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FINITE DIFFERENCE SOLUTION TO THE THREE-DIMENSIONAL,
INCOMPRESSIBLE THERMAL ENERGY BOUNDARY-LAYER EQUATION

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SUMMARY

An implicit numerical method has been adopted for the solution to the three-dimensional, energy boundary-layer equation. The energy equation is written in terms of a dimensionless temperature function, the relative stagnation-enthalpy difference, and transformed by the introduction of a Blasius-type transformation of coordinates as well as dimensionless stream functions. The method is applied to the problem consisting of an infinite cylinder joined with its axis perpendicular to a thin, flat, heated plate. A Prandtl number equal to one is simply considered in this report.

INTRODUCTION

In a previous report, NGR-22-010-019/4/71, we considered the application of a basically implicit finite difference method to the solution of the three-dimensional momentum boundary layer equations. In this report we extend the method to the solution of the three-dimensional thermal boundary layer equation.

As for the formulation of the basic equations, we consider steady, laminar flow of a viscous fluid; and for simplicity, the flow is assumed to be incompressible. The broadest possible interpretation of incompressible is adopted, so that low-speed flow of a gas as well as of a liquid is admitted. The requirement is that the momentum and continuity equations are uncoupled from the thermal energy equation. This is the situation if the Mach number is low and the relative variation in temperature small, so that the heat added by dissipation and by conduction is slight (see Ref.2). Consequently, the velocity and pressure fields are governed by the Navier-Stokes equations and on application of the boundary layer approximations subsequently determined from the boundary layer and Bernoulli equations. Having determined the velocity field, the temperature can be calculated from the thermal energy equation. Again we employ the boundary layer approximation to simplify the energy equation.

As in the previous report, a rectangular co-ordinate system is adopted. For numerical purposes a Blasius-type coordinate transformation is introduced so that the boundary layer is of nearly uniform thickness. In the difference approximations a rectangular mesh with equal elements

is adopted. Pivotal points for the computations are taken to be at the midpoints of the mesh. To approximate the dimensionless temperature functions, and derivatives, central differences are introduced. Since the energy equation has a second-order derivative, solution of the algebraic finite difference equations requires the inversion of a tri-diagonal matrix.

The test case which we have chosen is shown in Fig. (1). The geometries of the bodies in the figure consist of an infinite cylinder joined with its axis perpendicular to a thin flat heated plate. The velocity field was determined in Ref. (1). For simplicity a uniform, constant plate temperature is assumed.

BOUNDARY-LAYER EQUATIONS AND BOUNDARY CONDITIONS

In rectangular coordinates (x,y,z) the equations of motion and that governing the thermal field for a steady, laminar, incompressible and heat-conducting three-dimensional boundary layer over the surface $y = 0$ may be written as

$$u_x + v_y + w_z = 0 \quad (1)$$

$$uu_x + vu_y + wu_z = -1/\rho p_x + \nu u_{yy} \quad (2)$$

$$uw_x + vw_y + ww_z = 1/\rho p_z + \nu w_{yy} \quad (3)$$

$$uT_x + vT_y + wT_z = \alpha T_{yy} + 1/\rho c_p (up_x + vp_y) + u/\rho c_p (u_y^2 + w_y^2) \quad (4)$$

An alternative form of the energy equation, Eq. (4), in terms of stagnation enthalpy is

$$u \left[T + 1/2 c_p (u^2 + w^2) \right]_x + v \left[T + 1/2 c_p (u^2 + w^2) \right]_y + w \left[T + 1/2 c_p (u^2 + w^2) \right]_z =$$

$$v \left[\sigma^{-1} T + 1/2 c_p (u^2 + w^2) \right]_{yy} \quad (5)$$

This form of the energy equation proves to be more convenient in handling the outer boundary condition on the temperature.

(All symbols are defined in the section following the References).

The boundary conditions at the surface, and at the outer edge of the boundary layer are

$$y = 0 : u = v = w = 0, T = T_w(x, z), T_y = 0 \text{ or prescribed heat flux} \quad (6a)$$

$$y \rightarrow \infty : u \rightarrow U(x, z), w \rightarrow W(x, z), T \rightarrow H_1(0) - 1/2 \rho c_p U^2(x, z) \quad (6b)$$

The pressure gradients, p_x and p_z are related to the inviscid velocity components, U and W , via the Euler equations:

$$- 1/\rho p_x = UU_x + WW_z \quad (7a)$$

$$- 1/\rho p_z = UW_x + WW_z \quad (7b)$$

Initial conditions will be discussed in a later section.

COORDINATE TRANSFORMATION

Introducing two stream functions, ψ and ϕ , such that the continuity equation is identically satisfied, as well as a dimensionless temperature function (relative stagnation-enthalpy difference), S , where

$$u \equiv \psi_y, v \equiv -(\psi_x + \phi_z), w \equiv \phi_y \quad (8)$$

$$S \equiv \frac{T + 1/2 c_p (u^2 + w^2)}{H_1(0)} - 1 \quad (9)$$

Eqs. (1-3) and (5) become

$$\psi_y \psi_{xy} - (\psi_x + \phi_z) \psi_{yy} + \phi_y \psi_{zy} = -1/\rho p_x + \nu \psi_{yy} \quad (10)$$

$$\psi_y \phi_{xy} - (\psi_x + \phi_z) \phi_{yy} + \phi_y \phi_{zy} = -1/\rho p_z + \nu \phi_{yy} \quad (11)$$

$$\psi_y S_x - (\psi_x + \phi_z) S_y + \phi_y S_z = \nu \left[\frac{1}{\sigma} S + 1/2 c_p H_1(0) \left(\frac{\sigma-1}{\sigma} \right) (u^2 + w^2) \right]_{yy} \quad (12)$$

In terms of the Blasius-type transformation, η , dimensionless stream functions f and g , and dimensionless temperature function, θ , where

$$\xi = x, \zeta = z, \eta = \left(\frac{U}{2\nu\xi} \right)^{1/2} y \quad (13)$$

$$\text{and } \psi \equiv (2\nu\xi U)^{1/2} f(\xi, \eta, \zeta), \phi \equiv W \left(\frac{2\nu\xi}{U} \right)^{1/2} g(\xi, \eta, \zeta) \quad (14)$$

$$S = S_w \theta(\xi, \eta, \zeta) \quad (15)$$

Eqs. (10)-(12) become

$$f_{\eta\eta\eta} + (1+\beta/2)ff_{\eta\eta} + \beta(1-f^2)_{\eta} + k(1-fg_{\eta}) + (\beta^*-k/2)gf_{\eta\eta} + 2\xi(f_{\xi}f_{\eta\eta} - f_{\eta}f_{\eta\xi}) \\ + 2\xi W/U (g_{\zeta}f_{\eta\eta} - g_{\eta}f_{\eta\zeta}) = 0 \quad (16)$$

$$g_{\eta\eta\eta} + (1+\beta/2)gg_{\eta\eta} - \beta(1-g^2)_{\eta} + k^*(1-fg_{\eta}) + (\beta^*-k/2)gg_{\eta\eta} + 2\xi(f_{\xi}g_{\eta\eta} - f_{\eta}g_{\eta\xi}) \\ + 2\xi W/U (g_{\zeta}g_{\eta\eta} - g_{\eta}g_{\eta\zeta}) = 0 \quad (17)$$

$$\theta_{\eta\eta} + \sigma[(1+\beta/2)f + (\beta^*-k/2)g]\theta_{\eta} + E/2(\sigma-1) \left[\left(\frac{U}{U_{\infty}} f_{\eta} \right)^2 + \left(\frac{W}{U_{\infty}} g_{\eta} \right)^2 \right]_{\eta\eta} \\ + 2\xi\sigma(f_{\xi}\theta_{\eta} - f_{\eta}\theta_{\xi}) + 2\xi W/U \sigma(g_{\zeta}\theta_{\eta} - g_{\eta}\theta_{\zeta}) = 0 \quad (18)$$

BOUNDARY AND INITIAL CONDITIONS

The boundary conditions at the surface, and at the outer edge of the boundary layer, in terms of the transformed variables, are

$$\eta = 0: f = f_\eta = f_\xi = 0; g = g_\eta = g_\zeta = 0; \theta = 1 \quad (19)$$

$$\eta \rightarrow \infty: f_\eta \rightarrow 1; g_\eta \rightarrow 1; \theta \rightarrow 0 \quad (20)$$

To complete the specification of the problem, initial conditions are necessary, i.e., conditions along $\xi = 0, \zeta \neq 0$ and $\xi \neq 0, \zeta = 0$. As for the velocity problem, considerable discussion is given to these conditions in Ref. (1) so that we shall simply write them down without further discussion. The temperature conditions are determined in a straight forward manner, again found as limiting solution to Eqs. (16) - (18).

$$A. \quad \xi = 0, \zeta \neq 0: f_{\eta\eta\eta} + ff_{\eta\eta} = 0 \quad (21)$$

$$g_{\eta\eta\eta} + fg_{\eta\eta} = 0 \quad (22)$$

$$\theta_{\eta\eta} + \sigma f \theta_\eta + E/2(\sigma-1) \left[(U/U_\infty f_\eta)^2 + (W/U_\infty g_\eta)^2 \right] \quad (23)$$

subject to boundary conditions

$$\eta = 0: f = f_\eta = 0; g = g_\eta = 0; \theta = 1 \quad (24a)$$

$$\eta \rightarrow \infty: f_\eta \rightarrow 1; g_\eta \rightarrow 1; \theta \rightarrow 0 \quad (24b)$$

Eqs. (21) and (22) along with the boundary conditions reveal that $f = g$, the Blasius flat-plate solutions, so that Eq. (23) may be written as

$$\theta_{\eta\eta} + \sigma f \theta_\eta + E/2 (\sigma-1) \left[(U/U_\infty)^2 + (W/U_\infty)^2 \right] f_\eta^2 = 0 \quad (25)$$

It is convenient to represent the general solution of Eq. (25) by the superposition of two solutions of the form:

$$\theta(0, \eta, \zeta) = C\theta_1(\eta) + E/2 [(U/U_\infty)^2 + (W/U_\infty)^2] \theta_2(\eta) \quad (26)$$

Here $\theta_1(\eta)$ denotes the general solution of the homogeneous equation and $\theta_2(\eta)$ denotes a particular solution of the non-homogeneous equation. In this report we simply consider $\sigma = 1$ so that Eq. (25) reduces to

$$\theta_{\eta\eta} + f\theta_\eta = 0 \quad (27)$$

the solution of which is $\theta = 1 - f_\eta$ - the "flat-plate cooling problem solution". Since we are evolving a numerical method to take account of three-dimensional effects, the assumption that the Prandtl number equal one is not overly restrictive in order to bring out the three-dimensional effects, and any problems associated with taking account of the third dimension, ζ .

$$B. \quad \xi \neq 0; \quad \zeta = 0: \quad f_{\eta\eta\eta} + (1+\beta/2)ff_{\eta\eta} + \beta(1-f_\eta^2) + \beta*gf_{\eta\eta} + 2\xi(f_\xi f_{\eta\eta} - f_\eta f_{\eta\xi}) = 0 \quad (28)$$

$$g_{\eta\eta\eta} + (1+\beta/2)fg_{\eta\eta} - \beta(1-g_\eta^2) + k*(1-g_\eta f_\eta) + \beta*gg_{\eta\eta} + 2\xi(f_\xi g_{\eta\eta} - f_\eta g_{\eta\xi}) = 0 \quad (29)$$

$$\theta_{\eta\eta} + \sigma [(1+\beta/2)f + \beta*g]\theta_\eta + E/2(\sigma-1)(U/U_\infty f_\eta)^2 + 2\xi\sigma(f_\xi\theta_\eta - f_\eta\theta_\xi) = 0 \quad (30)$$

and for $\sigma = 1$, Eq. (30) is simply

$$\theta_{\eta\eta} + [(1+\beta/2)f + \beta*g]\theta_\eta + 2\xi(f_\xi\theta_\eta - f_\eta\theta_\xi) = 0 \quad (31)$$

subject to boundary conditions

$$\eta = 0: \quad f = f_\eta = f_\xi = 0; \quad g = g_\eta = 0; \quad \theta = 1 \quad (32a)$$

$$\eta \rightarrow \infty: \quad f_\eta \rightarrow 1, \quad g_\eta \rightarrow 1, \quad \theta \rightarrow 0 \quad (32b)$$

In Ref. (1) we noted that Case (B) represents the line of symmetry condition so that along this line $w = W = 0$, but w_ζ and $w_s \neq 0$. Consequently, for this condition $g_\eta = w_\zeta/W_\zeta$ (determined by L'Hôpital's rule).

OUTLINE OF CALCULATIONAL PROCEDURE

The values of f and g and derivatives of these functions have been determined for the problem depicted in Fig. (1) and reported in Ref. (1). Consequently, the calculations for θ are begun either along the edge ($\xi = 0, \zeta \neq 0$) or the line of symmetry ($\xi \neq 0, \zeta = 0$). The solution for θ along $\zeta = 0, \xi \neq 0$ is given by Eq. (27).

The calculations of θ along the line of symmetry, Eq. (31) proceed until flow reversal appears in the u -component of velocity, or, in other words, until f_η becomes negative near the surface.

Knowing the edge and line of symmetry solution, Eqs. (16)-(18) we solved for $\xi \neq 0, \zeta \neq 0$ ($\sigma = 1$). Calculations for f and g again proceed until flow reversal. We have not, though, taken the θ calculations all the way to the flow reversal line. A separate difference formulation is required near flow reversal but this does not introduce any added difficulties as shown in Ref. (1). Appendices A and B contain the difference equations while in Appendix C is the flow chart describing the manner in which the θ calculations are to be performed.

SOLUTION OF DIFFERENCE EQUATIONS

In contrast to the solutions for f and g which involve the solution of non-linear, coupled algebraic equations, the difference equations for

θ are linear. Considering a step-size $\Delta\eta = 0.2$ ($\Delta\xi = \Delta\zeta = 0.61$) and $N = 30$ the set of algebraic equations, Eqs. (5A) and (6B) involve the inversion of tri-diagonal matrices when solving for θ .

DISCUSSION

An implicit numerical method, developed in Ref. (1) for the three-dimensional momentum boundary layer equations, has been adopted for the solution to the three-dimensional energy boundary layer equation, Figs. (4)-(6) are plots of the dimensionless temperature function, defined by Eqs. (9) and (15). What is of interest, at least with respect to the particular problem under investigation, is that for a given ζ station, the θ profiles are virtually similar for different ξ stations. Table I, values of $-\theta_\eta$ at $\eta = 0$, show that actually for a given ξ station, the θ profiles show greater similarity for a ζ variation. This suggests that it may be possible to develop a simplified analytical procedure when calculating temperature profiles as, for example, the local similarity methods developed for two-dimensional boundary layer calculations.

APPENDIX A

FINITE DIFFERENCE EQUATIONS - LINE OF SYMMETRY

For the difference approximations to the two-dimensional line of symmetry equations, a rectangular mesh as illustrated in Fig. (2) is adopted. The basic step in the computation is to calculate conditions at station 2 (see Fig. 2) from known conditions at stations 1.

To approximate derivatives of θ central differences are chosen. The quantities are evaluated midway between the element.

$$\theta_c = 1/2(\theta_{\ell+1,n} + \theta_{\ell,n}) \quad (1A)$$

$$\theta_\xi)_c = 1/\Delta\xi (\theta_{\ell+1,n} - \theta_{\ell,n}) \quad (2A)$$

$$\theta_\eta)_c = 1/4\Delta\eta (\theta_{\ell+1,n+1} - \theta_{\ell+1,n-1} + \theta_{\ell,n+1} - \theta_{\ell,n-1}) \quad (3A)$$

$$\theta_{\eta\eta})_c = 1/2(\Delta\eta)^2 (\theta_{\ell+1,n+1} - 2\theta_{\ell+1,n} + \theta_{\ell+1,n-1} + \theta_{\ell,n+1} - 2\theta_{\ell,n} + \theta_{\ell,n-1}) \quad (4A)$$

Substituting into Eq. (31) yields

$$a_n \theta_{\ell+1,n+1} + b_n \theta_{\ell+1,n} + c_n \theta_{\ell+1,n-1} = d_n \quad (5A)$$

where

$$a_n = \frac{1}{2(\Delta\eta)^2} + \frac{1}{4\Delta\eta} \left[(1+\beta/2) f_c + \beta g_c + 2\xi f_\xi)_c \right] \quad (6A)$$

$$b_n = -\frac{1}{(\Delta\eta)^2} - \frac{2\xi}{\Delta\xi} F_c \quad (7A)$$

$$c_n = \frac{1}{(\Delta\eta)^2} - a_n \quad (8A)$$

$$\begin{aligned} -d_n &= \frac{1}{2(\Delta\eta)^2} (\theta_{\ell,n+1} - 2\theta_{\ell,n} + \theta_{\ell,n-1}) + \left[a_n - \frac{1}{2(\Delta\eta)^2} \right] (\theta_{\ell,n+1} - \theta_{\ell,n-1}) \\ &+ \frac{2\xi}{\Delta\xi} F_c \theta_{\ell,n} \end{aligned} \quad (9A)$$

Setting $n = 1, 2, \dots, N-1$, where N denotes the outer edge of the boundary layer in Eqs. (5A) - (9A) we obtain the sets of difference equations to be solved simultaneously for the values of θ at station $\ell+1$ from known values of θ (as well as f and g) at station ℓ .

APPENDIX B

FINITE DIFFERENCE EQUATIONS - OFF THE LINE OF SYMMETRY

Formulating the difference equation to Eq. (18), $\sigma = 1$, a mesh with sides $\Delta\xi$, $\Delta\zeta$ and $2\Delta\eta$ is chosen (Fig. 3). The pivotal points for the computation are taken to be at the midpoints of the corners of the mesh. To denote position a treble-suffix notation is employed.

As in the two-dimensional computation we approximate θ and its derivatives (as well as f and g) by central differences. The quantities are evaluated at the center of the element.

$$\theta_c = \frac{1}{4} (\theta_{\ell+1,m+1,n} + \theta_{\ell+1,m,n} + \theta_{\ell,m+1,n} + \theta_{\ell,m,n}) \quad (1B)$$

$$(\theta_\xi)_c = \frac{1}{2\Delta\xi} (\theta_{\ell+1,m+1,n} + \theta_{\ell+1,m,n} - \theta_{\ell,m+1,n} - \theta_{\ell,m,n}) \quad (2B)$$

$$(\theta_\zeta)_c = \frac{1}{2\Delta\zeta} (\theta_{\ell+1,m+1,n} + \theta_{\ell,m+1,n} - \theta_{\ell+1,m,n} - \theta_{\ell,m,n}) \quad (3B)$$

$$(\theta_\eta)_c = \frac{1}{4\Delta\eta} (\theta_{\ell+1,m+1,n+1} - \theta_{\ell+1,m+1,n-1} + \theta_{\ell,m,n+1} - \theta_{\ell,m,n-1}) \quad (4B)$$

$$\begin{aligned} (\theta_{\eta\eta})_c &= \frac{1}{2(\Delta\eta)^2} (\theta_{\ell+1,m+1,n+1} - 2\theta_{\ell+1,m+1,n} + \theta_{\ell+1,m+1,n-1} \\ &\quad + \theta_{\ell,m,n+1} - 2\theta_{\ell,m,n} + \theta_{\ell,m,n-1}) \end{aligned} \quad (5B)$$

Substituting into Eq. (18) yields

$$a_n \theta_{\ell+1,m+1,n+1} + b_n \theta_{\ell+1,m+1,n} + c_n \theta_{\ell+1,m+1,n-1} = d_n \quad (6B)$$

where

$$a_n = \frac{1}{2(\Delta\eta)^2} + \frac{1}{4\Delta\eta} \left[(1+\beta/2)f_c + (\beta^*-k/2)g_c + 2\xi f_\xi)_c + 2\xi \frac{W}{U} g_\zeta)_c \right] \quad (7B)$$

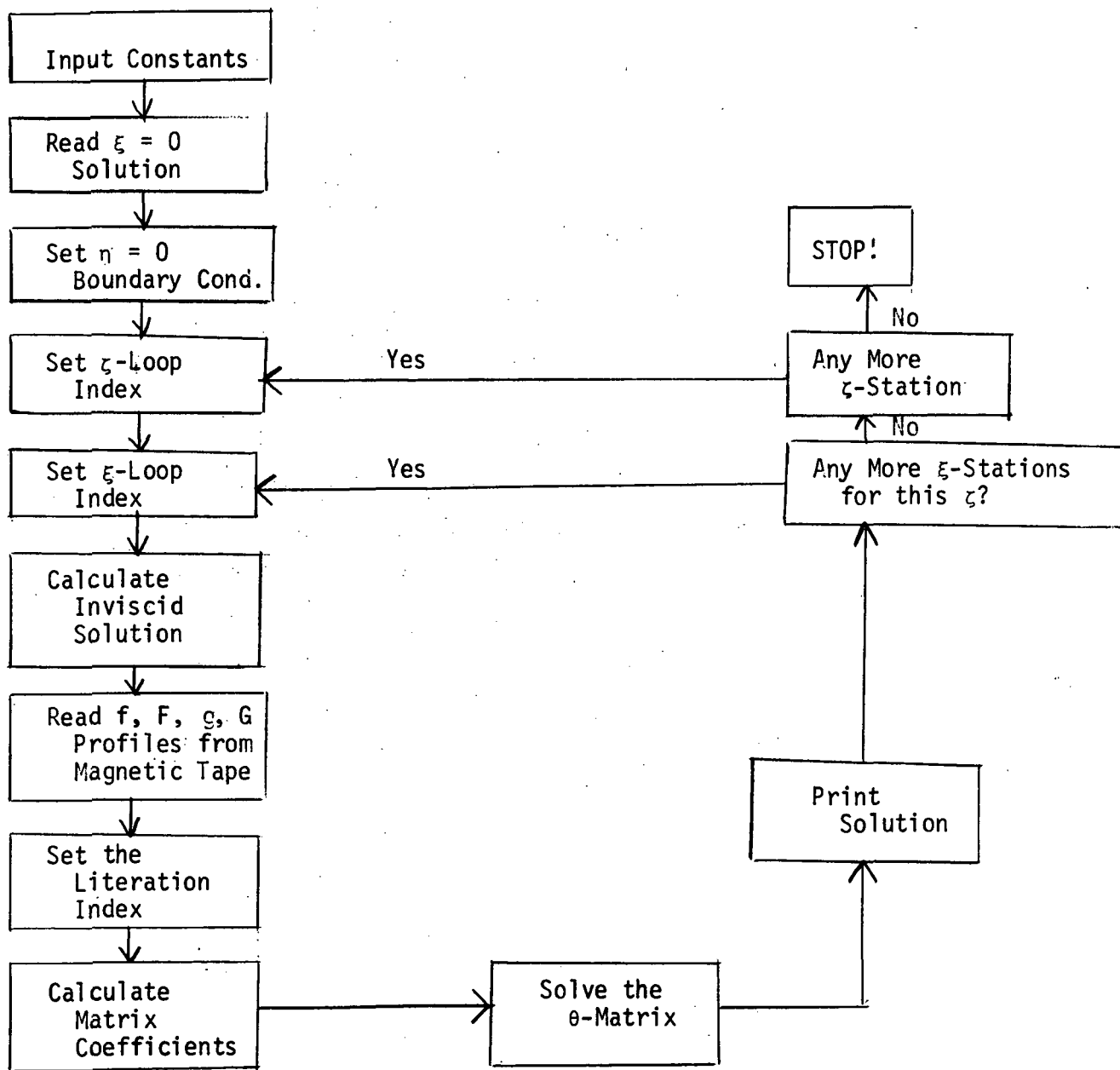
$$b_n = - \frac{1}{(\Delta\eta)^2} - \frac{\xi}{\Delta\xi} F_c - \frac{\xi}{\Delta\zeta} \frac{W}{U} G_c \quad (8B)$$

(9B)

$$\begin{aligned}
c_n &= \frac{1}{(\Delta n)^2} - a_n \\
-d_n &= \frac{1}{2(\Delta n)^2} (\theta_{\ell, m, n+1} - 2\theta_{\ell, m, n} + \theta_{\ell, m, n-1}) + \left[a_n - \frac{1}{2(\Delta n)^2} \right] (\theta_{\ell, m, n+1} \\
&\quad - \theta_{\ell, m, n-1}) - \frac{\xi F_c}{\Delta \xi} (\theta_{\ell+1, m, n} - \theta_{\ell, m+1, n} - \theta_{\ell, m, n}) - \frac{\xi}{\Delta \xi} G_c \frac{W}{U} (\theta_{\ell, m+1, n} \\
&\quad - \theta_{\ell+1, m, n} - \theta_{\ell, m, n})
\end{aligned}
\tag{10B}$$

As in Appendix A, setting $n = 1, 2, \dots, N-1$ in Eqs. (1B)-(10B) we obtain the sets of difference equations to be solved simultaneously.

APPENDIX C
SAMPLE FLOW CHART



REFERENCES

1. Fillo, J. A. and Burbank, R. A., "Finite Difference Solution to the Three-Dimensional, Incompressible Laminar Boundary-Layer Equations", NASA Report, NGR-22-010-019/4/71, Sept. (1971).
2. Van Dyke, M., "Higher Approximations in Boundary-Layer Theory, Part 1, General Analysis", J. Fluid Mech. 14:161, (1962).

SYMBOLS

a = cylinder radius

c_p = specific heat (constant pressure)

E = Eckert number $E = \frac{U_\infty^2}{c_p H_1(0)}$

$f = \int_0^n F(\xi, \lambda, \zeta) d\lambda$

$F = u/U$

$g = \int_0^n G(\xi, \lambda, \zeta) d\lambda$

$G = w/W$

$H_1(0)$ = stagnation enthalpy constant

p = pressure

S = relative stagnation enthalpy function - Eq. (9)

T = temperature

U = x-component of inviscid flow

W = z-component of inviscid flow

u, v, w = velocity components in the x, y, z directions

$\bar{x}$ = distance from cylinder axis to the leading edge of the flat plate (=45.7) cm

x, y, z = Cartesian coordinates

α = thermal diffusivity

ξ, η, ζ = transformed coordinates

λ = dummy variable of integration

ψ, ϕ = streamfunctions

ρ = density

θ = defined by Eq. (15) temperature function

$$\beta = 2\xi/U \, U_\xi$$

$$\beta^* = 2\xi/U \, W_\zeta$$

$$k = 2\xi W/U^2 \, U_\zeta$$

$$k^* = 2\xi/W \, W_\xi$$

$$\sigma = \nu/\alpha = \text{Prandtl number}$$

$$\mu = \text{dynamic viscosity}$$

$$\nu = \text{kinematic viscosity}$$

Subscripts

$$x, y, z, \eta, \dots = \text{differentiation (except on } \tau)$$

$$w = \text{wall condition}$$

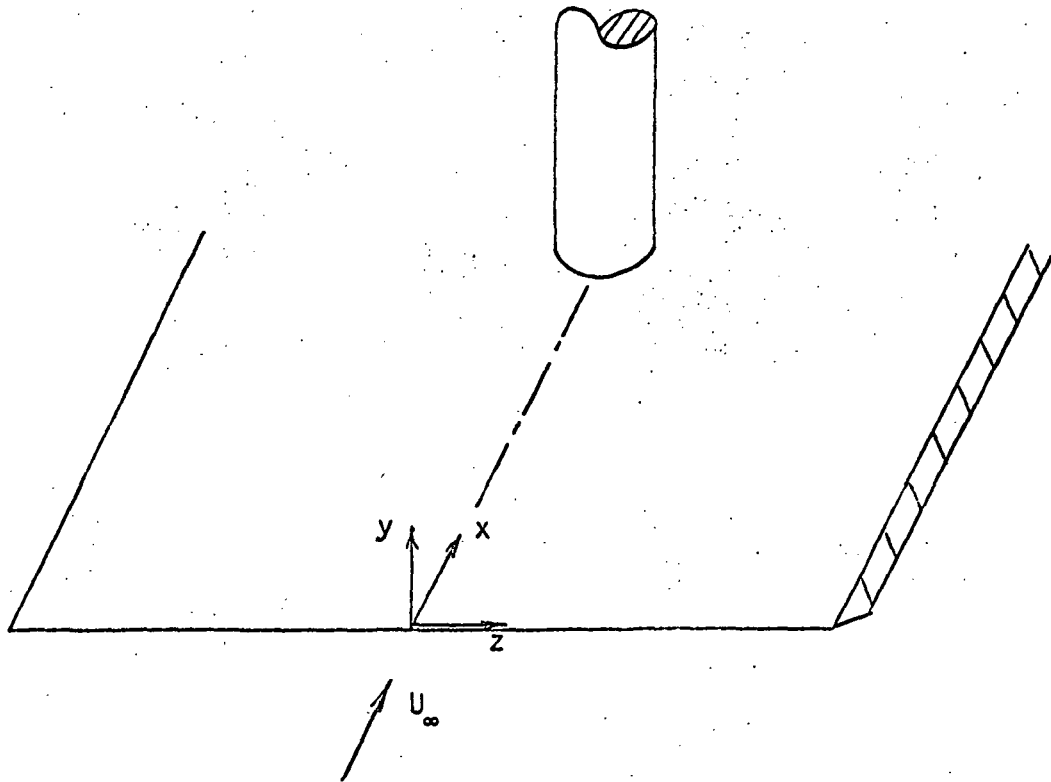


Fig. 1 Geometry of Flow Problem

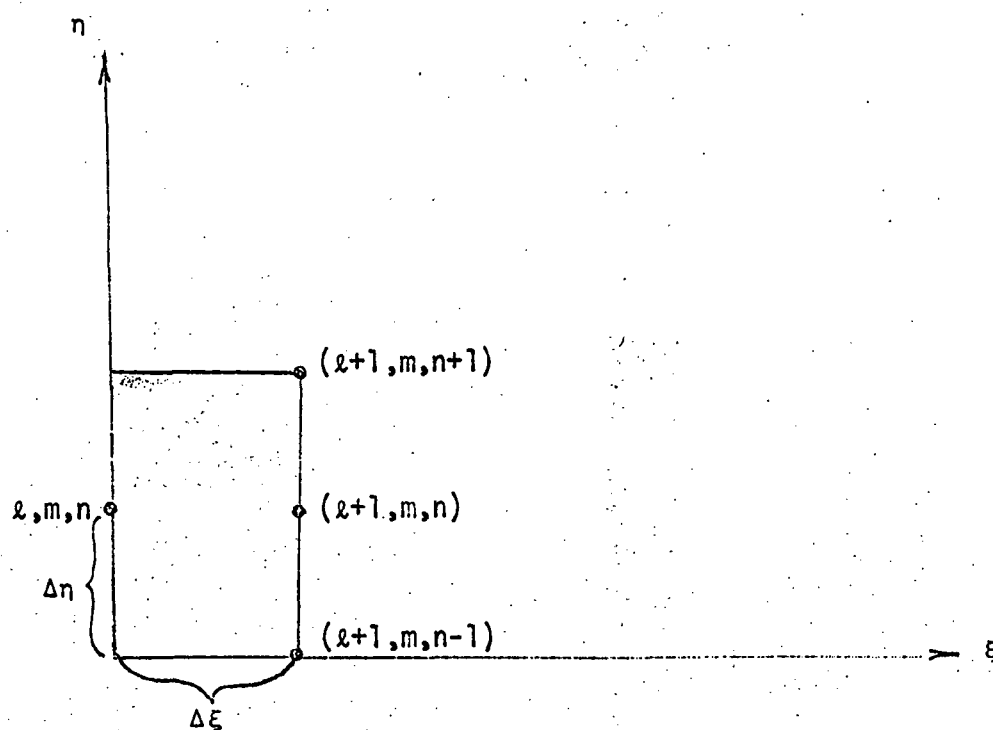


Fig. 2 Finite Difference Mesh in the $\zeta = 0$ Plane of Symmetry

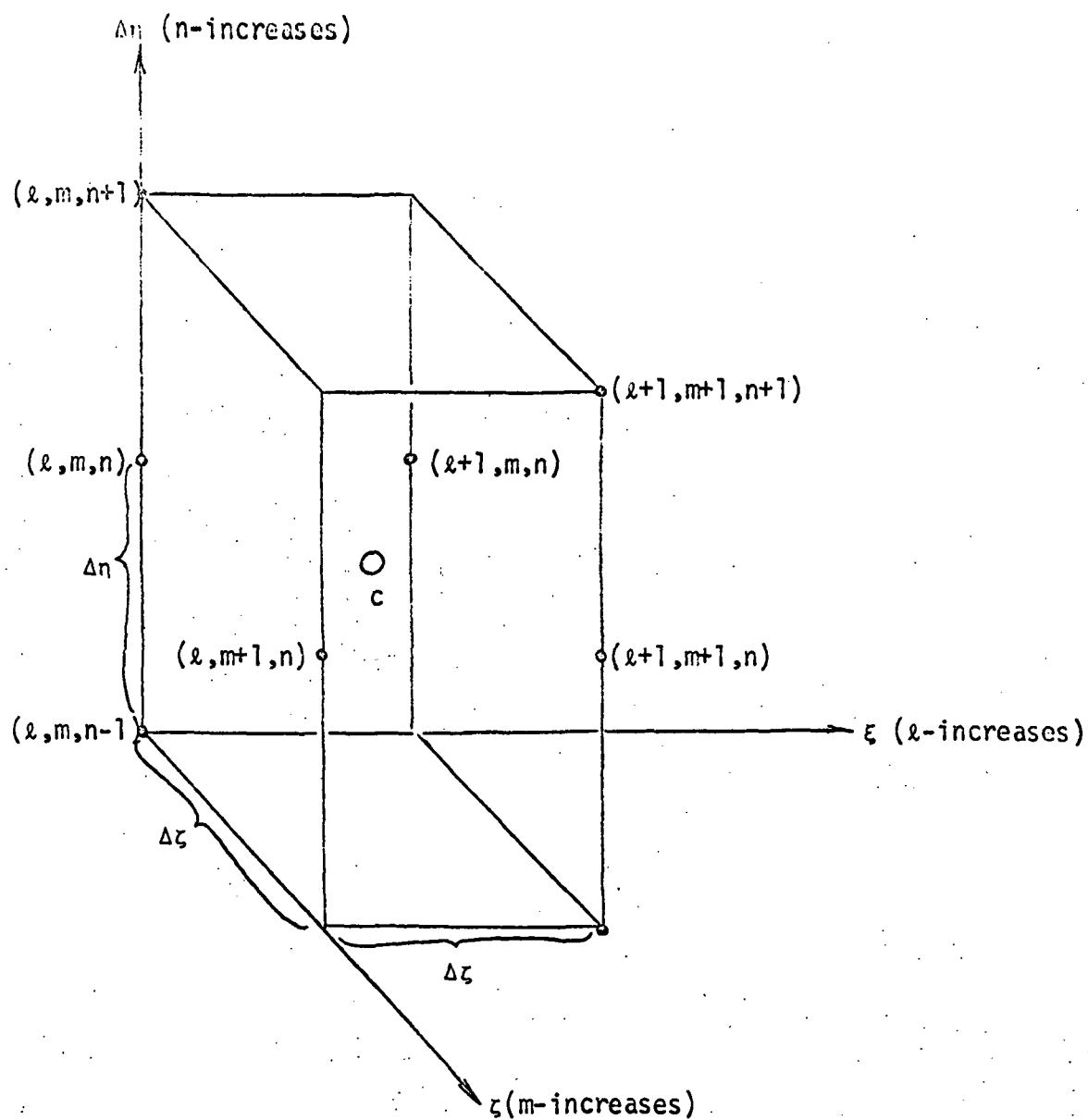


Fig. 3 The Finite Difference Mesh in Three Dimensions.

The following pages contain plots of the dimensionless relative stagnation-enthalpy function for the flow problem of Fig. (1). All dimensional distances are in centimeters and the following constants have been used:

cylinder radius, 6.1 cm.

freestream velocity, 3050 cm/sec.

distance of cylinder axis from leading edge, 45.7 cm.

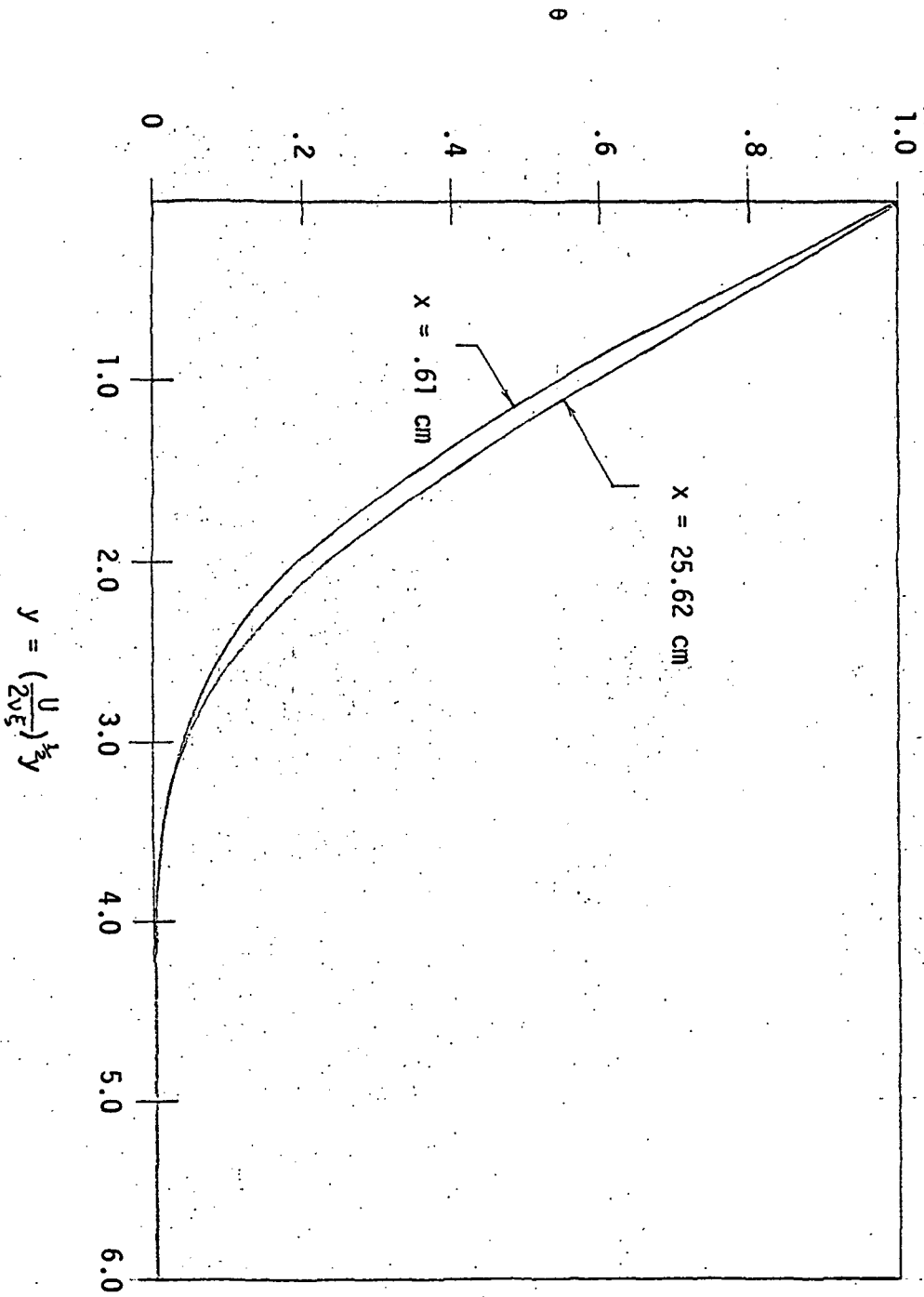


Fig. 4 Development of the Non-Dimensional Temperature along the Line of Symmetry.

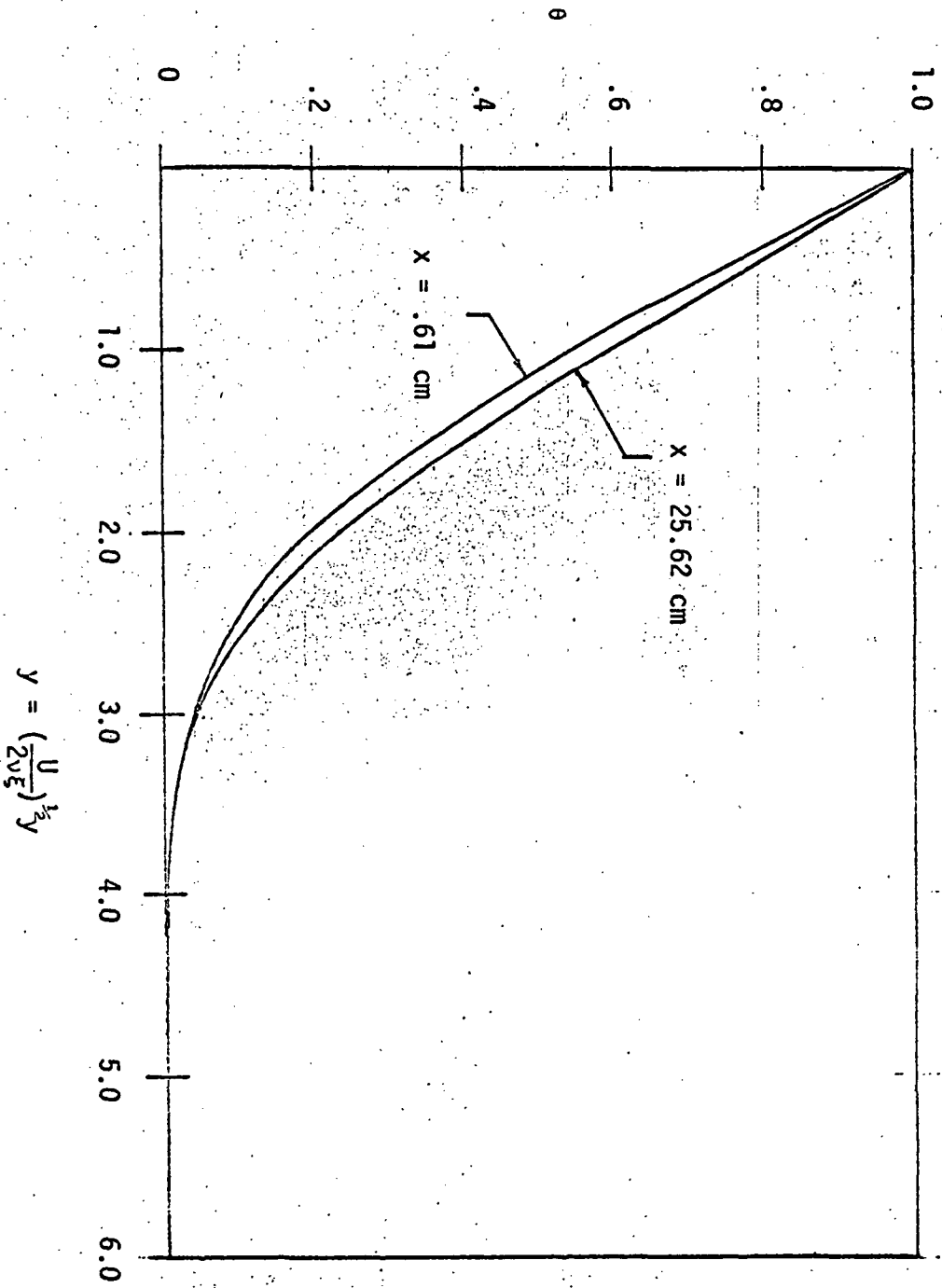


Fig. 5 Development of the Non-Dimensional Temperature along the Line $z = .61 \text{ cm}$.

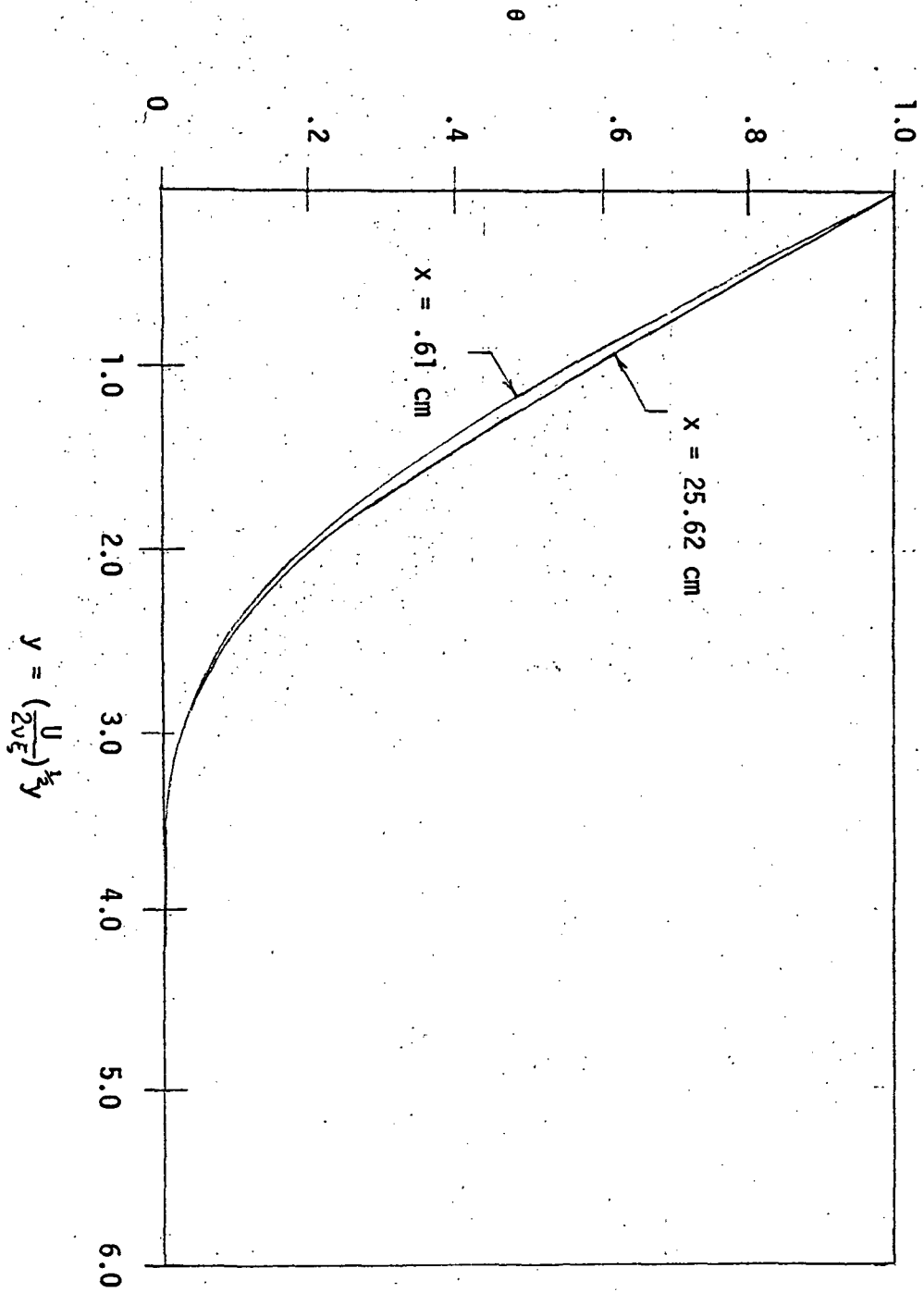


Fig. 6 Development of the Non-Dimensional Temperature along the Line $z = 6.1 \text{ cm}$.

TABLE I - SELECTED VALUES OF $-\theta_n(0)$

x-cm	z = 0	z = .61	z = 3.05	z = 6.10	z = 8.54
0	.4692	.4692	.4692	.4692	.4692
3.05	.4683	.4683	.4684	.4684	.4685
6.10	.4673	.4673	.4674	.4676	.4678
9.15	.4656	.4656	.4657	.4661	.4665
12.20	.4635	.4635	.4637	.4643	.4651
15.25	.4606	.4607	.4610	.4620	.4631
18.30	.4564	.4564	.4570	.4586	.4604
21.35	.4486	.4487	.4497	.4527	.4559
24.40	.4276	.4277	.4310	.4392	.4466
25.62	.3977	.3980	.4075	.4262	.4391