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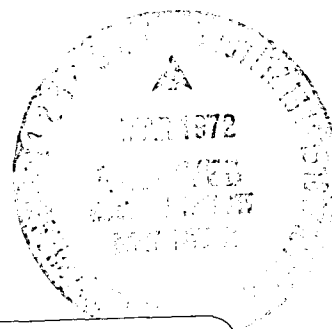
MEMORANDUM

A MODEL FOR CALCULATING EXPECTED
PERFORMANCE OF THE APOLLO UNIFIED
S-BAND (USB) COMMUNICATION SYSTEM

Bellcomm

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ABSTRACT

Predictions of communications performance under specified link configurations play a major role in pre-mission planning to assure that adequate communications support of the spacecraft is maintained during all phases of an Apollo lunar mission. This paper describes the general organization of the Apollo USB system and reviews the analysis used to derive a mathematical model that approximates the performance of this system. The expressions for performance measures contained in this paper have been implemented in a computer program at Bellcomm, Inc.

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TECHNICAL MEMORANDUM

I.   INTRODUCTION

Except during the near-earth phases of the mission, altitudes below 20,000 km, Ref. 1, where some VHF links are also used, all communication and tracking functions between the spacecraft and the ground support stations required during an Apollo Lunar Landing Mission are provided by the Apollo Unified S-Band (USB) System. The unified systems concept basic to the USB system design is based on techniques developed by the Jet Propulsion Laboratory for the unmanned lunar and planetary programs. In this system the ground station to vehicle link (up-link) consists of a single S-Band carrier that carries all up-link information to a given vehicle; the vehicle to ground station link (down-link) consists of one or two S-Band carriers, depending on the vehicle considered,

that carry all the down-link information. The earth-based
portion of the USB system is a world-wide network comprised
of ground stations, aircraft, and ships. The ground facil-
ities used are primarily those of the Manned Spaceflight Net-
work (MSFN), but during critical lunar surface phases of
past Apollo missions, the MSFN has been augmented by the
210 foot diameter receiving antennas of the Deep Space
Network (DSN). The spacecraft portion of the USB system
is carried in three vehicles - the Command and Service
Module (CSM), the Lunar Module (LM), and the final stage
of the launch vehicle (S-IVB).

As in most space missions, the communications sup-
port requirements of the Apollo spacecraft change as the
mission progresses. These changing requirements are reflected
by the varied capabilities that exist in the possible com-
munications links between an Apollo vehicle and the ground
support network. Each of the three vehicles is equipped with
both wide and narrow beamwidth antennas to permit communica-
tions during both the high acceleration and the distant coast
phases of the mission respectively. Since each of the ground
stations can provide communication support during only
those periods when the spacecraft is traversing the field
of its respective ground station antenna, the MSFN

consists of two types of stations. The first type is equipped with 85 foot diameter antennas; three of these stations have been built at sites nearly equidistant around the world - in California, Spain, and Australia - to provide continuous communications contact for vehicles at altitudes above about 10,000 N.mi., Ref. 1. The second type is equipped with 30 foot diameter antennas; these stations are positioned between the stations discussed above to provide communications contact during launch and other critical maneuvers performed while the vehicles are in earth orbit at about 100 N.mi. altitude or in near earth phases of a coast trajectory.

Basic to the design of the Apollo USB system is the technique used to measure the instantaneous range and velocity of the spacecraft because the technique requires a transponder-type receiver/transmitter on board the spacecraft. Range is determined by measuring the round-trip propagation time of a signal transmitted from the ground, retransmitted from the vehicle, and then received again at the ground. The signal used for this round-trip transmission is a binary sequence of 5,456,682 bits that is referred to as the pseudo-random noise (PRN) ranging code. This ranging code is transmitted at a one megabit rate and appears as a symmetrical $(\sin^2 x)/x^2$ power

distribution centered about the carrier frequency. Utilizing

the statistical properties of binary sequences discussed in

Ref. 2, the ranging system has been implemented as defined in

Ref. 3 and 4. Velocity is determined by measuring the doppler

shift in the down-carrier frequency, using the up-link carrier

frequency as a reference. This measurement is possible because

the spacecraft transponders are equipped to transmit a phase

modulated, down-link carrier that is coherently related to the

received up-link carrier by the frequency ratio of 240/221.

Several modulation schemes are used to place the

mission data on the S-Band carriers. Voice, up-link command,

emergency key, and down-link PCM telemetry data modulate sub-

carriers which in turn modulate respective S-Band carriers.

Back-up voice and the ranging code modulate a carrier phase

directly, and television modulates a carrier frequency directly.

The LM is equipped to transmit only one down-link carrier;

therefore, during LM television transmissions the LM PCM

telemetry and down-link voice subcarriers also modulate the

down-carrier frequency. When no LM television is transmitted,

the LM down-carrier is coherently related in frequency to the

LM up-link carrier; under this condition the subcarriers phase

modulate the LM down-link carrier. The CSM, however, is equipped

to transmit two down-link carriers simultaneously. One of
these carriers is coherently related in frequency to the
CSM up-link carrier, and on this are placed the subcarriers
and range code by phase modulation. The second CSM down-
link carrier is independent of the first carrier and separated
in frequency from it by 15 MHz. The CSM television signal
frequency modulates the second carrier. The S-IVB stage is
equipped to transmit only one down-link carrier which is phase
modulated by a PCM telemetry subcarrier and by the ranging
code. For a general description of some of the hardware used
in the USB system the discussion in Ref. 5 is suggested to
the reader. In the discussion that follows only those trans-
mission modes where the mission data is placed on the S-Band
carrier by phase modulation, as shown in Figs. 1 and 2, will
be considered further.

In the following discussion, three types of measures
are used to provide an estimate of the performance of the
communication links; these measures are: (1) Circuit Margin,
(2) Power Margin, and (3) Maximum Range. A circuit margin of
"C" db indicates that the expected signal to noise power ratio
$(S/N)_{exp}$ exceeds by C dB, a reference signal to noise ratio
$(S/N)_{ref}$ required for receiving minimally acceptable data.
The circuit locations in the receivers where these reference
ratios are defined by performance specifications are shown
in Figs. 3 and 4 and Refs. 6 and 7. Although circuit margins

are useful in predicting data quality under conditions
where link parameters are fixed, there exist cases espe-
cially during mission planning where it is more useful to
know the magnitude by which the receiver input signal power
could be reduced to achieve a circuit margin just equal to
zero; this permissible reduction in signal power is defined
as the Power Margin. Circuit Margins and Power Margins are
not directly related because the noise in the USB receivers
is a function of the receiver input signal power. A maximum
range of "R" nautical miles indicates that the circuit margin
will be negative, and consequently the data quality is expected
to be degraded, for vehicle ranges in excess of R n.mi.

II. BANDPASS LIMITERS

In the USB receivers bandpass limiters as shown in
Figs. 3 and 4 are used in cascade with a product detector.
This configuration has been discussed by W. D. Wynn in Ref. 8
using a detailed statistical analysis. The approximation of
the limiter (S/N) transfer characteristic used in the following
analysis is less accurate than the analysis presented by Wynn;
however, for the cases expected during an Apollo mission this
approximation characterized in Fig. 5 provides useful results
and requires minimal computations.

Jaffe and Rechtin (Ref. 9) and Viterbi (Ref. 10) show the input and output signal to noise power ratios of a bandpass limiter to be related by

$$S_{out}/N_{out} = k \ S_{in}/N_{in} \qquad (1)$$

Assuming that the total power output of the limiter is L^2, then

$$L^2 = S_{out} + N_{out} \text{ watts.} \qquad (2)$$

Thus the total narrow-band signal power at the limiter output is

$$S_{out} = L^2 (S_{in}/N_{in})/[(1/k) + (S_{in}/N_{in})] \text{ watts,} \qquad (3)$$

and consequently the noise power in a finite bandwidth at the limiter output in watts is

$$N_{out} = \left\{ L^2/[(k \ S_{in}/N_{in}) + 1] \right\} \left\{ \frac{\text{Bandwidth of Output}}{\text{Bandwidth of Input}} \right\} \qquad (4)$$

On the basis of the above relations, if the signal output of a bandpass limiter in the absence of input noise is

$$s_{out}(S_{in}) = \sqrt{2} \ L \sin (\omega_o t + \phi) \qquad \text{volts.} \qquad (5)$$

then the signal output in the presence of input noise is reduced to

$$s_{out}(S_{in}+N_{in}) = \sqrt{2} \; L \; f[k,(S/N)_{in}] \; \sin(\omega_o t+\phi) \qquad \text{volts} \qquad (6)$$

for

$$f[k,(S/N)_{in}] = \left\{(S_{in}/N_{in})/[(1/k) + (S_{in}/N_{in})]\right\}^{1/2}$$

Inherent in the above discussion is the assumption that the limiter power output is constant. This condition is satisfied only when the input signal plus noise process bandwidth is narrow compared to the carrier frequency; therefore, expression (6) is an approximation of the limiters used in the USB receivers to the extent that the actual limiter input processes differ from the narrow-band condition. The parameter "k" is a function of the input S/N power ratio as defined in Fig. 5.

III. UP-LINK CHANNELS

Modulating the phase of the up-link carrier are two types of signals - the two subcarriers and the ranging code. The up-link signal and noise that is received by the vehicle transponder is, therefore, assumed to be described by

$$S_u(t) + n_u(t) = \sqrt{2P_{sr}} \; \sin\left[\omega_{uc}t+m_1\sin \omega_1 t+m_2\sin \omega_2 t+\theta\right] + n_u(t) \qquad (7)$$

for

P_{sr}* = Total signal power in receiver IF in watts

ω_{uc} = Up-carrier frequency in radians/second

$\omega_{1,2}$ = Up-subcarrier frequencies in radians/second

$m_{1,2}$ = Up-subcarrier peak modulation indices in radians

θ = Ranging code peak modulation index in radians

$n_u(t)$ = Thermal noise added to up-link signal by transponder

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*In general the expected receiver input signal power in dBW is calculated from the relation

$$P_r = P_t + G_t + G_r - L_{tc} - L_{rc} - L_p - L_s \text{ dBW}$$

P_t = Transmitter power in dBW

G_t, G_r = Transmitting and receiving antenna gains in dB

L_{tc}, L_{rc} = Transmitting and receiving circuit losses in dB

L_p = Polarization loss in dB

$L_s = 20 \log_{10} R_{nm} + 20 \log_{10} f_{mhz} + 37.8 \text{ dB}$

R_{nm} = Range in nautical miles

f_{mhz} = carrier frequency in megahertz

3.1 Up-Carrier Circuit Margin

Required signal to noise power ratios $(S/N)_{ucr}$ are defined for the carrier channel at the input to the carrier tracking loop of the receiver shown in Figure 3. To derive an expression for the up-carrier circuit margin, it is necessary to determine which portion of the signal and noise expressed by (7) is passed by the carrier channel bandpass limiter and a tracking loop filter with a bandwidth of B_{uc} Hz.

The signal expressed in (7) is equivalent to

$$S_u(t) = \sqrt{2P_{sr}} \; Im \left\{ \exp \left[j(\omega_{uc}t + m_1 \sin \omega_1 t + m_2 \sin \omega_2 t + \theta] \right] \right\} \qquad (8)$$

$$= \sqrt{2P_{sr}} \; Im \left\{ \exp \left[j(\omega_{uc}t + \theta) \right] \; f(\omega_1, \omega_2) \right\} \qquad (9)$$

$$f(\omega_1, \omega_2) = \sum_{N_1 = -\infty}^{\infty} J_{N_1}(m_1) \exp(jN_1 \omega_1 t) \sum_{N_2 = -\infty}^{\infty} J_{N_2}(m_2) \exp(jN_2 \omega_2 t)$$

Setting $N_1 = N_2 = 0$ gives an expression for the narrow band component of the above expression that is centered about ω_{uc}. This reduces (9) to

$$S_{uc}(t) = \sqrt{2P_{sr}} \; J_o(m_1) \; J_o(m_2) \; (\sin \omega_{uc}t \cos \theta + \cos \omega_{uc}t \sin\theta). \qquad (10)$$

The first term of (10) is the carrier signal component, and the second term is an interference signal caused by the presence of the ranging code signal. Using the analysis of the ranging code signal spectrum presented by Benden in Reference 11, it can be shown that the combined magnitude of the ranging code spectrum components that lie in the carrier tracking loop bandwidth is negligible.

Having defined an expression for the carrier signal components at the input to the receiver, it is now necessary to determine how much this signal is attenuated by the carrier channel bandpass limiter. Since the signal power at the input to the limiter is P_{sr} and a flat thermal noise power spectrum across the IF bandwidth B_t is assumed, the limiter input signal to noise power ratio is

$$(S/N)_{IFu} = P_{sr}/B_t \, K_o \, T_u \qquad (11)$$

$K_o = 1.38 \times 10^{-23}$ watts/°Kelvin · Hz = Boltzmann constant

T_u = noise temperature referred to spacecraft receiver input in degrees Kelvin

The system noise temperature - T_u, as a consequence of the automatic gain control (AGC) function implemented in the USB receivers, is a function of the carrier signal power. Selden in Ref. 12 gives the following approximation for this relationship:

Noise temperature = A + B (Carrier Signal Power in Watts) (12)

Test data has been used to establish the magnitudes of the two constants A and B for each of the USB receivers.

From (6), (10) and (11) the carrier signal at the limiter output, in volts is

$$S_{ucp}(t) = \sqrt{2} L_{uc} \, f\left[k_{uc}, \, (S/N)_{IFu}\right] J_o(m_1) J_o(m_2) \cos \theta \sin \omega_{uc} t \qquad (13)$$

$$f\left[k_{uc}, \, (S/N)_{IFu}\right] = \left\{(S/N)_{IFu}\Big/\left[(1/k_{uc}) + (S/N)_{IFu}\right]\right\}^{1/2}$$

k_{uc} is evaluated from $(S/N)_{IFu}$ and Figure 5.

From (4) and (11) the noise power at the limiter output is

$$P_n(uc) = \left\{ L_{uc}^2 / \left[k_{uc}\ (S/N)_{IFu} + 1 \right] \right\} \left\{ B_{uc}/B_t \right\} \quad \text{watts} \quad (14)$$

Then the desired expression for the predicted signal to noise power ratio at the input to the carrier tracking loop is obtained from (13) and (14)

$$(S/N)_{ucp} = \alpha_{uc}\ k_{uc}\ P_{sr} / \left[B_{uc}\ K_o\ T_u \right]$$

$$\alpha_{uc} = J_o^2(m_1) J_o^2(m_2)\ \cos^2\theta \quad (15)$$

and finally the up-carrier circuit margin is expressed by

$$\text{Circuit Margin (uc)} = 10\ \log_{10} \left[(S/N)_{ucp} \right] - (S/N)_{ucr} \quad \text{decibels} \quad (16)$$

3.2 Up-Carrier Power Margin

As the signal power P_{sr} is reduced, the circuit margin calculated by (16) reduces also; however, the calculation of the minimum power ($P_{min}(uc)$) necessary for the up-carrier circuit margin to just equal zero is complicated by two factors --

A. The noise temperature is a function of the carrier signal power

$$T_u = A_u + B_u P_{sr} \alpha_{uc}$$

B. The limiter factor k_{uc} is a function
    of the signal to noise ratio at the
    input to the carrier channel limiter
    as shown in Figure 5
    for $(S/N)_{in} = P_{sr}/K_o T_u B_t$
        and $k = k_{uc}$

Because of these factors the calculation of $P_{min}(uc)$ requires
an iterative process that is discussed in Reference 14.

Setting (16) equal to zero and defining

$$10 \log_{10} (\gamma_{uc}) = (S/N)_{ucr}$$

$$P_{min}(uc) = P_{sr}\Big|_{\text{circuit margin} = 0}$$

gives the following expression for $P_{min}(uc)$

$$P_{min}(uc) = A_u B_{uc} \gamma_{uc} K_o \Big/ \left[\alpha_{uc}(k_{uc} - B_u B_{uc}\gamma_{uc}K_o)\right] \qquad (17)$$

After a solution for (17) is obtained, the up-carrier power
margin is calculated directly from

$$\text{Power Margin (uc)} = 10 \log_{10} \left[P_{sr}/P_{min}(uc)\right] \quad \text{decibels} \qquad (18)$$

3.3 Up-Subcarrier Circuit Margins

Required signal to noise power ratios $(S/N)_{u\omega_i r}$ are defined for the subcarrier channels at the input to the respective subcarrier demodulators as shown in Figure 3. To derive an expression for the up-subcarrier circuit margin, it is necessary to determine which portion of the signal and noise expressed by (7) is passed by the cascade comprised of the video-channel bandpass limiter, the wide-band product detector, and the subcarrier demodulator input bandpass filter $-B(\omega_i)$.

Returning to (9) and rearranging the terms the total signal at the input to the spacecraft receiver can be expressed by

$$S_u(t) = \sqrt{2P_{sr}}\; Im\left\{\exp\left[j(\omega_{uc}t+\theta)\right]\; f(\omega_1 t)f(\omega_2 t)\right. \tag{19}$$

$$f(\omega_1 t) = J_0(m_1) + 2\sum_{N_1=1}^{\infty}\left[J_{2N_1}(m_1)\cos 2N_1\omega_1 t\right.$$

$$\left. +jJ_{2N_1-1}(m_1)\sin(2N_1-1)\omega_1 t\right]$$

$$f(\omega_2 t) = J_0(m_2) + 2\sum_{N_2=1}^{\infty}\left[J_{2N_2}(m_2)\cos 2N_2\omega_2 t\right.$$

$$\left. +jJ_{2N_2-1}(m_2)\sin(2N_2-1)\omega_2 t\right]$$

Neglecting the higher order and cross frequency terms which lie outside the subcarrier demodulator input bandwidths, equation (19) reduces to

$$S_{uv}(t) = \sqrt{2P_{sr}} \left[J_o(m_1) J_o(m_2) \sin(\omega_{uc}t + \Theta) \right.$$

$$+2J_1(m_1) J_o(m_2) \sin \omega_1 t \cos(\omega_{uc}t+\Theta)$$

$$\left. +2J_o(m_1) J_1(m_2) \sin \omega_2 t \cos(\omega_{uc}t+\Theta) \right] \tag{20}$$

The second and third terms of (20) are the signal components of the ω_1 and ω_2 subcarriers, respectively; the first term is an interference signal caused by the presence of the ranging signal on the carrier.

From (11), the input signal to noise power ratio of the video-channel limiter is given by

$$(S/N)_{IFu} = P_{sr}/B_t K_o T_u \tag{11}$$

and consequently the signal is attenuated in the limiter by the factor

$$f\left[k_{uv}, (S/N)_{IFu}\right] = \left\{(S/N)_{IFu}/\left[(1/k_{uv} + (S/N)_{IFu}\right]\right\}^{1/2} \tag{21}$$

k_{uv} is evaluated from $(S/N)_{IFu}$ and Figure 5.

In the product detector the input signal is multiplied by the derived carrier ($B \cos \omega_{uc} t$), and the resulting sum frequencies are removed by filtering. After detection then the signal is expressed by

$$S_{ud}(t) = \left\{ \sqrt{2}\, L_{uv} B/2 \right\} \left\{ (S/N)_{IFu} \middle/ \left[(1/k_{uv}) + (S/N)_{IFu} \right] \right\}^{1/2} f_{ud}(t) \quad (22)$$

for

$$f_{ud}(t) = J_o(m_1) J_o(m_2) \sin \theta$$

$$+ 2J_1(m_1) J_o(m_2) \cos \theta \sin \omega_1 t$$

$$+ 2J_o(m_1) J_1(m_2) \cos \theta \sin \omega_2 t$$

In general then, the up-subcarrier signal power is expressed by

$$P_s(\omega_i) = \left\{ L_{uv}^2 B^2/2 \right\} \left\{ (S/N)_{IFu} \middle/ \left[(1/k_{uv}) + (S/N)_{IFu} \right] \right\} \alpha_{u\omega_i} \quad (23)$$

for
$$i = 1 \text{ or } 2$$
$$j = 1 \text{ or } 2$$
$$i \neq j$$

$$\alpha_{u\omega_i} = 2J_1^2(m_i) J_o^2(m_j) \cos^2\theta$$

Assuming a flat thermal noise spectrum across the low pass one-sided bandwidth of $B_t/2$, the noise power in the input bandwidth $B(\omega_i)$, of a subcarrier detector is

$$P_n(\omega_i) = \left\{ (L_{uv}B/2) \Big/ \left[1+k_{uv}(S/N)_{IFu}\right]^{1/2} \right\}^2 \cdot 2B(\omega_i)/B_t) \quad \text{watts} \quad (24)$$

Although the ranging code signal is negligible in the narrow-band carrier tracking loop, this signal presents significant interference to the up-subcarrier signals.

Modulating a carrier with the digital ranging code results in a power distribution that is symmetrical about the carrier frequency and is bounded approximately by a $\sin^2 x/x^2$ envelope. Modulating a carrier with both the ranging code and subcarriers results in a power distribution comprised of an infinite number of $\sin^2 x/x^2$ envelopes that are centered about each of the higher order and cross frequency terms defined by (19) in addition to the first order term centered about the carrier frequency given as the first term in (20). Analysis shows that for the Apollo cases, only the first order term of the ranging signal spectrum need be considered; therefore, the interference to the up-subcarrier caused by the presence of the ranging signal can be approximated using the analysis presented by Hill in Reference 13.

From (22) the total first order detected power of the ranging code signal is

$$P_C(T) = \left\{ \sqrt{2} \, L_{uv}(B/2) \right\}^2 \left\{ (S/N)_{IFu} / \left[(1/k_{uv}) + (S/N)_{IFu} \right] \right\} \alpha_{uco} \quad (25)$$

$$\alpha_{uco} = J_0^2(m_1) \, J_0^2(m_2) \, \sin^2 \theta$$

Using the $\sin^2 x / x^2$ approximation for the envelope of this power spectrum, shown in Figure 6, the power in (25) can be expressed by

$$P_C(T) = \int_0^\infty M \, \frac{\sin^2 x}{x^2} \, dx = M \, \frac{\pi}{2} \quad (26)$$

and

$$M = \frac{2}{\pi} \, P_C(T)$$

$$x = k_c f$$

$$k_c = \frac{\pi}{10^6} \quad \text{for a ranging signal transmitted at 1 MHz}$$

$$dx = k_c df \approx k_c B(\omega_i)$$

Assuming that the ranging signal spectrum is essentially constant over the bandwidth $B(\omega_i)$, the interference power is expressed by

$$P_C(\omega_i) = M \, \frac{\sin^2 k_c f_i}{(k_c f_i)^2} \, k_c B(\omega_i) \quad (27)$$

$$= \frac{1}{10^6} \, L_{uv}^2 \, B^2 \, f^2 \left[k_{uv}, \, (S/N)_{IFu} \right] \alpha_{uco} \, \frac{\sin^2 k_c f_i}{(k_c f_i)^2} \, B(\omega_i)$$

Now combining (11), (23), (24) and (27), the predicted signal to noise power ratio for the up-subcarrier is

$$(S/N)_{u\omega_i p} = P_s(\omega_i) \Big/ \Big\{ P_n(\omega_i) + P_c(\omega_i) \Big\} \tag{28}$$

$$= \frac{\frac{1}{2} k_{uv} P_{sr} \alpha_{u\omega_i}}{\left[\frac{1}{2} K_o T_u + \frac{1}{10^6} k_{uv} P_{sr} \alpha_{uco} \frac{\sin^2 k_c f_i}{(k_c f_i)^2} \right] B(\omega_i)}$$

and, consequently, the expression for the predicted circuit margin of an up-subcarrier is

$$\text{Circuit Margin } (u\omega_i) = 10 \log \left[(S/N)_{u\omega_i p} \right] - \left[(S/N)_{u\omega_i r} \right] \text{decibels.} \tag{29}$$

3.4 Up-Subcarrier Power Margins

Setting (29) equal to zero and defining

$$T_u = A_u + B_u P_{sr} \alpha_{uc}$$

$$10 \log_{10} (\gamma_{u\omega_i}) = (S/N)_{u\omega_i r}$$

$$\eta_{ui} = \frac{\alpha_{uco}}{10^6} \frac{\sin^2 k_c f_i}{(k_c f_i)^2}$$

$$P_{min}(u\omega_i) = P_{sr} \Big|_{\text{circuit margin} = 0}$$

$$P_{min}(u\omega_i) = \frac{A_u B(\omega_i) \gamma_{u\omega_i} K_o}{k_{uv} \left[\alpha_{u\omega_i} - 2B_{(\omega_i)} \gamma_{u\omega_i} n_{ui}\right] - \alpha_{uc} B_u B(\omega_i) \gamma_{u\omega_i} K_o} \quad (30)$$

After a solution for (30) has been obtained by iteration, the up-subcarrier power margin is calculated directly from

$$\text{Power Margin } (u\omega_i) = 10 \log_{10} \left[P_{sr}/P_{min}(u\omega_i)\right] \text{ decibels.} \quad (31)$$

IV. DOWN-LINK CHANNELS

Modulating the phase of the down-link carrier are two classes of signals -- the signal representing the composite base-band spectrum demodulated from the up-link carrier and the signals representing the mission data generated in the spacecraft. The down-link signal and noise that is received by the ground station receiver is therefore assumed to be described by

$$S_d(t) + n_d(t) = \sqrt{2P_{gr}} \sin \left[\omega_{dc}t + R_g \sin \omega_x t \right. \quad (32)$$

$$+ m_3 \sin \omega_3 t$$

$$\left. + m_4 \sin \omega_4 t + m_5 + n_r \right] + n_d(t),$$

for

P_{gr} = Total signal power in the receiver IF in watts

ω_{dc} = Down-carrier frequency in radians/second

$$R_g = \left[\sqrt{2}L_{uv}(B/2)G(S/N)^{1/2}_{IFu}\right] / \left[(1/k_{uv}) + (S/N)_{IFu}\right]^{1/2}$$ = Gain of the video channel in the transponder used to retransmit the up-link spectrum (see Equation 22).

G = Combined gain of the transponder IF and modulator

$\omega_x t = m_1 \sin \omega_1 t + m_2 \sin \omega_2 t + \Theta$ = up-link base-band signal

$\omega_{3,4}$ = Up-subcarrier frequencies in radians/second

$m_{3,4}$ = Up-subcarrier peak modulation indices in radians

m_5 = Effective peak modulation index for back-up voice modulated directly on the carrier

n_r = Thermal noise detected by spacecraft receiver and retransmitted to ground station

$n_d(t)$ = Thermal noise added to down-link signal by ground station receiver.

4.1 Down-Carrier Circuit Margins

Required signal to noise power ratios $(S/N)_{dcr}$ are defined for the carrier channel at the input to the carrier tracking loop of the receiver shown in Figure 4. To derive an expression for the down-carrier circuit margin, it is necessary to determine which portion of the signal and noise expressed by (32) is passed by the carrier channel band-pass limiter and a tracking loop filter with a bandwidth of B_{dc} Hz.

The signal expressed in (32) is equivalent to

$$S_d(t) = \sqrt{2P_{gr}}\ Im\left\{exp\left[j(\omega_{dc}t + x + \lambda)\right]\right\}$$

$$x = R_g \sin \omega_x t = R_g \sin\left[m_1\sin\omega_1 t + m_2\sin\omega_2 t + \theta\right]$$

$$\lambda = m_3 \sin \omega_3 t + m_4 \sin \omega_4 t + m_5 + n_r$$

then

$$S_d(t) = \sqrt{2P_{gr}}\ Im\left\{exp\left[j(\omega_{dc}t+m_5+n_r)\right] \sum_{N=-\infty}^{\infty} J_N(R_g)\exp(jN\omega_x t)f(\omega_{3,4})\right\} \quad (34)$$

$$= \sqrt{2P_{gr}}\ Im\left\{exp\left[j(\omega_{dc}t+m_5+n_r)\right]f(\omega_x)f(\omega_{3,4})\right\} \quad (35)$$

given that

$$f(\omega_x) = \sum_{N=-\infty}^{\infty} J_N(R_g)\exp(jN\theta)\ f_N(\omega_1,\omega_2)$$

$$f_N(\omega_1,\omega_2) = \sum_{N_1=-\infty}^{\infty} J_{N_1}(Nm_1)\exp(jN_1\omega_1 t) \sum_{N_2=-\infty}^{\infty} J_{N_2}(Nm_2)\exp(jN_2\omega_2 t)$$

$$f(\omega_{3,4}) = \sum_{N_3=-\infty}^{\infty} J_{N_3}(m_3)\exp(jN_3\omega_3 t) \sum_{N_4=-\infty}^{\infty} J_{N_4}(m_4)\exp(jN_4\omega_4 t)$$

Setting $N_1 = N_2 = N_3 = N_4 = 0$ gives an expression for the narrow-band component of the above expression that is centered about ω_{dc}. This reduces (35) to

$$S_{dn}(t) = \sqrt{2P_{gr}}\left\{\sum_{N=-\infty}^{\infty} J_N(R_g)J_o(Nm_1)J_o(Nm_2)J_o(m_3)J_o(m_4)\right. \quad (36)$$

$$\left.\left[\sin(\omega_{dc}t+N\theta+m_5+n_r)\right]\right\}$$

A method for approximating the retransmitted noise (n_r) as an additional subcarrier is given in References 13 and 14; however, for the Apollo cases the noise power calculated using this method is not significant when compared to the noise power $(K_o T_d)$ added to the signal in the ground receiver. The noise -- n_r -- will therefore be neglected here.

Expanding the sine term of (36) gives

$$S_{dn}(t) = \sqrt{2P_{gr}} \sum_{N=-\infty}^{\infty} J_N(R_g) J_o(Nm_1) J_o(Nm_2) J_o(m_3) J_o(m_4)$$

$$\begin{aligned}
& \overset{\text{(carrier)}}{[\sin \omega_{dc} t \cos N\theta \cos m_5} \\[6pt]
&+ \overset{\text{(ranging code)}}{\cos \omega_{dc} t \sin N\theta \cos m_5} \\[6pt]
&+ \overset{\text{(back-up voice)}}{\cos \omega_{dc} t \cos N\theta \sin m_5} \\[6pt]
&- \overset{\text{(code x voice)}}{\sin \omega_{dc} t \sin N\theta \sin m_5]}
\end{aligned} \qquad (37)$$

The respective signal components can be identified as shown in the parentheses above each term.

Using the identities

$$J_{-x}(m) = (-1)^x J_x(m)$$

$$J_x(-m) = (-1)^x J_x(m) \qquad (38)$$

$$\cos (\pm x) = \cos x$$

and

$$J_o(0) = 1$$

the first term of (37) can be expressed by

$$S_{dc}(t) = \sqrt{2P_{gr}}\, J_o(m_3)J_o(m_4) \left[J_o(R_g) + f_N(R_g) \right] \cos m_5 \sin \omega_{dc}t \qquad (39)$$

for

$$f_N(R_g) = 2 \sum_{N=1}^{\infty} J_{2N}(R_g)J_o(2Nm_1)J_o(2Nm_2) \cos 2N\theta$$

For the Apollo cases the series terms can be neglected; thus (39) reduces to

$$S_{dc}(t) \simeq \sqrt{2P_{gr}}\, J_o(R_g)J_o(m_3)J_o(m_4) \cos m_5 \sin \omega_{dc}t \qquad (40)$$

Having defined an expression for the carrier signal component at the input to the receiver, it is now necessary to determine how much this signal is attenuated by the carrier channel bandpass limiter.

Since the signal power at the input to the limiter is P_{gr} and a flat thermal noise power spectrum is assumed across the IF bandwidth B_g, the limiter input signal to noise power ratio is

$$(S/N)_{IFd} = P_{gr} / \left[B_g K_o T_d \right] \qquad (41)$$

T_d = Noise temperature referred to
    ground station receiver input
    in degrees Kelvin.

From (6), (40) and (41) the carrier signal at the limiter output is

$$S_{dc\ell}(t)=\sqrt{2}\ L_{dc}f\ k_{dc},(S/N)_{IFd}\ J_o(R_g)J_c(m_3)J_o(m_4)\cos\ m_5\sin\omega_{dc}t \qquad (42)$$

for

$$f\ k_{dc},\ (S/N)_{IFd} = \left\{(S/N)_{IFd}\Big/(1/k_{dc}) + (S/N)_{IFd}\right\}^{1/2}$$

k_{dc} is evaluated from $(S/N)_{IFd}$ and Figure 5

From 4 and (41) the noise power at the limiter output is

$$P_{n(dc)} = \left\{L_{dc}^2\Big/\left[k_{dc}(S/N)_{IFd} + 1\right]\right\}\left\{B_{dc}\Big/B_g\right\} \qquad (43)$$

Then the desired expression for the predicted signal to noise power ratio at the input to the carrier tracking loop is obtained from (42) and (43) to be

$$(S/N)_{dcp} = k_{dc}\ P_{gr}\ \alpha_{dc}\Big/B_{dc}\ K_o\ T_d \qquad (44)$$

$$\alpha_{dc} = J_o^2(R_g)J_o^2(m_3)J_o^2(m_4)\ \cos\ m_5$$

And finally the down-carrier circuit margin is expressed by

$$\text{Circuit Margin (dc)} = 10\ \log_{10}\ \left[(S/N)_{dcp}\right] - (S/N)_{dcr}\ \text{decibels} \qquad (45)$$

4.2 Down-Carrier Power Margin

Setting (45) equal to zero and defining

$$T_d = A_d + B_d \, P_{gr} \, \alpha_{dc}$$

$$10 \, \log_{10} \, (\gamma_{dc}) = (S/N)_{dcr}$$

$$P_{min}(dc) = P_{gr}\Big|_{\text{circuit margin} = 0}$$

gives the following expression for $P_{min}(dc)$

$$P_{min}(dc) = A_d B_{dc} \gamma_{dc} K_o / \alpha_{dc} (k_{dc} - B_d B_{dc} \gamma_{dc} K_o) \tag{46}$$

After a solution for (46) is obtained by iteration, the down-carrier power margin is calculated directly from

$$\text{Power margin (dc)} = 10 \, \log_{10} \left[P_{gr} / P_{min}(dc) \right] \quad \text{decibels} \tag{47}$$

4.3 Down-Subcarrier Circuit-Margins

Required signal to noise power ratios $(S/N)_{d\omega_k r}$ are defined for the subcarrier channels at the input to the respective subcarrier demodulators as shown in Figure 4. To derive an expression for the down-subcarrier circuit margin, it is necessary to determine which portion of the signal and noise expressed by

(32) is passed by the cascade comprised of the video-channel bandpass limiter, the wide-band product detector, and the subcarrier demodulator input bandpass filter - $B(\omega_k)$.

Returning to (35) and rearranging the terms the total signal at the input to the ground receiver can be expressed by

$$S_d(t) = \sqrt{2P_{gr}}\ Im\left\{f(N,\omega_1,\omega_2)\,f(\omega_3)\,f(\omega_4)\right\}$$

for

$$f(N,\omega_1,\omega_2) = \sum_{N=-\infty}^{\infty}\left[J_N(R_g)\sum_{N_1=-\infty}^{\infty}J_{N_1}(Nm_1)\sum_{N_2=-\infty}^{\infty}J_{N_2}(Nm_2)\exp(jZ)\right] \quad (48)$$

$$Z = \omega_{dc}t + m_5 + N\theta + N_1\omega_1 t + N_2\omega_2 t$$

$$f(\omega_3)=J_o(m_3)+2\sum_{N_3=1}^{\infty}\left[J_{2N_3}(m_3)\cos 2N_3\omega_3 t+jJ_{(2N_3-1)}(m_3)\sin(2N_3-1)\omega_3 t\right]$$

$$f(\omega_4)=J_o(m_4)+2\sum_{N_4=1}^{\infty}\left[J_{2N_4}(m_4)\cos 2N_4\omega_4 t+jJ_{(2N_4-1)}(m_4)\sin(2N_4-1)\omega_4 t\right]$$

Neglecting the higher order and cross frequency terms and the terms resulting from the ω_1 and ω_2 up-subcarriers $(N_1=N_2=0)$, which lie outside the subcarrier demodulator input bandwidths, Equation (48) reduces to

$$S_{dv}(t) = \sqrt{2P_{gr}} \sum_{N=-\infty}^{\infty} \left\{ J_N(R_g) J_o(Nm_1) J_o(Nm_2) \right. \tag{49}$$

$$\left[J_o(m_3) J_o(m_4) \sin(\omega_{dc}t + N\theta + m_5) \right.$$

$$+ 2J_1(m_3) J_o(m_4) \cos(\omega_{dc}t + N\theta + m_5) \sin \omega_3 t$$

$$\left. \left. + 2J_o(m_3) J_1(m_4) \cos(\omega_{dc}t + N\theta + m_5) \sin \omega_4 t \right] \right\}$$

The second and third terms of (49) contain the signal components of the ω_3 and ω_4 subcarriers, respectively; the first term contains the signal components of both the ranging code and the back-up voice.

From (41) the input signal to noise power ratio of the video-channel limiter is given by

$$(S/N)_{IFd} = P_{gr} / B_g K_o T_d \tag{41}$$

and consequently the signal is attenuated in the limiter by the factor

$$f\left[k_{dv}, (S/N)_{IFd}\right] = \left\{ (S/N)_{IFd} / \left[(1/k_{dv}) + (S/N)_{IFd}\right] \right\}^{1/2} \tag{50}$$

k_{dv} is evaluated from $(S/N)_{IFd}$ and Figure 5.

In the product detector the input signal is multiplied by the derived carrier $B \cos \omega_{dc}t$, and the resulting sum frequencies are removed by filtering. After detection the signal is expressed by

$$S_{dd}(t) = \sqrt{2}L_{dv}\,\frac{B}{2}\,f\,k_{dv}\cdot (S/N)_{IFd}\,\sum_{N=-\infty}^{\infty}\,J_N(R_g)\,J_o(Nm_1)\,J_o(Nm_2)\,f_{dd}(t)$$

for

$$f_{dd}(t) = J_o(m_3)\,J_o(m_4)\,\sin(N\theta+m_5) \tag{51}$$

$$+2J_1(m_3)\,J_o(m_4)\,\cos(N\theta+m_5)\,\sin\omega_3 t$$

$$+2J_o(m_3)\,J_1(m_4)\,\cos(N\theta+m_5)\,\sin\omega_4 t$$

Expanding the cosine term of (51) gives

$$\cos(N\theta + m_5) = \cos N\theta \cos m_5 - \sin N\theta \sin m_5$$

The first term of this expansion represents a desired signal term, and the second represents a negligible second order interference signal consisting of a cross-product between the ranging code and back-up voice signals.

Using (38), (50) and (51), the general expression for the down-subcarrier signal power is

$$P_s(\omega_k) = \left\{(L_{dv}B)^2/2\right\}\left\{(S/N)_{IFd}\Big/\left[(1/k_{dv}) + (S/N)_{IFd}\right]\right\}\alpha_{d\omega_k} \quad \text{watts}$$

for

$$\alpha_{d\omega_k} = 2J_o^2(R_g)\,J_1^2(m_k)\,J_o^2(m_\ell)\cos^2 m_5 \tag{52}$$

$$k = 3 \text{ or } 4$$
$$\ell = 3 \text{ or } 4 \qquad k \neq \ell$$

Assuming a flat thermal noise spectrum across the low pass one-sided bandwidth of $B_{gr}/2$, the noise power in the input bandwidth $B(\omega_k)$ of the subcarrier detector is

$$P_n(\omega_k) = \left\{ (L_{dv}B/2 \Big/ \left[1+k_{dv}(S/N)_{IFd} \right]^{1/2} \right\}^2 \left\{ 2B(\omega_k)/B_g \right\} \quad \text{watts} \quad (53)$$

Then the predicted signal to noise power ratio for the down-subcarrier is

$$(S/N)_{d\omega_k p} = P_s(\omega_k) \Big/ P_n(\omega_k) \tag{54}$$

$$= k_{dv}P_{gr}\alpha_{d\omega_k} \Big/ \left[B(\omega_k)K_o T_d \right]$$

and consequently the expression for the predicted down-subcarrier circuit margin is

$$\tag{55}$$
$$\text{Circuit Margin } (d\omega_k) = 10 \log_{10} \left[(S/N)_{d\omega_k p} \right] - (S/N)_{d\omega_k r} \quad \text{decibels.}$$

4.4   Down-Subcarrier Power Margins

Setting (55) equal to zero and defining

$$T_d = A_d + B_d\, P_{gr}\, \alpha_{dc}$$

$$10 \log_{10} (\gamma_{d\omega_k}) = (S/N)_{d\omega_k r}$$

$$P_{min}(d\omega_k) = P_{gr}\Big|_{\text{circuit margin} = 0}$$

gives the following expression for $P_{min}(d\omega_k)$

$$P_{min}(d\omega_k) = A_d B(\omega_k) \gamma_{d\omega_k} K_o / k_{dv} \alpha_{d\omega_k} - \alpha_{dc}(B_d B(\omega_k) \gamma_{d\omega_k} K_o) \qquad (56)$$

After a solution for (56) has been obtained by iteration, the down-subcarrier power margin is calculated directly from

$$\text{Power Margin } (d\omega_k) = 10 \log_{10} P_{gr}/P_{min}(d\omega_k) \quad \text{decibels.} \qquad (57)$$

4.5 Ranging Code Circuit Margin

Required performance is specified for the ranging receiver in a slightly different manner than it is for the carrier and subcarrier channels that have been discussed so far. An adaptation of a maximum likelihood detection process has been implemented in the ranging receiver. The specified performance of this receiver is based on the ratio of available ranging code signal power to noise power density $(S/N)_{dcor}$ at the input to the ranging code receiver as shown in Figure 4. To derive an expression for the ranging code circuit margin, it is necessary now only to determine the portion of the IF signal (32) which comprises the ranging signal. The noise power density is simply $K_o T_d$.

Returning to the second term of (37), the first order portion of the ranging code signal is defined by

$$S_{dco}(t) = \sqrt{2P_{gr}} \sum_{N=-\infty}^{\infty} \{ J_N(R_g) J_o(Nm_1) J_o(Nm_2) J_o(m_3) J_o(m_4) \qquad (58)$$

$$\sin N\theta \cos m_5 \cos \omega_{dc} t \}$$

Using the identities

$$J_{-x}(m) = (-1)^x J_x(m)$$

$$J_x(-m) = (-1)^x J_x(m)$$

$$\sin(\pm x) = \pm\sin(x)$$

$$J_o(0) = 1$$

$$\sin(0) = 0 \tag{59}$$

the ranging code signal power can be expressed by

$$P_{dco} = P_{gr}J_o^2(m_3)J_o^2(m_4)\left[2\sum_{N=1}^{\infty}\left\{J_{2N-1}(R_g)J_o\left[(2N-1)m_1\right]\right.\right. \tag{60}$$

$$\left.\left.J_o\left[(2N-1)m_2\right]\cdot\sin(2N-1)\theta\right\}\right]^2\cos^2 m_5$$

For the Apollo cases the higher order terms can be neglected; thus (60) reduces to

$$P_{dco} = P_{gr}\alpha_{dco}$$

for

$$\alpha_{dco} = 4J_1^2(R_g)\ J_o^2(m_1)J_o^2(m_2)\ J_o^2(m_3)\ J_o^2(m_4)\ \cos^2 m_5\ \sin^2\theta$$

Thus the predicted input signal power to noise power density ratio of the ranging receiver is

$$(S/N)_{dcop} = P_{gr}\alpha_{dco}/K_oT_d \qquad (62)$$

and the desired expression for the circuit margin of the ranging channel is

$$(63)$$

$$\text{Circuit Margin (dco)} = 10 \log_{10} \left[(S/N)_{dcop}\right] - (S/N)_{dcor} \text{ decibels.}$$

4.6 Ranging Code Power Margin

Setting (63) equal to zero and defining

$$T_d = A_d + B_d P_{gr} \alpha_{dc}$$

$$10 \log_{10} (\gamma_{dco}) = (S/N)_{dcor}$$

$$P_{min}(dco) = P_{gr}\Big|_{\text{circuit margin} = 0}$$

gives the following expression for $P_{min}(dco)$

$$P_{min}(dco) = A_d\gamma_{dco}K_o/\left[\alpha_{dco} - \alpha_{dc}B_d\gamma_{dco}K_o\right] \qquad (64)$$

Since there is no limiter factor in the above expression, $P_{min}(dco)$ can be calculated directly without requiring an iteration process. Power margin for the ranging system then is defined by

Power Margin (dco) = $10 \log_{10} \left[P_{gr}/P_{min} (dco) \right]$ decibels   (65)

4.7 Back-Up Voice Circuit Margins

Detection of the back-up voice signal is performed by filtering with a low pass filter the output of the multiplier in the carrier tracking phase-lock loop of the ground station receiver. Required signal to noise power ratios, $(S/N)_{bvr}$, for this back-up voice channel are defined at the output of the low pass filter as shown in Figure 4. To derive an expression for the back-up voice circuit margin, it is necessary to determine which portion of the signal and noise expressed by (32) is passed by a cascade comprised of the carrier channel bandpass limiter, carrier tracking phase-lock loop multiplier, and low pass filter (B_{bv}).

Returning to (37) and (38), the back-up voice signal at the input to the limiter is expressed by

$$S_{bv}(t) = \sqrt{2P_{gr}} \; J_o(R_g) \; J_o(m_3) \; J_o(m_4) \; \sin m_5 \; \cos \omega_{dc}t \qquad (66)$$

From (6) and (41) this signal is attenuated in the limiter by the factor

$$f\left[k_{dc}, (S/N)_{IFd}\right] = \left\{ (S/N)_{IFd} \Big/ \left[(1/k_{dc}) + (S/N)_{IFd}\right] \right\}^{1/2} \qquad (67)$$

k_{dc} is evaluated from $(S/N)_{IFd}$ and Figure 5.

In the multiplier the input signal is multiplied by the derived carrier - B cos $\omega_{dc}t$, which after low-pass filtering gives a back-up voice signal of

$$S_{dv\ell}(t) = \left\{ \sqrt{2}L_{dc} \; B/2 \right\} f\left[k_{dc}, (S/N)_{IFd} \; J_o(R_g)J_o(m_3)J_o(m_4)\sin m_5 \right. \tag{68}$$

For an output low pass filter bandwidth of B_{bv} and an input one-sided low pass IF bandwidth of $B_g/2$, the noise power to be considered here is

$$P_n(bv) = \left\{ (L_{dc}B/2)/\left[1+k_{dc}(S/N)_{IFd} \right]^{1/2} \right\}^2 \; 2B_{bv}/B_g \tag{69}$$

Then the expression for the predicted signal to noise power ratio for the back-up voice channel is

$$(S/N)_{bvp} = k_{dc}P_{gr}\alpha_{bv}/\left[B_{bv}K_oT_d \right] \tag{70}$$

for

$$\alpha_{bv} = J_o(R_g)J_o(m_3)J_o(m_4)\sin m_5$$

and consequently the expression for the predicted circuit margin of the voice channel is

$$\text{Circuit Margin (bv)} = 10 \log_{10}\left[(S/N)_{bvp} \right] - (S/N)_{bvr} \text{ decibels.} \tag{71}$$

4.8 Back-up Voice Power Margin

Setting (71) equal to zero and defining

$$T_d = A_d + B_d \, P_{gr} \, \alpha_{dc}$$

$$10 \, \log_{10} \, (\gamma_{bv}) = (S/N)_{bvr}$$

$$P_{min}(bv) = P_{gr}\Big|_{circuit \; margin \; = \; 0}$$

gives the following expression for $P_{min}(bv)$

$$P_{min}(bv) = \frac{A_d \; B_{bv} \; \gamma_{bv} \; K_o}{\alpha_{bv} k_{dc} - \alpha_{dc} \; B_d \; B_{dv} \; \gamma_{bv} \; K_o} \quad . \tag{72}$$

After a solution for (72) has been obtained by iteration, the back-up voice power margin is calculated directly from

$$\text{Power Margin (bv)} = 10 \, \log_{10} \left[P_{gr} \Big/ P_{min}(bv) \right] \quad \text{decibels .} \tag{73}$$

V. MAXIMUM RANGE CALCULATIONS

Range is a factor used in calculating the total receiver power expected at the input to a receiver as shown by (7). Therefore, if the signal power (P_o) is known at a given range (R_o), then the signal power (P_r) at any range (R) is given by

$$P_r = P_o + 20 \, \log_{10} \, (R_o) - 20 \, \log_{10}(R) \quad . \tag{74}$$

Solving this expression for R gives

$$R = \text{Antilog}_{10} \left\{ (P_r - P_o - 20 \log_{10} R_o)/20 \right\} \quad . \tag{75}$$

For the case where P_r equals the minimum required signal power $P_{min}(x)$, calculated from the expressions in the last section, R equals the maximum permissible range of a vehicle. The maximum range for which non-negative circuit margins can be expected for channel x_i, is then expressed by

$$\tag{76}$$

$$R_{max} = \text{Antilog}_{10} \left\{ \left[P_{min}(x_i) - P_o(\text{at range } R_o) - 20 \log_{10} R_o \right] / 20 \right\} \quad .$$

$P_{min}(x_i)$ and P_o must both be expressed in decibels using a common reference level such as dBW or dBm.

VI. USING THE MODEL

Communications performance predictions have been calculated at Bellcomm, Inc. for the Apollo spacecraft using the MARGINS computer program. This program, written in FORTRAN V for a UNIVAC 1108 computer, provides a capability for calculating the three performance measures discussed above and also provides a capability for storing the S-Band system parameters used in these calculations.

A summary of the down-link performance expected on the APOLLO 14 mission is given in Tables I and II which have been taken from reference 15. The two spacecraft antennas, HGA(N) and Omni, listed in the tables are respectively the narrow beam and wide beam antennas of the CSM. Figure 2

shows the possible signals that can be placed on the down-link
carrier, but these need not all be present simultaneously. The
term - FULL MODE - in the tables refers to the case where the
down-link carrier is modulated simultaneously by a voice sub-
carrier, a telemetry subcarrier, and the ranging code. The
S-Band system provides a capability for transmitting telemetry
data at two bit rates. The high bit rate is 51.2 kilo-bits/
second and the low bit rate is 1.6 kilo-bits/second; the ex-
pected performance at each bit rate is shown in the tables.

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2034-NWS-tls

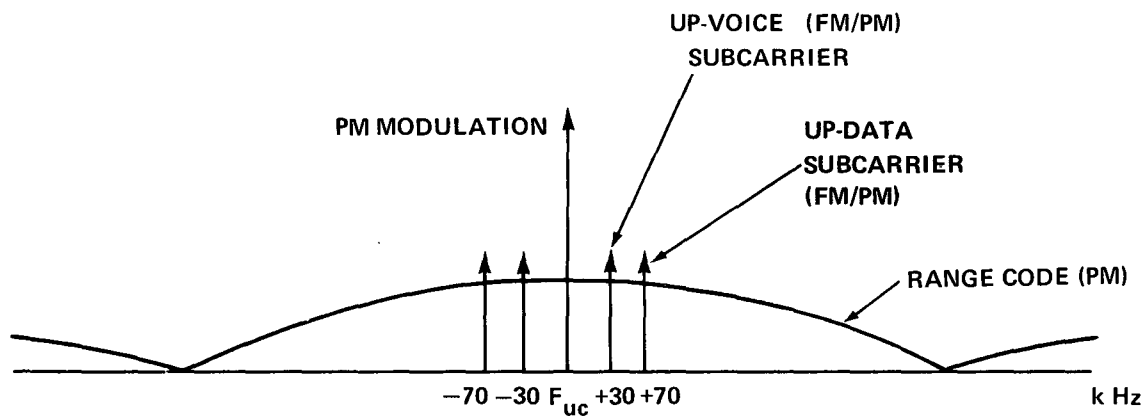Attachments
Figures 1 - 6
Tables I and II

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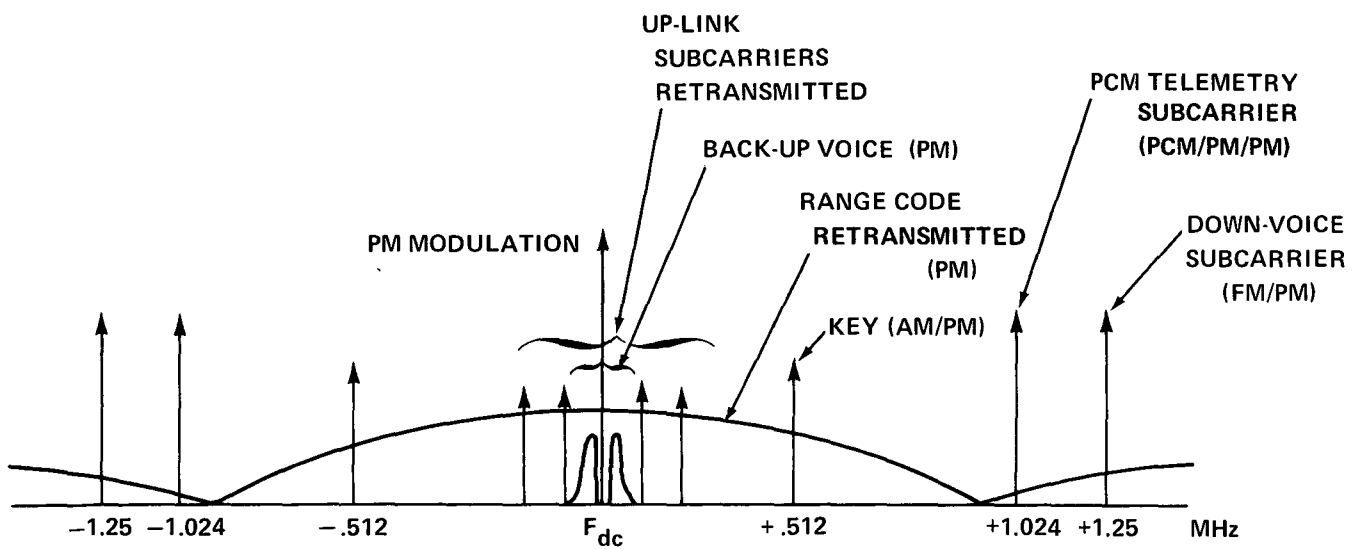   North American Rockwell Corporation, SID 64-1613B.

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PM MODULATION

UP-VOICE (FM/PM)
SUBCARRIER

UP-DATA
SUBCARRIER
(FM/PM)

RANGE CODE (PM)

−70 −30 F_{uc} +30 +70   k Hz

$F_{uc} = \begin{cases} 2106.4 \text{ MHz FOR COMMAND AND SERVICE MODULE (CSM)} \\ 2101.8 \text{ MHz FOR LUNAR MODULE (LM) AND LAUNCH VEHICLE (S-IVB)} \end{cases}$

FIGURE 1 - UP-LINK SIGNAL SPECTRUM

UP-LINK
SUBCARRIERS
RETRANSMITTED

PCM TELEMETRY
SUBCARRIER
(PCM/PM/PM)

BACK-UP VOICE (PM)

RANGE CODE
RETRANSMITTED
(PM)

DOWN-VOICE
SUBCARRIER
(FM/PM)

PM MODULATION

KEY (AM/PM)

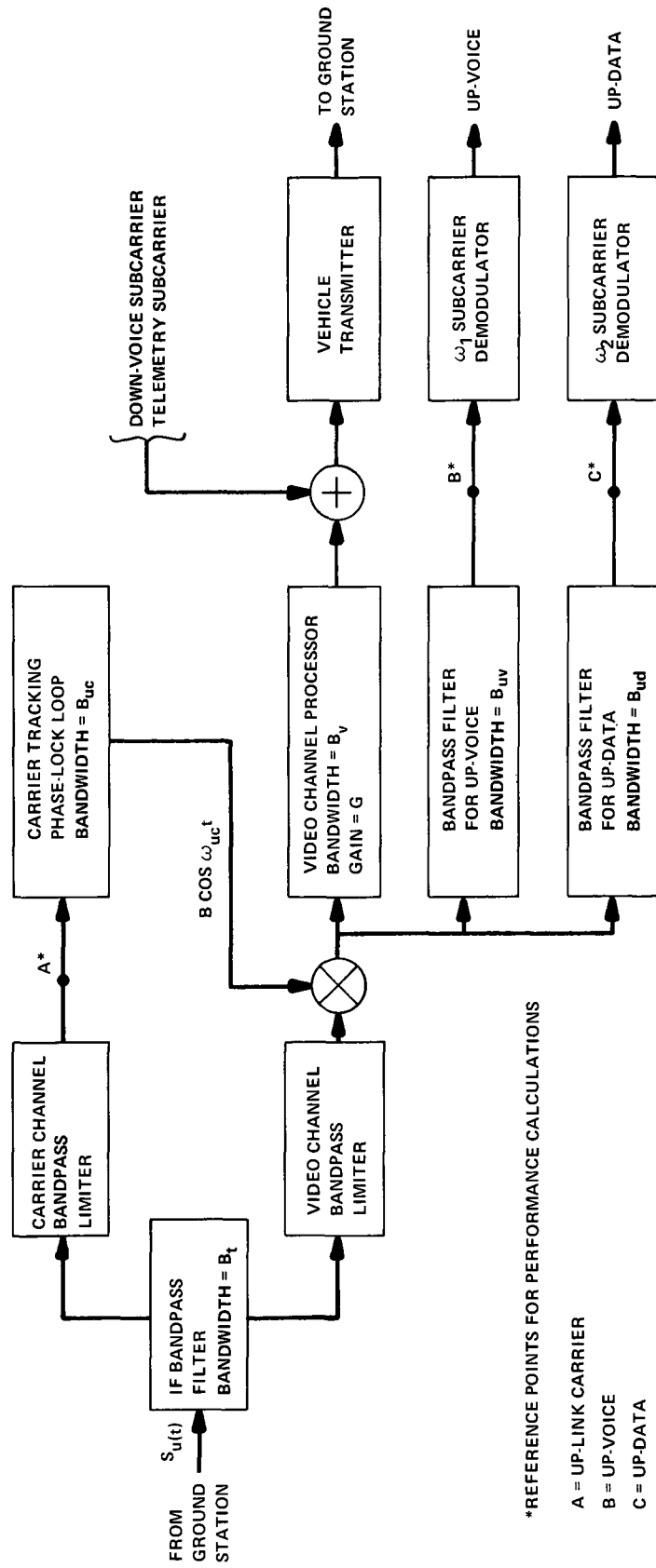−1.25 −1.024   −.512 F_{dc} + .512   +1.024 +1.25   MHz

$F_{dc} = \begin{cases} 2287.5 \text{ MHz FOR COMMAND AND SERVICE MODULE (CSM)} \\ 2282.5 \text{ MHz FOR LUNAR MODULE (LM) AND LAUNCH VEHICLE (S-IVB)} \end{cases}$

FIGURE 2 - DOWN-LINK SIGNAL SPECTRUM

FIGURE 3 - SPACECRAFT TRANSPONDER

FROM GROUND STATION

$S_{u(t)}$

IF BANDPASS FILTER BANDWIDTH = B_t

CARRIER CHANNEL BANDPASS LIMITER

VIDEO CHANNEL BANDPASS LIMITER

CARRIER TRACKING PHASE-LOCK LOOP BANDWIDTH = B_{uc}

A^*

$B \cos \omega_{uc} t$

VIDEO CHANNEL PROCESSOR BANDWIDTH = B_v GAIN = G

BANDPASS FILTER FOR UP-VOICE BANDWIDTH = B_{uv}

BANDPASS FILTER FOR UP-DATA BANDWIDTH = B_{ud}

DOWN-VOICE SUBCARRIER TELEMETRY SUBCARRIER

VEHICLE TRANSMITTER

TO GROUND STATION

ω_1 SUBCARRIER DEMODULATOR

UP-VOICE

B^*

ω_2 SUBCARRIER DEMODULATOR

UP-DATA

C^*

*REFERENCE POINTS FOR PERFORMANCE CALCULATIONS

A = UP-LINK CARRIER

B = UP-VOICE

C = UP-DATA

FIGURE 4 - GROUND STATION RECEIVER

* REFERENCE POINTS FOR PERFORMANCE CALCULATIONS

A = DOWN-LINK CARRIER
B = DOWN-VOICE
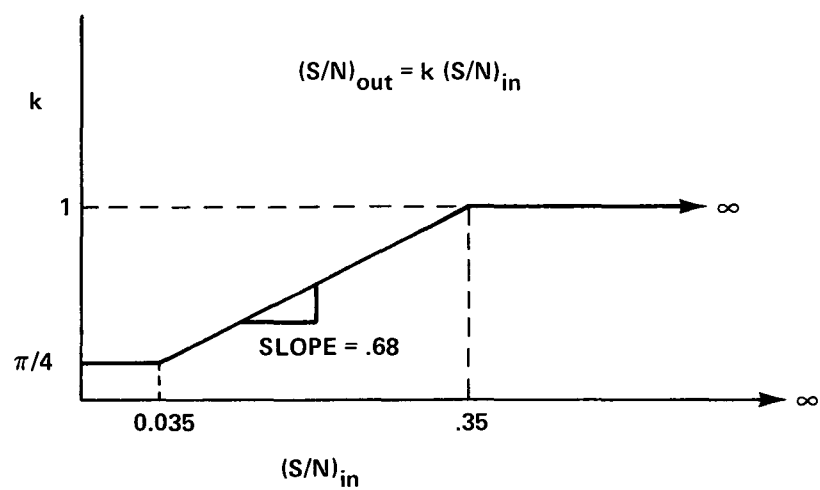C = TELEMETRY
D = RANGING CODE
E = BACK-UP VOICE

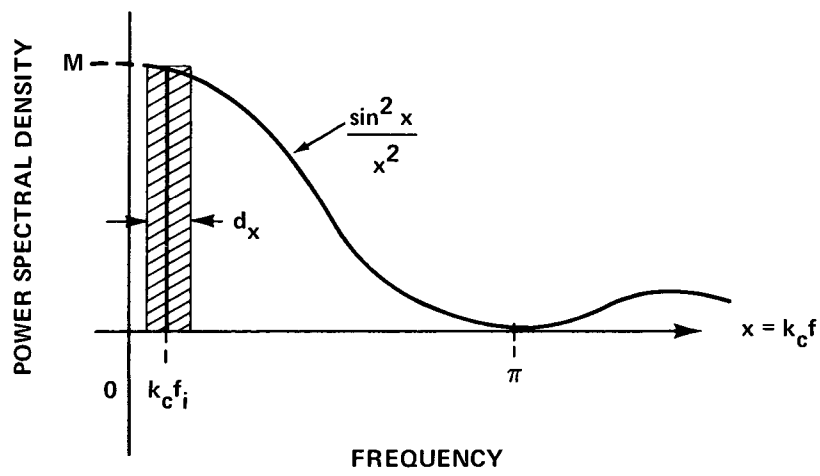FIGURE 5 - APPROXIMATION OF BANDPASS LIMITER $(S/N)_{out}$ CHARACTERISTIC

FIGURE 6 - APPROXIMATION OF RANGING CODE POWER SPECTRUM

TABLE I (PHASE MODULATED MODE)
SUMMARY OF MAXIMUM RANGES† FOR COMMUNICATIONS
FROM A CSM

SPACECRAFT ANTENNA / GROUND STATION ANTENNA	STATE OF POWER AMP	VOICE & BIOMED DATA			TELEMETRY (BER = 10^{-6})		
		FULL MODE HBR	WITH TELEMETRY		FULL MODE HBR	WITH VOICE	
			HBR	LBR		HBR	LBR
P_{min} (dBW) →		−141.7	−142.2	−146.4	−138.4	−139.0	−150.0
P_r (dBW) →							
HGA (N)/85'	OFF	117,000	123,000	198,000	81,000	85,000	+
HGA (N)/85'	ON	+	+	+	+	+	+
HGA (N)/210'	OFF	+	+	+	203,000	213,000	+
HGA (N)/210'	ON	+	+	+	+	+	+
OMNI*/85'	OFF	9,000	9,000	15,000	6,000	6,000	22,000
OMNI*/85'	ON	89,000	93,000	150,000	61,000	64,000	+
OMNI*/210'	OFF	22,000	23,000	38,000	15,000	16,000	56,000
OMNI*/210'	ON	+	+	+	154,000	161,000	+

P_r (dBW) values for rows: −147.1, −127.1, −139.1, −119.1, −169.5, −149.5, −161.5, −141.5

† RANGES GIVEN IN NAUTICAL MILES

* CSM OMNI ANTENNA GAINS = 0 db

+ MAXIMUM RANGE EXCEEDS 215,000 NAUTICAL MILES

TABLE II (PHASE MODULATION MODE)
SUMMARY OF POWER MARGINS† FOR COMMUNICATIONS FROM A CSM AT LUNAR RANGE (R = 215,000 N. MI.)

SPACECRAFT ANTENNA / GROUND STATION ANTENNA	STATE OF POWER AMP	VOICE & BIOMED DATA			TELEMETRY (BER = 10⁻⁶)		
		FULL MODE HBR	WITH TELEMETRY		FULL MODE HBR	WITH VOICE	
			HBR	LBR		HBR	LBR
P_{min} (dBW) →		−141.7	−142.2	−146.4	−138.4	−139.0	−150.0
	P_r (dBW) →						
HGA (N)/85'	OFF	− 5.4	− 4.9	− 0.7	− 8.7	− 8.1	+
HGA (N)/85'	ON	+	+	+	+	+	+
HGA (N)/210'	OFF	+	+	+	− 0.7	− 0.1	+
HGA (N)/210'	ON	+	+	+	+	+	+
OMNI/85'	OFF	−27.8	−27.3	−23.1	−31.1	−30.5	−19.5
OMNI/85'	ON	− 7.8	− 7.3	− 3.1*	−11.1	−10.5	+
OMNI/210'	OFF	−19.8	−19.3	−15.1	−23.1	−22.5	−11.5
OMNI/210'	ON	+	+	+	− 3.1*	− 2.5*	+

† POWER MARGINS ARE GIVEN IN DECIBELS

* MARGINS WOULD BE POSITIVE ASSUMING A + 5 db (BORE SIGHT) GAIN FOR THE CSM OMNI ANTENNAS

+ POWER MARGIN EXCEED ZERO ANTENNAS

Pr (dBW) →

Spacecraft/Ground	P_r (dBW)
HGA (N)/85'	−147.1
HGA (N)/85'	−127.1
HGA (N)/210'	−139.1
HGA (N)/210'	−119.1
OMNI/85'	−169.5
OMNI/85'	−149.5
OMNI/210'	−161.5
OMNI/210'	−141.5