to solve this type of equation have unusual stability properties, in some cases being stable for all step sizes whatever. This increases the computing time per step, and may adversely affect the accuracy, but it is possible to take large steps (limited by the smallest eigenvalues rather than the largest), so that the response at the lower frequencies may be studied more or less as if the high-frequency phenomena were not present. In our problem, this would mean that the step size would be determined by the rigid-body modes, rather than the highest elastic mode.

This sounds quite attractive, but there are some problems with it, for the current application. The first is that these methods have apparently not been applied to systems of equations as large as needed here. Probably more serious is the fact that they yield very little information about the higher-frequency motions of the system. This might not be especially serious in computing a transient response to a deterministic input, where the higher-frequency motion is excited only once, and dies out subsequently, leaving only the low-frequency motion. In our problem, however, the turbulence is continually exciting the high-frequency response. Large steps, then, can inherently not yield information about these higher frequencies.

Further research may show that these higher-frequency components have a negligible effect on the resulting criterion value, but it would be dangerous to make this assumption at the outset. In the analog simulation, this point could be investigated. If the higher frequencies do have little effect, then a switch to digital computation could be made, making use of these specially adapted numerical integration methods.

CONCLUSIONS

In any type of design when safety is considered, some model of the threat must be postulated, and the design technique should demonstrate that the resulting design will meet the threat to the degree desired. In the turbulence problem considered here, perhaps the central difficulty is in defining the threat. After reviewing the data available on the gust-related threat, the conclusion was reached that present knowledge is quite imperfect.

The principal problem seems to be not so much in the amount and type of data taken, as in the way it was reduced, and the conclusions drawn from it. The way in which data are reduced should depend rather strongly on what use is to be made of the data. It is one thing to set out to determine a power spectrum. It might be something quite different to attempt to determine how safe flight in a given turbulence patch would be. Indeed, in planning the flight measurement program, account should be taken of the ultimate estimation problem which the data are intended for. A power spectrum or an autocorrelation are of limited interest per se to the student of flight safety. They are but means to an end, and in fact may not be an optimum intermediate step.

The study by Skelton (ref. 73) is one in which an attempt was made, prior to planning the measurement program, to determine just what aspects of the turbulence made a contribution to the ultimate quantities of interest. Something similar should be done relative to thunderstorm and mountain wave turbulence (and their effect on flight safety) before undertaking any new measurement program or re-evaluation of data from past programs.

In the absence of such new data or re-evaluations, it is necessary to proceed with present information. It is proposed that the threat model should consist of a finite-size patch of severe turbulence, whose parameters are based on existing measurements, augmented by additional convenient assumptions where the data are insufficient or inconclusive.

Unsatisfactory though this is, it is felt to be much more realistic than existing procedures which either ignore the stochastic character of the threat (as the 1-cosine gust does) or the nonlinearity of the problem (as the traditional power spectral approach does).

It is concluded that essentially all the threat comes from the severe turbulence environments, in which large gusts are comparatively common. Therefore, it is not necessary to include a range of gust intensities in the analysis. It is proposed that a single type of turbulence encounter will be adequate to test the capability of an aircraft and control system to survive the gust-related threat.

Since the threat is stochastic, the survival capacity must be stochastically expressed also. The most straightforward measure, the probability of surviving an encounter with a "standard patch" of turbulence, seems at present to be the best.

The computational problem of determining this probability can be solved only by a Monte Carlo simulation. For an adequately comprehensive model of the airframe, control system, and turbulence, this simulation is quite difficult. The preferred approach is by use of an analog or hybrid computer. If this is done, it appears that evaluation of the survival probability will be reasonably economical.

The logical next step is to put this to a practical test. Such a test would accomplish several things: (1) it would show conclusively how much effort is required to evaluate the criterion, (2) it would permit selection of some of the parameters of the standard turbulence patch which cannot be fixed at present, and (3) it would provide a test of certain simplifications which might make it possible to ultimately carry out the evaluation on an all-digital computer.

The approach suggested here is capable of almost indefinite refinement without essential change in either point of view or computational methods. Arbitrarily complex models of aircraft, control system, and turbulence can be introduced into the Monte Carlo simulation, the major limitation being the running time required. On an analog or hybrid computer, some types of model complexity would increase running time (e.g., inclusion of higher order normal modes) while others would not (more complex models of turbulence).

As such, the proposed approach seems to offer the beginnings of an extremely flexible and comprehensive tool for studying and specifying the gust-related threat.

APPENDIX A

CONFIDENCE BOUNDS FOR EXCEEDANCE CURVES

While it is seldom stated explicitly, exceedance curves of the type shown in Figures 1, 2, and 3 make use of estimates of certain unknown parameters. These estimates are based on a finite amount of data, and even if those data are free of bias and measurement error, there will still be uncertainties in the result, simply based on the limited extent of the data.

Aside from some early work by Press (ref. 74) there does not seem to have been any consideration of the problem of confidence bounds on exceedance curves. Press was applying the extreme value theory of Gumbel (ref. 75) to the problem of estimating the probable range of the exceedance curve in regions beyond the largest data point. The same theory has been used more recently (ref. 76) to predict the peak wind loads on buildings. This is an approach which could usefully be carried further for aircraft gusts.

For our purposes, however, a simpler result will be sufficient: placing confidence bounds on the data points which have been obtained. In particular, the data points associated with the largest measured magnitudes are of greatest interest. Usually there are only a few exceedances of these high levels, so large-sample approximations cannot be used.

In order to derive confidence limits, it is necessary to make certain postulates about the underlying statistical structure. The principal postulate used here is that exceedances of the higher levels are Poisson-distributed within the flight experience of the aircraft. This will be true provided 1) the probability of an exceedance occurring with a small time interval Δt is proportional to Δt, 2) the probability of two or more exceedances in Δt is negligible compared with the probability of one, and 3) the number of exceedances in a given time interval is independent of the number in any non-overlapping interval. In addition, we make the assumption that the Poisson process is stationary, that is the probability of finding an exceedance in Δt is $\lambda\Delta t$, where λ is a constant.

No conclusive test of this assumption has been made, but it is a reasonable one. It seems intuitively clear that any given point of the flight experience, picked at random will have about the same tendency for exceedances, and that two exceedances in a small time interval are much less likely than one. The independence of non-overlapping intervals is less clear, as there might be some tendency to find the exceedances bunched in certain portions of the record. If one has been found at a certain t, then it might be that neighboring values of t would be more likely than distant ones.

However, even in a patch of severe turbulence, the Poisson distribution might be justified for crossings of levels sufficiently high. It is known, for example, that in a stationary Gaussian process, the crossings of levels sufficiently high are Poisson-distributed in time (ref. 77) (ref. 78).

Accordingly, at least in the limit of large amplitudes, it appears that the Poisson hypothesis would hold even in severe patches of turbulence.

If this hypothesis is accepted, the probability of having k exceedances in a time period T would be

$$p(k,T) = \frac{(\lambda T)^k e^{-\lambda T}}{k!} \tag{A-1}$$

and the expected number of exceedances is λT.

In the measurement program, a certain amount of flight time T_o is analyzed, and a certain number k_o of exceedances (of some fixed level) are observed. The problem is to estimate λ from these data. The maximum-likelihood estimator is

$$\lambda_o = \frac{k_o}{T_o} \quad .$$

The next question is to estimate how close λ_o would be to λ.

Hald (ref. 79), p 722 ff, gives a procedure for computing confidence bounds for this estimator. It is desired to compute two quantities λ_- and λ_+ which have the property that there is a given probability P_2 that $\lambda > \lambda_-$ and there is a given probability P_1 that $\lambda > \lambda_+$, given the observed k_o exceedances in period T. This may be done by use of the chi-squared distribution as follows:

$$\lambda_+ = \frac{1}{2T}\chi^2_{1-P_1}; \quad f = 2(k_o + 1) \tag{A-3}$$

$$\lambda_- = \frac{1}{2T}\chi^2_{1-P_2}; \quad f = 2k_o \tag{A-4}$$

where f is the number of degrees of freedom. These limits may be evaluated from standard tables, once a decision is made on what P_1 and P_2 are desired.

For the confidence bounds of Figure 1, it was decided to set $P_1 = 0.2$ and $P_2 = 0.8$, so that there is a 60 percent probability that the true value of λ lies between λ_- and λ_+. It is more common to use 95 percent or 99 percent levels, but the objective here was to give perhaps a better indication of what the errors would actually be. Forty percent of the time, the errors would be bigger than the limits shown.

Strictly speaking, this procedure applies only to a single data point on the exceedance curve. The various data points are strongly coupled, since the exceedance count for the 1.0 g level contains all the exceedances of 1.2 g's, etc. The data which determine the various points are overlapping. The confidence bounds are shown for several points, but no statement is made concerning whether being within the bound at one point implies being within at another.

The Kolmogoroff-Smirnoff type of test would permit a statement concerning the entire curve, rather than a single point. This test, however, tends to give looser bounds in the region of prime concern: the high-level exceedances, even though it does permit a stronger statement about the curve as a whole.

APPENDIX B

TURBULENCE IN THE ATMOSPHERE

The atmosphere is subject to a considerable variety of types of motion. Not all of them are important to flight safety, and within the category that is important, only a few types relate to the central problem of this study. In this appendix several types of turbulence are reviewed briefly in terms of their relevance to catastrophic occurrences in cruise flight.

General Reviews of Aircraft-Related Turbulence

The types of turbulence which are important to aircraft designers can be specified primarily in terms of frequency. Gust frequencies lower than perhaps once cycle per 400 seconds (as seen in the flying aircraft) will have little importance, even though the power present in the lower frequencies is substantial. High frequencies are important, possibly all the way up to infinity, but there is very little power present above a few cycles per second. Accordingly, it is the higher frequency portion of the atmospheric motion which is important.

For reviews of the overall flight-related turbulence problem, see the books by Taylor (ref. 80), Dobrolensky (ref. 81), and the paper by O'Hara and Burnham (ref. 82). The principal sources of concern are (1) thermal and mechanical turbulence in the first few thousand feet above the ground, (2) turbulence generated by shear in a stable layer, usually designed as CAT, (3) turbulence in storms of all kinds, and (4) turbulence generated by airflow over mountains.

Since this study is concerned primarily with cruise flight, the first of these is not relevant. Accordingly, low-altitude turbulence will not be considered further here, despite the fact that one of the best turbulence measurement and data reduction programs has recently been completed in the very low-altitude regime (ref. 83).

In the following sections, the remaining turbulence sources are discussed, and references are provided for more detailed information.

Thunderstorm Studies

The importance of specialized thunderstorm studies has long been recognized. Flight test programs have been under way more or less continuously for more than twenty years. See Sinclair (ref. 84), Burnham and Lee (ref. 58), and Steiner and Rhyne (ref. 57), for some of the more recent examples. As a result of these flight programs (some correlated with radar measurements) and other studies, a reasonably clear picture exists of the processes at work. References 85 and 86 discuss the general thunderstorm structure. Thunderstormes may contain updrafts of 200 feet per second and downdrafts of somewhat smaller, but still sizable magnitude. The transition from updraft to downdraft may be rather abrupt, and substantial turbulence may be present throughout, of the order of 15 ft/sec rms or more. Incremental accelerations of $\pm$ 3 g have been observed.

Once precipitation develops, the storm can be readily detected by radar, and avoidance should be relatively simple, if proper equipment is available. Early in the storm development, however, there may be substantial air motions, but minor precipitation and radar reflectivity.

Accordingly, storm avoidance in commercial operations is not perfect, but with most operators, it is a high-priority objective. It was estimated some years ago, see Reference (57), that commercial transports spend something between 0.1 percent and 0.01 percent of their flight time in thunderstorms. Probably the smaller figure is more representative of current operations.

There is probably no single area of study which is more significant in gust-related fatal accidents than that of thunderstorms. Burnham, see Reference (3), in surveying the gust-related fatal accidents in commercial passenger operations over a 16-year span, concludes that virtually all are associated with thunderstorms. Despite its critical importance, and despite the extensive flight programs which have been carried out, and despite the numerous thunderstorm encounters in routine operations, both civil and military, it does not appear that there exists a statistically validated model of sufficient accuracy for setting aircraft specifications.

Perhaps the nearest thing is the work of Pritchard, et al, see Reference (59), who correlate the spectra resulting from 88 thunderstorm penetrations. The data show such scatter that few conclusions can be drawn. The largest rms gust velocity in this group is just over 26 ft/sec.

Mountain Lee Waves

Probably the second most serious source of large gusts is the flow pattern set up by mountains. A recent survey by Miles (ref. 87) summarizes the history and current status of the subject, and gives numerous further references. Briefly, it is possible to have disturbed flows when a wind blows over a mountain range or over an isolated peak. The disturbances are essentially gravity waves, and can extend in altitude several times the height of the mountain. The horizontal extent of the disturbed flow can be many times the size of the mountains, and in some cases is of continental dimensions. The lower part of the flow pattern may contain regions of almost purely rotary flow (closed streamlines) or heavy turbulence. Under some conditions, the wave pattern can produce patches of turbulence at almost any altitude.

In terms of the effect on aircraft, these waves can be quite serious. At least one fatal accident (near Mt. Fuji in 1966) has been attributed to this cause. Also, a number of incidents relating to the phenomenon have been reported. Burnham, see Reference (30), cites one case in which the indicated airspeed changed 50 knots in one second. Military aircraft in Colorado have turned up even more imporessive examples. In 1964 a B-52 aircraft had 85 percent of its vertical tail sheared off by a gust later estimated at 100 ft/sec.

Subsequent flights in a fighter aircraft in the same region turned up 25 gusts in excess of 100 ft/sec, with the largest at 175 ft/sec, see Reference (34). While such violent disturbances are relatively rare, they have been observed, and theoretical studies have shown that these occurrences can be explained in terms of the mountain wave phenomenon.

Most of the studies have been theoretical or combinations of theory and laboratory work. There have been, however, a number of studies on full-scale phenomena, using either cloud patterns or aircraft and balloon measurements (ref. 88). Sometimes the wave patterns seem to be quite stable. At other times, they fluctuate rapidly from one state to another.

While it has not been possible to predict the full-scale flow field in detail, it does seem possible to give the general conditions under which the wave phenomena will be found, and the approximate volume affected. As with other gust phenomena, the severe manifestations are comparatively rare, hence difficult to study. Again, a comprehensive and validated statistical description of mountain lee waves is not available.

Clear Air Turbulence

During the past 10 years or so, no aircraft-related atmospheric motion has been studied more thoroughly than the pnenomenon known as clear-air turbulence, usually called CAT. The name is slightly misleading in that not all turbulence occurring in clear air is CAT, as the term is currently used. For example, the turbulence in clear air below or above a storm cloud is not included, nor is turbulence encountered in the lee of mountains. Also, the ubiquitous thermal and mechanical turbulence near the ground is not considered CAT, however clear the air.

CAT has come to mean turbulence generated in a specific way: the Kelvin-Helmholtz instability arising from shear in a stable layer. For surveys, see Dutton and Panofsky (ref. 89), Saxton (ref. 90), Veazey (ref. 91), or Smith and Boyer (ref. 92). When the shear becomes sufficiently strong, a divergent circulatory motion begins. This motion eventually breaks up into random turbulence and decays. CAT tends to occur in flat sheets, a few hundred feet thick and perhaps a few miles in lateral extent. It may be triggered by wave motion or other disturbances. It may also generate wave motions itself. In any event, CAT and gravity waves are frequently associated (ref. 93). There are observational problems in distinguishing between wave motions and CAT (ref. 94), so it is not certain which is the primary effect.

While the fundamental mechanism of CAT has been understood for some years, only recently have observational methods been refined to the point of studying the evolution and decay. Woods' work in the ocean (ref. 95) with dye tracers and the radar observations in the atmosphere by Atlas, Metcalf, and Gossard (ref. 96) have given a very interesting picture of the life cycle of CAT. Thus, it appears that the phenominological aspects of the problem are now in good order. It remains to consider the effect on aircraft.

During the past few years, flight programs have yielded substantial amounts of data on CAT. The most comprehensive of these is the U.S. Air Force program called HICAT. A summary report on the study is given by Crooks, et al (ref. 97), though summaries and partial reporting of results have appeared elsewhere (refs. 98-101). This program involved some 1200 flight hours in twelve different goegraphical areas. Over 50 hours of flight in CAT were recorded. This was not representative of typical operation in that many of the flights were flown specifically to find CAT at times and places where it is likely to occur. Still, a considerable volume of data was gathered, and analyzed in a number of ways. Both c.g. accelerations and absolute gust velocities (three components) were recorded, in addition to meteorological and other variables. Exceedance statistics and power spectra were computed and correlated with various quantities.

Briefly, the picture of CAT which emerges from this and other studies is as follows. CAT occurs in patches. HICAT results (ref. 99) indicate that about half of all patches are less than 100 sec, 90 percent less than 400 seconds. Patches of heavy turbulence tend to be longer than patches of light turbulence, though the statistical standing of this trend is not established. For severe patches alone, half are less than 200 sec. Burnham, see Reference (30), gives a result for severe patches obtained in worldwide civil jet operations of 50 percent less than about 60 seconds. In neither result are confidence bounds given, so it is not possible to state the extent of the disagreement, if any. Most of the true rms gust velocities observed were less than 5 ft/sec.

It was also concluded that a stationary Gaussian model for the turbulence within a single patch was not consistent with the observations. Not only were there changes in rms magnitude, readily detectable in the traces, but the exceedance data were also inconsistent with those predicted with the stationary. Gaussian hypothesis. Non-stationarity is felt to be significant even within a single patch of turbulence.

While the statistical confidence of the presently available data are not all that might be desired, it seems probable that CAT is not a critical threat to flight safety in the sense that storms and mountain waves are. The highest single peak in the HICAT program, for example, gave an incremental 1.1 g at the c.g. with a U_{de} of 22 ft/sec. At no time in the program did the U_{de} exceed about 85 percent of the design gust velocity for the existing altitude, and this, of course, is far removed from the ultimate. While CAT is certainly significant in structural fatigue and passenger comfort, it seems that the risk of disaster from this source is minimal.

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