"WAGGING TAIL" VIBRATION ABSORBER

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Spacecraft often have long cantilevered booms which project outward from the main body of the spacecraft. In the low-damping environment of space undesired lateral vibration of these booms may build up. As a damping mechanism it is postulated that a relatively short piece of boom material can be attached to the tip of the main boom by a hinge so that it will "wag" so-to-speak when the main boom vibrates. Viscous damping is built into the hinge so that this "wagging" action dissipates energy and damps the vibration. Spring-return properties at the hinge are also postulated so that the tail tends to return to a "straight out" position. The main question to be answered is whether such a damping device of "reasonable" size will result in "reasonable" damping performance. The answer to this question, based on a modest analog computer study in two dimensions, is that "reasonable" performance does result; "reasonable" at least in the special application studied.

In this preliminary study, a 750-foot cantilever length of extendible-tape boom (very low stiffness) is considered as the main system to be damped. A number of "tail" lengths were tried from 20 feet to 80 feet after which 40 feet was investigated further as a desirable compromise between performance and practical lengths. A 40-foot damping tail produced a damping effect on the main boom for the first mode equivalent in decay rate to 3.1% of critical damping. In this case the spring-hinge and tail were tuned to the main boom first mode frequency and the hinge damping was set at 30% of critical based on the tail properties. With this same setting, damping of the second mode was .4% and the third mode .1%.

Symbols

C_c	Critical viscous damping factor
C_T	Viscous damping factor for hinge
E	Tensile modulus of elasticity
f_n	Frequency in cycles per second
I	Cross sectional area moment of inertia
I_{1T}	Mass moment of inertia of tail about hinge
K_T	Hinge spring constant
ℓ	Length of tail
L	Length of main boom
m_T	Mass of tail
M	Moment
V	Shear force
W	Lateral deflections
β	Time scaling factor (greater than 1 means computer running faster than real time)
ΔX	Length of main boom segments (see Figure 1)
θ	See Figure 1
ρ	Mass per unit length
ω	Frequency in radians per second

INTRODUCTION

On July 4, 1968, the Radio Astronomy Explorer satellite was launched. Its principal radio astronomy antennas consisted of four

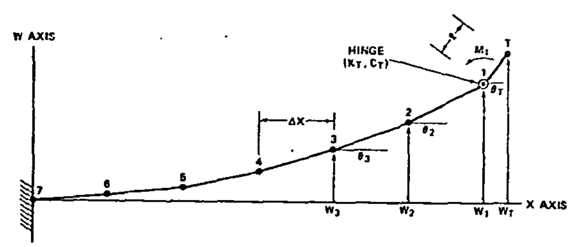

Fig. 1 - Boom, Hinge and Tail Nomenclature

750-foot tubular booms in an "X" configuration. Each tube was formed using beryllium copper tape which had been prestressed to curl lenghwise into a tube. For storage the tape was forced flat and wound onto its drum. In orbit the end of the tape is dispensed from the storage drum whereupon it immediately assumes its tubular shape. There is a seam running the length of each boom which contains interlocking teeth to prevent overlapping and so improve torsional rigidity. This example illustrates the general situation with regard to spacecraft booms.

1. Stored and then deployed in space.
2. Minimal lateral stiffness.
3. Space environment (low damping due to vacuum).

Although the lateral vibration of the Radio Astronomy Explorer booms has remained within tolerable limits without any auxiliary damping the question of how to damp in the above situation remains of interest for future spacecraft and has led to this study of the wagging-tail damper.

DESCRIPTION OF WAGGING TAIL DAMPER

The wagging-tail damper is, strictly speaking, a damped dynamic vibration absorber. A cantilever boom (i.e., cantilever beam) in lateral vibration is the system-to-be-damped. Figure 1 shows the built-in end of the cantilever at point 7. Point 1 is the free end of the cantilever. At this point there is a hinge to which is attached a piece of the boom material. This piece of boom material has been dubbed the "tail" and extends from point 1 to point T. The hinge rotation is equipped with a restoring spring (K_T) such that the tail tends to lie along the line from 2 to 1 extended. Viscous damping (C_T) is also a property of the hinge rotation so that when the tail "wags" relative to the main boom energy is dissipated.

METHOD OF ANALYSIS

The principal question to be answered is whether this type of absorber is effective. The fact that the absorber is acted on both by lateral accelerations at point 1 and by rotations at point 1 complicates the system beyond the classical one-dimensional analysis of vibration absorbers. In order to obtain a quick preliminary appraisal it was decided to make trial runs on an analog computer in two dimensions by merely deflecting the systems and observing its decay.

The finite difference representation of the boom follows Clymer, reference (1). The finite difference steps are noted on Figure 2. The progression through the circuit is from the left; deflections (W) to moments (M) to shear forces (V). These forces act on the masses involved to give accelerations which are then integrated first to velocity and then to deflections on the right which are, of course, connected to the deflection inputs on the left in the usual analog computer back-to-the-beginning-again operation.

To connect the tail to the boom it is necessary to determine what moments and forces are produced by its action. Since the tail is connected to boom only by a hinge having spring and damper properties the moment transmitted from the tail to the boom is; for small deflections:

$$M_1 = -K_T \left[\left(\frac{W_T - W_1}{\ell}\right) - \left(\frac{W_1 - W_2}{\Delta X}\right)\right]$$

$$- C_T \left[\left(\frac{\dot{W}_T - \dot{W}_1}{\ell}\right) - \left(\frac{\dot{W}_1 - \dot{W}_2}{\Delta X}\right)\right]$$

as

$$\theta_T = \frac{W_T - W_1}{\ell} \quad \text{and} \quad \theta_2 = \frac{W_1 - W_2}{\Delta X}$$

Now consider the vertical force acting on the tail at the hinge point. This force is $-V_{1/2}$ in the notation of Figure 2.

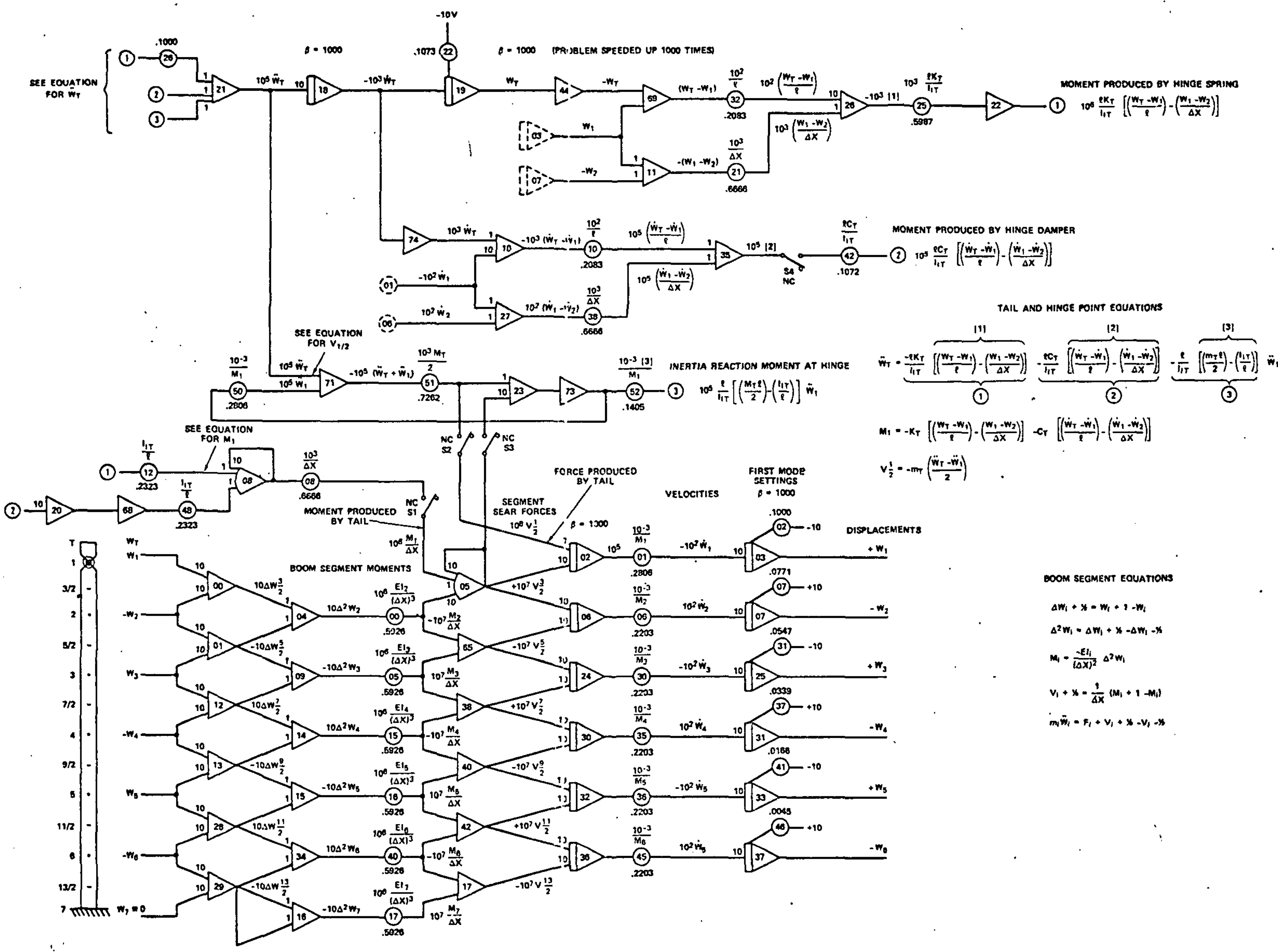

Fig. 2 - Boom and Tail Dynamics (Patching Diagram)

$$-V_{1/2} = \int_0^{\ell} (\ddot{W}_1 + x\,\ddot{\theta}_T)\,(\rho\,dx)$$

ρ = Mass per unit length

$$-V_{1/2} = \rho\ell\,\ddot{W}_1 + \rho\ell\frac{\ell}{2}\,\ddot{\theta}_T$$

$$m_T = \rho\ell$$

$$\ddot{\theta}_T = \frac{\ddot{W}_T - \ddot{W}_1}{\ell}$$

$$-V_{1/2} = m_T\,\ddot{W}_1 + \frac{m_T}{2}\,(\ddot{W}_T - \ddot{W}_1)$$

$$V_{1/2} = -\,m_T\left(\frac{\ddot{W}_T + \ddot{W}_1}{2}\right)$$

The next step is to express the dynamics of the tail so we may compute W_T, $\dot{W}_T$ and $\ddot{W}_T$ which we see appearing in the M_1 and $V_{1/2}$ expressions.

With reference to Figure 1 the summation of moments acting on the tail are given below,

$$0 = K_T\,(\theta_T - \theta_2) + C_T\,(\dot{\theta}_T - \dot{\theta}_2)$$

$$+ \int_0^{\ell} (\ddot{W}_1 + x\,\ddot{\theta}_T)\,(x)\,(\rho\,dx)$$

Upon integration and substitution of the limits of integration the integral term on the right becomes

$$\ddot{W}_1\,\rho\ell\left(\frac{\ell}{2}\right) + \ddot{\theta}_T\,\rho\frac{\ell^3}{3}$$

since $\rho\ell = m_T$, the mass of the tail, and $\rho\ell^3/3 = I_{1T}$, the moment of inertia of the tail about point 1.

The complete expression becomes

$$0 = K_T\,(\theta_T - \theta_2) + C_T\,(\dot{\theta}_T - \dot{\theta}_2)$$

$$+ \ddot{W}_1\,m_T\left(\frac{\ell}{2}\right) + I_{1T}\,\ddot{\theta}_T$$

assuming small deflections

$$\theta_T = \frac{W_T - W_1}{\ell},\quad \theta_2 = \frac{W_1 - W_2}{\Delta X}$$

substituting the above and solving for $\ddot{W}_T$

$$\ddot{W}_T = \underbrace{-\frac{\ell K_T}{I_{1T}}\overbrace{\left[\left(\frac{W_T - W_1}{\ell}\right) - \left(\frac{W_1 - W_2}{\Delta X}\right)\right]}^{[1]}}_{①}$$

$$\underbrace{-\frac{\ell C_T}{I_{1T}}\overbrace{\left[\left(\frac{\dot{W}_T - \dot{W}_1}{\ell}\right) - \left(\frac{\dot{W}_1 - \dot{W}_2}{\Delta X}\right)\right]}^{[2]}}_{②}$$

$$\underbrace{-\frac{\ell}{I_{1T}}\overbrace{\left[m_T\left(\frac{\ell}{2}\right) - \frac{I_{1T}}{\ell}\right]}^{[3]}\ddot{W}_1}_{③}$$

This expression is used to obtain the response of the tail, (i.e., $\ddot{W}_T$, $\dot{W}_T$, W_T). The brackets indicate abbreviations which are employed in Figure 2.

ANALOG COMPUTATION

The annotated patching diagram, Figure 2, shows how the computations were made. The problem was speeded up on the computer by a factor of 1000. The interface between the tail and the boom appears inconsistent. However, this is the result of the sign convention adopted in the tail analysis compared to the standard shear and bending moment conventions used in the boom analysis.

The analog computer used was made up of two TR-48 units and one Digital Expansion Unit, DES 30. An 8875 Recorder, 8 channel strip type, was used for output. This equipment was manufactured by Electronic Associates, Inc., West Long Branch, New Jersey 07764. In addition, a pictorial output was displayed on a 564 Tektronix oscilloscope (S. W. Millikan Way, P. O. Box 500, Beaverton, Oregon 97005). The details of the oscilloscope displayed are covered in Appendix 2.

RESULTS

Tuning. The spring constant at the hinge spring was chosen for these tests so that the natural frequency of the tail (assumed rigid) wagging at its hinge was the same as the first lateral vibration mode frequency of the boom (without the tail). A few trials were conducted which seemed to indicate that this tuning was not critical. There seemed to be little effect

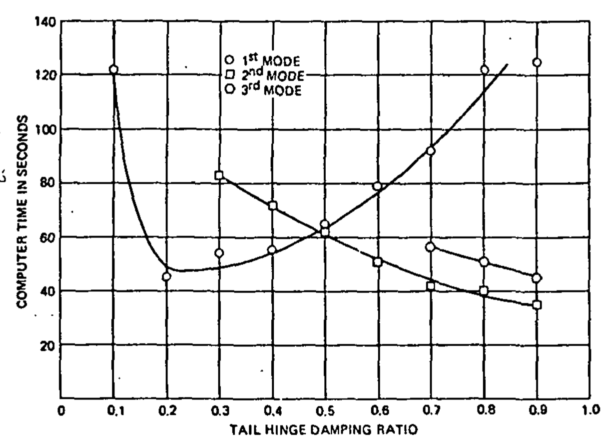

Fig. 3.0 - Decay time to 1/3 amplitude; 240-inch tail (20 feet)

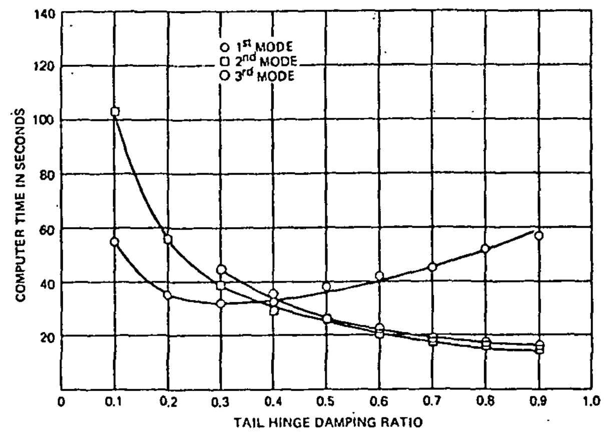

Fig. 3.1 - Decay time to 1/3 amplitude; 480-inch tail (40 feet)

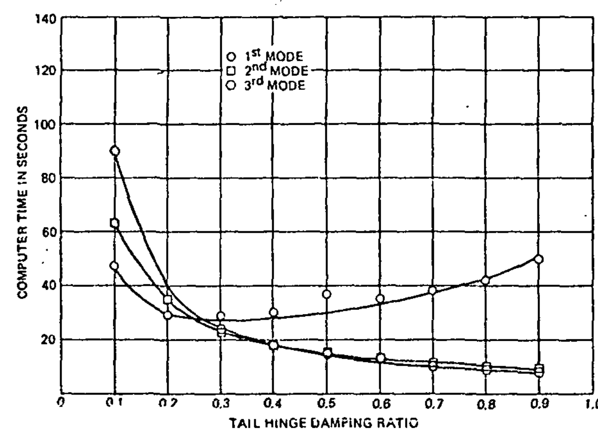

Fig. 3.2 - Decay time to 1/3 amplitude; 720-inch tail (60 feet)

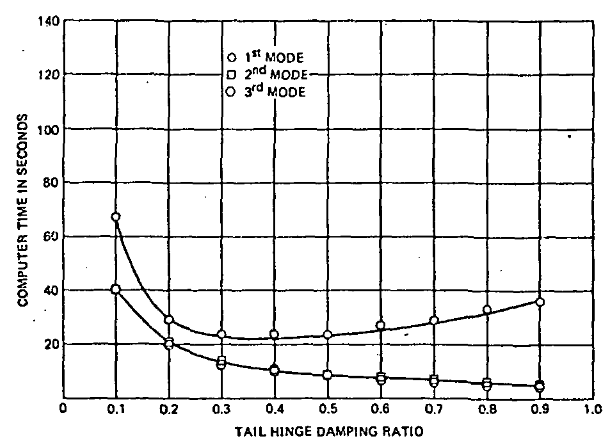

Fig. 3.3 - Decay time to 1/2 amplitude; 960-inch tail (80 feet)

on the first mode damping when the tail tuning was changed by ±10%.

Tail Damping Constant. Figures 3.0, 3.1, 3.2, and 3.3 show the decay time in seconds as a function of damping at the tail hinge. The first three modes of lateral boom vibration were investigated for 20, 40, 60 and 80-foot tail lengths. In each case the boom was given the proper initial modal deflection and the time to decay to one third the initial deflection noted.

responds to both these mechanisms so that the damper is really a deflection damper and a rotation damper at the same time. In the case studied here the tail was merely a piece of the beam material with bending of the tail not permitted. It might be interesting to see how "tails" with other characteristic might perform.

For the special case of damping spacecraft flexible booms the damping tail seems to be promising.

System Damping Results

Tail Length		Optimum Hinge Damping Ratio $\frac{(C)}{(C_c)}$	Resulting System Damping Ratio $\frac{(C)}{(C_c)}$		
inches and (feet)	% of main boom length 9000 inches (750 feet)		1st Mode	2nd Mode	3rd Mode
240 (20)	2.7	.2	.022	No Data	No Data
480 (40)	5.3	.3	.031	.0040	.0012
720 (60)	8.0	.3	.035	.0068	.0023
960 (80)	10.7	.4	.045	.016	.0056

Discussion of Results

With a first-mode-tuned tail there seems to be a definite optimum choice for damping the system (boom) first mode. In damping higher modes with this same tail tuning, larger damping appears desirable.

CONCLUSIONS

A cantilever beam in lateral vibration undergoes both lateral displacement and slope changes which are the usual beam-type deflecting mechanisms. A wagging tail damper

REFERENCES

1. A. Ben Clymer, Operational Analog Simulation of the Vibration of a Beam and a Rectangular Multicellular Structure, IRE Transactions on Electronic Computers, Vol. EG-8, pp. 381, Sept. 1959

2. J. P. Den Hartog, Mechanical Vibrations, pp. 119-132, Third Edition, McGraw-Hill, New York, 1947.

3. C. R. Freberg and E. N. Kemler, Elements of Mechanical Vibration, pp. 142. Second Edition, John Wiley and Sons, New York, 1949

APPENDIX 1 – CALCULATIONS

Main Boom

ρ mass per unit length, 3.026×10^{-6} (slugs/inch)

EI stiffness, 2000.0 (pound-inches squared)

L length, 9000.0 (inches)

f_n frequency (cycles per second)

From Reference 3 pp. 142

$$f_n = A\sqrt{\frac{EI}{\rho L^4}}$$

where

A = .560 for the first mode

A = 3.51 for the second mode

A = 9.82 for the third mode

Frequencies
- 1st mode $.1777 \times 10^{-3}$ (cps) 1.117×10^{-3} r/sec
- 2nd mode 1.0074×10^{-3} (cps) 6.330×10^{-3} r/sec
- 3rd mode 3.1169×10^{-3} (cps) 19.584×10^{-3} r/sec

Periods
- 1st mode 5627.46 sec. = 93.79 min. = 1.56 hours
- 2nd mode 992.65 sec. = 16.54 min. = .276 hours
- 3rd mode 320.83 sec. = 5.35 min. = .089 hours

Tail Calculations

- ρ mass per unit length, 3.026×10^{-6} (slugs/inch)
- ℓ length (inches) and (feet)
- m_T mass of tail (slugs)
- K_T hinge spring constant (inch-pounds/radian)
- C_T hinge damping constant (inch-pounds/radian/sec)
- C_c critical damping constant at hinge (inch-pounds/radian/sec)
- I_{1T} mass moment of inertia of tail about hinge point (slug-inches squared)
- ω_1 1st boom mode frequency = tail frequency = 1.117×10^{-3} (radians/sec)

System Decay

$\frac{C}{C_c}$ damping ratio

$\frac{y_n}{y_1}$ amplitude ratio of nth cycle to first cycle

assuming exponential decay

$$\ell n \frac{y_n}{y_1} = -n\,(2\pi)\frac{C}{C_c}$$

consider decay to 1/3 amplitude

$$\ell n\ .3333 = -1.1$$

$$\frac{C}{C_c} = \frac{1.1}{2\pi n} = \frac{.175}{n}$$

Length (ℓ)	240 (20)	480 (40)	720 (60)	960 (80)
$m_T = \rho\ \ell$	$.726 \times 10^{-3}$	1.452×10^{-3}	2.178×10^{-3}	2.905×10^{-3}
$I_{1T} = m_T \frac{\ell^2}{3}$	1.394×10^{1}	1.115×10^{2}	3.764×10^{2}	8.924×10^{2}
$C_c = 2\, I_{1T}\, \omega_1$	.031	.249	.841	1.99
$K_T = \omega_1^2\, I_{1T}$	1.739×10^{-5}	1.390×10^{-4}	4.694×10^{-4}	11.128×10^{-4}

APPENDIX 2 – OSCILLOSCOPE DISPLAY

The main purpose of an oscilloscope display was so that a movie record could be made by photographing the scope face. The picture looks approximately like Figure 5 with lateral vibration up and down. The numbers do not appear on the scope but are in the figure for better description. The persistence of the screen, in a sense, recorded the past amplitude of the vibration and is indicated by the shading in Figure 5. The lateral movement was greatly magnified and leads to tail-length distortion when the tail deflection is large.

The patching diagram for the scope display is shown in Figure 4. Using a 1-Kc clock and flip-slops in the Digital Expansion Unit a multivibrator was constructed which puts out a sawtooth voltage. See upper left of Figure 4. This voltage, suitably amplified and switched, is used as a basic input by both the horizontal deflection circuit and the vertical deflection circuit.

The work of the horizontal circuit is to deflect the electron beam in the scope from point 7 to 6 to 5 to 4 etc. Actually what is done is to create, simultaneously, the 7 to 6, 6 to 5, 5 to 4 etc. deflection voltages and to switch each into the scope one at a time in proper sequence. Potentiometers 00, 01, 02, etc., on the right hand side of the diagram, are used to set the horizontal deflection voltages for the points 7, 6, 5, 4, 3, etc. To those starting voltages (each in a separate circuit) the properly amplified sawtooth is added. Thus, in circuit 7 (summer 10) a voltage is created every sawtooth cycle that, if switched in, would deflect the scope beam through the horizontal distance from point 7 to point 6. Simultaneously, in summer 11 a voltage is created that, if switched in, would deflect the scope beam from 6 to 5. In summer 12 we have 5 to 4 capability and in 13 we have 4 to 3 capability and so on up the line. The rise time of the saw tooth fixes the time used for each deflection.

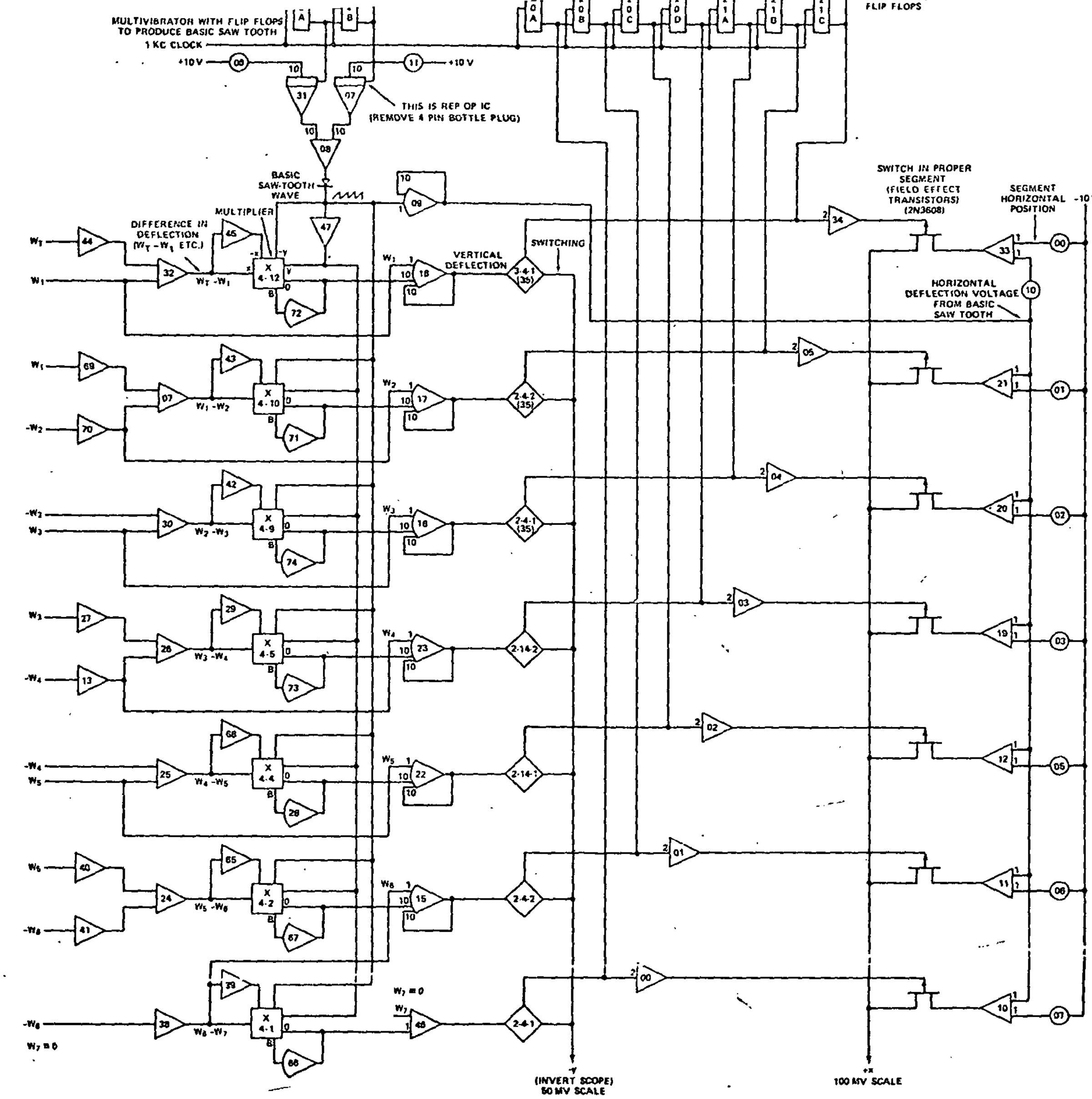

Fig. 4 - Scope Display (Patching Diagram)

The next step in arranging the horizontal output of the scope is to provide the switching (mentioned in the previous paragraph). This switching is accomplished by using the flip-flop chain (upper right in figure 4) to put the field-effect transistors (right side of figure 4) into conductivity in sequence. This connects the horizontal deflection voltage to the scope. For instance, on one saw-tooth cycle the scope beam uses the 7 to 6 deflection voltage and deflects from 7 to 6. The next cycle it goes from 6 to 5 and so on.

This same flip flop chain activates switches in the center of Figure 4 which switch in the corresponding vertical deflection voltage to go with the horizontal deflection voltage. To generate the vertical deflection voltage a similar technique to that used in the horizontal case is employed in that there is a starting point for each segment. In the horizontal case these starting points were fixed voltages as the segment ends, 7, 6, 5, 4, etc., do not move horizontally. In the vertical case these points do move but their positions are known from the

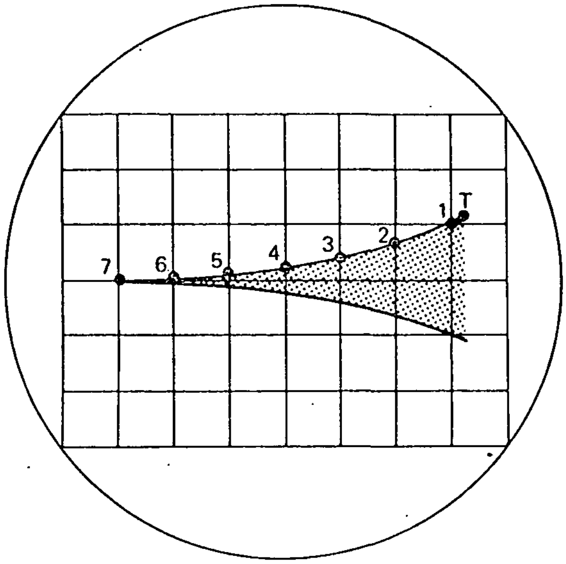

Fig. 5 - Oscilloscope Presentation

computer dynamic simulation, namely W_7, W_6, W_5, W_4 etc. Thus, we see these values fed into summer 46, 15, 22, 23 etc., as starting points. Now, in addition, the scope beam must have the proper vertical movement as it goes from say point 6 to point 5 so that it reaches point 5 with the deflection equal to that of point 5. This is done by forming the difference $W_5 - W_6$ and multiplying it by the saw-tooth voltage. These multipliers are the square symbols about 1/3 of the-figure-width in from the left side of Figure 4. In-other-words if the maximum of the saw tooth is taken as unit it can be seen that starting at $W_6 + (1)(W_5 - W_6) = W_5$. Once again the time base for this deflection is furnished by the saw tooth.

The main advantage of this circuit is that it draws the whole beam in 7 milliseconds which is more in keeping with scope display persistance and motion picture photography than the slower moving display that one might drive from computer integrator time bases and logic switching.

DISCUSSION

Mr. Ruzicka (Barry Controls): What are the frequencies in hertz, or other dimensions if you like?

Mr. Barclay: The period is ninety minutes, so it is pretty slow stuff. In the paper I gave all the minutes and seconds and hours. I have to speed it up by a factor of one thousand and put it on the computer.

Mr. Verga (Hazeltine Corp.): You said that these things have been applied to spacecraft. Obviously, since the booms are so long they are deployed after the spacecraft has reached a space position. Can you tell us why it is that they still get excited? Also, you have listed the damping coefficients for the hinge. Can you tell us a little bit about any special properties of that hinge and how you obtain the specified coefficients?

Mr. Barclay: The radio astronomy explorer did not have this on because it was very well behaved without it. We are going to send the next radio astronomy explorer to the moon and things may be a little bit tougher there. We have very good circularity of orbit around the earth which is a disturbing factor for the gravity gradient satellites. This leads to the kind of excitation that you get. When the sun shines on one side and you do not always have that side to the sun, the solar pressure sort of bends the structure back and forth. The pressure is not very large, but it exists. Further, if there is any eccentricity in the orbit, when the satellite is closer to the earth it is pulled in more or let out more. The gravity gradient effect which is used to stabilize this thing in a local vertical position as it goes around the earth is very weak. If you think the structure is weak, the gravity gradient effect is far weaker. That is why you get excitation. With respect to the hinge, it is created by twisting a wire. Bearings could not be used because the friction is too much or too unpredictable. The twisted wire creates the hinge and also a spring returning effect. These things have been built and there is really not much of a problem getting them into hardware.