OPTIMAL SHIELD MASS DISTRIBUTION
FOR SPACE RADIATION PROTECTION*

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ABSTRACT

Computational methods have been developed and successfully used for determining the optimum distribution of space radiation shielding on geometrically complex space vehicles. These methods have been incorporated in computer program SWORD which uses the full capability of state-of-the-art methods for dose evaluation in complex geometry, and iteratively calculates the optimum distribution of (minimum) shield mass satisfying multiple acute and protracted dose constraints associated with each of several body organs. The unique and effective technique used to accommodate multiple constraints, eliminates the awkward discontinuities associated with the formulation of inequality constraints, and produces a result meeting mathematical tests for optimality.

INTRODUCTION

The capability to compute space radiation doses at specified points within a vehicle and shield of fixed geometry is provided by a number of computer programs, e.g., SIGMA (Ref. 1), MEVDP (Ref. 2), and LSVDC4 (Ref. 3). An additional capability is provided by techniques developed at the McDonnell Douglas Astronautics Company and incorporated in the SWORD program (Ref. 4), this being the automated computation of the optimal shield mass distribution that meets a set of radiobiological dose criteria associated with the specified vehicle configuration and mission profile. It is emphasized that this optimization function is accomplished without necessitating any simplifying assumptions regarding geometrical framework, radiation transmission evaluation, etc., that are not also commonly invoked in programs which perform dose evaluation only. In fact, SWORD incorporates the dose analysis framework of the SIGMA program referred to above. To a great extent, the capacity of SWORD to accommodate a variety of complicating factors influencing shield mass distribution is due to the efficacy, yet simplicity, of the optimization technique employed.

The role of the SWORD program in performing space radiation shielding analyses is indicated in Figure 1, together with those of other space radiation analysis programs also developed at the McDonnell Douglas Astronautics Company. Overall capabilities are summarized in Figure 2. SWORD uses basic dose transmission data, in the form of one-dimensional point kernel functions, typically calculated by the CHARGE program (Ref. 5) for the total space radiation environment defined by the OGRE program (Ref. 6) for the specified mission. These dose transmission data are applied in conjunction with ray tracing computations performed on a generalized quadric surface representation of the vehicle, to compute dose levels to specified items, usually the critical organs of crew members. The derivative of total dose from all radiation sources, with respect to the thicknesses of specified candidate shield regions located on various surfaces of the vehicle, is also computed. This dose derivative information is processed by SWORD in an iterative procedure to determine the optimal distribution of shield material among such candidate locations as wall structure, biowell for solar cosmic ray protection, and personal shields. The optimization technique employed is based on a particular formulation of the Lagrange multiplier constraint equations. Shield shaping over extended surface areas is accomplished by subdividing them into a number of smaller areas over which shield thickness is uniform. SWORD can treat the effect of (1) multiple dose constraints (separate constraints for each organ), (2) time-dependent astronaut locations (the work-rest cycle influence), (3) organ-dependent quality factors or dose distribution factors, and (4) direct and scattered neutron and gamma radiations from on-board nuclear power sources. The geometric framework, numerical integration schemes, and optimization procedures are sufficiently flexible and efficient to allow the analysis of any practical space vehicle configuration.
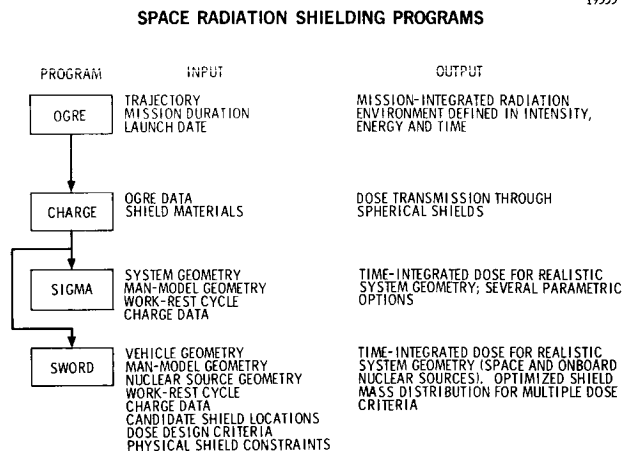
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SPACE RADIATION SHIELDING PROGRAMS

PROGRAM	INPUT	OUTPUT
OGRE	TRAJECTORY MISSION DURATION LAUNCH DATE	MISSION-INTEGRATED RADIATION ENVIRONMENT DEFINED IN INTENSITY, ENERGY AND TIME
CHARGE	OGRE DATA SHIELD MATERIALS	DOSE TRANSMISSION THROUGH SPHERICAL SHIELDS
SIGMA	SYSTEM GEOMETRY MAN-MODEL GEOMETRY WORK-REST CYCLE CHARGE DATA	TIME-INTEGRATED DOSE FOR REALISTIC SYSTEM GEOMETRY; SEVERAL PARAMETRIC OPTIONS
SWORD	VEHICLE GEOMETRY MAN-MODEL GEOMETRY NUCLEAR SOURCE GEOMETRY WORK-REST CYCLE CHARGE DATA CANDIDATE SHIELD LOCATIONS DOSE DESIGN CRITERIA PHYSICAL SHIELD CONSTRAINTS	TIME-INTEGRATED DOSE FOR REALISTIC SYSTEM GEOMETRY (SPACE AND ONBOARD NUCLEAR SOURCES). OPTIMIZED SHIELD MASS DISTRIBUTION FOR MULTIPLE DOSE CRITERIA

FIGURE 1

SUMMARY OF SWORD CAPABILITIES

- DESCRIBES SPACE VEHICLE IN A GENERALIZED QUADRATIC SURFACE GEOMETRY

- USES TABULAR DOSE-SHIELD THICKNESS DATA FROM DETAILED SHIELDING PHYSICS CALCULATIONS

- EVALUATES MISSION-INTEGRATED CRITICAL ORGAN DOSE USING FLEXIBLE NUMERICAL ANGULAR INTEGRATION SCHEME

- CALCULATES OPTIMAL DISTRIBUTION OF SHIELD MATERIAL SUBJECT TO MULTIPLE DOSE CRITERIA

- USES EFFICIENT, ACCURATE OPTIMIZATION TECHNIQUE

- DISTRIBUTES SHIELD MATERIAL AMONG NESTED SURFACES OF VEHICLE, BIOWELL, AND PERSONAL AND PORTABLE SHIELDS

- SHAPES SHIELD THICKNESS OVER EXTENDED SHIELD SURFACE AREAS

FIGURE 2

CRITICAL ORGAN DOSE EVALUATION

- GEOMETRY MODEL—GENERALIZED QUADRIC SURFACES

$$G(X, Y, Z) = A_0 + A_1 X + A_2 Y + A_3 Z + A_4 X^2 + A_5 Y^2 + A_6 Z^2 + A_7 XY + A_8 YZ + A_9 ZX = 0$$

- SPACE RADIATION DOSE—EVALUATED FOR EACH SOURCE AND SUMMED OVER VARIOUS TIMEWEIGHTED ASTRONAUT LOCATIONS

  - $D = \frac{1}{4\pi} \left(\frac{\Delta\theta}{3}\right)\left(\frac{\Delta\mu}{3}\right) \sum_{I\theta} A_i \sum_{M\mu} B_m \; K\left[\rho\left(\theta_i \; \mu_m\right)\right]$

  - $K(\rho)$ IS TABULAR, INTERPOLATED BY POWER LAW BETWEEN ENTRIES

  - $\frac{\partial D}{\partial T_i} = \frac{1}{4\pi} \left(\frac{\Delta\theta}{3}\right) \left(\frac{\Delta\mu}{3}\right) \sum_{I\theta} A_i \sum_{M\mu} B_m \; \frac{\partial K(\rho)}{\partial T_i}$

  - DATA ARE SAVED FOR EACH RAY DEFINING ITS SOLID-ANGLE WEIGHT, MASS TRAVERSED, SHIELDS CROSSED, AND COSINE OF ANGLE AT EACH CROSSING

- NUCLEAR RADIATION DOSE—EVALUATED CONCURRENTLY WITH SPACE RADIATION DOSE EVALUATION USING POINT KERNEL METHOD, CALCULATING SCATTERED DOSE FOR EACH RAY TRACED IN 4π INTEGRATION AS WELL AS DIRECT DOSE

FIGURE 3

DOSE AND SHIELD MASS EVALUATION

The significance of an optimum solution to a given space radiation protection problem is dependent on a number of factors. These include: (1) the accuracy with which dose and shield mass data, and their derivatives with respect to the variables of the problems, can be determined, (2) the suitability of the variables for characterizing the shield mass distribution, and (3) the extent to which all relevant constraints are suitably imposed and satisfied.

Dose Evaluation

The procedures for dose evaluation that have been incorporated in SWORD are outlined in Figure 3. They have been adapted from the SIGMA code and are representative of the state-of-the-art of space-radiation-dose analysis. The geometric description of the vehicle structure, equipment, stores, fixed shields, astronauts, etc., is accomplished using generalized quadric surfaces which form the boundaries of homogeneous, contiguous, non-overlapping regions. Doses are evaluated at any number of points representing the locations of radiation-sensitive body organs by repetitively tracing rays from each dose point through the surrounding materials, employing the resultant data on mass distributions along each ray to determine a differential dose contribution for the path. Total, mission-integrated critical organ doses are obtained by integrating the differential dose contributions over solid angle ($d\theta \; d\mu$) about each dose point and over various time-weighted astronaut locations within the vehicle. The integration is accomplished numerically using Simpson's Rule; A_i and B_m in Figure 3 are the weighting factors associated with the particular values of the variables (θ, μ).

The specific dose transmission kernels used in an analysis are conveniently supplied in tabular form for an arbitrary number of distinct, mission-dependent sources (e.g., trapped protons, trapped electrons, bremsstrahlung, solar cosmic rays); the data are interpolated by power law between entries to obtain dose levels for various values of equivalent shield thickness. For virtually all analyses it is satisfactory to account generally for the material-dependence of the dose attenuation kernels by applying mass density scaling factors based on known radiation-interaction properties. This approach is satisfactory for secondary nucleon dose as well as for primary proton dose (Ref. 6). Since the origin of the dose attenuation kernels is irrelevant to the operation of SWORD, it is not restricted to use with a specific one-dimensional, dose-analysis computer program.

For each ray traced in the initial evaluation of mission-integrated critical organ dose, data are stored to facilitate efficient re-evaluation of the dose associated with the ray as shield thicknesses are iteratively modified. These data include the equivalent thickness of all fixed regions traversed, the solid angle weight of the ray (incorporating Simpson's rule coefficients for the particular values of polar and azimuthal angles defining the ray), the indices of all shields crossed, and the cosine of the angle at the crossing. Information is also saved regarding the neutron and gamma differential doses associated with the ray; such data represent the dose either transmitted directly from an on-board nuclear power source or single-scattered in regions lying along the ray. The scattered dose calculation is limited to scattering in regions external to the outermost shielded volume, based on the assumption that, for scattering in materials within inhabited regions to be significant, the unscattered dose would be prohibitive.

Shield System Variables

A framework defining candidate shield locations is superimposed on the basic vehicle geometry by designating that shield material be placed, as required, at a specific bounding surface of a specific region. Shield material is assumed to be uniformly distributed over such shield areas, such that a single value of the shield thickness measured along a surface normal, characterizes the mass of each such candidate shield. The relationship of shield mass and the derivative of shield mass with respect to the thicknesses are then quite simple:

$$W = \sum_i A_i t_i \; ; \quad \frac{\partial W}{\partial t_i} = A_i$$

In SWORD the areas of each shield, A_i, may be specified by input or may be estimated from data obtained during ray tracing computations associated with dose evaluation. The latter is convenient for complex shield shapes, but the former is usually more accurate.

This technique for defining shield variables is summarized in Figure 4 and illustrated in Figure 5. In the illustration, a total of seven variables have been used to define a shield system consisting of an external shell completely enclosing inhabited areas (t_1, t_2, t_3) and an internal biowell for solar flare protection (t_4, t_5, t_6, t_7). Several more shields could have been specified if, for example, it had appeared potentially rewarding to use non-uniform shield thicknesses over large areas such as those spanned by shields 1, 2 and 3. In the actual problem from which this illustration was taken, however, the location of fixed equipment along the vehicle walls was not well defined, and it was necessary to assume it to be uniformly distributed; in such a circumstance, total shield mass is best estimated by also assuming a uniform distribution of shield mass over the same areas.

VEHICLE CALCULATIONAL MODEL

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A-A

SURFACES DESIGNATED BY FOLLOWING SYMBOLS
○ CYLINDER   △ CONE   ⊛ SPHERE   ▢ PLANE

SHIELDS DESIGNATED BY T_i

FIGURE 5

In the specification of shield variables for a given problem, two capabilities not illustrated by Figure 5 are available: (1) shields can be triply-nested (doubly-nested being illustrated), and (2) shields can be portable. Both capabilities might be employed if, for example, an astronaut wore a personal portable shield for portions of the mission, including some time in the biowell. Then the effectiveness of such a shield would be incorporated in dose and dose derivative calculations at all work and rest stations by describing the shield geometry at each man-model location, but counting its weight only once.

Constraints on Mass Distribution

Two types of constraints on shield mass distribution can be explicitly included: those dealing with maximum levels of radiation exposure and those dealing with minimum and maximum values of the variables (shield thicknesses). Several other constraints are potentially significant, but are of such a nature that they can be expressed within the overall framework; i.e., restriction to non-zero shield thicknesses, restriction of crew movement during solar flares, anisotropic radiation leakage from on-board nuclear power sources.

The constraints which can be imposed are listed in Figure 4. Those constraints pertaining to radiation exposure are expressed by allowing both acute and protracted dose criteria for each of several radiation sensitive body organs. These dose criteria are understood to be mission-integrated criteria and are compared, at each step of an optimization computation, with dose values determined by summing dose contributions for all crew stations, explicitly incorporating time factors expressing the exposure to each radiation source at each station.

VARIABLES AND CONSTRAINTS

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VARIABLES

- CHARACTERISTIC THICKNESS (g/cm²) AT CANDIDATE SHIELD LOCATIONS
- THICKNESSES MEASURED ALONG SURFACE NORMALS
- SHIELD WEIGHT · THICKNESS x AREA
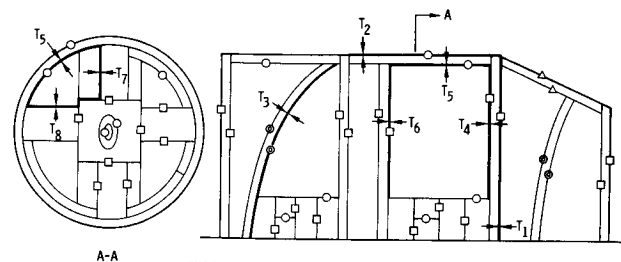- TRIPLY-NESTED SHIELDS ALLOWED

CONSTRAINTS

- MAXIMUM ALLOWABLE DOSE TO CRITICAL BODY ORGANS (I.E., BFO, SKIN, EYES)
  DOSE FOR PROTRACTED EXPOSURE COMPARED WITH MISSION-INTEGRATED DOSE (ORGAN DOSE SUMMED OVER VARIOUS TIME-WEIGHTED ASTRONAUT LOCATIONS)
  DOSE FOR ACUTE EXPOSURE COMPARED WITH DOSE FROM A MAJOR FLARE (ASTRONAUT LOCATION MAY BE RESTRICTED DURING EVENT, I.E., TO A BIOWELL)
- MINIMUM SHIELD THICKNESS VALUES (≥0)
- MAXIMUM SHIELD THICKNESS VALUES CORRESPONDING TO VOLUME OR WEIGHT RESTRICTIONS AT ANY CANDIDATE SHIELD LOCATION

FIGURE 4

OPTIMIZATION METHOD

Numerous mathematical techniques exist for computing optimal solutions to engineering problems. For many problems, the solutions can be obtained directly by solving the set of simultaneous equations for the values of the variables satisfying the constraints imposed. In other cases, and particularly for geometrically realistic shielding problems, the complexity of the functional relationships is such that the equations can only be solved by iteration on the many variables, hopefully converging on a solution that is optimum and that can be ascertained for a realistic expenditure of engineering labor and computer time.

Capabilities Required

Among the approaches to optimization of radiation shield systems, per se, those that have been reported have dealt predominantly with optimization of shields for nuclear power systems, rather than with shields for space radiations. An application of the gradient nonlinear programming technique to optimization of a divided shield system for a nuclear powered aircraft was reported in References 8, 9 and 10; the problem was considerably simplified by the assumption that the source could be adequately represented by a point source. Another approach to the same optimization problem was reported in Reference 11, in which case a restricted representation of shield system variables was used to facilitate iteration on the shield thicknesses and on the two Lagrange multipliers associated with the neutron and gamma dose constraints. A shield synthesis technique was reported for optimization of compact power reactor shields in Reference 12 and an application of the technique to optimization of proton shields was presented in Reference 13; both applications restricted shield geometry to convex shapes.

Such shield optimization approaches generally relaxed one or more important aspects in order to facilitate determination of a minimum weight solution, i.e., the detail with which the optimization problem can be characterized, the number and nature of the constraints which can be simultaneously imposed, or the accuracy of dose and dose derivative evaluations. The usual difficulty was that the optimization formulation did not have the capacity to allow for much detail in either problem geometry (basic system and shield) or design criteria. In developing SWORD, however, a determined attempt was made to avoid such simplifications; rather, a number of capabilities required of the overall program were identified as requirements to be satisfied by the optimization procedure. These included:

o Compatibility with a detailed geometric description of the vehicle, equipment, stores, etc., preferably the same generalized geometry framework now commonly employed in space radiation dose evaluation programs and with dose transmission data provided by detailed physics computations.

o A flexible system for defining shield system variables in a framework which can be superimposed on the detailed fixed geometry of the vehicle and its contents.

o Capability to accommodate multiple radiation level criteria with no a priori knowledge of their interdependence or independence. The latter implies that the design values for some constraints may necessarily be exceeded in meeting the specified values for others. Also the relative importance of each constraint should be ascertained automatically and its effect diminished automatically if it becomes an inequality constraint.

o Internal calculation of radiation levels, shield weights, and their derivatives, to minimize data handling and to permit a detailed re-evaluation of all dose and weight values as required throughout the iterative operations.

o Procedures for controlling values of the variables during iterations so as to minimize the quantity of data to be updated at each step, as this is potentially time consuming.

Candidate Techniques

Two optimization techniques potentially able to satisfy these requirements were identified. One consisted of a conventional formulation of a multiple-constraint problem using the Lagrange multiplier technique and used an iterative procedure suggested by Arrow and Hurwicz (Ref. 14); this method is outlined in Figure 6. The other method, developed and applied at McDonnell Douglas, reduced the multiple-constraint problem so that it can be handled operationally as a single-constraint problem. It used a single approximate, continuous, functional combination of the multiple constraints, which forces the result to converge to the solution for the exact original problem. This approach is outlined in Figure 7.

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W_i = WEIGHT OF ith SHIELD REGION
D_k = RADIATION LEVEL AT k^{th} CRITICAL LOCATION
C_k = CONSTRAINT ON RADIATION LEVEL AT k^{th} LOCATION
λ_k = LAGRANGE MULTIPLIER
t_i = CHARACTERISTIC THICKNESS OF ith SHIELD REGION

OPTIMIZATION METHOD
MULTIPLE CONSTRAINT
FORMULATION

1. DEFINE (INEQUALITY) CONSTRAINTS

$$G_k = D_k(\vec{t}) - C_k$$

2. LAGRANGE FORMULATION IS:

$$V(\vec{t}, \vec{\lambda}) = W(\vec{t}) + \lambda_1 G_1 + \lambda_2 G_2 + \cdots + \lambda_k G_k$$

$$\frac{\partial V}{\partial t_i} = 0 ; \quad \frac{\partial V}{\partial \lambda_k} = G_k \leq 0$$

3. FIND THE SET $(\vec{t}, \vec{\lambda})$ WHICH IS A SADDLE POINT OF V

$$\begin{array}{cc} \text{MIN} & \text{MAX} \\ t \geq 0 & \lambda \geq 0 \end{array} V(\vec{t}, \vec{\lambda})$$

USING ARROW AND HURWICZ ITERATIVE PROCEDURE

$$\Delta t_i = \frac{\Delta V}{|\text{GRAD } V|^2} \frac{\partial V}{\partial t_i}$$

$$\Delta \lambda_k = \frac{\Delta V}{|\text{GRAD } V|^2} \frac{\partial V}{\partial \lambda_k}$$

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W_i • WEIGHT OF ith SHIELD REGION

D_k • RADIATION LEVEL AT kth CRITICAL LOCATION

C_k • CONSTRAINT ON RADIATION LEVEL AT kth LOCATION

λ • LAGRANGE MULTIPLIER

n • EXPONENT SUFFICIENTLY LARGE TO EMPHASIZE DOMINANT CONSTRAINTS

t_i • CHARACTERISTIC THICKNESS OF ith SHIELD REGION

OPTIMIZATION METHOD

SINGLE CONSTRAINT
FORMULATION

1. COMBINE CONSTRAINTS INTO SINGLE RELATIONSHIP

$$U = \left(\frac{D_1}{C_1}\right)^n + \left(\frac{D_2}{C_2}\right)^n + \cdots + \left(\frac{D_k}{C_k}\right)^n$$

2. LAGRANGE FORMULATION IS

$$V = W(\vec{t}) + \lambda U(\vec{t}); \quad \frac{\partial V}{\partial t_i} = 0$$

3. SINCE THERE IS ONLY ONE LAGRANGE MULTIPLIER (λ), FIND THE SET ($\vec{t}$) SUCH THAT

$$-\frac{1}{\lambda} = \frac{\partial U/\partial t_1}{\partial W/\partial t_1} = \frac{\partial U/\partial t_2}{\partial W/\partial t_2} = \cdots = \frac{\partial U/\partial t_I}{\partial W/\partial t_I}$$

THEREFORE, REPETITIVELY MODIFY t_m IN THE SET ($\vec{t}$) WHERE m IS SUCH THAT

$$\frac{\partial U/\partial t_m}{\partial W/\partial t_m} = \underset{i}{MAX} \frac{\partial U/\partial t_i}{\partial W/\partial t_i} \quad OR \quad \frac{\partial U/\partial t_m}{\partial W/\partial t_m} = MIN \frac{\partial U/\partial t_i}{\partial W/\partial t_i}$$

FIGURE 7

Experience with these methods on a variety of shield optimization problems led to a preference for the latter, single-constraint approach. One significant advantage was due to altering only one thickness value at each step of the iterative process, because this approach then reduced the amount of data that had to be updated prior to the next step. Hence, the single constraint approach was generally more efficient. It is likely that this advantage would be attainable in the multiple constraint approach if only the few largest Δt_i were implemented at each step.

The major objection to the conventional multiple constraint formulation lay in the fact that, for shielding problems, the measure of importance of a constraint is not fairly represented by its linear distance from the criterion (as indicated in Item (1) of Figure 2). The exponential nature of the constraint functions results in the importance of each constraint being much more accurately expressed by the ratio of current value to criterion (as is utilized in the single constraint formulation). This failure to incorporate the exponential character of the function leads to severe difficulties in converging on an optimum solution. The rate of convergence is necessarily dependent on the step size (i.e., ΔV) at each iteration and this is limited by the range of (Δt_i, $\Delta \lambda_k$) over which the partial derivatives can be applied. For the linear formulation indicated in the figure, this range is quite small. While some alteration in the expression of the constraining relationships would presumably alleviate this difficulty, it was not pursued.

Rather than modify the multiple constraint formulation to achieve a more effective technique for the kinds of shield optimization analyses of interest, the activity centered instead on exploitation of the single constraint approach. This latter technique, having fewer variables, was considerably simpler to apply and, where comparisons were made of results obtained with both techniques, provided results that satisfied tests for optimality.

Optimality of Preferred Method

The capacity of the single constraint method to produce a result which not only meets the criteria at low total weight, but is truly optimum, has been demonstrated numerically. The criteria for optimality are indicated for the conventional multiple constraint formulation in Figure 6, i.e.,

$$\frac{\partial V}{\partial t_i} = \frac{\partial W}{\partial t_i} + \lambda_1 \frac{\partial D_1}{\partial t_i} + \lambda_2 \frac{\partial D_2}{\partial t_i} + \text{---}$$

$$+ \lambda_k \frac{\partial D_k}{\partial t_i} = 0; \quad i = 1, 2, \ldots I$$

These equations can be evaluated using the set of thicknesses, t_i, provided by the single constraint method directly and using a set of multipliers, λ_k, constructed from other data determined in the analysis:

$$\lambda_k = \frac{n\lambda_i}{C_k} \left(\frac{D_k}{C_k}\right)^{n-1}; \quad \lambda_i = \frac{\partial W/\partial t_i}{\partial U/\partial t_i}$$

There obviously are a total of I values of each λ_k, these values agreeing with one another to the extent that the values of λ_i are in agreement. Since the essence of the single-constraint formulation is to align the values λ_i, the several values for each λ_k can be made to agree quite well by aligning the λ_i within a very small difference. This is accomplished by continually reducing the increment (or decrement) to each variable once all constraints have been satisfied, iterating in the neighborhood of the optimum solution until satisfactory convergence is attained. This procedure, however, can be extremely time consuming and is of almost no benefit in terms of reducing the total shield weight from the value determined with a relatively coarse optimization criterion.

Typically, the convergence of thickness values to the extent required to prove optimality doubles or triples the time required to determine a shield condition to within one percent of that optimum weight.

SAMPLE OPTIMIZATION ANALYSIS

Results from a simplified application of the SWORD program are presented in Figure 8 to illustrate some of the points made in the preceding discussion. This sample problem involves optimization of the shield system illustrated in Figure 5.

The problem included constraints on acute dose from solar cosmic rays (SCR) and on protracted dose from geomagnetically trapped radiations and their secondaries, and from neutrons and gamma rays from an on-board isotopic power system. The total number of dose constraints was six, these being separate acute and protracted dose criteria for each of three critical organs: lens of the eye, skin, and blood forming organs.

There were three man-model locations, representing crew stations in a command area, an experimental area and a living area. The command area was specified to be within a biowell for SCR protection, and all SCR exposure was assumed to be taken at this location. Two of the man-model locations were on centerline and, with the vehicle being reasonably symmetrical cylindrically, a single dose point in the man-model was used to represent the location of each of the three organs. The man-model position in the biowell, however, was sufficiently off axis that some recognition of the resultant asymmetry was required. This was accomplished by using two dose points to represent each organ, one at the appropriate depth below the surface of the skin facing the outer wall and the other 180° opposite. The time-weights associated with the biowell locations were halved to compensate for the extra dose point.

The value of making even token recognition of system asymmetry has been proven in a number of shield optimization problems. When dose can be delivered asymmetrically, as at the biowell location, it is particularly important that the dose point(s) be representative; otherwise, introduction of an appreciable bias in the discrete and discontinuous representation of critical organ location can lead to false concentrations of shield mass. Ideally, the dose would be evaluated by sampling from a continuous timeline of astronaut position within the vehicle, sampling also from the solid angle at each position along the timeline; the effects of astronaut orientation at each position might also be diminished, since these are not necessarily real. This approach to dose integration has been incorporated in a recent version of the SIGMA program, as reported in Reference 6, but is not yet incorporated in SWORD.

There were eight variables, three of which defined a cylindrically symmetric shield system enclosing all inhabited areas, and five of which defined the biowell.

Dose transmission data were furnished for a synchronous orbit mission of several months duration, these then being trapped electron and secondary bremsstrahlung dose, and SCR dose. The geometry, materials, and radiation source characteristics for the on-board isotopic power system were also defined and the direct and scattered doses computed therefrom. Only the direct dose proved to be of any significance and it had relatively little influence on the optimum shield determination, because this was dominated by SCR considerations; i.e., the acute dose criteria were the most stringent for the particular mission.

Mission-integrated critical organ doses were calculated using a relatively coarse integration grid, namely four intervals in μ (cosine of polar angle) and six intervals in θ (azimuthal angle) or a total of 21 rays for each of the twelve dose points. The 252 rays traced made a total of 382 shield crossings because of the nesting of the biowell shield within the main shield. The computer time required to perform the initial dose evaluation, in preparation for the shield optimization computation, was 21 seconds on the CDC 6500 computer. (Much more detailed integration grids can be used; problems involving up to 3000 rays have been solved.)

The optimization history for this problem is shown in Figure 8. A total of 106 iterations on weight were accomplished in 40 seconds, each iteration initially involving weight increments of up to 250 Kg, decreasing to 15 Kg as the solution converged. It can be seen that two of the constraints dominated the optimization, the domination being particularly evident when the ratios of dose to constraint value decreased to approximately 0.9 after iteration 80, and when these ratios were then raised to an exponent of 100. As noted on the figure, essentially the same result was obtained when the exponent was 4 rather than 100; there is no particular reason not to use a large value of the exponent, however, other than to avoid overflow when the ratios are large, and SWORD internally increases the value to 100 as rapidly as the values of the ratios permit.

TYPICAL SHIELD OPTIMIZATION PROBLEM 19559
(ITERATION HISTORY)

• SIX DOSE CRITERIA
• THREE TIME-WEIGHTED MAN MODEL LOCATIONS
• EIGHT SHIELD THICKNESS VARIABLES, FIVE DEFINING BIOWELL
• ONBOARD ISOTOPIC POWER SYSTEM
• DOSE RATIO EXPONENT = 100
• TWO CRITERIA DOMINATE; IDENTICAL RESULT WHEN FOUR IN-EQUALITY CONSTRAINTS DELETED
• DOSE EVALUATED WITH 252 RAYS IN 21 SEC (CDC 6500)
• OPTIMUM CONDITION DETERMINED WITH WEIGHT INCREMENT OF 250 kg IN 40 SEC
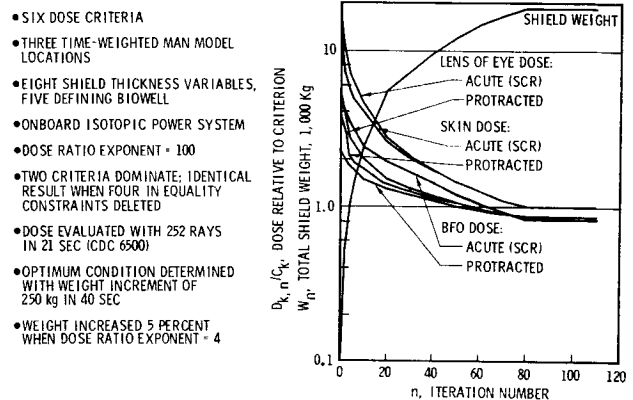• WEIGHT INCREASED 5 PERCENT WHEN DOSE RATIO EXPONENT = 4

FIGURE 8

The optimization of the solution was checked at the 106th iteration by calculating the eight possible values (i.e., eight variables) of each of the six Lagrange multipliers (i.e., six dose criteria) from the several values of the multiplier determined in the single constraint solution. The set of multipliers obtained from each of the I equations was used to evaluate all I equations. Actually, each equation was divided through by A_i so that the magnitude of the remainder was referenced to the magnitude of the numbers involved, i.e.,

$$R_i = \frac{\partial V/\partial t_i}{A_i} = 1 + \frac{1}{A_i} \sum_k \lambda_{ik} G_k \overset{?}{=} 0$$

The iteration was then continued for another 41 steps, progressively decreasing the weight increment to 1 Kg, with these results then being compared to the optimality as evaluated after 106 steps. While the total shield weight decreased only 0.14%, the small changes in values of each of the variables led to a dramatic change in

apparent optimality: the λ_{ik} for the two significant constraints agreed within 0.4% and the maximum value of R_i for any set (λ_{ik}) was 0.001. This experience is typical and serves to illustrate that the shield mass converges much more rapidly than the variables or the multipliers, and that the convergence criterion for iteration cutoff should be based on mass (as it is) rather than on some optimization test.

CONCLUSIONS

The work reported herein was initiated when a need arose to analyze the dependence of space vehicle shield weight requirements on various system parameters. The space vehicle in question was to fly long duration missions in synchronous orbit and hence a considerable mass of supplementary shielding appeared necessary. In order for the shield parametric data to be sufficiently self-consistent that the dependence on system parameters be meaningful, it was evident that a systematic procedure was required to produce it. The optimization technique, and the implementation thereof, as described, proved to be extremely effective, both in terms of the scope of the problems which could be analyzed and optimized, and in terms of the significance of the results. Results from some production applications of SWORD were reported in References 15 and 16.

The applicability of the optimization approach embodied in the single constraining function is much broader than that reviewed here. It is certainly worthy of consideration for, and has in fact been applied to, other shielding optimization problems. It could also be effectively applied in other, non-shielding efforts, possibly with some modification of the constraining functions to best approximate the functional relationships of constrained quantities to their specific constraint values.

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