INTEGRAL DOSE DURING CONSTANT VELOCITY MOTION NEAR A SPACE POWER REACTOR

H. S. Manning, NASA/MSFC

The dose rate in a rotationally-symmetric radiation field near a space power reactor is assumed to be inversely proportional to separation distance squared and directly proportional either to a constant or a constant times the sine-cubed of a polar angle. Constant velocity motion (constant speed and direction) is examined in both cases in both two- and three-dimensional geometries. The two-dimensional geometry occurs when the line of motion and the field axis of symmetry are co-planar. The dose integral in the sine-cubed region may be integrated directly, but a more useful form is obtained after a change of variable. A coordinate system rotation greatly simplifies the results. The three-dimensional problem is integrated after a change of variable. Finally, tables of normalized functions are presented and discussed.

INTRODUCTION

A key problem associated with the study of nuclear energy sources in space is the evaluation of the leakage radiation reaching the astronauts or other radio-sensitive detectors. This paper discusses a solution to the problem of constant velocity motion near a space power reactor with a specific leakage radiation pattern. The term "constant velocity" is intended to be understood in the vector sense implying constant speed and constant direction.

The radiation field description was derived from material presented by Atomics International in References 1 and 2 and is illustrated in Figures 1 and 2. Figure 3 depicts an isodose contour in the (x, +y) half-plane. The radiation source is represented as a point source located at the origin O. The radiation field is assumed to be comprised of three regions. Region I represents the field due to an unshielded isotropic source or leaking through a relatively thin shield. Region III represents the field behind a thick "biological shield" of half-angle ϵ. In both regions, the shield is assumed to be of approximately uniform attenuation with azimuth angle so the dose rate is a function of distance alone. Region II represents the transition between regions I and III. Because of variations in attenuation and/or source strength, the dose rate has an azimuthal dependence. The field studied here varies as the sine-cubed of a polar angle ϕ measured from an abscissa inclined an angle δ to the x-axis.

MOTION WITHIN A SINGLE PLANE

Consider first the case where the detector moves with constant velocity along a line p contained within a nonrotating plane which also contains the x-axis. Let us first examine the case where all motion is in region II, or

$$(\epsilon - \delta) \le \phi \le \frac{\pi}{2} \quad .$$

Let the motion be outward along p from an initial position P_1. Then, from Figure 4,

$$r(P_1) = r_1 = \text{constant} \quad ,$$

$$\xi = \text{constant} \quad ,$$

$$\alpha_1 = \text{constant} \quad ,$$

$$\phi = \text{constant} \quad ,$$

and

$$\frac{dp}{dt} = V = \text{constant} \quad .$$

In region II, the dose rate is

$$\dot{D}_{II} = \frac{K_1 \sin^3 \phi}{R_{II}^2} \quad ,$$

but

$$\phi = (\alpha - \xi) \quad ;$$

thus,

$$\dot{D}_{II} = A\,\frac{\sin^3 \alpha}{r^2} + B\,\frac{\sin \alpha}{r^2} + C\,\frac{\sin \alpha'}{r^2} + D\,\frac{\sin^3 \alpha'}{r^2}$$

where

$$A = K_1 \cos 3\delta \quad ,$$

$$B = K_1 \cos \delta \sin^2 \delta \quad ,$$

$$C = K_1 \sin \delta \cos^2 \delta \quad ,$$

$$D = K_1 \sin 3\delta \quad ,$$

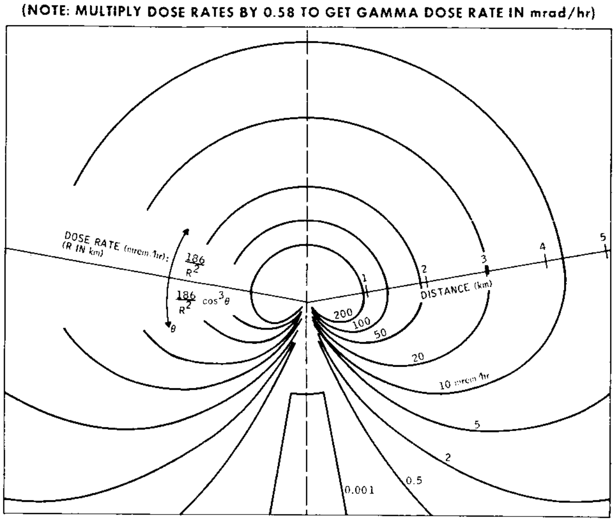
$$\alpha' = \frac{\pi}{2} - \alpha \quad ,$$

and

$$r = R_{II}$$

Consider the first term. Referring back to Figure 4 and introducing initial conditions,

(NOTE: MULTIPLY DOSE RATES BY 0.58 TO GET GAMMA DOSE RATE IN mrad/hr)



8-D18-173-16

FIGURE 1. REACTOR INDUCED RADIATION FIELD ABOUT 25 kwe REACTOR-TE POWER SYSTEM

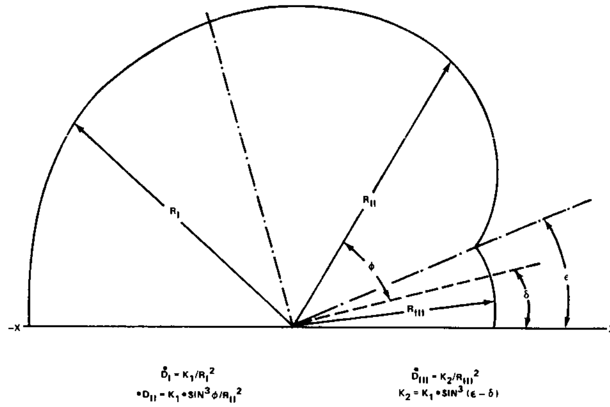


FIGURE 3. GEOMETRY OF A PLANAR ISODOSE CONTOUR

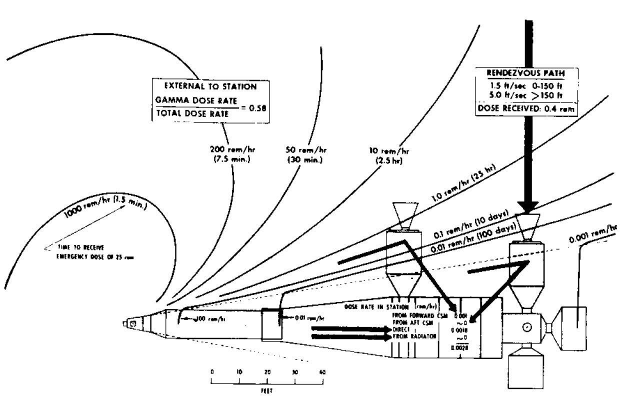


FIGURE 2. SPACE STATION RADIATION ENVIRONMENT FROM NUCLEAR REACTOR

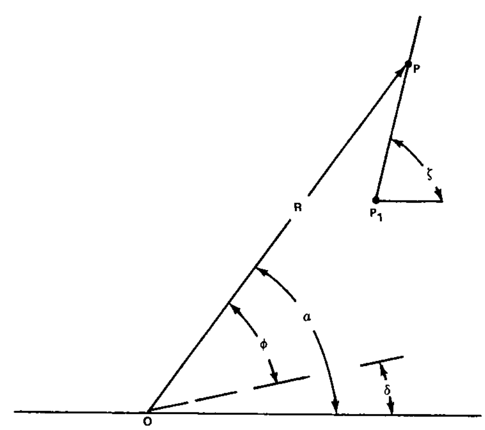


FIGURE 4. CONSTANT VELOCITY MOTION IN A PLANE

$$r \sin \alpha = r_1 \sin \alpha_1 + p \sin \zeta \quad .$$

Also,

$$r^2 = p^2 + r_1^2 + 2pr_1 \cos(\zeta - \alpha_1) \quad .$$

We now write

$$\dot{D}_{IIA} = \frac{A}{V} \frac{(r_1 \sin \alpha_1 + p \sin \zeta)^3}{[r_1^2 + 2pr_1 \cos(\zeta - \alpha_1) + p^2]^{5/2}} \frac{dp}{dt} \quad .$$

The limits for the complete integral along the line are:

$$p(1) = 0$$

and

$$p(2) = \infty \quad .$$

By integrating and inserting the limits and simplifying, the equation above eventually yields

$$D_{IIA} = \frac{A}{3r_1V} \left\{ \frac{(\sin^3 \alpha_1 + \sin^3 \zeta)}{[1 + \cos(\zeta - \alpha_1)]} + \frac{(\sin \alpha_1 + \sin \zeta)^3}{[1 + \cos(\zeta - \alpha_1)^2]} \right\} \quad .$$

CHANGE OF VARIABLE

This general form is somewhat cumbersome to handle. A more useful, if somewhat restricted, form may be obtained through a change of variable before integration. Referring again to Figure 4, we observe that

$$r \cos \alpha = r_1 \cos \alpha_1 + p \cos \zeta \quad .$$

Taking this in conjunction with the expressions above, we find that

$$r = r_1 \frac{\sin(\zeta - \alpha_1)}{\sin(\zeta - \alpha)} \quad ,$$

$$p = r_1 \frac{\sin(\alpha - \alpha_1)}{\sin(\zeta - \alpha)} \quad ,$$

and

$$\frac{dp}{dt} = \frac{\sin(\zeta - \alpha_1)}{\sin^2(\zeta - \alpha)} \frac{d\alpha}{dt} \quad .$$

Substituting into the dose rate expression,

$$\dot{D}_{IIA} = \frac{\sin^3 \alpha}{\sin(\zeta - \alpha_1)} \frac{d\alpha}{dt} \quad .$$

The limits will be in general

$$\alpha(1) = \alpha_1$$

and

$$\alpha(2) = \alpha_2 \quad .$$

By integrating and inserting the limits, we have the complete first term,

$$D_{IIA} = \frac{A}{3r_1V} \left[\frac{3(\cos \alpha_1 - \cos \alpha_2) - (\cos^3 \alpha_1 - \cos^3 \alpha_2)}{\sin(\zeta - \alpha_1)} \right] \quad .$$

The second term of the expanded dose rate equation is

$$\dot{D}_{IIB} = \frac{B \sin \alpha}{r^2}$$

which yields immediately

$$D_{IIB} = \frac{B}{r_1V} \frac{(\cos \alpha_1 - \cos \alpha_2)}{\sin(\zeta - \alpha_1)}$$

using the limits above.

The doses D_{IIC} and D_{IID} may be evaluated using the arguments above but with the complementary angle α'. The complete dose expression can now be written as

$$\begin{aligned} D_{II} = \frac{K_1}{3r_1V \sin(\zeta - \alpha_1)} \{ & \cos 3\delta [3(\cos \alpha_1 - \cos \alpha_2) \\ & - (\cos^3 \alpha_1 - \cos^3 \alpha_2)] \\ & + 3 \cos \delta \sin^2 \delta (\cos \alpha_1 - \cos \alpha_2) \\ & - 3 \sin \delta \cos^2 \delta (\cos \alpha_1' - \cos \alpha_2') \\ & + \sin 3\delta [3(\cos \alpha_1' - \cos \alpha_2') \\ & - (\cos^3 \alpha_1' - \cos^3 \alpha_2')]\} \end{aligned}$$

This form is valid except in the case of radial motion discussed below. An additional minor constraint is that $\alpha_1 \geq \epsilon$.

This expression can be greatly simplified by performing a coordinate system rotation through an angle δ. Thus,

$$\alpha* = \alpha - \delta = \phi \quad ,$$

$$\zeta* = \zeta - \delta \quad ,$$

$$\delta* = 0 \quad ,$$

and, constraining the point P_1 , to lie along the $\alpha = \delta$ line,

$$D_{II}(0, \phi) = \frac{K_1}{3r_1V} \left[\frac{\cos^3 \phi - 3 \cos \phi + 2}{\sin(\zeta - \delta)} \right]$$

where the parentheses on the left are used to indicate the angular limits. The radial case may be integrated directly from the dose rate equation. It is unaffected by the coordinate system rotation;

$$D_{II}(\phi, \phi) = \frac{K}{V} \left(\frac{1}{r_1} - \frac{1}{r} \right) \sin^3 \phi \quad .$$

The case of constant velocity co-planar motion in an inverse square field has been solved before and is included here only for completeness and notational consistency. Referring to Figure 5, we again let the motion be along a line p , and, as before,

$$r(P_1) = r_1 = \text{constant} \quad ,$$

$$\zeta = \text{constant} \quad ,$$

$$\alpha_1 = \text{constant} \quad ,$$

$$\phi_1 = \text{constant} \quad ,$$

and

$$\frac{dp}{dt} = \text{constant} = V \quad .$$

The dose rate is given by

$$\dot{D} = \frac{K}{r^2} \quad .$$

We observe that

$$\eta = \frac{\pi}{2} - (\zeta - \alpha) \quad ,$$

$$r = r_1 \cos \eta_1 \sec \eta \quad ,$$

$$p = r_1 \cos \eta_1 (\tan \eta - \tan \eta_1) \quad ,$$

and

$$\frac{dp}{dt} = r_1 \cos \eta_1 \sec^2 \eta \frac{d\eta}{dt} \ ;$$

thus,

$$D = \frac{K}{r_1 V} \left[\frac{\alpha_2 - \alpha_1}{\sin (\zeta - \alpha_1)} \right] \quad .$$

MOTION IN THREE DIMENSIONS

If isodose contours such as in Figure 3 are rotated through 2π radians around the x-axis, a complex three-dimensional radiation field will be produced. Assume constant velocity motion along an arbitrarily oriented straight line, as in Figure 6. Again, let us first examine the case of motion within region II along the line $P_1P = p$ outward from an initial position P_1. The radiation field polar angle is again

$$\phi = \alpha - \delta \quad ,$$

and, as before,

$$\dot{D}_{II} = A \frac{\sin^3 \alpha}{r^2} + B \frac{\sin \alpha}{r^2} + C \frac{\sin \alpha'}{r^2} + D \frac{\sin^3 \alpha'}{r^2}$$

The direct integral of the cubic term has not been obtained; however, the term has been successfully integrated after performing a variable change similar to that in the restricted solution of the co-planar case.

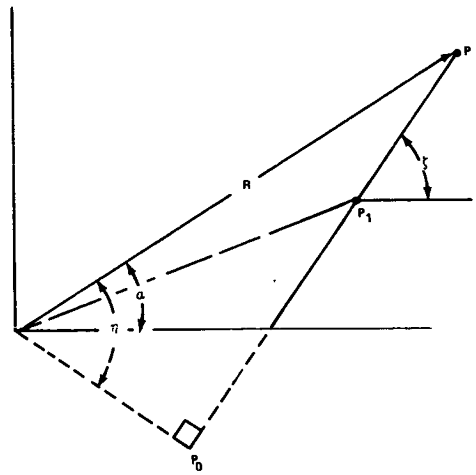


FIGURE 5. CONSTANT VELOCITY COPLANAR MOTION IN AN INVERSE SQUARE FIELD

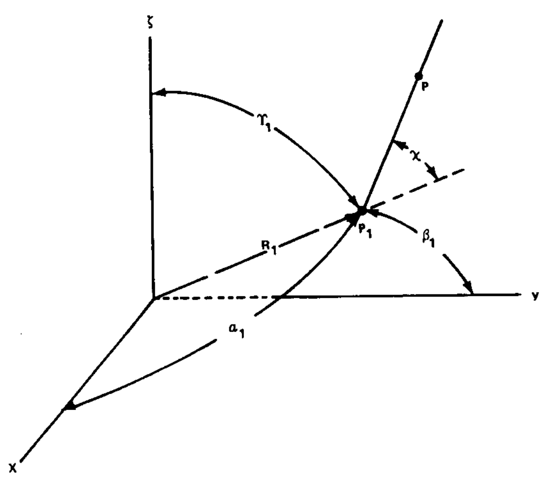


FIGURE 6. CONSTANT VELOCITY MOTION IN THREE DIMENSIONS

The radius vector to the initial point can be written

$$\bar{r}_1 = (\hat{i}\cos\alpha_1 + \hat{j}\cos\beta_1 + \hat{k}\cos\gamma_1) r_1 \quad ,$$

where $r_1 = |\bar{r}_1|$ and $\cos\alpha_1$, $\cos\beta_1$, and $\cos\gamma_1$ are direction cosines. Similarly, the vector traced by the moving point is

$$\bar{p} = (\hat{i}\cos\alpha_p + \hat{j}\cos\beta_p + \hat{k}\cos\gamma_p)\dot{p} \quad .$$

Since the direction is constant, $\bar{p}/p$ is constant. The angle between these two vectors is also a constant,

$$\cos\chi = \cos\alpha_1\cos\alpha_p + \cos\beta_1\cos\beta_p + \cos\gamma_1\cos\gamma_p \quad .$$

Figure 7 illustrates the nonrotating plane containing the vectors $\bar{r}$, $\bar{r}_1$, and $\bar{p}$. This plane contains the variable angle θ between the vector $\bar{r}$ and the (x, y) plane. Consider the nonrotating plane containing the x-axis and $\bar{r}_1$ and the rotating plane containing the x-axis and $\bar{r}$. In the former, there is the constant angle α_1 and in the latter the variable angle α. If a unit sphere is constructed at the origin, these three angles become the sides of a spherical triangle. If the face angle opposite side α is called λ, then

$$\cos\alpha = \cos\alpha_1\cos(\theta - \theta_1) + \sin\alpha_1\sin(\theta - \theta_1)\cos\lambda \quad .$$

But, λ is also the angle between the planes containing POP_1 and P_1OX_1. Thus,

$$\cos(\pi - \lambda) = \left(\frac{\bar{r}_1 \times \bar{p}}{r_1 p}\right) \cdot \left(\frac{\hat{i} \times \bar{r}_1}{r_1}\right) \frac{1}{\sin\chi\sin\alpha_1}$$

and

$$\cos\lambda = \frac{\cos\alpha_p\sin^2\alpha_1 - \cos\alpha_1(\cos\beta_p\cos\beta_1 + \cos\gamma_p\cos\gamma_1)}{\sin\chi\sin\alpha_1} \quad .$$

Continuing, we simplify

$$\cos\alpha = k\cos[\theta - (\theta_1 - \psi)] \quad ,$$

where the phase angle ψ is defined by

$$\tan\psi = \tan\alpha_1\cos\lambda$$

and the modulus k is found from

$$k^2 = \cos^2\alpha_1 + \sin^2\alpha_1\cos^2\lambda \quad .$$

We note in passing that θ_1, ψ, and k are defined by initial conditions only and that $k^2 \le 1$.

Returning to Figure 7, we may write

$$r = r_1 \frac{\sin(\zeta - \theta_1)}{\sin(\zeta - \theta)} \quad ,$$

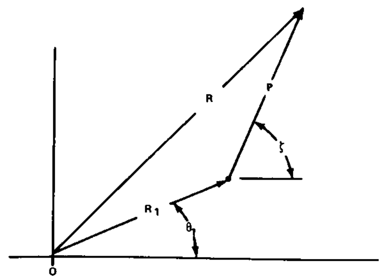


FIGURE 7. PLANE OF MOTION IN THREE-DIMENSIONAL CASE

$$p = r_1 \frac{\sin(\theta - \theta_1)}{\sin(\zeta - \theta)} \quad ,$$

and

$$\frac{dp}{dt} = r_1 \frac{\sin(\zeta - \theta_1)}{\sin^2(\zeta - \theta)}\frac{d\theta}{dt} \quad .$$

The dose rate is now

$$\dot{D}_{IIA} = \frac{A}{r_1 V} \frac{\{1 - k^2\cos^2[\theta - (\theta_1 + \psi)]\}^{3/2}}{\sin(\zeta - \theta)} \frac{d\theta}{dt} \quad .$$

Let

$$\sin^2\mu = \cos^2[\theta - (\theta_1 + \psi)]$$

and

$$\Delta = (1 - k^2\sin^2\mu)^{1/2} \quad , \qquad k^2 \le 1$$

and the expression integrates to

$$D_{IIA} = \frac{A}{3 r_1 V \sin(\zeta - \theta_1)} [k^2\Delta\sin\mu\cos\mu - (1 - k^2)F(k,\mu) + 2(2 - k^2)E(k,\mu)]_{\mu(1)}^{\mu(2)} \quad ,$$

where $F(k,\mu)$ is the incomplete elliptic integral of the first kind and $E(k,\mu)$ is the incomplete elliptic integral of the second kind. The limits of integration are indicated outside the bracket on the right.

Using the substitutions above, the second term in the expanded dose rate equation becomes

$$\dot{D}_{IIB} = \frac{B}{r_1 V \sin(\zeta - \theta_1)} \Delta \frac{d\mu}{dt} \quad ;$$

thus,

$$D_{IIB} = \frac{B}{r_1 V \sin(\zeta - \theta_1)} [E(k, \mu)]_{\mu(1)}^{\mu(2)} \quad .$$

The third term may be written as

$$\dot{D}_{IIC} = \frac{Ck}{r_1 V \sin(\zeta - \theta_1)} \sin\mu \frac{d\mu}{dt} \quad ;$$

and

$$D_{IIC} = \frac{Ck}{r_1 V \sin(\zeta - \theta_1)} (\cos\mu)_{\mu(2)}^{\mu(1)} \quad .$$

The last term is

$$\dot{D}_{IID} = \frac{Dk^3}{r_1 V \sin(\zeta - \theta_1)} \sin^3\mu \frac{d\mu}{dt}$$

which yields

$$D_{IID} = \frac{Dk^3}{3 r_1 V \sin(\zeta - \theta_1)} (3\cos\mu - \cos^3\mu)_{\mu(2)}^{\mu(1)} \quad .$$

The complete expression for the three-dimensional case could now be constructed using the components above. Regrettably, no simplifying step such as the coordinate system rotation used in the planar case has been found.

The restriction against radial motion applies here as in the planar case. However, as radial motion is necessarily co-planar with the x-axis, the special form given earlier will apply.

APPLICATIONS

Many problems involved with close-in operations near a nuclear power source can be adequately approximated by assuming constant velocity co-planar motion. The integral dose in this case can most easily be found from the expression for $D_{II}(0, \phi)$ developed above. The expression can be normalized as

$$D_{II}(0, \phi) \frac{r_1 V \sin(\zeta - \delta)}{K_1} = \frac{(\cos^3\phi - 3\cos\phi + 2)}{3} \quad .$$

This expression is tabulated as a function of the angle ϕ in Table 1. This table is organized in standard math table format and allows five significant figure values of the function to be read corresponding to three significant figure values for the angle ϕ. Linear interpolation is used for intermediate values.

TABLE 1. NORMALIZED DOSE INTEGRALS FOR A TWO-DIMENSIONAL CASE

	0	1	2	3	4	5	6	7	8	9
-.0	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000
1.0	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000
2.0	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000
3.0	00000	00000	00000	00000	00000	00000	00000	00000	00000	00001
4.0	00001	00001	00001	00001	00001	00001	00001	00001	00001	00001
5.0	00001	00002	00002	00002	00002	00002	00002	00002	00003	00003
6.0	00003	00003	00003	00004	00004	00004	00004	00005	00005	00005
7.0	00006	00006	00006	00007	00007	00007	00008	00008	00009	00009
8.0	00009	00010	00010	00011	00011	00012	00013	00013	00014	00014
9.0	00015	00016	00016	00017	00018	00019	00020	00020	00021	00022
10.0	00023	00024	00025	00026	00027	00028	00029	00030	00031	00032
11.0	00034	00035	00036	00037	00039	00040	00041	00043	00044	00046
12.0	00047	00049	00051	00052	00054	00056	00058	00059	00061	00063
13.0	00065	00067	00069	00071	00073	00076	00078	00080	00083	00085
14.0	00087	00090	00092	00095	00098	00100	00103	00106	00109	00112
15.0	00115	00118	00121	00124	00127	00131	00134	00137	00141	00145
16.0	00148	00152	00156	00159	00163	00167	00171	00175	00180	00184
17.0	00188	00193	00197	00202	00206	00211	00216	00221	00226	00231
18.0	00236	00241	00246	00251	00257	00262	00268	00274	00280	00285
19.0	00291	00298	00304	00310	00316	00323	00329	00336	00343	00349
20.0	00356	00363	00371	00378	00385	00393	00400	00408	00416	00423
21.0	00431	00439	00448	00456	00464	00473	00482	00490	00499	00508
22.0	00517	00527	00536	00545	00555	00565	00575	00585	00595	00605
23.0	00615	00626	00636	00647	00658	00669	00680	00691	00703	00714
24.0	00726	00738	00750	00762	00774	00786	00799	00812	00824	00837
25.0	00850	00864	00877	00891	00904	00918	00932	00946	00961	00975
26.0	00990	01004	01019	01035	01050	01065	01081	01097	01112	01129
27.0	01145	01161	01178	01195	01211	01229	01246	01263	01281	01299
28.0	01317	01335	01353	01372	01390	01409	01428	01447	01467	01487
29.0	01506	01526	01546	01567	01587	01608	01629	01650	01672	01693
30.0	01715	01737	01759	01781	01804	01826	01849	01872	01896	01919
31.0	01943	01967	01991	02015	02040	02065	02090	02115	02140	02166
32.0	02192	02218	02244	02271	02298	02325	02352	02379	02407	02435
33.0	02463	02491	02520	02548	02577	02607	02636	02666	02696	02726
34.0	02756	02787	02818	02849	02880	02912	02944	02976	03008	03041
35.0	03073	03106	03140	03173	03207	03241	03276	03310	03345	03380
36.0	03415	03451	03487	03523	03559	03596	03633	03670	03707	03745
37.0	03783	03821	03859	03898	03937	03976	04016	04055	04095	04136
38.0	04176	04217	04258	04300	04341	04383	04426	04468	04511	04554
39.0	04597	04641	04685	04729	04774	04818	04864	04909	04955	05000
40.0	05047	05093	05140	05187	05234	05282	05330	05378	05427	05476
41.0	05525	05574	05624	05674	05724	05775	05826	05877	05929	05980
42.0	06033	06085	06138	06191	06244	06298	06352	06406	06461	06516
43.0	06571	06626	06682	06738	06795	06852	06909	06966	07024	07082
44.0	07140	07199	07258	07317	07377	07437	07497	07557	07618	07680
	0	1	2	3	4	5	6	7	8	9

TABLE 1. (Concluded)

	0	1	2	3	4	5	6	7	8	9
45.0	07741	07803	07865	07928	07991	08054	08117	08181	08245	08310
46.0	08374	08440	08505	08571	08637	08703	08770	08837	08905	08972
47.0	09041	09109	09178	09247	09316	09386	09456	09527	09597	09669
48.0	09740	09812	09884	09956	10029	10102	10176	10250	10324	10398
49.0	10473	10549	10624	10700	10776	10853	10930	11007	11084	11162
50.0	11241	11319	11398	11478	11557	11637	11718	11798	11879	11961
51.0	12043	12125	12207	12290	12373	12457	12540	12625	12709	12794
52.0	12879	12965	13051	13137	13224	13311	13398	13486	13574	13662
53.0	13751	13840	13929	14019	14109	14200	14290	14382	14473	14565
54.0	14657	14750	14843	14936	15030	15124	15218	15313	15408	15503
55.0	15599	15695	15792	15888	15986	16083	16181	16279	16378	16477
56.0	16576	16676	16776	16876	16977	17078	17179	17281	17383	17485
57.0	17588	17691	17795	17898	18003	18107	18212	18317	18423	18529
58.0	18635	18742	18849	18956	19064	19172	19280	19389	19498	19607
59.0	19717	19827	19937	20048	20159	20271	20383	20495	20607	20720
60.0	20833	20947	21061	21175	21290	21404	21520	21635	21751	21867
61.0	21984	22101	22218	22336	22454	22572	22691	22810	22929	23049
62.0	23169	23289	23410	23531	23652	23773	23895	24018	24140	24263
63.0	24387	24510	24634	24758	24883	25008	25133	25259	25385	25511
64.0	25638	25764	25892	26019	26147	26275	26404	26533	26662	26791
65.0	26921	27051	27181	27312	27443	27575	27706	27838	27970	28103
66.0	28236	28369	28503	28637	28771	28905	29040	29175	29310	29446
67.0	29582	29718	29855	29992	30129	30266	30404	30542	30681	30819
68.0	30958	31098	31237	31377	31517	31658	31798	31939	32081	32222
69.0	32364	32506	32649	32791	32934	33078	33221	33365	33509	33654
70.0	33798	33943	34088	34234	34380	34526	34672	34819	34966	35113
71.0	35260	35408	35556	35704	35852	36001	36150	36299	36449	36599
72.0	36749	36899	37049	37200	37351	37502	37654	37806	37958	38110
73.0	38263	38415	38568	38722	38875	39029	39183	39337	39491	39646
74.0	39801	39956	40112	40267	40423	40579	40735	40892	41049	41205
75.0	41363	41520	41678	41836	41994	42152	42310	42469	42628	42787
76.0	42946	43106	43266	43426	43586	43746	43907	44068	44228	44390
77.0	44551	44713	44874	45036	45198	45361	45523	45686	45849	46012
78.0	46175	46339	46502	46666	46830	46994	47158	47323	47487	47652
79.0	47817	47983	48148	48313	48479	48645	48811	48977	49143	49310
80.0	49476	49643	49810	49977	50144	50312	50479	50647	50815	50983
81.0	51151	51319	51487	51656	51825	51993	52162	52331	52500	52670
82.0	52839	53009	53178	53348	53518	53688	53858	54029	54199	54369
83.0	54540	54711	54882	55053	55224	55395	55566	55737	55909	56080
84.0	56252	56424	56595	56767	56939	57111	57284	57456	57628	57801
85.0	57973	58146	58318	58491	58664	58837	59010	59183	59356	59529
86.0	59702	59876	60049	60222	60396	60569	60743	60917	61090	61264
87.0	61438	61612	61786	61960	62133	62307	62482	62656	62830	63004
88.0	63178	63352	63527	63701	63875	64050	64224	64398	64573	64747
89.0	64922	65096	65271	65445	65620	65794	65969	66143	66318	66492
	0	1	2	3	4	5	6	7	8	9

Values from this table give the integral dose along a line originating on the $\phi = 0$ or $\alpha_1 = \delta$ line and proceeding outward until $\alpha - \delta = \phi$. The inclination of the line is accounted for in the normalizing constants. Integral dose accrued between the points at ϕ_1 and ϕ_2 on the same line as $[D_{II}(0, \phi_2) - D_{II}(0 - \phi_1)]$.

This allows one to correct for the small difference between $\alpha = \epsilon$, the closest the axis may be approached in region II, and $\alpha = \delta$, the abscissa of the extended field contours.

Another useful form results from constraining P_1 to lie along the $\phi = \frac{\pi}{2}$ line; thus,

$$D_{II}\left(\frac{\pi}{2}, \phi\right) = \frac{K_1}{3r_1V}\left[\frac{3 \cos \phi - \cos^3 \phi}{\cos (\zeta - \delta)}\right] .$$

This dose is related to the tabulated dose by

$$D_{II}\left(\frac{\pi}{2}, \phi\right) = \left[D_{II}\left(0, \frac{\pi}{2}\right) - D_{II}(0, \phi)\right] \tan (\zeta - \delta) \quad .$$

The three-dimensional problem in region II contains too many variables to allow simple parametric representation. To obtain sample results, a specific field pattern was chosen and values of the integral above were obtained. The field axis offset angle, $\delta = 8.663$ degrees, corresponds to the field illustrated in Figures 1 and 2. Generalized values of the resulting integral are shown in Table 2 as functions of the modulus k and the field angle μ defined above. The table is read in a manner similar to Table 1; thus,

$$D_{II}(k, \mu_1, \mu_2) = D_{II}(k, \mu_2) - D_{II}(k, \mu_1) \quad ,$$

where it is understood that the normalizing factor $[r_1V \sin (\zeta - \theta)/k]$ has been divided out of both sides of the expression.

TABLE 2. NORMALIZED DOSE INTEGRALS FOR A THREE-DIMENSIONAL CASE, $\delta = 8.667$ DEGREES

Range Angle μ, Degrees	Modulus Angle $\sin^{-1}$ k, Degrees									
	00	10	20	30	40	50	60	70	80	90
00	0.0	0.0271	0.0620	0.1101	0.1722	0.2441	0.3172	0.3807	0.4240	0.4393
10	0.1605	0.1874	0.2218	0.2692	0.3308	0.4017	0.4743	0.5373	0.5802	0.5954
20	0.3216	0.3464	0.3784	0.4224	0.4800	0.5473	0.6162	0.6764	0.7173	0.7320
30	0.4824	0.5030	0.5302	0.5665	0.6149	0.6727	0.7328	0.7859	0.8223	0.8353
40	0.6432	0.6600	0.6764	0.6990	0.7312	0.7720	0.8165	0.8569	0.8852	0.8954
50	0.8040	0.8139	0.8161	0.8185	0.8266	0.8425	0.8639	0.8859	0.9024	0.9085
60	0.9649	0.9610	0.9499	0.9255	0.9018	0.8847	0.8763	0.8748	0.8766	0.8777
70	1.1256	1.1142	1.0782	1.0215	0.9594	0.9029	0.8592	0.8308	0.8159	0.8115
80	1.2864	1.2663	1.2027	1.1130	1.0040	0.9036	0.8214	0.7640	0.7320	0.7220
90	1.4539	1.4152	1.3250	1.1930	1.0418	0.8962	0.7728	0.6859	0.6369	0.6217

REFERENCES

1. "Joint AEC/NASA/MSFC Study of Reactor-Thermoelectric Powered Space Station," FY-1969 Summary, Atomics International.

2. Glyfe, J. D. and R. A. Johnson, "Reactor-Thermoelectric System for NASA Space Station," AI-AEC-12839, Atomics International, July 1, 1969.

BIBLIOGRAPHY

Petit Bois, G., Tables of Indefinite Integrals, Dover Publications, New York, N. Y., 1961.

Hancock, H., Elliptic Integrals, Dover Publications, New York, N.Y., 1958.