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**NOISE FROM TWO-DIMENSIONAL VORTICES**

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TECHNICAL PAPER proposed for presentation at  
Tenth Aerospace Sciences Meeting sponsored by the  
American Institute of Aeronautics and Astronautics  
San Diego, California, January 17-19, 1972

## NOISE FROM TWO-DIMENSIONAL VORTICES

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### Abstract

The fluctuating flow in an idealized model of a turbulent shear layer composed of many discrete vortices is analyzed. Computer solutions reveal irregular motions which are similar in many respects to observed flows in turbulent three-dimensional layers. The model is further simplified to a pair of equal co-rotating vortices and the noise generation is analyzed in terms of equivalent quadrupole oscillations. Results of the analysis in a uniform medium are consistent with Lighthill's results. New results are obtained for the effects of mean velocity gradients, compressibility, temperature inhomogeneities, and gradients of the mean Mach number.

### Introduction

Lighthill<sup>(1)</sup> relates the sound radiation from a region of time varying flow to the fluctuating shearing stresses. Intervening steps of the analysis relate the shearing stresses to the strengths of the quadrupole radiators and then relate the quadrupole strengths to the sound field. Experimental evaluations of the shearing stresses require difficult and almost impossible measurements of high order correlations. The complicated nature of the equations is a serious bar to engineering judgment and intuition in the estimates of the effects of important variables on noise and the reduction of noise. A number of investigators have attempted to reduce the complexity of the equations by simplifying assumptions or by postulating simplified models of the flow. Usually, the analyses using these simplifications fail to predict the effects of velocity gradients, compressibility, temperature gradients, and Mach number gradients on the sound power from jets.

This paper presents a model flow with the objective of partially clarifying the effects just mentioned. The model consists of freely moving vortices. This selection of a model follows the lead of Powell<sup>(2)</sup> with the exception that the motions are restricted to two dimensions. An early part of the paper shows how an assembly of freely moving, two-dimensional vortices generates a turbulent shear layer having some of the properties of real turbulent shear layers. Subsequently, the model is limited to two vortices thereby permitting a simplified analysis. In the simplified analysis, the sound power is estimated using quadrupole formulas. Following this, the effects of velocity gradients, compressibility, temperature discontinuities, and Mach number discontinuities on sound power are discussed.

The model given here differs from real flows in many significant ways. However, the model does provide a time varying flow which obeys the Euler equations without the addition or subtraction of mass from the system, and without the application of external forces to maintain the time varying velocities.

### Shear Layer Model

A vortex sheet of uniform strength extending to infinity in both directions is shown in Fig. 1(a) along with the profile of velocities induced by the vorticity. The addition of a uniform flow equal and opposite to the induced velocities below the vortex sheet produces the velocities shown in Fig. 1(b). This vortex sheet is unstable and, in time, begins to roll up as shown in Fig. 1(c). Finally, the vortex sheet divides into discrete vortices as shown in Fig. 1(d). Each of these vortices moves with the field induced by all of the other vortices in the flow. As a consequence, each vortex moves in an irregular and somewhat random path. The velocities in the shear layer fluctuate irregularly with time. These fluctuating velocities produce noise in the far field.

Motion-pictures of the motions of two-dimensional vortices in a shear layer were produced from computer solutions of the equations of motion of the vortices. The model used for the computer solution is shown in Fig. 2. A uniform steady vortex sheet extends from minus infinity to zero; a steady vortex field fans out from  $X = 0$  to  $X = 1$ ; between  $X = 1$  and  $X = 3$  the vorticity is concentrated in 50 quasi randomly placed vortices; and steady vorticity is distributed in a fan-shaped region extending from  $X = 3$  to plus infinity. Other significant assumptions for the computer solutions are:

1. Mean flow velocity is parallel with the X-axis in all parts of the field.
2. Each vortex moves with the local velocity of the fluid.
3. Each vortex has a viscous core whose radius increases with the square-root of time.
4. Outside the vortex cores, the effects of viscosity are zero.
5. When two vortices come close together so that the center of one vortex lies within the core of the other, the pair of vortices is replaced by a single vortex whose strength is the sum of the two vortex strengths and whose core has an area equal to the sum of the areas of the two vortices.
6. When a vortex passes through the boundary at  $X = 3$ , it is removed from the ensemble and new vortices having the same total strength are introduced at  $X = 1$ .
7. The initial radii of the cores as they are introduced at the left boundary were chosen by a trial procedure such that the number of vortices crossing the left boundary was three times the number crossing the right boundary. This choice established similarity of flow at the two stations.

After the computer solution proceeds for a time, the initial distribution of vortices disappears and a stationary statistical distribution is approached.

An instantaneous distribution after the initial distribution has disappeared is shown in Fig. 2. The vortices are represented by discs whose diameters are equal to the core diameters. Trajectories of three selected vortices are shown in Fig. 3. The three vortices coalesce and form one vortex.

The motion pictures show an irregularly swirling flow field which is similar in many respects to high-speed shadowgraph motion pictures of turbulent shear flow. We see therefore, that an assembly of freely moving two-dimensional vortices can generate a shear layer with fluctuating velocities; the shear layer grows as it moves downstream; and similarity can be maintained in the growth. This shear layer has many of the properties of real turbulent shear layers.

The near sound field associated with the model can be calculated by the computer in a straightforward manner. With the introduction of retarded time, the far sound field can be calculated by the computer also.

A closed form analysis of the motions and sound field of the large assembly of vortices is not possible. By restricting the model to two vortices, closed form analysis is possible. Such analysis would be helpful in understanding the noise production by the more complete model. In the next section, we will analyze the sound generated by the interaction between two individual vortices.

#### Twin Vortex Model

The noise from two three-dimensional vortices was the subject of a very elegant investigation by Powell.<sup>(2)</sup> A much less elegant investigation of two-dimensional vortices follows. The motions of a pair of two-dimensional vortices shown in Fig. 4 can be described analytically. The two vortices rotate in the same sense, have equal strengths  $B/2$ , and are separated by distance  $d$ . Each vortex moves with the induced field of the other vortex. The pair move around a common circle with diameter  $d$  and with an angular frequency  $\omega$  given by

$$\omega = \frac{B}{2\pi d^2} \quad (1)$$

This simplified model produces time-varying velocities, obeys Euler's equations, and does not require external forces or flows to maintain it.

Our next task is to develop analytical expressions relating the vortex velocities to the radiated sound power. The first step in the analysis is assuming that the orbital diameter  $d$  is small compared to the radius of the near-field at the frequency  $\omega$ . The radius of the near-field is the reciprocal of the wave number  $k$  (or, equivalently, the radius is equal to  $c/\omega$  where  $c$  is the speed of sound). We then place a control cylinder having radius  $r$  concentric with the orbit and surrounding the region in which the vortices move (Fig. 4). An expression for the fluctuating tangential velocities at the control cylinder will be written. A quadrupole will be placed at the center of the cylinder with orientation and strength required to duplicate the fluctuating velocities produced by the rotating vortices. It is then a simple matter to relate the radiated acoustic power to the quadrupole strength, frequency, and fluid properties. Matching

of the quadrupole to the vortices could have been accomplished just as well using radial velocities, velocity potentials, or pressures.

The tangential velocities at the surface of the control cylinder at an instant of time are given by the following series expression:

$$U_t = \frac{B}{2\pi r} + \frac{Bd^2}{8\pi r^3} \cos 2\theta \dots \quad (2)$$

where

$U_t$  tangential velocities at the surface of the cylinder

$B$  sum of the two vortex strengths

$d$  diameter of the orbit

$r$  radius of the control cylinder

$\theta$  angular orientation as shown in Fig. 4

The first term in the equation is simple vortex motion; the second term is the deviation from the simple vortex motion.

A two-dimensional, steady-state quadrupole with strength  $A$  at the center of the cylinder will produce the tangential velocities at the control surface according to the following approximate near-field equation:

$$U_t = \frac{A}{r^3} \cos 2\theta \quad (3)$$

This equation is derived from the equation for velocity potential as given by Rayleigh.<sup>(3)</sup> By comparing equations (3) and (2), we see that if we choose the quadrupole strength such that

$$A = \frac{Bd^2}{8\pi} \quad (4)$$

then the quadrupole velocities will match the velocity deviations from simple vortex velocities at the control surface.

Equations (2) and (3) describe velocity patterns at the instant of time that the vortices are in the position shown in Fig. 4. The vortex pair rotates with the angular velocity  $\omega$  as described earlier and, consequently, the pattern described by equation (2) rotates at the same angular velocity. The velocities at the control surface can be approximated by giving the matching quadrupole the same angular velocity  $\omega$ .

Equations relating the near- and far-field velocities of rotating two-dimensional quadrupoles are not well known. A rotating quadrupole can be synthesized from two equal non-rotating quadrupoles whose axes are displaced 45 degrees in azimuth and displaced 90 degrees in phase. The equation for the near-field velocities of the rotating quadrupole is

$$U_t = \frac{A}{r^3} \left[ e^{i\omega t} \cos 2\theta + e^{i(\omega t + \pi/2)} \sin 2\theta \right] \quad (5)$$

The corresponding acoustic velocities in the far field are given by the following equation which can be derived from Rayleigh:(3)

$$U_f = -(1 - i) \frac{\sqrt{\pi}}{8} \frac{Ak^{5/2}}{\sqrt{R}} \left[ e^{i(\omega t - kR)} \sin 2\theta - e^{i(\omega t - kR + \pi/2)} \cos 2\theta \right] \quad (6)$$

where

- $U_f$  acoustic velocity in the far field  
 $i$   $\sqrt{-1}$   
 $A$  quadrupole strength  
 $k$   $\omega/c$   
 $\omega$  angular frequency  
 $c$  speed of sound  
 $R$  distance from quadrupole to point where  $U_f$  exists  
 $t$  time  
 $\theta$  azimuthal direction of  $R$

This equation represents a uniformly rotating pattern and, consequently, the acoustic radiation is omni-directional. The equation can be used to calculate sound intensities in the far field and the result can be integrated to give acoustic power. The result, stated in terms of near field tangential velocity  $U_t$  at distance  $r$  is

$$\text{Power} = \left[ \frac{\pi^2}{16} (\rho c) \left( \frac{\omega r}{c} \right)^5 r \right] U_t^2 \quad (7)$$

where  $(\rho c)$  is the characteristic impedance of the surrounding medium. The quantity in brackets is the acoustic resistance of the surrounding medium to the velocity perturbation  $U_t$ .

The frequency  $\omega$  is proportional to velocity and equation (7) becomes

$$\text{Power} \approx \frac{\rho U_t^7}{4}$$

Thus, the acoustic power from a rotating two-dimensional quadrupole is omni-directional, is proportional to density and the seventh power of the velocity, and inversely proportional to the 4th power of the speed of sound.

In the three-dimensional case, equation (3) becomes

$$U_t = K_2 \frac{A}{r^4} \quad (8)$$

and equation (7) becomes

$$\text{Power} = \left[ K_3 (\rho c) \left( \frac{\omega r}{c} \right)^6 r^2 \right] U_t^2 \quad (9)$$

where  $K_2$  and  $K_3$  are directionality and proportionality parameters, respectively, for quadrupoles.

Again, the frequency  $\omega$  is proportional to velocity. Equation (9) becomes

$$\text{Power} \approx \frac{\rho U_t^8}{c^5}$$

In this three-dimensional case, acoustic power is proportional to density, the eighth power of velocity, and inversely proportional to the fifth power of the speed of sound. This is Lighthill's famous eighth power law. Powell(2) obtained the same result with vortices.

An equation for the average acoustic power from the twin two-dimensional vortices can be derived from equations (1), (5), and (7):

$$\text{Average Power} = \frac{1}{2} \frac{16}{\pi^5} \frac{(\rho c)}{c^5} \frac{B^7}{d^6} \quad (10)$$

We see, therefore, that two equal two-dimensional vortices generate acoustic power proportional to the seventh power of the vortex strength and inversely proportional to the sixth power of the separation distance.

#### Some Significant Characteristics of the Model

##### Velocity Gradient Effects

The addition of a uniform mean velocity to the rotating vortex pair produces translation of the orbit but the total acoustic power radiated is unaffected. The addition of a gradient of mean velocity will elongate the orbit and reduce the angular frequency  $\omega$  as shown in Fig. 5. The abscissa is the velocity gradient non-dimensionalized so that at a value of 1.0, the gradient produces a velocity differential across the orbit that matches the orbital velocities. The symbols  $U_0$  and  $\omega_0$  represent orbital linear velocity and orbital angular velocity, respectively, for the reference circular orbit when no mean velocity gradient is present. The elongation of the orbit increases the acoustic power, and this increase more than offsets the decrease in power accompanying the reduction in orbital frequency. The elongated orbit produces directional variations in radiation with maximum radiation aligned parallel to the mean flow. Fig. 6 shows the variation of total acoustic power and the maximum far field intensity with increasing non-dimensional velocity gradient. Both the total power and the intensity in the direction of maximum radiation increases rapidly with increasing velocity gradient. When the value of the non-dimensional gradient is 0.25, the total power is doubled (3 dB) and the maximum intensity is quadrupled (6 dB).

These results are similar in some respects to the results of Ribner's(4) analysis of "self" and "shear" noise in real turbulent flow.

## Compressibility Effects

The strengths of each of the two vortices is constant with time and, therefore, there is no "dilatation" or net flow into or out of the control cylinder with time. By assumption, the separation between the two vortices is small compared to the near field radius  $c/\omega$  and, therefore, the motions near the vortices are nearly incompressible. As with all compact source regions, compressibility of the fluid in the near field is not essential to sound power generation. On the other hand, compressibility of the fluid in the far field is essential to the generation of sound power. Acoustic power is proportional to the square of a characteristic near-field velocity,  $U$ , in equation (7), and the acoustic resistance of the surrounding fluid. The acoustic resistance is dependent upon the speed of sound  $c$  and, therefore, upon the compressibility in the far field.

## Temperature discontinuities

A temperature discontinuity which is separated from the vortex pair by a large distance compared to the near-field radius  $c/\omega$  will not change the total sound power radiation. The discontinuity will redirect the sound by reflection and refraction.

If a temperature discontinuity passes through the near field of the vortex pair, the acoustic impedance and total noise generation will be modified. In Fig. 7, a plane surface of discontinuity passes through the near field. The equations for the reflection and transmission were used to analyze the effects of the temperature discontinuity on the acoustic resistance. Results of the analysis, Fig. 7, show that as the temperature  $T_2$  of the fluid on the remote side of the discontinuity is raised above the ambient temperature  $T_1$ , the acoustic resistance falls and, at very high temperatures, varies inversely with the square root of the temperature. If the temperature  $T_2$  is decreased toward zero, the acoustic resistance approaches double the value for a uniform medium. Identical results are obtained if the rotating vortex pair is placed in the variable temperature fluid.

In the range of temperature ratios between 1.0 and 10, the acoustic resistance, and acoustic power, vary approximately with the -0.35 power of the temperature ratio.

We conclude that acoustic power from the vortex model is a complicated function of the temperature ratio and the geometry of the discontinuity. Also, the temperature effects are not large.

## Mach Number Discontinuities

Strong Mach number gradients and discontinuities in the far field of any source will have negligible effect on the total sound power radiated; such gradients and discontinuities will, however, redirect the sound by reflection and refraction. Strong Mach number gradients in the near field will increase the acoustic power from compact source regions such as the spinning vortex model. A moderately extreme case is shown in Fig. 8 in which a source is placed close to a surface of discontinuity across which the Mach number differential approaches unity. The near-field radius  $c/\omega$  is as-

sumed to be large compared to the distance between the source and the discontinuity. According to Blokhintsev<sup>(5)</sup>, the pressures accompanying a given velocity perturbation varies with a complicated function of Mach number including a factor  $(1 - M^2)^{-1/2}$  where  $M$  is the Mach number differential. Therefore, as  $M$  approaches unity, the pressures required to distort the discontinuity approach infinity. At  $M = 1$ , the factor  $(1 - M^2)^{-1/2}$  becomes infinity and the discontinuity acts like a rigid wall. An image of the source appears on the opposite side of the discontinuity and is equal in strength and sign to the real source. Consequently, the pressures in the stationary fluid are doubled, the acoustic intensity is quadrupled, but, because radiation is into half of the total space, the acoustic power is doubled (3 dB).

In a more extreme situation, Fig. 9, the source is between two Mach number discontinuities. If the Mach number differentials across each discontinuity is unity, all perturbations are confined to the region between the discontinuities and the radiation is one-dimensional and parallel with the mean flow. The acoustic resistance is approximately inversely proportional to the separation  $S$  between the two discontinuities. This analysis fails when  $S$  is reduced to values close to the diameter of the orbit.

We conclude that a single Mach number discontinuity in the near field of a source may double the acoustic power; two discontinuities may cause much larger increases.

## Conclusions

The following conclusions apply to the two-dimensional vortex model of this report and do not necessarily apply to real turbulent flows:

1. A two-dimensional model of a flow field consisting of a combination of freely moving discrete vortices and a distributed fixed field of vorticity generates a shear layer with fluctuating velocities. The flow maintains statistical similarity and grows as it moves downstream. The layer has some other properties of real turbulent layers.

2. A simplified model consisting of two equal two-dimensional vortices moving in a common orbit is shown to generate noise by a quadrupole mechanism. The noise from the two-dimensional quadrupole varies with the seventh power of velocity. In contrast, a three-dimensional quadrupole typical of real turbulence radiates noise in proportion to the eighth power of the velocity.

3. Although the total acoustic power is independent of mean translational velocity, a gradient of mean velocity will cause a marked increase in total power and a marked departure from omni-directional radiation. Quantitative effects of the gradient of mean velocity are shown.

4. The model does not produce a "dilatation" effect. Compressibility of the fluid in the near field is not essential to sound generation.

5. Temperature discontinuities in the far field do not influence the total acoustic power but do redirect the sound by reflection and refraction. Temperature discontinuities in the near field have minor effects on the acoustic power; the effects are

a complicated function of the geometry and temperature ratio of the discontinuity. In one extreme case, the total acoustic power varied approximately with the  $-0.35$  power of the temperature ratio over a range of ratios between 1.0 and 10.

6. A strong gradient or discontinuity of the mean Mach number in the near field will increase the total acoustic power. The maximum increase from a single plane discontinuity is 3 decibels. Two parallel plane discontinuities on opposite sides of the vortex pair can cause greater increases in acoustic power.

#### References

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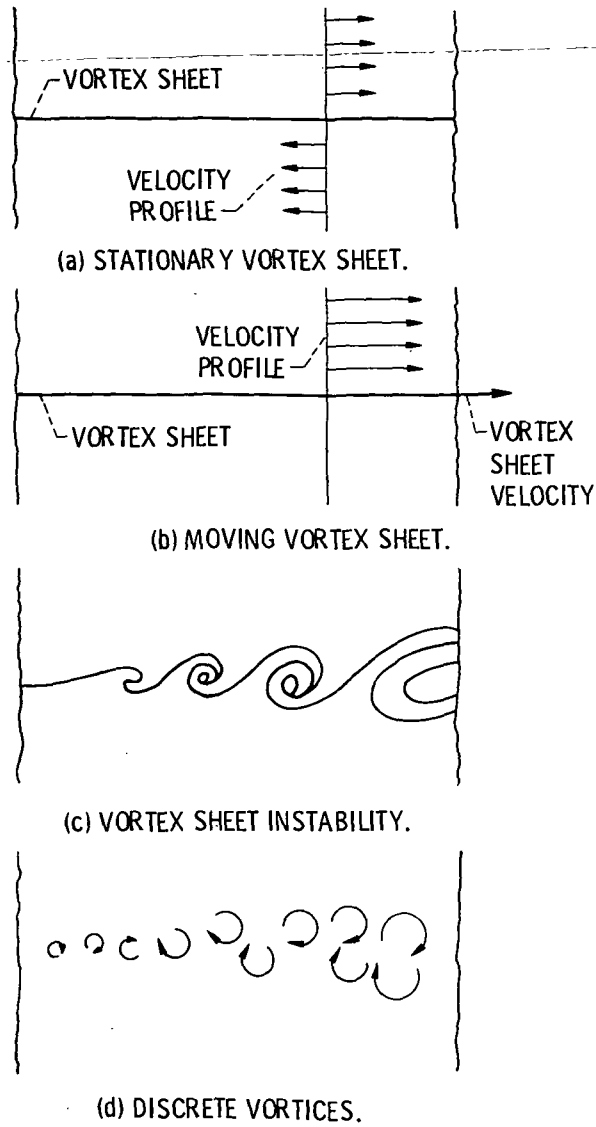


Figure 1. - Vortex representation of a shear layer.

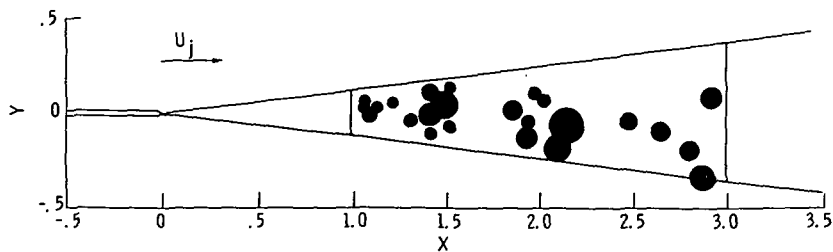


Figure 2. - Model shear layer.

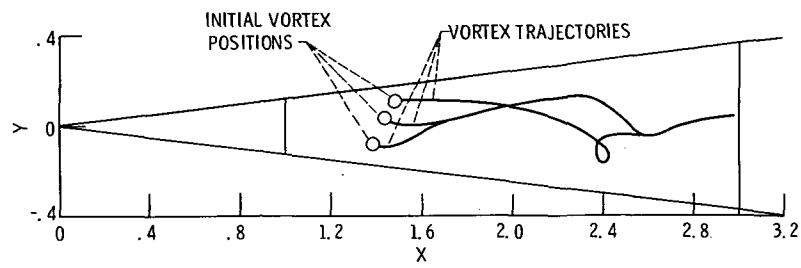


Figure 3. - Trajectories of three vortices.

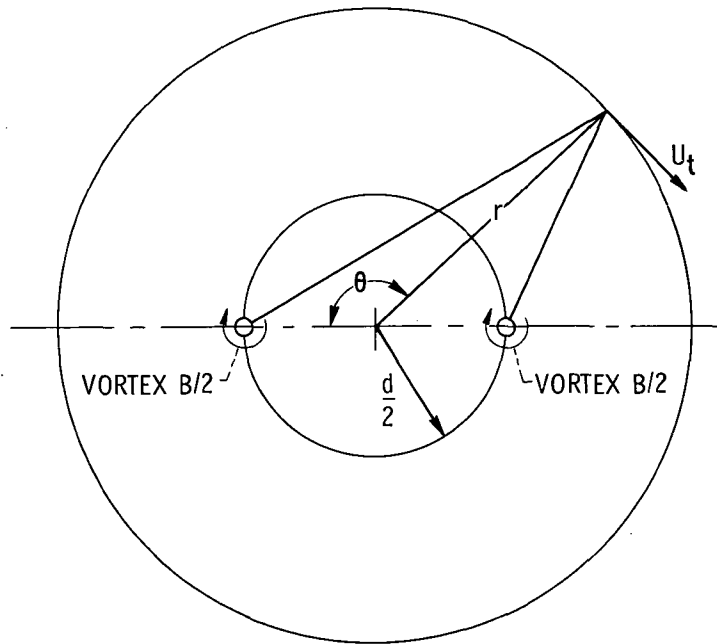


Figure 4. - Two-dimensional twin vortex model.



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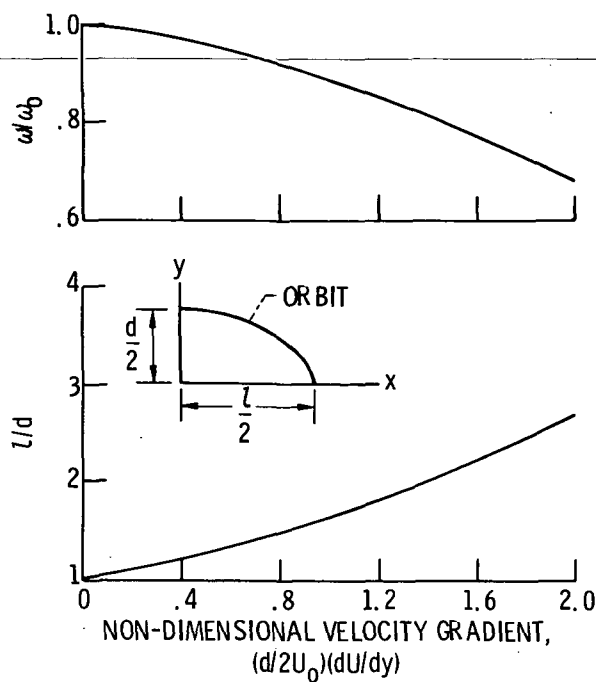


Figure 5. - Effect of velocity gradient on orbit of vortex pair.

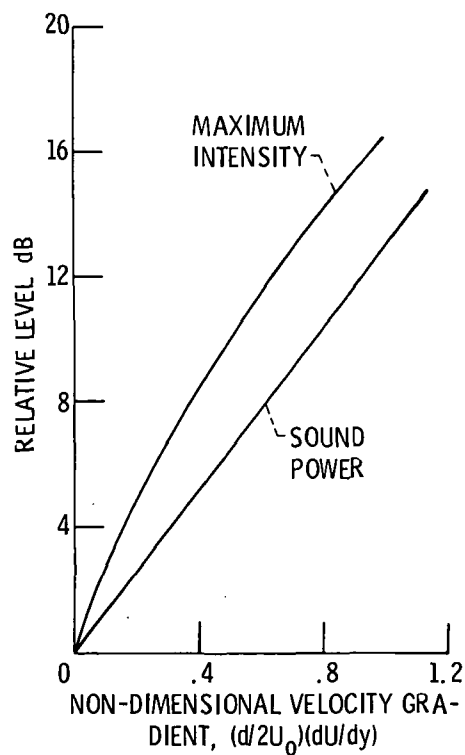


Figure 6. - Effect of velocity gradient on sound generation.

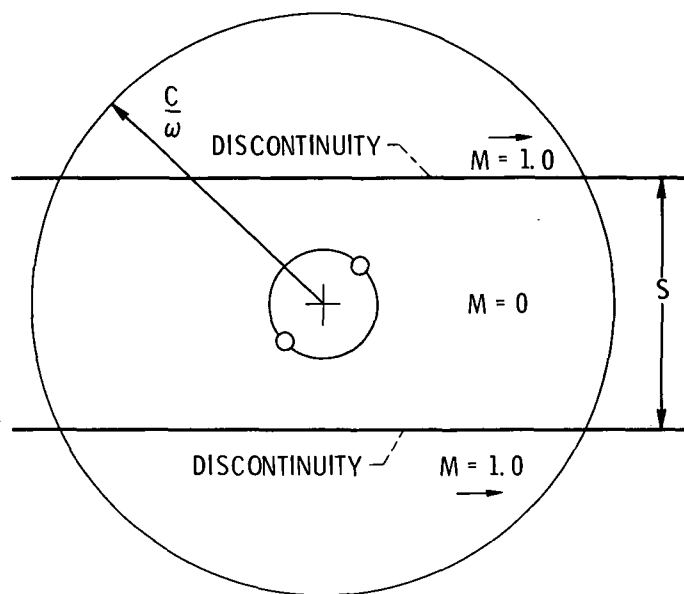


Figure 9. - Vortices in a region between streams moving at sonic velocities.