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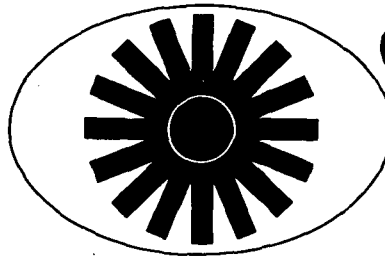
HEAT ADDITION TO A SUBSONIC BOUNDARY LAYER

A PRELIMINARY ANALYTICAL STUDY

TEES-1178-TR-71-01

by

JOHN M. MACHA AND DAVID J. NORTON



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September 1971

Prepared for

NASA Manned Spacecraft Center

Aerodynamics and Entry Section

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**ABSTRACT**

This report is a preliminary analytical study of the effects of heat addition to the subsonic boundary layer flow over a typical airfoil shape. This phenomenon becomes of interest in the space shuttle mission since heat absorbed by the wing structure during re-entry will be rejected to the boundary layer during the subsequent low speed maneuvering and landing phase. A survey of existing literature and analytical solutions for both laminar and turbulent flow indicate that a heated surface generally destabilizes the boundary layer. Specifically, the boundary layer thickness is increased, the skin friction at the surface is decreased and the point of flow separation is moved forward. In addition, limited analytical results predict that the angle of attack at which a heated airfoil will stall is significantly less than the stall angle of an unheated wing. These effects could adversely affect the lift and drag, and thus the maneuvering capabilities of booster and orbiter shuttle vehicles. A final report will be submitted upon completion of wind tunnel testing of a heated, symmetrical airfoil section.

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## NOMENCLATURE

|            |                                                    |
|------------|----------------------------------------------------|
| a          | speed of sound                                     |
| C          | chord length                                       |
| $C_p$      | specific heat at constant pressure                 |
| H          | shape parameter ( $\delta^*/\theta$ )              |
| M          | Mach number                                        |
| P          | pressure                                           |
| Pr         | Prandtl number                                     |
| R          | Reynolds number                                    |
| T          | absolute temperature                               |
| U          | velocity                                           |
| u          | streamwise component of velocity in boundary layer |
| v          | normal component of velocity in boundary layer     |
| x          | streamwise spacial coordinate                      |
| y          | normal spacial coordinate                          |
| $\gamma$   | ratio of specific heats                            |
| $\delta$   | velocity boundary layer thickness                  |
| $\delta^*$ | displacement boundary layer thickness              |
| $\theta$   | momentum boundary layer thickness                  |
| $\mu$      | coefficient of absolute viscosity                  |
| $\nu$      | coefficient of kinematic viscosity                 |
| $\rho$     | density                                            |
| $\tau$     | shearing stress                                    |

### Superscripts

' denotes differentiation with respect to  $x$

### Subscripts

H total or stagnated conditions

w conditions at wall

1 conditions at outer edge of boundary layer

$\infty$  freestream conditions

## HEAT ADDITION TO A SUBSONIC BOUNDARY LAYER -

### A PRELIMINARY ANALYTICAL STUDY

#### 1. Introduction

The purpose of this report is to investigate the effects of heat addition to subsonic boundary layers, especially those of external flows over aerodynamic surfaces. This will be accomplished by studying the literature and the available analytical methods for both laminar and turbulent flow. The prime variable that makes the subject investigation different from conventional flow is the ratio of the wall temperature to the free stream temperature,  $T_w/T_\infty$ . Several laminar and turbulent flow solution techniques have been programed for numerical solution and are presented for linearly retarded flows, as well as for typical airfoil flows.

In the space shuttle mission both the orbiter and the booster will be subjected to a high speed re-entry. The re-entry will be followed by pitch over maneuver and subsequent subsonic maneuvering prior to a conventional aircraft landing sequence. The booster re-entry may be at a sufficiently low altitude to warrant a passive, heat sink, thermal protection system. The orbiter, however, will almost certainly require an ablative-type thermal protection system. Thus, both vehicles will be subjected to appreciable heat loads which will soak into the structure and the aerodynamic surfaces. After the pitch over maneuver the heated surfaces will begin

rejecting heat to the boundary layer.

The effects on the boundary layer due to the stagnation of supersonic and hypersonic flow have received a good deal of attention in recent years. Both experimental and analytical data are available to predict the heat transfer to the surfaces and the behavior of the boundary layer. The inverse problem, that of heat transfer from the wall to the boundary layer, has been largely ignored.

A survey of the available literature, which deals primarily with laminar flow, indicated that stability of the boundary layer is significantly reduced as  $T_w/T_\infty$  becomes greater than unity. Thus it may be anticipated that transition from laminar to turbulent flow and separation are hastened by heat transfer to the boundary layer. It is possible, therefore, that the pressure drag of the airfoil may be increased and its useful angle of attack range may be decreased. Thus, the maneuvering capabilities of the booster and orbiter vehicles may be adversely affected during the subsonic cruise and landing phases. The following sections will investigate the qualitative and quantitative effects of heat transfer to a subsonic boundary layer in laminar and turbulent flow.

## 2. Qualitative Consideration of the Effects of

### Wall Temperature on Laminar Flow

The following qualitative conclusions on the effects of wall temperature on laminar flows have been reached by various



investigators:

1. heating the wall tends to increase the direct\* effect of a pressure gradient;
2. heating the wall tends to move the point of separation upstream; and
3. heating the wall tends to diminish skin friction in an adverse pressure gradient.

These conclusions have been reached on the basis of a Karman-Pohlhausen type of analysis by Morduchow and Grape (2), on the basis of similarity solutions by Cohen and Reshotko (3) and Li and Nagamatsu (4), and by use of the Illingworth-Stewartson transformation (5). While the just cited references establish conclusions 1, 2, and 3 in detail, it is possible to show here how they might be reached.

The first two conclusions are usually explained by the argument that the decrease in the density due to heating makes the fluid more susceptible to an adverse pressure gradient. This argument, however, is not complete, since not only are the density dependent inertial forces reduced, but also the viscous forces at the wall. It is necessary, therefore, to delve somewhat deeper into boundary layer theory to make any valid conclusions.

---

\*According to Morduchow (1), by "direct" effect is meant the influence of the gradient term  $\partial P / \partial x$  proportional to  $dU_1/dx$ . The influence of a pressure gradient also appears "indirectly" through the dependence of  $U_1$  and  $T_1$  on  $X$ .

In what follows it will be shown how heating of the boundary layer effects the pressure gradient term. This will be accomplished by transforming the boundary layer equations to an incompressible form. The results yield an equation with the pressure gradient term multiplied by a function  $S$ , where  $S$  is the ratio of local stagnation enthalpy to the stagnation enthalpy at the edge of the boundary layer.

The momentum boundary layer equation for steady two-dimensional compressible flow may be expressed as

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_1 U_1 \frac{dU_1}{dx} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) \quad (2-1)$$

Now define the stream function  $\Psi$  by

$$\rho u = \rho_\infty \frac{\partial \Psi}{\partial y}, \quad \rho v = -\rho_\infty \frac{\partial \Psi}{\partial x} \quad (2-2)$$

The Dorodnitsyn-Howarth transformation (6) is used to remove the density from the equation, and introduces a new variable

$$\bar{Y} = \int_0^y \frac{\rho dy}{\rho_\infty} \quad (2-3)$$

instead of  $y$ . The velocity components can now be given in terms of the stream function by

$$u = \frac{\partial \Psi}{\partial \bar{Y}}, \quad v = -\frac{\rho_\infty}{\rho} \left\{ \frac{\partial \Psi}{\partial x} + u \left( \frac{\partial \bar{Y}}{\partial x} \right)_y \right\} \quad (2-4)$$

Now the momentum equation can be written as

$$\frac{\partial \Psi}{\partial \bar{Y}} \frac{\partial^2 \Psi}{\partial x \partial \bar{Y}} - \frac{\partial \Psi}{\partial x} \frac{\partial^2 \Psi}{\partial \bar{Y}^2} = \frac{\rho_1}{\rho} U_1 \frac{dU_1}{dx} + \frac{\partial}{\partial \bar{Y}} \left( \frac{\mu \rho}{\rho_\infty} \frac{\partial^2 \Psi}{\partial \bar{Y}^2} \right) \quad (2-5)$$

Using the equation of state in the forms

$$\frac{\rho_1}{\rho} = \frac{T}{T_1}, \quad \frac{P_1}{P_\infty} = \frac{\rho}{\rho_\infty} \frac{T}{T_\infty} \quad (2-6)$$

$\rho$  can be removed explicitly from the momentum equation giving

$$\frac{\partial \Psi}{\partial \bar{Y}} \frac{\partial^2 \Psi}{\partial x \partial \bar{Y}} - \frac{\partial \Psi}{\partial x} \frac{\partial^2 \Psi}{\partial \bar{Y}^2} = \frac{T}{T_1} U_1 \frac{dU_1}{dx} + \frac{P_1 T_\infty}{\rho_\infty P_\infty} C(x) \frac{\partial^3 \Psi}{\partial \bar{Y}^3} \quad (2-7)$$

where  $C(x) = \frac{\mu}{T}$  is the Chapman (7) viscosity law. (For a model fluid, it is assumed that  $C(x) = \mu_\infty/T_\infty$ ). If the mainstream is homoenergetic, then

$$\frac{1}{2} U_1^2 + \frac{a_1^2}{\gamma-1} = \frac{1}{2} U_\infty^2 + \frac{a_\infty^2}{\gamma-1} \quad (2-8)$$

Assuming  $Pr = 1$ , the total temperature at the outer edge of the boundary layer is constant and is expressed as

$$T_{H_1} = T_\infty \left( 1 + \frac{\gamma-1}{2} M_\infty^2 \right) \quad (2-9)$$

Now write the total temperature anywhere in the boundary layer as

$$T_H(x, \bar{Y}) = T_\infty \left( 1 + \frac{\gamma-1}{2} M_\infty^2 \right) \cdot S(x, \bar{Y}) \quad (2-10)$$

and the temperature as

$$T(x, \bar{Y}) = T_\infty \left( 1 + \frac{\gamma-1}{2} M_\infty^2 \right) \cdot S(x, \bar{Y}) - \frac{\gamma-1}{2a_\infty^2} T_\infty \left( \frac{\partial \Psi}{\partial \bar{Y}} \right)^2 \quad (2-11)$$

where the function  $S(x, \bar{Y})$  is to be determined. The momentum equation

(2-7) now reduces to

$$\frac{\partial \Psi}{\partial \bar{Y}} \frac{\partial^2 x}{\partial x \partial \bar{Y}} - \frac{\partial \Psi}{\partial x} \frac{\partial^2 x}{\partial \bar{Y}^2} = U_1 \frac{dU_1}{dx} \frac{a_\infty^2}{a_1^2} \left\{ \left[ 1 + \frac{\gamma-1}{2} M_\infty^2 \right] S \right. \\ \left. - \frac{\gamma-1}{2a_\infty^2} \left( \frac{\partial \Psi}{\partial \bar{Y}} \right)^2 \right\} + \frac{P_1 T_\infty}{\rho_\infty P_\infty} C(x) \frac{\partial^3 \Psi}{\partial \bar{Y}^3} \quad (2-12)$$

The next step is to remove the  $\left( \frac{\partial \Psi}{\partial \bar{Y}} \right)^2$  term by stretching the  $\bar{Y}$  coordinate such that

$$Y = \frac{a_1}{a_\infty} \bar{Y} = \frac{a_1}{a_\infty} \int_0^Y \frac{\rho dy}{\rho_\infty} \quad (2-13)$$

Equation (2-12) now reduces to

$$\frac{\partial \Psi}{\partial Y} \frac{\partial^2 \Psi}{\partial \Psi \partial Y} - \frac{\partial \Psi}{\partial x} \frac{\partial^2 \Psi}{\partial Y^2} = \left( \frac{a_\infty}{a_1} \right)^4 U_1 \frac{dU_1}{dx} \left( 1 + \frac{\gamma-1}{2M_\infty^2} \right) S + \frac{P_1 T_\infty a_1}{\rho_\infty P_\infty a_\infty} \frac{\partial^3 \Psi}{\partial Y^3} \quad (2-14)$$

Stretching the X coordinate by writing

$$X = \int_0^x \frac{P_1 a_1}{P_\infty a_\infty} dx = \int_0^x \left( \frac{a_1}{a_\infty} \right)^{\frac{3\gamma-1}{\gamma-1}} dx \quad (2-15)$$

equation (2-14) reduces to

$$\frac{\partial \Psi}{\partial Y} \frac{\partial^2 \Psi}{\partial X \partial Y} - \frac{\partial \Psi}{\partial X} \frac{\partial^2 \Psi}{\partial Y^2} = V_1 \frac{dV_1}{dX} S + C(X) \frac{T_\infty}{\rho_\infty} \frac{\partial^3 \Psi}{\partial Y^3} \quad (2-16)$$

where

$$V_1(X) = \frac{a_\infty V_1}{a_1} = a_\infty M_1(X) \quad (2-17)$$

For an impermeable wall, the boundary conditions are

$$\psi = \frac{\partial \psi}{\partial Y} = 0 \quad \text{at} \quad Y = 0 \quad (2-18)$$

$$\frac{\partial \psi}{\partial Y} \rightarrow V_1 \quad \text{as} \quad Y \rightarrow \infty$$

Imposing these conditions on equation (2-11) and solving for S,

$$S = S_w(X) = \frac{T_w(X)}{T_\infty [1 + \frac{\gamma-1}{2} M_\infty^2]} \quad \text{at} \quad Y = 0 \quad (2-19)$$

$$S \rightarrow S_1 = 1 \quad \text{as} \quad Y \rightarrow \infty$$

Thus, S varies from one at the outer edge of the boundary layer to a value h at the wall, where  $h = T_w/T_\infty$  for the case  $M_\infty \rightarrow 0$ . From equation (2-16) it can be seen that heating the wall ( $h > 1$ ) will tend to increase the magnitude of the pressure gradient term in the momentum equation. Since the magnitude of the adverse pressure gradient directly affects how long the flow is able to remain attached to the surface, it can be expected that heating the surface will cause the separation point to move upstream.

Next consider the effect of heat on the viscous forces at the surface. First examine the viscous term of the momentum equation in the physical coordinate system:

$$\frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) \quad (2-20)$$

Now write a Dorodnitsyn-Howarth transformation such that

$$y = \int_0^t \frac{T}{T_1} dt \quad (2-21)$$

Assuming that  $\frac{T}{T_1} = f[y(t)]$ , then it follows that

$$\frac{dy}{dt} = \frac{T}{T_1}, \quad \frac{dt}{dy} = \frac{T_1}{T} \quad (2-22)$$

and, from this

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial t} \frac{dt}{dy} = \frac{T_1}{T} \frac{\partial}{\partial t} \quad (2-23)$$

Expanding equation (2-20), write

$$\mu \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial \mu}{\partial y} \quad (2-24)$$

or

$$\mu \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) + \frac{T_1}{T} \frac{\partial u}{\partial t} \frac{T_1}{T} \frac{\partial \mu}{\partial t} \quad (2-25)$$

and

$$\mu \frac{T_1}{T} \frac{\partial}{\partial t} \left( \frac{T_1}{T} \frac{\partial u}{\partial t} \right) + \left( \frac{T_1}{T} \right)^2 \frac{\partial u}{\partial t} \frac{\partial \mu}{\partial t} \quad (2-26)$$

Now, expanding the first term, write

$$\mu \frac{T_1}{T} \left[ \frac{T_1}{T} \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} \frac{\partial}{\partial t} \left( \frac{T_1}{T} \right) \right] + \left( \frac{T_1}{T} \right)^2 \frac{\partial u}{\partial t} \frac{\partial \mu}{\partial t} \quad (2-27)$$

Substituting  $\frac{\mu}{\mu_1} = \frac{T}{T_1}$ ,

$$\mu_1 \left[ \frac{T_1}{T} \frac{\partial^2 u}{\partial t^2} \right] + \mu_1 \frac{\partial u}{\partial t} \left[ \frac{\partial}{\partial t} \left( \frac{T_1}{T} \right) + \left( \frac{T_1}{T} \right)^2 \frac{\partial}{\partial t} \left( \frac{T}{T_1} \right) \right] \quad (2-28)$$

or,

$$\mu_1 \left[ \frac{T_1}{T} \frac{\partial^2 u}{\partial t^2} \right] + \mu_1 \frac{\partial u}{\partial t} \left[ -\frac{T_1}{T^2} \frac{\partial T}{\partial t} + \frac{T_1}{T^2} \frac{\partial T}{\partial t} \right] \quad (2-29)$$

and finally,

$$\mu_1 \left[ \frac{T_1}{T} \frac{\partial^2 u}{\partial t^2} \right] \quad (2-30)$$

Assuming that  $\frac{T_1}{T} = \frac{\rho}{\rho_1}$ , the viscous term becomes

$$\mu_1 \left[ \frac{\rho}{\rho_1} \frac{\partial^2 u}{\partial t^2} \right] \quad (2-31)$$

The above equation shows that the viscous term is proportional to density and thus can be expected to decrease with heating.

### 3. Effect of Temperature on Boundary Layer Thickness

When considering the effects of temperature on the boundary layer properties, it is instructive to reexamine the fundamental definitions of the various boundary layer thicknesses.

#### Boundary Layer Thickness

The boundary layer thickness  $\delta$  is defined as the distance from the wall at which the stream-wise component of velocity in the boundary layer is equal to the velocity of the inviscid flow outside the boundary layer. (Since this definition would make  $\delta \rightarrow \infty$ , for practical purposes,  $\delta$  can be defined as the distance at which  $u \approx 0.99U_1$ .)

For laminar flow over a flat plate

$$\frac{\delta}{x} \propto R_x^{-1/2} \quad (3-1)$$

This can also be written as

$$\frac{\delta}{x} \propto \left( \frac{\mu}{\rho U_\infty x} \right)^{1/2} \quad (3-2)$$

Noting that for a flat plate with  $M_\infty \rightarrow 0$

$$\frac{\rho_\infty}{\rho} = \frac{T}{T_\infty} \quad (3-3)$$

and assuming the viscosity-temperature relation

$$\frac{\mu}{\mu_\infty} = \frac{T}{T_\infty} \quad (3-4)$$

the temperature dependent boundary layer thickness becomes

$$\left( \frac{\delta}{x} \right)_T \propto \left( \frac{\mu_\infty}{\rho_\infty U_\infty x} \right)^{1/2} \frac{T}{T_\infty} \quad (3-5)$$

or,

$$\left( \frac{\delta}{x} \right)_T = \left( \frac{\delta}{x} \right) \frac{T}{T_\infty} \quad (3-6)$$

From equation (3-6) it is evident that heating the flow ( $T/T_\infty > 1$ ) will increase the boundary layer thickness  $\delta$ .

Since  $T$  varies continuously through the boundary layer, some average value of temperature must be determined. Define an average temperature by

$$\left( \frac{T}{T_\infty} \right)_{\text{avg}} = \frac{1}{\delta} \int_0^\delta \left( \frac{T}{T_\infty} \right) dy \quad (3-7)$$



Now assume that the velocity and thermal boundary layers are identical over a flat plate at zero Mach number. That is,

$$\frac{T - T_w}{T_\infty - T_w} = \frac{u}{U_\infty} \quad (3-8)$$

or,

$$\frac{T}{T_\infty} = \frac{T_w}{T_\infty} + \frac{u}{U_\infty} \left(1 - \frac{T_w}{T_\infty}\right) \quad (3-9)$$

From Pohlhausen, the velocity profile over a flat plate can be represented by the fourth-degree polynomial

$$\frac{u}{U_\infty} = 2\eta - 2\eta^3 + \eta^4 \quad (3-10)$$

where  $\eta = y/\delta(x)$ . Substituting this new expression for  $T/T_\infty$ , equation (3-7) becomes

$$\frac{T_{avg}}{T_\infty} = \int_0^1 \left[ \frac{T_w}{T_\infty} + \left(1 - \frac{T_w}{T_\infty}\right) \frac{u}{U_\infty} (\eta) \right] d\eta \quad (3-11)$$

which can be readily integrated to give

$$\frac{T_{avg}}{T_\infty} = 0.7 + 0.3 \frac{T_w}{T_\infty} \quad (3-12)$$

Thus, the expression for boundary layer thickness becomes

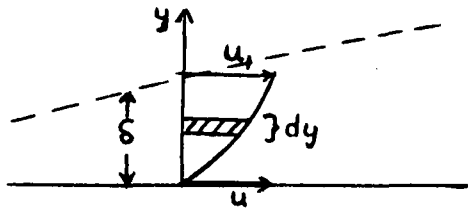
$$\left(\frac{\delta}{x}\right)_T = \frac{\delta}{x} \left[0.7 + 0.3 \frac{T_w}{T_\infty}\right] \quad (3-13)$$

or,

$$\frac{\delta_T}{\delta} = 0.7 + 0.3 \left( \frac{T_w}{T_\infty} \right) \quad (3-14)$$

### Displacement Thickness

Consider an elemental mass flow rate of  $\rho u dy$ .



The total mass flow rate of the boundary layer is

$$\dot{m} = \int_0^\delta \rho u dy \quad (3-15)$$

If the flow was totally inviscid, the mass flow rate would be

$$\dot{m}_{\text{inviscid}} = \int_0^\delta \rho_1 U_1 dy > \int_0^\delta \rho u dy \quad (3-16)$$

Thus, there is a "displaced" mass flow rate

$$\dot{m}_d = \int_0^\delta (\rho_1 U_1 - \rho u) dy \quad (3-17)$$

Defining a "displacement thickness",  $\delta^*$  by

$$\rho_1 U_1 \delta^* = \int_0^\delta (\rho_1 U_1 - \rho u) dy \quad (3-18)$$

then,

$$\delta^* = \int_0^\delta \left( 1 - \frac{\rho u}{\rho_1 U_1} \right) dy \quad (3-19)$$

For a flat plate at zero Mach number,

$$\rho_1 = \rho_\infty, \quad U_1 = U_\infty \quad (3-20)$$

and equation (3-19) becomes

$$\delta^* = \int_0^\delta \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy \quad (3-21)$$

For incompressible flow, or flow with no temperature gradient through the boundary layer, the density is constant and equal to  $\rho_\infty$  yielding

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy \quad (3-22)$$

If a temperature gradient exists, the density in equation (3-21) is not constant. However, from equation (3-9),

$$\frac{\rho}{\rho_\infty} = \frac{T_\infty}{T} = \left[ \frac{T_w}{T_\infty} + \frac{u}{U_\infty} \left(1 - \frac{T_w}{T_\infty}\right) \right]^{-1} \quad (3-23)$$

Substituting this into equation (3-21) and rearranging, the temperature dependent displacement thickness becomes

$$\delta_T^* = \int_0^\delta \frac{\left(1 - \frac{u}{U_\infty}\right)}{\left(1 - \frac{u}{U_\infty}\right) + \frac{u}{U_\infty} \frac{T_\infty}{T_w}} dy \quad (3-24)$$

Assuming Pohlhausen's fourth-degree velocity profile, the expression for displacement thickness can finally be written as

$$\left(\frac{\delta^*}{\delta}\right)_T = \int_0^1 \frac{[1 - \frac{u}{U_\infty}(\eta)]}{[1 - \frac{u}{U_\infty}(\eta)] + \frac{T_\infty}{T_w} \frac{u}{U_\infty}(\eta)} d\eta \quad (3-25)$$

The above equation can be integrated for any assumed form of the velocity profile.

### Momentum Thickness

Now consider an elemental momentum flux in the boundary layer of

$$\rho u^2 dy \quad (3-26)$$

The total momentum flux of the boundary layer from the wall to the outer edge is

$$\int_0^\delta \rho u^2 dy \quad (3-27)$$

Again, if the flow was completely inviscid, the momentum flux would be

$$U_1 \int_0^\delta \rho u dy > \int_0^\delta \rho u^2 dy \quad (3-28)$$

There is a "lost" momentum flux for the viscous flow then, of

$$U_1 \int_0^\delta \rho u dy - \int_0^\delta \rho u^2 dy \quad (3-29)$$

Defining a "momentum thickness",  $\theta$ , by

$$\rho_1 U_1^2 \theta = U_1 \int_0^\delta \rho u dy - \int_0^\delta \rho u^2 dy \quad (3-30)$$

then

$$\theta = \int_0^\delta \frac{\rho u}{\rho_1 u_1} \left(1 - \frac{u}{U_1}\right) dy \quad (3-31)$$

For a flat plate at zero Mach number, this becomes

$$\theta = \int_0^\delta \frac{\rho u}{\rho_\infty U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy \quad (3-32)$$

Again assuming identical velocity and temperature profiles and

$$\frac{\rho}{\rho_\infty} = \frac{T_\infty}{T} \quad (3-3)$$

the expression for temperature dependent momentum thickness becomes

$$\theta_T = \int_0^\delta \frac{\frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right)}{\frac{u}{U_\infty} + \frac{T_w}{T_\infty} \left(1 - \frac{u}{U_\infty}\right)} dy \quad (3-33)$$

or, with Pohlhausen's velocity profile,

$$\left(\frac{\theta}{\delta}\right)_T = \int_0^1 \frac{\frac{u}{U_\infty}(\eta) \left[1 - \frac{u}{U_\infty}(\eta)\right]}{\frac{u}{U_\infty}(\eta) + \frac{T_w}{T_\infty} \left[1 - \frac{u}{U_\infty}(\eta)\right]} d\eta \quad (3-34)$$

The ratios of the heated boundary layer thicknesses to the unheated thicknesses can be expressed as

$$\frac{\delta_T^*}{\delta^*} = \left(\frac{\delta^*}{\delta}\right)_T \left(\frac{\delta_T}{\delta}\right) \left(1/\frac{\delta^*}{\delta}\right) \quad (3-35)$$

and

$$\frac{\theta_T}{\theta} = \left(\frac{\theta}{\delta}\right)_T \left(\frac{\delta_T}{\delta}\right) \left(1/\frac{\theta}{\delta}\right) \quad (3-36)$$

These ratios are plotted in Figures 1 and 2 for a range of wall-to-freestream temperature ratios. It can be seen from the figures that the displacement thickness increase is almost strictly proportional to temperature increase, while momentum thickness is relatively independent of temperature.

It can be readily shown by an alternate method that displacement thickness is dependent on wall temperature while momentum thickness is not.

Momentum thickness, in the physical spacial coordinates is defined as

$$\theta = \int_0^{\infty} \frac{\rho u}{\rho_1 U_1} \left(1 - \frac{u}{U_1}\right) dy \quad (3-37)$$

Defining a Dorodnitsyn-Howarth transformation such that

$$\bar{Y} = \int_0^y \frac{\rho}{\rho_{\infty}} dy \quad (3-38)$$

equation (3-37) becomes

$$\theta = \int_0^{\infty} \frac{\rho_{\infty} u}{\rho_1 U_1} \left(1 - \frac{u}{U_1}\right) d\bar{Y} \quad (3-39)$$

Now define a new variable  $\bar{\eta}$  such that

$$\bar{\eta} = \bar{Y} \left(\frac{\rho_1 U_1}{2\mu_1 x}\right)^{1/2} \quad (3-40)$$

and

$$d\bar{Y} = d\bar{\eta} \left(\frac{\rho_1 U_1}{2\mu_1 x}\right)^{-1/2} \quad (3-41)$$

Now,

$$\theta = x \left( \frac{2}{R_x} \right)^{1/2} \int_0^\infty \frac{u}{U_1} \left( 1 - \frac{u}{U_1} \right) d\bar{\eta} \quad (3-42)$$

and is independent of temperature.

Now consider the displacement thickness

$$\delta^* = \int_0^\infty \left( 1 - \frac{\rho u}{\rho_1 U_1} \right) dy \quad (3-43)$$

Performing the same transformations to  $\bar{Y}$  and then to  $\bar{\eta}$ , this becomes

$$\delta^* = x \left( \frac{2}{R_x} \right)^{1/2} \int_0^\infty \left( \frac{T}{T_1} - \frac{u}{U_1} \right) d\bar{\eta} \quad (3-44)$$

Making the assumptions of zero Mach number and identical temperature and velocity profiles,

$$\delta^* = x \left( \frac{2}{R_x} \right)^{1/2} \int_0^\infty \frac{T}{T_1} \left( 1 - \frac{u}{U_1} \right) d\bar{\eta} \quad (3-45)$$

and is a function of wall temperature.

Thus far, this analysis of the effect of temperature on boundary layer thickness has considered laminar flow only. That is

$$\frac{u}{U_1} = 2\eta - 2\eta^3 + \eta^4$$

and

$$\frac{\delta}{x} \propto R_x^{-1/2}$$

are laminar relations. However, equations (3-42) and (3-45) hold for boundary layer flows in general. It can be expected, then, that qualitatively, the effect of temperature on the boundary

layer thickness of turbulent flows should be similar to its effect on laminar flows. Figures 1 and 2 also present data from the methods of Cebeci, Smith and Wang (15) for the flow

$$U_1/U_\infty = 1 - x/C, M_\infty \approx 0$$

It can be seen from the figures that for both laminar and turbulent flow of this type, the increase in displacement thickness and momentum thickness with temperature is of the same order as the increase analytically derived for the flat plate laminar flow. Also, it can be concluded that the increase in boundary layer thickness with temperature is relatively independent of whether the flow is laminar or turbulent.

#### 4. Review of Various Methods for Calculating the Effects of Heat Addition to Laminar Flow

Numerous methods are in existence that will compute the compressible laminar boundary layer with heat transfer. Following is a review of several methods which are applicable to the problem being considered.

Li and Nagamatsu (4) extended the work of Hartree (8) on boundary layer equations which reduce to the similar solutions of Falkner-Skan (9) for flow past a wedge. They generalized the equations to include the case of a constant temperature wall which is different from the freestream. The temperature difference is



accounted for in the differential equations

$$f''' = -ff'' + \beta[f'^2 - G] \quad (4-1)$$

$$G'' + fG' = 0 \quad (4-2)$$

by the term  $G = H/H_1$ , the ratio of local stagnation enthalpy to the stagnation enthalpy at the outer edge of the boundary layer. Here,  $f' = u/U_1$  and  $\beta$  is related to the wedge angle and, therefore, to  $dU_1/dx$ .

The condition for flow separation is that  $f''$  at the wall be equal to zero. The value of  $\beta$  required to satisfy this condition for zero temperature gradient through the boundary layer is  $\beta_0 = -0.199$ . For the case of  $G \neq 1$ , consider that  $G = 1 + \Delta G$  for  $\Delta G \ll 1$ . Since for  $G = 1$ ,  $f''(0)$  vanishes when  $\beta = \beta_0$ , it is expected that for  $G = 1 + \Delta G$ ,  $f''(0)$  vanishes when  $\beta = \beta_0 + \Delta\beta$ . Li and Nagamatsu deduce that

$$\Delta\beta = -\beta_0 \Delta G \quad (4-3)$$

Thus, for a heated surface ( $\Delta G > 0$ ),  $-\beta_0 \cdot (\Delta G) > 0$  and  $\beta = \beta_0 + \Delta\beta > \beta_0$ . This means that the boundary layer over a heated surface will separate under a comparatively weaker adverse pressure gradient. Quantitatively, Li and Nagamatsu found that for  $T_w/T_\infty = 2.0$ , separation occurs for a value of  $\beta_0 = -0.124$ .

Curle (10) has developed an approximate method for computing the compressible laminar boundary layer with pressure gradient and uniform wall temperature based on the incompressible work of Thwaites (11). Thwaites examined all of the known solutions of laminar

boundary layer flows and recognized that they could be represented in general by two parameters,  $\ell(m)$  and  $H(m)$  where

$$m = \frac{U_1' \theta^2}{\nu}, \quad H = \frac{\delta^*}{\theta} \quad (4-4)$$

and  $\ell(m)$  is defined by

$$\left( \frac{\partial u}{\partial y} \right)_{y=0} = \frac{U_1'}{\theta} \ell(m) \quad (4-5)$$

Thwaites then integrated the momentum equation to give

$$\frac{\theta^2}{\nu} = 0.45 U_1'^{-6} \int_0^x U_1'^5 dx \quad (4-6)$$

Curle extended this work to cover compressible flows by introducing an approximate integral of the energy equation based on an approximate temperature profile. A transformation of the normal coordinate partially reduces the momentum equation to an incompressible form. With the approximate temperature profile, the integration of the momentum equation is reduced to a form requiring only the two quadratures

$$G_1(x) = \exp \left\{ 2 \int_0^x \left( \frac{T_w}{T_1} + 2-1/2 M_1^2 + K \right) \frac{U_1'}{U_1} dx \right\} \quad (4-7)$$

and

$$\frac{\theta^2}{\nu_1} = \frac{0.45}{G_1} \int_0^x \frac{G_1}{U_1} dx \quad (4-8)$$

where

$$K = K(T)$$

Curle's method has been applied to the problem of linearly retarded flow and the results indicate a forward movement of separation for

the case of the heated wall.

Morduchow and Grape (2) present a theoretical analysis of the effect of pressure gradient and wall temperature on laminar boundary layer characteristics and, in particular, on the separation point in an adverse pressure gradient. A simple method of locating the separation point in a compressible flow with heat transfer is developed. Assumptions made are that the Prandtl number is unity and the viscosity is proportional to temperature by a factor derived from the Sutherland relation. The usual transformation of the normal coordinate is performed and the velocity and stagnation-enthalpy profiles are represented, respectively, by sixth and seventh degree polynomials. Concerning the point of flow separation, Morduchow and Grape show, by differentiating the momentum partial differential equation, that at the point of separation the additional boundary condition at the wall  $(\partial^4 u / \partial y^4)_w = 0$  must be satisfied. A seventh-degree velocity profile is chosen to satisfy this new boundary condition in addition to those satisfied by the sixth-degree profiles. From this, Morduchow and Grape develop an expression for the point of separation which requires a single quadrature. Results for the linearly retarded flow

$$U_1/U_\infty = 1 - x/C$$

at zero Mach number and a range of temperature ration are presented in figure (3). It is evident from the figure that separation occurs earlier for the case of the heated wall.

Illingworth (12) has developed an approximate method to treat laminar boundary layers with pressure gradient and large wall to

free stream temperature differences. Applying von Mises's transformation, he writes the momentum and energy boundary layer equations in terms of the coordinate  $x$  and the stream function  $\Psi$ . Following Lighthill(13), he assumes the approximate expression for the shear stress at the wall

$$\tau_w(x) \approx \tau(x, \Psi) = \mu \rho u \frac{\partial u}{\partial \Psi} \quad (4-9)$$

It should be noted that this equation underestimates the shear stress at the wall for most flows. Allowing this error and assuming that viscosity is proportional to absolute temperature by

$$\frac{\mu}{\mu_1} = \frac{T}{T_1} = \frac{\rho_1}{\rho} \quad (4-10)$$

equation (4-9) can be integrated to yield the approximate expression for streamwise velocity

$$u = \left[ \frac{2\tau_w \Psi}{\mu_1 \rho_1} \right]^{1/2} \quad (4-11)$$

This expression is most accurate near the wall, but overestimates  $u$  in the outer regions of the boundary layer. However, cofactors of  $u$  in the momentum and energy equations tend to lessen the seriousness of this inaccuracy as  $\Psi \rightarrow \infty$ . Using this approximation for  $u$ , Illingworth finds similar solutions of the boundary layer equations for flows defined by a power series  $U\alpha(x^n)$  and flows of the linearly retarded type. For the flow

$$\frac{U_1}{U_\infty} = 1 - x/C$$

at zero Mach number but with varying wall temperature, Illingworth concluded that the point of laminar flow separation is given by

$$(x/C)_{\text{sep}} = 0.12 \left[ \frac{T_\infty}{T_w} \right]^{1/2} \quad (4-12)$$

Results for this flow are compared with those of Morduchow and Grape in figure 3.

Two other approaches to the calculation of the laminar boundary layer with a heated wall are the integral relation method of Walz (14) and the finite-difference, eddy viscosity method of Cebeci, Smith and Wang (15). Since these methods are also applicable to turbulent flow, they are of particular interest and will be considered in greater detail later. It is sufficient to mention here that both of these methods predict the same trend as the previously cited methods on the forward movement of the separation point for flow over a heated surface.

For a comparison of the various procedures reviewed, figure 3 presents results for the linearly retarded flow

$$\frac{U_1}{U_\infty} = 1 - x/C$$

at zero Mach number and for a range of surface temperatures. It can be seen from the results that all of the methods investigated predict a forward movement of the point of laminar separation for the case of the heated wall.

### 5. Turbulent Boundary Layer Flow

Consideration of the separation point of turbulent flow over a heated surface has received little attention in the literature. Some knowledge of this phenomenon however, can be obtained by investigating methods that solve the turbulent boundary layer equations with heat transfer. Two methods -- completely different from one another in theoretical approach -- are the integral relation method of Walz (14) and the finite-difference, eddy conductivity, eddy viscosity method of Cebeci, Smith and Wang (15). Both of these techniques were rated in the upper one third of the 1968 Stanford Conference on turbulent boundary layers. (16)

#### Method of Walz

The method of Walz is based on the integral relations for momentum and energy,

$$\frac{d\theta}{dx} + \theta \frac{1}{U_1} \frac{dU_1}{dx} \left[ 2 + \frac{\delta^*}{\theta} - M_1^2 \right] - \frac{\tau_w}{\rho_1 U_1^2} = 0 \quad (5-1)$$

$$\frac{d\delta_3}{dx} + \delta_3 \frac{1}{U_1} \frac{dU_1}{dx} \left[ 3 + 2 \frac{\delta_4}{\delta_3} - M_1^2 \right] - \frac{2}{\rho_1 U_1^3} \int_0^{U_1} \tau du = 0 \quad (5-2)$$

where

$$\theta = \int_0^\infty \frac{\rho u}{\rho_1 U_1} \left( 1 - \frac{u}{U_1} \right) dy \quad (5-3)$$

$$\delta_3 = \int_0^\infty \frac{\rho u}{\rho_1 U_1} \left[ 1 - \left( \frac{u}{U_1} \right)^2 \right] dy \quad (5-4)$$

$$\delta_4 = \int_0^\infty \frac{\rho u}{\rho_1 U_1} \left( \frac{\rho}{\rho_1} - 1 \right) dy \quad (5-5)$$

The last terms in equations (5-1) and (5-2) are, respectively, the wall shearing stress and the dissipation integral. These quantities are determined empirically. Walz then reduces equations (5-1) and (5-2) to two first order, ordinary differential equations

$$\frac{dZ}{dx} + Z \frac{1}{U_1} \frac{dU_1}{dx} F_1^* - F_2 = 0 \quad (5-6)$$

and,

$$\frac{dH^*}{dx} + H^* \frac{1}{U_1} \frac{dU_1}{dx} F_3 - \frac{F_4}{Z} = 0 \quad (5-7)$$

where

$$Z = \theta R_\theta^n, \quad H^* = \frac{\delta_3}{\theta} \quad (5-8)$$

These equations are then amenable to numerical integration by a finite difference technique.

#### Cebeci, Smith and Wang

The approach of Cebeci, Smith and Wang is to solve directly the exact turbulent boundary layer equations for continuity, momentum and energy

$$\frac{\partial}{\partial x} (\rho u + \overline{\rho' U'}) + \frac{\partial}{\partial y} (\rho v + \overline{\rho' v'}) = 0 \quad (5-9)$$

$$\rho u \frac{\partial u}{\partial x} (\rho v + \overline{\rho' v'}) \frac{\partial u}{\partial y} = - \frac{dp}{dx} + \frac{\partial}{\partial y} \left[ \mu \frac{\partial u}{\partial y} \right] \left[ \mu \frac{\partial u}{\partial y} - \overline{\rho v' u'} \right] \quad (5-10)$$

$$\rho u \frac{\partial H}{\partial x} + (\rho v + \overline{\rho' v'}) \frac{\partial H}{\partial y} = \frac{\partial}{\partial y} \left[ \frac{\mu}{Pr} \frac{\partial H}{\partial y} - \overline{\rho v' H'} \right] + \mu \left( 1 - \frac{1}{Pr} \right) u \frac{\partial u}{\partial y} \quad (5-11)$$

This is possible after eddy viscosity and eddy conductivity terms are introduced. These terms are defined as

$$\epsilon \equiv \frac{\overline{-U' v'}}{\frac{\partial u}{\partial y}} \quad \text{eddy viscosity} \quad (5-12)$$

$$\lambda_t \equiv - \frac{C_p \overline{v' H'}}{\frac{\partial H}{\partial y}} \quad \text{eddy conductivity} \quad (5-13)$$

In the region near the wall, the eddy viscosity varies linearly with  $y$  and is given by

$$\epsilon_o = k_2 U_1 \delta^* \quad (5-14)$$

In the outer region of the boundary layer, the eddy viscosity is represented by

$$\epsilon_1 = \ell^2 \left| \frac{\partial u}{\partial y} \right| \quad (5-15)$$

where  $\ell$  is Prandtl's mixing length.



### Numerical Results for Turbulent Flow

As a check for agreement between the methods of Walz, and Cebeci, Smith and Wang, Figure 4 compares results for the shape parameter  $H = \frac{\delta^*}{\theta}$  for a linearly retarded flow. From the figure agreement appears good to a temperature ratio of  $T_w/T_\infty = 2$ . Figure 5 illustrates the effect of wall temperature on the separation point of turbulent linearly retarded flow according to the two methods. Both methods indicate a tendency for the point of separation to move upstream for the case of the heated wall. As was the case for the laminar flow, Walz's method seems to be conservative, predicting separation later than Cebeci, Smith and Wang.

### 6. Flow Over a Two-Dimensional Airfoil

Since it has now been shown that wall temperature significantly affects simple linearly retarded boundary layer flows, attention is given to a two dimensional airfoil shape. Similar effects of temperature on the flow over an airfoil could adversely affect the vehicle's maneuvering capabilities. For this purpose, wind tunnel test data (17) for a 0012-64 airfoil section was employed as input into the numerical methods of Walz, and Cebeci, Smith and Wang. Calculations were made using freestream conditions existing at an altitude of 50,000 feet and Mach number of 0.25. Wall to freestream temperature ratios varied from one to four at angles of attack of 0, 4, 8, 12 and 16 degrees.

Figure 6 illustrates results from the method of Walz. This figure indicates a substantial decrease in the angle of attack at which stall occurs with an increase in surface temperature. Here, stall is considered to be turbulent flow separation within the first ten percent of the chord length. The method of Cebeci, Smith and Wang is slower in predicting a decrease in stall angle of attack with surface temperature. According to their method, a temperature ratio of four is required to reduce the stall angle from 16 degrees to 12 degrees.

## 7. Conclusions

Based on the work done thus far, the following tentative conclusions can be made:

1. Displacement thickness is proportional to wall temperature ( $\delta^* \propto T_w/T_\infty$ ). The increase in boundary layer thickness over a heated surface may be significant enough to alter the velocity-pressure distribution and thus increase the pressure drag. However, since the skin friction drag decreases with heating, further study and experiment is necessary before final conclusions concerning drag can be made.
2. Laminar separation occurs earlier for flow over a heated surface. Based on the several methods for laminar boundary layers investigated, it can be concluded that separation occurs earlier for the case of the heated surface. This

phenomenon could be applicable to high altitude flight

where the flow over a large portion of the airfoil may be laminar. Under these conditions, earlier separation would increase the drag and possibly affect the lift.

4. Turbulent separation occurs earlier for flow over a heated surface. Based on limited results, it can be concluded that turbulent boundary layer flow--as is the case at low altitude and high angle of attack--will separate sooner over a heated airfoil.

5. A heated airfoil may stall at a lower angle of attack.

Based on results of the method of Walz, it appears that surface heating reduces the angle of attack at which an airfoil will stall. The method of Cebeci, Smith and Wang predict the same trend but to a lesser degree than Walz's. Since both methods break down at the point of separation and there also exists the possibility of re-attachment of flow, more conclusive arguments will require experimentation with a heated model.

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**FIGURES**

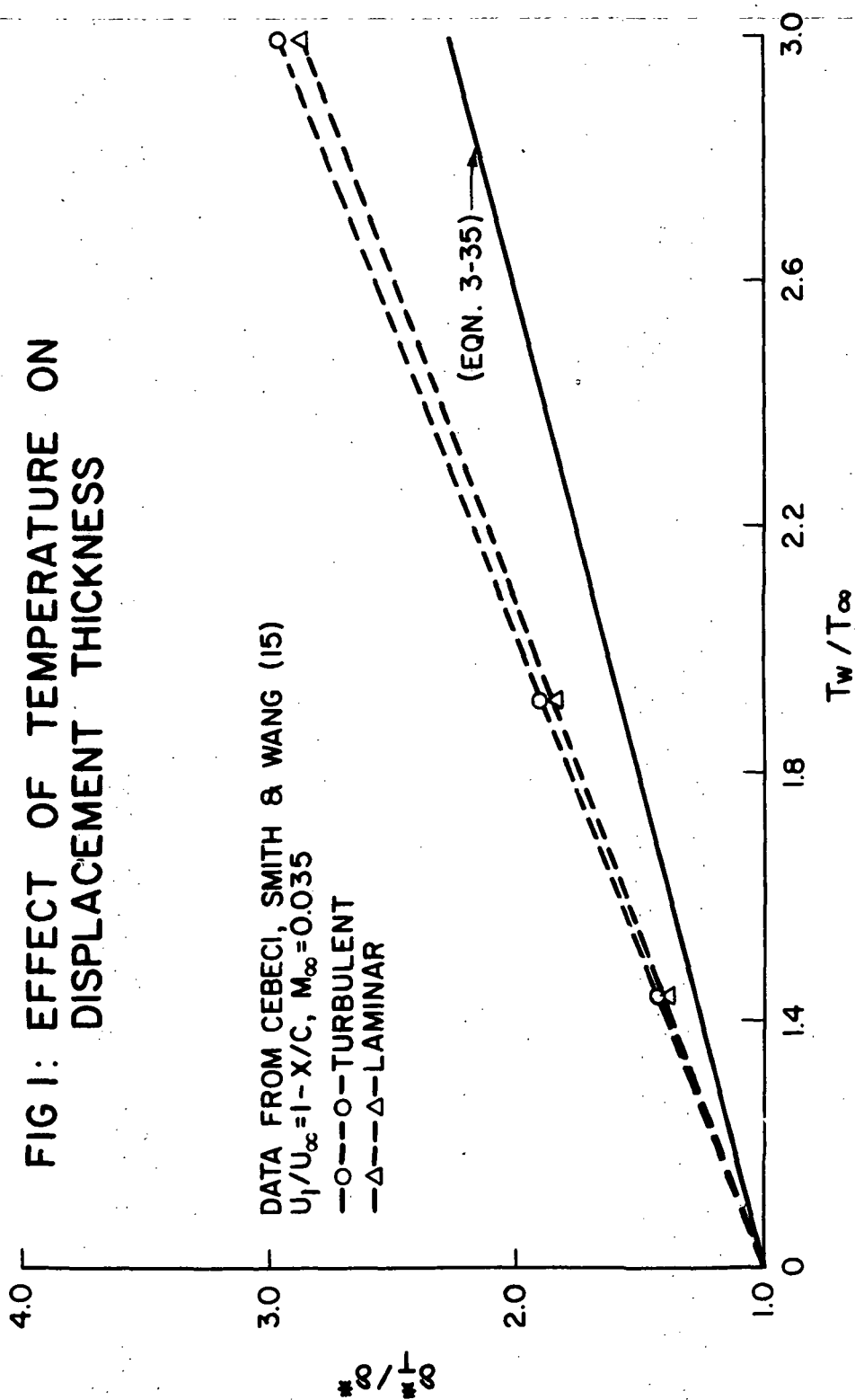


FIG 2: EFFECT OF TEMPERATURE ON  
MOMENTUM THICKNESS

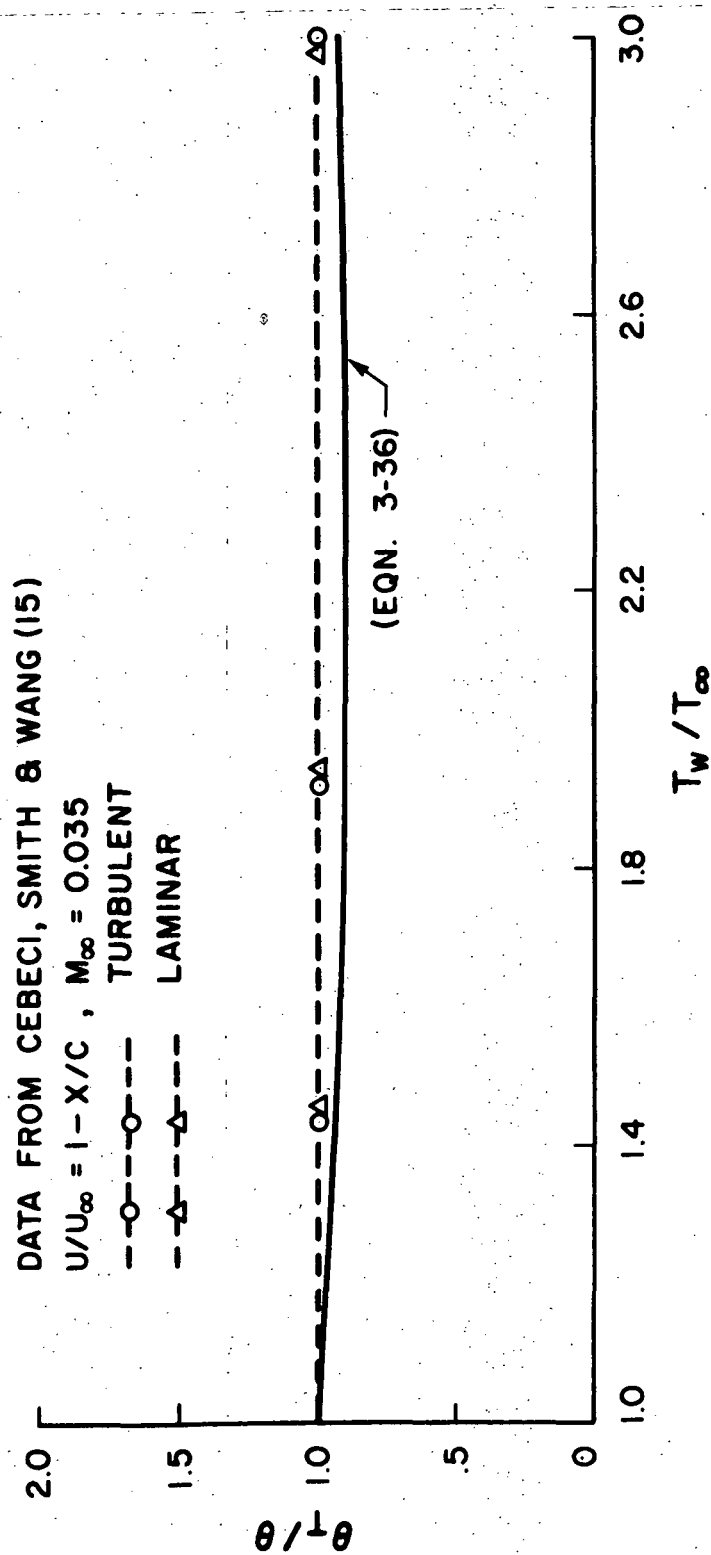




FIGURE 3: LAMINAR SEPERATION POINT AS A  
FUNCTION OF WALL TEMPERATURE  
 $U/U_\infty = 1 - X/C$  ,  $M_\infty = 0$

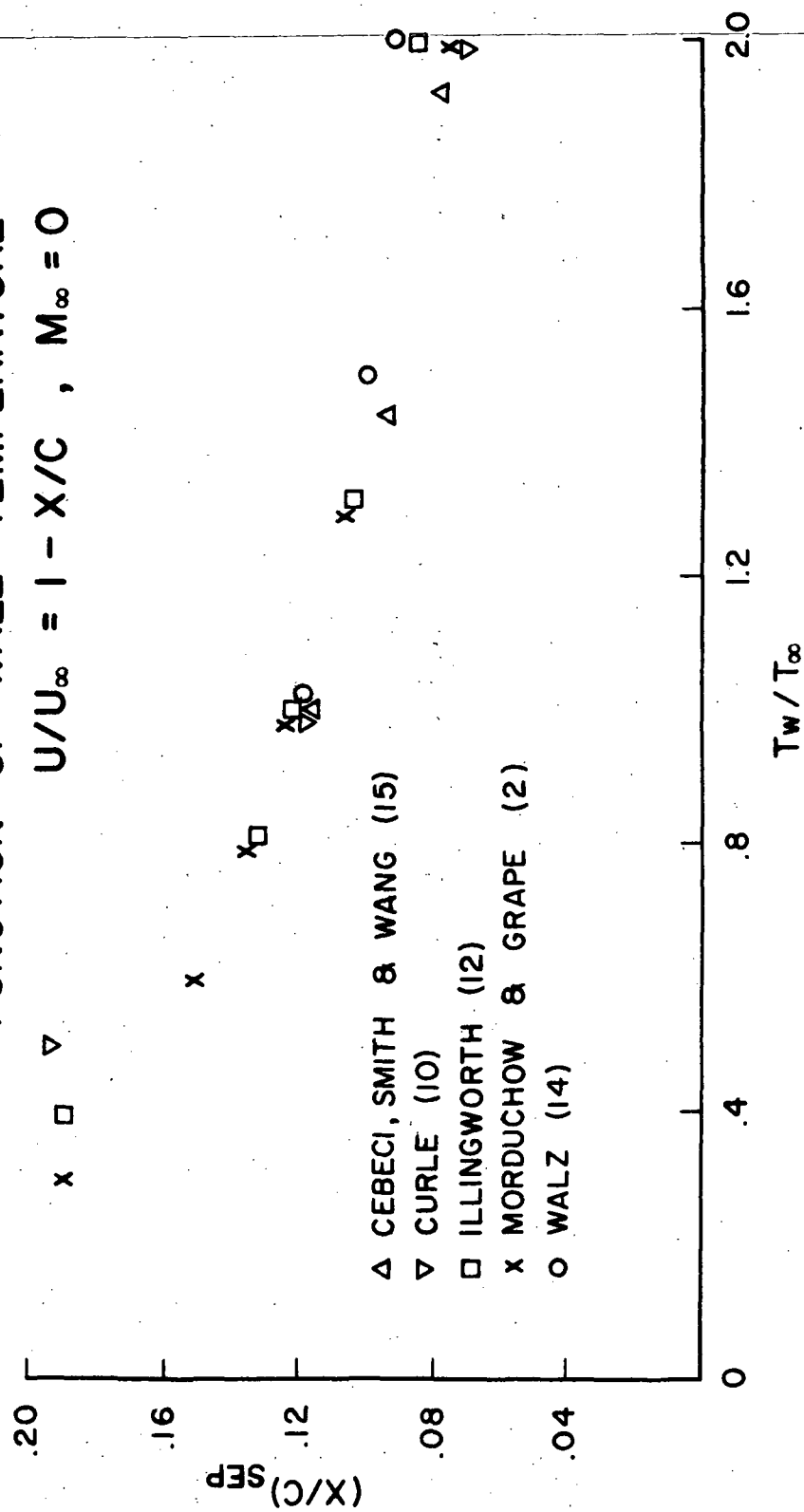


FIGURE 4 SHAPE PARAMETER  $H_{12}$  AS A FUNCTION OF CHORD

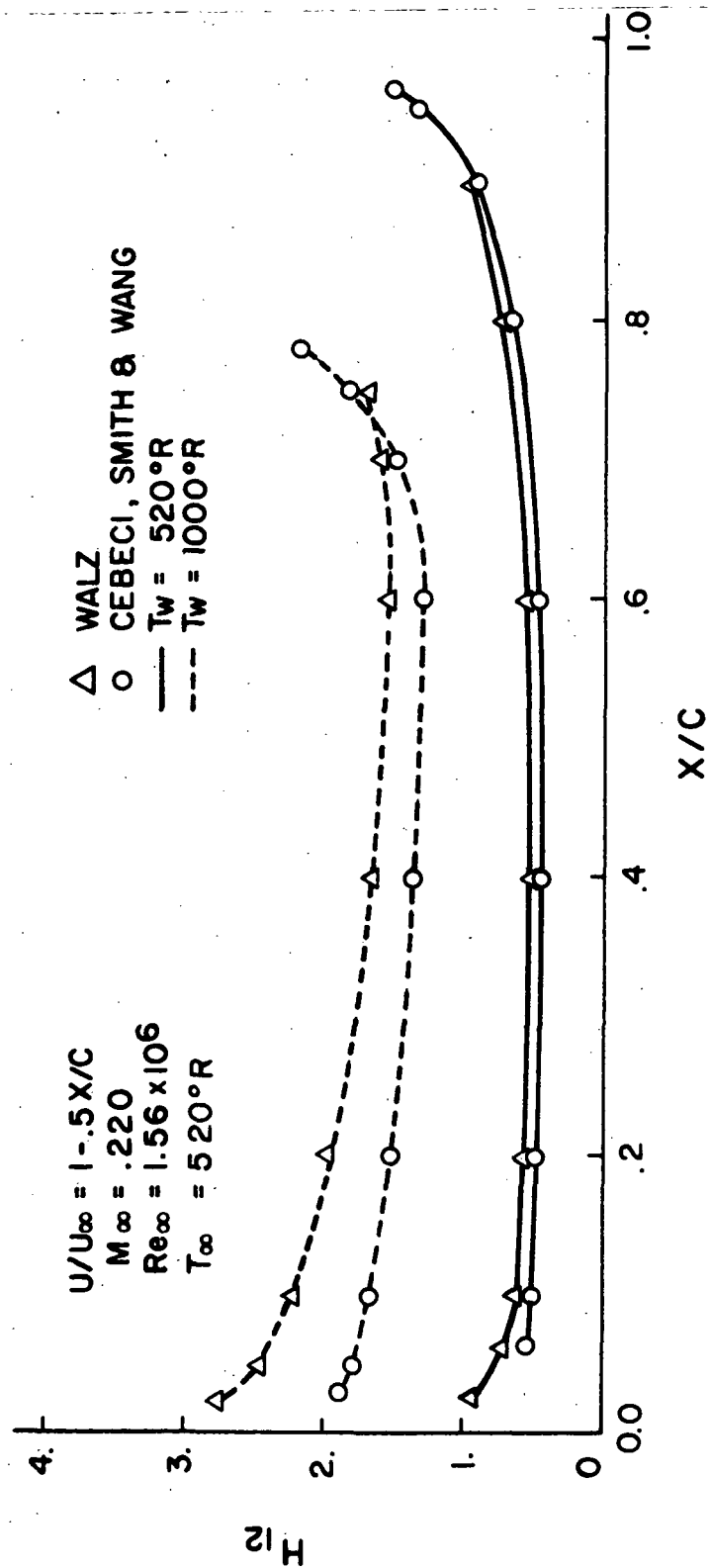
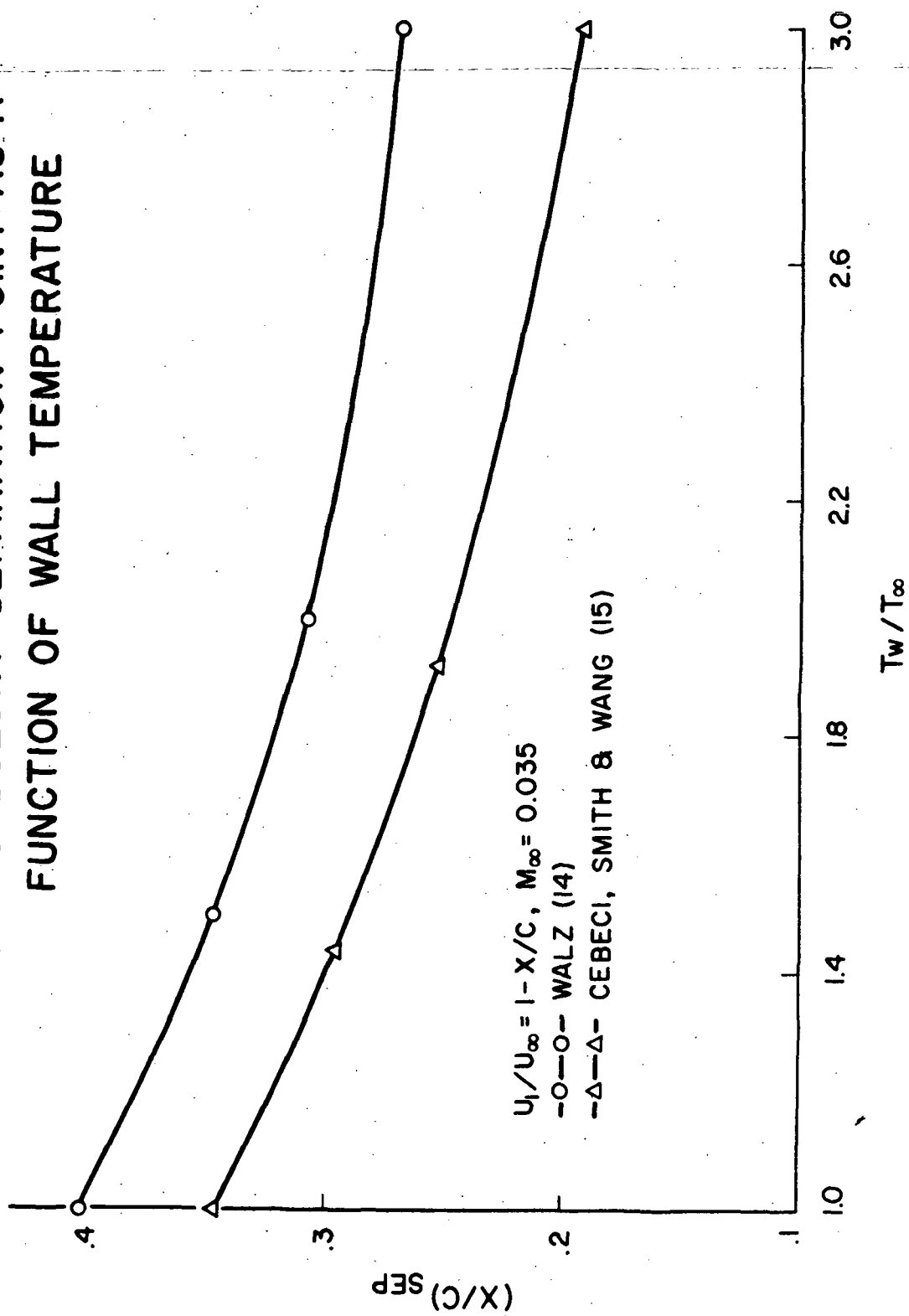


FIGURE 5: TURBULENT SEPARATION POINT AS A  
FUNCTION OF WALL TEMPERATURE



**FIGURE 6: STALL ANGLE-OF-ATTACK AS A FUNCTION  
OF WING SURFACE TEMPERATURE**

