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THE PROBLEM OF UNIQUENESS IN THERMOSPHERE DYNAMICS

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Abstract

The unique solution of tidal wave propagation within the thermosphere depends on the boundary conditions of the model. It is shown that the radiation condition leads to such a unique solution. Any other boundary values give rise to deviations from the physical solution with unrealistic physical parameters in the environment of the boundaries. The thickness of these boundary layers with unrealistic solutions is a few scale heights below the upper boundary of the model and a few tens of scale heights above the lower boundary of the model.

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CONTENTS

1. INTRODUCTION

Since the pioneering work of Harris and Priester (1962), much has been written in the last decade about the diurnal variation of the neutral thermosphere. Selfconsistent models considering the solar XUV-heat input as the generation mechanism have calculated the corresponding pressure and wind fields at thermospheric heights. They include one-dimensional models (Harris and Priester, 1962, 1965; Lagos and Mahoney, 1967; Blum, 1968; and Chandra and Stubbe, 1970), two-dimensional models (Dickinson et al, 1968; Volland and Mayr, 1970; Isakov, 1971) and three-dimensional models (Lindzen, 1971; Volland and Mayr, 1971). The choice of the number of dimensions as well as the neglect of the one or the other physical process like horizontal heat advection, ion-neutral collisions, Coriolis force or viscosity naturally led to inconsistencies between the various models. However, since computers are available to day large enough to calculate systems of coupled differential equations, these differences should be overcome some day if the same assumptions are made in the various models.

Even then, however, there remains a big problem. It is the selection of the boundary conditions for the model thermosphere. As it is well known from the theory of ordinary differential equations, a system of m differential equations of first order needs exactly m boundary conditions for its unique solution.

The dynamic behavior of the three dimensional thermosphere taking into account heat conduction and viscosity can be treated in the simplest form in terms of a system of eight complex coupled linear inhomogeneous differential equations of first order for the calculation of the height functions of the various physical parameters like pressure p, temperature T and winds $\vec{v} = (u, v, w)$. This requires therefore the determination of eight complex or sixteen real boundary values for the thermosphere model.

For convenience the thermosphere models are usually limited between the heights of about 100 and 1000 km. These boundaries are completely artificial and must be considered as open boundaries through which energy can be transported in each direction. The question arises therefore whether the boundary values at these artificial boundaries can be determined uniquely or at least whether they can be approximately selected such that the solution is consistent with physical conditions.

We shall emphasize in this paper that the radiation conditions can provide such a unique solution for the problem. A simple analytic solution for the generation and propagation of tidal waves at thermospheric heights will be employed to demonstrate the implications and the fall stricks when more or less arbitrary boundary conditions are used that violate these radiation conditions.

2. GENERATION AND PROPAGATION OF TIDAL WAVES

In order to show the influence of the boundary values on thermosphere dynamics we shall select a very simple model. It is strictly valid for tidal wave propagation within the lower nondissipative isothermal atmosphere. It is however also approximately valid within an isothermal thermosphere and reflects essential features of tidal and planetary wave propagation at thermospheric heights. Our assumptions are

a) Isothermy

b) Validity of perturbation theory

c) Heat input proportional to the mean pressure

d) Consideration of gravity waves only.

For convenience we furthermore shall consider only tidal waves with the period of one solar day which is however in no way restrictive to the conclusions to be drawn in the following.

From the equations of mass, momentum and energy, one can derive a system of two linear inhomgeneous differential equations of first order for the various Hough-functions of order n (e.g., Chapman and Lindzen, 1970; Volland and Mayr, 1971)

$$\frac{1}{j\,k_0}\,\frac{d}{d\,z}\left[\frac{w_n}{c_0}\right] + 2\,j\,A\,(1-\kappa)\,\frac{w_n}{c_0} + \left(1-\kappa-\frac{H_0}{h_n}\right)\frac{p_n}{p_0} = -\,j\,J_n$$

$$\frac{1}{j\,k_0}\,\frac{d}{d\,z}\left[\frac{p_n}{p_0}\right] - 4\,A^2\,\kappa\,\frac{w_n}{c_0} + 2\,j\,A\,\kappa\,\frac{p_n}{p_0} = -\,2\,A\,J_n. \tag{1}$$

Here, the following abbreviations have been used:

w_n vertical wind velocity
p_n pressure amplitude
} of the Hough-functions θ_n with frequency $\Omega = 7.27 \times 10^{-5}\ s^{-1}$

c_0 velocity of sound

p_0 mean pressure of the isothermal atmosphere

J_n $= \kappa\,\dot{Q}_n/\Omega\,p_0$ normalized heat function

$\dot{Q}_n$ component of the solar heat input generating the Hough-function θ_n

$\kappa = (\gamma - 1)/\gamma$

γ ratio between the specific heats at constant volumn and at constant pressure

$A = \gamma\, g/2\, c_0\, \Omega$

g gravitational constant

h_n equivalent depth of the Hough-function θ_n

$H_0 = 1/2\, k_0\, A$ scale of the isothermal atmosphere

$k_0 = \Omega/c_0$

z height above ground

We ask for solutions of (1) assuming a constant normalized heat input within the height range between z_0 and z_1 and zero heat input outside this region:

$$J_n = \begin{cases} \text{constant} & \text{for } z_0 \leqq z \leqq z_1 \\ 0 & \phantom{\text{for }} z < z_0;\ z > z_1 \end{cases} \tag{2}$$

and write the general solution for this problem as

$$\frac{w_n}{c_0} = a + b$$
$$\frac{p_n}{p_0} = F_a\, a + F_b\, b \tag{3}$$

with

$$a\,(z) = G_a + C_a \exp\{-\lambda_a\,(z - z_0)\}$$
$$\text{for } z_0 \leqq z \leqq z_1 \tag{3a}$$
$$b\,(z) = G_b + C_b \exp\{\lambda_b\,(z - z_1)\}$$

and

$$a(z) = D_a \exp(-\lambda_a z)$$

$$\text{for } z \geqq z_0; \; z \leqq z_1 \tag{3b}$$

$$b(z) = D_b \exp(\lambda_b z)$$

Here it is

$$\lambda_a = (\beta - 1)/2 H_0$$

$$\lambda_b = (\beta + 1)/2 H_0 \tag{4}$$

$$\beta = \sqrt{1 - \frac{4 \kappa H_0}{h_n}}$$

$$F_a = - j A \frac{(1 + \beta - 2 \kappa)}{(1 - \kappa - H_0/h_n)}; \quad F_b = - j A \frac{(1 - \beta - 2 \kappa)}{(1 - \kappa - H_0/h_n)}$$

$$G_a = - \frac{J_n (1 + \beta - 2 H_0/h_n)}{2 A \beta (1 - \beta)}; \quad G_b = \frac{J_n (1 - \beta - 2 H_0/h_n)}{2 A \beta (1 + \beta)}.$$

C_a, C_b, D_a and D_b are integration constant to be determined below.

Evidently, solution (3b) is the solution of the homogeneous system in (1) while (3a) is the solution of the inhomogeneous system (1) consisting of the sum of the homogeneous solution (3b) and a particular solution of (1).

We want to consider furthermore only so-called trapped or evanescent modes with negative equivalent depths h_n. The fundamental symmetric diurnal tidal mode (1, -1) discovered by Kato (1966) and by Lindzen (1966) is such a trapped mode with $h_1 = - 12.3$ km. Therefore, the exponential factor β in (4) is real and greater than one. λ_a and λ_b become positive and the discrimination of the solution of the physical parameters in (3) is such that it consists of a term a which decreases with altitude and a term b which increases with altitude.

It has been shown (Volland and Mayr, 1971) that at thermospheric heights above about 160 km, nearly all important tidal and planetary waves of gravity wave type are quasi-evanescent. That means, their equivalent depths h_n are complex containing a large negative real part. Therefore, the solutions for these waves at thermospheric heights do not differ very much from (3) apart from the fact that now β in (4) contains a small imaginary part which is responsible for a small phase change with height in the physical parameters w_n and p_n. Thus, the solutions obtain wave properties. The quantities a and b in (3b) can be considered as the up- and downgoing characteristic waves of the problem, with the square root in (4) being their eigen value. For tidal waves at thermospheric heights above 160 km, β is of the order $1 < |\beta| \lesssim 2$, while for planetary waves it is $1.5 \lesssim |\beta| \lesssim 5$.

3. THE BOUNDARY CONDITIONS

3.a The Radiation Condition

For a unique solution of our problem, we have to determine the integration constant C and D in (3a) and (3b). Since the amplitudes of the characteristic waves in (3b) decrease rapidly in the direction of propagation, we can consider a thermosphere model as imbedded within an atmosphere infinitely extended in the vertical direction. That means, we can neglect the earth's surface as well as the outermost atmosphere above the upper boundary of the model. According to our assumptions, the waves are generated exclusively within the height range between z_0 and z_1. Therefore, the waves generated there can only leave the boundaries. No upgoing wave can exist below z_0 and no downgoing wave can exist above z_1 because no energy source is available to generate these waves outside our model. This leads in a straightforward manner to the formulation of the boundary conditions in terms of the radiation conditions:

$$
\begin{aligned}
a &= 0 \quad \text{for} \quad z \leqq z_0 \\
b &= 0 \quad \text{for} \quad z \geqq z_1 .
\end{aligned} \tag{5}
$$

Since at the boundaries the physical parameters w_n and p_n must be contineous according to well known hydrodynamic principles, a and b must be contineous too at z_0 and z_1 within the isothermal atmosphere, and we find from (3a), (3b) and (5) the solution which fulfills the radiation condition:

$$
\left.
\begin{aligned}
a(z) &= G_a \{1 - \exp[-\lambda_a (z - z_0)]\} \\
b(z) &= G_b \{1 - \exp[\lambda_b (z - z_1)]\}
\end{aligned}
\right\} \quad \text{for } z_0 \leqq z \leqq z_1 \tag{6a}
$$

$$
\begin{aligned}
a = 0;\ b &= b(z_0) \exp[\lambda_b (z - z_0)] \quad \text{for } z \leqq z_0 \\
b = 0;\ a &= a(z_1) \exp[-\lambda_a (z - z_1)] \quad \text{for } z \geqq z_1
\end{aligned} \tag{6b}
$$

In order to show the general behavior of a and b, we made numerical calculations which are based on the following somewhat arbitrary data set:

$$
\kappa = 0.3; \ \beta = 3; \ \lambda_a (z_1 - z_0) = 10.
$$

While κ reflects mean lower atmospheric properties, β (and therefore H_n/h_n) has been chosen for convenience such that we obtain values for a and b which are both of the same order of magnitude. β is a measure for the penetration depth of the free internal waves. It is also responsible for the relative importance of the upgoing wave a with respect to the downgoing wave b (see Eqs. (3) and (4)). With decreasing number β, the upgoing wave becomes increasingly larger than the downgoing wave (see Eq. (8)). The specific number of β is irrelevant however in the following qualitative discussions due to the normalizations which we introduce in order to become independent on atmospheric data. The thickness of the layer was chosen large enough so that the two boundaries become practically decoupled from each other. For a normalization we use the quantities

$$\tilde{a} = \frac{A}{J_n} a; \ \tilde{w} = \tilde{a} + \tilde{b} = \frac{A}{J_n} \frac{w_n}{c_0}$$

$$\tilde{b} = \frac{A}{J_n} b; \ \tilde{p} = \tilde{F}_a \tilde{a} + \tilde{F}_b \tilde{b} = \frac{j}{J_n} \frac{p_n}{p_0}$$

$$\tilde{G}_a = \frac{A}{J_n} G_a; \ \tilde{F}_a = \frac{j}{A} F_a \tag{7}$$

$$\tilde{G}_b = \frac{A}{J_n} G_b; \ \tilde{F}_b = \frac{j}{A} F_b$$

$$\tilde{z} = \lambda_a (z - z_0); \ \tilde{\xi} = \lambda_b (z_1 - z)$$

The imaginary number j in $\tilde{p}$ corresponds to a phase shift of 6 h for p_n with respect to w_n in the case of the diurnal variations. The vertical wind w_n peaks at the same time as the heat input J_n. Within the dissipative thermosphere this phase relationship becomes more complicated (Volland and Mayr, 1971).

In Fig. 1, we plotted as solid lines $\tilde{a}$ and $\tilde{b}$ versus $\tilde{z}$. The right ordinate is scaled according to $\tilde{\xi}$. As it is evident from (6), $\tilde{a}$ increases from zero at $\tilde{z} \leq 0$ to an assymptotic value $\tilde{G}$ above $\tilde{z} \sim 5$ which is the particular solution of (3). $\tilde{b}$, on the other hand, increases from zero at $\tilde{\xi} \leq 0$ to $\tilde{G}_b$ above $\tilde{\xi} \sim 5$. Both $\tilde{a}$ and $\tilde{b}$, decrease exponentially outside the boundaries at $\tilde{z} = 10$ and $\tilde{\xi} = 20$ respectively and become insignificant after distances of $\Delta \tilde{z}$, $\Delta \tilde{\xi} \sim 5$ from these boundaries. This strong decay of the free internal waves justifies our assumption about the infinitely extended atmosphere in our thermosphere model.

Evidently, the constant particular solutions $\tilde{G}_a$ and $\tilde{G}_b$ are the solutions for an infinitely extended thermosphere model with heat inputs everywhere ($z_0 \to -\infty$; $z_1 \to \infty$). These solutions are reached within the model at distances of $\Delta \tilde{z}$, $\Delta \tilde{\xi} \sim 5$ from both boundaries and indicate final stages where up- and down-going waves are generated simultaneously at each height. The waves propagate up- and downward away from the respective sources, however without changing their phases. The sums of all these phase locked waves give two resultant up- and downgoing waves of constant amplitude. Here it is always a > b (see (4)) because of

$$\frac{G_a}{G_b} = \frac{(\beta + 1)^2 \, (\beta - 1 + 2\,\kappa)}{(\beta - 1)^2 \, (\beta + 1 - 2\,\kappa)} > 1 \text{ for } (\beta > 1). \tag{8}$$

This ratio decreases with β and is infinite for $\beta \to 1$. It shows that the heat source predominantly generates upgoing waves which propagate into a region of decreasing density.

If we consider an isothermal atmosphere bounded by the rigid earth's surface and extended infinitely into the vertical direction we have to modify our radiation condition (5) because the downgoing waves are totally reflected at the earth's surface (a (0) = - b (0)) which is equivalent to the condition that the vertical velocity disappears there. Together with $z_1 \to \infty$ we obtain from (3)

$$a(z) = G_a - (G_a + G_b)\exp(-\lambda_a z) \qquad (z \geqq \phi) \tag{9}$$
$$b(z) = G_b$$

Here again we can conclude that the influence of the earth's surface is significant only within a distance of $\tilde{z} \lesssim 5$ from below.

We now turn to the discussion of the physical parameters $\tilde{w}$ and $\tilde{p}$ from (3) and (7). These parameters are plotted as solid lines in Fig. 2. $\tilde{w}$ and $\tilde{p}$ have finite values at the boundaries of the model and disappear at infinity. While $\tilde{w}$ is positive everywhere, $\tilde{p}$ is negative below the height $\tilde{z}_{i\,sob} = 0.3$ and it is positive above that height. The height $\tilde{z}_{i\,sob}$ therefore is an isobaric layer where the phase of $\tilde{p}$ jumps by 180°. Since the horizontal winds are proportional to the pressure, they too become zero there and jump in phase. This means that $\tilde{z}_{i\,sob}$ is the center of the circulation cell of the associated wind system. The winds blow away from the maximum of the heat source above $\tilde{z}_{i\,sob}$ and blow toward the maximum of the heat source below $\tilde{z}_{i\,sob}$ in the same manner as in the case of thermally driven winds (e.g. land-sea-breezes).

Pressure and vertical wind become constant far enough away from the boundaries ($\Delta\tilde{z}$, $\Delta\tilde{\xi} \gtrsim 5$) and obtain the values

$$\tilde{w}_\infty = \tilde{G}_a + \tilde{G}_b = \frac{\beta^2 - 1 + 4\kappa}{2\kappa(\beta^2 - 1)} \qquad (\beta > 1) \tag{10}$$
$$\tilde{p}_\infty = \tilde{F}_a \tilde{G}_a + \tilde{F}_b \tilde{G}_b = \frac{4}{\beta^2 - 1}$$

Because of our assumption about the validity of perturbation theory, solutions for Eq. (6) can be determined for various regions with different mean temperature and different heat input and they can be superimposed so that realistic temperature and heat input distribution of the atmosphere can be simulated to a sufficient degree of accuracy (see e.g., Volland and Mayr, 1970).

Since the thermosphere above 200 km is nearly isothermal and since the heat input can be considered as proportional to the mean pressure in a first order approximation (Hays, 1970), solution (10) gives already a fair estimate of thermosphere dynamics. For the implications due to the dissipation effects within thermospheric heights see Volland and Mayr, 1971.

3b. Arbitrary Boundary Conditions

We have seen in section 3a that the unique solution of our problem involves the radiation condition which necessarily needs a discrimination of solution (3) in terms of the eigenfunctions or characteristic waves a and b. This is possible in the simplest case considered in this paper. It is also possible to find analytic expressions for the characteristic waves if heat conduction is taken into account (Volland and Mayr, 1970). However, in the general case which includes heat conduction and viscosity at thermospheric heights, no analytic solutions for the eigen values and the characteristic waves are known.

For this and other reasons most authors started from system (1) or an equivalent more complicated system and solved this system numerically. Therefore, they were forced to select more or less arbitrary boundary values for the physical parameters at the lower or (and) the upper boundary of thier models. A very common boundary condition is

$$p_n = T_n = u_n = v_n = 0 \quad \text{at} \quad z_0$$

$$\rho_0 \, w_n = d\, T_n/d\, z = d\, u_n/d\, z = d\, v_n/d\, z = 0 \quad \text{at} \quad z_1 \tag{11}$$

if heat conduction and viscosity are included in the model. The arguing is that according to (11) the fluxes of mass, momentum and energy vanish at the upper boundary z_1 , and the hydrodynamic fluxes of energy disappear at the lower boundary z_0 (Dickinson et al., 1968).

In the case that only heat conduction is taken into account, the number of boundary values reduces to four. Following Harris and Priester (1962), they are very often taken as

$$w_n = p_n = T_n = 0 \quad \text{at} \quad z_0$$

$$d\, T_n/d\, z = 0 \quad \text{at} \; z_1 . \tag{12}$$

In the case of the nondissipative lower atmosphere, the lower boundary condition is at the earth's surface:

$$w_n = 0 \quad \text{at} \quad z_0 = 0, \tag{13}$$

while the upper boundary condition is the radiation condition according to Wilkes (1949) or is taken from the condition that the vertical energy flow vanishes (Siebert, 1961).

We want now to simulate these boundary conditions and to discuss their physical meaning. Since we have only the two physical parameters w_n and p_n available in our simplified model we first set

$$w_n = p_n = 0 \quad \text{at} \quad z_0 \;\; \text{or} \; z_1 . \tag{14}$$

It follows immediately from (3) that strictly speaking no realistic solution is consistent with that boundary condition because it violates a general principle of wave propagation namely that the time averaged wave energy $\overline{w_n \; p_n}$ must be different from zero within the entire propagation range

Next we consider the condition

$$\begin{matrix} w_n = 0 & & z_0 \\ & \text{at} & \\ p_n = 0 & & z_1 \end{matrix} \qquad (15)$$

which has some similarity with tidal water waves in an ocean of finite deepness. It means, the solid bottom of the ocean leading to the boundary condition $w_n = 0$ corresponds to the bottom of our model while the open surface of the ocean to the air which gives $p_n = 0$ is the top of our model. There exists also a correspondence between this model and electromagnetic waves ducted within a wave guide with one electric and one magnetic wall (Budden, 1961). The lower boundary condition in (15), by the way, is identical with the boundary condition at the earth's surface used in (9) leading to the same solution in (16a) far enough away from the upper boundary.

The resultant up- and downgoing waves corresponding to this boundary condition (15) have been determined from (3) and become (assuming $\exp\{-\lambda_i \; (z_1 - z_0)\} << 1$)

$$a \sim G_a - (G_a + G_b) \exp \{-\lambda_a (z - z_0)\}$$

$$\text{for } z_0 \leqq z \leqq z_1 \qquad (16a)$$

$$b \sim G_b - \frac{(F_a G_a + F_b G_b)}{F_b} \exp \{\lambda_b (z - z_1)\}$$

$$a = a_1 \sim - G_b \exp \{-\lambda \quad (z - z_0)\}$$

$$\text{for } z \leqq z_0 \qquad (16b)$$

$$b \sim G_b \exp \{\lambda_b (z - z_0)\}$$

$$a \sim G_a \exp \{-\lambda_a (z - z_1)\}$$

$$\text{for } z \geqq z_1. \qquad (16c)$$

$$b = b_1 \sim -\frac{F_a G_a}{F_b} \exp \{\lambda_b (z - z_1)\}$$

The normalized values $\tilde{a}$ and $\tilde{b}$ are plotted as dashed lines Fig. 1. Here we notice that in order to maintain condition (15), an upgoing wave $\tilde{a}_1$ must exist below z_0 (see Eq. (16a)) and a downgoing wave $\tilde{b}_1$ must exist above z_1 (see Eq. (16c)). Thus, condition (15) can only be obtained by the superposition of the waves generated by the internal heat source within the model (z_0, z_1) and of two additional waves generated by two respective sources outside of the model. Due to the strong decay of these waves in the direction of propagation, the influence of those fictive sources diminishes with increasing distance from the boundary and virtually disappears at distances of $\Delta\tilde{z}, \Delta\tilde{\xi} \gtrsim 5$.

The corresponding physical parameters $\tilde{w}$ and $\tilde{p}$ are plotted as dashed curves in Fig. 2. Here too, outside the model, pressure and vertical wind increase exponentially due to the corresponding up- and downgoing waves and give rise to an increasingly unrealistic situation outside the model.

An equivalent explanation for the situation which is caused by the boundary condition (15) is the following: The boundaries of the model are a solid boundary at z_0 and an open boundary to a vacuum at z_1. The waves generated within the model by the internal heat source are reflected at both boundaries. The reflected waves interfer with the primary waves which lead to the deviations in Fig. 1 and 2 (the dashed curves) from the undisturbed conditions (the solid lines). The source outside the model can be considered then as fictive sources in "mirror points" in a similar sense as in the case of an electromagnetic dipole field above an electric wall with infinitely large electric conductivity.

If we assume any other combination of w_n and p_n at the boundaries except the unique one derived from the radiation condition

$$\frac{p_n}{p_0} \Big/ \frac{w_n}{c_0} = \begin{cases} F_b & \text{at} \quad z_0 \\ F_a & \phantom{\text{at}} \quad z_1 \end{cases} \tag{17}$$

we obtain equivalent results. We conclude therefore: The introduction of arbitrary boundary values (not consistent with the radiation condition) leads to a wave structure which is unrealistic outside the boundaries of the model and within distances of

$$\Delta z_0 = \frac{10\, H_0}{\beta - 1} \quad \text{from the lower boundary } z_0$$

and (18)

$$\Delta z_1 = \frac{10\, H_0}{\beta + 1} \quad \text{from the upper boundary } z_1.$$

However, inside this region, that is, far enough away from the boundaries within the model, the wave structure becomes identical with the exact physical solution for our problem.

The same conclusion can be drawn for the more sophisticated models which include heat conduction and viscosity. Application of boundary values of the form of (11) or (12) necessarily leads to "reflections" at the boundaries and thus to deviations from the realistic solutions. Because of the strong decay of the downgoing waves, the errors introduced by the upper boundary conditions are not so significant if one considers only those results as physically acceptable which are below a few scale hieghts from the height of the upper boundary (see Eq. (18)). However, in the case of the lower boundary, the penetration depth of the waves from below may be much greater than a few scale heights (e.g. about $20H_0$ for the diurnal tidal (1, -1) mode in the height range between 180 and 200 km). Therefore a tidal wave generated within the lower atmosphere below $z_0 = 100$ km may penetrate into the thermosphere and may contribute significantly to the density amplitude even at 200 km altitude. A unique separation of this wave from the waves generated within the thermosphere above z_0 by the internal heat input is only possible via the application of the radiation conditions.

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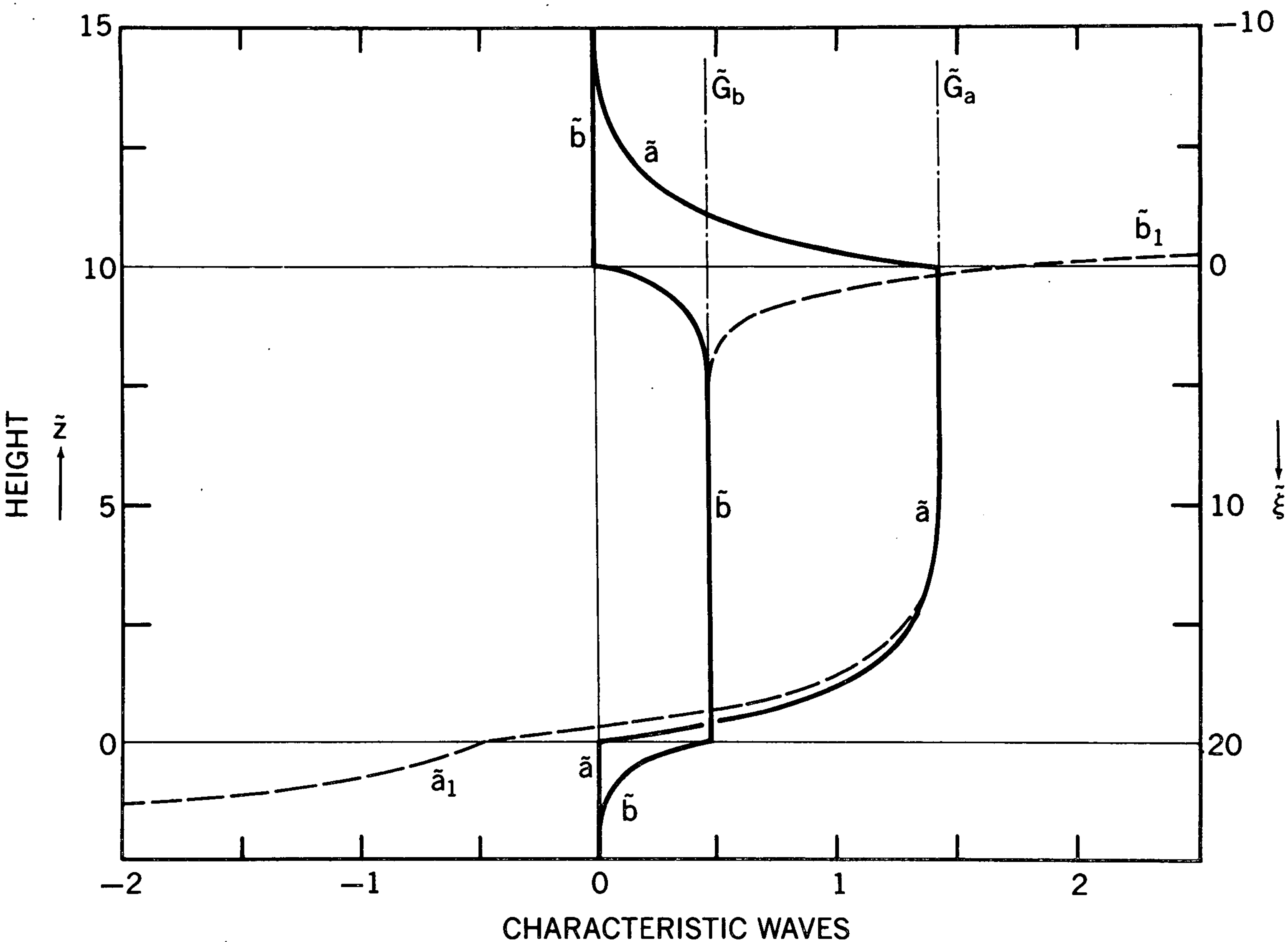

Figure 1. Normalized upgoing (a) and downgoing (b) characteristic waves versus altitude in a model thermosphere. Solid Lines: Radiation condition. Dashed: Arbitrary boundary conditions.

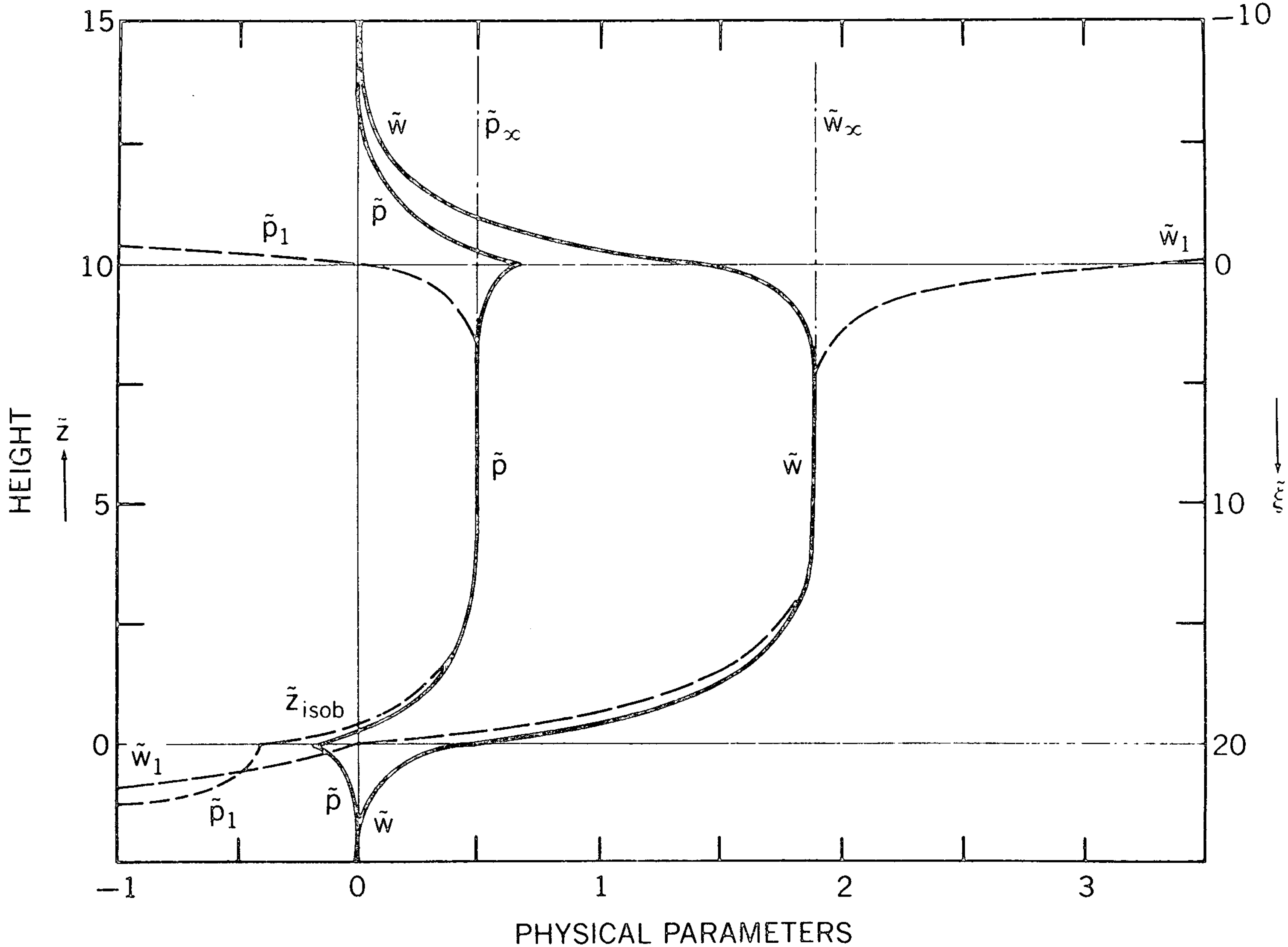

Figure 2. Normalized pressure p and vertical wind w versus altitude in a model thermosphere. Solid lines: Radiation condition. Dashed lines: Arbitrary boundary conditions.