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STABILIZING A SPINNING SKYLAB

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SUMMARY

This paper presents the results of a study of the dynamics of a spinning Skylab space station. The stability of motion of several simplified models with flexible appendages was investigated. A digital simulation model that more accurately portrays the complex Skylab vehicle is described, and simulation results are compared with analytically derived results.

INTRODUCTION

In 1970 NASA's Marshall Space Flight Center initiated a study to determine the possibility of spinning the Skylab (the first U. S. manned orbiting space station). The purpose of the spin would be to provide an artificial gravity environment to assess and compare the physiological and mental ramifications of prolonged zero-gravity and artificial-gravity environments. Several study teams were formed at the Marshall Space Flight Center. The coauthors of this paper comprise one of the study teams that have been designated to conduct this study until conclusive results have been obtained. It is anticipated that the results of the study will be appropriate not only for Skylab but also for analysis of future spacecraft, particularly those that are composed of several connected bodies with attached flexible appendages.

In spinning the Skylab, it will be necessary to spin the vehicle about the principal axis of intermediate moment of inertia (in the original nonspinning Skylab configuration) to keep the solar panels pointed nominally toward the sun. Since it was hoped that passive stability could be achieved, it became necessary to consider deploying masses either on cables or extendable booms so that the principal axis of maximum moment of inertia could point in the same direction as the solar panels.

The approach followed was to attack the problem from two extremes. The first extreme was to study the stability of motion of a simplified model composed of a single rigid core mass with two attached rigid appendages lying in the same plane of rotation. The approach involved adding complexity to the simple model, one step at a time, in an attempt to achieve a more realistic model and still obtain results that would be amenable to interpretation in terms of physical parameters. The second model studied consisted of the same model but with flexible, rather than rigid, appendages in an attempt to obtain a closed-form solution indicating the effect of appendage flexibility on the motion of the system.

Geometric asymmetries of the original Skylab require that the extendable booms be attached out of the original plane of rotation to make the desired rotation axis an axis of principal inertia. However, the out-of-plane distance of the booms is small compared with their lengths, so it is neglected in the first two models. Subsequently, its influence is investigated. A third model reported in this paper consisted of a core mass with two tip masses connected to it by flexible massless beams lying in two different planes. The stability study involved the use of the linearized equations of motion of the system under consideration.

The fourth and final model consists of a detailed simulation model for the complex Skylab vehicle (Fig. 1). The simulation is being developed at the Marshall Space Flight Center with the assistance and advice of Professor Peter Likins of the University of California, Los Angeles. The stability of the motion and transient response of the spinning Skylab is being investigated by numerical integration of the equations of motion. The hybrid coordinate method developed by Likins[1] is used in analyzing structural characteristics. The appropriate matrices characterizing the flexible appendages and the general equations of motion of the entire Skylab have been developed and programed for in-house use. Applicability of the analysis is not restricted to a spinning vehicle since the development of matrix equations is adaptable to the nonspin case.

SIMPLIFIED MODELS

The third simplified model (referred to previously) chosen to represent the spinning Skylab vehicle is portrayed in Figure 2. The angular velocity vector of the vehicle may be written in body-fixed coordinates 1, 2, 3 as $\omega = [w_1, w_2, w_3 + \Omega]$, where $|w_i| \ll 1$ $(i = 1, 2, 3)$ represent small perturbations about the steady state velocity Ω. Two tip masses m are attached to the core body by flexible massless booms. The variables u_i^k $(i = 1, 2, 3;\ k = 1, 2)$ represent small displacements of the tip masses from the steady state. In the steady state, the principal axes of inertia of the total vehicle coincide with the 1, 2, 3 axes and the principal moments of inertia are I_1, I_2, I_3, respectively, with $I_1 < I_2 < I_3$. The stiffness of the booms is characterized by the stiffness coefficients k_i of the nonrotating booms and the contribution of the geometric stiffness terms $m\Omega^2$ introduced by spin. Hence, the overall boom stiffness in the 1, 2, 3 directions is represented by $k_1 + m\Omega^2$, k_2, and $k_3 + m\Omega^2$, respectively. Structural damping is proportional to elastic deformation velocities in the 1, 2, 3 directions and is denoted by coefficients d_i $(i = 1, 2, 3,)$. The coordinates of the two tip masses in the equilibrium state are described by distances $[\Gamma_i]$ and $[-\Gamma_i]$, respectively. The rotational dynamics of the vehicle may be represented by a set of nine differential equations written in the variables u_i^k, w_i. The set may be reduced to six equations by making either of the substitutions, $u_i = u_i^1 - u_i^2$ or $v_i = u_i^1 + u_i^2$. Physically, the u_i's represent the skew symmetric mode of the elastic deformations and hence cause angular disturbances about the vehicle's steady state. The v_i's represent the symmetric mode and only cause the vehicle to translate. Since stability of rotational motion is of interest, only the skew symmetric mode is considered. The corresponding linearized equations of motion are

$$
\begin{aligned}
& I_1 \dot{w}_1 + (I_3 - I_2)\Omega w_2 + m\Gamma_2 (\ddot{u}_3 + \Omega^2 u_3) \\
& -m\Gamma_3 (2\Omega\dot{u}_1 + \ddot{u}_2 - \Omega^2 u_2) = T_1 \\
& (I_1 - I_3)\,\Omega w_1 + I_2 \dot{w}_2 + m\Gamma_3 (\ddot{u}_1 - \Omega^2 u_1 - 2\Omega\dot{u}_2) = T_2 \\
& 2m\Gamma_2 (\dot{w}_1 + \Omega w_2) + m\ddot{u}_3 + d_3 \dot{u}_3 + (k_3 + m\Omega^2)\, u_3 = 0
\end{aligned}
\qquad (1)
$$

and

$$
\begin{aligned}
& I_3\dot{w}_3 - m\Gamma_2 (\ddot{u}_1 - 2\Omega\dot{u}_2) = T_3 \\
& 2m\Gamma_3 (\Omega w_1 + \dot{w}_2) - 2m\Gamma_2\dot{w}_3 + m\ddot{u}_1 \\
& + d_1 \dot{u}_1 + k_1 u_1 - 2m\Omega\dot{u}_2 = 0 \\
& 2m\Gamma_3 (-\dot{w}_1 + \Omega w_2) - 4m\Gamma_2\Omega w_3 + 2m\Omega\dot{u}_1 + m\ddot{u}_2 \\
& + d_2 \dot{u}_2 + (k_2 - m\Omega^2)u_2 = 0,
\end{aligned}
\qquad (2)
$$

where T_i's represent applied torques about the body-fixed axes. When Γ_3 is set equal to zero, equations (1) become uncoupled from equations (2) with the former governing wobble motion and the latter describing variations in spin velocity.

Passive Stability

The motion of the vehicle can be described by a nutation about the axis of angular momentum which is inertially fixed when no external torques act on the vehicle. The motion is called passively stable if the nutation damps out and the vehicle rotates only about the axis of angular momentum. Any change of attitude of the axis of angular momentum can occur only by the application of external torques. Therefore, to achieve attitude stabilization of the vehicle, an active control system providing stabilizing torques must be added. Because the 3-axis is the axis of intermediate principal moment of inertia of the original vehicle (represented by the core body of the simplified model), the vehicle cannot be stably spun about that axis. It is of interest to determine if the spinning vehicle can be passively stabilized by adding extendable booms with tip masses so that the condition $I_1 < I_2 < I_3$ is met for the entire vehicle.

An analysis of the stability of wobble motion may be made by obtaining the characteristic equation associated with equations (1),

$$
\begin{aligned}
& (1 - \gamma_1)\,\lambda^4 + \Delta_3\lambda^3 - (K_1K_2 + \gamma_1K_2 + \gamma_1 - \sigma_3^2 - 1)\lambda^2 \\
& - \Delta_3K_1K_2\lambda - [(\sigma_3^2 + 1)\ \ K_1K_2 + \gamma_1K_2] = 0.
\end{aligned}
\qquad (3)
$$

where $\lambda = s/\Omega$, $\sigma_i^2 = k_i/m\Omega^2$, $\gamma_i = 2m\Gamma_2^2/I_i$, $\Delta_i = d_i/m\Omega$, $K_1 = (I_2 - I_3)/I_1$, $K_2 = (I_3 - I_1)/I_2$, and s is the Laplace operator. For physical reasons, $|K_1| < 1$, $|K_2| < 1$, $0 < \gamma_1 < 1$, and $d_i > 0$. When D-decomposition[2] or other stability determination techniques are applied, regions of stability can be obtained analytically and shown on the K_2, K_1K_2 parameter plane. The stability region (Fig. 3) is bounded by straight lines,

$$K_2 = -[(\sigma_3^2 + 1)/\gamma_1]K_1K_2, \qquad (4)$$

$$K_2 = K_1K_2, \qquad (5)$$

$$K_1K_2 = -1. \qquad (6)$$

As shown in Figure 3, the effect of the flexibility of the booms is to alter the region of stability from that of a rigid vehicle, but for sufficient boom stiffness it is possible to stabilize the wobble motion of the vehicle. These results corroborate those found and reported by Frank Barbera[3]. Although not included in this paper (for the sake of brevity), the transient response may be determined by mapping contours of constant damping ratio, plotted as functions of natural frequency, on the K_2, K_1K_2 plane, using the parameter plane technique[4].

The stability of the spin velocity motion is determined from the characteristic equation obtained from equations (2). Analysis shows that stability exists as long as the booms have a finite stiffness in the 1-direction and are sufficiently stiff to withstand the centrifugal force in the 2-direction, i.e., $\sigma_1 > 0$ and $\sigma_2^2 > 1 - 4\gamma_3$.

To model the vehicle more accurately, the influence of the asymmetrical arrangement of the booms is investigated. With $\Gamma_3 \neq 0$, equations (1) and (2) are coupled. If numerical values are chosen to represent Skylab characteristics (Table 1), the effect of varying σ_3 (characterizing boom stiffness) may be determined. This is done by solving equations (1) and (2) for their eigenvalues and plotting a root locus (Fig. 4), using σ_3 as the variable parameter. The asymmetry Γ_3 has a negligible effect on the locations of the roots for all physically possible values of Γ_3, and, therefore, on the stability of the vehicle motion. However, there is a constraint on the minimum magnitude of σ_1^2. A necessary stability condition for the limit case consisting of a high value of stiffness in the 2-direction ($\sigma_2 \rightarrow \infty$) results in the constraints,

$$\sigma_1^2 > (2m\Gamma_3^2/I_2)/K_2,$$
$$\sigma_3^2 > -(2m\Gamma_2^2/I_1)/K_1. \qquad (7)$$

Active Attitude Control

The mission of a spinning Skylab makes it necessary to point the 3-axis at the sun rather than to passively stabilize the steady state rotation of the vehicle about its 3-axis. This will place the solar panels (lying in the 1-2 plane of the vehicle) normal to the impinging rays of the sun, making maximum use of solar energy. To maintain the 3-axis inertially fixed (moving it slightly at discrete intervals to realign it toward the sun), attitude control torques must be applied to the vehicle to compensate for the effect of disturbance torques.

The control torques must in some manner depend on error signals that are proportional to the angle between the 3-axis and the solar vector. Additionally, it may be desirable to add a damping term that depends on the time rate of change of the error signals. Because of existing hardware on the present Skylab configuration[5], these signals are readily available from sun sensors and rate gyros. The sun sensors resolve the small angle between the solar vector and the 3-axis into rotations about the 1 and 2 axes, φ_1 and φ_2 (Fig. 5). Angular velocities w_1, w_2 of the vehicle are measured by the rate gyros. The variables φ_j and w_j are shown in Figure 5. The control torques may be provided by three control moment gyros (CMG's) that also exist on the present Skylab. It is technically feasible to use a mass expulsion system to provide these torques, and such a system does exist on the Skylab. However, in an attempt to prevent impairing optical experiments and minimize contamination of the Skylab environment, it is desired to use the CMG's to the greatest extent possible to provide control torques.

If the CMG's are used to provide control torques T_1 and T_2, a linear control postulate may be formulated as

$$T_n = \alpha_{nj}\varphi_j + \beta_{nj} w_j, \quad n = 1, 2, \; j = 1, 2. \qquad (8)$$

If first the value Γ_3 is neglected, wobble motion can again be treated separately from spin velocity motion. The spin velocity Ω and its perturbation w_3 are controlled separately and are not considered in this paper. Using equations (1) and (8) and the kinematic relationships,

$$w_1 = -\dot{\varphi}_1 + \Omega\varphi_2,$$
$$w_2 = -\dot{\varphi}_2 - \Omega\varphi_1, \qquad (9)$$

one obtains the attitude control equations in matrix form,

$$A\ddot{x} + B\dot{x} + Cx = 0, \qquad (10)$$

where

$$x = [\varphi_1, \varphi_2, u_3]^T,$$

$$A = \begin{bmatrix} I_1 & 0 & -m\Gamma_2 \\ 0 & I_2 & 0 \\ -m\Gamma_2 & 0 & m/2 \end{bmatrix}, \quad B = \begin{bmatrix} -\beta_{11} & -(I_1 + I_2 - I_3)\Omega - \beta_{12} & 0 \\ (I_1 + I_2 - I_3)\Omega - \beta_{21} & -\beta_{22} & 0 \\ 0 & 0 & d_3/2 \end{bmatrix}$$

$$C = \begin{bmatrix} (I_3 - I_2)\Omega^2 - \beta_{12}\Omega + \alpha_{11} & \beta_{11}\Omega + \alpha_{12} & -m\Gamma_2\Omega^2 \\ -\beta_{22}\Omega + \alpha_{21} & -(I_1 - I_3)\Omega^2 + \beta_{21}\Omega + \alpha_{22} & 0 \\ -m\Gamma_2\Omega^2 & 0 & (k_3 + m\Omega^2)/2 \end{bmatrix}$$

A simplification is obtained by retaining only the torque coefficients α_{12} and β_{11}. This may be justified by applying general stability criteria[6] which require that the damping forces of a linear system such as equation (10) be greater than the destabilizing forces. This condition is sufficiently met if the symmetric part of the matrix B is positive definite, which leads to the consideration of only the more essential diagonal terms β_{11}, β_{22}, in the matrix B. Furthermore, the matrix C must be positive definite to avoid instability. A physical explanation for retaining α_{12} and α_{21} is that a control torque must be applied about the 1-axis to correct for an attitude error angle about the 2-axis of the spinning vehicle and vice versa. A further simplification is achieved if the control torques are applied about a single body axis. If the 1-axis is chosen, α_{12} and β_{11} are retained. Then the stability boundary associated with the imaginary axis in the complex plane may be obtained by setting λ equal to its imaginary part, $i\upsilon$, in the characteristic equation obtained from equation (10). Setting the real and imaginary parts of the resulting equation equal to zero separately, one obtains two simultaneous equations,

$$(1 - \upsilon^2)\,\delta + (1 - K_2)\,\epsilon$$
$$= -(1 - \upsilon^2)[-\gamma_1 (1 - \upsilon^2)(K_2 - \upsilon^2) + (\upsilon^2 - \sigma_3^2 - 1)$$
$$(\upsilon^2 + K_1K_2)]/\Delta\upsilon^2 \qquad (11a)$$

and

$$(1 - \upsilon^2)\delta + (1 - K_2)\epsilon = \Delta(K_1K_2 + \upsilon^2)$$

$$(1 - \upsilon^2)/(\upsilon^2 - \sigma_3^2 - 1), \qquad (11b)$$

respectively, where $\delta = \beta_{11}/I_1\Omega$ and $\epsilon = \alpha_{12}/I_1\Omega^2$. From the structure of equations (11), it is observed that the right-hand sides must be set equal to each other for a common solution to exist. The result of this equality is a cubic equation in υ^2, which may be solved for its three roots.

Only positive real roots for υ^2 have any physical significance. An approximate solution of the cubic equation may be obtained if the stiffness of the boom is sufficiently high so that one of the roots can be approximately associated with the natural frequency of the boom, i.e., made equal to $\upsilon^2 \approx \sigma_3^2 + 1$. For the very flexible booms of the Skylab, the numerical values of the approximate roots differ by less than 5 percent from the exact values. Substitution of these roots into either of equations (11) yields three straight lines in the parameter plane for control coefficients ϵ and δ, one of which is a stability boundary. Other stability constraints are $\epsilon > 0$ and $K_2(\gamma_1 + \Omega K_1) < 0$. These results are portrayed on the ϵ, δ parameter plane (Fig. 6), using the numerical parameters representative of the actual Skylab. The stability boundary line $\epsilon = -[(1 + K_1K_2)/(1 - K_2)]\delta$ indicates the limit case where the total vehicle is considered to be rigid. In general, flexibility influences the stability region in terms of admissible control torque; for the Skylab example (Fig. 6), the ϵ, δ stability region is increased. Contrary to the rigid body case, small values of destabilizing damping torques (δ>0) can be tolerated without destabilizing the system because of the passive stability of the wobble motion, which depends on the structural damping of the elastic boom.

The transient response may be determined on the ϵ, δ plane by applying the parameter plane technique. It shows that a considerably improved response can be obtained by a proper choice of the control terms. The stability of the spin velocity motion is determined from equations (2) and therefore yields the same stability requirements as the passive case.

As with the passive case, the influence of the asymmetrical arrangement of the booms is investigated. The effect of boom asymmetry ($\Gamma_3 \neq 0$) on attitude stability is found to be negligibly small for Skylab parameters.

Control system efficiency, in terms of vehicle dynamic response and ease of implementing a CMG control law should be improved by providing control torques about two body-fixed axes. This case is presently being investigated. Also under investigation is the characterization of suitable performance criteria for the selection of control parameters and the development of other candidate control laws. Other areas to be explored include the modeling and analysis of significant hardware nonlinearities, control of the spin motion, and an analysis of the nonstationary equations of motion associated with changes in spin rate, either deliberate or unexpected.

DETAILED SIMULATION MODEL

Modern space vehicles can be described as a combination of essentially rigid bodies and relatively flexible bodies. The analytical modeling of extremely flexible appendages requires that they be considered to consist of small rigid bodies interconnected by elastic massless bodies. At present there appear to be two different methods of writing the equations of motion for spinning vehicles with appendages: the discrete parameter approach and the hybrid coordinate method. In the discrete parameter approach, equations of motion of various subsets of bodies or individual bodies are obtained by the direct use of Newton-Euler equations of translation and rotation. To develop an effective digital simulation, it is necessary to model the flexible appendage as many small rigid bodies which will yield a large number of first- and second-order differential equations of motion. Since there is no way of reducing the number of variables, it is difficult to numerically integrate these equations. However, the hybrid coordinate method removes this difficulty by transforming a set of discrete coordinates into a smaller set of distributed coordinates, each of which is associated with the normal mode of a spinning or nonspinning appendage. In the hybrid coordinate approach, first the equations of motion of the flexible appendages are written in discrete coordinates, separately from those of the entire vehicle. Then the discrete motion of idealized bodies of the appendage is transformed into a fewer number of modal coordinates, thereby reducing the total number of equations of motion to be numerically integrated.

The effect of spin is induced in the development of equations of motion despite the great added complexity introduced. This is necessary because the results of several cases studied here indicated the noticeable and possibly severe effect of spin on the modes of vibration.

Flexible Appendage Equations

The linearized set of equations of motion of one or more flexible appendages attached to a single body, after removing the rigid-body degrees-of-freedom, can be written as

$$M'\ddot{u} + G'\dot{u} + K'u = F, \text{ where } K' = K + K_e + K_g \qquad (12)$$

and u is the vector of the discrete generalized displacements of the masses of the appendages with respect to the central rigid body. M' is the symmetric nondiagonal mass matrix, G' is the skew symmetric matrix of the coriolis accelerations, K is the symmetric matrix of the centripetal accelerations, K_e is the elastic stiffness matrix of the appendages; and K_g is the geometric (or differential) stiffness matrix showing the effect of stretching caused by spin.

The vector F represents a function of the angular velocities of the central rigid body and their time derivatives and couples the motion of the flexible appendages to the motion of the rigid body. Structural damping is added after truncation has taken place. The relations $y = Du$, $M' = MD$, $G' = GD$, $K = K''D$ may be used to rewrite equations (12) in the form

$$M\ddot{y} + G\dot{y} + [K'' + (K_e + K_g)\, D^{-1}]\, y = F, \tag{13}$$

where y is the vector representing the displacement of the masses, with respect to the center of mass of the system. M is a diagonal mass matrix, G is the tridiagonal skew symmetric matrix, and K'' is a tridiagonal symmetric matrix. Matrices D and D^{-1} can be written explicitly.

Truncation

The set of equation (13) is being used because it has the advantage of requiring less computer storage core than the set of equations (12). However, this approach has the disadvantage that the matrix $[K_e + K_g]\,[D]^{-1}$ is nonsymmetric. The eigenvectors of equation (13) are obtained and transformed to yield the eigenvectors of equation (12).

Since it is not possible to uncouple either the set of equations (12) or (13), they are reduced to the state space formulation in several steps. This leads to the uncoupled set of equation (15), which is amenable to truncation. The first step is to write the state space equation,

$$P\dot{q} + Qq = L, \tag{14}$$

where

$$P = \begin{bmatrix} M' & 0 \\ 0 & K' \end{bmatrix}, \quad Q = \begin{bmatrix} G' & K' \\ -K' & 0 \end{bmatrix}, \quad L = \begin{bmatrix} F \\ 0 \end{bmatrix},$$

$$\{q\} = \begin{Bmatrix} \dot{u} \\ u \end{Bmatrix} .$$

The eigenvectors of equation (14) are constructed from those of equation (13).

Let Φ_j be the jth eigenvector of equation (14) and Φ'_j be the jth eigenvector of the adjoint of equation (14). Eigenvector Φ'_j is the complex conjugate of eigenvector Φ_j, since P is symmetric and Q is skew symmetric. Using the transformation $q = \Phi z$ and premultiplying equation (14) by the matrix Φ'^T, one finally obtains a set of uncoupled equations

$$P^D\dot{z} + Q^D z = \Phi'^T L \ , \tag{15}$$

where P^D and Q^D are diagonal matrices defined by $P^D = \Phi'^T P\Phi$, $Q^D = \Phi'^T Q\Phi$, $\Phi = [\Phi_1 \,|\Phi_2|\, \dots \,|\Phi_n]$, $\Phi' = [\Phi'_1 \,|\Phi'_2|\, \dots |\, \Phi'_n]$, where n denotes the number of eigenvectors retained after truncation, using the criterion described below.

Truncation Criterion

Normally for structural dynamical analysis, the first few lower frequency modes and the modes whose frequencies are near the frequencies of applied loads are selected for simulation. However, for stability analysis, only those modes which might cause instability are retained. A criterion for such a selection described by Likins[1] is obtained by constructing a square matrix of the order three for each mode shape. This matrix has the dimensions of the moment of inertia. If the matrix contains relatively large terms, it indicates that the corresponding modes may contribute to rotational motion instability, and only such modes are retained. This may eliminate the consideration of some low-frequency modes that otherwise would have been retained.

Vehicle Equations

For the entire vehicle, equations of deplacement and rotation are written using Newton-Euler relations. Retaining the rotational relations and neglecting the effect of the center of mass motion with respect to the body, one obtains

$$T = I \cdot \dot{\omega} + \omega \times I \cdot \omega + \overset{o}{I} \cdot \omega + \frac{d}{dt} \int p \times \dot{p}\, dm, \tag{16}$$

where I is the moment of inertia of the entire vehicle, and ω is the rotation vector. Since the flexible appendages vibrate, I varies with time. Superscript ^{o} indicates differentiation with respect to time in body coordinates. The generic vector p joins a fixed point on the central rigid body to each of the masses on the flexible appendage.

The complete set of equations used to describe the Skylab consists of a set of n equations (15) describing the flexible appendage, a set of three equations (16) describing the entire vehicle, a set of three equations describing the effect of relative rotation of the Command Service Module (CSM) elastically attached to the core body composed of the Orbital Workshop (OWS) and Apollo Telescope Mount (ATM), and a set of three equations describing the skew symmetric mode of the attached booms.

Digital Program

A general-purpose finite element structural analysis program, called NASTRAN and developed for all the NASA Centers, is modified to obtain eigenvalues and eigenvectors of any given spinning structure. The existing program can generate the elastic and geometric stiffness matrices, K_e and K_g, respectively, and either the diagonal mass matrix M or the nondiagonal consistent mass matrix for any structure. Matrices D, G, K'' are generated for any arbitrary structure in the empty modules available in NASTRAN. The complex eigenvalue subroutine available in NASTRAN can solve either of the sets of equations (12) or (13).

The initial model of the system of solar panels and beam fairing mounted on each side of the OWS had 2100 degrees-of-freedom and was reduced to 90 degrees-of-freedom for dynamic analysis. This 90 degrees-of-freedom model was further reduced to 27 degrees-of-freedom in such a way that the first five natural frequencies of both the models closely matched. Each of the four solar panels on the ATM was idealized as 40 degrees-of-freedom models and subsequently reduced to a 12 degrees-of-freedom system. The final stiffness matrix of the entire system of solar panels was regarded as one flexible appendage with 102 degrees-of-freedom.

The detailed Skylab simulation is programed on a Univac 1108 digital computer with a 64,000 word core storage. Whereas equations (12) would have required 31,000 words of storage, the equations of motion expressed as equation (13) require 11,000.

CONCLUSION

It has been shown that it is possible to passively and actively stabilize the motion of a simplified model of a Skylab spinning about its intermediate axis of inertia. This is accomplished by deploying flexible booms, thus changing the inertias, and applying suitable control torques. Analytical results indicate the required boom stiffness properties for given vehicle mass distribution and spin rates to achieve passive stability. Furthermore, the use of a simplified model leads to results which are amenable to physical interpretation.

To gain confidence that these results will apply to the actual Skylab, an additional step is being implemented. A detailed digital simulation model of the spinning Skylab vehicle has been developed. Results obtained from the simulation model compared favorably with those of the simplified models.

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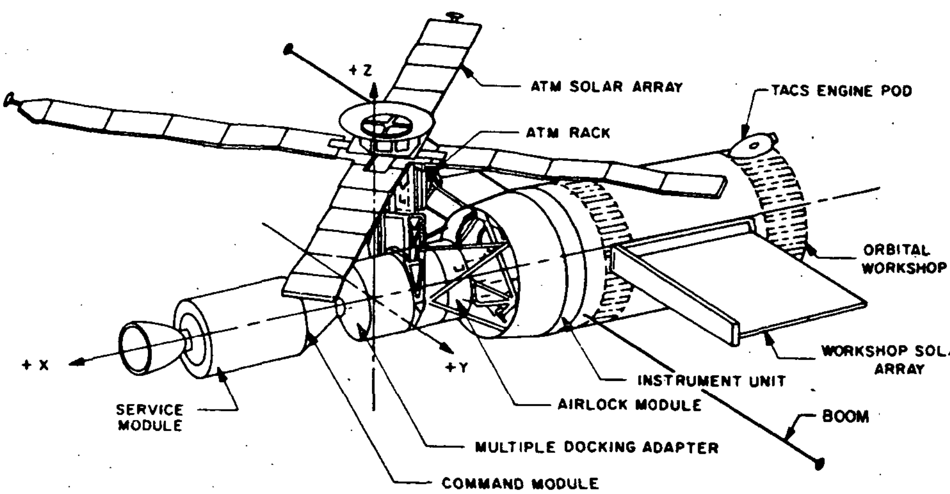

Figure 1. Skylab

Table 1. Physical Characteristics of Skylab

$I_1 = 1.25 \times 10^6$ kg m^2

$I_2 = 6.90 \times 10^6$ kg m^2

$I_3 = 7.10 \times 10^6$ kg m^2

$\Gamma_1 = 0$

$\Gamma_2 = 23.3$ m

$\Gamma_3 = -1.53$ m

$m = 227$ kg

$k_1 = k_3 = 146$ N/m

$k_2 = 7.4 \times 10^4$ N/m

$d_1 = d_3 = 0.04\ (k_3 m)^{1/2}$

$d_3 = 0.04\ (k_2 m)^{1/2}$

$\Omega = 0.6\ s^{-1}$

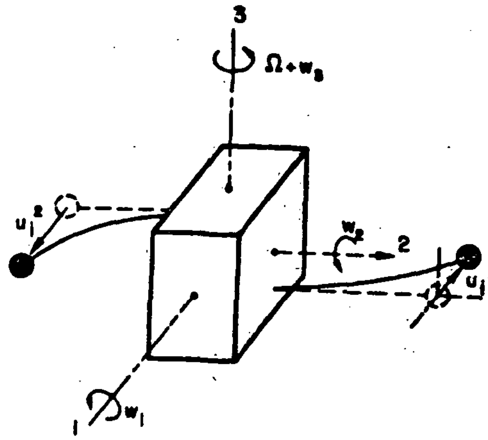

Figure 2. Simplified Model

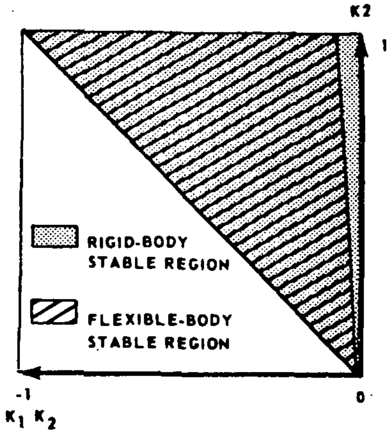

Figure 3. Stability Region for Wobble Motion, Passive Case

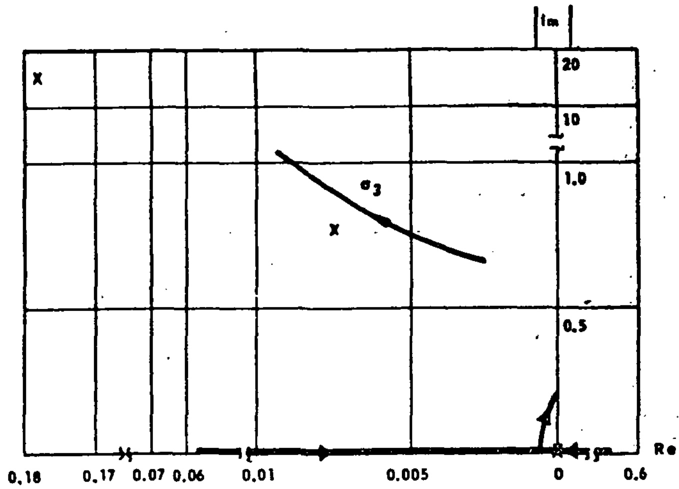

Figure 4. Root Locus for Varying Boom Stiffness ($0.4 \leq \sigma_3 \leq 1.4$)

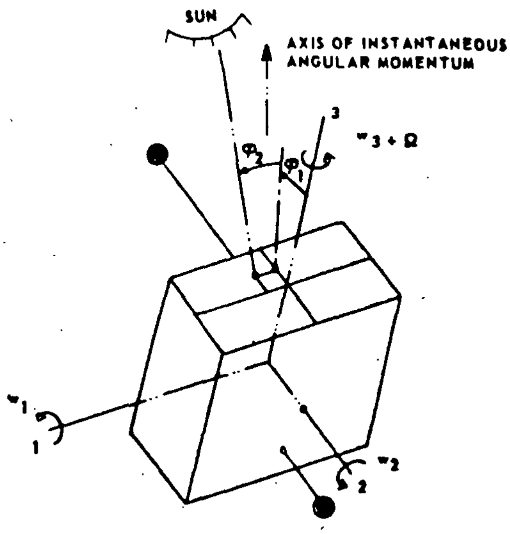

Figure 5. Variables for Attitude Control

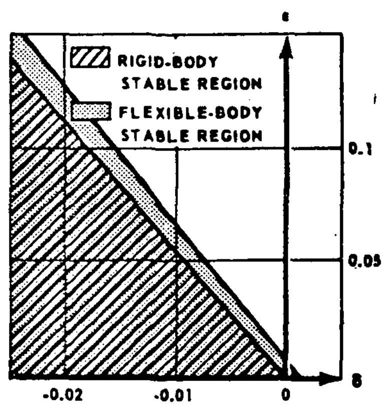

Figure 6. Stability Region for Wobble Motion in the Control Parameter Plane

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STABILIZING A SPINNING SKYLAB

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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