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X-641-71-321
PREPRINT

NASA TM X-65802

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SEPTEMBER 1971

GSFC GODDARD SPACE FLIGHT CENTER

(NASA-TM-X-65802) WAVE GROWTH IN A STRONGLY TURBULENT PLASMA T.J. Birmingham, et al (NASA) Sep. 1971 19 p CSCL 20I

N72-16610

Unclas
G3/25 17034

FACILITY FORM 602

(ACCESSION NUMBER) (THRU)
19 G3
(PAGES) (CODE)
TMX 65802 25
(NASA CR OR TMX OR AD NUMBER) (CATEGORY)

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X-641-71-321
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WAVE GROWTH IN A STRONGLY TURBULENT PLASMA

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WAVE GROWTH IN A STRONGLY TURBULENT PLASMA

ABSTRACT

We derive an equation for the average non-linear growth (damping) rate $\langle\gamma_{\mathbf{k}}\rangle$ of an electrostatic mode with wave vector $\mathbf{k}$ for an ensemble of electrostatically turbulent, unmagnetized plasmas. Strong turbulence alters particle orbits during the course of wave growth and hence brings particles in and out of resonance with the wave. The wave growth rate in a strongly turbulent plasma therefore depends on the initial velocity distribution of particles not only at $u = \hat{k} \cdot \mathbf{v} = \alpha_{\mathbf{k}}/k$, the wave speed, but in the vicinity of this speed as well. We consider two orbital effects: turbulence modification of ensemble average orbits and turbulence produced orbital dispersion about the average orbits. The first of these leads to a shift in the central u of the resonance. Both contribute to a resonance broadening.

WAVE GROWTH IN A STRONGLY TURBULENT PLASMA

INTRODUCTION

The diffusion of resonant particles in the presence of strong plasma turbulence was the subject of a previous paper[1] (hereafter referred to as I). We considered in I an ensemble of plasmas and found that strongly turbulent fluctuations modify the statistical orbits of resonant particles and hence affect the ensemble average diffusion rate. The fluctuations (1) modify the ensemble average orbits of resonant particles and (2) introduce dispersion in orbits about the averages. Both effects are quadratic in the magnitude of the fluctuations.

In strong turbulence the coherence time between resonant particles and turbulence depends on the strength of these two effects. Initially resonant particles may be driven from resonance, or conversely initially non-resonant particles may be made more resonant by the turbulence. The more rapidly that these processes occur, the more significantly is the diffusion tensor altered.

Turning now to the topic of wave growth (damping), we recall that in the Landau[2] analysis γ_k, the growth (damping) factor for an electrostatic wave of frequency α_k and wave number k in an unmagnetized, one dimensional electron plasma is

$$\gamma_k = \pi/2 \, \frac{\alpha_k \, \omega_e^2}{k^2} \, f'\left(\frac{\alpha_k}{k}\right). \tag{1}$$

proportional to the slope of the electron one particle distribution function f at the resonant velocity $v = \alpha_k / k$ (ω_e is the electron plasma frequency). Tacit in Equation (1) is the assumption that resonant electrons move in straight lines over the time interval $|\gamma_k|^{-1}$. Large amplitude fluctuations, as may occur in an unstable plasma, lead to orbit modifications which violate this straight line approximation.

In the strongly turbulent regime we consider an ensemble of three dimensional plasmas just as in I. We seek here the value of the ensemble average growth rate $\langle \gamma_k \rangle$ for an electrostatic wave of frequency α_k and wave vector $\mathbf{k}$.

Both statistical orbit modifications discussed in I also affect the form of $\langle \gamma_k \rangle$. Because the average velocity of each particle is changing and its velocity is also dispersing during the course of wave growth, a wave of phase velocity α_k / k interacts not just with particles whose parallel velocity $u = \hat{k} \cdot \mathbf{v}$ equals α_k / k at the beginning of growth; it interacts also with particles whose initial u's lie near α_k / k and which thus have a high probability of resonating (i.e., having $u = \alpha_k / k$) <u>during the growth period.</u>

In a strongly turbulent three dimensional plasma $\langle \gamma_k \rangle$ thus depends on the value of $\langle \tilde{f} \rangle$ over a band of u. Here $\langle \tilde{f} \rangle$ is the ensemble averaged one particle distribution function integrated over velocity components perpendicular to $\mathbf{k}$. If the wave-particle interaction were the same for all resonant particles, one would expect $\langle \gamma_k \rangle$ to be of the form

$$\langle\gamma_k\rangle = \frac{\pi}{2} \frac{\alpha_k \omega_e^2}{k^2} \frac{1}{g_1 + g_2} \left\{ \langle \tilde{f} \rangle \left[\frac{\alpha_k}{k} + g_1 (\delta E^2) \right] - \langle \tilde{f} \rangle \left[\frac{\alpha_k}{k} - g_2 (\delta E^2) \right] \right\}$$

the distortion of the resonance being of second order in the fluctuation amplitude δE and having contributions arising from both average orbit modification and orbit dispersion.

In this paper we calculate $\langle \gamma_k \rangle$ for electrostatic waves in a spatially homogeneous, three dimensional, multi-component plasma. The turbulence which deforms particle orbits is likewise electrostatic. (Indeed, the mode whose stability we are considering is one of the waves comprising the turbulence.) The importance of average orbit distortion and orbit dispersion here lies in the fact that these effects can act as an amplitude-dependent stabilizing factor for waves which are linearly unstable

$$\left[\langle \tilde{f}' \rangle \left(\frac{\alpha_k}{k} \right) > 0 \text{ for } \frac{\alpha_k}{k} > 0 \right]$$

Dupree[3] has already included the effect of orbit dispersion in his one dimensional calculation. His pioneering work has been amplified and applied to specific linearly unstable modes by several authors.[4-12] We here add the effect on the growth rate of average orbit alteration, an effect, like dispersion, quadratic in δE.

As in I our approach is via Dupree's[3] statistical orbits technique. We make frequent use of results derived and approximations discussed in I. The reader who is interested in the details of this calculation is thus advised to study the papers in conjunction with one another.

CALCULATION OF $\langle \gamma_k \rangle$

In each realization of the ensemble, Poisson's Equation relates the fluctuating electrostatic field $\delta \mathbf{E}$ to the fluctuating charge density $\delta \rho$ and thus to δf_i, the fluctuating part of the one particle distribution function for each species i

$$\nabla \cdot \delta \mathbf{E} = 4\pi \, \delta \rho = 4\pi \sum_i n_i \, q_i \int d^3 v \, \delta f_i \, (\mathbf{x}, \mathbf{v}, t) \tag{2}$$

In the weak coupling approximation to strong plasma turbulence, δf_i is in turn related to its initial value $\delta f_i(\mathbf{x}, \mathbf{v}, t_0)$, the ensemble average one particle distribution function $\langle f \rangle_i \, (\mathbf{v}, t)$ for species i, and $\delta \mathbf{E}(\mathbf{x}, t)$ itself by

$$\begin{aligned} \delta f_i \, (\mathbf{x}, \mathbf{v}, t) = {} & \langle U_i \, (t, t_0) \rangle \, \delta f_i \, (\mathbf{x}, \mathbf{v}, t_0) \\ & - \frac{q_i}{m_i} \int_{t_0}^{t} d\tau \, \langle U_i \, (t, \tau) \rangle \, \delta \mathbf{E} \, (\mathbf{x}, \tau) \cdot \frac{\partial \langle f \rangle_i}{\partial \mathbf{v}} (\mathbf{v}, \tau) \end{aligned} \tag{3}$$

The propagator $\langle U_i(t, t_0) \rangle$ is the ensemble average Vlasov propagator for particles of species i. $\langle U(t,t_0) \rangle$ is defined by Equations (9) and (10) of I and contains the effects of the turbulent electrostatic waves on particle trajectories. The relationship between $\langle U(t,t_0) \rangle$ and the characteristic phase space trajectories of plasma particles is developed by Birmingham and Bornatici.[13] The weak coupling approximation is discussed in detail in I.

We next plug Equation (3) into Equation (2). The initial value contribution serves as a driving source for the waves. Since we are here concerned only with the dielectric properties of the plasma, we assume that $\delta f_i(\mathbf{x}, \mathbf{v}, t_0)$ is a well behaved function and simply represent its effect in Poisson's Equation by $S(\mathbf{x}, t; t_0)$. Thus

$$\nabla \cdot \delta \mathbf{E} = -\sum_i \omega_i^2 \int d^3 v \int_{t_0}^{t} d\tau \, \langle U_i(t, \tau) \rangle \, \delta \mathbf{E}(\mathbf{x}, \tau) \cdot \frac{\partial \langle f \rangle_i (\mathbf{v}, \tau)}{\partial \mathbf{v}} + S(\mathbf{x}, t; t_0) \quad (4)$$

where

$$\omega_i^2 = \frac{4 \pi n_i q_i^2}{m_i}$$

is the plasma frequency associated with the i^{th} species.

In Equation (4) $\langle U_i(t, \tau) \rangle$ operates on both $\delta \mathbf{E}$ and $\partial \langle f \rangle_i / \partial \mathbf{v}$. We shall, however, assume (1) that the scale length of $\langle f \rangle_i$ in $\mathbf{v}$ is large compared to the incremental change in velocity coordinates of particles in the growth time $|\gamma_k|^{-1}$ of any mode and (2) that the time scale on which $\langle f \rangle_i$ changes is likewise long compared with $|\gamma_k|^{-1}$. [In the important resonant particle region of phase space, $\langle f \rangle_i$ evolves according to the diffusion equation derived in I on the time scale of several correlation times τ_c.] Corrections introduced by including the finite velocity and time scales of $\langle f \rangle_i$ are of the same order of magnitude as those encountered in considering higher order terms in the weak coupling approximation.

A cumulant expansion of $\langle U_i(t,\tau)\rangle$[14,15] (cf. Equation 14 of I) puts Equation (4) in the form

$$\nabla\cdot\delta\mathbf{E} = -\sum_i \omega_i^2 \int d^3 v \int_{t_0}^{t} d\tau \exp\left\{\langle\Delta\mathbf{x}_i^*(\tau)\rangle\cdot\nabla + \frac{1}{2}\left[\langle\Delta\mathbf{x}_i^*(\tau)\,\Delta\mathbf{x}_i^*(\tau)\rangle - \langle\Delta\mathbf{x}_i^*(\tau)\rangle\langle\Delta\mathbf{x}_i^*(\tau)\rangle\right] : \nabla\nabla\right\}\delta\mathbf{E}(\mathbf{x},\tau)\cdot\frac{\partial\langle f\rangle_i}{\partial\mathbf{v}} + S(\mathbf{x},t;t_0) \tag{5}$$

$\mathbf{x} + \langle\Delta\mathbf{x}_i^*(\tau)\rangle$ is the ensemble average phase space position coordinate at time τ of a particle of species i which in each realization of the ensemble is at the point $\mathbf{x}$, $\mathbf{v}$ at time t. Similarly,

$$\underset{\approx}{I} = \left[\langle\Delta\mathbf{x}_i^*(\tau)\,\Delta\,\mathbf{x}_i^*(\tau)\rangle - \langle\Delta\mathbf{x}_i^*(\tau)\rangle\langle\Delta\mathbf{x}_i^*(\tau)\rangle\right]$$

is the position dispersion tensor for such a particle.

We have derived in I expressions relating $\langle\Delta x^*(\tau)\rangle$ and $\underset{\approx}{I}$ to the particle diffusion tensor $\underset{\approx}{D}$ for an unmagnetized, spatially homogeneous, and quasi-stationary (in the sense that $|\gamma_k|^{-1}$ is longer than the cut-off time of autocorrelation functions), but strongly turbulent plasma. For particles of species i these expressions are

$$\langle\Delta\mathbf{x}_i^*(\tau)\rangle = -\mathbf{v}(t-\tau) - \frac{(t-\tau)^2}{2}\frac{\partial}{\partial\mathbf{v}}\mathrm{Tr}\,\{\underset{\approx}{D}_i(\mathbf{v},t-\tau)\} \tag{6}$$

$$\underset{\approx}{I}_i = \frac{2}{3}(t-\tau)^3\,\underset{\approx}{D}_i(\mathbf{v},t-\tau) \tag{7}$$

In Equation (6) Tr $\{\underset{\approx}{D}_i(v, t-\tau)\}$ is the trace of the matrix of $\underset{\approx}{D}_i$. The diffusion tensor is essentially zero for all particles except those resonant with the turbulence. For resonant particles $\underset{\approx}{D}_i$ is time dependent, building up from zero until the time when fluctuation correlations (as observed by the resonant particles) have died out. Beyond this time $\underset{\approx}{D}_i$ has the asymptotic value given by Equation (28) of I.

Let us now introduce the forms Equations (6) and (7) into Equation (5) and at the same time Fourier transform in **x**:

$$\delta E_{\mathbf{k}}(t) = \frac{i}{k^2}\sum_i \omega_i^2 \int d^3 v \int_{t_0}^{t} d\tau \exp\left\{-i\,\mathbf{k}\cdot\left[\mathbf{v}(t-\tau) + \frac{(t-\tau)^2}{2}\right.\right.$$

$$\left.\left.\frac{\partial}{\partial \mathbf{v}} \operatorname{Tr}\{\underset{\approx}{D}_i(\mathbf{v}, t-\tau)\}\right] - \frac{1}{3}(t-\tau)^3\,\mathbf{k}\,\mathbf{k} : \underset{\approx}{D}_i(\mathbf{v}, t-\tau)\right\}\delta E_{\mathbf{k}}(\tau) \tag{8}$$

$$\mathbf{k}\cdot\frac{\partial\langle f\rangle_i}{\partial \mathbf{v}} - \frac{i\,S_k(t;t_0)}{k^2}$$

Asymptotically in time ($t_0 \to -\infty$) we expect only the natural oscillation modes of the system to persist and write $\delta E_{\mathbf{k}}(\tau) \propto \exp -i\,\langle\omega_{\mathbf{k}}\rangle\,\tau$. The ensemble average eigenfrequency $\langle\omega_{\mathbf{k}}\rangle$ occurs because in Equation (8) the dynamic response of the plasma to all waves has been averaged over and represented by the $\underset{\approx}{D}_i$'s We thus obtain the dispersion relation

$$1 = \frac{i}{k^2}\sum_i \omega_i^2 \int d^3 v \int_0^{\infty} dz \exp\left\{i\left[(\langle\omega_{\mathbf{k}}\rangle - \mathbf{k}\cdot\mathbf{v})\,z\right.\right.$$

$$\left.\left.-\frac{z^2}{2}\,\mathbf{k}\cdot\frac{\partial}{\partial \mathbf{v}}\operatorname{Tr}\{\underset{\approx}{D}_i(\mathbf{v}, z)\}\right] - \frac{1}{3}z^3\,\mathbf{k}\,\mathbf{k} : \underset{\approx}{D}_i(\mathbf{v}, z)\right\}\mathbf{k}\cdot\frac{\partial\langle f\rangle_i}{\partial \mathbf{v}} \tag{9}$$

For slowly growing (decaying) waves,

$$|\langle\gamma_{\mathbf{k}}\rangle| = |\mathrm{Im}\,(\langle\omega_{\mathbf{k}}\rangle)| \ll |\langle\alpha_{\mathbf{k}}\rangle| = |\mathrm{Re}\,(\langle\omega_{\mathbf{k}}\rangle)|,$$

we obtain by taking real and imaginary parts of Equation (9):

$$1 = -\frac{1}{k^2}\sum_i \omega_i^2 \int d^3 v\, \mathbf{k}\cdot\frac{\partial \langle f\rangle_i}{\partial \mathbf{v}} \int_0^\infty dz\, \sin\left[(\langle\alpha_{\mathbf{k}}\rangle - \mathbf{k}\cdot\mathbf{v})\, z - z^2 A_i(z)\right] \exp - z^3 C_i(z) \tag{10a}$$

$$\langle\gamma_{\mathbf{k}}\rangle = \frac{\sum_i \omega_i^2 \int d^3 v\, \mathbf{k}\cdot\frac{\partial \langle f\rangle_i}{\partial \mathbf{v}} \int_0^\infty dz\, \cos\left[(\langle\alpha_{\mathbf{k}}\rangle - \mathbf{k}\cdot\mathbf{v})\, z - z^2 A_i(z)\right] \exp - z^3 C_i(z)}{\frac{\partial}{\partial\langle\alpha_{\mathbf{k}}\rangle}\sum_i \omega_i^2 \int d^3 v\, \mathbf{k}\cdot\frac{\partial \langle f\rangle_i}{\partial \mathbf{v}} \int_0^\infty dz\, \sin\left[(\langle\alpha_{\mathbf{k}}\rangle - \mathbf{k}\cdot\mathbf{v})\, z - z^2 A_i(z)\right] \exp - z^3 C_i(z)} \tag{10b}$$

We have introduced here the definitions

$$A_i(z) = \frac{1}{2}\mathbf{k}\cdot\frac{\partial}{\partial \mathbf{v}}\,\mathrm{Tr}\,\{\underset{\approx}{D}_i(\mathbf{v}, z)\}$$

and

$$C_i(z) = \frac{1}{3}\mathbf{k}\,\mathbf{k} : \underset{\approx}{D}_i(\mathbf{v}, z)$$

for notational convenience. Equation (10a) determines $\langle\gamma_{\mathbf{k}}\rangle$ which when inserted into Equation (10b) yields $\langle\alpha_{\mathbf{k}}\rangle$.

We assert that in Equations (10) A_i and C_i may be neglected in the integrals involving the sine function. The main contributions to these integrals comes from the non-resonant region of velocity space which is not greatly affected by the turbulence. Equation (10a) thus reduces to

$$1 = -\frac{1}{k^2}\sum_i \omega_i^2 \, P \int d^3 v \, \frac{\mathbf{k}\cdot\dfrac{\partial \langle f\rangle_i}{\partial \mathbf{v}}}{\alpha_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v}} \tag{11}$$

the form obtained in linear theory. (Since $\alpha_{\mathbf{k}}$ is not affected by the turbulence we drop the $< >$.)

On the other hand resonant particles play the dominant role in the integral appearing in the numerator of Equation (10b). Recall that

$$\lim_{d\to 0} \frac{1}{\pi} \int_0^\infty dz \cos\left[(\alpha_k - \mathbf{k}\cdot\mathbf{v})\, z\right] \exp - dz = \lim_{d\to 0} \frac{1}{\pi} \frac{d}{d^2 + (\alpha_k - \mathbf{k}\cdot\mathbf{v})^2}$$

is one representation of $\delta(\alpha_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v})$. We thus obtain

$$\langle\gamma_{\mathbf{k}}\rangle = \frac{\sum_i \omega_i^2 \int d^3 v \, \mathbf{k}\cdot \dfrac{\partial \langle f\rangle_i}{\partial \mathbf{v}} \int_0^\infty dz \cos\left[(\alpha_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v})\, z - z^2 A_i(z)\right] \exp - z^3 C_i(z)}{\partial/\partial \alpha_k \sum_i \omega_i^2 \, P \int d^3 v \, \mathbf{k}\cdot\partial\langle f\rangle_i/\partial \mathbf{v} \, (\alpha_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v})^{-1}} \tag{12}$$

Note that for an electron plasma with no streaming, Equation (11) predicts $\alpha_{\mathbf{k}} \cong \pm\, \omega_e$ and that in the absence of turbulence Equation (12) then yields

$$\langle\gamma_{\mathbf{k}}\rangle = \frac{\pi\, \alpha_{\mathbf{k}}\, \omega_e^2}{2\, k^2} \tilde{f}_e' \left(\frac{\alpha_{\mathbf{k}}}{k}\right) \tag{13}$$

in agreement with Equation (1).

As in I the strongly damped nature of the z-integrand in Equation (12) allows us to approximate $A_i(z)$ and $C_i(z)$ by their small z expansions

$$A_i\ (z) \cong \frac{1}{6}\, z^3\, \frac{q_i^2}{m_i^2}\, \mathbf{k} \cdot \sum_{\mathbf{k}'} \mathbf{k}'\ (\alpha_{\mathbf{k}'} - \mathbf{k}' \cdot \mathbf{v}) \left\langle |\delta\, \mathbf{E}_{\mathbf{k}'}|^2 \right\rangle = z^3\, \tilde{A}_i \tag{14}$$

$$\begin{aligned} C_i\ (z) &\cong \frac{1}{3}\, z\, \frac{q_i^2}{m_i^2}\, \mathbf{k}\, \mathbf{k} : \sum_{\mathbf{k}'} \hat{k}'\, \hat{k}' \left\langle |\delta\, \mathbf{E}_{\mathbf{k}'}|^2 \right\rangle \\ &= \frac{1}{3}\, z\, \frac{q_i^2}{m_i^2}\, \mathbf{k}\, \mathbf{k} : \langle \delta\, \mathbf{E}\, (\mathbf{x},\ t)\quad \delta\, \mathbf{E}\, (\mathbf{x},\ t) \rangle = z\, \tilde{C}_i \end{aligned} \tag{15}$$

Here $\delta\, \mathbf{E}_{\mathbf{k}'}$ is the amplitude of the electrostatic turbulence mode with wave vector $\mathbf{k}'$.

We now plug the forms Equations (14) and (15) into Equation (12) and obtain

$$\langle \gamma_{\mathbf{k}} \rangle = \frac{\sum_i \omega_i^2 \int d^3\, v\, \mathbf{k} \cdot \frac{\partial \langle f \rangle_i}{\partial\, \mathbf{v}} \int_0^{\infty} d\, z \cos\, [(\alpha_{\mathbf{k}} - \mathbf{k} \cdot \mathbf{v})\, z - z^5\, \tilde{A}_i]\ \exp - z^4\, \tilde{C}_i}{\partial / \partial\, \alpha_{\mathbf{k}} \sum_i \omega_i^2\, P \int d^3\, v\, \mathbf{k} \cdot \partial \langle f \rangle_i / \partial\, \mathbf{v}\, (\alpha_{\mathbf{k}} - \mathbf{k} \cdot \mathbf{v})^{-1}} \tag{16}$$

Equation 16 is the main result of this paper. Effects of the strong turbulence are explicitly manifest: alteration of ensemble averaged orbits is the source of the z^5 - term in the cosine function and dispersion in orbits about the ensemble average results in the z^4 - exponential damping.

The non-linear wave-particle interaction for species i is contained in the function

$$G_i\ (\mathbf{k},\ \alpha_{\mathbf{k}},\ u = \hat{k} \cdot \mathbf{v},\ \mathbf{v}_{\perp}) = \int_0^{\infty} d\, z \cos\, [(\alpha_{\mathbf{k}} - k\, u)\, z - z^5\, \tilde{A}_i]\ \exp - z^4\, \tilde{C}_i \tag{17}$$

which in the absence of turbulence is just $\pi\delta(\alpha_k - ku)$. The effects of turbulence, as we shall see, are to broaden this resonance and to shift its peak from $u = \alpha_k/k$.

Note from Eqs. 14 and 15 that $\tilde{A}_i$ has dimensions $(\text{time})^{-5}$ and $\tilde{C}_i$ has dimensions $(\text{time})^{-4}$. In order of magnitude $\tilde{A}_i \sim O(k/k_0\tau_{2i}^4\tau_c)$ and $\tilde{C}_i \sim O(k^2/k_0^2\tau_{2i}^4)$, where k_0 is a typical wave number of the turbulent spectrum. τ_c has been defined in I as the autocorrelation time of the fluctuations as observed by a resonant particle moving as if there were no fluctuations. (τ_c is approximately the same for all resonant particles, irrespective of species.) Also from I, τ_2 is the time for a resonant particle to diffuse (because of the turbulence) the distance $2\pi/k_0$ (τ_{2i} is τ_2 for particles of species i.)

As stressed in I the weak coupling approximation to strong plasma turbulence limits us to a consideration of the case $\tau_c << \tau_2$. The ratio $\tilde{A}_i{}^{5/4}/\tilde{C}_i$ is hence in order of magnitude greater than unity. We conclude therefore that average orbit distortion is an important factor in determining the nature of the resonance function G_i, since over the decay time of the exponential the z^5 factor in the cosine function has produced a considerable oscillation.

The fact that $\tilde{A}_i{}^{5/4}/\tilde{C}_i \gtrsim 1$ prohibits us from expanding the integrand in Eq. (17) with respect to small $\tilde{A}_i$. Consequently there is not much that can be done analytically with the resonance function $\tilde{G}_i$. To see the qualitative effects of finite $\tilde{A}_i$ and $\tilde{C}_i$ we have therefore performed a numerical integration of Eq. 17 for a one dimensional example. The turbulence spectrum that we have chosen is the same one discussed in I except that we here have taken $\Delta k/k_0 = 1/2$ and $\epsilon = 10^{-5}$.

Plotted in Figure I is $\tilde{G}_i$ in units of α_k against $\tilde{v} = kv/\alpha_k$. Recall that α_k and k are the frequency and wave number of the mode whose stability is being studied. In Curve I we have taken $\tilde{A}_i = 0$ so this trace illustrates the resonance broadening about $\tilde{v} = 1$ due to orbit dispersion as predicted and discussed by Dupree.[3] In Curve II we have added the effect of average orbit distortion. The resonance is altered asymmetrically: its maximum is shifted to $\tilde{v} > 1$ and it acquires an additional broadening primarily on the large $\tilde{v}$ side of maximum. The height of the maximum G_i is also diminished. Were we to replace the resonance function Curve II by a rectangular function (in the spirit of Dupree's[3] treatment of all resonant particle interactions with the turbulence on an equal footing) the rectangular resonance function should thus be asymmetrically positioned with respect to $\tilde{v} = 1$. This result is the basis of our assumptions in the Introduction of a shift in the "center of mass" of the resonance to $v = (\alpha_k/k) + (g_1 - g_2)/2$.

Recall now that a resonance function of the form Curve II for each species when folded with $\partial\langle f\rangle_i/\partial v$ into a v integration determines the sign of $\langle\gamma_k\rangle$ and thus the stable or unstable nature of the plasma. Near marginal stability $\langle\gamma_k\rangle \approx 0$ and hence we here expect that the deviations from asymmetry in the G_i's are decisive in determining the stability character of the plasma.

SUMMARY

We have derived an expression for $\langle \gamma_k \rangle$, the average non-linear growth (damping) rate of an electrostatic mode of wave vector **k**, for an ensemble of electrostatically turbulent, unmagnetized plasmas. The non-linearity arises because in the course of wave growth turbulence alters the orbits of particles from straight lines. The non-linearity manifests itself as both a shift in the central $u = \hat{k} \cdot \mathbf{v}$ for the Landau resonance and a broadening of this resonance in u.

Two characteristic times enter the problem: τ_c, the correlation time of the fluctuations as observed by resonant particles moving as if there were no turbulence, and τ_{2i}, the time for a resonant particle of species i to diffuse one typical wavelength $2\pi/k_0$ of the turbulence. In this theory we consider the weak coupling approximation to strong plasma turbulence so that $\tau_c << \tau_{2i}$. Average orbit distortion then becomes a significant factor in determining the form of $\langle \gamma_k \rangle$.

Average orbit distortion leads to both a shift in the Landau resonance and a broadening of its width. We feel that the resonance shift is especially important when the modes are linearly near marginal stability.

Acknowledgment

We thank P. Palmedesso of Goddard Space Flight Center for helpful discussions and E. Monesterski of Goddard Space Flight Center for assistance in the numerical computation.

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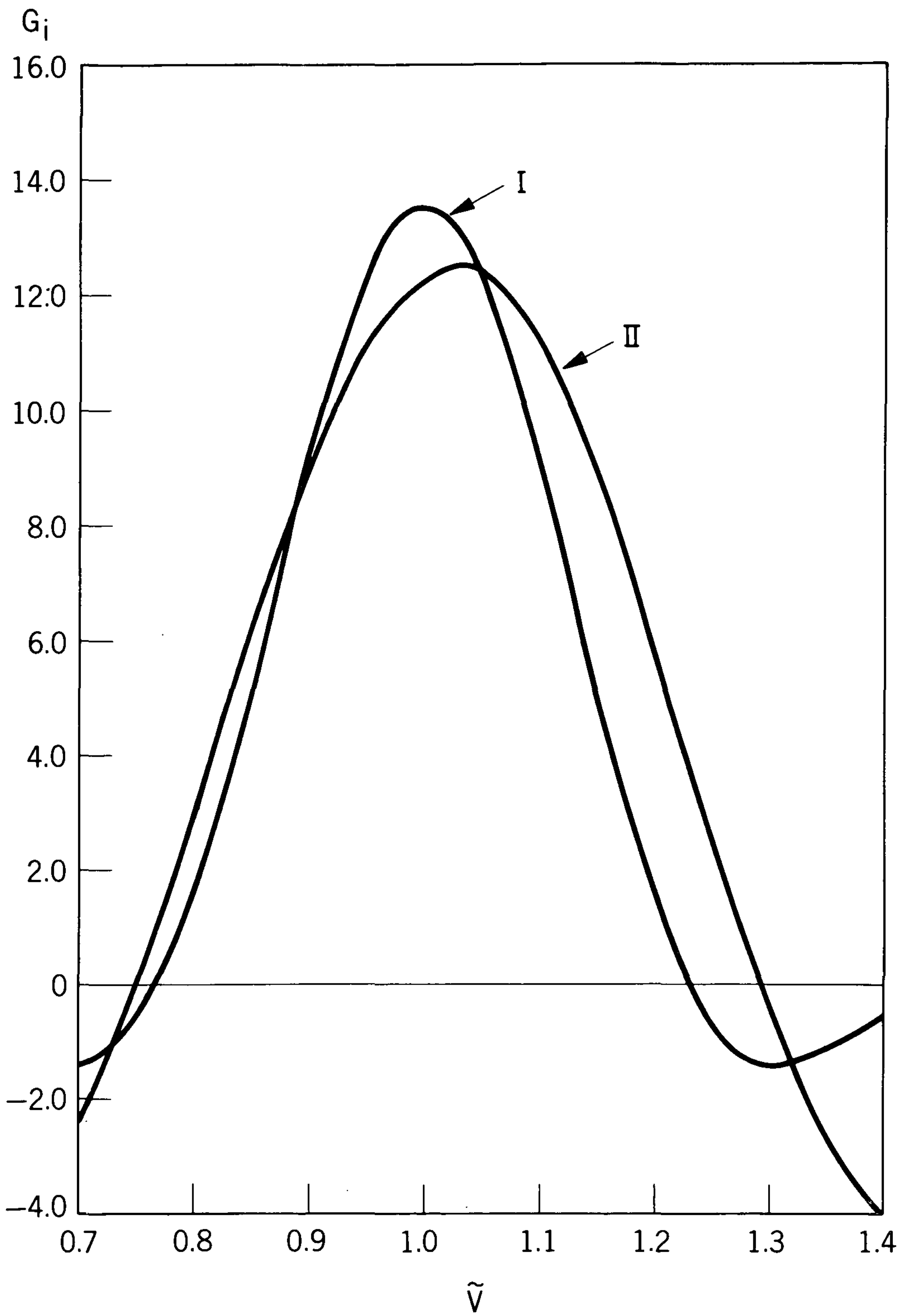
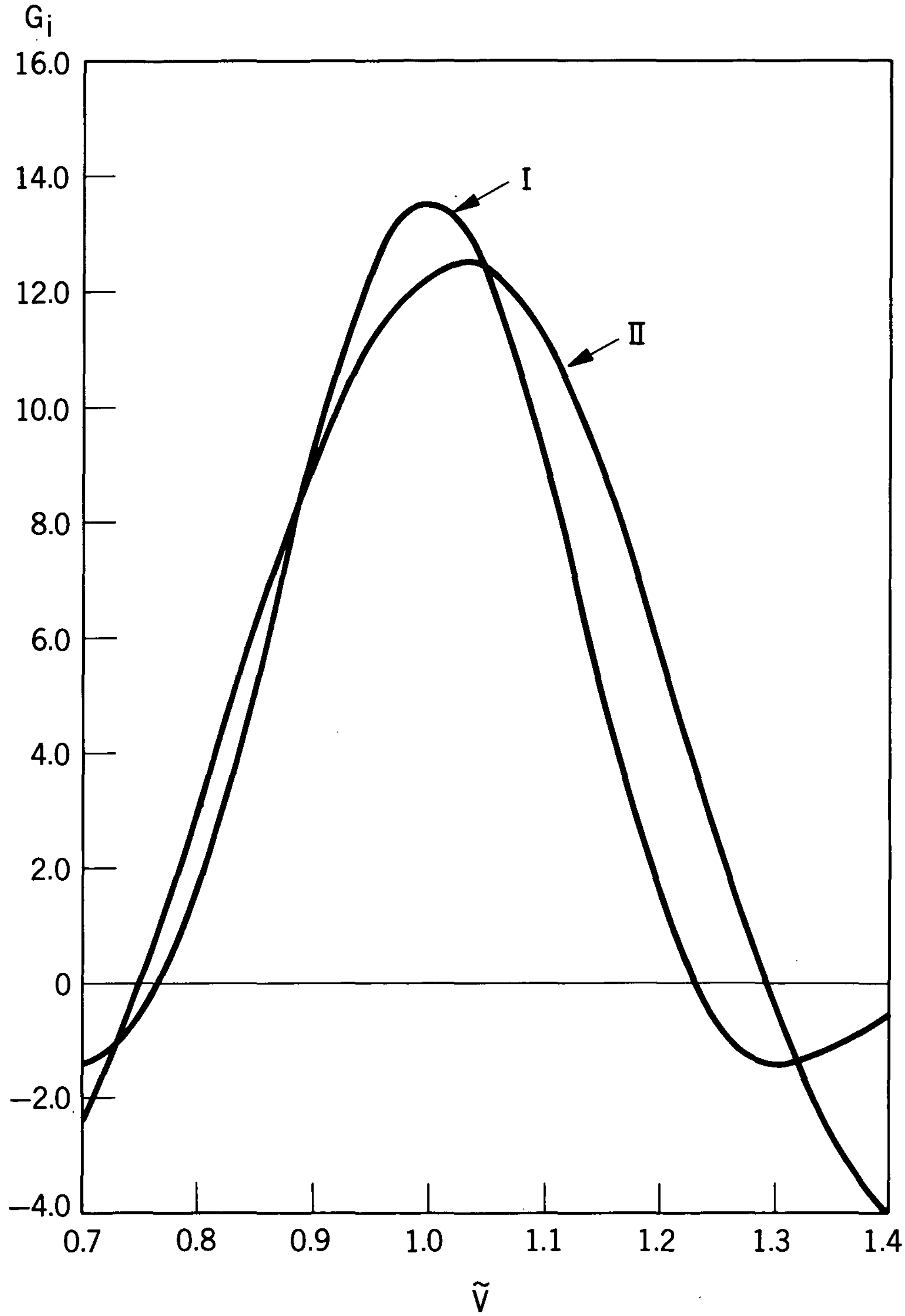


The resonance function G_i with orbit dispersion (Curve I) and both orbit dispersion and average orbit distortion (Curve II).

Figure Caption

Figure 1. The resonance function G_i with orbit dispersion (Curve I) and both orbit dispersion and average orbit distortion (Curve II).



G_i
16.0
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0
−2.0
−4.0
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
$\tilde{V}$
I
II