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SYSTEM EVALUATION FROM IMPULSE RESPONSE FRAGMENTS

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16. ABSTRACT

Statistical estimation or identification procedures often describe a dynamical system by its sampled impulse response. This is not always useful or desirable for further data processing. It is largely necessary to provide information on the z- or s- transfer function of the system. As an additional hardship, the impulse response is sometimes known only over a limited time interval because of an insignificantly damped system or a limited computer capacity. By use of rational approximation, a procedure has been found which allows calculation of the z-transfer function from a restricted number of time sequence values. The order of the system need not be known. Transformation to the s-domain is easily provided.

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INTRODUCTION

Estimation or identification of transfer functions is often required in control techniques or any other field in which the knowledge of system dynamics is a necessity. In aerospace techniques, it might be desirable to identify the flight dynamics of a vehicle. The two main cases of application are (1) identification of vehicle dynamics in self-adapting or self-learning control procedures and (2) confirmation of vehicle dynamics in flight testing of newly developed aerospace vehicles.

Different approaches of estimation and identification have been made, but all of them are based on statistical methods. A whole family of procedures ends up with the impulse response of a system to be identified. Because of the inherent characteristic of these procedures, the impulse response is represented by an equally sampled time series. The description of the system by its impulse response is often not desirable for data handling. In many cases, e.g., in control law optimization, it is more convenient to have the vehicle dynamic described by a rational expression of polynominals [1].

Furthermore, the impulse response represented by the time series is often available only for a limited time interval. This can occur if the system under investigation is either instable, indifferent, or only slightly damped. In these cases the calculation of the impulse response has to be stopped after a time interval to be defined. Another reason for only a limited knowledge of the impulse response might be a limited computer capacity available for its calculation.

Given a sampled record of a portion of the impulse response of a linear system, estimate the transfer functions, G (S) and G (z), of the system. A procedure which covers the preceding requirements is found by use of rational approximation. The impulse response described by a time series is converted to a rational expression in z. After determination of the poles and zeros of the function, a transformation to the s-domain can then be provided if required. In the following section, the appropriate procedure is described, and restrictions are discussed. A digital program for implementation of the method is attached.

ACKNOWLEDGMENT

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TASK DESCRIPTION

By use of some identification or estimation procedure based on statistical theory, up-to-then totally unknown systems have to be identified by its impulse response. The impulse response is represented by a time series equally sampled by a sampling interval T, which is known from the identification procedure (Fig. 1). As a shortcoming, the impulse response is known only over a limited time interval.

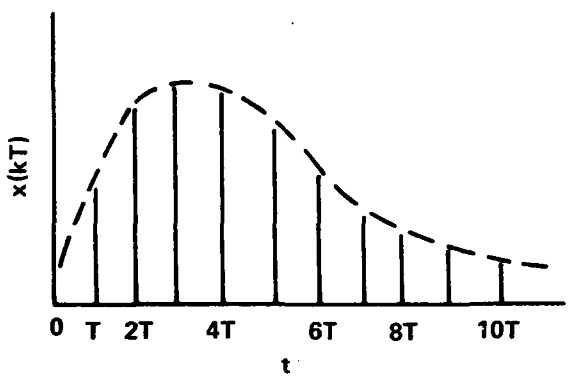

Figure 1. Time series representation of impulse response.

The available data must be converted to a form which describes the vehicle dynamic completely in the sampling moments and which can be used for further data processing.

THE RATIONAL APPROXIMATION

The z-transform of the time series x(kT) is defined to be

$$G(z) = c_0 + c_1 z^{-1} + \dots + c_k z^{-k} + \dots \tag{1}$$

where $c_k = x(kT)$ with sampling interval T.

The desired equivalent rational expression in z is of the form

$$F(z) = \frac{a_0 + a_1 z^{-1} + \dots + a_m z^{-m}}{b_0 + b_1 z^{-1} + \dots + b_n z^{-n}} . \tag{2}$$

If (1) is any series in z^{-1}, there exists a unique rational "approximation" of the form (2) for each pair of integers m and n that agrees when expanded term by term with the series for more terms than any other rational expression with smaller or equal m and n [2]. It follows that, if G(z) has a rational expression, as is the case with most engineering systems, the exact rational expression should be found when m and n are chosen large enough. The choice of m and n does create a problem which will be discussed later.

Once m and n are chosen, the rational approximation is found in multiplying G(z) by the denominator of F(z) and collecting the like powers of z^{-1}:

$$
\begin{array}{l}
c_0 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_k z^{-k} + \ldots \\[1em]
b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_n z^{-n} \\
\hline
b_0 c_0 + b_0 c_1 z^{-1} + b_0 c_2 z^{-2} + \ldots + b_0 c_k z^{-k} \\[1em]
\qquad b_1 c_0 z^{-1} + b_1 c_1 z^{-2} + b_1 c_2 z^{-3} + \ldots + b_1 c_k z^{-(k+1)} \\
\qquad \vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
\hline
b_0 c_0 + (b_0 c_1 + b_1 c_0) z^{-1} + (b_0 c_2 + b_1 c_1 + b_2 c_0) z^{-2} \ldots \qquad .
\end{array}
\tag{3}
$$

The first m+1 powers of z^{-1} can be forced to agree with the numerator of F(z), and the next n can be forced to vanish.

The equations for this are

$$
a_k = \sum_{i+j=k} c_i b_i \qquad , \qquad k \leq m \quad ; \tag{4}
$$

thus,

$$
\begin{aligned}
a_0 &= b_0 c_0 \qquad , \\
a_1 &= b_0 c_1 + b_1 c_0 \qquad ,
\end{aligned}
\tag{5}
$$

and

$$\sum_{\substack{i+j=k \\ i \leq n}} b_i c_j = 0 \quad , \qquad k = m+1,\ m+2,\ \ldots\ m+n \tag{6}$$

with $n \geq m$;

thus,

$$
\begin{aligned}
& b_0 c_{m+1} + b_1 c_m + \ldots + b_{m+1} c_0 = 0 \\
& b_0 c_{m+2} + b_1 c_{m+1} + \ldots + b_{m+2} c_0 = 0 \\
& \quad \vdots \\
& b_0 c_n + b_1 c_{n-1} + \ldots + b_n c_0 = 0 \\
& b_0 c_{n+1} + b_1 c_n + \ldots + b_n c_1 = 0 \\
& \quad \vdots \\
& b_0 c_{n+m} + b_1 c_{n+m-1} + \ldots\ b_n c_m = 0 \quad .
\end{aligned} \tag{7}
$$

The last equations are solved for b_0, b_1, ... b_n and then substituted in the first m+1 to obtain a_0, a_1, ... a_m. Clearly, one of the b_i can be chosen arbitrarily (anything except 0), and the rest are unique if the system of equations is not singular. If m and n are chosen too large, the system of equations will be singular.

If the rank of the system is r, the denominator of F(z) should be of degree r in z^{-1}. If the system is solved using n too large, theoretically the numerator and denominator of F(z) will have common factors which could be factored out. In practice, these factors may not be exactly the same because of the roundoff errors.

Some procedure for checking whether the value of n is too large should be used to prevent the introduction of zeros in the denominator and also to prevent the solution of a singular system of equations.

SCALING

Assuming that the time sequence $x(n\Delta t)$ has been received from the time function $x(t)$ through an ideal sampler, $x(n\Delta t)$ is attenuated by a factor $\frac{1}{\Delta t}$ [3]. For correct scaling, the numerator of $f(z)$ must be multiplied by Δt.

RECONVERSION TO THE TIME DOMAIN

For later use in the realization of the procedure described previously, it is presumed [4] that $F(z)$ is reconverted to the time domain by

$$y_k = a_0 x_k + a_1 x_{k-T} + \dots + a_m x_{k-mT} - b_1 y_{k-T} \dots - b_m y_{k-mT} \quad (8)$$

where x_k and y_k are the input and output time sequences of the digital filter, and $x_k - mT$ and $y_k - mT$ are the $m \cdot T$ earlier values of the input and output time sequences. Applying an impulse as an input to equation (8), the impulse response of $F(z)$ can be calculated as a time series representation.

IMPLEMENTATION OF THE PROCEDURE IN A DIGITAL COMPUTER

For implementation of the method just described, it has to be realized that, according to the task description, the vehicle dynamics are completely unknown; thus, information on the order of the rational expression in both numerator and denominator is not available. Consequently, a scheme must be established to evaluate m and n. A suitable scheme has been found by taking logical steps as follows:

1. The order of the numerator and denominator of equation (2) is assumed to be equal; thus $m = n$. As later shown, this assumption does not introduce a significant error.

2. An initial estimate on m is made assuring that

$$m_{estimate} \geq m_{actual} \ .$$

3. With this estimate in m, the procedure previously described will be executed. If the estimate was too high, the matrix which has to be solved in the process of the procedure will be ill conditioned or will provide an otherwise insignificant solution.

4. To confirm the result received from step 3, the coefficients of $F(z)$ will be checked out by calculating its impulse response according to equation (8). The $n+m+1$ values of the impulse response time series will be calculated. If these values disagree with the input time series, m is reduced by 1 and the complete procedure is repeated until correspondence between both time series is received.

5. If we assume that the sampling rate has been chosen at least four times higher than the highest natural mode of the system, all roots of $F(z)$ will be located in the right half of the z plane. This will result in alternative signs for the polynomial of $F(z)$. Thus, an alternative sign condition might be established and introduced prior to the implementation of step 4. However, this condition is not necessarily acceptable or useful in all cases of application.

This scheme is, of course, sensitive to the accuracy of the calculation. Thus, the number of digits which have to agree has to be evaluated taking into account the number of significant digits through the whole sequence of calculations. Furthermore, in following this procedure, possible disturbances by noise of the initial input data have to be considered.

Additional confidence in this described checkout procedure may be established by extending the number of values from the time series to be compared, e.g., $2 \cdot (n + m + 1)$.

A digital program performing the procedure described is presented in the appendix.

APPLICATION OF THE DIGITAL PROGRAM

As an example and to show the significance of the method, the digital program is applied to an impulse response of which the transfer function in s is known. To provide exact knowledge to the input data, the impulse response has been received by inverse Laplace transformation of the transfer function in s.

The example has been chosen to be

$$F_1(s) = \frac{\left(1 + \frac{1}{31.4}\, s\right)}{\left(1 + \frac{2}{6.28}\, s + \frac{1}{6.28^2}\, s^2\right)} \tag{9}$$

with $\omega = 6.28$ and $\xi = 1$.

The impulse response is printed in Table 1 for $t = 0.01$ sec. Figure 2 shows the impulse response as a curve.

From the given example, we know that $m = 2$. Assuming that this is not known, we make an initial estimate of $m = 4$; thus, the first $m + n + 1 = 9$ values are used as an input to the program. Futhermore, $m = n$ and $t = 0.01$ sec.

Executing the program results in

$$F_1(z) = \frac{0.1256 \cdot 10^{-1} - 0.882432663955 \cdot 10^{-2} z^{-1} + 0.133715714656 \cdot 10^{-5} z^{-2}}{0.1 \cdot 10^{-1} - 0.187758651589 \cdot 10^{1} z^{-1}\ 0.881226462775 \cdot 10^{0} z^{-2}}$$

with gain factor $GF = 1.02666622147$.

Table 2 shows the impulse response derived from the calculated $F_1(z)$ according to equation (8). Comparing with the input response of Table 1 and Figure 2, it can be seen that a slight deviation builds up with increasing numbers of time sequence values, due to roundoff errors, but that it still does not exceed 15 percent of the hundredth already very small input value.

TABLE 1. IMPULSE RESPONSE VALUES AND ROOTS FOR EXAMPLE $F_1(s)$

NUMERATOR ROOTS
S=-0.31400000E 02 + AND - J 0.00000000E-38

DENOMINATOR ROOTS
S=-0.62800000E 01 + AND - J 0.00000000E-38
S=-0.62800000E 01 + AND - J-0.00000000E-38

TIME	FUNCTION
0.000000	1.25600000E 00
0.010000	1.47585160E 00
0.020000	1.66428550E 00
0.030000	1.82431190E 00
0.040000	1.95869100E 00
0.050000	2.06995200E 00
0.060000	2.16041140E 00
0.070000	2.23218930E 00
0.080000	2.28722490E 00
0.090000	2.32729030E 00
0.100000	2.35400410E 00
0.110000	2.36884320E 00
0.120000	2.37315430E 00
0.130000	2.36816390E 00
0.140000	2.35498850E 00
0.150000	2.33464300E 00
0.160000	2.30804910E 00
0.170000	2.27604780E 00
0.180000	2.23938160E 00
0.190000	2.19875070E 00
0.200000	2.15476910E 00
0.210000	2.10799540E 00
0.220000	2.05893240E 00
0.230000	2.00803200E 00
0.240000	1.95569980E 00
0.250000	1.90229860E 00
0.260000	1.84815250E 00
0.270000	1.79355000E 00
0.280000	1.73874720E 00
0.290000	1.68397090E 00
0.300000	1.62942080E 00
0.310000	1.57527250E 00
0.320000	1.52167900E 00
0.330000	1.46877340E 00
0.340000	1.41667070E 00
0.350000	1.36546890E 00
0.360000	1.31525160E 00
0.370000	1.26608850E 00
0.380000	1.21803750E 00
0.390000	1.17114510E 00
0.400000	1.12544850E 00
0.410000	1.08097580E 00
0.420000	1.03774730E 00
0.430000	9.95776310E-01
0.440000	9.55069940E-01
0.450000	9.15629670E-01
0.460000	8.77452220E-01
0.470000	8.40529990E-01
0.480000	8.04851610E-01
0.490000	7.70402480E-01
0.500000	7.37165110E-01
0.510000	7.05119640E-01
0.520000	6.74244100E-01
0.530000	6.44514800E-01
0.540000	6.15906610E-01
0.550000	5.88393170E-01
0.560000	5.61947200E-01
0.570000	5.36540700E-01
0.580000	5.12145120E-01
0.590000	4.88731510E-01
0.600000	4.66270700E-01
0.610000	4.44733470E-01
0.620000	4.24090590E-01
0.630000	4.04313000E-01
0.640000	3.85371630E-01
0.650000	3.67238520E-01
0.660000	3.49884900E-01
0.670000	3.33283250E-01
0.680000	3.17406320E-01
0.690000	3.02227390E-01
0.700000	2.87720320E-01
0.710000	2.73859550E-01
0.720000	2.60620140E-01
0.730000	2.47977820E-01
0.740000	2.35908950E-01
0.750000	2.24390550E-01
0.760000	2.13400320E-01
0.770000	2.02916660E-01
0.780000	1.92918690E-01
0.790000	1.83385880E-01
0.800000	1.74298870E-01
0.810000	1.65638660E-01
0.820000	1.57386930E-01
0.830000	1.49526060E-01
0.840000	1.42039030E-01
0.850000	1.34909460E-01
0.860000	1.28121570E-01
0.870000	1.21660160E-01
0.880000	1.15510690E-01
0.890000	1.09659080E-01
0.900000	1.04091860E-01
0.910000	9.87960900E-02
0.920000	9.37593530E-02
0.930000	8.89697370E-02
0.940000	8.44158180E-02
0.950000	8.00866480E-02
0.960000	7.59717490E-02
0.970000	7.20610530E-02
0.980000	6.83449650E-02
0.990000	6.48142770E-02
1.000000	6.14601870E-02

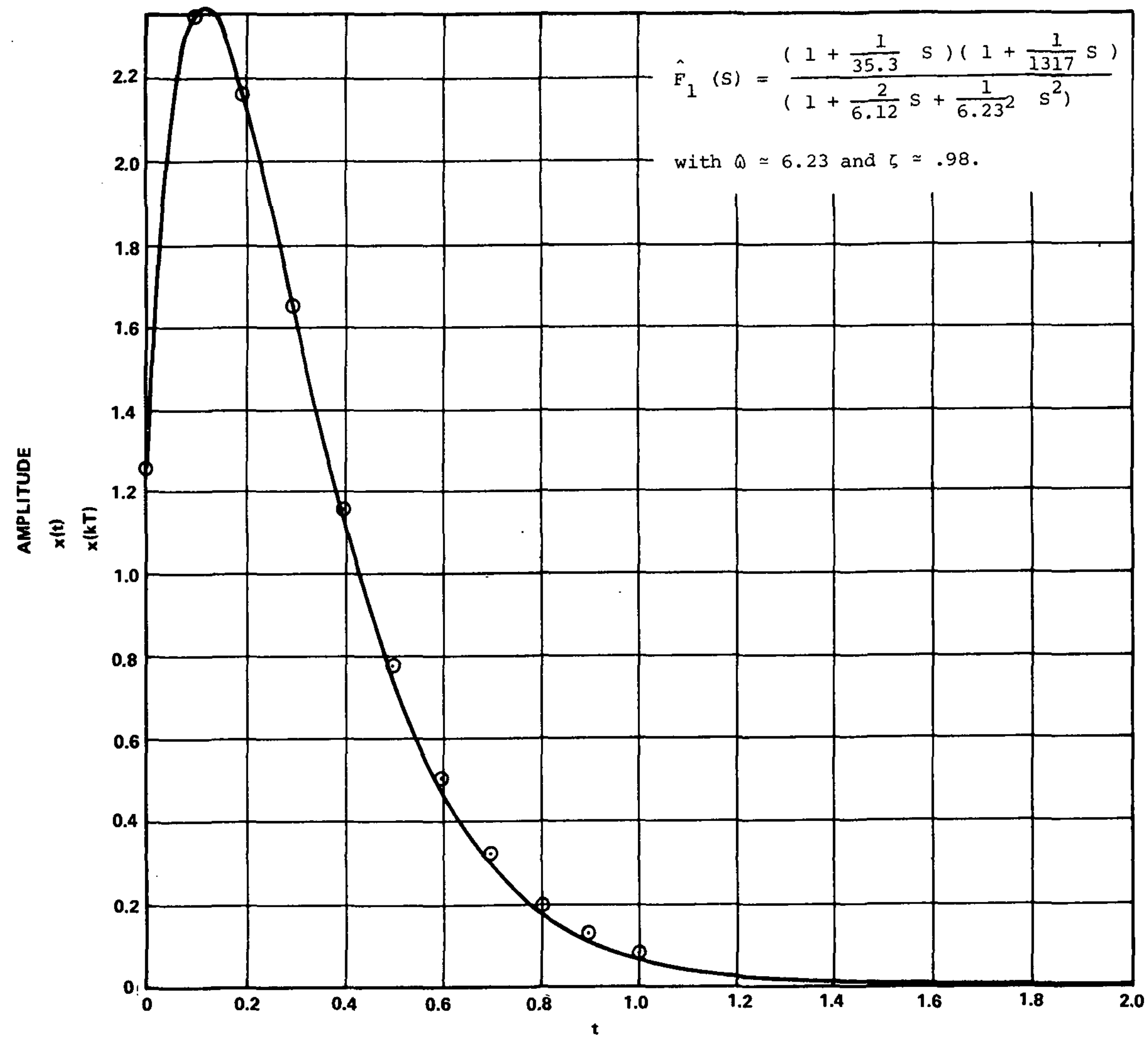

Figure 2. Comparison between continuous impulse response of $F_1(s)$ and sample values of impulse response time sequence reconverted from $F_1(z)$

TABLE 2. IMPULSE RESPONSE TIME SEQUENCE, RECONVERTED FROM $F_1(z)$. POLES AND ZEROS OF $F_1(z)$ AND CORRESPONDING ROOTS IN s-DOMAIN

D IMPULSE

0	.125600000000+001	.147581600000+001	.166428550000+001	.182431190000+001	.195869100000+001
	.206997988775+001	.216051598381+001	.223243462406+001	.228768528954+001	.232804658511+001
10	.235514006091+001	.237044296402+001	.237530000071+001	.237093418413+001	.235845683652+001
	.233887681000+001	.231310898512+001	.228198210210+001	.224624597542+001	.220657813888+001
20	.216358996436+001	.211783229483+001	.206980062850+001	.201993988872+001	.196864881146+001
	.191628397976+001	.186316353244+001	.180957057225+001	.175576629673+001	.170194287341+001
30	.164832607905+001	.159507772164+001	.154234786180+001	.149026684963+001	.143894719131+001
	.138848525900+001	.133896285628+001	.129044865082+001	.124299948460+001	.119666157159+001
40	.115147159185+001	.110745769041+001	.106464038856+001	.102303341464+001	.982644460886+000
	.943475872389+000	.905525273631+000	.868786137826+000	.833248303736+000	.798898444336+000
50	.765720491334+000	.733696019250+000	.702804592451+000	.673024078293+000	.644330929243+000
	.616700436639+000	.590106958643+000	.564524123919+000	.539926016228+000	.516282331311+000
60	.493568532326+000	.471755968348+000	.450816993080+000	.430724064060+000	.411449830577+000
	.392967210451+000	.375249456720+000	.358270215179+000	.342003573661+000	.326424103851+000
70	.311506896378+000	.297227589844+000	.283562394415+000	.270480110529+000	.257982143231+000
	.246022512606+000	.234587860736+000	.223657455561+000	.213211192013+000	.203229590722+000
80	.193693794608+000	.184585563603+000	.175887267765+000	.167581878979+000	.159652961458+000
	.152084661217+000	.144861694677+000	.137969336548+000	.131393407117+000	.125120259061+000
90	.119136763892+000	.113430298116+000	.107988729204+000	.102800401442+000	.978541217189-001
	.931391453208-001	.886451618203-001	.843622809485-001	.802810187654-001	.763922838837-001
	.726873639433-001				

NUM ROOTS Z

Z=	.190323182071-005	+ AND - J	.000000000000
Z=	~~.702571873802+000~~	~~+ AND - J~~	~~.000000000000~~

NUM ROOTS S

S=	-.131719571585+004	+ AND - J	.000000000000
S=	-.353007571545+002	+ AND - J	.000000000000

DENOM ROOTS Z

Z=	.928482177234+000	+ AND - J	.000000000000
Z=	~~.949104338653+000~~	~~+ AND - J~~	~~.000000000000~~

DENOM ROOTS S

S=	-.742040935979+001	+ AND - J	.000000000000
S=	-.522365405219+001	+ AND - J	.000000000000

RELATION BETWEEN z- AND s-TRANSFER FUNCTION

Considering the theoretical approach of the procedure and its execution as previously described, it can be stated that an exact z-transform has been provided with respect to $F_{(s)}$. The accuracy is limited only by the number of significant digits used in the course of the calculation.

Assuming that the system to be identified has been a digital one, it is totally described by the time series which has been used as input. Yet it has to be realized that, in most of all cases, specifically in identifying the dynamic of a vehicle, the system is an analog one. Thus, the dynamic of the calculated $F_{(z)}$ differs from the actual $F_{(s)}$ represented by the impulse response.

For further consideration, we assume that the sampling rate has been chosen as high and that with respect to the natural frequency of the system, its dynamical behavior between sampling instances can be neglected.

Any remaining difference in the dynamical behavior between the calculated $F_{(z)}$ and the actual $F_{(s)}$ is largely dependant on the sampling rate. If the sampling rate has been selected sufficiently high with respect to the natural modes of the system and the interesting frequency range, these differences might be negligible.

To cover more demanding requirements, a transformation to the s-domain has to be provided.

Thus, after having solved F(z) for poles and zeros, the appropriate roots in s are found according to Figure 3 as

$$|z| = \sqrt{(\mathrm{Re}\, z)^2 + (\mathrm{Im}\, z)^2}$$

$$\mathrm{Re}\, s = \frac{1}{T} \cdot \ln |z|$$

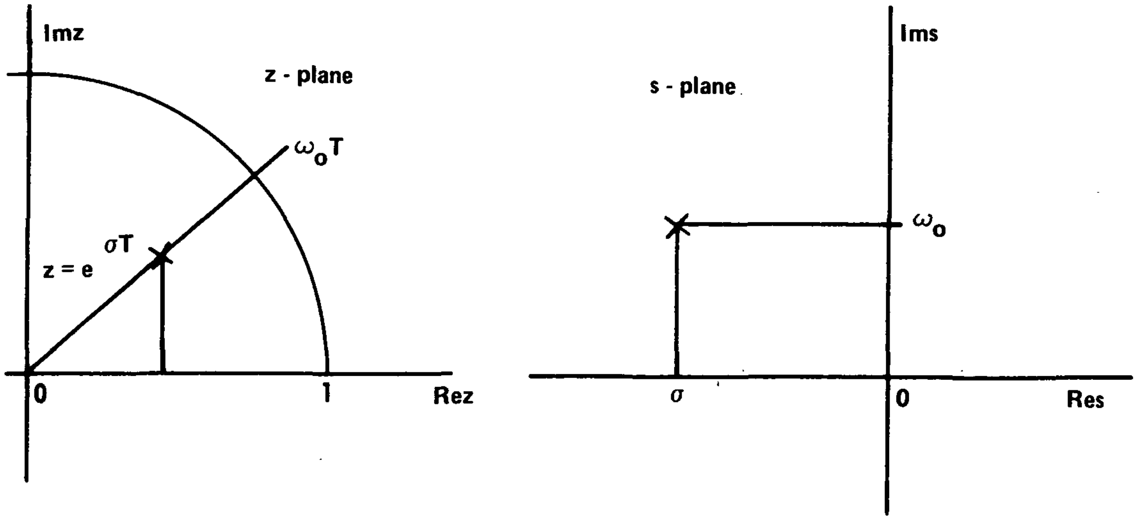

Figure 3. Correspondence between z- and s-domain.

and

$$\text{Im } S = \frac{\tan^{-1} \frac{\text{Im } z}{\text{Re } z}}{T} .$$

The gain factor of the system is received out of F(z) by letting

$$z \longrightarrow 1 .$$

The executive of the transformation to the s-domain has been provided for the example previously discussed. The printout in Table 2 shows that some deviation from the original transfer function in s exists. This deviation, however, would not be significant in a practical case of application. Furthermore, it can be seen that the assumption m = n does not result in any significant error. The erroneously introduced second zero at -1317 ... has no practical effect on the dynamic of the system.

ACCURACY

In working with the procedure outlined in this report, it must be considered that it is sensitive

1. To noise superimposed to the impulse response as a result of errors in the identification procedure, and

2. To the accuracy of the execution in the digital program.

Both effects become more effective with decreasing T because of the decreasing gradient between sample values. Thus, noise becomes more and more significant and may create necessity for the application of smoothing procedures. Furthermore, decreasing T requires increasing accuracy, i.e., number of significant digits of the execution procedure.

Inaccuracy effects become even more significant for more complex transfer functions where the number of operations, necessary for the execution of the program, increases.

CONCLUSION

The procedure outlined in this report allows the exact z transformation from a continuous time function presented by an equally spaced time series and conversion of the resulting power series in z to a rational expression in z. By a proper choice of the sampling interval, the dynamical behavior of the transfer function in s can be significantly approximated by the transfer function in z with respect to practical application. If necessary, however, a transformation to the s-domain is easily provided. By introduction of a special program subroutine and by taking an initial estimate on the order of the transfer function in s, it is possible to apply the procedure even if the order of the system under investigation is not known. It has to be made sure only that the estimate is greater or at least equal to the actual value. For the implementation of the procedure, only a limited number of values of the impulse response time series have to be known. In the ideal case, this number equals 1 plus twice the order of the denominator of the transfer function in s. For practical application, it might be useful to have a slightly higher number for checkout purposes. In any case, however, only a fraction of the complete impulse response of the system under investigation has to be known. Thus, the procedure is suitable for data processing in connection with the identification of unknown plants.

It must be realized that the digital program in the Appendix has been developed to prove the practical feasibility of the described method. It is in no way optimal with respect to the required computer capacity. If the practical application of the described method is envisaged, it is suggested that the digital program be reviewed. It is especially recommended to increase the accuracy of the procedure. Furthermore, it might be useful to extend the checkout procedure with respect to the number of time series values to be compared. In addition, the integration of a smoothing procedure for smoothing of the input values as a fixed subroutine of the program should be considered.

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Marshall Space Flight Center, Alabama 35812, June 1, 1971
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APPENDIX

DIGITAL PROGRAM

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      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION C(500),A(500),B(500)
      DIMENSION E(800),D(102)
      DIMENSION F(800)
      DIMENSION RTR(500),RTI(500),TG(500),G(500),X(500),Y(500)
      DIMENSION TGG(500),GG(500),XX(500),YY(500)
C     RATIONAL APPROXIMATION IN DOUBLE PRECESION
      NAME LIST /INPUT/ M,N,DT,C
    1 READ(5,INPUT)
      GO TO 3
    2 N=N-1
      M=N-1
      DO4 I=1,30
      B(I)=0.0D0
    4 CONTINUE
      N2=N2+2
    3 MN=M+N
      M1=M+1
      N1=N+1
      N2=M+N+1
      WRITE(6,15)M,N
   15 FORMAT(//3H M=I5,/,3H N=I5/)
      WRITE(6,123) DT
  123 FORMAT(4H DT=10F4/)
      IF(M+N.EQ.0) GO TO 50
      CALL PRINT(12HINPUT DATA   ,N2,C)
C     SIMULTANEOUS EQUATIONS ARE FORMED
      MN=M+N
      K=0
      MM=2
    5 LL=(MN/2)+MM
      DO25 J=1,N1
      K=K+1
      E(K)=C(LL)
      LL=LL-1
   25 CONTINUE
      IF(MM.NE.N1) GO TO 9
      GO TO 8
    9 MM=MM+1
      GO TO 5
    8 CALL PRINT(12H      E     ,30,E)
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C    COEF. OF B ARE STORED IN E, AND PUT IN THE FORM REQUIRED BY DSPE
C    SUBROUTINE
        K=0
        DO 20 J=1,M
        L=-M+J
        DO20 I=1,M
        K=K+1
        L=L+M+1
 20     F(K)=E(L)
        L=-M
        DO300 J=1,M
        K=K+1
        L=L+M+1
 300    F(K)=-E(L)
        CALL PRINT (12H  F           ,30,F)
        K=M**2+M
        CALL DPSE(F,N,N,DET)
C    B IS CHANGED INTO DOUBLE PRECESION
  100 DO304 I=1,N
        J=N*N-N*(I-1)
 304    B(I)=F(J)
        DO45 I=1,N
        K=N-I+1
   45 B(K+1)=B(K)
        B(1)=1.0
        CALL PRINT(12H B ARRAY     ,N1,B)
C     A ARRAY IS CALCULATED
        A(1)=C(1)
        B(1)=1.0
        DO40 I=2,M1
        II=I
        T=0.0
        DO30 J=1,I
        T=T+B(J)*C(II)
   30 II=II-1
   40 A(I)=T
C   IMPULSE IS CALCULATED AS A CHECKOUT FOR THE PROGRAM
        D(102)=0.0
        D(1)=A(1)
        DO999 I=2,101
        L=I
        T=0.0
        DO888 J=2,N1
        L=L-1
        K=L
        IF(J.GT.I) K=102
        T=-B(J)*D(K)+T
  888 CONTINUE
        IF(I.GT.N1) A(I)=0.0
        D(I)=A(I)+T
  999 CONTINUE
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      DO777 I=1,N1
      A(I)=A(I)*DT
  777 CONTINUE
      CALL PRINT(12HA ARRAY      ,N1,A)
      R=0.0
      T=0.0
      DO1222 I=1,N1
      T=T+A(I)
      R=R+B(I)
 1222 CONTINUE
      GF=T/R
      WRITE(6,901) GF
  901 FORMAT(4H GF=,D24.12/)
      TOL=0.0
      DO555 I=1,N1,2
      IF(B(I).LT.TOL.OR.B(I+1).GT.TOL) GO TO 2
  555 CONTINUE
      CALL PRINT(12H   B STABLE ,N1,B)
C  THE IMPULSE IS CHECKED WITH THE IMPULSE-RESPONSE INPUT
      DO111 I=1,N2
      IF(ABS(D(I)-C(I)).GE.0.00001) GO TO 2
  111 CONTINUE
   29 CALL PRINT(12H D IMPULSE  ,101,D)
      K=N
      DO10 I=1,N
      A(K)=A(K)/A(1)
   10 K=K-1
      CALL RTPOLY(N,A,50,1.E-25,RTR,RTI,CONV,A,B,C,D,E)
      WRITE(6,332)
  332 FORMAT(' NUM ROOTS Z')
      DO442I=1,N
      WRITE(6,441) RTR(I),RTI(I)
  441 FORMAT(3H Z= D24.12,11H + AND - J D24.12)
  442 CONTINUE
      DO443 I=1,N
      TG(I)=DSQRT(RTR(I)*RTR(I)+RTI(I)*RTI(I))
      G(I)=RTI(I)/RTR(I)
      X(I)=DATAN(G(I))/TG(I)*DT
      Y(I)=DLOG(TG(I))/DT
  443 CONTINUE
      WRITE(6,445)
  445 FORMAT(' NUM ROOTS S')
      DO446 I=1,N
      WRITE(6,447) Y(I),X(I)
  447 FORMAT(3H S= D24.12,11H + AND - J D24.12)
  446 CONTINUE
      CALL RTPOLY(N,B,50,1.E-25,RTR,RTI,CONV,A,B,C,D,E)
      WRITE(6,448)
  448 FORMAT(' DENOM ROOTS Z')
      DO449 I=1,N
      WRITE(6,500) RTR(I),RTI(I)
  500 FORMAT(3H Z= D24.12,11H + AND - J D24.12)
  449 CONTINUE
      DO502 I=1,N
      TGG(I)=DSQRT(RTR(I)*RTR(I)+RTI(I)*RTI(I))
      GG(I)=RTI(I)/RTR(I)
      XX(I)=DATAN(GG(I))/TGG(I)*DT
      YY(I)=DLOG(TGG(I))/DT
  502 CONTINUE
      WRITE(6,503)
  503 FORMAT(' DENOM ROOTS S')
      DO504 I=1,N
      WRITE(6,506) YY(I),XX(I)
  506 FORMAT(3H S= D24.12,11H + AND - J D24.12)
  504 CONTINUE
      GO TO 1
   50 CALL PRINT(12HEND OF RUN  ,1,A)
      STOP
      END
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